Calculus β

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Opskriv det generelle differentiale for de følgende funktioner og angiv differentialet i punktet (1,2):

0.1 a

$$f(x, y) = xy^2.$$

First off the partial derivatives are determined

$$\frac{\partial f}{\partial x} = y^2.$$

$$\frac{\partial f}{\partial y} = 2xy.$$

The general differential is therefore

$$df = \left(y^2\right) dx \, (2xy) \, dy.$$

the differential in (1,2) is therefore

$$df = 2^2 dx + 2 \cdot 1 \cdot 2dy = 4dx + 4dy.$$

0.2 b

Given the function

$$x^2 \sin\left(\pi y^2\right).$$

the partial derivative for a given variable is determined by differentiating the original function with the respective variable and keeping all other variables constant f_x is therefore

$$f_x = \frac{\partial}{\partial x} = 2x \sin\left(\pi y^2\right).$$

Differentiating $f(x, y)$ for y requires differentiating the function $\sin(\pi y^2)$. Since this is a composite function the chain rule is used

$$(f(g(x)))' = f'(g(x)) \cdot g'(x).$$

the following functions are chosen as f and g respectively

$$f = \sin(y), \;\; g = \pi y^2.$$

this means f' and g' become

$$f' = \cos(y), \;\; g' = 2\pi y.$$

these can now all be inserted into the chain rule

$$2\pi y \cos\left(\pi y^2\right).$$

then multiply with the constant x^2 to obtain the full expression for f_y

$$f_y = \frac{\partial}{\partial y} = 2\pi x^2 y \cos\left(\pi y^2\right).$$

Finally inputting the original equation in a CAS program and finding the partially derived for y returns

$$f_y = \frac{\partial}{\partial y} = 2\pi x^2 y \cos\left(\pi y^2\right).$$

which suggests the method used was correct the generel differential can now be found by insering f_x and f_y

$$2dxx \sin\left(\pi y^2\right) + 2\pi dyx^2 y \cos\left(\pi y^2\right).$$

and inserting the point

$$4\pi dy.$$

the differential in (1,2) is therefore

$$4\pi dy.$$