Calculus β

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1 U17

The function in question is

$$f(x,y) = 2x^3 + y^2 - 24x - 6y + 5.$$

2 a

Finding the partial derivatives by differentiating with respect to x and y respectively

$$f_x = 6x^2 - 24.$$

and

$$f_y = 2y - 6.$$

3 b

since the objective is to find critical points the gradient must be $\nabla f(x,y) = (0,0)$ which results in the following two equations

$$\begin{aligned} 6x^2 - 24 &= 0 \\ 2y - 6 &= 0. \end{aligned}$$

Since they only happen to contain one variable each they can easily be solved for their respective variable

$$\begin{aligned} x &= \sqrt{4} \\ y &= 3. \end{aligned}$$

y_0 is therefore 3

4 c

The critical value is found by inserting the critical points in the function

$$f(2,3) = 2 * 2^3 + 3^2 - 24 * 2 - 6 * 3 + 5 = -36.$$

and

$$f(-2,3) = 2(-2)^3 + 3^2 - 24 * (-2) - 6 * 3 + 5 = 28.$$

the greatest critical value is therefore 28

5 d

$$(x_1, y_0) = (-2, 3).$$

the partial derivatives of second order of f is

$$\frac{\partial^2}{\partial^2 x} = 12x \frac{\partial^2}{\partial^2 y} = 2.$$

and inserting (x_1, y_0)

$$\frac{\partial^2}{\partial^2 x} = -24 \frac{\partial^2}{\partial^2 y} = 2.$$

6 e

The determinant of a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

is defined as

$$D = ad - bc.$$

For the Hessian matrix used in the second order criteria

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2}(x_0, y_0) & \frac{\partial^2 f}{\partial x, \partial y}(x_0, y_0) \\ \frac{\partial^2 f}{\partial x, \partial y}(x_0, y_0) & \frac{\partial^2 f}{\partial y^2}(x_0, y_0) \end{bmatrix}.$$

$$D = \frac{\partial^2 f}{\partial x^2}(x_0, y_0) \frac{\partial^2 f}{\partial y^2}(x_0, y_0) - \left(\frac{\partial^2 f}{\partial x, \partial y}(x_0, y_0) \right)^2.$$

$\frac{\partial^2 f}{\partial x, \partial y}$ of the function in this case is

$$\frac{\partial^2 f}{\partial x, \partial y} = 0.$$

The determinant of the Hessian matrix in the point $(-2, 3)$ is therefore

$$D = -48.$$

7 f

According to the result in section e the critical point $(-2, 3)$ is a saddle point since $d < 0$

8 g

Since inserting $y = \frac{5}{2}$ in f_y returns -1, x must be 0 as per Pythagoras

$$1^2 = \sqrt{x^2 + (-1)^2}.$$

$$x = 0.$$