Calculus β

Rasmus Crolly

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1 up

given the function

$$f(x, y) = 4x^2y + 5xy^2 + x^3.$$

1.1 a

f_y is determined by differentiating with respect to y

$$f_y = 4x^2 + 10xy.$$

1.2 b

the gradient is found by finding the partially derived with respect to x and y respectively. Since f_y is already found in a) the only one left to find is

$$f_x = 8xy + 5y^2 + 3x^2.$$

the gradient is therefore

$$\nabla f(x, y) = (8xy + 5y^2 + 3x^2, 4x^2 + 10xy).$$

1.3 c

calculating the unit vector from the given vector by using Pythagoras to find the length of the vector

$$\mid \vec{v} \mid = \sqrt{8^2 + (-6)^2} = 10.$$

then dividing the vector $\vec{v}$ by its length to get the unit vector in the direction

$$\overline{u} = \begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix}.$$

1.4 d

according to sentence 3.10:

$$D_{\overline{u}}f(x, y) = \frac{\partial f}{\partial x}(x, y)a + \frac{\partial f}{\partial y}(x, y)b.$$

where a and b are the x and y values of the unit vector
since the partial derivatives are already known from the gradient the values can all be inserted which results in

$$D_{\overline{u}}f(2, 1) = (3 \cdot 2^2 + 8 \cdot 2 \cdot 1 + 5 \cdot 1^2) \cdot 0.8 + (4 \cdot 2^2 + 10 \cdot 2 \cdot 1) \cdot (-0.6) = 0.48.$$

1.5 e

the greatest directional derivative in a point is the same as the length of the gradient in the point which means using Pythagoras

$$\mid \overline{\nabla} \mid = \sqrt{\left(4 \cdot 2^2 + 10 \cdot 2 \cdot 1\right)^2 + \left(3 \cdot 2^2 + 8 \cdot 2 \cdot 1 + 5 \cdot 1^2\right)^2} = \sqrt{36^2 + 33^2} = \sqrt{2385}.$$

by dividing the gradient by the length the unit vector $\overline{v}$ is found

$$\begin{pmatrix} \frac{33}{\sqrt{2385}} \\ \frac{36}{\sqrt{2385}} \end{pmatrix}.$$

$$\overline{v} = \begin{pmatrix} 11\frac{\sqrt{265}}{265} \\ 12\frac{\sqrt{265}}{265} \end{pmatrix}.$$

1.6 f

the value of the greatest directional derivative is equivalent to the length of the gradient

$$\mid \overline{\nabla} \mid = \sqrt{\left(4 \cdot 2^2 + 10 \cdot 2 \cdot 1\right)^2 + \left(3 \cdot 2^2 + 8 \cdot 2 \cdot 1 + 5 \cdot 1^2\right)^2} = \sqrt{36^2 + 33^2} = \sqrt{2385}.$$

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