

# Calculus $\beta$

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## 1 U17

The function in question is

$$f(x, y) = 2x^3 + y^2 - 24x - 6y + 5.$$

## 2 a

Finding the partial derivatives by differentiating with respect to x and y respectively

$$f_x = 6x^2 - 24.$$

and

$$f_y = 2y - 6.$$

## 3 b

since the objective is to find critical points the gradient must be  $\nabla f(x, y) = (0, 0)$  which results in the following two equations

$$6x^2 - 24 = 0$$

$$2y - 6 = 0.$$

Since they only happen to contain one variable each they can easily be solved for their respective variable

$$x = \sqrt{4}$$

$$y = 3.$$

$y_0$  is therefore 3

## 4 c

The critical value is found by inserting the critical points in the function

$$f(2, 3) = 2 * 2^3 + 3^2 - 24 * 2 - 6 * 3 + 5 = -36.$$

and

$$f(-2, 3) = 2(-2)^3 + 3^2 - 24 * (-2) - 6 * 3 + 5 = 28.$$

the greatest critical value is therefore 28

## 5 d

$$(x_1, y_0) = (-2, 3).$$

the partial derivatives of second order of f is

$$\frac{\partial^2}{\partial^2 x} = 12x \frac{\partial^2}{\partial^2 y} = 2.$$

and inserting  $(x_1, y_0)$

$$\frac{\partial^2}{\partial^2 x} = -24 \frac{\partial^2}{\partial^2 y} = 2.$$

## 6 e

The determinant of a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

is defined as

$$D = ad - bc.$$

For the Hessian matrix used in the second order criteria

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2}(x_0, y_0) & \frac{\partial^2 f}{\partial x, \partial y}(x_0, y_0) \\ \frac{\partial^2 f}{\partial x, \partial y}(x_0, y_0) & \frac{\partial^2 f}{\partial y^2}(x_0, y_0) \end{bmatrix}.$$

$$D = \frac{\partial^2 f}{\partial x^2}(x_0, y_0) \frac{\partial^2 f}{\partial y^2}(x_0, y_0) - \left( \frac{\partial^2 f}{\partial x, \partial y}(x_0, y_0) \right)^2.$$

$\frac{\partial^2 f}{\partial x, \partial y}$  of the function in this case is

$$\frac{\partial^2 f}{\partial x, \partial y} = 0.$$

The determinant of the Hessian matrix in the point  $(-2, 3)$  is therefore

$$D = -48.$$

## 7 f

According to the result in section e the critical point  $(-2, 3)$  is a saddle point since  $d < 0$

## 8 g

the gradient in  $(a, \frac{5}{2})$  is  $(6a^2 - 24, 2 * \frac{5}{2} - 6) = (6a^2 - 24, -1)$

the length of the gradient is 1 since its a unit vector

$$(6a^2 - 24)^2 + (-1)^2 = 1$$

thus  $(6a^2 - 24)^2 = 0$

$$a^2 = 4.$$

$$a = 2.$$