Calculus β 2b

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September 16, 2021

1 2.12)

1.1 introduction

For a function $f : D(f) \to \mathbb{R}$ the range is the values that f hits. A real number t is said to be in the range of f if there exists an element $z \in D(f)$ where $t = f(z)$ (from 2.1 "Terminologi og notation" in the book) [**calcbeta2.1**]

1.2 a

$$f : \mathbb{R}^2 \to \mathbb{R} \tag{1}$$

where

$$f(x, y) = x - y \tag{2}$$

Since both x and y can assume any real value the range of the function is

$$Vm(f) = (-\infty, \infty) = t \in \mathbb{R} : -\infty < t < \infty \tag{3}$$

Note that the use of soft brackets is because infinity is not technically a real number and therefore not within the range

1.3 b

$$f : \mathbb{R}^2 \to \mathbb{R} \tag{4}$$

where

$$f(x, y) = x^2 + y^2 \tag{5}$$

Since both of the variables have 2 as exponent neither of them can ever return a value below 0 and since they are added together they can return any positive value or 0

$$Vm(f) = [0, \infty) = \{t \in \mathbb{R} : 0 < t < \infty\} \tag{6}$$

1.4 c

$f : D(f) \to \mathbb{R}$ given that $f(x, y) = \frac{1}{x^2+y^2}$, where

$$D(f) = (x, y) \in \mathbb{R}^2 : (x, y) \neq (0, 0) \tag{7}$$

When

$$\lim_{x \wedge y \to 0} f(x, y) \Rightarrow \lim_{f \to \infty} \tag{8}$$

When

$$\lim_{x \vee y \to \infty} f(x, y) \lim_{f \to 0} \tag{9}$$

$$Vm(f) = (0, \infty) = \{t \in \mathbb{R} : 0 < t < \infty\} \tag{10}$$

1.5 d

$$f : \mathbb{R}^2 \to \mathbb{R} \text{ where } f(x, y) = \frac{7}{x^2+y^2+1}$$

We first observe that the function is continious as per Sentence (1.16) [**calcbeta1.16**] in the book 7 and 1 are both continious functions, as is x^2 and y^2 and since both x^2 and y^2 will return positive numbers or 0 the denominator must be different from 0 since it also contains a +1

If both x and y are 0 the function will return 7, when they are very large the function will tend towards 0 and the range of the function is therefore

$$Vm(f) = (0, 7] \tag{11}$$