Challenging Independence of NPV Valuations

Should We Assume Independence?

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STAT535: Statistical Computing, Fall 2019

Original Lab Purpose

Lab 2: Company Valuation

- Introduces probabilistic valuation of a company.
- Goal of original lab: Put valuation of a company based on a sequence of uncertain future events.

$$NPV = B_0 - C_0 + \sum_{n=1}^{N} \frac{B_n - C_n}{(1+\delta_n)^n} \tag{1}$$

- B_k=revenues in the kth period, C_k=costs in the kth period, δ_k=discount rate (fixed at 4%) in all periods for our model.
- B_k and C_k are assumed to be independent random variables.
- B_k is assumed to be a Bernoulli random variable.

The Lab Extension

- Many methods of company valuation, NPV fairly simple.
  - i.e. times revenue method, discounted cash flow method, earnings multiplier etc.
- Consulting firms use Autoregressive Models to predict firm performance based on past behavior.
  - Computationally complex
- Lab assumes independence of variables
  - Much simpler model
- Extension & main claim:
  - Assuming time-related independence among future revenues and costs can bias the results of probabilistic company valuation.

Methodology

Some background

- NPV:

$$NPV = B_0 - C_0 + \sum_{n=1}^{N} \frac{B_n - C_n}{(1+\delta_n)^n} \qquad (2)$$

- Probability of observing a NPV for n periods:

$$P(NPV) = \left[P_{s1}^{B} * P_{s2|s1}^{B} * \ldots * P_{sn|sn-1}^{B}\right] * \left[P_{s1}^{C} * P_{s2|s1}^{C} * \ldots * P_{sn|sn-1}^{C}\right]$$

- Assuming independence:

$$P(NPV) = \left[P_{s1}^{B} * P_{s2}^{B} * \ldots * P_{sn}^{B}\right] * \left[P_{s1}^{C} * P_{s2}^{C} * \ldots * P_{sn}^{C}\right]$$

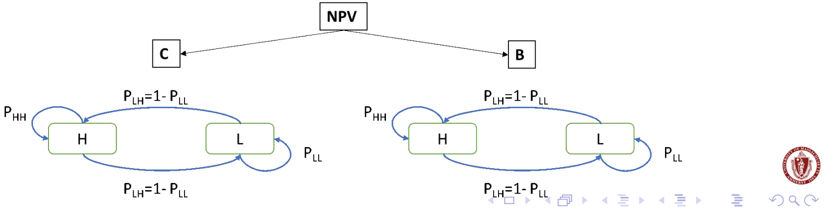
Three steps:

Data simulation, Monte Carlo maximum likelihood, Test model estimation.

Methodology

Data Simulation:

- Set three parameters: P_H, P_{HH} and P_{LL}.
- For $t = 1$ get random variable X, if $X < P_H$ then $B_1 = H$, else $B_1 = L$.
- For t_n, with $n > 1$: if $B_{n-1} = H$ use P_{HH}, else use P_{LL}. Get random variable Y, if $Y < P_{HH}$ then $B_n = H$, else $B_n = L$.
- Repeat this process for Cost.
- Calculate NPV using simulated B, and C, and the probability of that NPV P_{npv}.



Methodology

Monte Carlo Maximum Likelihood:

- Using different values of P_H get random vectors for B, C.
- For each value of P_H, check how many times the simulated NPV is observed.
- Choose the value of P_H with the most observations of the simulated NPV.
- Calculate the probability of the simulated NPV $\hat{P}_{npv}$ using the selected $\hat{P}_H$.

$$P(NPV) = \left[\hat{P}^B_{s1} * \hat{P}^B_{s2} * ... * \hat{P}^B_{sn}\right] * \left[\hat{P}^C_{s1} * \hat{P}^C_{s2} * ... * \hat{P}^C_{sn}\right]$$

Methodology

Test Model Estimation:

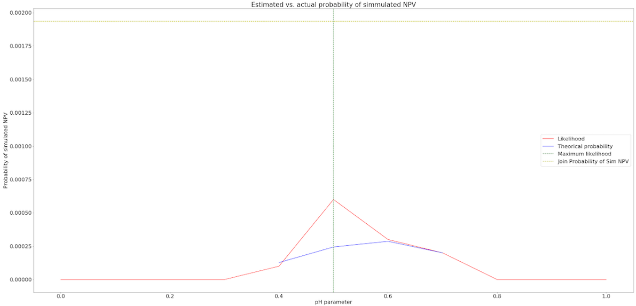
- Repeat the previous two steps 1,000 times.
- For each repetition compute the perceptual difference between P_{npv} and $\hat{P}_{npv}$.

$$diff = \frac{\hat{P}_{npv} - P_{npv}}{P_{npv}}$$

- Construction of the empirical probability function using Kernel Density estimation.
- If the time-independent model produces a consistent estimator for the P_{npv}, then the density probability function should be centered at 0.

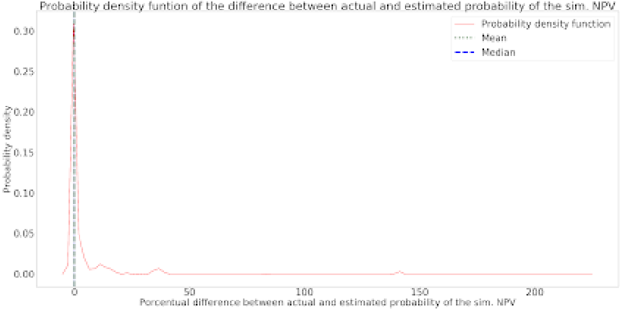
Results

- Figure (1) below is an example of the project's Monte Carlo simulation. We can conclude that assuming independence in the likelihood estimation model is useful as long as there is substantial data to estimate the model. With a higher number of simulations (say 1,000,000) we would eventually see the peaks of the likelihood and theoretical probability line up.



Results

- In Figure (2) below, the probability density function allocates probability to positive differences between $\hat{P}_{npv}$ and P_{npv} as a result of our chosen parameters.
- There is generally no porcentual difference between the actual probability of the simulated NPV and the estimated probability of the simulated NPV. If choosing between models to use: choose **independent** model since it is much quicker, less complex



Summary

- We began with our claim $\rightarrow$ Assuming intertemporal independence of revenues and costs can bias the results of a probabilistic company valuation.
- Steps to reaching conclusions:
  - Simulated a set of revenues and costs with intertemporal dependence and recovered the corresponding NPV.
  - Conducted the simulation of revenues and costs assuming independence.
  - Performed Monte Carlo simulation to recover the maximum likelihood estimator
- Compared maximum likelihood estimator with actual observed probability
- Conducted the above steps many times where we observed results in density function of Figure (2).