EXAM 2

Name (Print): ______________________________

MTH 197
Spring 2022

Instructor: Brody Erlandson

This exam contains 9 pages (including this cover page) and 7 problems. Print your name legibly in the space above.

Read and follow the instructions below. FAILURE TO FOLLOW THESE INSTRUCTIONS WILL AFFECT YOUR GRADE.

1. **Do not tear the exam pages apart.**
2. **Show all work for full credit.** Answers without supporting work will not receive full credit. Also, indicate what you are doing on your calculator.
3. **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.
4. **Write all work and answers on the exam pages.** You may use scrap paper to work out a solution, but the final draft of the work must be written on the exam in the space provided after each question. **Do not turn in scrap paper with the exam.**
5. **Use exact values for all numbers unless explicitly told to approximate.** Do not approximate irrational numbers on your calculator unless the problem clearly tells you to. **Decimal approximations will be marked wrong.** Exact decimal numbers are acceptable.

Problem	Points	Score
1	15	
2	30	
3	20	
4	10	
5	15	
6	65	
7	10	
Total:	165	

1. Given the following set,

$$S = \left\{ x, x^2, 1 + x^3, x^3 \right\}$$

(a) (5 points) Provide a basis of $\langle S \rangle$, such that each vector in the basis you provide is contained in S.

(b) (5 points) What is the dimension of $\langle S \rangle$.

(c) (5 points) Let $\mathbf{A}$ be a matrix where the columns are the coordinate vectors of the elements of S. What is the rank and nullity of $\mathbf{A}$?

2. Let $T : \mathbb{C}^4 \to \mathbb{C}^3$ and $S : \mathbb{C}^3 \to \mathbb{C}^4$ where,

$$T(\mathbf{u}) = \begin{bmatrix} u_1 + u_2 \\ u_4 \\ u_2 + u_3 \end{bmatrix} \text{ and } S(\mathbf{v}) = \begin{bmatrix} v_1 \\ v_3 \\ v_2 + v_3 \end{bmatrix}$$

(a) (10 points) Show this is a T linear transformation.

(b) (10 points) Give the matrix transformation for $2(S \circ T)(\mathbf{x})$

(c) (10 points) Use the matrix transformation to find the kernel and range of T.

3. Given $T : \mathbb{P}_2 \to \mathbb{C}^3$ where

$$T(\mathbf{p}) = \begin{bmatrix} p(0) \\ p(1) \\ p(2) \end{bmatrix}$$

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(a) (5 points) What is the kernel of T?

(b) (5 points) What is the range of T?

(c) (10 points) Check if T is an isomorphism by checking both injective and surjective.

4. (10 points) Given $T : \mathbb{P}_2 \to \mathbb{P}_3$,

$$T(\mathbf{p}) = xp(0) + p(x) + x^3 p(1)$$

Find the matrix representation $M^T_{B,C}$, where B and C are the standard bases for $\mathbb{P}_2$ and $\mathbb{P}_3$ respectively. Use this representation to find $T(x + 3x^2)$.

5. Given

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(a) (10 points) Find $\det(\mathbf{A})$.

(b) (5 points) Given the $\det(\mathbf{A})$, is $\mathbf{A}$ invertible?

6. Given $T : \mathbb{P}_2 \to \mathbb{P}_2$,

$$T(a + bx + cx^2) = (a + c) + (2b)x + (3c)x^2$$

Find $M^T_{B,B} = C^{-1}_{B,C} M^T_{C,C} C_{B,C}$, where $M^T_{C,C}$ is diagonal and B is the standard basis. Use this to find $(M^T_{B,B})^3$.

(a) (10 points) First find $M^T_{B,B}$.

(b) (10 points) Now find the eigenvalues of $M^T_{B,B}$.

(c) (10 points) Now find the bases for the eigenspaces of $M^T_{B,B}$.

(d) (5 points) Use the above to find the eigenvectors of T and form the basis C using these.

(e) (10 points) Form $M^T_{C,C}$.

(f) (10 points) Form $C_{B,C}$ and find $C^{-1}_{B,C}$.

(g) (10 points) Finally find $(M^T_{B,B})^3$

7. (10 points) Prove that the coordinate vector transformation (ρ_B) is linear.