

# 1.3 EXERCISES

In Exercises 1 and 2, compute  $\mathbf{u} + \mathbf{v}$  and  $\mathbf{u} - 2\mathbf{v}$ .

1.  $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$       2.  $\mathbf{u} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

In Exercises 3 and 4, display the following vectors using arrows on an  $xy$ -graph:  $\mathbf{u}, \mathbf{v}, -\mathbf{v}, -2\mathbf{v}, \mathbf{u} + \mathbf{v}, \mathbf{u} - \mathbf{v}$ , and  $\mathbf{u} - 2\mathbf{v}$ . Notice that  $\mathbf{u} - \mathbf{v}$  is the vertex of a parallelogram whose other vertices are  $\mathbf{u}, \mathbf{0}$ , and  $-\mathbf{v}$ .

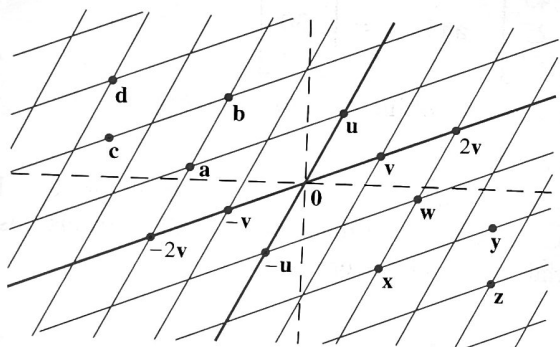
3.  $\mathbf{u}$  and  $\mathbf{v}$  as in Exercise 1      4.  $\mathbf{u}$  and  $\mathbf{v}$  as in Exercise 2

In Exercises 5 and 6, write a system of equations that is equivalent to the given vector equation.

5.  $x_1 \begin{bmatrix} 3 \\ -2 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 0 \\ -9 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$

6.  $x_1 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Use the accompanying figure to write each vector listed in Exercises 7 and 8 as a linear combination of  $\mathbf{u}$  and  $\mathbf{v}$ . Is every vector in  $\mathbb{R}^2$  a linear combination of  $\mathbf{u}$  and  $\mathbf{v}$ ?



7. Vectors  $\mathbf{a}, \mathbf{b}, \mathbf{c}$ , and  $\mathbf{d}$

8. Vectors  $\mathbf{w}, \mathbf{x}, \mathbf{y}$ , and  $\mathbf{z}$

In Exercises 9 and 10, write a vector equation that is equivalent to the given system of equations.

9.  $x_2 + 5x_3 = 0$       10.  $3x_1 - 2x_2 + 4x_3 = 3$   
 $4x_1 + 6x_2 - x_3 = 0$        $-2x_1 - 7x_2 + 5x_3 = 1$   
 $-x_1 + 3x_2 - 8x_3 = 0$        $5x_1 + 4x_2 - 3x_3 = 2$

In Exercises 11 and 12, determine if  $\mathbf{b}$  is a linear combination of  $\mathbf{a}_1, \mathbf{a}_2$ , and  $\mathbf{a}_3$ .

11.  $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

12.  $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$

In Exercises 13 and 14, determine if  $\mathbf{b}$  is a linear combination of the vectors formed from the columns of the matrix  $A$ .

13.  $A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$

14.  $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

15. Let  $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$ , and  $\mathbf{b} = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$ . For what value(s) of  $h$  is  $\mathbf{b}$  in the plane spanned by  $\mathbf{a}_1$  and  $\mathbf{a}_2$ ?

16. Let  $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$ , and  $\mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$ . For what value(s) of  $h$  is  $\mathbf{y}$  in the plane generated by  $\mathbf{v}_1$  and  $\mathbf{v}_2$ ?

In Exercises 17 and 18, list five vectors in  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ . For each vector, show the weights on  $\mathbf{v}_1$  and  $\mathbf{v}_2$  used to generate the vector and list the three entries of the vector. Do not make a sketch.

17.  $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$

18.  $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

19. Give a geometric description of  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$  for the vectors  $\mathbf{v}_1 = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix}$  and  $\mathbf{v}_2 = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}$ .

20. Give a geometric description of  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$  for the vectors in Exercise 18.

21. Let  $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$  and  $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ . Show that  $\begin{bmatrix} h \\ k \end{bmatrix}$  is in  $\text{Span}\{\mathbf{u}, \mathbf{v}\}$  for all  $h$  and  $k$ .

22. Construct a  $3 \times 3$  matrix  $A$ , with nonzero entries, and a vector  $\mathbf{b}$  in  $\mathbb{R}^3$  such that  $\mathbf{b}$  is *not* in the set spanned by the columns of  $A$ .

In Exercises 23 and 24, mark each statement True or False. Justify each answer.

23. a. Another notation for the vector  $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$  is  $[-4 \ 3]$ .

b. The points in the plane corresponding to  $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$  and  $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$  lie on a line through the origin.

c. An example of a linear combination of vectors  $\mathbf{v}_1$  and  $\mathbf{v}_2$  is the vector  $\frac{1}{2}\mathbf{v}_1$ .