Practice Exam 1

Name (Print): ____________________

MTH 197
Spring 2022
Instructor: Brody Erlandson

This exam contains 8 pages (including this cover page) and 9 problems. Print your name legibly in the space above.

Read and follow the instructions below. FAILURE TO FOLLOW THESE INSTRUCTIONS WILL AFFECT YOUR GRADE.

1. **Do not tear the exam pages apart.**
2. **Show all work for full credit.** Answers without supporting work will not receive full credit.
3. **Organize your work** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.
4. **Write all work and answers on the exam pages.** You may use scrap paper to work out a solution, but the final draft of the work must be written on the exam in the space provided after each question. **Do not turn in scrap paper with the exam.**
5. **Use exact values for all numbers unless explicitly told to approximate.** Do not approximate irrational numbers on your calculator unless the problem clearly tells you to. **Decimal approximations will be marked wrong.** Exact decimal numbers are acceptable.

Problem	Points	Score
1	20	
2	15	
3	10	
4	25	
5	20	
6	10	
7	10	
8	15	
9	15	
Total:	140	

1. Given the following system of equations,

$$\begin{aligned} 2x_1 + 6x_2 + 2x_3 + 9x_4 &= 8 \\ 0x_1 + 7x_2 + 9x_3 + 6x_4 &= 2 \\ 1x_1 + 9x_2 + 9x_3 + 9x_4 &= 2 \end{aligned}$$

   (a) (5 points) Provide the column vector equation for the system.

   (b) (5 points) Provide the matrix equation for the system.

   (c) (10 points) Solve the system and provide the solution in parametric form.

2. Given,

$$S = \{ \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \}$$

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(a) (5 points) Is $\begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix} \in \langle S \rangle$? Why or why not?

(b) (10 points) Is S linearly independent set?

3. (10 points) Given,

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \text{ and } T = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right\}$$

which of the above sets span $\mathbb{C}^3$ and why?

4. Given the augmented matrix,

$$[\mathbf{A}|\mathbf{b}] = \begin{bmatrix} 3 & 6 & -3 \\ 6 & 12 & -6 \\ 7 & 14 & -7 \end{bmatrix}$$

(a) (10 points) Find the solution to $\mathbf{Ax} = \mathbf{b}$. (***No Calculator***)

(b) (5 points) Find the linearly independent spanning set for $\mathcal{N}(\mathbf{A})$.

(c) (5 points) Find the linearly independent spanning set for $\mathcal{C}(\mathbf{A})$.

(d) (5 points) Find the linearly independent spanning set for $\mathcal{R}(\mathbf{A})$.

5. Given the augmented matrix,

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 3 & 2 & -1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 2 & -3 \\ 8 & 2 & -6 \\ 1 & 4 & -7 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

(a) (10 points) Find $(-3\mathbf{A}+\mathbf{B})^T$. Is the solution symmetric? Why or Why not? (***No Calculator***)

(b) (5 points) Find $\mathbf{AC}$. Is the result invertible, why or why not? (***No Calculator***)

(c) (5 points) Find the $\mathbf{A}^{-1}$.

6. (10 points) Why is the following not a vector space? (Name the axiom it does not satisfy)

$$V = \{p(x) = x^2\}$$

$$F = \mathbb{R}^+ \text{ all real numbers greater than } 0$$

$$\text{Addition: } (p+q)(x) = p(x) + q(x)$$

$$\text{Scalar Multiplication: } (\alpha p)(x) = 1p(x)$$

Also, show or explain why the axiom is not satisfied.

7. (10 points) Is the following a subspace of $\mathbb{C}^4$?

$$H = \{\mathbf{v} \in \mathbb{C}^4 | \mathbf{v} = \begin{bmatrix} 1 \\ t \\ -t \\ s+t \end{bmatrix}, s, t \in \mathbb{C}\}$$

8. (15 points) Show that if $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{C}^m$, then we can write the inner product of $\mathbf{u}$ and $\mathbf{v}$ as a matrix product:

$$\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u}^* \mathbf{v}$$

where $\mathbf{u}^* = \bar{\mathbf{u}}^T$

9. (15 points) Given,

$$S = \{\mathbf{v}\}, T = \{\mathbf{u}\} \text{ where } \mathbf{v} \perp \mathbf{u}$$

Show that every vector in $\langle S \rangle$ is orthogonal to every vector in $\langle T \rangle$.
(Hint: What is the what is the vector form of $\langle S \rangle$ and $\langle T \rangle$?)