Reminder: The general form of the determinant is:

$$\det(A) = \sum_{(i \text{ or } j)=1}^{n} (-1)^{i+j} a_{ij} A(i|j)$$

Where you either go down a column or across a row.

1. Given the system:

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 5 \\ 2x_1 + x_2 - 3x_3 &= 13 \\ -x_1 + x_2 &= -8 \end{aligned}$$

(a) (5 points) Write the system in vector form.

(b) (5 points) Write the system in matrix form.

(c) (10 points) Solve the system and write the solution in parametric form.

2. Let $A = \begin{bmatrix} 1 & -3 & 4 & -1 \\ -2 & 6 & -6 & -1 \\ -3 & 9 & -6 & -6 \\ 3 & -9 & 4 & 9 \end{bmatrix}$

(a) (10 points) Find $\mathcal{N}(A)$.

(b) (10 points) Find $\mathcal{C}(A)$.

3. (10 points) Find the matrix representation for the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$, where

$$T(\mathbf{p}) = 1 + x\mathbf{p}'$$

$\mathbf{p}'$ is the first derivative of the polynomial.

4. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 3 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

(a) (5 points) Find the determinant of A by hand. Show all work.

(b) (5 points) Is A is invertible? Why or why not?

5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

   (a) (10 points) Find an orthonormal basis for $\mathcal{C}(A)$

(b) (10 points) Find a QR factorization of A.

6. Let $A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$.

(a) (5 points) Find the eigenvalues of A.

(b) (10 points) Find a basis for the eigenspace corresponding to each eigenvalue.

(c) (5 points) Orthogonally diagonalize A.

7. Let $A = \begin{bmatrix} 0 & 5 & 2 \\ 4 & 3 & 3 \\ 1 & -2 & 0 \\ 3 & 1 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 8 \\ 9 \\ 1 \\ 3 \end{bmatrix}$, find the normal equations and the least squares solution.