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COURSE NAME: APPLIED NUMERICAL

DA - 1

SUMMITED TO:

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Part A   18BML0104 AuA

(1) Solve the system of equation by Newton Raphson method $e^x - y = 0$ and $xy - e^x = 0$ correct to three decimal places by taking $x_0 = 0.95$, $y_0 = 2.7$

Solution, let $f(x,y) = 0$, & $g(x,y) = 0$

$x_1 = x_0 + h$, $y_1 = y_0 + k$

where $\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}^{-1}_{[x_0,y_0]} \begin{bmatrix} -f \\ -g \end{bmatrix}_{[x_0,y_0]}$

$f(x,y) = e^x - y = 0$, and $g(x,y) = xy - e^x = 0$

$f_x = e^x$; $g_x = y - e^x$

$f_y = -1$; $g_y = x$.

$$\begin{bmatrix} \Delta x^{[0]} \\ \Delta y^{[0]} \end{bmatrix} = \begin{bmatrix} e^x & -1 \\ y - e^x & x \end{bmatrix}^{-1}_{[0.95, 2.7]} \begin{bmatrix} -(e^x - y) \\ -(xy - e^x) \end{bmatrix}_{[0.95, 2.7]}$$

$$= \begin{bmatrix} e^{0.95} & -1 \\ 2.7 - e^{0.95} & 0.95 \end{bmatrix}^{-1} \begin{bmatrix} -(e^{0.95} - 2.7) \\ -((0.95)(2.7) - e^{0.95}) \end{bmatrix}$$

$$= \begin{bmatrix} 2.58571 & -1 \\ 0.11429 & 0.95 \end{bmatrix}^{-1} \begin{bmatrix} 0.114290 \\ 0.0207097 \end{bmatrix}$$

$$= \begin{bmatrix} 0.369547 & 0.388997 \\ -0.0444585 & 1.005833203 \end{bmatrix} \begin{bmatrix} 0.114290 \\ 0.0207097 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0502915 \\ 0.0157493 \end{bmatrix}$$

$x_1 = x_0 + \Delta x^{[0]} = 0.95 + 0.0502915 = 1.0002915$

$y_1 = y_0 + \Delta y^{[0]} = 2.7 + 0.0157493 = 2.715749$

$$\begin{bmatrix} \Delta x^{[1]} \\ \Delta y^{[1]} \end{bmatrix} = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}^{-1}_{[x_1,y_1]} \begin{bmatrix} -f \\ -g \end{bmatrix}_{[x_1,y_1]}$$

$$\frac{-}{\frac{-}{c}}\begin{bmatrix}\Delta x^{[1]}\\ \Delta y^{[1]}\end{bmatrix} = \begin{bmatrix} e^{1.0002915} & -1 \\ 2.715749-e^{1.0002915} & 1.0002915\end{bmatrix}^{-1}\begin{bmatrix}-\left(e^{1.0002915}-2.715749\right)\\ -\left(1.0002915\times 2.715749-e^{1.00029 15}\right)\end{bmatrix}$$

$$x_2 = \Delta x^{[1]} + x_1 = -3.87141\times10^{-4} + 1.0002915 = 0.999904359$$

$$y_2 = \Delta y^{[1]} + y_1 = 2.53166\times10^{-3} + 2.715749 = 2.71828$$

$\therefore$ to three significent digit

$$\left.\begin{aligned} x_2 &= 1.000 \\ y_2 &= \underline{\underline{2.72}} \end{aligned}\right\}$$

② **Solve** the system of equation by Thomas Algorithm

$$\begin{cases} 3x_1 - x_2 = 5 \\ 2x_1 - 3x_2 + 2x_3 = 5 \\ x_2 + 2x_3 + 5x_4 = 10 \\ x_3 - x_4 = 1 \end{cases}$$

$\boxed{\text{Solution}}$

$$\begin{bmatrix} b_1 & c_1 & 0 & 0 \\ a_2 & b_2 & c_2 & 0 \\ 0 & a_3 & b_3 & c_3 \\ 0 & 0 & a_4 & b_4 \end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}R_1\\R_2\\R_3\\R_4\end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 & 0 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}5\\5\\10\\1\end{bmatrix}$$

we have $b_1 = 3$ $a_2 = 2$ $c_1 = -1$

$b_2 = -3$ $a_3 = 1$ $c_2 = 2$

$b_3 = 2$ $a_4 = 1$ $c = 5$

$b_4 = -1$

Index	r	s
1	$r_1 = -1/3$	$s_1 = +5/3$
2	$r_2 = -6/7$	$s_2 = -5/7$
3	$r_3 = \cancel{7/4}$	$s_3 = 15/4$
		$s_4 = 1$

$s_1 = \frac{R_1}{b_1}$

$r_1 = \frac{c_1}{b_1}$

$$\begin{bmatrix} 1 & r_1 & 0 & 0 \\ 0 & 1 & r_2 & 0 \\ 0 & 0 & 1 & r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1/3 & 0 & 0 \\ 0 & 1 & -6/7 & 0 \\ 0 & 0 & 1 & 7/4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} +5/3 \\ -5/7 \\ 15/4 \\ 1 \end{bmatrix}$$

$x_4 = 1$ $\quad x_3 + 7/4 x_4 = 15/4$ $\quad x_3 = 15/4 - 7/4 = 8/4 = 2$

$x_2 - 6/7 x_3 = -5/7 \quad = x_2 = -5/7 + 6/7 x_3 = -5/7 + \frac{12}{7} = 1$

$x_1 - 1/3 x_2 = -5/3 \quad = x_1 = +5/3 + 1/3 x_2 = +5/3 + 1/3 = 6/3 = 2$

$\therefore \{x_1, x_2, x_3, x_4\} = \{2, 1, 2, 1\}$

(13) From the ff data estimate the value of $\int_0^1 \frac{dx}{1+x^2}$ using trapezoid rule & simpson's one-third rule. Hence compute the approximate value of π.

X	0	0.25	0.05	0.75	1.00
$Y = \frac{1}{1+x^2}$	1	0.9412	0.8000	0.64	0.500
	y_0	y_1	y_2	y_3	y_4

soln → som Trapezoid rule $\int_a^b f(x)dx = \frac{h}{2}((y_0+y_n) + 2(y_1+y_2+y_3+ \cdots y_{n-1}))$

we have $h = 0.25$

$\Rightarrow \frac{0.25}{2}((1+0.5) + 2(0.9412 + 0.8 + 0.64))$

$= 0.7828$, the value of $\pi = 4 \times 0.7828 = 3.1312$

Using simpson's 1/3 rule $= h/3((y_0+y_n) + 4(y_1+y_3+y_5 \cdots y_{n-1})$
$+ 2(y_2+y_4+ \cdots y_{n-2}))$

$= \frac{0.25}{3}((1+0.5) + 4(0.9412 + 0.64) + 2(0.8))$

$= 0.7854$, The value of $\pi = 4 \times 0.7854 = 3.1416$

④/ compute $I = \int_0^{1/2} \cos x^2 dx$ using trapezoidal rule with $h = 1/2$, $h = 1/4$, $1/8$, & Romberg's method

Solution

$x =$	0	1/2
$y = \cos x^2$	1	0.968912
	y_0	y_1

$h = 0.5 = 1/2$

$I_0 = \frac{h}{2}(y_0 + y_1) = \frac{0.5}{2}(1 + 0.968912) = 0.492228$

when $h = 0.25$

x	0	0.25	0.5
$y = \cos x^2$	1	0.998048	0.968912
	y_0	y_1	y_2

$I_0 = \frac{h}{2}((y_0 + y_2) + 2(y_1)) = \frac{0.25}{2}((1 + 0.968912) + 2(0.998048))$

$= 0.495626$

when $h = 0.125$

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
$y = \cos x^2$	1	0.999878	0.998048	0.990129	0.968912				
	y_0	y_1	y_2	y_3	y_4				

$I = \frac{h}{2}((y_0 + y_4) + 2(y_1 + y_2 + y_3))$

$= \frac{0.125}{2}((1 + 0.968912) + 2(0.999878 + 0.998048 + 0.990129))$

$= 0.496564$

, By Romberg's method

step index	I value		
$h = 1/2$	0.492228	$\frac{4(0.495626) - 0.492228}{3} = 0.496159$	$\frac{16(0.496877) - 0.496159}{15} = 0.496885$
$h = 1/4$	0.495626	$\frac{4(0.496564) - 0.495626}{3} = 0.496877$	
$h = 1/8$	0.496564		

$\therefore$ So, $I = 0.496885$

(5) The velocity V of a particle at distance 's' from the point on its linear path is given by the ff data. Estimate the time taken by the particle to traverse the distance of 20m using simpson's one-third rule.

S(m)	0	2.5	5	7.5	10	12.5	15	17.5	20
V(m/sec)	16	19	21	22	20	17	13	11	9

Solution. Wkt $V = \frac{dV}{dt} \Rightarrow t = \int_0^{20} \frac{1}{V} ds$

let $Y = 1/V$

V(m/s)	16	19	21	22	20	17	13	11	9
$Y = 1/V$	$1/16 = 0.0625$	$1/19$	$1/21$	0.04545	0.05	$1/17$	$1/13$	$1/11$	$1/9$
	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8

We have simpson's 1/3 rule formula $\frac{h}{3}[(Y_0+Y_8)+4(Y_1+Y_3+Y_5+Y_7)+2(Y_2+Y_4+Y_6)]$

& $h = 2.5$

So, $\frac{2.5}{3}\left((0.0625+\frac{1}{9})+4(\frac{1}{19}+\frac{1}{22}+\frac{1}{17}+\frac{1}{11})+2(\frac{1}{21}+0.05+\frac{1}{13})\right)$

$t = 1.26164$ sec

$t = 1.26164$ sec for distance of 20 m.

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6) Find $f(10)$ by Newton's divided difference formula

	x	$F(x)$ $f(x_0)$	$\Delta f(x_0,x_1)$ $f(x_0,x_1)$	$\Delta^2 f(x_0)$ $f(x_0,x_1,x_2)$	3rd $f(x_0,x_1,x_2,x_3)$	4th $f(x_0,x_1,x_2,x_3,x_4)$	5th $f(x_0,x_1,x_2,x_3,x_4,x_5)$
x_0	1	6	1.5	0.1	0.02857143	-0.0121166	2.07141×10^{-3}
x_1	3	9	2	0.3	-0.107411	0.0168832	
x_2	6	15	3.5	-0.6667	0.0978873		
x_3	8	22	-0.5	0.214286			
x_4	12	20	-1				
x_5	15	17					

$f(x) = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2)$
$f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)f(x_0,x_1,x_2,x_3,x_4) + (x-x_0)(x-x_1)(x-x_2)$
$(x-x_3)(x-x_4)\,f(x_0,x_1,x_2,x_3,x_4,x_5)$

So, $f(10) = f(1) + (10-1)(1.5) + (10-1)(10-3)(0.1) + (10-1)(10-3)(10-6)$
$(0.02857143) + (10-1)(10-3)(10-6)(10-8)(-0.0121166)$
$+ (10-1)(10-3)(10-6)(10-8)(10-12)(2.07141\times10^{-3})$

$= 6 + (9\times1.5) + (9\times7\times0.1) + (9\times7\times4)(0.02857143) + (9\times7\times4\times2)($
$-0.0121166) + (9\times7\times4\times2\times-2)(2.07141\times10^{-3})$

$f(10) = 24.805253$

7) Find the cubic spline for the ff data

	x_0	x_1	x_2	x_3	x_4
x	1	2	5	6	8
$f(x)$	6	9	15	12	17
	f_0	f_1	f_2	f_3	f_4

$h_1 = 1$
$h_2 = 3$
$h_3 = 1$
$h_4 = 2$

Soln/ $s_i(x) = \frac{1}{6h_i}\left((x_i-x)^3 M_{i-1} + (x-x_{i-1})^3 M_i\right] + 1/h_i\,[x_i-x)\left(f_{i-1} - \frac{h_i^2}{6}M_{i-1}\right)$
$+ 1/h_i\,[x-x_{i-1}]\left[f_i - \frac{h_i^2}{6}M_i\right]$ where $h = x_i - x_{i-1}$, $i = 1, 2, 3 \ldots$

$h_i M_{i-1} + 2M_i(h_i + h_{i+1}) + h_{i+1}M_{i+1} = \frac{6}{h_{i+1}}(f_{i+1} - f_i) - \frac{6}{h_i}(f_i - f_{i-1})$ $i = 1, 2, \ldots n-1$

for $i=1$ $h_1 M_0 + 2M_1(h_1 + h_2) + h_2 M_2 = \frac{6}{h_2}(f_2 - f_1) - \frac{6}{h_1}(f_1 - f_0)$

$M_0 = M_n = 0 \Rightarrow M_0 = M_4 = 0$

for $i=1$ $\Rightarrow \cancel{M_0}_0 + 2M_1(4) + 3M_2 = \frac{6}{3}(6) - 6(3) = -6$ $4 - 18$

$8M_1 + 3M_2 = 1$ — (1)

for $i=2$ $h_2 M_1 + 2M_2(h_2 + h_3) + h_3 M_3 = \frac{6}{h_3}(f_3 - f_2) - \frac{6}{h_2}(f_2 - f_1)$

$3M_1 + 8M_2 + M_3 = 6(-3) - \frac{16}{3}(6) = -18 - 12 = -30$

$3M_1 + 8M_2 + M_3 = -20$ --- (2)

for $i=3$ $h_3 M_2 + 2M_3(h_3+h_4) + \cancel{h_4} M_4 = \frac{6}{h_4}(f_4 - f_3)$

$- \frac{6}{h_3}(f_3 - f_2)$

$M_2 + 2M_3(3) + 0 = \frac{6}{2}(5) - \frac{6}{1}(-3)$

$M_2 + 6M_3 = 33$ --- (3)

Solve equation 1, 2 & 3

$8M_1 + 3M_2 = -6$ -(1)

$3M_1 + 8M_2 + M_3 = -30$ -(2)

$M_2 + 6M_3 = 33$ --(3)

$M_1 = 51/46 = 1.109$

$M_2 = -4.951$

$M_3 = 6.326$

for $i=1$ $S_1(x) = \frac{1}{6h_1}[(x_1-x)^3 \cancel{M_0} + (x-x_0)^3 M_1] + \frac{1}{h_1}[x_1-x][f_0 - \cancel{\frac{h_1^2}{6}M_0}]$

$+ \frac{1}{h_1}[x-x_0][f_1 - \frac{h_1^2}{6}M_1]$

$= \frac{1}{6}(x-1)^3(1.109) + [2-x](6) + [x-2](9 - \frac{1}{6}(1.109))$

$= 0.1848(x^3-3x^2+3x-1) + 12-12x + [x-2](9-0.1848)$

$= 0.1848x^3 - 0.5544x^2 + 0.5544x - 0.1848 + 12 - 12x + [x-2][8.8152]$

$= 0.1848x^3 - 0.5544x^2 + 2.6304x - 17.8152$

for $i=2$ $S_2(x) = \frac{1}{6h_2}((x_2-x)^3 M_1 + (x-x_1)^3 M_2) + \frac{1}{h_2}(x_2-x)(f_1 - \frac{h_2^2}{6}M_1)$

$+ \frac{1}{h_2}(x-x_1)(f_2 - \frac{h_2^2}{6}M_2)$

$= \frac{1}{6\times3}((5-x)^3(1.109) + (x-2)^3(-4.957)) + \frac{1}{3}(5-x)(9 - \frac{9}{6}\times1.109)$

$+ \frac{1}{3}(x-2)(15 - \frac{9}{6}(-4.957))$

$= \frac{1.109}{18}(5-x)^3 - \frac{4.957}{18}(x-2)^3 + \frac{7.3365}{3}(5-x) + \frac{22.4355}{3}(x-2)$

$=$

for $i=3$

$S_3(x) = \frac{1}{6h_3}((x_3-x)^3 M_2 + (x-x_2)^3 M_3) + \frac{1}{h_3}(x_3-x)(f_2 - \frac{h_3^2}{6}M_2)$

$+ \frac{1}{h_3}(x-x_2)(f_3 - \frac{h_3^2}{6}M_3)$

$= \frac{1}{6}((6-x)^3(-4.957) + \frac{1}{6}(6.326)(x-5)^3 + (6-x)(15 - \frac{1}{6}(-4.957))$

$+ 1(x-5)(12 - \frac{1}{6}(6.326))$

$= -0.8262(6-x)^3 + 1.0543(x-5)^3 + 15.8262(6-x) + 10.9457(x-5)$

for $i=4$ $S_4(x) = \frac{1}{6h_4}((x_4-x)^3 M_3 + \cancel{(x-x_3)^3 M_4}) + \frac{1}{h_4}(x_4-x)(f_3 - \frac{h_4^2}{6}M_3)$

$+ \frac{1}{h_4}(x-x_3)(f_4 - \cancel{\frac{h_4^2}{6}M_4})$

$= \frac{1}{12}((8-x)^3(6.326) + \frac{1}{2}(8-x)(12 - \frac{4}{6}(6.326))$

$+ \frac{1}{2}(x-6)(17)$

$= 0.5272(8-x)^3 + 3.8913(8-x) + 8.5(x-6)$

$=$

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8. Find the maxima / minima of

S(m)	0	2.5	5	7.5	10	12.5	15	17.5	20
V(m/sec)	16	19	21	22	20	17	13	11	9
t (sec)	0	0.1316	0.2381	0.3409	0.5	0.7353	1.1538	1.5809	2.2222

Sol/n

$V = \frac{ds}{dt}$ | max speed occur at $t = 0.3409$

	x	y (m/s)	Δy_0	Δy_1	Δy_2	Δy_3	Δy_4	Δy_5	Δy_6	Δy_7
1	0	16	3	-1	0	-2	6	-12	23	-47
2	2.5	19	2	-1	-2	4	-6	11	-24	
3	5	21	1	-3	2	-2	5	-13		
4	7.5	22	-2	-1	0	3	-8			
5	10	20	-3	-1	3	-5				
6	12.5	17	-4	2	-2					
7	15	13	-2	0						
8	17.5	11	-2							
9	20	9								

$\rightarrow P = \frac{x - x_0}{h} \Rightarrow h = 2.5$

$y = y_0 + P\Delta y_0 + \frac{P(P-1)\Delta^2 y_0}{2!} = 16 + P(3) + \frac{P(P-1)}{2}(-1)$

$\rightarrow \frac{dy}{dP} = 0, \quad 3 + \frac{2P-1}{2}(-1) = 0$

$-2P + 1 = -6 \qquad P = 3.5$

$\rightarrow$ point of max, $x = x_0 + Ph = 0 + 3.5 \times 2.5 = 8.75$

$\rightarrow$ max value $\Rightarrow 16 + (3.5 \times 3) + \frac{3.5(3.5-1)}{2}(-1) = 22.125$

part B 18BML0104 Ans

1) Solve $y' = 2e^{x} - y$ given that $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$ to get $y(0.4)$ using Adams- Bashforth method

given

$x_{-3} = 0 \rightarrow y_{-3} = 2 \rightarrow f(x_{-3}, y_{-3}) = 2e^{0} - 2 = 0$

$x_{-2} = 0.1 \rightarrow y_{-2} = 2.010 \rightarrow f(x_{-2}, y_{-2}) = 2e^{0.1} - 2.010 = 0.200342$

$x_{-1} = 0.2 \rightarrow y_{-1} = 2.040 \rightarrow f(x_{-1}, y_{-1}) = 2e^{0.2} - 2.040 = 0.40280552$

$x_{0} = 0.3 \rightarrow y_{0} = 2.090 \rightarrow f(x_{0}, y_{0}) = 2e^{0.3} - 2.090 = 0.609718$

$y_1^{P} = y_0 + \frac{h}{24}[55f_0 - 59f_{-1} + 37f_{-2} - 9f_{-3}]$

$= 2.090 + \frac{0.1}{24}(55 \times 0.609718 - 59(0.40280552) + 37(0.200342) - 9(0))$

$= 2.16159$

$f_1 = f(x_1, y_1^{P}) = 2e^{0.4} - 2.16159 = f(0.4, 2.16159) = 0.8220594$

$y_1^{C} = y_0 + \frac{h}{24}[9f_1 + 19f_0 - 5f_{-1} + f_{-2}]$

$= 2.090 + \frac{0.1}{24}(9 \times 0.8220594 + 19 \times 0.609718 - 5(0.40280552) + 0.200342)$

$= 2.161539546$

18BML0104 Anu A   Part B

2) Evaluate the integral $\int_4^{5.2} \log_e x \, dx$ using Trapezoidal rule, Simpsons 1/3 rule & simpsons 3/8 rule by taking 6 interval

Soln: $h = \frac{5.2-4}{6} = 0.2$

x	4	4.2	4.4	4.6	4.8	5	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

- By Trapezoidal rule

$$I = \frac{h}{2}\left((y_0+y_n) + 2(y_1+y_2+y_3+ \cdots y_{n-1})\right)$$

$$= \frac{0.2}{2}\left((1.3863+1.6487) + 2(1.4351+1.4816+1.5261+1.5686+1.6094)\right)$$

$$= \underline{\underline{1.82766}}$$

- By simpson 1/3: $I = \frac{h}{3}\left((y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right)$

$$= \frac{0.2}{3}\left((1.3863+1.6487) + 4(1.4351+1.5261+1.6094) + 2(1.4816+1.5686)\right)$$

$$= \underline{\underline{1.827853}}$$

By simpson's 3/8 rule: $I = \frac{3h}{8}\left((y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right)$

$$= \frac{3\times 0.2}{8}\left((1.3863+1.6487) + 3(1.4351+1.4816+1.5686+1.6094) + 2(1.5261)\right)$$

$$= \underline{\underline{1.8278415}}$$

3, Compute $I = \int_0^1 \left(1 + \frac{\sin x}{x}\right) dx$ using Simpson's rule with 3 & 5 nodes, improve result using Romberg integration.

Soln, 3 node

$h = 1/3$

X	0	1/3	2/3	1
Y	1	1.9816	1.9276	1.8415
	y_0	y_1	y_2	y_3

By Simpson's 1/3 = $\frac{h}{3}\left((y_0+y_3)+4(y_1)+2y_2\right) =$

$= \frac{1}{9}\left(1+1.8415+4(1.9816)+2(1.9276)\right)$

$I_1 = 1.6247889$ By Simpson's 1/3 rule

BY Romberg's Integration

$h = 1/3,\ h = h/2 \Rightarrow h/2 = \frac{1/3}{2} = 1/6$

X	0	1/6	1/3	1/2	2/3	5/6	1
Y	1	1.9954	1.9816	1.9589	1.9276	1.8882	1.8415
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$I_2 = \frac{1/6}{3}\left((y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right)$

$= \frac{1}{18}\left((1+1.8415)+4(1.9954+1.9589+1.8882)+2(1.9816+1.9276)\right)$

$= 1.89055$

$h/4 \Rightarrow \frac{1/3}{4} = 1/12$

X	0	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	1
Y	1	1.9988	1.9954	1.9896	1.9816	1.9713	1.9589	1.9442	1.9276	1.9089	1.8882	1.8657	1.8415
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}

$I_3 = \frac{1/12}{3}\left((y_0+y_{12})+4(y_1+y_3+y_5+y_7+y_9+y_{11})+2(y_2+y_4+y_6+y_8+y_{10})\right)$

$= \frac{1}{36}\left((1+1.8415)+4(1.9988+1.9896+1.9713+1.9442+1.9089+1.8657)+2(1.9954+1.9816+1.9589+1.9276+1.8882)\right)$

$= 1.91883$

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By Romberg's method

step size			
h	1.6247889	$\frac{4(1.89055)-1.6247889}{3} = 1.979137$	1.92486468
$h/2$	1.89055	$\frac{4(1.91883)-1.89055}{3} = 1.9282567$	
$h/4$	1.91883		

for 5 node

x	0	0.2	0.4	0.6	0.8	1
y	1	1.9933	1.9735	1.9411	1.8961	1.8415
	y_0	y_1	y_2	y_3	y_4	y_5

→ By simpson's 3/8 rule, $I = \frac{3h}{8}\left((y_0+y_5)+3(y_1+y_2+y_4)+2y_3\right)$

$= 1.823565$

→ By simpson's 1/3 rule $I = \frac{h}{3}\left((y_0+y_5)+4(y_1+y_3)+2(y_2+y_4)\right)$

$= \frac{0.2}{3}\left((1+1.8415)+4(1.9933+1.9411)+2(1.9735+1.8961)\right)$

$= \underline{\underline{1.754633}}$

4) Given $y'' + x^2 y' + y = 0$, $y(0) = 1$ $y'(0) = 0$ find the value of y at $x = 0.1$ by Runge-Kutta method of fourth order.

Let $\frac{dy}{dx} = z$, then $\frac{d^2y}{dx^2} = z'$

$y' - z = 0$, $z(0) = 0$ $x_0 = 0$

$z' + x^2 z + y = 0$ $\rightarrow$ $y(0) = 1$

$\frac{dy}{dx} = z \Rightarrow y' = z$ ---1, $f_1(x, y, z) = z$ ---1

$z' = -x^2 z - y$ ---2 $f_2(x, y, z) = -x^2 z - y$ ---2

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let $h = 0.1$, $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
$z_{i+1} = z_i + \frac{1}{6}(L_1 + 2L_2 + 2L_3 + L_4)$

$k_1 = h f_1(x_0, y_0, z_0) = 0.1 f_1(0, 1, 0) = 0.1 \times 0 = 0$

$L_1 = h f_2(x_0, y_0, z_0) = 0.1 f_2(0, 1, 0) = -(0)^2(0) - 1 = -1$

$k_2 = h f_1(x_0 + h/2, y_0 + \frac{k_1}{2}, z_0 + L_1/2) = 0.1 f_1(0 + 0.1/2, 1 + \frac{0}{2}, 0 + -1/2)$
$= 0.1 f_1(0.05, 1, -0.5)$
$= 0.1 \times (-0.5) = -0.05$

$L_2 = h f_2(x_0 + h/2, y_0 + k_1/2, z_0 + L_1/2) = h f_2(0.05, 1, -0.5)$
$= 0.1(-(0.05)^2(-0.5) - 1)$
$= -0.099875$

$k_3 = h f_1(x_0 + h/2, y_0 + \frac{k_2}{2}, z_0 + L_2/2)$
$= 0.1 f_1(0.05, 0.975, -0.0499375)$
$= 0.1 \times (-0.0499375) = -0.00499375$

$L_3 = h f_2(0.05, 0.975, -0.0499375)$
$= 0.1(-0.05^2 \times (-0.0499375) - 0.975) = -0.0974875$

$k_4 = h f_1(x_0 + h, y_0 + k_3, z_0 + L_3)$
$= 0.1 f_1(0.1, 0.99500625, -0.0974875)$
$= 0.1 \times (-0.0974875) = -0.00974875$

$L_4 = h f_2(0.1, 0.99500625, -0.0974875)$
$= 0.1[-(0.1)^2(-0.0974875) - 0.99500625)$
$= -0.0994031375$

$Y(0.1) = Y(0) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
$= 1 + \frac{1}{6}(0 + 2(-0.05) + 2(-0.0499375) + -0.00974875)$
$= 0.9650627083$

$z(0.1) = z(0) + \frac{1}{6}(L_1 + 2L_2 + 2L_3 + L_4)$
$= 0 + \frac{1}{6}(-1 + 2(-0.099875) + 2(-0.0974875) + -0.099940)$
$= -0.23411683$

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(5) Solve the equation $y''(x) - xy(x) = 0$ for $y(0), y(1/3), y(2/3)$ given that $y(0) + y'(0) = 1$ and $y(1) = 1$

Solution

x_0	x_1	x_2	x_3
0	$1/3$	$2/3$	1
y_0	$y(1/3)$	$y_{2/3}$	y_1
$=1$			$=1$

$h = 1/3$

$$y'' = \frac{y_{i+1} + y_{i-1} - 2y_i}{h^2}, \quad \boxed{y' = \frac{y_{i+1} - y_{i-1}}{2h}}, \quad y(0) + y'(0) = 1$$

→ (two point centeral difference)

→ By using finite difference.

So, we have $y''(x) - xy(x) = 0$

$$\frac{(y_{i+1} + y_{i-1} - 2y_i)}{h^2} - x_i y_i = 0$$

By two point forward difference

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

$$f_0' = \frac{f_1 - f_0}{h} \Rightarrow \frac{1 - f_0}{1/3} + f_0 = 1$$

$$3 - 3f_0 + f_0 = 1$$

$$-2f_0 = 1 - 3$$

$$\Rightarrow -2f_0 = -2$$

$$f_0 = 1$$

for $i = 1$,

$$\frac{y_2 + y_0 - 2y_1}{(1/3)^2} - x_1 y_1 = 0 \quad \text{and } y(0) = 1 = y_0$$

$$\Rightarrow 9y_2 + 9y_0 - 18y_1 - \tfrac{1}{3}y_1 = 0$$

$$\Rightarrow 9y_2 + 9y_0 - \frac{55}{3}y_1 = 0$$

$$\Rightarrow 27y_2 + 27y_0 - 55y_1 = 0$$

$$\Rightarrow 27y_2 - 55y_1 + 27y_0 = 0 \quad \text{--- (1)}$$

$$\Rightarrow 27y_2 - 55y_1 + 27 = 0 \quad \text{since } y_0 = 1$$

Since

$$y_0' = \frac{y_1 - y_0}{h}$$

$$\Rightarrow y(0) + \frac{y_1 - y_0}{1/3} = 1$$

$$y(0) + 3 - 3y_0 = 1$$

$$-2y_0 = -2$$

$$y_0 = 1$$

for $i = 2$,

$$\frac{(y_3 + y_1 - 2y_2)}{(1/3)^2} - x_2 y_2 = 0$$

$$\Rightarrow 9y_3 + 9y_1 - 18y_2 - \frac{2}{3}y_2 = 0$$

$$\Rightarrow 27y_3 + 27y_1 - 56y_2 = 0$$

$$\Rightarrow 27y_3 - 56y_2 + 27y_1 = 0 \quad \text{--- (2)}$$

we have $y_3 = 1$

$$\Rightarrow -56y_2 + 27y_1 = -27 \quad \text{--- (2)}$$

$$\begin{cases} -55y_1 + 27y_2 = -27 \\ 27y_1 - 56y_2 = -27 \end{cases}$$

$$\boxed{\begin{aligned} y(1/3) &= y_1 = 0.9532113984 \\ y(2/3) &= y_2 = 0.9417269247 \end{aligned}}$$

$- 1.81602$

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6) Use natural cubic spline to determine y at $x=1.5$

x	1	2	3	4	5
$f(x)$	0	1	0	1	0
	f_0	f_1	f_2	f_3	f_4

Sol/n Since it has uniform interval $h_i = h = 1$

$M_0 = 0,\ M_4 = 0$

we have $M_{i-1} + 4M_i + M_{i+1} = 6/h^2 (f_{i-1} - 2f_i + f_{i+1})$

put $\Rightarrow i=1$ $\quad M_0 + 4M_1 + M_2 = 6(f_0 - 2f_1 + f_2)$

$= 4M_1 + M_2 = 6(0 - 2(1) + 0) = -12$ --- (1)

put $i=2$ $\quad M_1 + 4M_2 + M_3 = 6(f_1 - 2f_2 + f_3)$

$M_1 + 4M_2 + M_3 = 6(1 - 0 + 1) = 12$ --- (2)

put $i=3$ $\quad M_2 + 4M_3 + \cancel{M_4}^{0} = 6(f_2 - 2f_3 + f_4)$

$M_2 + 4M_3 = 6(0 - 2(1)) = -12$ --- (3)

+ solve 1, 2 & 3 we get $M_1 = -30/7$, $M_2 = 36/7$, $M_3 = -30/7$

we have formula $y_i(x) = \frac{1}{6h_i}\left((x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right) + \frac{1}{h_i}$
$(x_i - x)\left(f_{i-1} - \frac{h_i^2}{6} M_{i-1}\right) + \frac{1}{h_i}(x - x_{i-1})$
$\left(f_i - \frac{h_i^2}{6} M_i\right)$

$\Rightarrow$ put $i=1 \Rightarrow \frac{1}{6}\left((x_1 - x)^3 \cancel{M_0}^{0} + (x - x_0)^3 M_1\right) + (x_1 - x)\left(f_0 - 1/6 \cancel{M_0}^{0}\right)$
$+ (x - x_0)(f_1 - 1/6 M_1)$

$\Rightarrow 1/6 (x-1)^3(-30/7) + (2-x)(0) + (x-1)(1 - 1/6(-30/7))$

$\Rightarrow -5/7 (x-1)^3 + (x-1)(1 + 5/7) = -5/7 (x-1)^3 + (x-1)\left(\frac{12}{7}\right)$

$\Rightarrow -5/7 (x-1)^3 + \frac{12}{7}(x-1)$ $\quad [x_0, x_1] = [1, 2]$

so/ y at $x = 1.5$, $\quad -5/7 (1.5-1)^3 + \frac{12}{7}(1.5-1)$

$= -5/7 (0.5)^3 + \frac{12}{7}(0.5) = \frac{43}{56}$

$= 0.7678571429$

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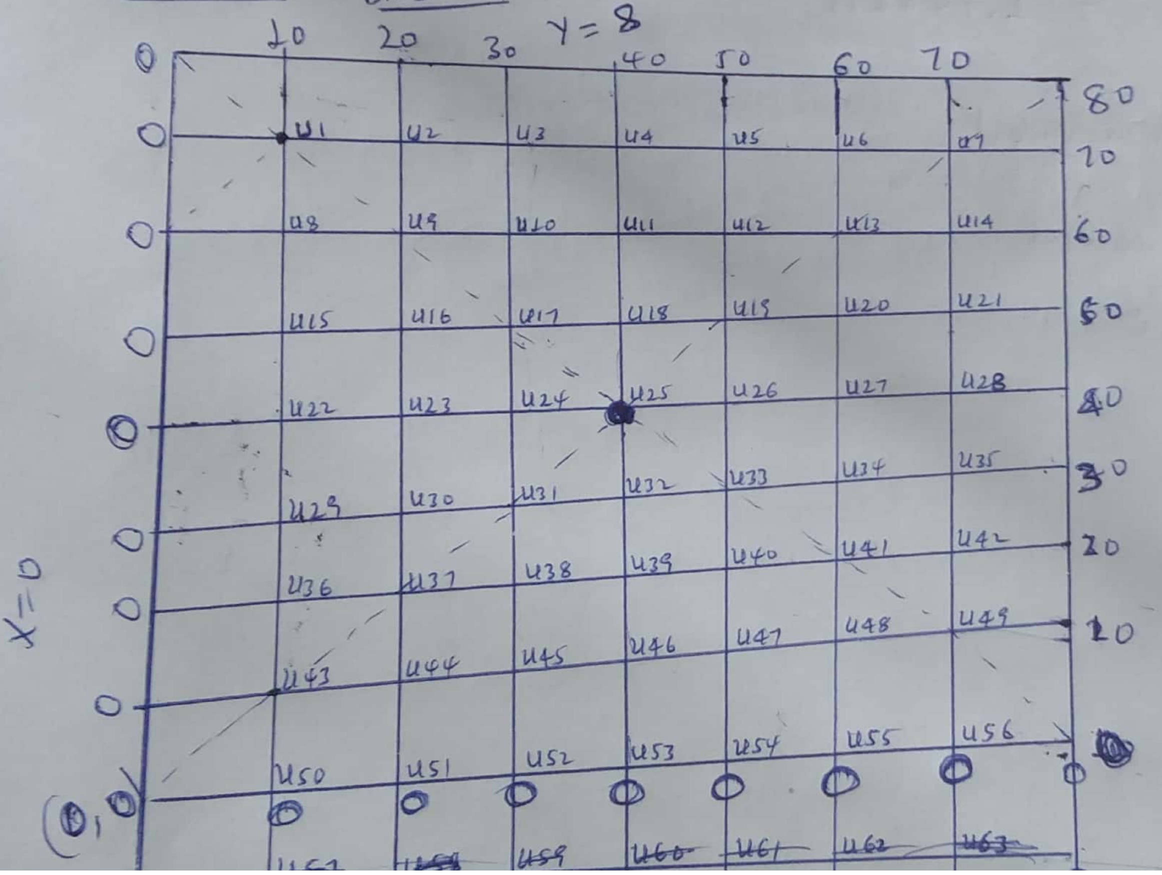
7 / Solve the laplace equation $u_{xx}+u_{yy}=0$ over a square region with side $x=0$ and $x=8$, $y=0$, $y=8$, taking the boundary condition $u(0,y)=0$, $u(x,0)=0$
$u(x,8)=10x$, $u(8,y)=10y$ and uniform spacing
$\Delta x = \Delta y = 1$

Solution:- 8x8 grid



$u_{25} = \frac{1}{4}(0+0+40+40)$

$= \frac{40+40}{4} = 20$

Using SFPF

X = 8

$u_9 = \frac{1}{4}(0+20+40+0)$

$= 15$

(diagonal formula)

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$u_{13} = \frac{1}{4}(20+40+80+40) = 45 \rightarrow$ diagonal formula

$u_{37} = \frac{1}{4}(0+0+0+0+20) = 5$ diagonal formula

$u_{41} = \frac{1}{4}(0+0+40+20) = 15 \rightarrow$ diagonal formula

$u_{11} = \frac{1}{4}(15+45+20+40) = 30 \rightarrow$ SFPF

$u_{39} = \frac{1}{4}(0+15+20+5) = 10 \rightarrow$ SFPF

$u_1 = \frac{1}{4}(0+15+20+0) = 8.75 \rightarrow$ diagonal formula

$u_3 = \frac{1}{4}(15+30+40+20) = 26.25 \rightarrow$ diagonal

$u_5 = \frac{1}{4}(30+45+60+40) = 43.75 \rightarrow$ diagonal

$u_7 = \frac{1}{4}(45+60+80+60) = 61.25 \rightarrow$ diagonal

$u_2 = \frac{1}{4}(8.75+15+26.25+20) = 17.5 \rightarrow$ standard SPF

$u_4 = \frac{1}{4}(26.25+30+43.75+40) = 35$ SFPF

$u_6 = \frac{1}{4}(43.75+45+61.25+60) = 52.5$ SFPF

$u_{23} = \frac{1}{4}(0+5+20+15) = 10 \rightarrow$ SFPF

$u_{15} = \frac{1}{4}(0+10+15+0) = 6.25 \rightarrow$ diagonal Formula

$u_{17} = \frac{1}{4}(20+30+15+10) = 18.75 \rightarrow$ diagonal

$u_{27} = \frac{1}{4}(15+40+45+20) = 30$ SFPF

$u_{19} = \frac{1}{4}(20+30+45+30) = 31.25 \rightarrow$ diagonal

$u_{21} = \frac{1}{4}(40+60+45+30) = 43.75 \rightarrow$ dF

$u_8 = \frac{1}{4}(15+6.25+0+8.75) = 7.5 \rightarrow$ SFPF

$u_{10} = \frac{1}{4}(15+30+26.25+18.75) = 22.5$ SFPF

$u_{12} = \frac{1}{4}(30+31.25+45+43.75) = 37.5 \rightarrow$ SFPF

$u_{16} = \frac{1}{4}(6.25+10+18.75+15) = 12.5$ SFPF

$u_{18} = \frac{1}{4}(18.75+20+31.25+30) = 25 \rightarrow$ SFPF

$u_{20} = \frac{1}{4}(31.5+30+43.75+45) = 37.5 \rightarrow$ SFPF

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$u_{29} = \frac{1}{4}(0+5+10+0) = 3.75 \rightarrow$ dF

$u_{31} = \frac{1}{4}(5+20+10+10) = 11.25 \rightarrow$ dF

$u_{33} = \frac{1}{4}(10+15+30+20) = 18.75 \rightarrow$ dF

$u_{35} = \frac{1}{4}(15+20+40+30) = 26.25 \rightarrow$ dF

$u_{22} = \frac{1}{4}(0+3.75+10+6.25) = 5 \rightarrow$ SFPF

$u_{24} = \frac{1}{4}(10+20+18.75+11.25) = 15 \rightarrow$ SFPF

$u_{26} = \frac{1}{4}(20+18.75+30+31.25) = 25 \rightarrow$ SFPF

$u_{28} = \frac{1}{4}(30+26.25+40+43.75) = 35 \rightarrow$ SFPF

$u_{30} = \frac{1}{4}(3.75+5+11.25+10) = 7.5 \rightarrow$ SFPF

$u_{32} = \frac{1}{4}(11.25+10+18.75+20) = 15 \rightarrow$ SFPF

$u_{34} = \frac{1}{4}(18.75+15+26.25+30) = 22.5 \rightarrow$ SFPF

$u_{43} = \frac{1}{4}(0+0+0+5) = 1.25$

$u_{45} = \frac{1}{4}(5+0+0+10) = 3.75$

$u_{47} = \frac{1}{4}(10+0+0+15) = 6.25$

$u_{49} = \frac{1}{4}(15+0+0+20) = 8.75$

dF

$u_{36} = \frac{1}{4}(0+1.25+5+3.75) = 2.5$

$u_{38} = \frac{1}{4}(5+3.75+10+11.25) = 7.5$

$u_{40} = \frac{1}{4}(10+6.25+15+18.75) = 12.5$

$u_{42} = \frac{1}{4}(15+8.75+20+26.25) = 17.5$

$u_{44} = \frac{1}{4}(1.25+0+3.75+5) = 2.5$

$u_{46} = \frac{1}{4}(3.75+0+6.25+10) = 5$

$u_{48} = \frac{1}{4}(6.25+0+8.75+15) = 7.5$

SFPF

$u_1^{(1)} = \frac{1}{4}(0 + 7.5 + 17.5 + 10) = 8.75$

$u_2^{(1)} = \frac{1}{4}(8.75 + 15 + 26.25 + 20) = 17.5$

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