Fluctuation-dissipation relations for thermodynamic distillation processes

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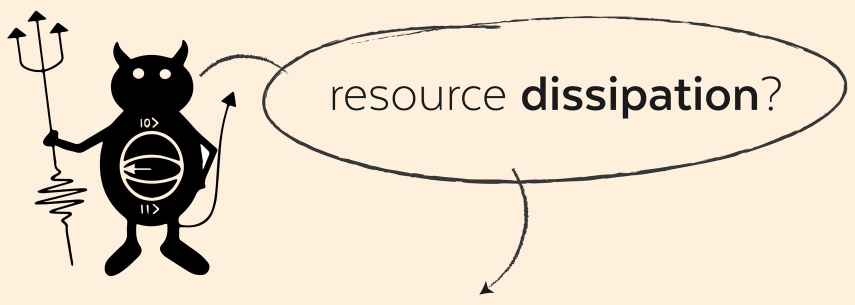
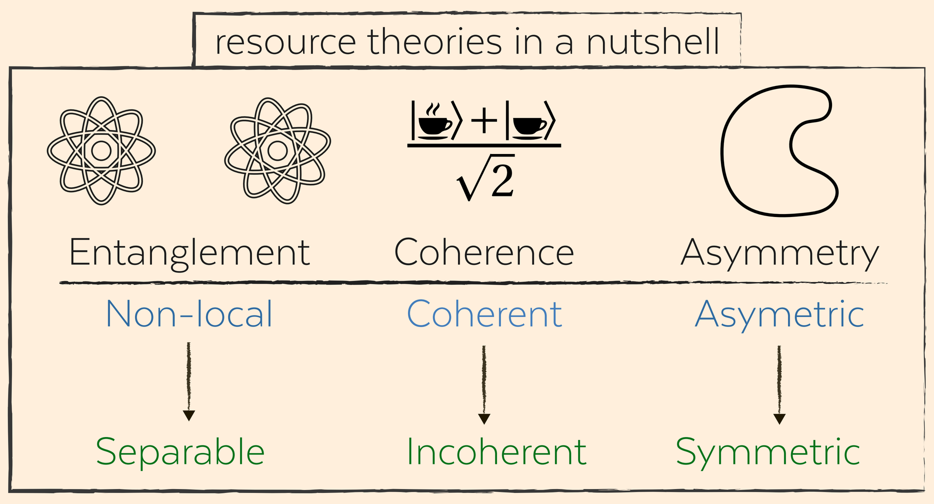
Background

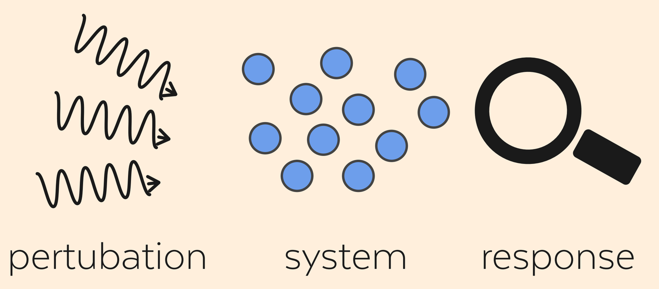
Goal: study resource dissipation and characterise optimal state transformation protocols.

Framework: resource-theoretic ordering of quantum states.





Fluctuation-dissipation relations



$\Longrightarrow$ fluctuation - dissipation

reference or **thermal state**

thermodynamic charge

Resource theory of thermodynamics

Indentifying the set of **thermodynamically-free states**

$$(\rho, H) + (\gamma_E, H_E) \Longrightarrow \gamma = \frac{e^{-\beta H}}{Z}, \quad Z = \mathrm{Tr}(e^{-\beta H})$$

Thermodynamic transformations are modelled by **thermal operations**

$$\mathcal{E}(\rho) = \mathrm{Tr}_E(U(\rho\otimes\gamma_E)U^\dagger) \quad \text{with} \quad [U, H\otimes\mathbb{1}_E + \mathbb{1}_E\otimes H_E] = 0$$

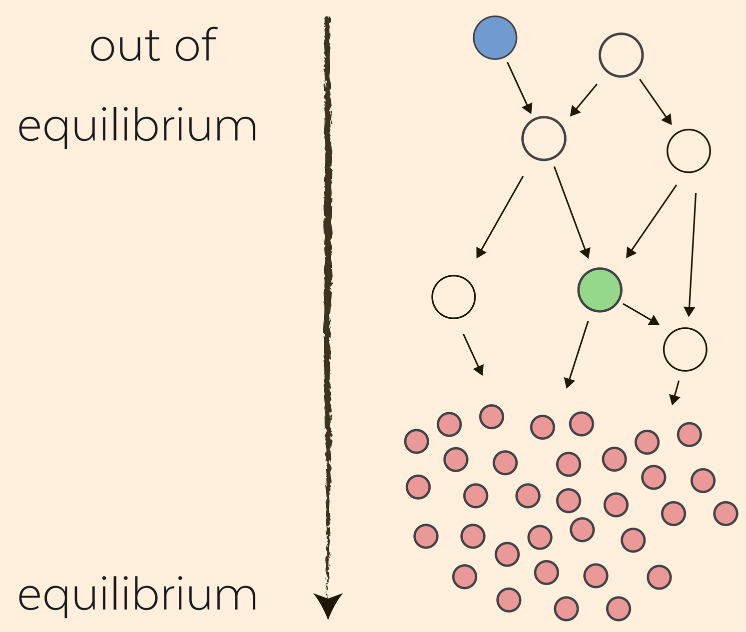
Relevant **information-theoretic** quantities

i. $D(\rho\|\gamma) = \mathrm{Tr}(\rho(\log\rho - \log\gamma))$ Generalised **free energy** (monotone)

ii. $V(\rho\|\gamma) = \mathrm{Tr}\left(\rho\,(\log\rho - \log\gamma - D(\rho\|\gamma))^2\right)$ **Fluctuations**

Thermodynamic distillation process

For initial state ρ, target state σ, thermal bath β $\Longrightarrow \mathcal{E}(\rho) = \sigma$.



! General answer not known beyond the simplest qubit case.

Phys. Rev. X 5, 021001 (2015)
Nat.Commun. 6, 7689 (2015)

! For **energy-incoherent** states.

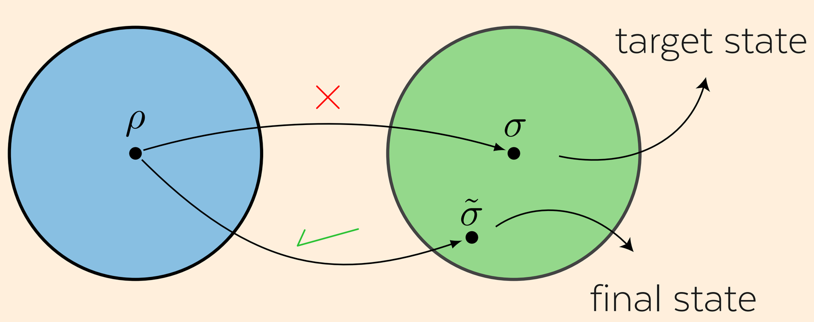
Nat.Commun. 4, 2059 (2013)

$$\boxed{\mathcal{E}(\rho) = \sigma : \mathbf{p} \succ^{\beta} \mathbf{q}}$$

ϵ -approximate **interconversion** problem:

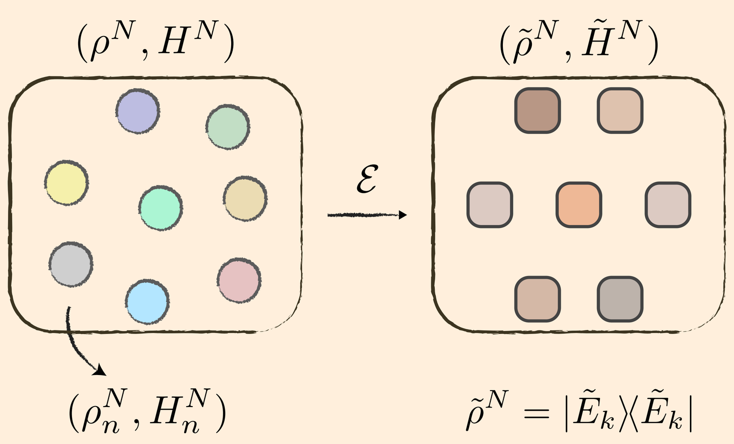
! Approximate interconversion problem with **finite** system.

Quantum, vol. 2, p.108, (2018)



? For **general** states. ? **Dissipation** of resources.
? **Different** subsystems. ? **Optimal** processes.

ϵ -approximate **thermodynamic distillation process**:



Relevant thermo **information-theoretic** quantities

iii. $\Delta F^N := \frac{1}{\beta}\left(\sum_{n=1}^{N} D(\rho_n^N\|\gamma_n^N) - D(\tilde{\rho}^N\|\tilde{\gamma}^N)\right)$

Free energy **difference**

iv. $\sigma^2(F^N) := \frac{1}{\beta^2}\sum_{n=1}^{N} V(\rho_n^N\|\gamma_n^N)$

Free energy **fluctuations**

Results

Theorem. For a distillation setting with energy incoherent or pure initial states, the transformation error of the approximate distillation process in the asymptotic limit is given by

$$\lim_{N\to\infty}\epsilon_N = \lim_{N\to\infty}\Phi\left(-\frac{\Delta F^N}{\sigma(F^N)}\right)$$

Moreover, for any N there exist an approximate distillation process with enegy incoherent initial state with the transformation error bounded by

$$\epsilon_N \leq \Phi\left(-\frac{\Delta F^N}{\sigma(F^N)}\right) + \frac{C\kappa^3(F^N)}{\sigma^3(F^N)}$$

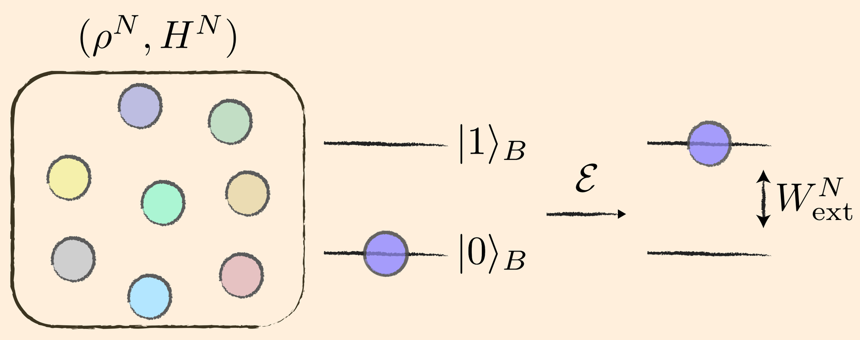
The amout of free energy dissipated in the above setting satisfies

$$F_{\text{diss}}^{\text{tot}} = a(\epsilon)\,\sigma^{\text{tot}}(F) \quad \text{with } a(\epsilon) = -\Phi^{-1}(\epsilon)(1-\epsilon) + \frac{\exp\left(\frac{-(\Phi^{-1}(\epsilon))^2}{2}\right)}{\sqrt{2\pi}}$$

with Φ^{-1} being the inverse of the Gaussian cumulative distribution

Aplications

Optimal work extraction



Applying the above *Theorem*, the extracted work for a given error is given by:

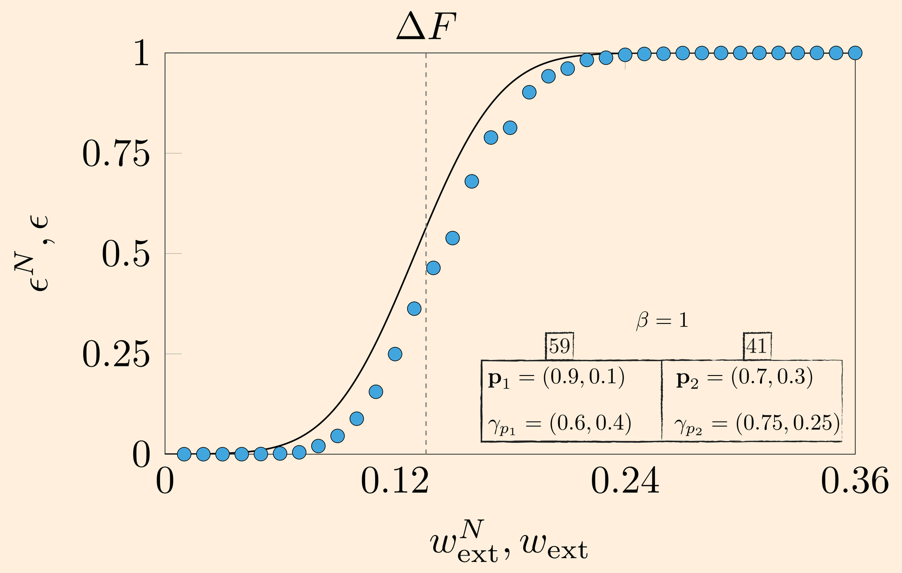
$$\boxed{W_{\text{ext}}^N \simeq F^N + \sigma(F^N)\Phi^{-1}(\epsilon)}$$

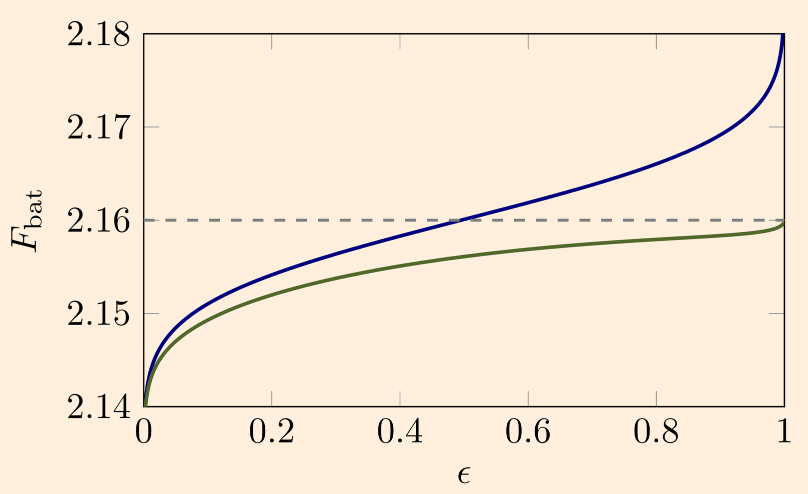
energy-incoherent states

The optimal amount of work extracted from N pure quantum systems up to second-order is given by:

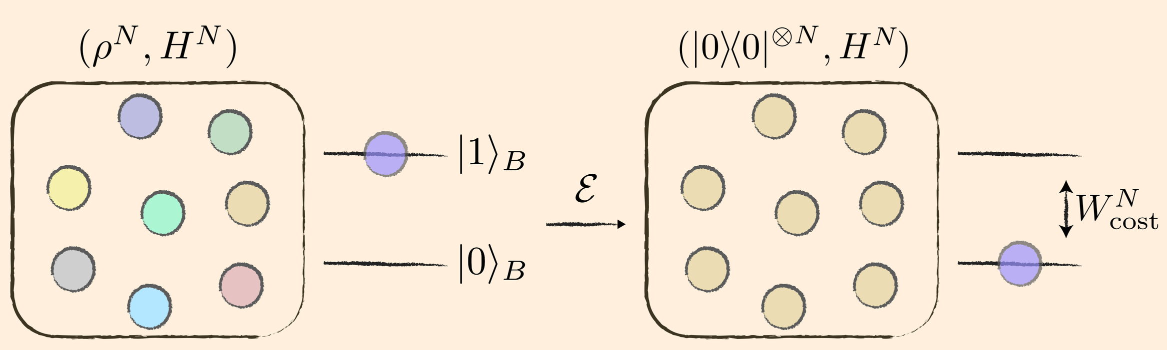
$$\boxed{W_{\text{ext}} \simeq N\left(\langle H\rangle_\psi + \frac{\log Z}{\beta} + \frac{\langle H^2\rangle_\psi - \langle H\rangle_\psi^2}{\sqrt{N}}\Phi^{-1}(\epsilon)\right).}$$

pure states





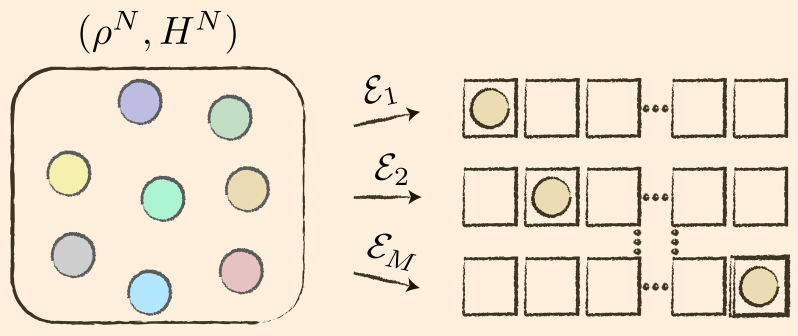
Optimal information erasure



The above *Theorem* also provides the second-order asymptotics for the cost of erasure:

$$\boxed{W_{\text{cost}}^N \simeq \frac{S(\rho^N)}{\beta} - \sigma(F^N)\Phi^{-1}(\epsilon).}$$

Optimal thermodynamically-free communication rate



The optimal number of messages that can be encoded into in a thermodynamically-free way:

! This result is valid for either a pure or incoherent state!

$$\boxed{R(\rho^{\otimes N}, \epsilon_{\text{d}}) \simeq D(\rho\|\gamma) + \frac{\sqrt{V(\rho\|\gamma)}}{\sqrt{N}}\Phi^{-1}(\epsilon_{\text{d}}).}$$

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