Quantum catalysis in cavity QED

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What if quantum catalysis could go beyond theory and step into practical context?

Main idea: A $\xrightarrow[\times]{\text{constraint}}$ B but A+c $\xrightarrow[\checkmark]{\text{constraint}}$ B+c ⟿ *Resource theories* have uncovered **fundamental** limits and revelead **properties** of c

catalyst

highly abstract + limited to special cases

❓ How to make it useful?

Setting the scene

Composite system: $\rho_S \otimes \chi_C \longrightarrow \rho_S \to \sigma_S := \mathrm{Tr}_C[U(\rho_S \otimes \chi_C)U^\dagger]$

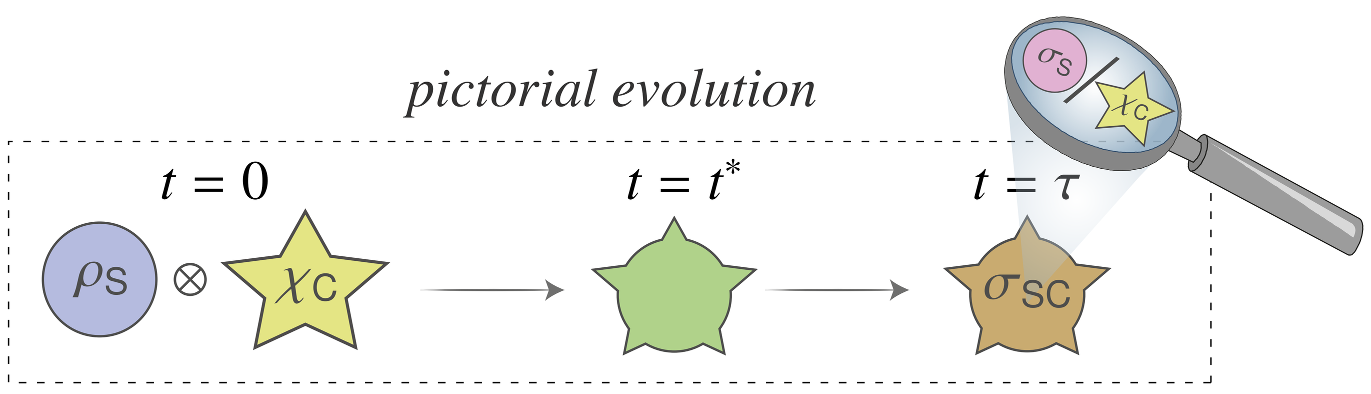
while the state of the catalyst **returns** to its initial state at time: τ

$$\sigma_C := \mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C \quad \boxed{\textbf{catalytic constraint}}$$

$U := U(\tau)$

can always be satisfied!

pictorial evolution

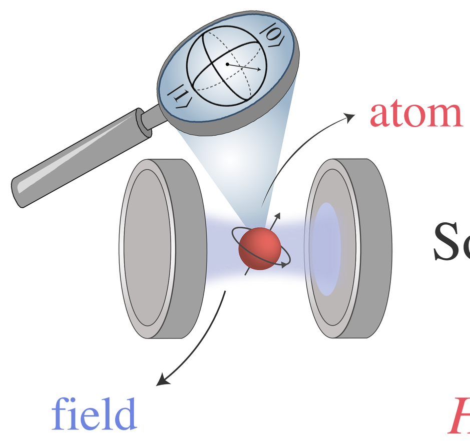


P. Lipka-Bartosik, H. Wilming, and N. H. Y. Ng, arXiv. 2306.00798

Model

Jaynes-Cummings model ($\hbar = 1$): $H_{SC} = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + g\left(\sigma_+ a + \sigma_- a^\dagger\right)$

only couple pairs of atom-field states: $\{|n+1, g\rangle, |n, e\rangle\}$



So, the eigenproblem is completely determined by $H_{SC} = \bigoplus_{n=0}^{\infty} H_{SC}^{(n)}$:

$$H_{SC}^{(n)} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix} = \begin{bmatrix} (n+1/2)\omega & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1/2)\omega \end{bmatrix} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix}$$

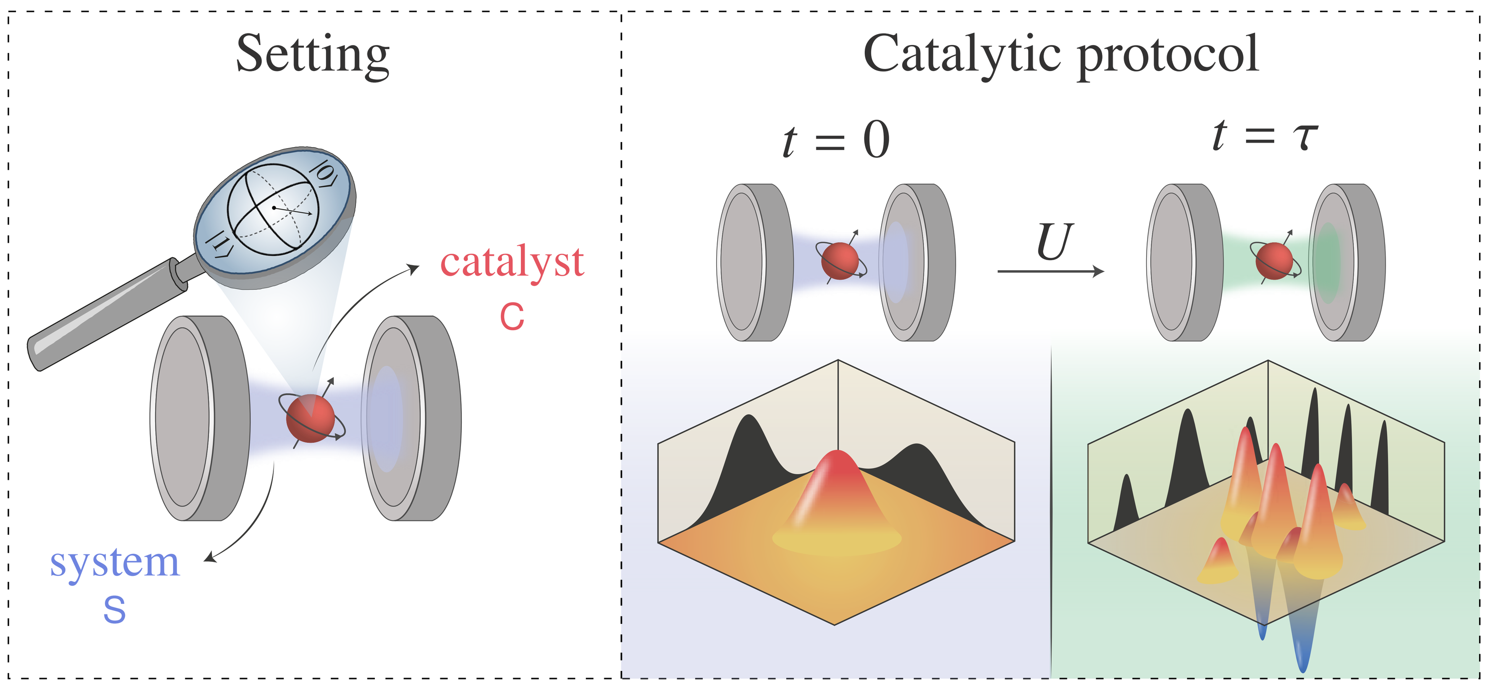
The eigenvalue problem yields the eigenfrequencies: $\omega_\pm^{(n)} = \left(n + \frac{1}{2}\right)\omega \pm g\sqrt{n+1}$

$$\sigma_{S/C} = \mathrm{Tr}_{C/S}[U(\rho_S \otimes \chi_C)U^\dagger]$$

E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963)

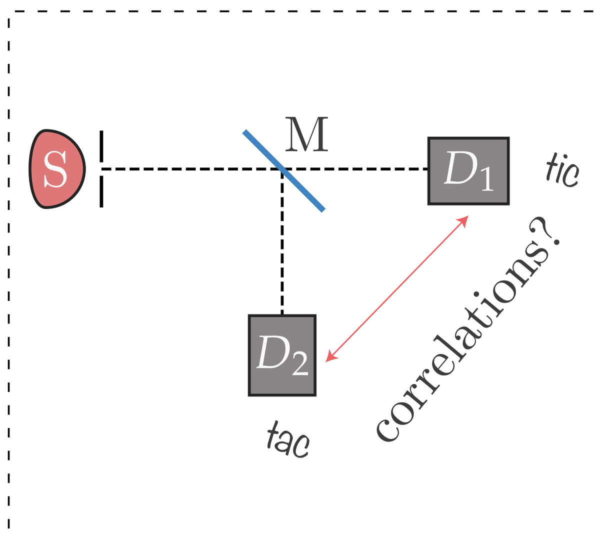
Statement of the problem

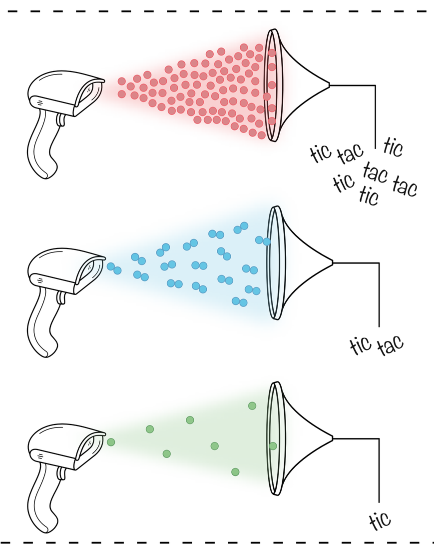
- **Task**: generation of non-classical light in a catalytic way
- **Consideration**: $\rho_S = |\alpha\rangle\langle\alpha|$, where $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$
- **Goal**: find χ_C and τ, such that $\mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C$ and the state of the cavity is non-classical



Figures of merit

i. Second-order coherence: $g^{(2)}(\sigma_S) = \dfrac{\langle n_S^2\rangle_\sigma - \langle n_S\rangle_\sigma}{\langle n_S\rangle_\sigma^2}$





Thermal state: $g^{(2)}(\sigma) = 2$ **(bunching)**

Coherent state: $g^{(2)}(\sigma) = 1$

Fock state: $g^{(2)}(\sigma) < 1$ **(anti-bunching)**

R. J. Glauber, Phys. Rev. 130, 2529 (1963)

ii. Wigner logarithimic negativity: $\mathsf{W}(\rho) := \log\left(\int \mathrm{d}x\,\mathrm{d}p\,\left|W_\rho(x,p)\right|\right)$ where W_ρ is the Wigner function:

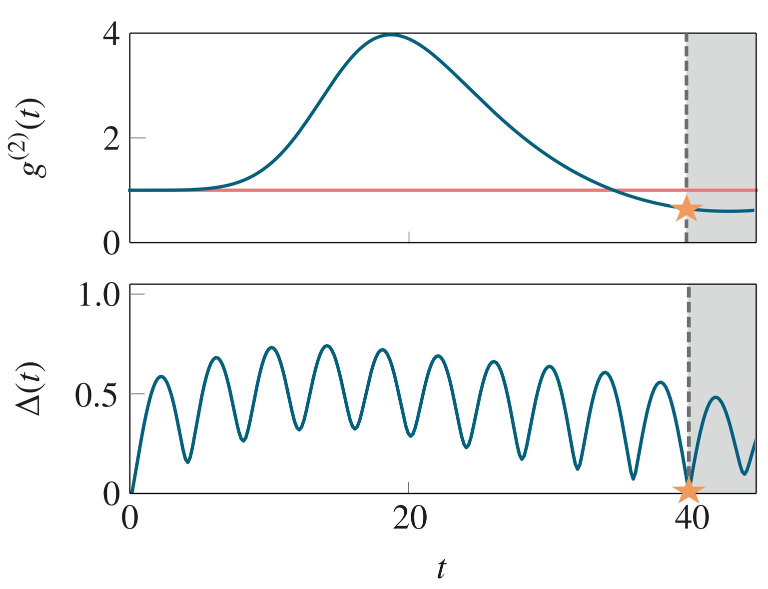
$$W_\sigma(x,p) = \frac{1}{\pi}\int e^{2ipx'}\langle x - x'|\sigma|x + x'\rangle\,\mathrm{d}x'$$

F. Albarelli, M G. Genoni, M G. A. Paris, A. Ferraro, Phys. Rev. A 98, 052350 (2018)

Results

- Generating non-classical states of light:

(a) Generating light with **sub-Poissonian** photon statistics:



★ Catalysis occurs at $\tau \approx 40$ for which $g^{(2)}(\tau) \approx 0.5$

Definitions

$g^{(2)}(t) := g^{(2)}[\sigma_S(t)]$

$\Delta(t) := \|\chi_C - \sigma_C\|_1$

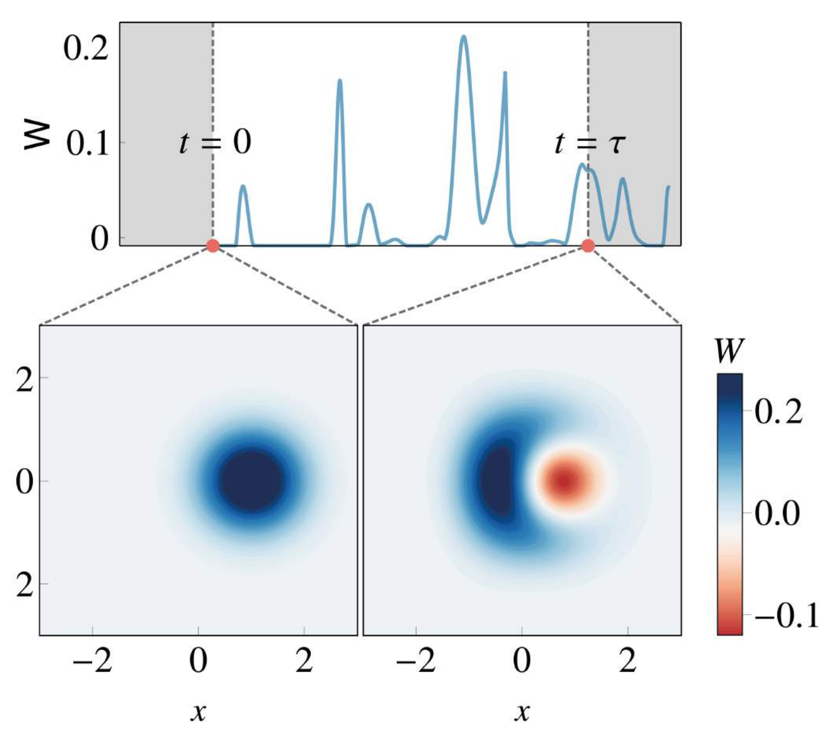
Parameters

$\alpha = 1/\sqrt{2}$

$\omega = 2\pi$

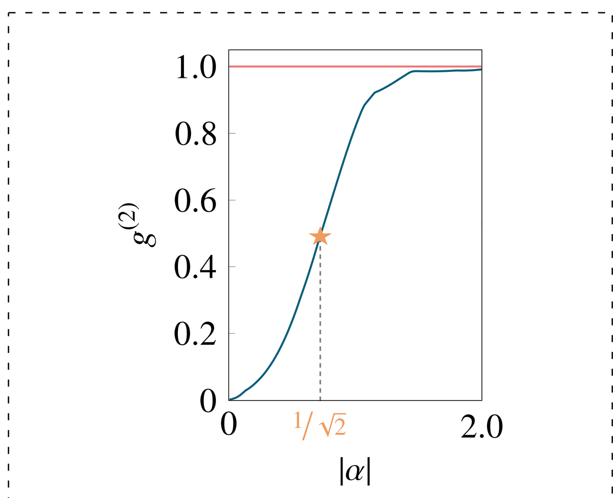
$g = \pi$

(b) Generating light with **negative** Wigner Function

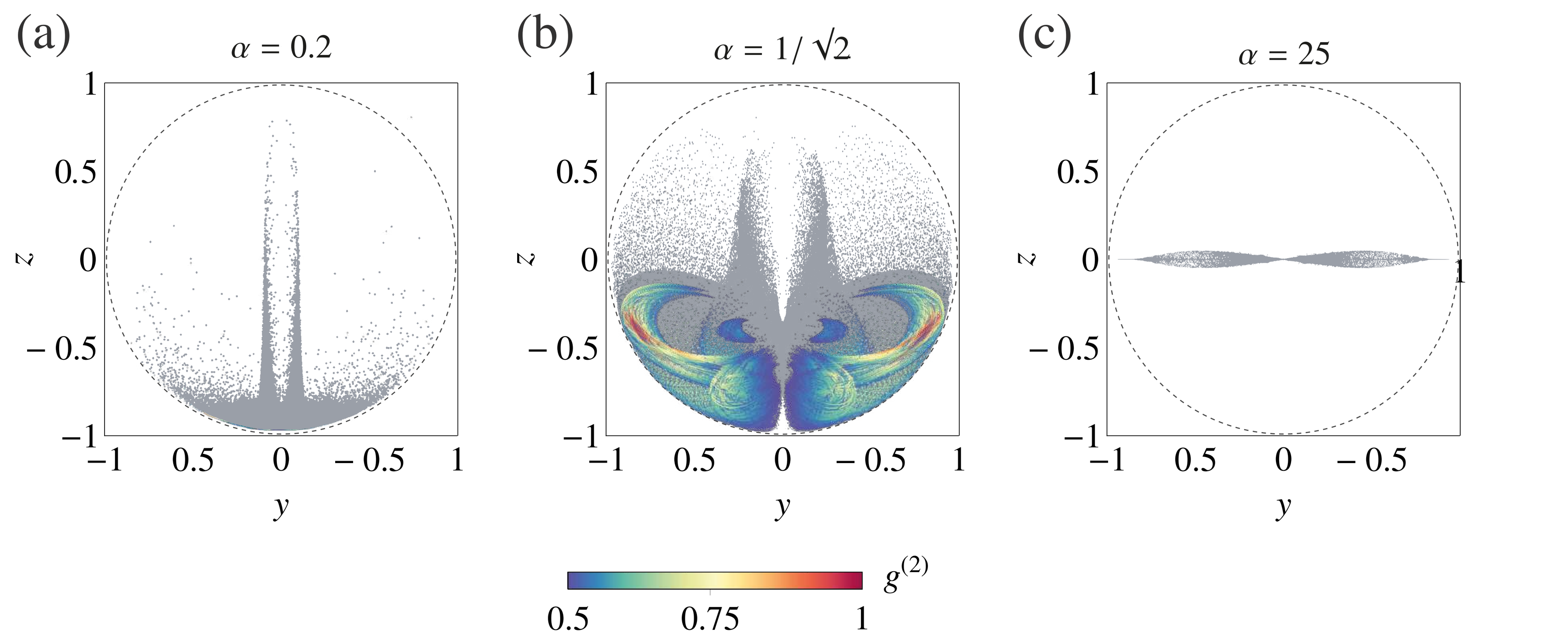


★ Catalysis occurs at $\tau \approx 5$ for which $\mathsf{W} \approx 0.1$

Engineering nonclassicality



- Which **states** lead to catalyst?



- **Mechanism** of catalysis?

i. Energy-preserving: $[U(t), n_S + n_C] = 0$

ii. Catalytic constraint $\longrightarrow$ all moments of n_C are preserved and $\langle n_S\rangle_\rho = \langle n_C\rangle_\sigma$:

$$\langle n_S^2\rangle_\sigma = \langle n_S^2\rangle_\rho + 2\left(\langle n_S\rangle_\sigma\langle n_C\rangle_\sigma - \langle n_S \otimes n_C\rangle_\sigma\right)$$

❗ Our results also holds in the presence of **dissipation**

❗ Our results also apply to multi-mode scenarios

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