Quantum catalysis in cavity QED

Alexssandre de Oliveira Junior

Faculty of Physics, Astronomy and Applied Computer Science,

Jagiellonian University

Quantum Information & Chaos

June 12, 2023

Outline

1. Introduction
2. Setting the scene
3. Results

In collaboration with:

Martí Perarnau-Llobet

Nicolas Brunner

Patryk Lipka-Bartosik

University of Geneva

Based on:
arXiv:2305.19324 - Framework & Applications

Introduction

$$A \xrightarrow{\text{Constraints}} B$$

- **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

  $A = 2H_2O_2$, B = $2H_2O + O_2$, Constrain = activation energy.

$$A \xrightarrow{\text{Constraints}} B$$

- **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

  $A = 2H_2O_2$, B = $2H_2O + O_2$, Constrain = activation energy.

$$A \;\not\xrightarrow{\text{Constraints}}\; B$$

- **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

  $A = 2H_2O_2$, B = $2H_2O + O_2$, Constrain = activation energy.

$$A \otimes c \xrightarrow{\text{Constraints}} B \otimes c$$

D. Jonathan and M. Plenio, Phys.Rev.Lett. 83,3566, (1999)
P. Lipka-Bartosik, H. Wilming, and N. H. Y. Ng, arXiv. 2306.00798

- **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

  $A = 2H_2O_2$, B = $2H_2O + O_2$, Constrain = activation energy.

$$A \otimes c \xrightarrow{\text{Constraints}} B \otimes c$$

★ Use **auxiliary degrees of freedom** to **lift** constraints!

D. Jonathan and M. Plenio, Phys.Rev.Lett. 83,3566, (1999)

P. Lipka-Bartosik, H. Wilming, and N. H. Y. Ng, arXiv. 2306.00798

- **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

  $A = 2H_2O_2$, B = $2H_2O + O_2$, Constrain = activation energy.

$$A \otimes c \xrightarrow{\text{Constraints}} B \otimes c$$

- Use **auxiliary degrees of freedom** to **lift** constraints!

- *Resource theories* have uncovered **fundamental** limits and revealed **properties** of *c!*

D. Jonathan and M. Plenio, Phys.Rev.Lett. 83,3566, (1999)

P. Lipka-Bartosik, H. Wilming, and N. H. Y. Ng, arXiv. 2306.00798

○ **Example:** $A = |\psi\rangle$, $B = |\phi\rangle$, Constrain = $LOCC$.

$A = 2H_2O_2$, B = $2H_2O + O_2$, Constrain = activation energy.

$$A \otimes c \xrightarrow{\text{Constraints}} B \otimes c$$

★ Use **auxiliary degrees of freedom** to **lift** constraints!

■ *Resource theories* have uncovered **fundamental** limits and revealed **properties** of c!

highly abstract + limited to special cases.

D. Jonathan and M. Plenio, Phys.Rev.Lett. 83,3566, (1999)

P. Lipka-Bartosik, H. Wilming, and N. H. Y. Ng, arXiv. 2306.00798

Q. Can we go beyond theory and step into **practical** contexts?

Setting the scene

Catalytic picture

Composite system: $\rho_{\mathrm{S}} \otimes \chi_{\mathrm{C}}$

Catalytic picture

Composite system: $\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}}$

$$\rho_{\mathsf{S}} \to \sigma_{\mathsf{S}} := \mathrm{Tr}_{\mathsf{C}}[U(\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}})U^\dagger]$$

Catalytic picture

Composite system: $\rho_S \otimes \chi_C$

$$\rho_S \to \sigma_S := \mathrm{Tr}_C[U(\rho_S \otimes \chi_C)U^\dagger]$$

while the state of the catalyst **returns** to its initial state at time τ:

can always be satisfied!

$$\sigma_C := \mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C$$

catalytic constrain

$U(\tau)$

stopping time: τ

Catalytic picture

Composite system: $\rho_S \otimes \chi_C$

$$\rho_S \to \sigma_S := \mathrm{Tr}_C[U(\rho_S \otimes \chi_C)U^\dagger]$$

while the state of the catalyst **returns** to its initial state at time τ:

can always be satisfied!

$$\sigma_C := \mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C$$

catalytic constrain

$U(\tau)$

stopping time: τ

pictorial evolution

$t = 0$

$\rho_S \otimes \chi_C$

Catalytic picture

Composite system: 

$$\rho_\mathsf{S} \to \sigma_\mathsf{S} := \mathrm{Tr}_\mathsf{C}[U(\rho_\mathsf{S} \otimes \chi_\mathsf{C})U^\dagger]$$

while the state of the catalyst **returns** to its initial state at time τ:

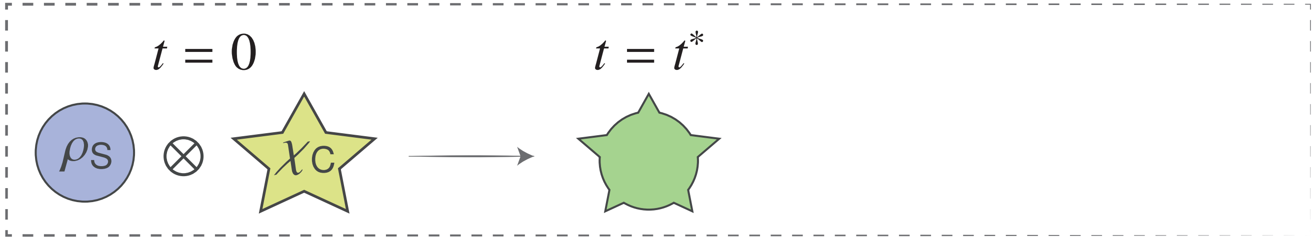
can always be satisfied!

$$\sigma_\mathsf{C} := \mathrm{Tr}_\mathsf{S}[U(\rho_\mathsf{S} \otimes \chi_\mathsf{C})U^\dagger] = \chi_\mathsf{C} \qquad \boxed{\textit{catalytic constrain}}$$

$U(\tau)$

stopping time: τ

pictorial evolution



Catalytic picture

Composite system: $\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}}$

$$\rho_{\mathsf{S}} \to \sigma_{\mathsf{S}} := \mathrm{Tr}_{\mathsf{C}}[U(\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}})U^\dagger]$$

while the state of the catalyst **returns** to its initial state at time τ:

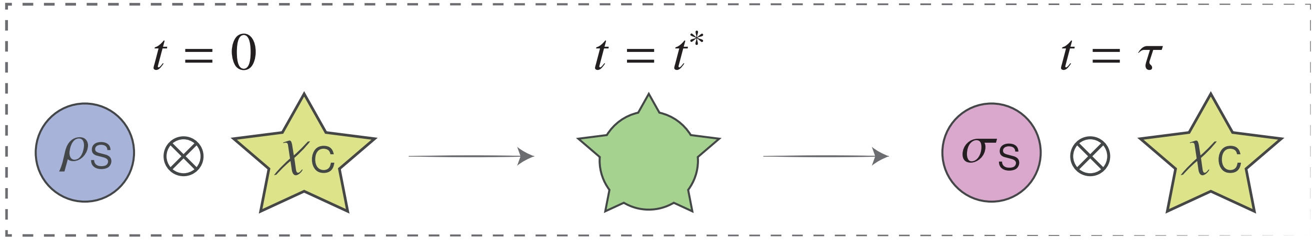
can always be satisfied!

$$\sigma_{\mathsf{C}} := \mathrm{Tr}_{\mathsf{S}}[U(\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}})U^\dagger] = \chi_{\mathsf{C}} \quad \boxed{\textbf{\textit{catalytic constrain}}}$$

$U(\tau)$

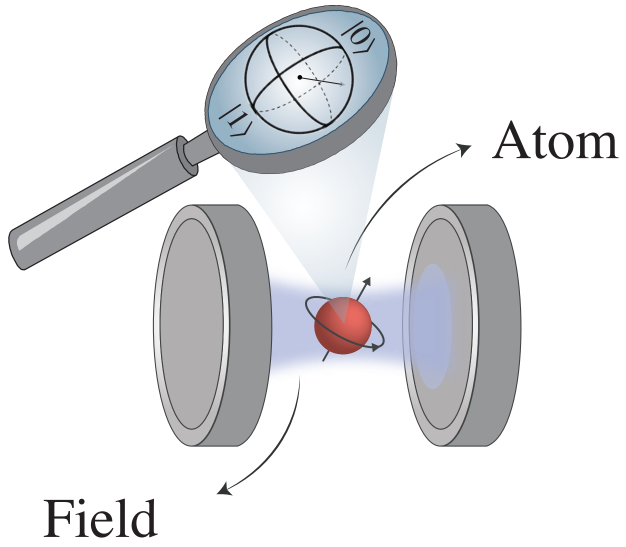
stopping time: τ

pictorial evolution



Model

Jaynes-Cummings model $(\hbar = 1)$: $H_{\mathrm{SC}} = \omega a^{\dagger} a + \frac{\omega}{2}\sigma_z + g\left(\sigma_+ a + \sigma_- a^{\dagger}\right)$

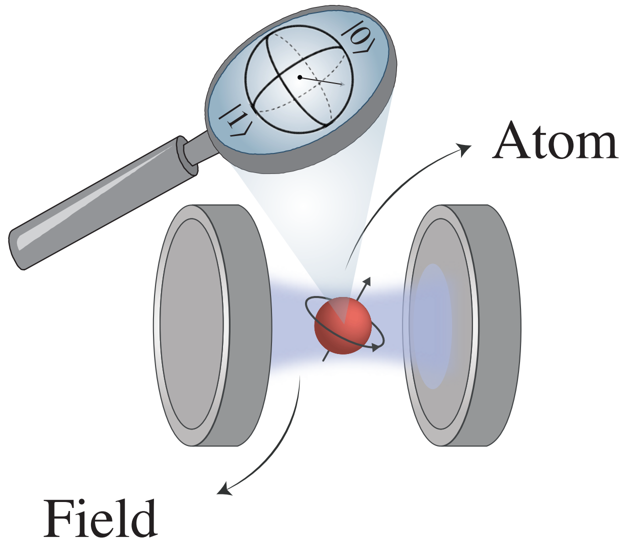


E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).

Model

Jaynes-Cummings model $(\hbar = 1)$: $H_{\mathrm{SC}} = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + g\left(\sigma_+ a + \sigma_- a^\dagger\right)$

only couple pairs of atom-field states: $\{|n+1, g\rangle, |n, e\rangle\}$

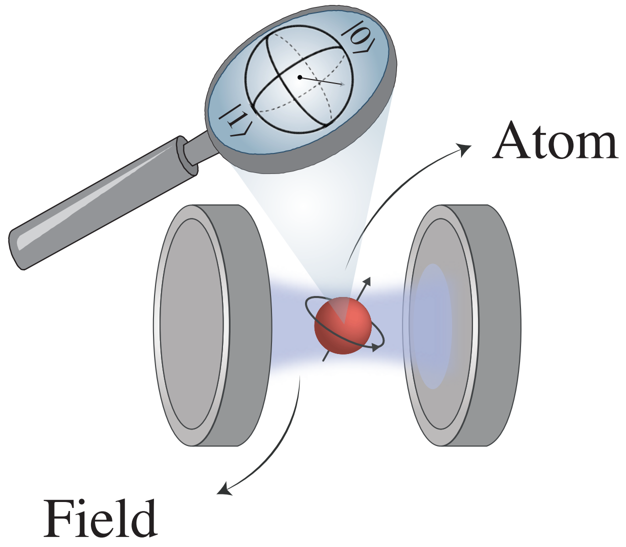


E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).

Model

Jaynes-Cummings model $(\hbar = 1)$: $H_{\mathsf{SC}} = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + g\left(\sigma_+ a + \sigma_- a^\dagger\right)$

only couple pairs of atom-field states: $\{|n+1, g\rangle, |n, e\rangle\}$



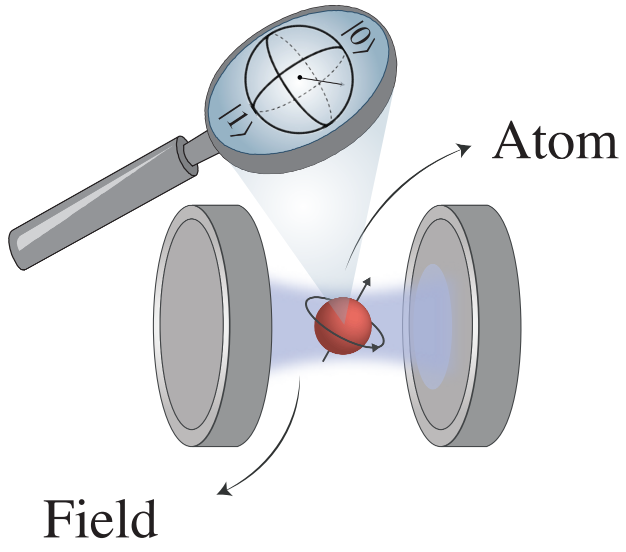
So, the eigenproblem is completely determined by $H_{\mathrm{SC}} = \bigoplus_{n=0}^{\infty} H_{\mathrm{SC}}^{(n)}$:

E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).

Model

Jaynes-Cummings model $(\hbar = 1)$: $H_{\mathrm{SC}} = \omega a^\dagger a + \dfrac{\omega}{2}\sigma_z + g\left(\sigma_+ a + \sigma_- a^\dagger\right)$

only couple pairs of atom-field states: $\{|n+1, g\rangle, |n, e\rangle\}$



So, the eigenproblem is completely determined by $H_{\mathrm{SC}} = \bigoplus_{n=0}^{\infty} H_{\mathrm{SC}}^{(n)}$:

$$H_{\mathrm{SC}}^{(n)} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix} = \begin{bmatrix} (n+1/2)\omega & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1/2)\omega \end{bmatrix} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix}$$

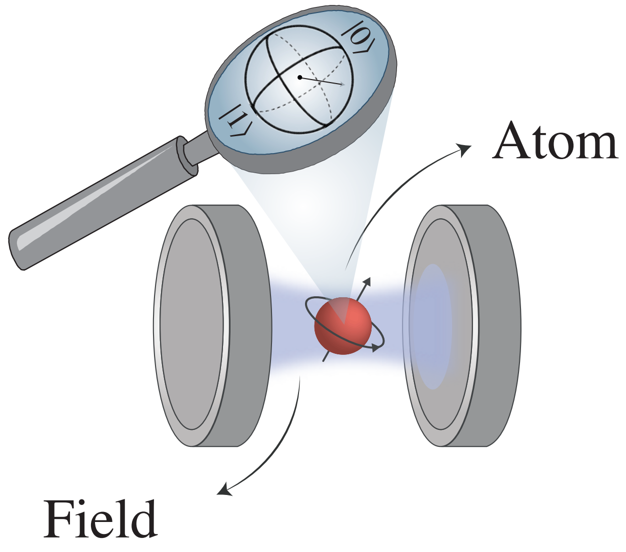
easily diagonalisable

E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).

Model

Jaynes-Cummings model $(\hbar = 1)$: $H_{\mathsf{SC}} = \omega a^\dagger a + \frac{\omega}{2}\sigma_z + g\left(\sigma_+ a + \sigma_- a^\dagger\right)$

only couple pairs of atom-field states: $\{|n+1, g\rangle, |n, e\rangle\}$



So, the eigenproblem is completely determined by $H_{\mathsf{SC}} = \bigoplus_{n=0}^{\infty} H_{\mathsf{SC}}^{(n)}$:

$$H_{\mathsf{SC}}^{(n)} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix} = \begin{bmatrix} (n+1/2)\omega & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1/2)\omega \end{bmatrix} \begin{bmatrix} |n+1, g\rangle \\ |n, e\rangle \end{bmatrix}$$

easily diagonalisable

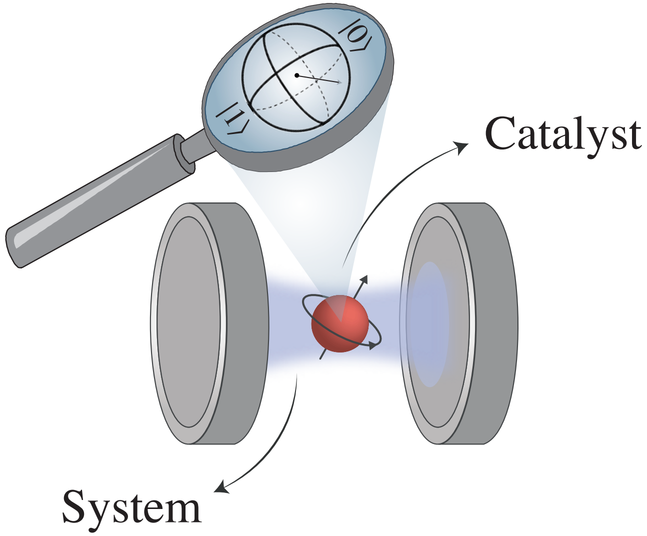
The eigenvalue problem yields the eigenfrequencies: $\omega_\pm^{(n)} = \left(n + \frac{1}{2}\right)\omega \pm 2g\sqrt{n+1}$ $\longrightarrow U$

$$\boxed{\sigma_{\mathsf{S/C}} = \mathrm{Tr}_{\mathsf{C/S}}[U(\rho_{\mathsf{S}} \otimes \omega_{\mathsf{C}})U^\dagger]}$$

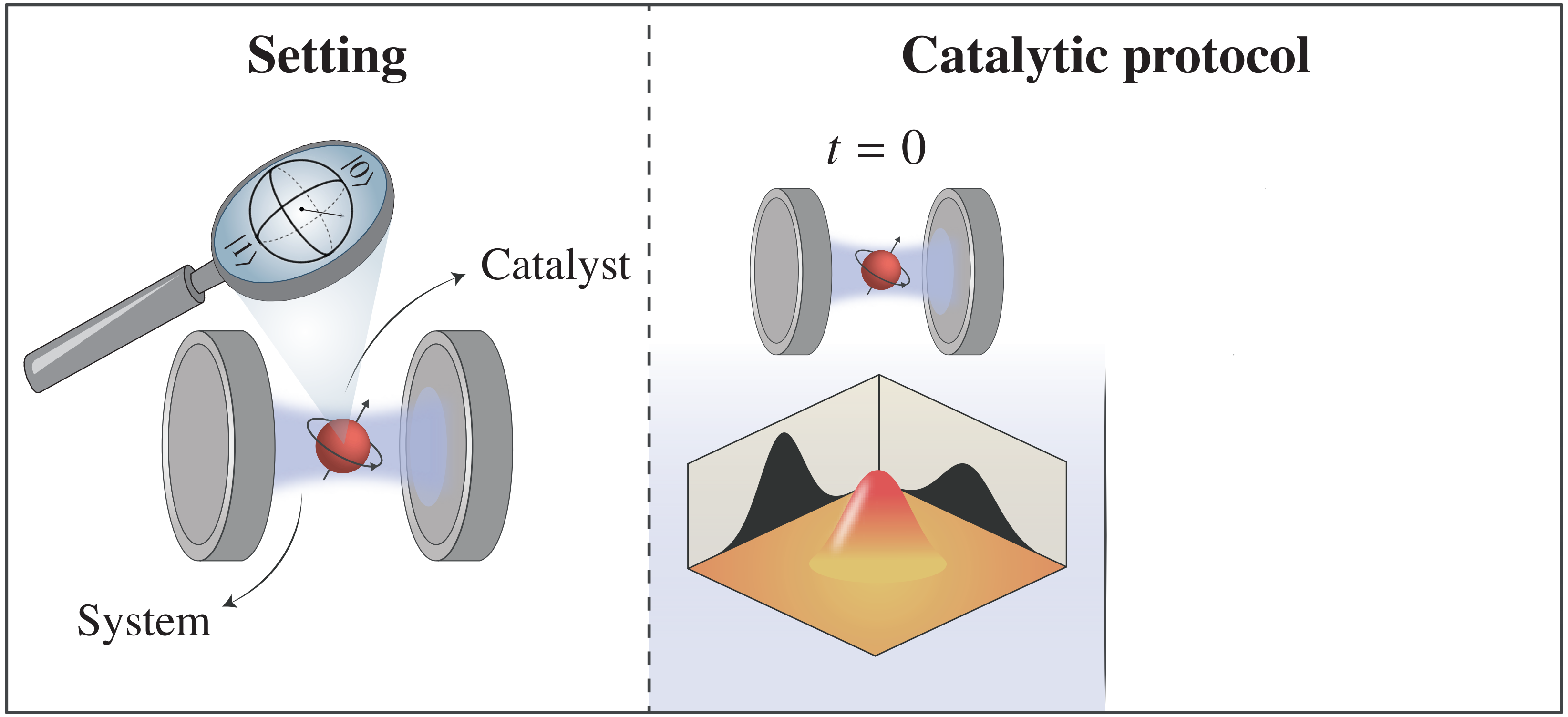
E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).

Spoiler

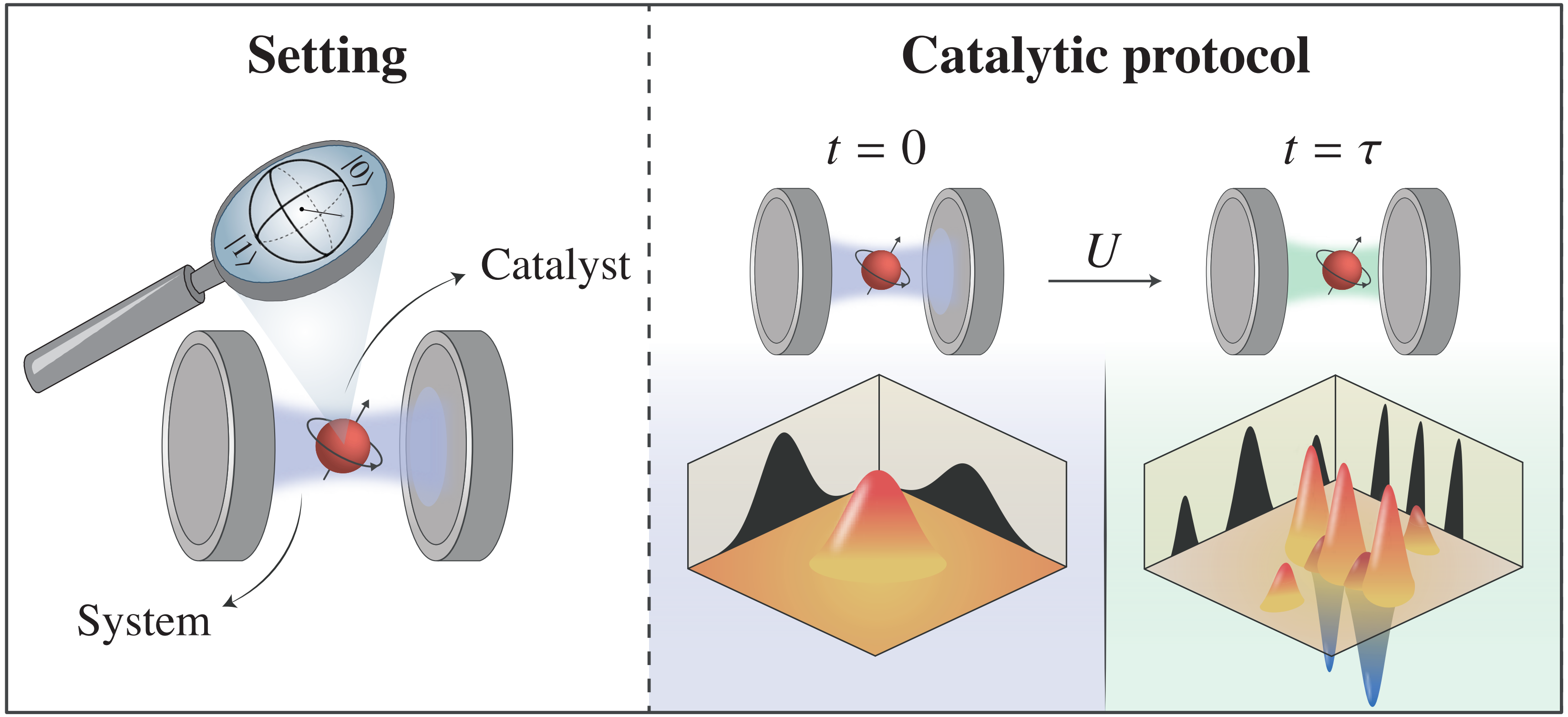
Setting



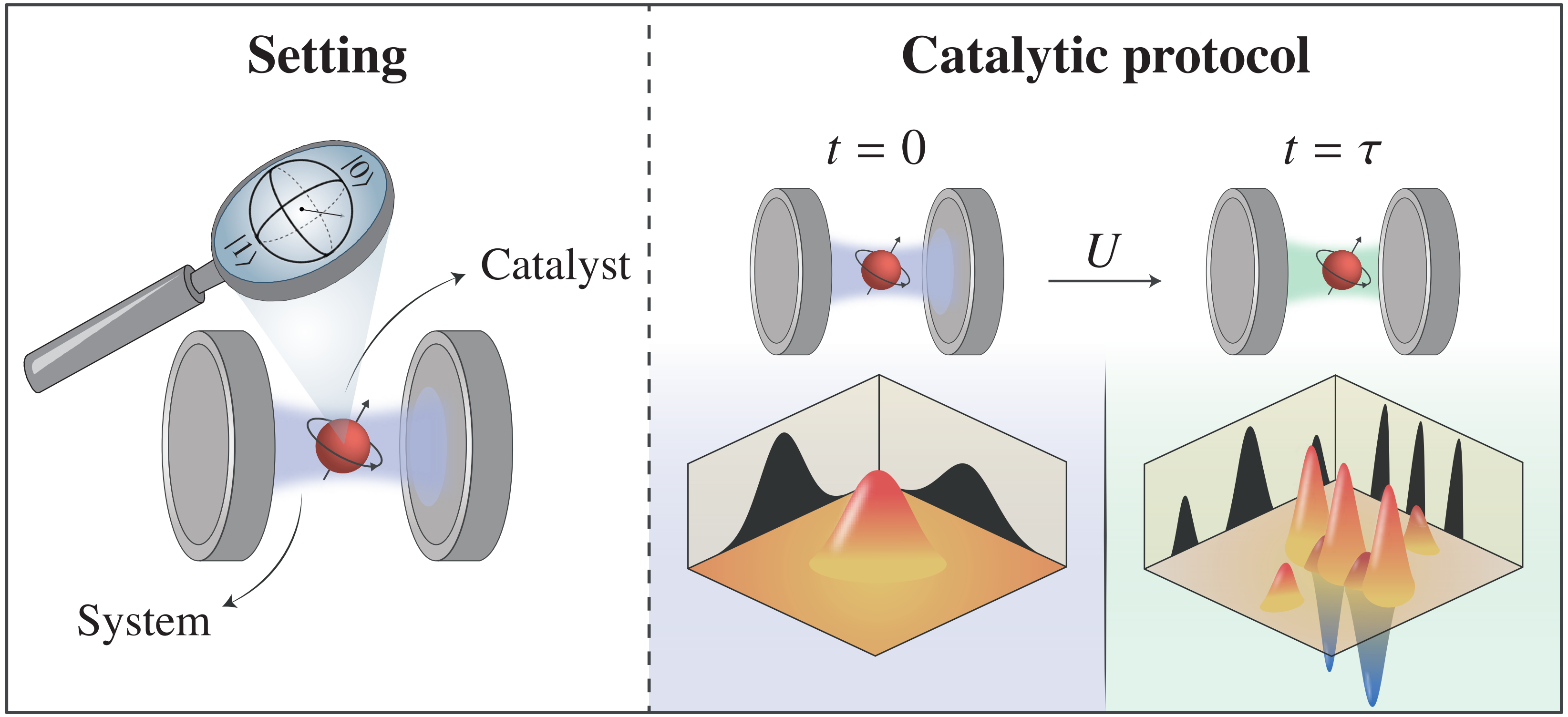
Spoiler



Spoiler


Setting
Catalytic protocol
$|0\rangle$
$|1\rangle$
Catalyst
System
$t = 0$
$t = \tau$
U

Spoiler



Q. Which notion of non-classicality?

Figures of merit

i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle a^{\dagger 2} a^2 \rangle_\sigma}{\langle a^\dagger a \rangle_\sigma^2}$$

measures the ‘probability’ of detecting two photons arriving at the same time at a photon detector.

R. J. Glauber, Phys. Rev. 130, 2529 (1963).

R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_{\mathsf{S}}^2 \rangle_\sigma - \langle n_{\mathsf{S}} \rangle_\sigma}{\langle n_{\mathsf{S}} \rangle_\sigma^2}$$

measures the 'probability' of detecting two photons arriving at the same time at a photon detector.

R. J. Glauber, Phys. Rev. 130, 2529 (1963).

R. Loudon, Phys. Bull. 2721 (1976)

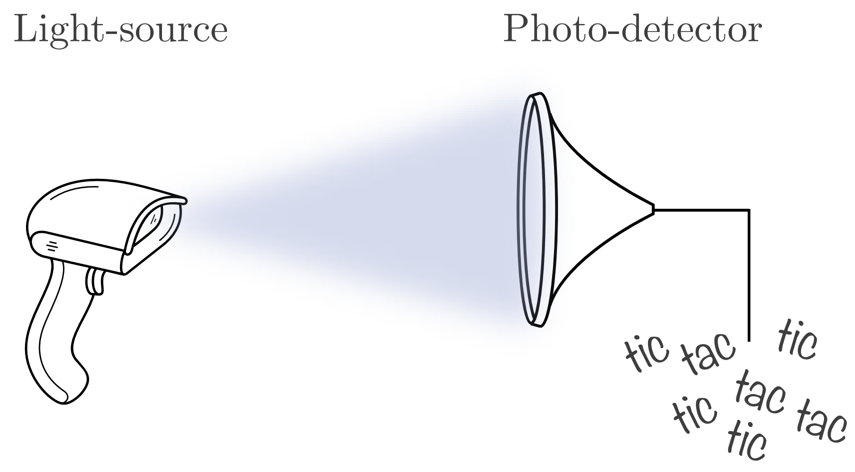
Figures of merit

i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_{\mathsf{S}}^2 \rangle_\sigma - \langle n_{\mathsf{S}} \rangle_\sigma}{\langle n_{\mathsf{S}} \rangle_\sigma^2}$$

measures the 'probability' of detecting two photons arriving at the same time at a photon detector.

○ **Example:**



R. J. Glauber, Phys. Rev. 130, 2529 (1963).
R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

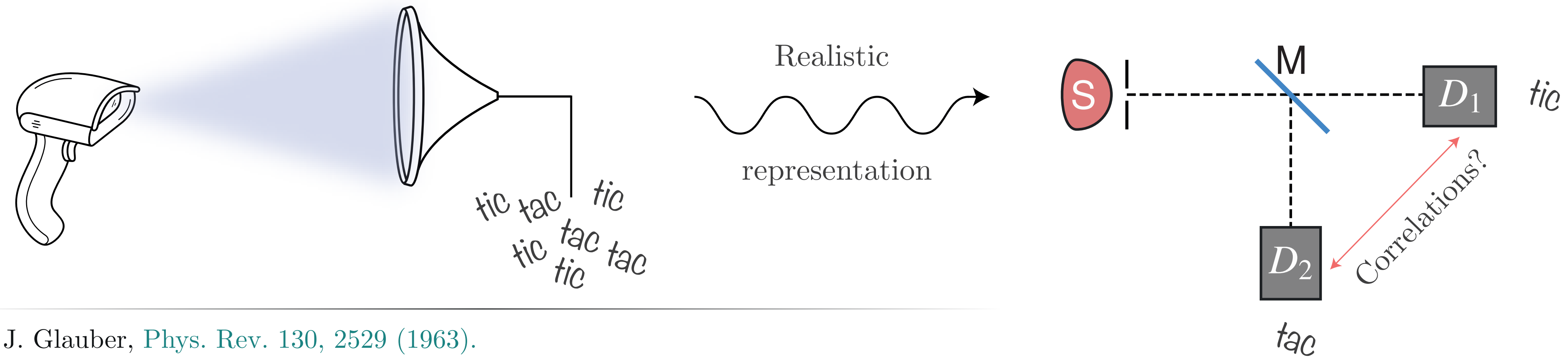
i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_{\mathsf{S}}^2 \rangle_\sigma - \langle n_{\mathsf{S}} \rangle_\sigma}{\langle n_{\mathsf{S}} \rangle_\sigma^2}$$

measures the ‘probability’ of detecting two photons arriving at the same time at a photon detector.

○ **Example:**

Light-source Photo-detector



R. J. Glauber, Phys. Rev. 130, 2529 (1963).
R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

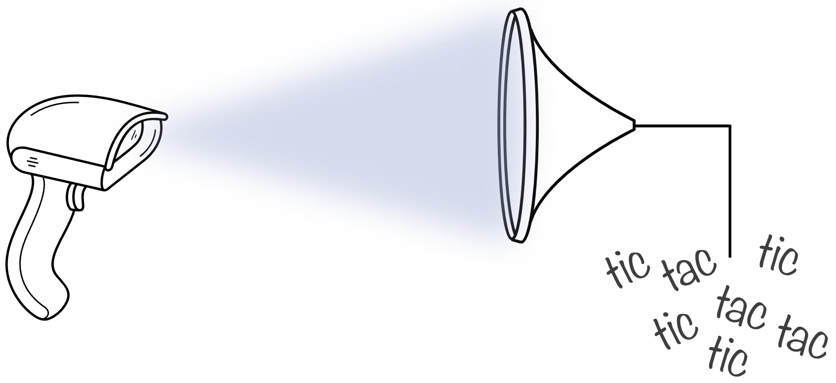
i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_{\mathsf{S}}^2\rangle_\sigma - \langle n_{\mathsf{S}}\rangle_\sigma}{\langle n_{\mathsf{S}}\rangle_\sigma^2}$$

measures the ‘probability’ of detecting two photons arriving at the same time at a photon detector.

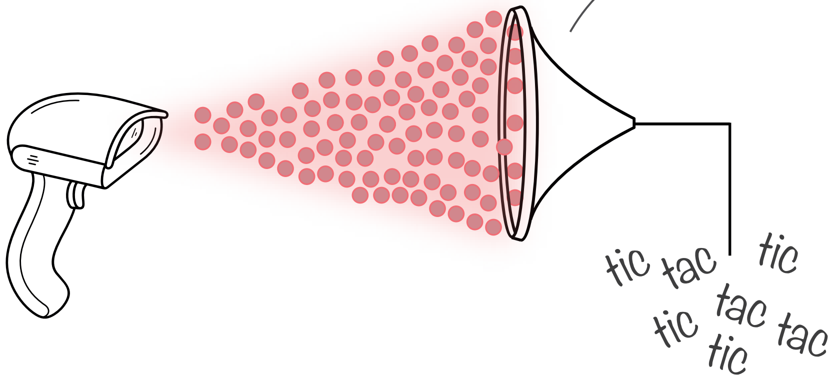
- **Example:**

Light-source Photo-detector



first case

thermal radiation



photons arrive in clusters!

$g^{(2)}(\sigma) = 2$

bunching

R. J. Glauber, Phys. Rev. 130, 2529 (1963).
R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

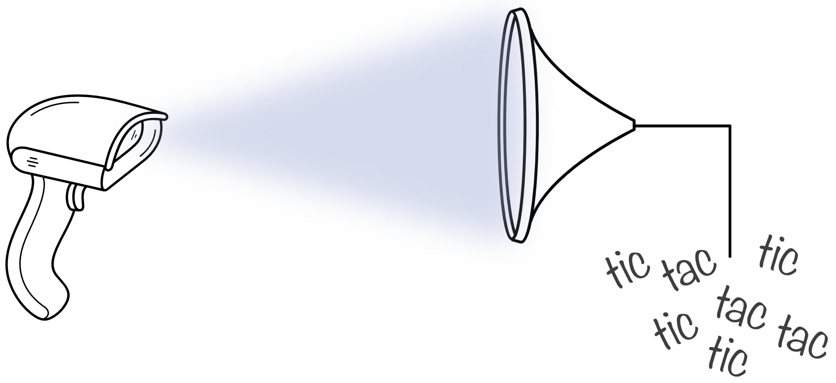
i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_{\mathsf{S}}^2\rangle_\sigma - \langle n_{\mathsf{S}}\rangle_\sigma}{\langle n_{\mathsf{S}}\rangle_\sigma^2}$$

measures the ‘probability’ of detecting two photons arriving at the same time at a photon detector.

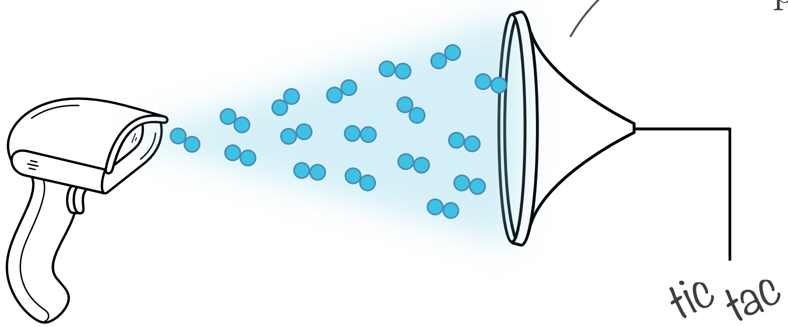
- **Example:**

Light-source Photo-detector



second case

coherent light



photons arrive in pairs!

$$g^{(2)}(\sigma) = 1$$

R. J. Glauber, Phys. Rev. 130, 2529 (1963).
R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

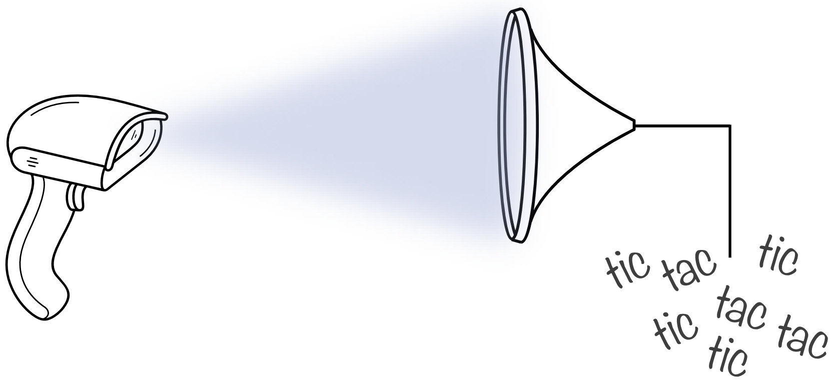
i. Second-order coherence

$$g^{(2)}(\sigma) = \frac{\langle n_{\mathsf{S}}^2\rangle_\sigma - \langle n_{\mathsf{S}}\rangle_\sigma}{\langle n_{\mathsf{S}}\rangle_\sigma^2}$$

measures the ‘probability’ of detecting two photons arriving at the same time at a photon detector.

- **Example:**

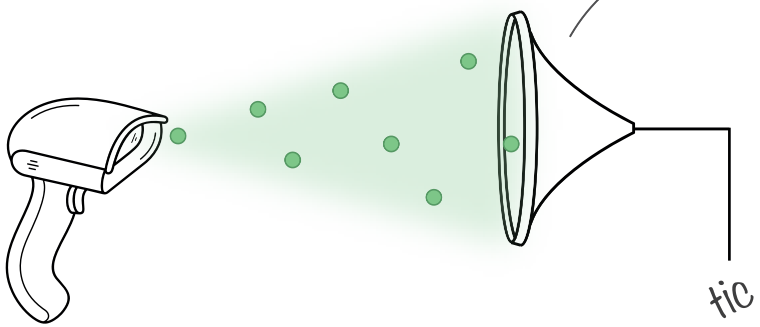
Light-source Photo-detector



third case

single-photon light

photons are never detected simultaneously!



$g^{(2)}(\sigma) < 1$

anti-bunching

R. J. Glauber, Phys. Rev. 130, 2529 (1963).

R. Loudon, Phys. Bull. 2721 (1976)

Figures of merit

ii. Wigner function:

$$W_\sigma(x, p) = \frac{1}{\pi} \int e^{2ipx'} \langle x - x' | \sigma | x + x' \rangle \, \mathrm{d}x'$$

E. Wigner, Phys. Rev. 40, 749 (1932)

A. Kenfack and K. Życzkowski, J. Opt., B Quantum Semiclass. Opt. 6, 396 (2004)

F. Albarelli, M G. Genoni, M G. A. Paris, A. Ferraro, Phys. Rev. A 98, 052350 (2018)

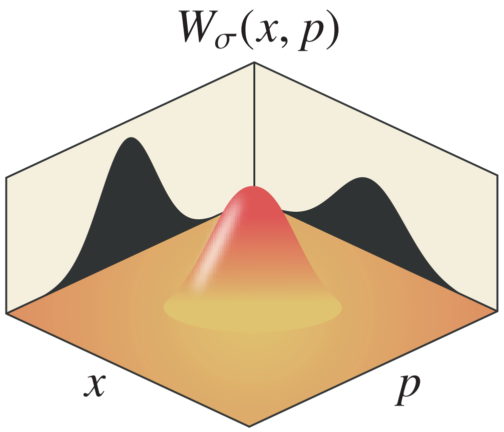
Figures of merit

ii. Wigner function:

$$W_\sigma(x,p) = \frac{1}{\pi} \int e^{2ipx'} \langle x - x' | \sigma | x + x' \rangle \, \mathrm{d}x'$$

- **Example:**

Coherent state



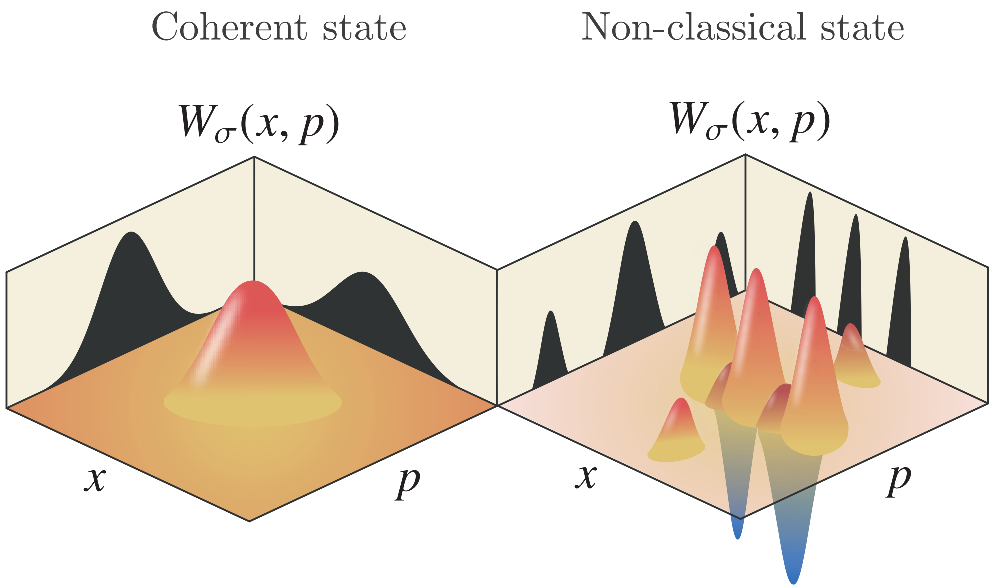
E. Wigner, Phys. Rev. 40, 749 (1932)
A. Kenfack and K. Życzkowski, J. Opt., B Quantum Semiclass. Opt. 6, 396 (2004)
F. Albarelli, M G. Genoni, M G. A. Paris, A. Ferraro, Phys. Rev. A 98, 052350 (2018)

Figures of merit

ii. Wigner function:

$$W_\sigma(x,p) = \frac{1}{\pi}\int e^{2ipx'}\langle x - x'|\sigma|x + x'\rangle\,\mathrm{d}x'$$

○ **Example:**



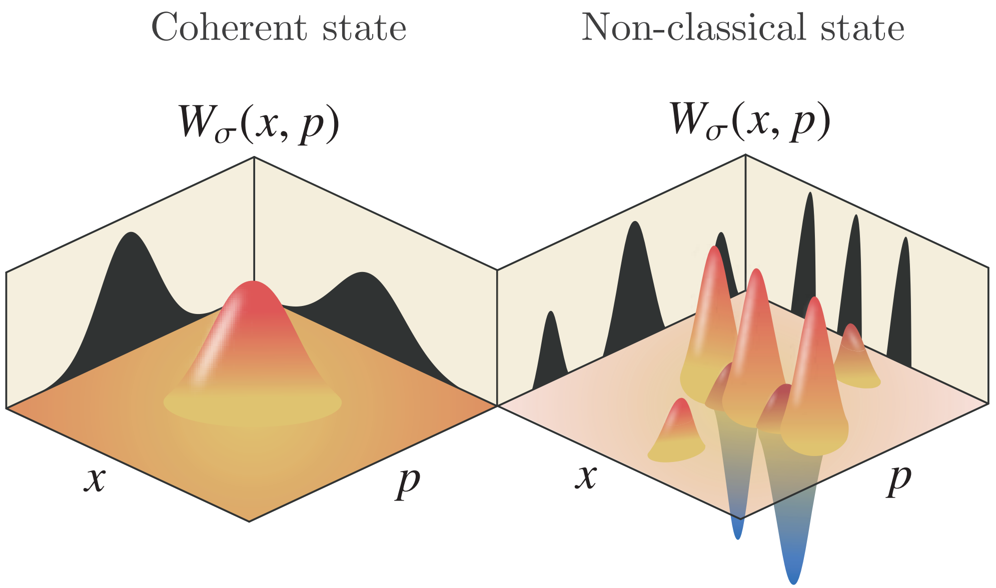
E. Wigner, Phys. Rev. 40, 749 (1932)
A. Kenfack and K. Życzkowski, J. Opt., B Quantum Semiclass. Opt. 6, 396 (2004)
F. Albarelli, M G. Genoni, M G. A. Paris, A. Ferraro, Phys. Rev. A 98, 052350 (2018)

Figures of merit

ii. Wigner function:

$$W_\sigma(x,p) = \frac{1}{\pi}\int e^{2ipx'}\langle x-x'|\sigma|x+x'\rangle\,\mathrm{d}x'$$

- **Example:**



Q. How to quantify the degree of non-classicality?

$$\mathsf{W}(\sigma) := \log\left(\int \mathrm{d}x\,\mathrm{d}p\,|W_\sigma(x,p)|\right)$$

E. Wigner, Phys. Rev. 40, 749 (1932)

A. Kenfack and K. Życzkowski, J. Opt., B Quantum Semiclass. Opt. 6, 396 (2004)

F. Albarelli, M G. Genoni, M G. A. Paris, A. Ferraro, Phys. Rev. A 98, 052350 (2018)

Results

Statement of the problem

Task: generation of non-classical light in a catalytic way.

Statement of the problem

Task: generation of non-classical light in a catalytic way.

Consideration: $\rho_{\mathsf{S}} = |\alpha\rangle\langle\alpha|$ where

$g^{(2)}(\rho_{\mathsf{S}}) = 1$ and $\mathsf{W} = 0$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} |n\rangle \,.$$

Statement of the problem

Task: generation of non-classical light in a catalytic way.

Consideration: $\rho_{\mathsf{S}} = |\alpha\rangle\langle\alpha|$ where

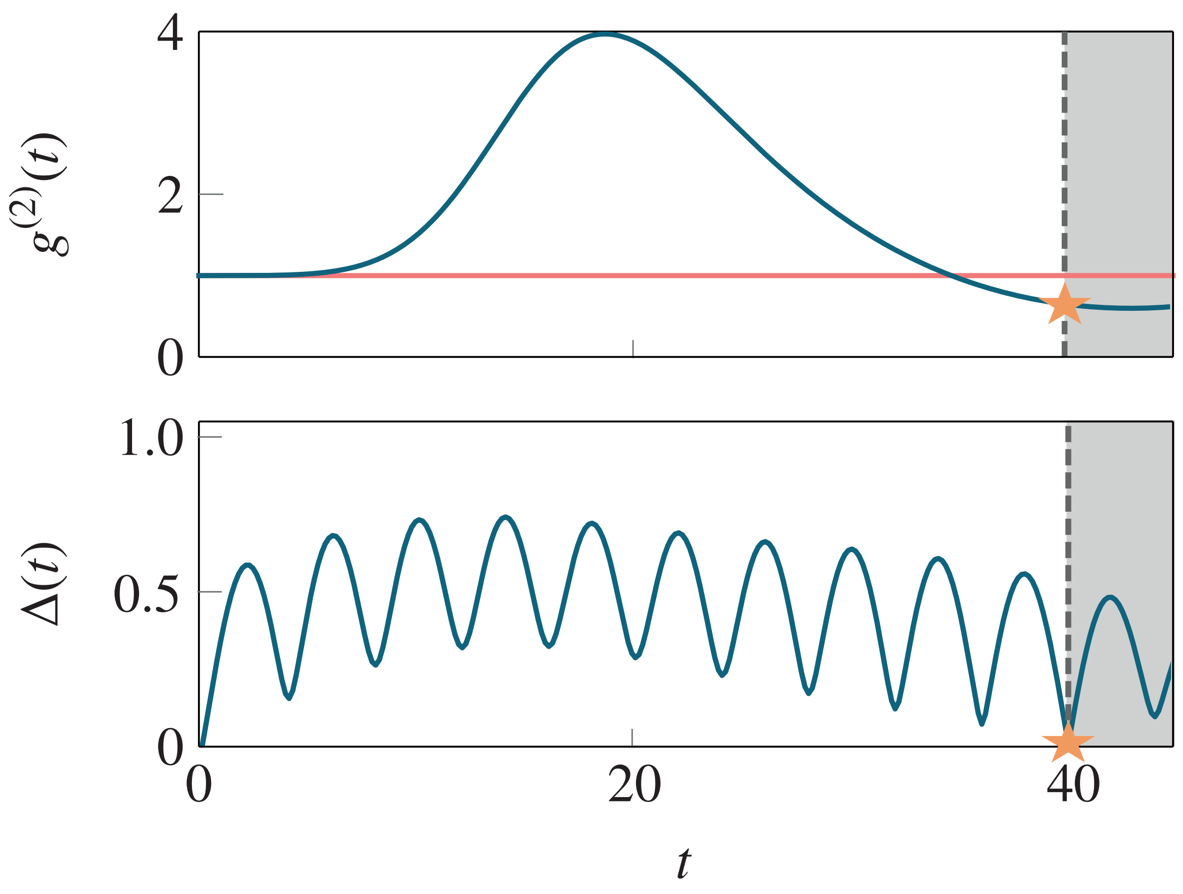
$g^{(2)}(\rho_{\mathsf{S}}) = 1$ and $\mathsf{W} = 0$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} |n\rangle \,.$$

Goal: find χ_{C} and τ, such that $\mathrm{Tr}\Big[U(\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}})U^{\dagger}\Big] = \chi_{\mathsf{C}}\,.$

First example

Generating light with **sub-Poissonian** photon statistics:



Definitions

$$g^{(2)}(t) := g^{(2)}[\sigma_{\mathsf{S}}(t)]$$

$$\Delta(t) := \|\chi_{\mathsf{C}} - \sigma_{\mathsf{C}}\|_1$$

Parameters

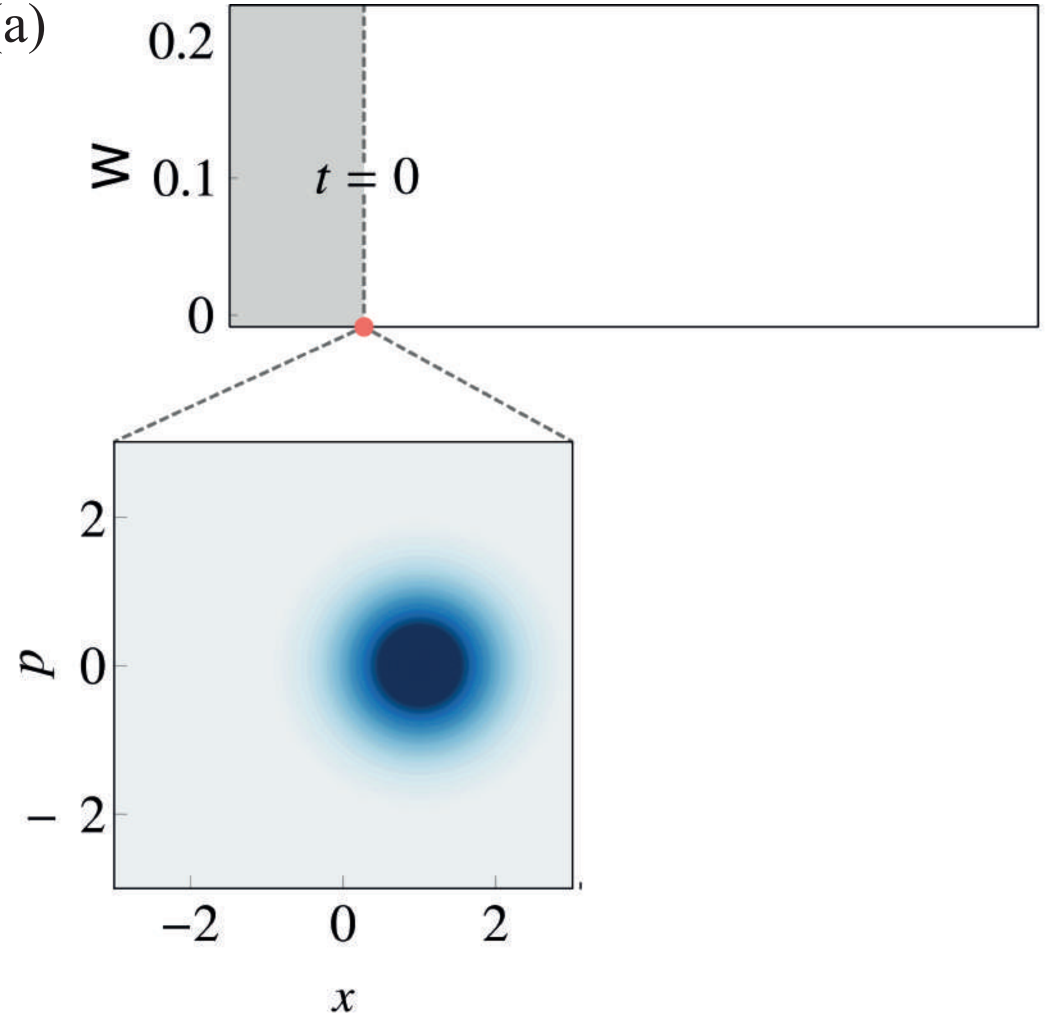
$$\alpha = 1/\sqrt{2}$$
$$\omega = 2\pi$$
$$g = \pi$$

★ Catalysis occurs at $\tau \approx 40$ for which $g^{(2)}(\tau) \approx 0.5$.

Second example

Generating light with **negative** Wigner function:

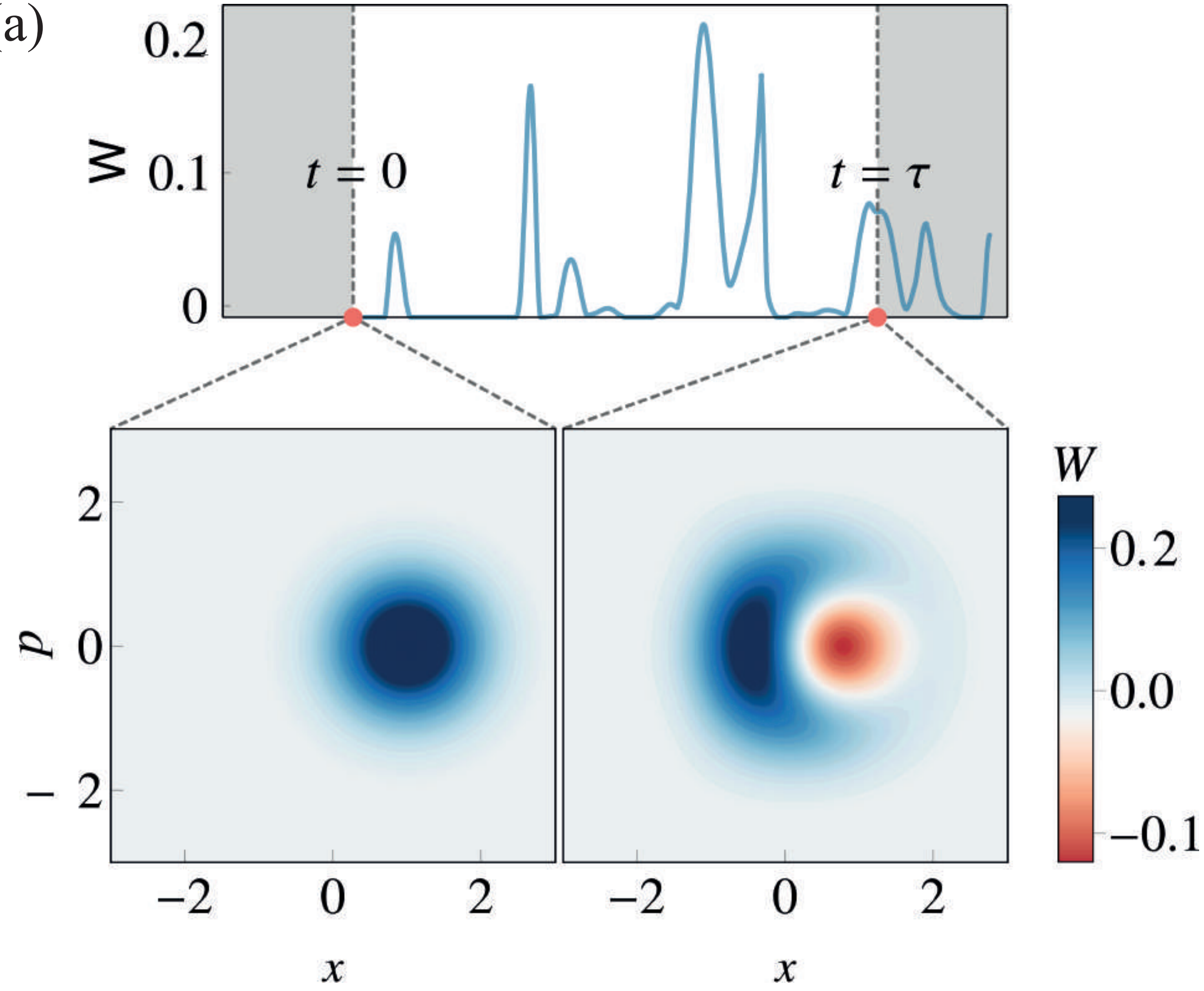
(a)



Second example

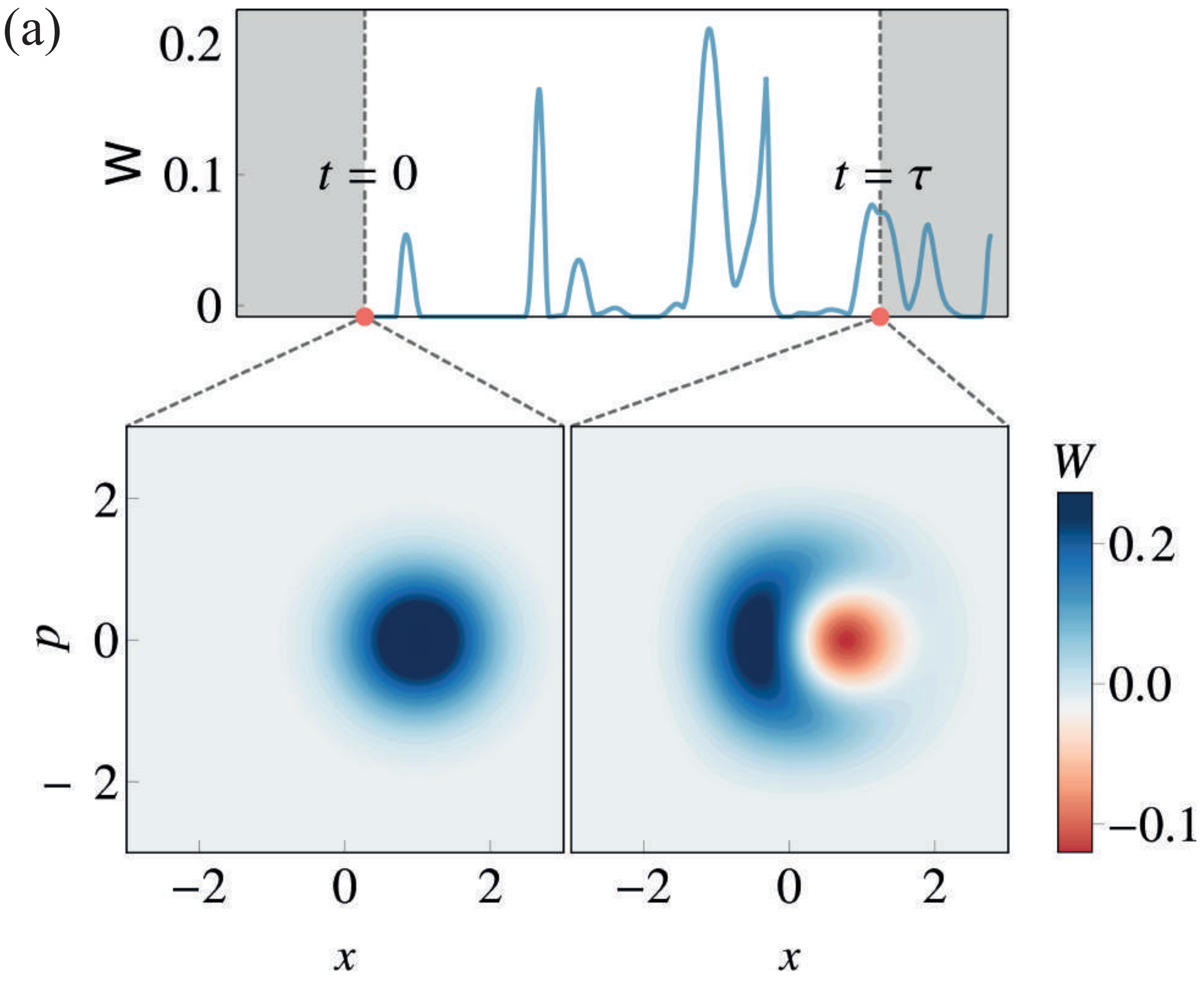
Generating light with **negative** Wigner function:

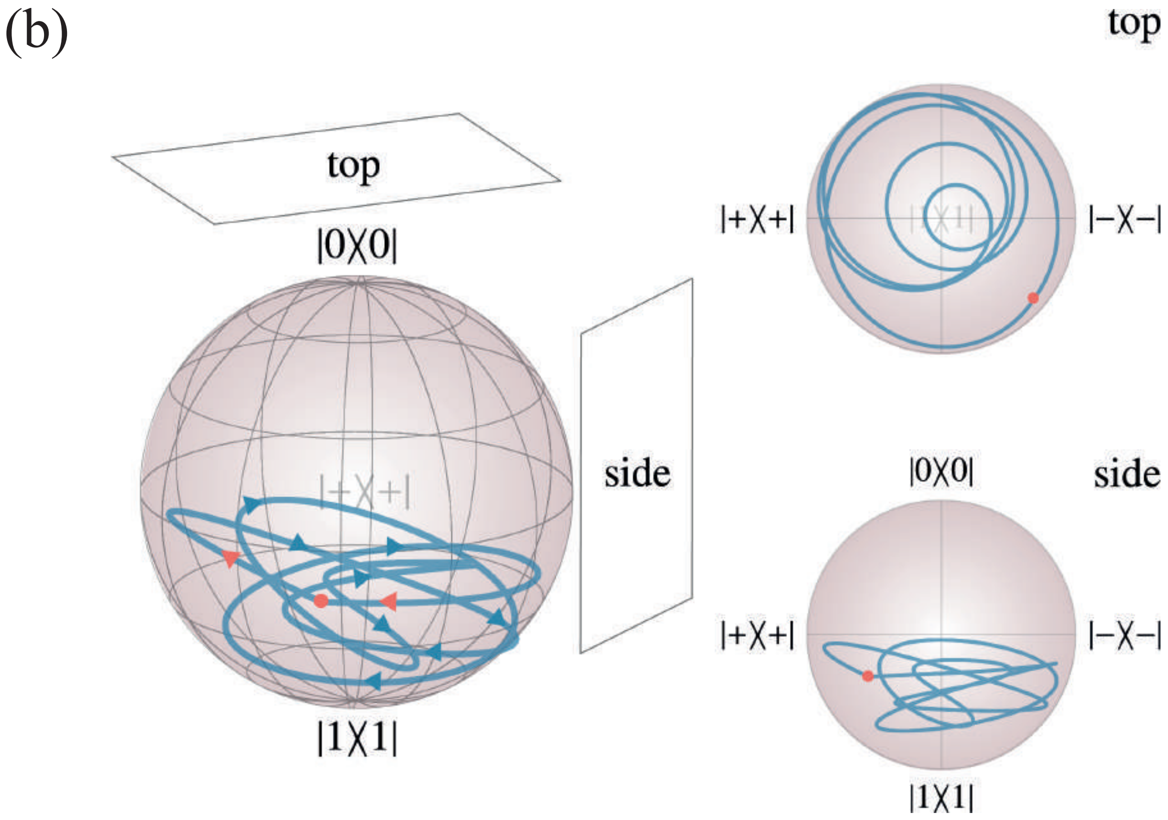
(a)



Second example

Generating light with **negative** Wigner function:





★ Catalysis occurs at $\tau \approx 5$ for which $\mathsf{W} \approx 0.1$.

Mechanism of catalysis

- **Observation 1:** Energy-preserving $[U(t), n_{\mathsf{S}} + n_{\mathsf{C}}] = 0$.

Mechanism of catalysis

- **Observation 1:** Energy-preserving $[U(t), n_\mathsf{S} + n_\mathsf{C}] = 0$.

- **Observation 2:** Catalytic constrain $\longrightarrow$ all moments of n_C are preserved and $\langle n_\mathsf{S} \rangle_\rho = \langle n_\mathsf{C} \rangle_\sigma$.

Mechanism of catalysis

- **Observation 1:** Energy-preserving $[U(t), n_\mathsf{S} + n_\mathsf{C}] = 0$.

- **Observation 2:** Catalytic constrain $\longrightarrow$ all moments of n_C are preserved and $\langle n_\mathsf{S}\rangle_\rho = \langle n_\mathsf{C}\rangle_\sigma$.

$$\langle n_\mathsf{S}^2\rangle_\sigma = \langle n_\mathsf{S}^2\rangle_\rho + 2\Big(\langle n_\mathsf{S}\rangle_\sigma\langle n_\mathsf{C}\rangle_\sigma - \langle n_\mathsf{S}\otimes n_\mathsf{C}\rangle_\sigma\Big)$$

measure of correlations

Mechanism of catalysis

- **Observation 1:** Energy-preserving $[U(t), n_{\mathsf{S}} + n_{\mathsf{C}}] = 0$.

- **Observation 2:** Catalytic constrain $\longrightarrow$ all moments of n_{C} are preserved and $\langle n_{\mathsf{S}} \rangle_\rho = \langle n_{\mathsf{C}} \rangle_\sigma$.

$$\langle n_{\mathsf{S}}^2 \rangle_\sigma = \langle n_{\mathsf{S}}^2 \rangle_\rho + 2 \underbrace{\Big(\langle n_{\mathsf{S}} \rangle_\sigma \langle n_{\mathsf{C}} \rangle_\sigma - \langle n_{\mathsf{S}} \otimes n_{\mathsf{C}} \rangle_\sigma \Big)}_{\text{measure of correlations}}$$

- **Observation 3:** The second moment changes only when the **system** and **catalyst** become **correlated**.

Mechanism of catalysis

- **Observation 1:** Energy-preserving $[U(t), n_{\mathsf{S}} + n_{\mathsf{C}}] = 0$.

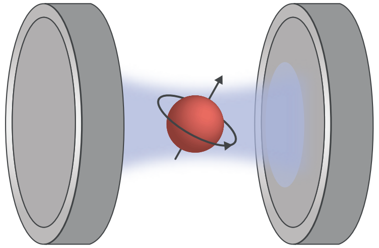
- **Observation 2:** Catalytic constrain $\longrightarrow$ all moments of n_{C} are preserved and $\langle n_{\mathsf{S}} \rangle_\rho = \langle n_{\mathsf{C}} \rangle_\sigma$.

$$\langle n_{\mathsf{S}}^2 \rangle_\sigma = \langle n_{\mathsf{S}}^2 \rangle_\rho + 2 \left(\underbrace{\langle n_{\mathsf{S}} \rangle_\sigma \langle n_{\mathsf{C}} \rangle_\sigma - \langle n_{\mathsf{S}} \otimes n_{\mathsf{C}} \rangle_\sigma}_{\text{measure of correlations}} \right)$$

- **Observation 3:** The second moment changes only when the **system** and **catalyst** become **correlated**.

Q. Which **atomic states** lead to catalyst?

Which states lead to catalyst?

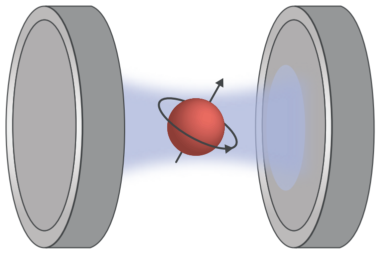


$\xrightarrow{\text{General case}}$ $\rho_S = \sum_{n,m}^{\infty} p_{n,m} |n\rangle\langle m|$, $\chi_C = q\,|g\rangle\langle g| + r\,|g\rangle\langle e| + r^*\,|e\rangle\langle g| + [1-q]\,|e\rangle\langle e|$

Recall:

$$\boxed{\mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C}$$

Which states lead to catalyst?



General case $\longrightarrow$ $\rho_{\mathsf{S}} = \sum_{n,m}^{\infty} p_{n,m} |n\rangle\langle m|$, $\chi_{\mathsf{C}} = q\,|g\rangle\langle g| + r\,|g\rangle\langle e| + r^*\,|e\rangle\langle g| + [1-q]\,|e\rangle\langle e|$

Recall:

$$\boxed{\mathrm{Tr}_{\mathsf{S}}[U(\rho_{\mathsf{S}} \otimes \chi_{\mathsf{C}})U^\dagger] = \chi_{\mathsf{C}}}$$

Follows that the ground-state occupation can be decomposed as $q = q_{\mathrm{inc}} + q_{\mathrm{coh}}$:

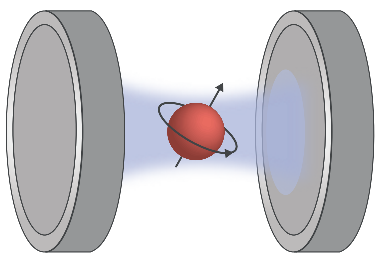
$$p_{n,n}$$

$$q_{\mathrm{inc}} = \frac{1}{Q} \sum_{n=0}^{\infty} p_n s_n^2$$

$$Q := \sum_{n=0}^{\infty} (p_n + p_{n+1}) s_n^2$$

$$s_n := \sin\left(gt\sqrt{n+1}\right)$$

Which states lead to catalyst?



$\xrightarrow{\text{General case}}$ $\rho_S = \sum_{n,m}^{\infty} p_{n,m} |n\rangle\langle m|$, $\chi_C = q\, |g\rangle\langle g| + r\, |g\rangle\langle e| + r^* \, |e\rangle\langle g| + [1-q]\, |e\rangle\langle e|$

Recall:

$$\boxed{\mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C}$$

Follows that the ground-state occupation can be decomposed as $q = q_{\mathrm{inc}} + q_{\mathrm{coh}}$:

$p_{n,n}$

$$q_{\mathrm{inc}} = \frac{1}{Q} \sum_{n=0}^{\infty} p_n s_n^2 \qquad \text{and} \qquad q_{\mathrm{coh}} = \frac{1}{Q} \sum_{n=0}^{\infty} y_n$$

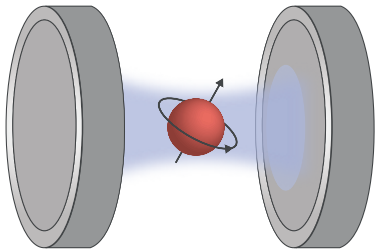
$c_n := \cos\left(gt\sqrt{n+1}\right)$

$Q := \sum_{n=0}^{\infty} (p_n + p_{n+1}) s_n^2$

$s_n := \sin\left(gt\sqrt{n+1}\right)$

$y_n := 2\,\mathrm{Im}\,[r p_{n+1,n}] s_n c_n$

Which states lead to catalyst?



General case $\longrightarrow$

$$\rho_S = \sum_{n,m}^{\infty} p_{n,m} |n\rangle\langle m| \quad , \quad \chi_C = q\,|g\rangle\langle g| + r\,|g\rangle\langle e| + r^*\,|e\rangle\langle g| + [1-q]\,|e\rangle\langle e|$$

Recall:

$$\boxed{\mathrm{Tr}_S[U(\rho_S \otimes \chi_C)U^\dagger] = \chi_C}$$

where the off-diagonal term r satisfies:

$$r = \frac{i(a_3 a_4^* + a_1^* a_4)}{|a_1|^2 - |a_3|^2} - \frac{i(a_3 a_2^* + a_1^* a_2)}{|a_1|^2 - |a_3|^2} q$$

with

$$a_1 = \sum_{n=0}^{\infty} p_{n,n} c_{n-1} c_n - e^{-i\omega\tau}, \qquad a_3 = \sum_{n=0}^{\infty} p_{n,n+2} s_n s_{n+1},$$

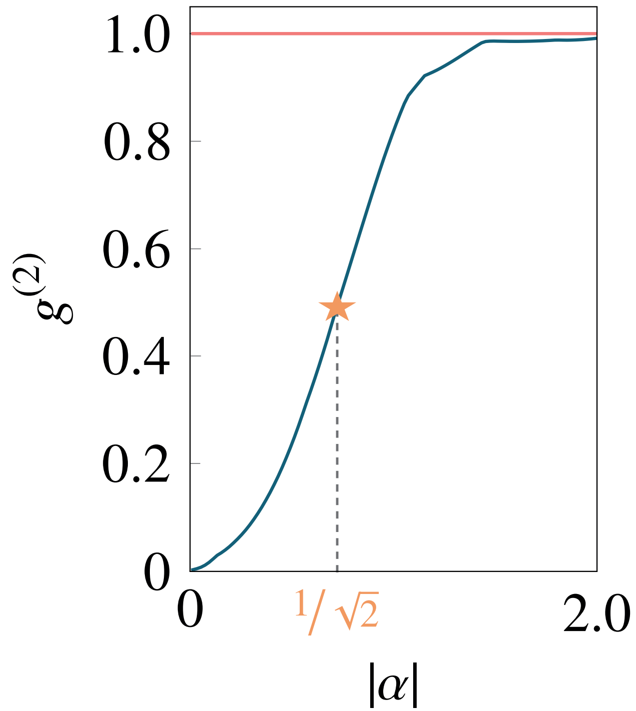
$$a_2 = \sum_{n=0}^{\infty} p_{n,n+1} s_n [c_{n-1} + c_{n+1}], \qquad a_4 = \sum_{n=0}^{\infty} p_{n,n+1} s_n c_{n+1}$$

How general is catalysis?

Q. How often a catalytic evolution leads to a non-classical state?

How general is catalysis?

Q. How often a catalytic evolution leads to a non-classical state?



$$g^{(2)}(\sigma_S) = g^{(2)}(\rho_S) - \frac{2}{\langle n_S\rangle_\rho^2}\left[\langle n_S \otimes n_C\rangle_\sigma - (1-q)\langle n_S\rangle_\rho\right]$$

where

$$\langle n_S \otimes n_C\rangle_\sigma = \sum_{n=0}^{\infty} n\left[(1-q)p_n c_n^2 + y_n + q p_{n+1} s_n^2\right]$$

Parameters

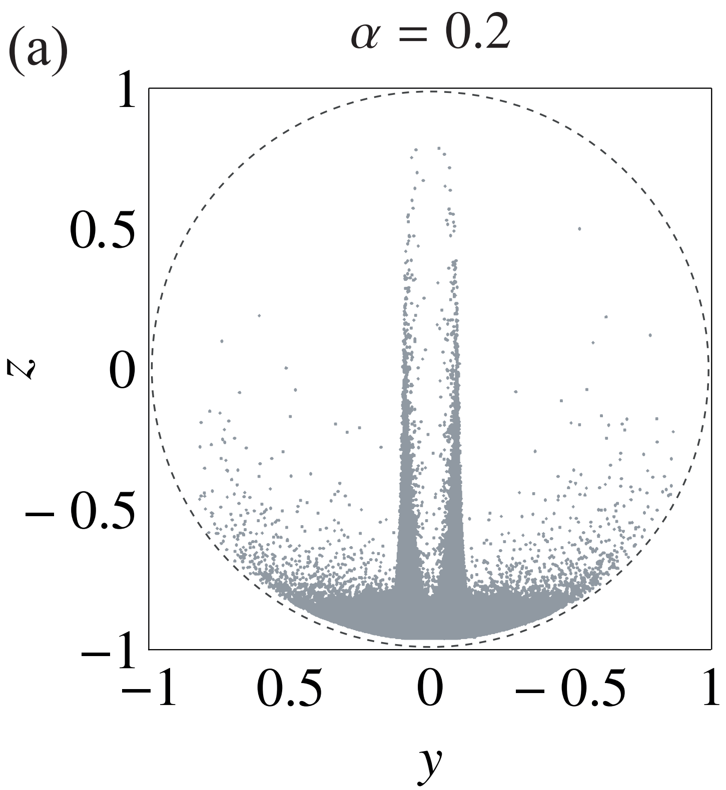
$\omega = 2\pi,\ g = \pi,\ g\tau \leq 100$

How general is catalysis?

Q. How does the set of catalytic states look like?

How general is catalysis?

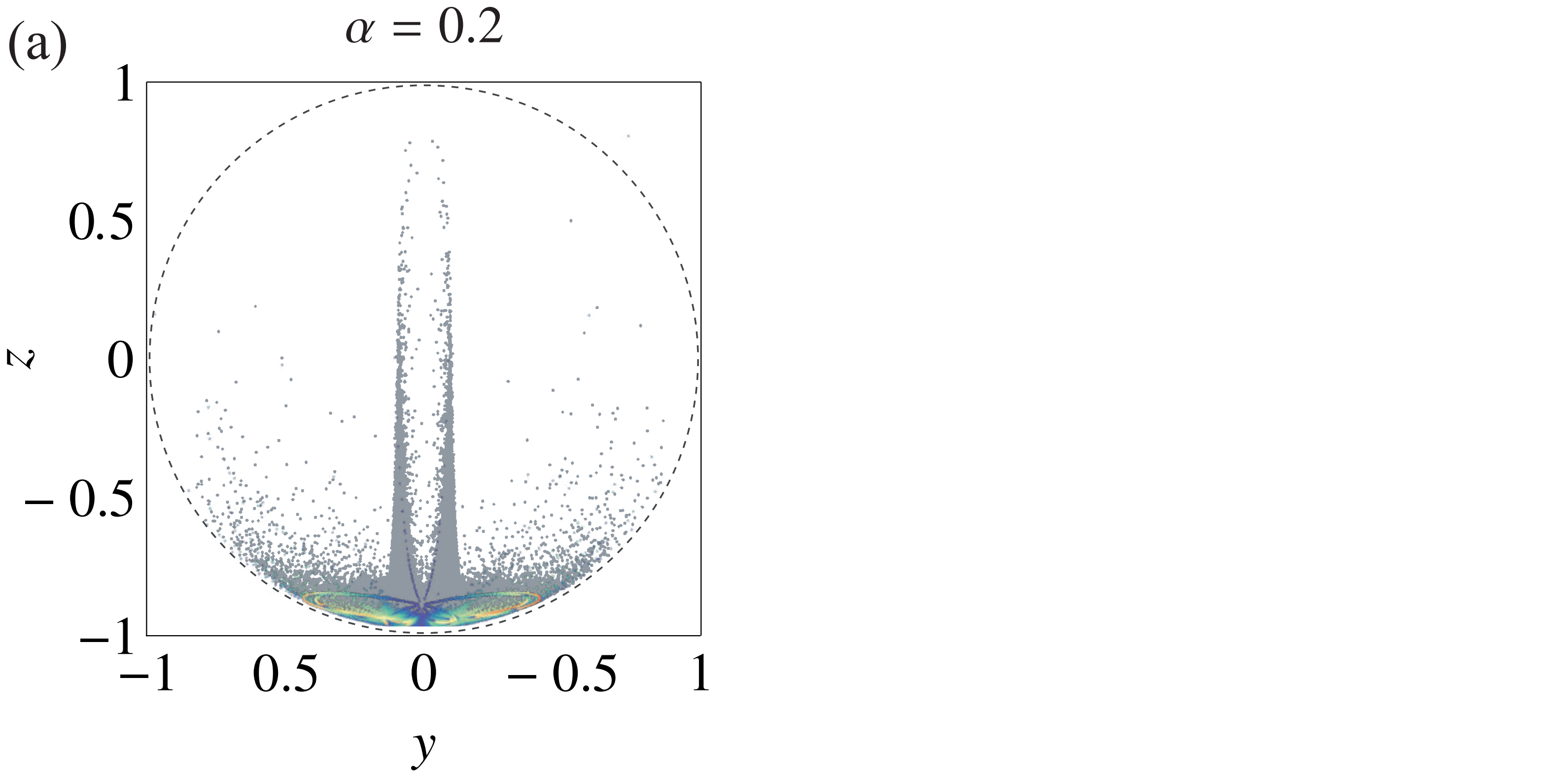
Q. How does the set of catalytic states look like? Highly non-trivial!



,

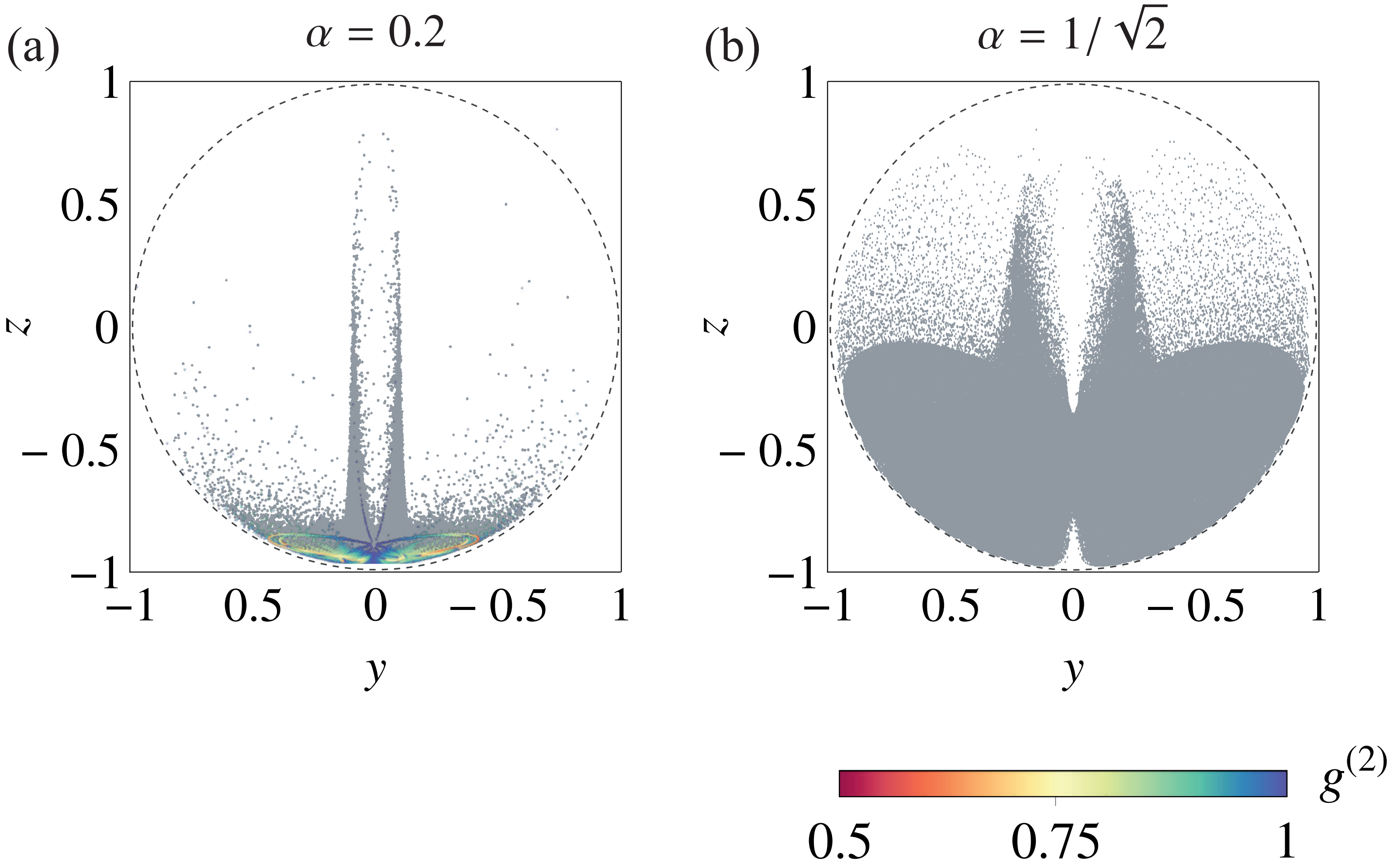
How general is catalysis?

Q. How does the set of catalytic states look like? Highly non-trivial!



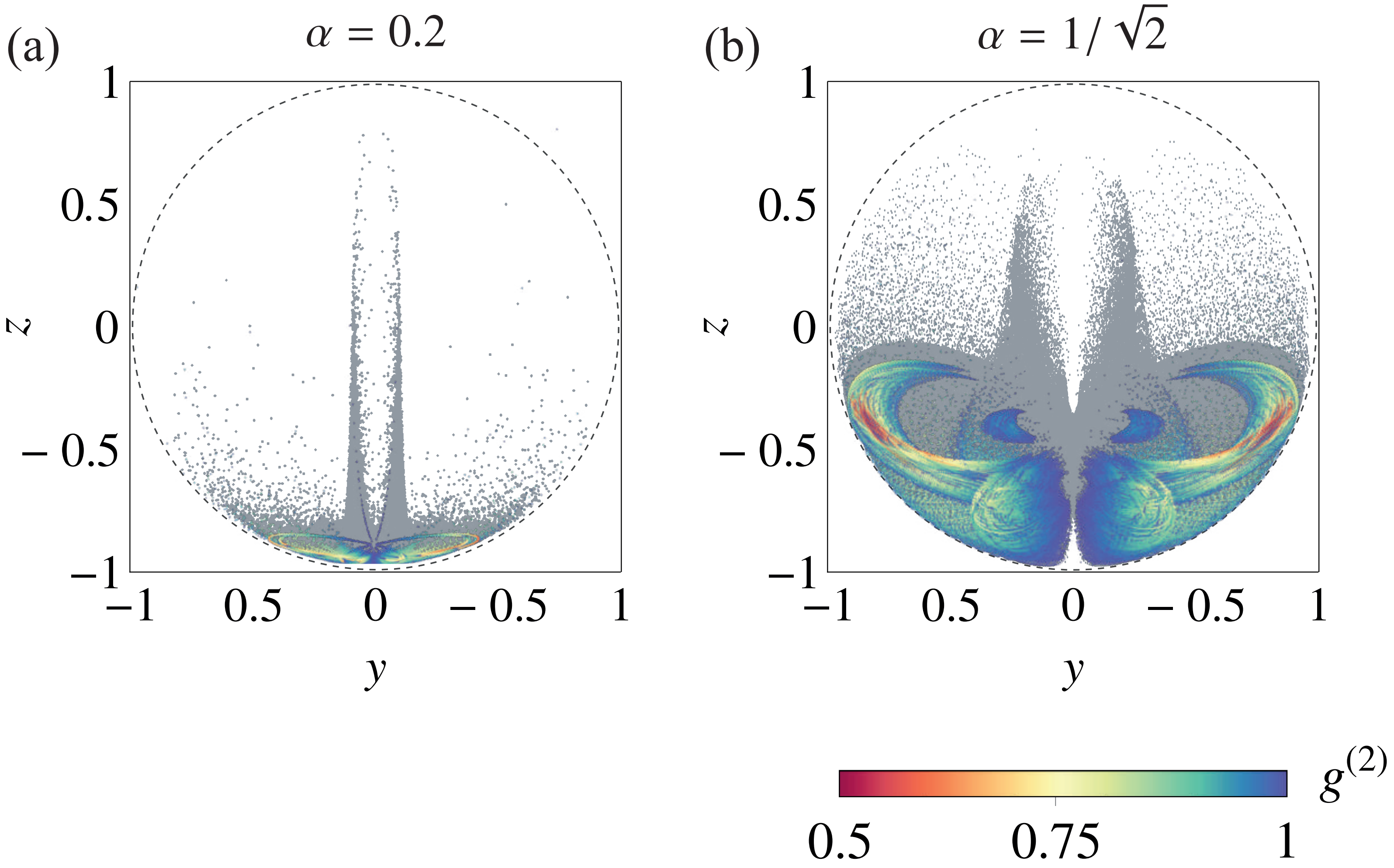
How general is catalysis?

Q. How does the set of catalytic states look like? Highly non-trivial!



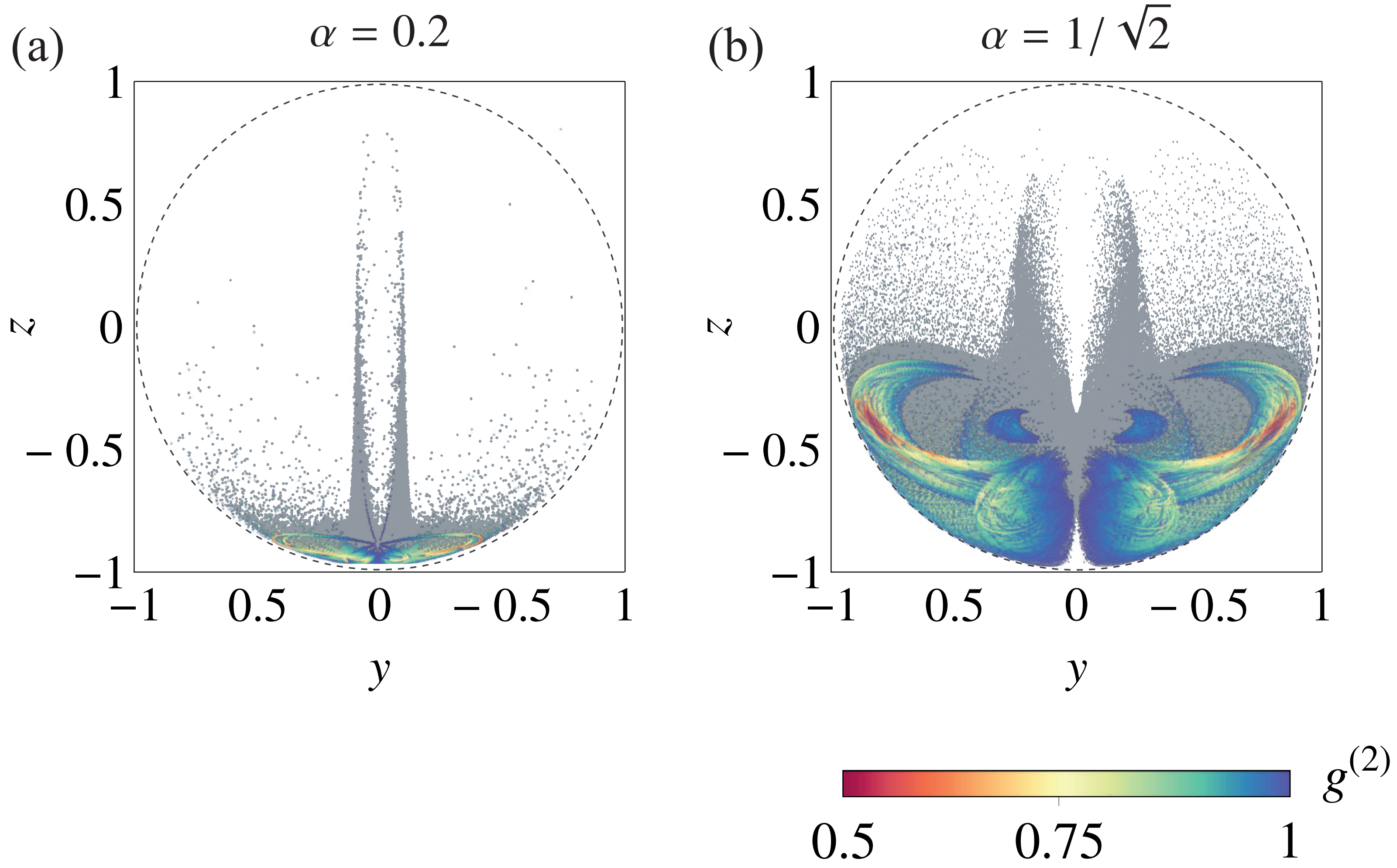
How general is catalysis?

Q. How does the set of catalytic states look like? Highly non-trivial!



How general is catalysis?

Q. How does the set of catalytic states look like? Highly non-trivial!



Outlook

Summary

! Catalytic process in a paradigmatic quantum optics setup:

- Generation of non-classical states of light.
- Mechanism behind the catalytic evolution.
- Atomic states.

Outlook

Summary

! Catalytic process in a paradigmatic quantum optics setup:

- Generation of non-classical states of light.
- Mechanism behind the catalytic evolution.
- Atomic states.

What's next?

? Change the role between main system and catalyst.

? Different models.

? Go beyond quantum optics.

Thank you!