Witnessing entanglement by measuring heat

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May 02, 2024

Outlook

I. Rough statement of the problem

II. Results

i. Protocol

ii. Optimal heat exchange

iii. Entanglement thermomether

In collaboration with

Jonatan Bohr Brask

Patryk Lipka Bartosik

Based on:

arXiv: xxxx.xxxx - Framework & Applications

I. Rough statement of the problem

How do ***quantum features*** affect thermodynamic processes?

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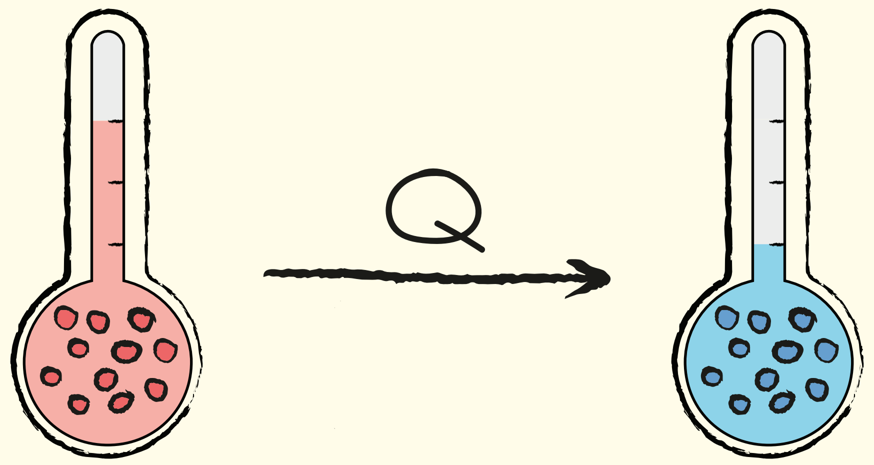
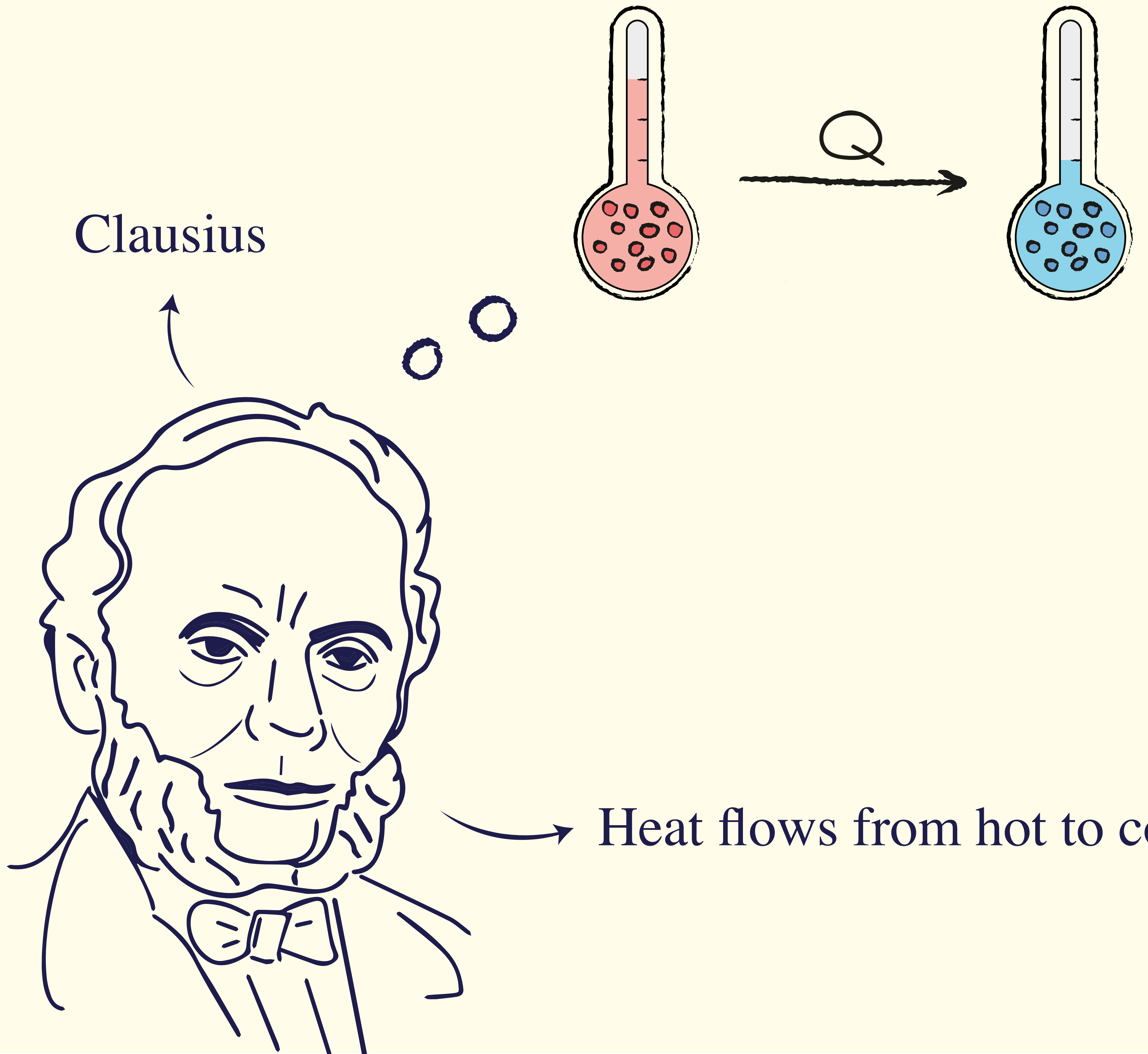


Clausius

Heat flows from hot to cold

How do ***quantum features*** affect thermodynamic processes?

Clausius



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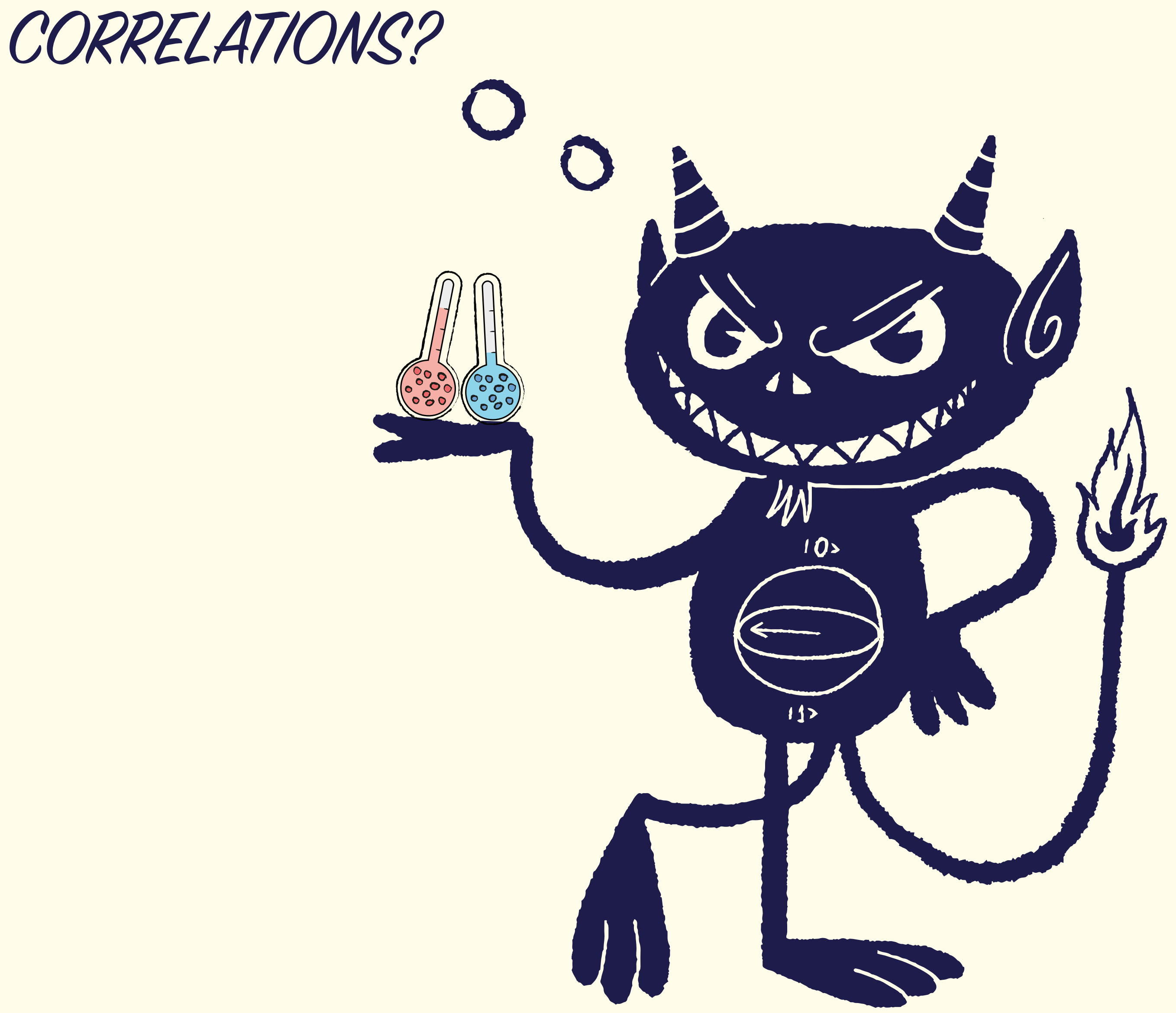
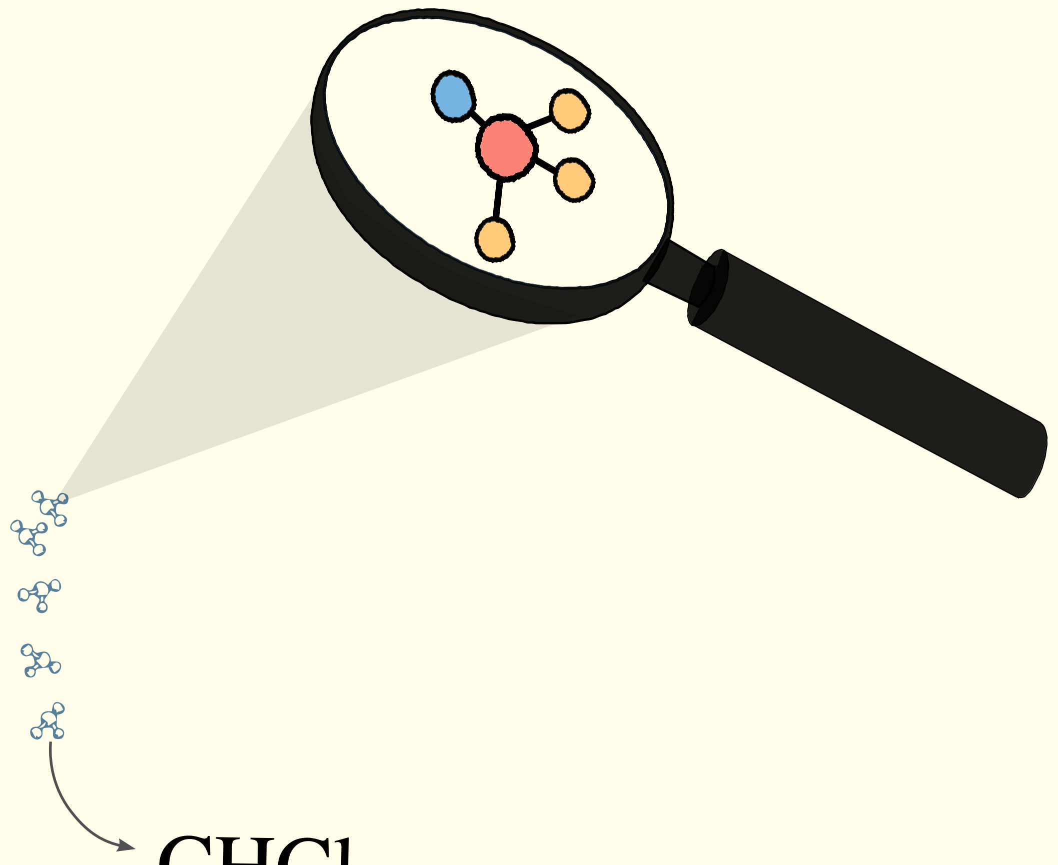


M. H . Partovi, *Phys. Rev. E* ***77***, *021110 (2008)*
D. Jennings & T. Rudolph, *Phys. Rev. E* ***81***, *061130 (2010)*
S .Jevtic, D. Jennings & T. Rudolph, *Phys. Rev. Lett.* ***108***, *110403 (2012)*

- **Composite system:** ^{13}C (system A) and H (system B)



$CHCl_3$

K. Micadei, J. P. S. Peterson, A. M. Souza, R. S. Sarthour, I. S. Oliveira, G. T. Landi, T. B. Batalhão, R. M. Serra & E. Lutz
Nat. Commun. **10**, 2456 (2019)

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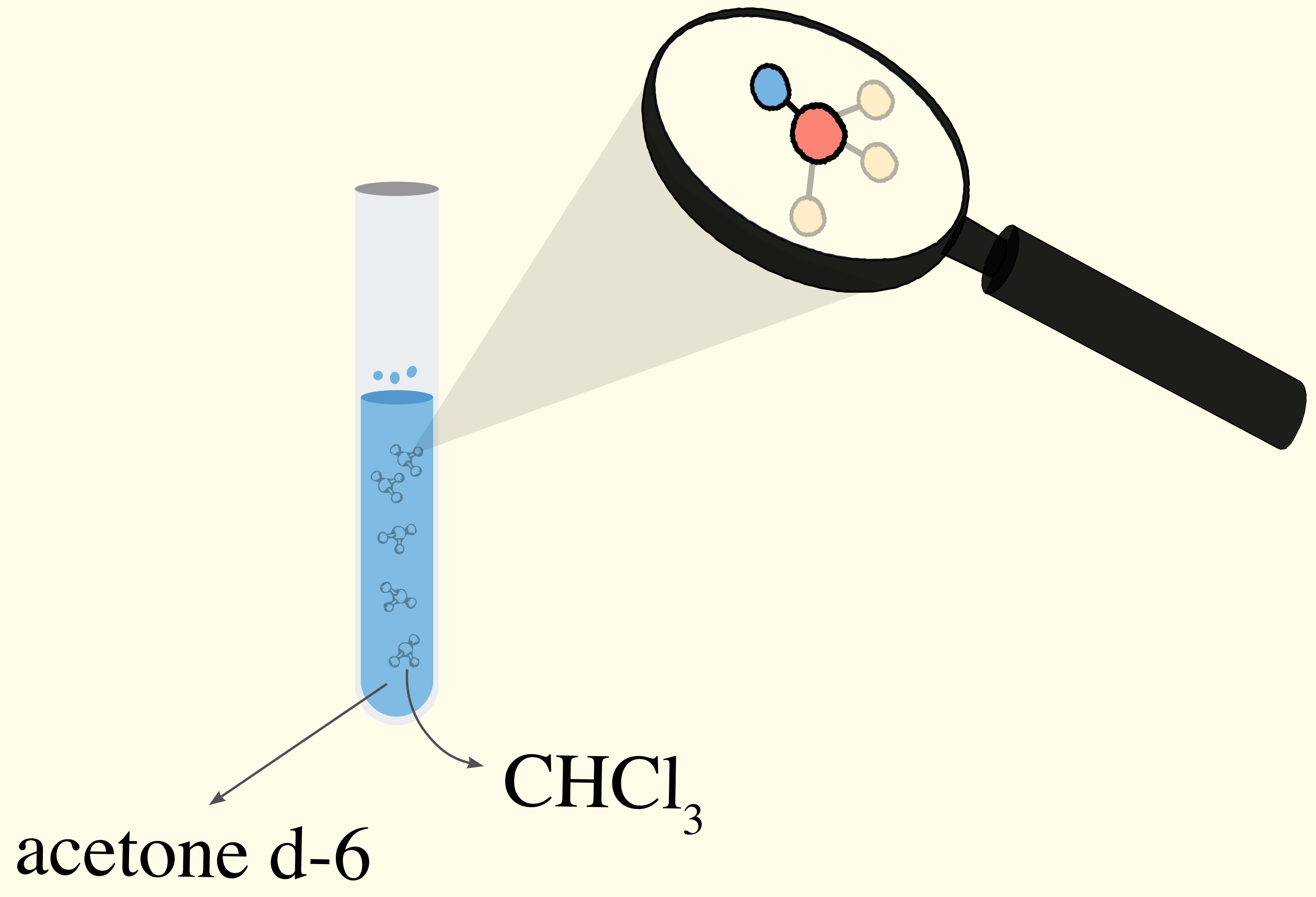

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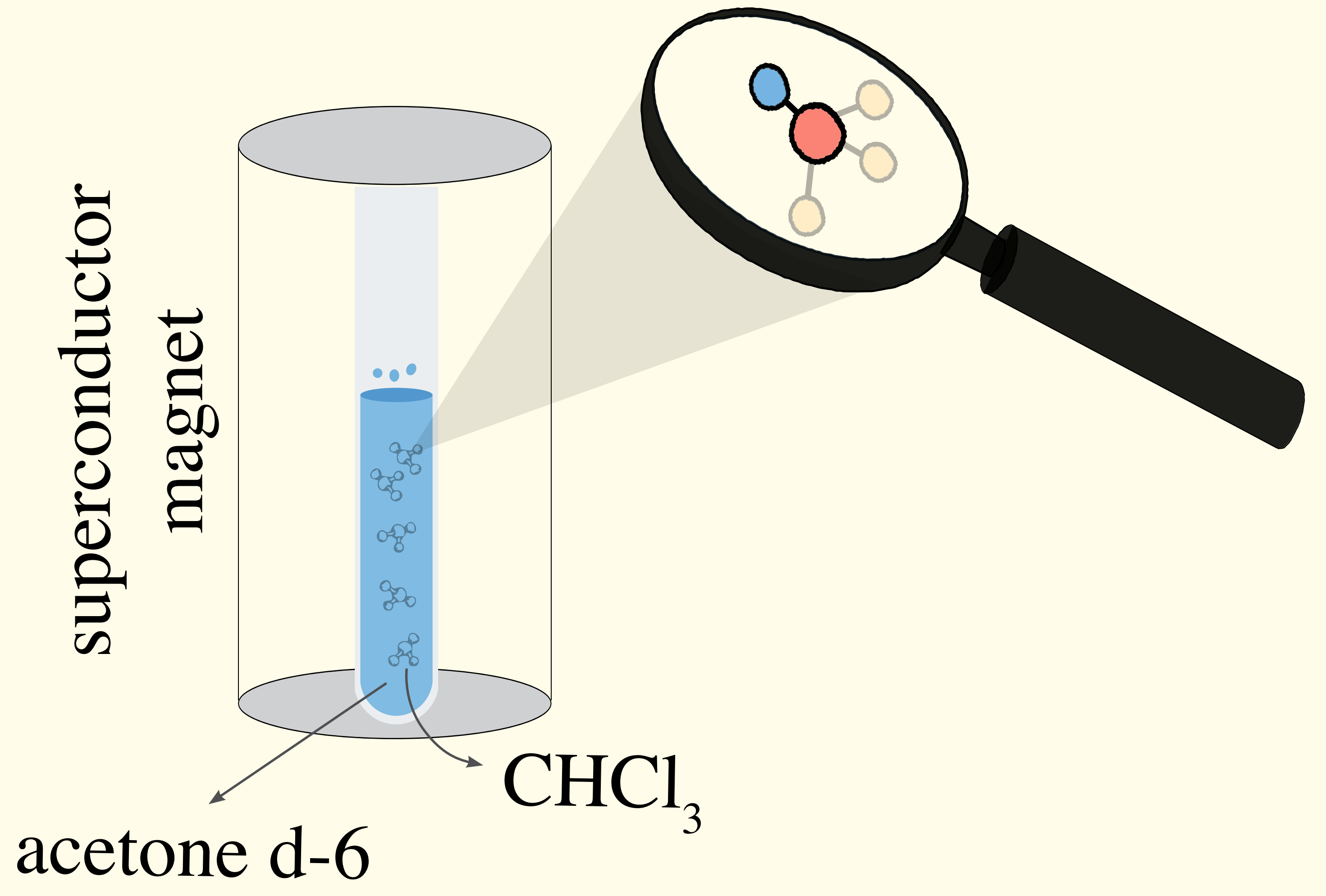

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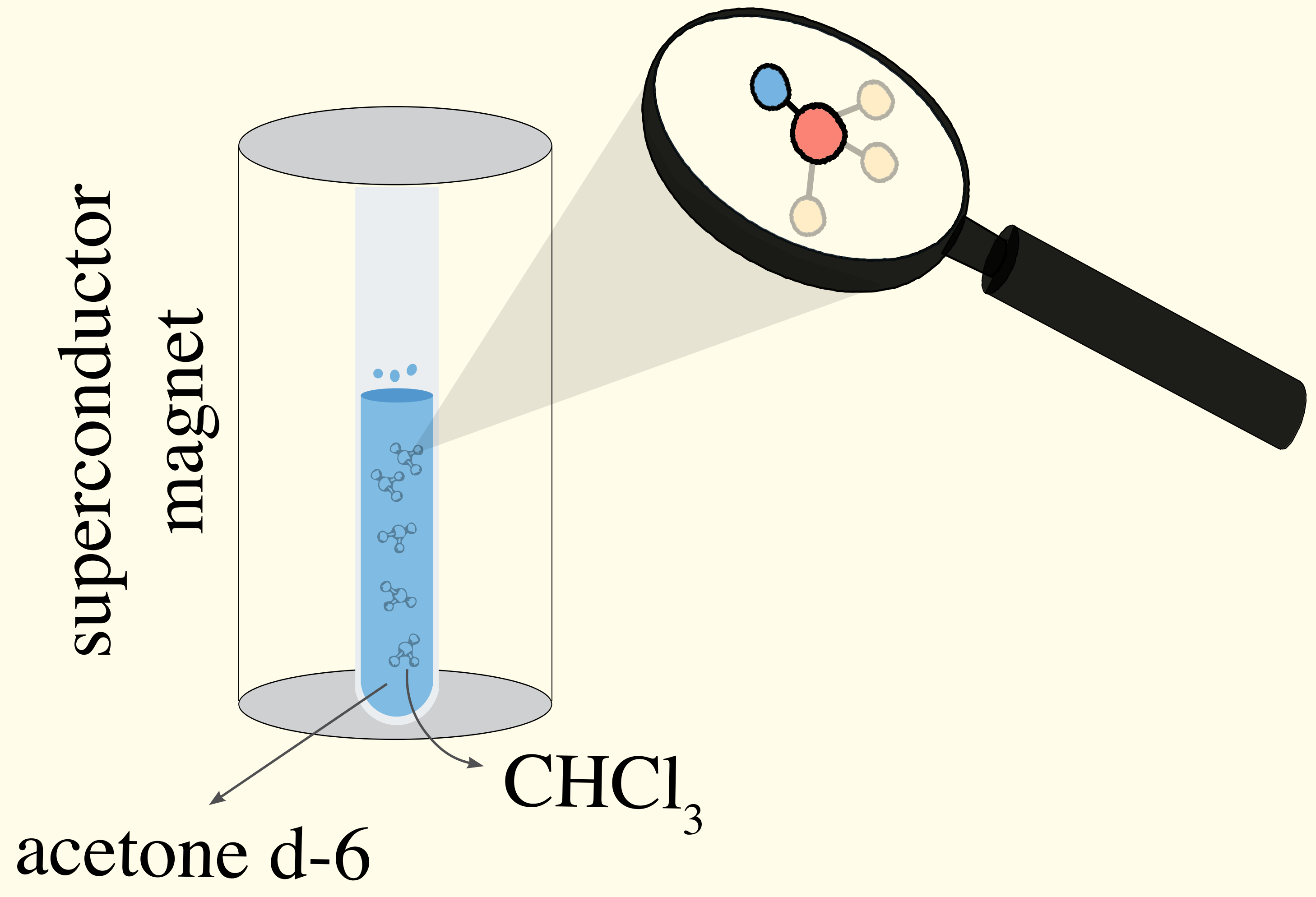
- **Composite system:** ^{13}C (system A) and H (system B): $H = \underbrace{\frac{1}{2}\sum_{i\in\{\mathsf{A,B}\}}(\mathbb{1}-\sigma_z^{(i)})}_{H_0} + \underbrace{\frac{\pi J}{2}(\sigma_x^{\mathsf{A}}\sigma_y^{\mathsf{B}} - \sigma_y^{\mathsf{A}}\sigma_x^{\mathsf{B}})}_{\mathcal{V}}$



where $[H_0, \mathcal{V}] = 0$ (energy-preserving interaction)

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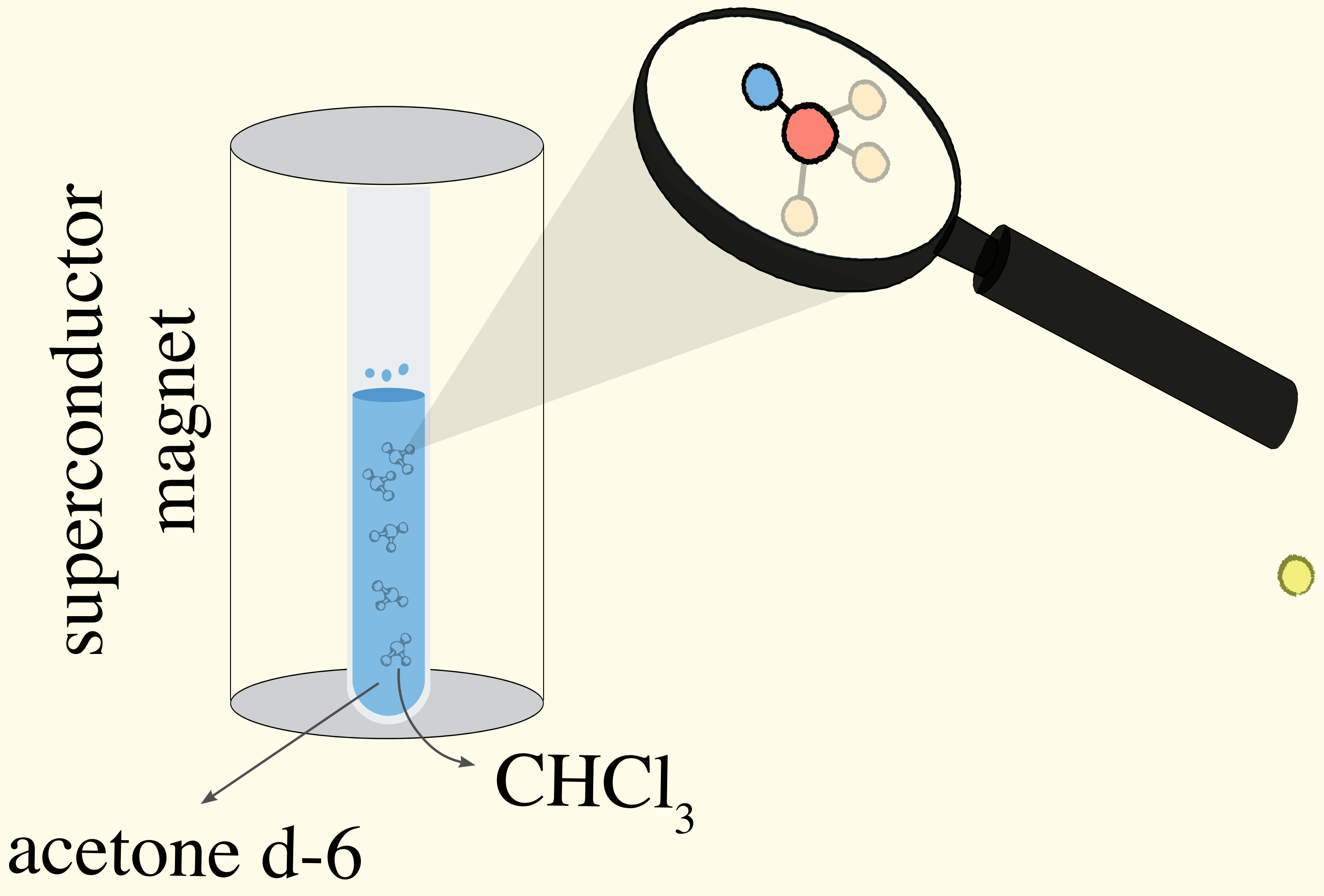

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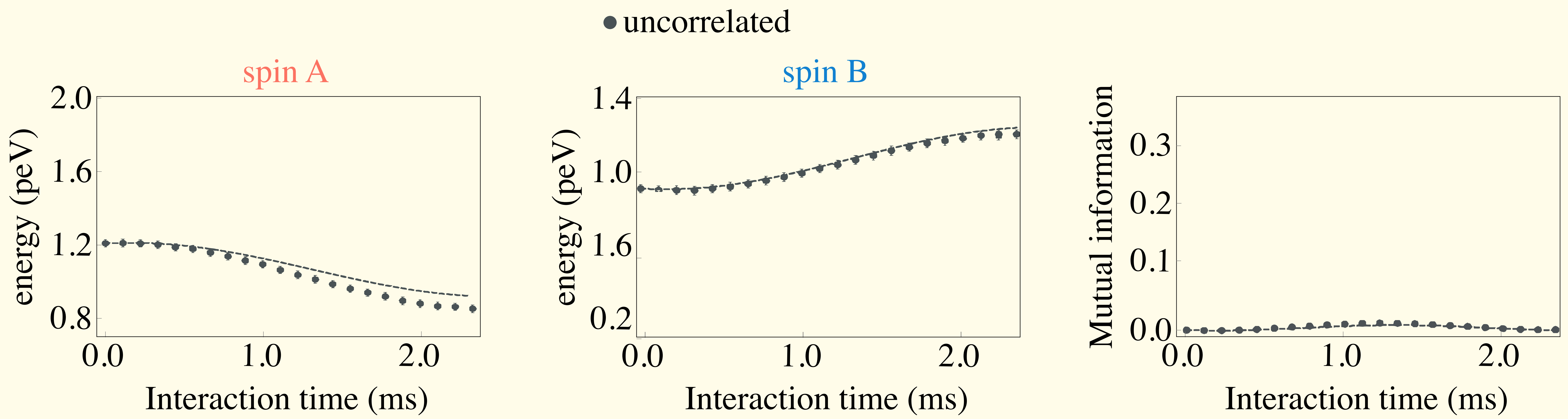
Initial state: $$\rho_{\mathsf{AB}} = \frac{e^{-\beta_{\mathsf{A}}H_{\mathsf{A}}}}{\mathrm{tr}(e^{-\beta_{\mathsf{A}}H_{\mathsf{A}}})} \otimes \frac{e^{-\beta_{\mathsf{B}}H_{\mathsf{B}}}}{\mathrm{tr}(e^{-\beta_{\mathsf{B}}H_{\mathsf{B}}})}$$



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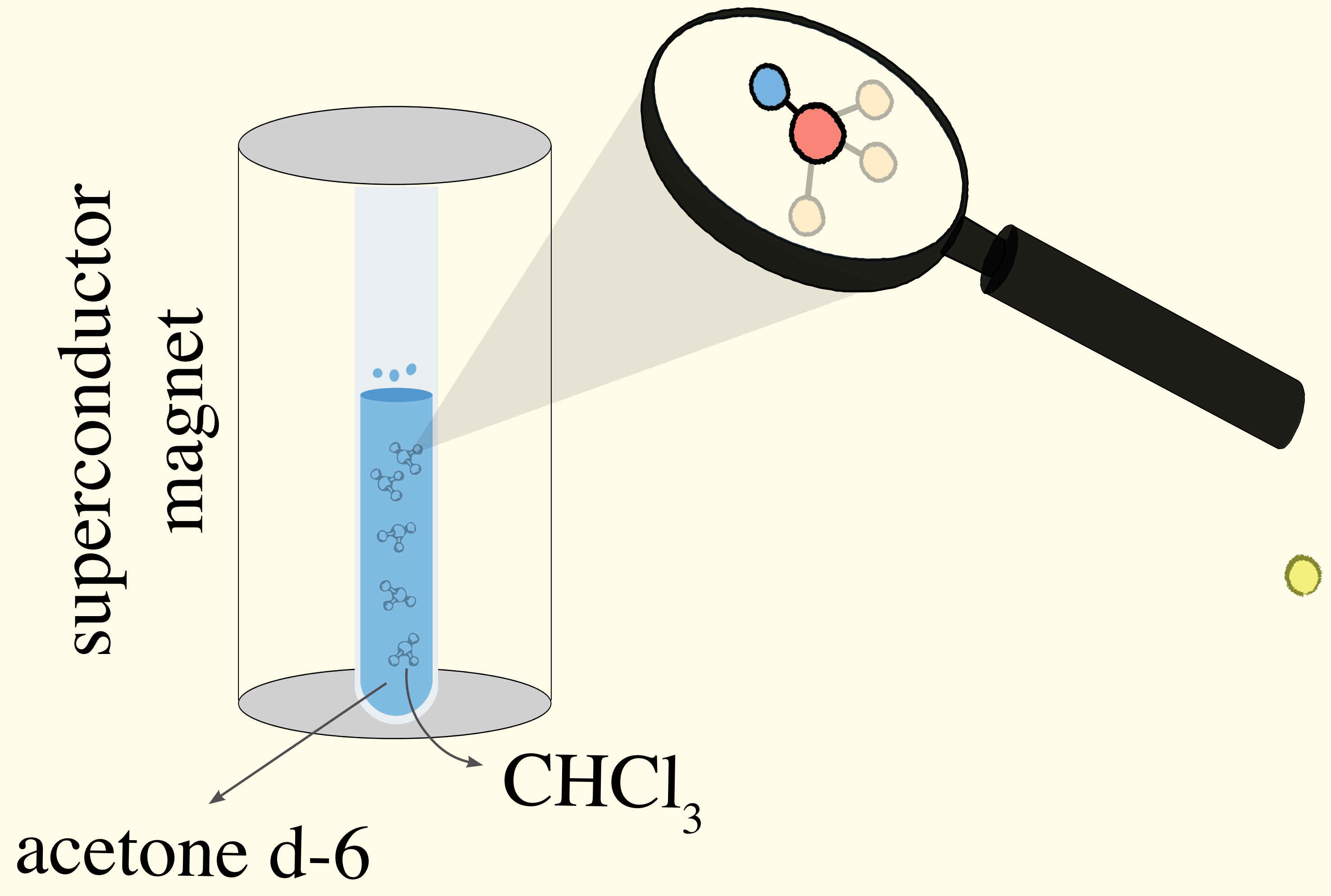

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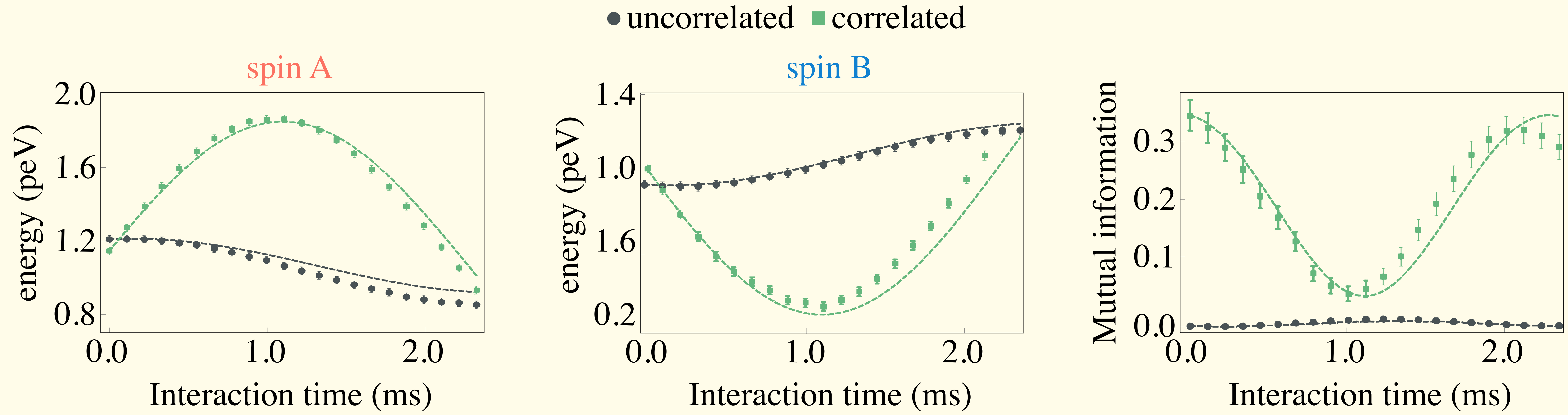
$\chi_{\mathrm{AB}} = \alpha\,|01\rangle\langle 10| + \alpha^*\,|10\rangle\langle 01|$



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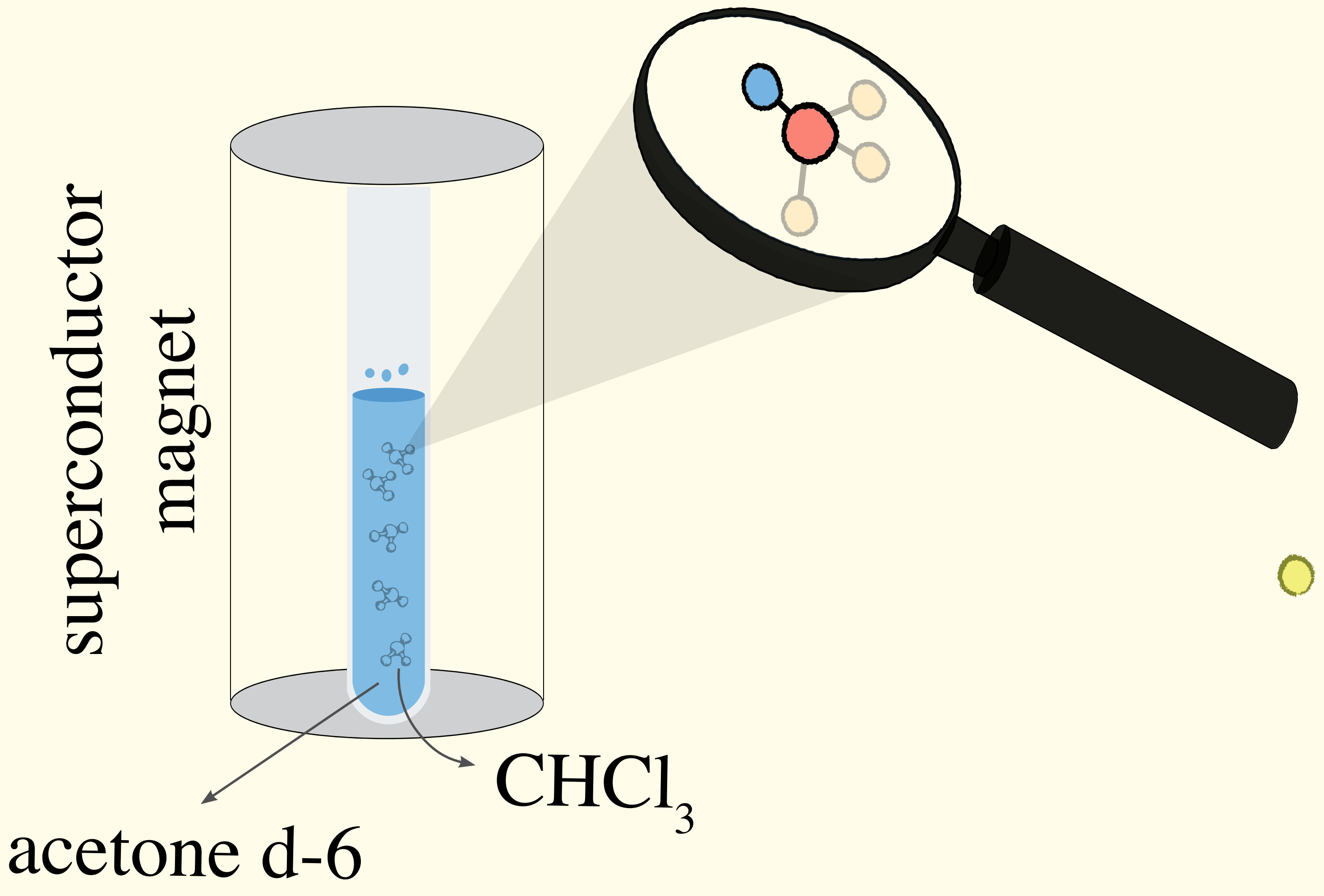

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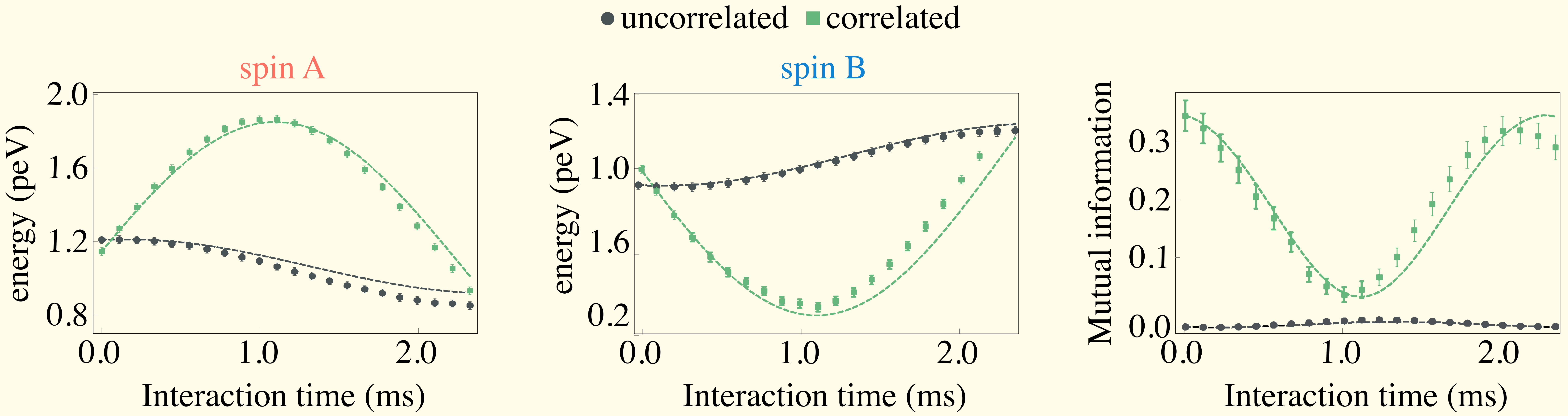

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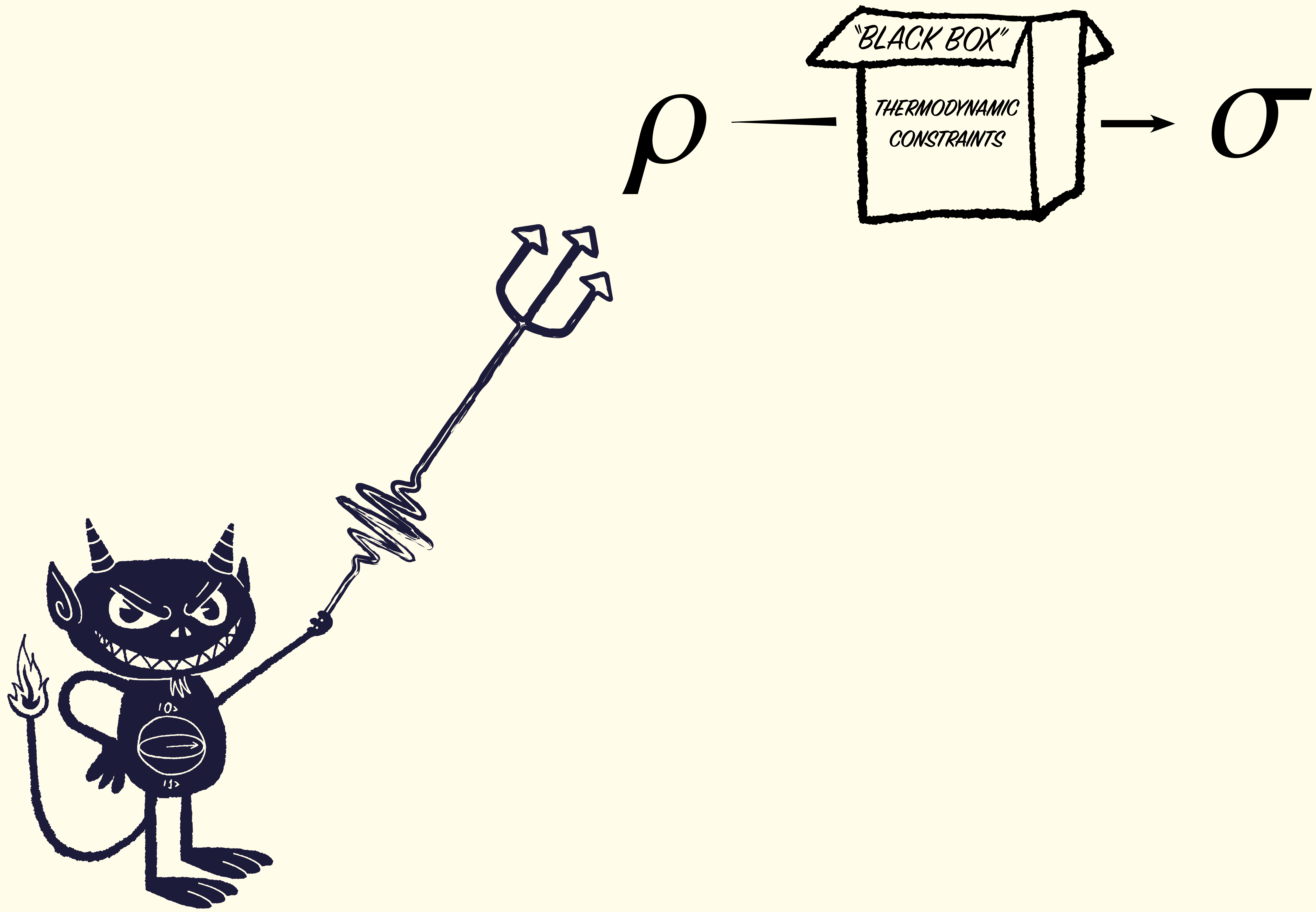
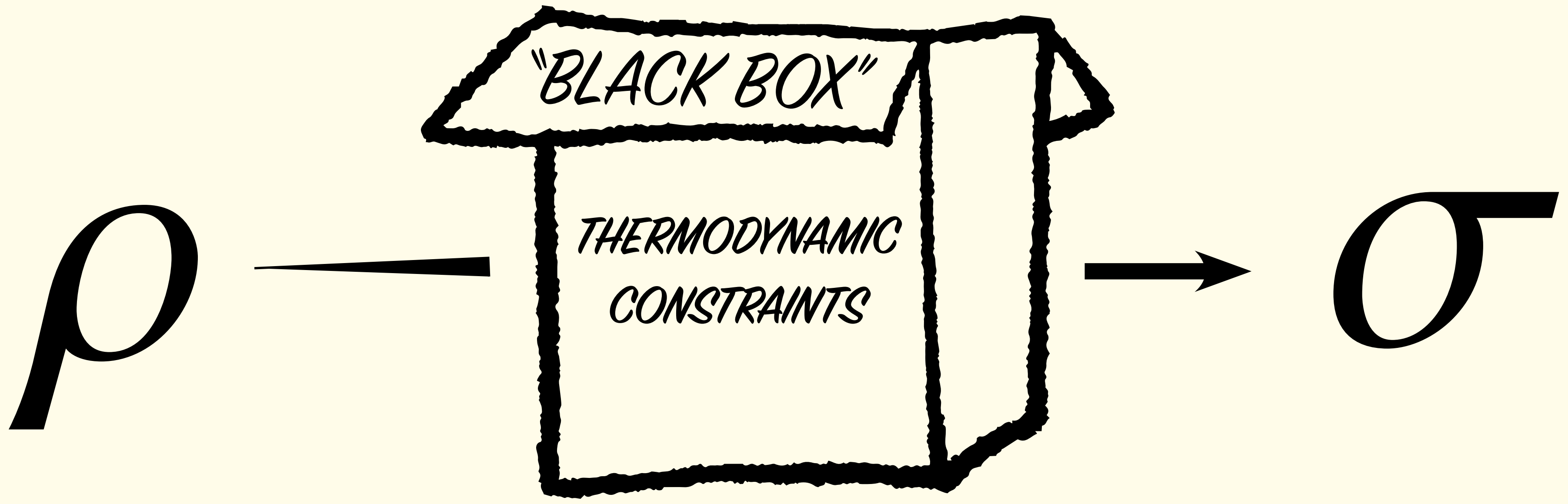
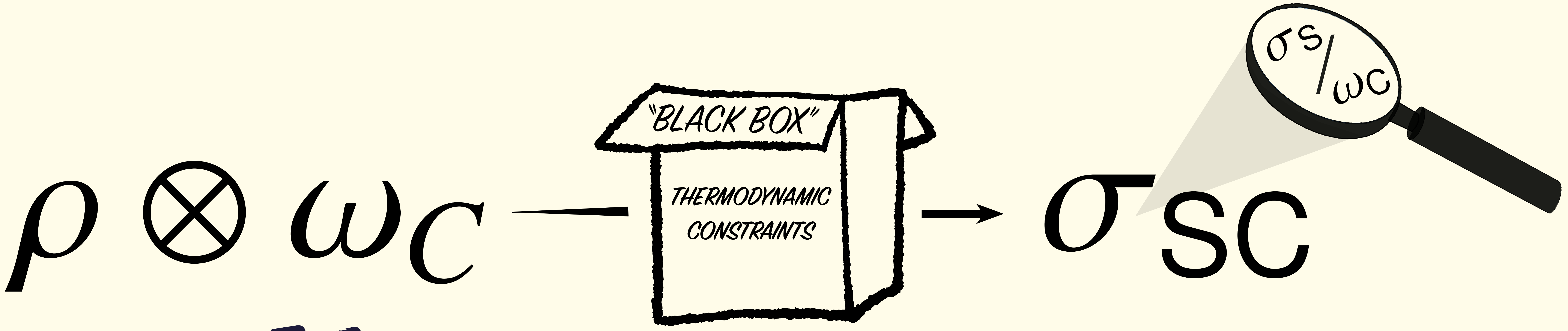

ρ
"BLACK BOX"
THERMODYNAMIC
CONSTRAINTS
σ



ALEX'S WISH LIST

✓ The 'thermodynamic blackbox' can be arbitrary



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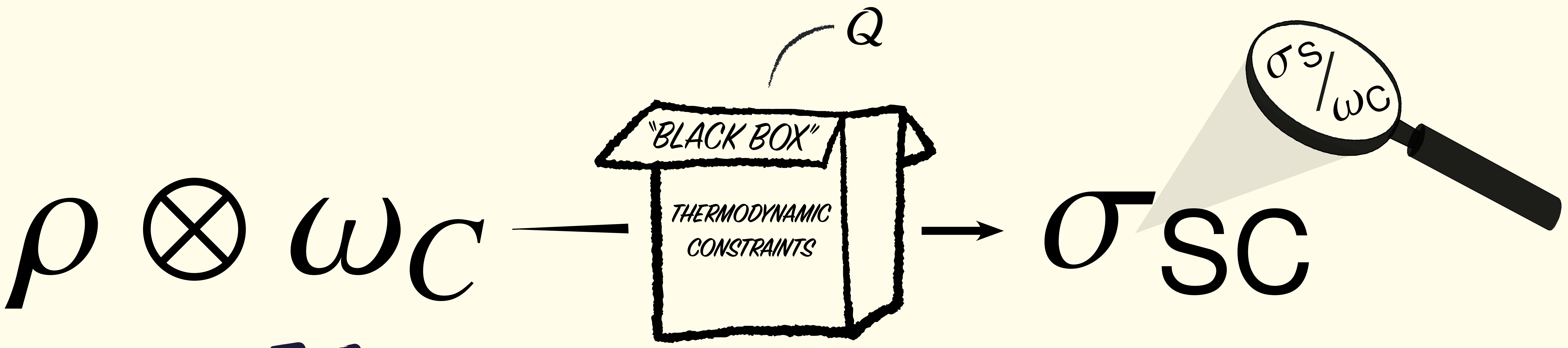
- ✓ The ‘thermodynamic blackbox’ can be arbitrary
- ✓ We can borrow auxiliary systems



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- ✓ The 'thermodynamic blackbox' can be arbitrary
- ✓ We can borrow auxiliary systems
- ✓ We have access to thermodynamic charges

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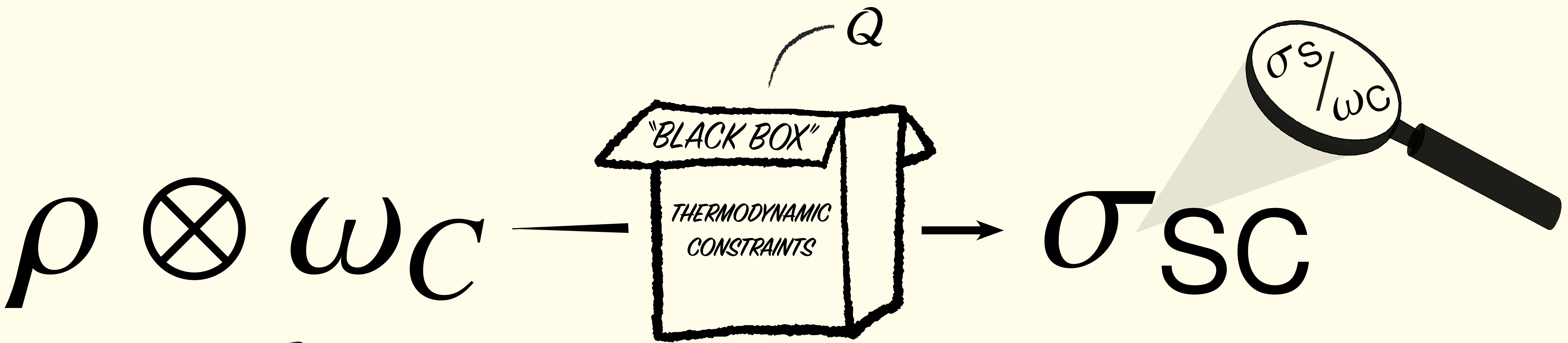

ALEX'S WISH LIST

- ✓ The 'thermodynamic blackbox' can be arbitrary
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- ✓ We have access to thermodynamic charges
- ✓ We can certify such a quantumness

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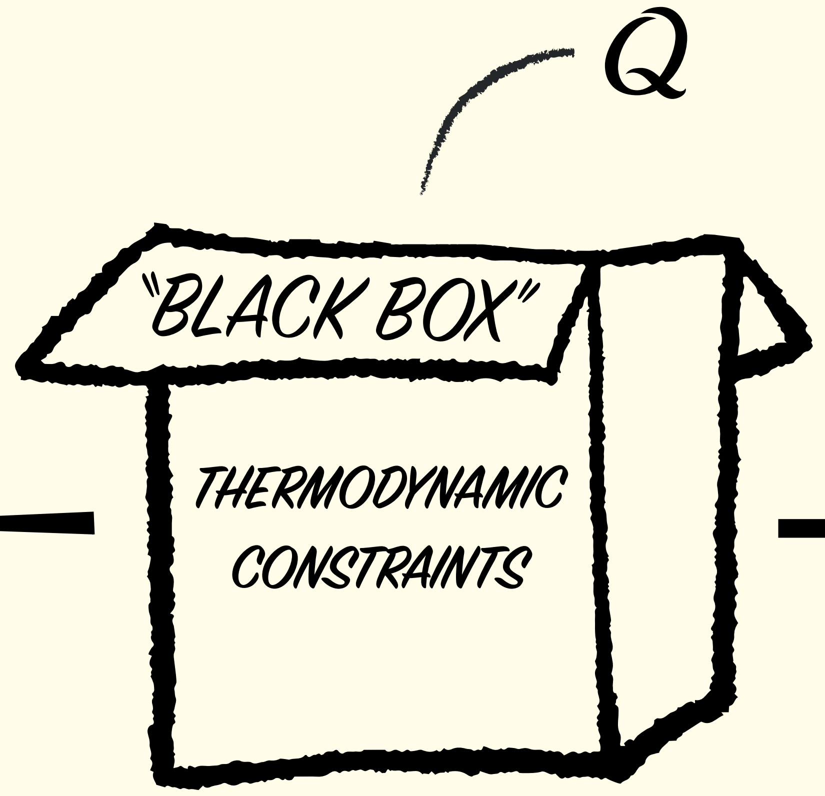
$$\rho \otimes \omega_C \longrightarrow$$



$$\longrightarrow \sigma_{SC}$$

Is it possible?

ALEX'S WISH LIST

- ✓ The 'thermodynamic blackbox' can be arbitrary
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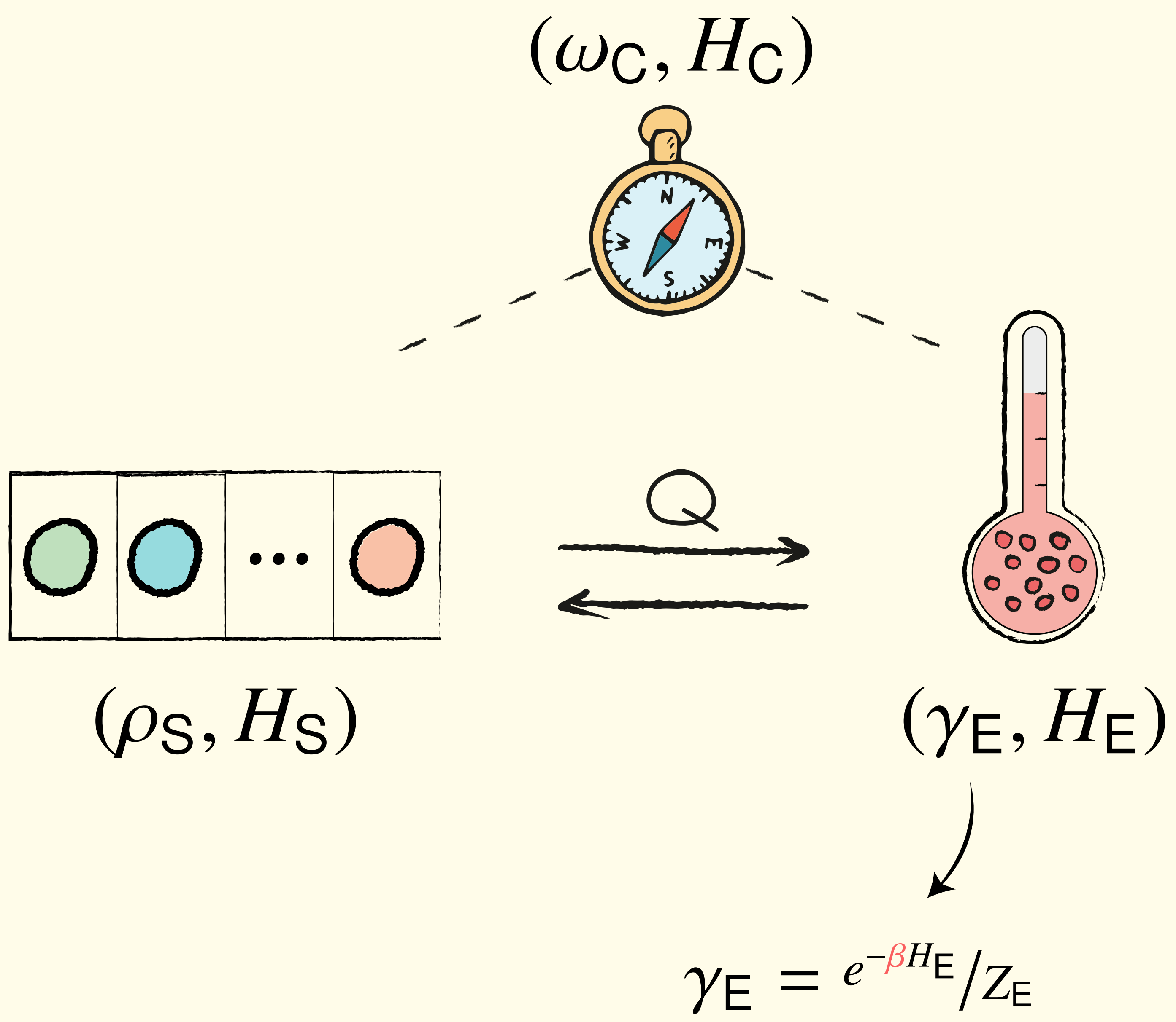
II. Results

Can we infer the presence of **quantum correlations** via **thermodynamic quantities**?

i. Protocol

Protocol

- **Composite system:** S, C and E



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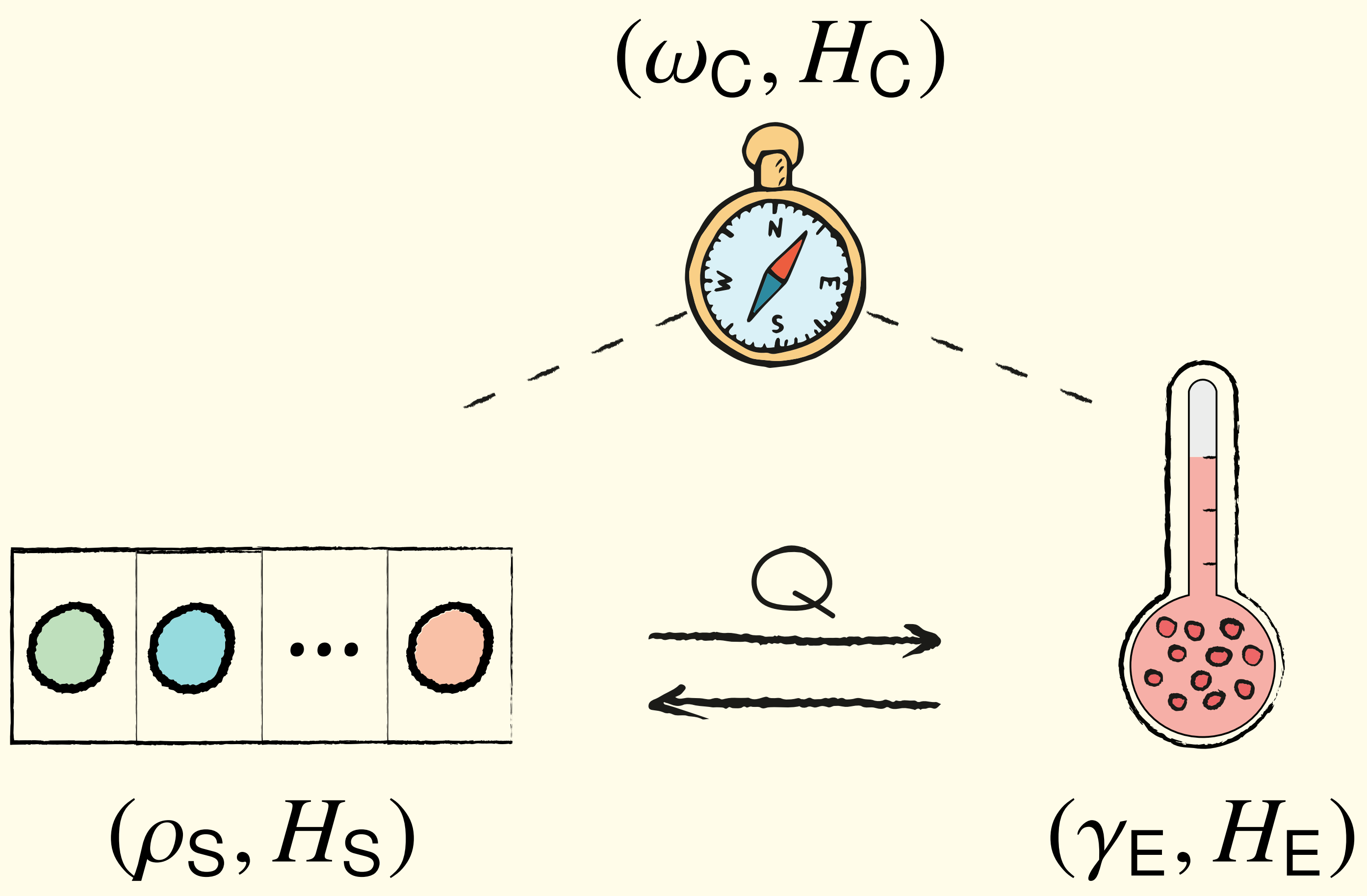

No assumptions about

- Interaction's strength, complexity, duration
- Only heat exchange

i. S, C and E evolves under an energy-preserving unitary:

$$\sigma_{\mathsf{SCE}} = U(\rho_{\mathsf{S}} \otimes \omega_{\mathsf{C}} \otimes \gamma_{\mathsf{E}})U^{\dagger}$$

where $[U, H_{\mathsf{S}} + H_{\mathsf{C}} + H_{\mathsf{E}}] = 0$

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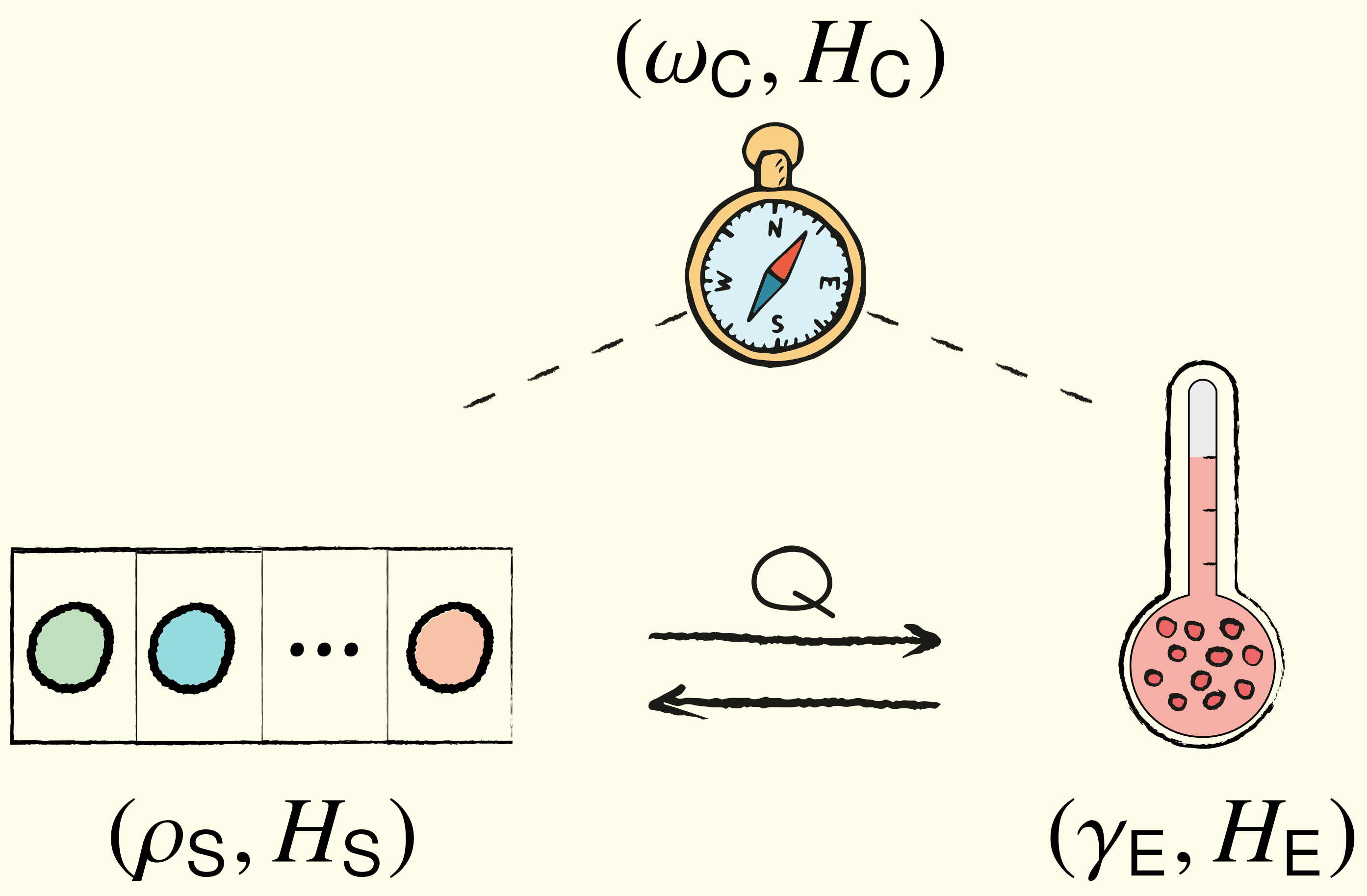

No assumptions about

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No heat from C to SE

- Reference frame is a catalyst
- Access locked correlations

i. S, C and E evolves under an energy-preserving unitary:

$$\sigma_{\mathsf{SCE}} = U(\rho_\mathsf{S} \otimes \omega_\mathsf{C} \otimes \gamma_\mathsf{E})U^\dagger$$

where $[U, H_\mathsf{S} + H_\mathsf{C} + H_\mathsf{E}] = 0$

ii. The state of the reference frame is fixed by the dynamics:

$$\sigma_\mathsf{C} := \mathrm{tr}_{\mathsf{SE}}[U(\rho_\mathsf{S} \otimes \omega_\mathsf{C} \otimes \gamma_\mathsf{E})U^\dagger] = \omega_\mathsf{C}$$

P. Lipka-Bartosik, G.F Diotallevi, P. Bakhshinezhad, arXiv:2307.03828 (2024)
P. Lipka-Bartosik, M. Perarnau-Llobet, N. Brunner, *Phys. Rev. Lett.* **130**, 040401 (2023)

Protocol

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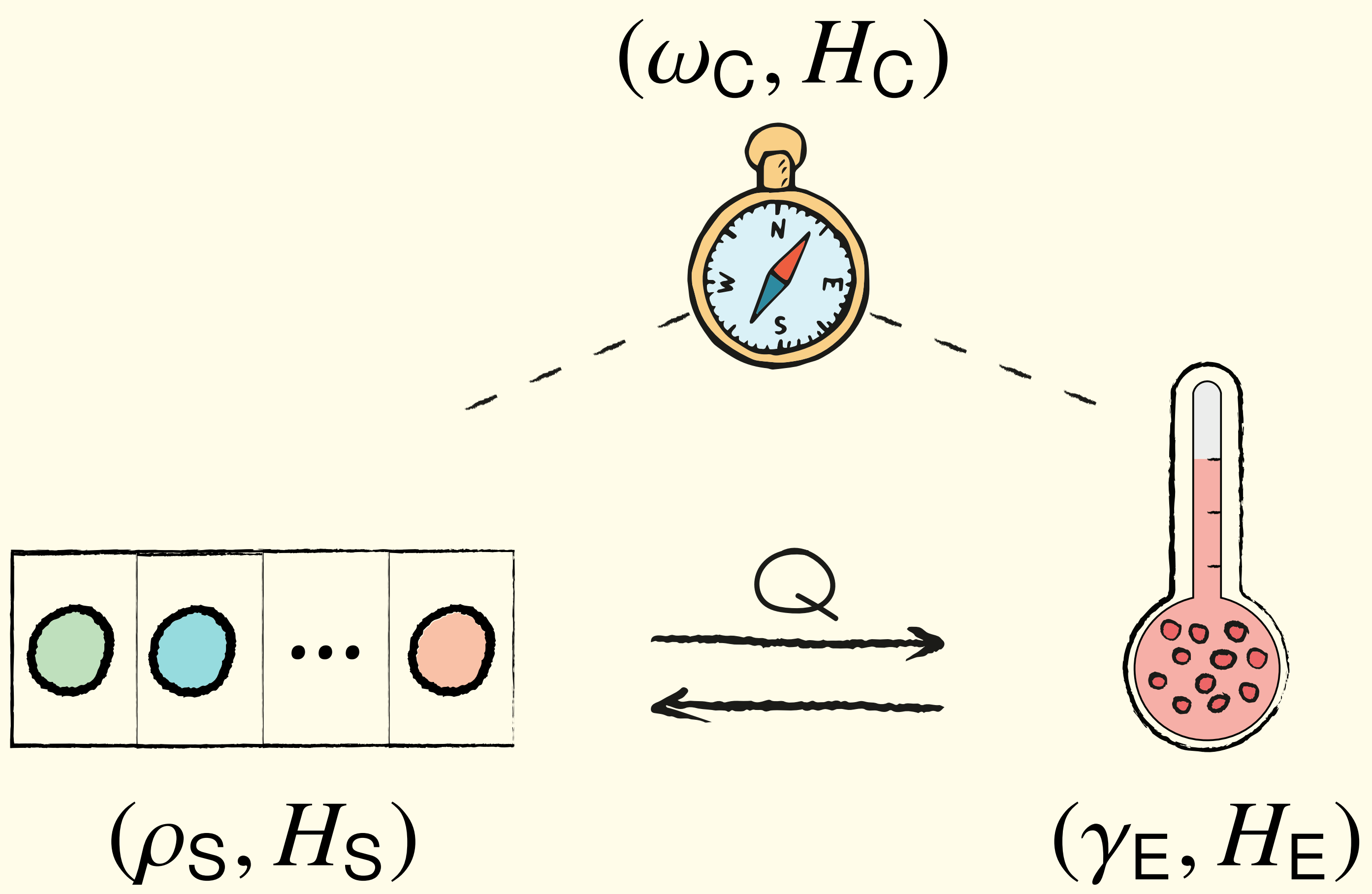


No assumptions about

- Interaction's strength, complexity, duration
- Only heat exchange

No heat from C to SE

- Reference frame is a catalyst
- Access locked correlations

No complicated way to infer Q

- Just measuring the thermal ancilla

i. S, C and E evolves under an energy-preserving unitary:

$$\sigma_{\mathsf{SCE}} = U(\rho_{\mathsf{S}} \otimes \omega_{\mathsf{C}} \otimes \gamma_{\mathsf{E}})U^\dagger$$

where $[U, H_{\mathsf{S}} + H_{\mathsf{C}} + H_{\mathsf{E}}] = 0$

ii. The state of the reference frame is fixed by the dynamics:

$$\sigma_{\mathsf{C}} := \mathrm{tr}_{\mathsf{SE}}[U(\rho_{\mathsf{S}} \otimes \omega_{\mathsf{C}} \otimes \gamma_{\mathsf{E}})U^\dagger] = \omega_{\mathsf{C}}$$

iii. Probe the heat exchange between S and E:

$$Q_{\mathsf{E}}(\beta, H_{\mathsf{E}}, U, \omega_{\mathsf{C}}, H_{\mathsf{C}}) := \mathrm{tr}\left[H_{\mathsf{E}}(\sigma_{\mathsf{E}} - \gamma_{\mathsf{E}})\right]$$

where $\sigma_{\mathsf{E}} = \mathrm{tr}_{\mathsf{SC}}(\sigma_{\mathsf{SCE}})$

What's the **optimal** heat flow?

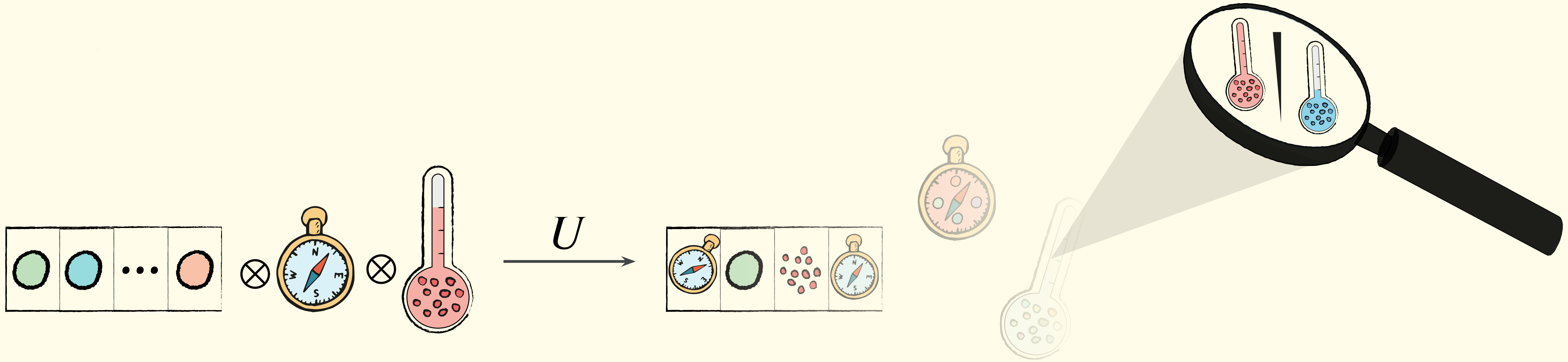
P. Lipka-Bartosik, G.F Diotallevi, P. Bakhshinezhad, arXiv:2307.03828 (2024)

P. Lipka-Bartosik, M. Perarnau-Llobet, N. Brunner, *Phys. Rev. Lett.* **130**, 040401 (2023)

ii. Optimal heat exchange

Optimal heat exchange

- **Pictorial representation:**



Optimal heat exchange

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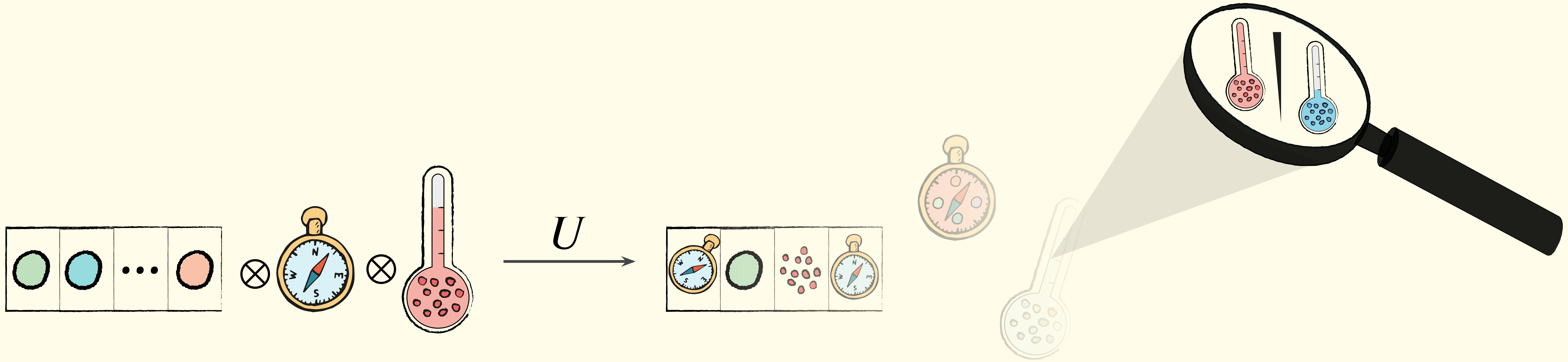


- **Figure of merit:**

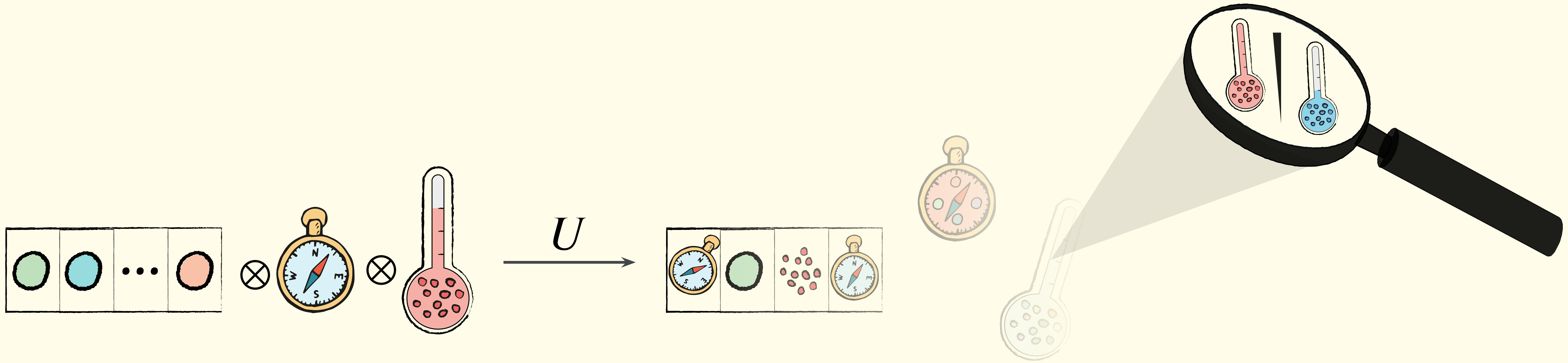
$$Q_{c/h}(\rho_{\mathsf{S}}) := \min_{H_{\mathsf{E}},H_{\mathsf{C}},U,\omega_{\mathsf{C}}} / \max \; Q_{\mathsf{E}}(\beta, H_{\mathsf{E}}, U, \omega_{\mathsf{C}}, H_{\mathsf{C}}),$$

$$\text{s.t.} \quad \sigma_{\mathsf{C}} = \omega_{\mathsf{C}},$$

$$[U, H_{\mathsf{AB}} + H_{\mathsf{C}} + H_{\mathsf{E}}] = 0$$

Optimal heat exchange

- **Pictorial representation:**



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$$\begin{aligned} Q_{c/h}(\rho_{\mathsf{S}}) &:= \min_{H_{\mathsf{E}},H_{\mathsf{C}},U,\omega_{\mathsf{C}}} / \max \; Q_{\mathsf{E}}(\beta, H_{\mathsf{E}}, U, \omega_{\mathsf{C}}, H_{\mathsf{C}}), \\ \text{s.t.} &\quad \sigma_{\mathsf{C}} = \omega_{\mathsf{C}}, \\ &\quad [U, H_{\mathsf{AB}} + H_{\mathsf{C}} + H_{\mathsf{E}}] = 0 \end{aligned}$$

$$\Longleftrightarrow$$

$$\begin{aligned} Q_{c/h}(\rho_{\mathsf{S}}) &:= -\max_{\sigma_{\mathsf{S}}} / \min \; \mathrm{tr}[H_{\mathsf{S}}(\sigma_{\mathsf{S}} - \rho_{\mathsf{S}})], \\ \text{s.t.} &\quad F_\beta(\rho_{\mathsf{S}}) \geq F_\beta(\sigma_{\mathsf{S}}) \end{aligned}$$

$$F_\beta(\rho) := \left[\mathrm{tr}(H\rho) - \frac{1}{\beta} S(\rho)\right] - \frac{1}{\beta}\log Z$$

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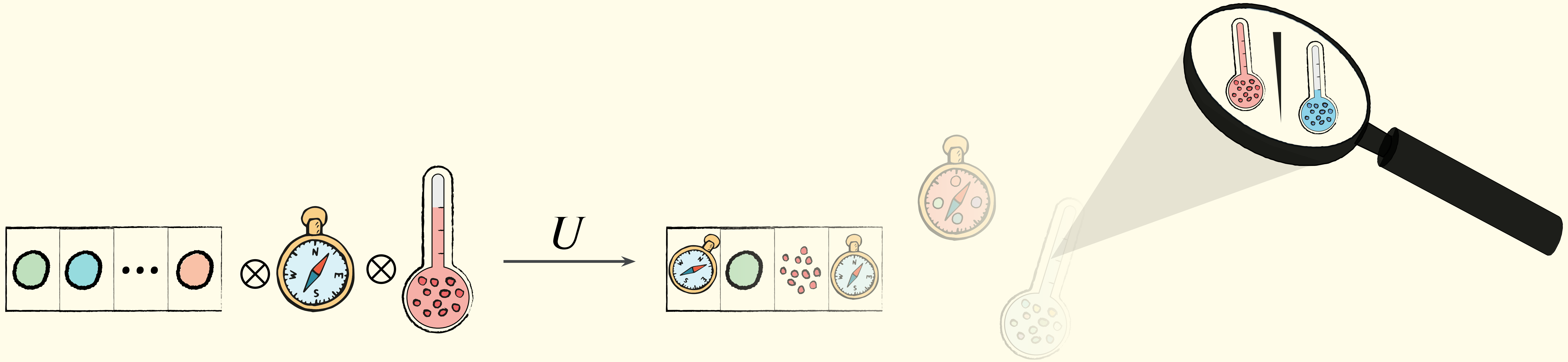
Optimal heat exchange

- **Pictorial representation:**



- **Figure of merit:**

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$$\iff$$

$$Q_{c/h}(\rho_{\mathsf{S}}) := -\max_{\sigma_{\mathsf{S}}} / \min \; \mathrm{tr}[H_{\mathsf{S}}(\sigma_{\mathsf{S}} - \rho_{\mathsf{S}})], \quad \text{s.t.} \quad F_\beta(\rho_{\mathsf{S}}) \geq F_\beta(\sigma_{\mathsf{S}})$$

- **"simple" solution:**

$$Q_{c/h}(\rho_{\mathsf{S}}) = \frac{1}{\beta}[S(\rho_{\mathsf{S}}) - S(\gamma_{\mathsf{S}}^{(c/h)})]$$

$$\gamma_{\mathsf{S}}^{(c/h)} = \frac{e^{-\beta_{c/h} H_{\mathsf{S}}}}{\mathrm{tr}[e^{-\beta_{c/h} H_{\mathsf{S}}}]}$$

easy to determine!

Optimal heat exchange

- **Given ρ_{S} and H_{S}:**

$$\beta_{c/h} = \min_{\tilde{\beta}} / \max \frac{\partial}{\partial\tilde{\beta}} \log \tilde{Z}_{\mathsf{S}} + E_{\mathsf{S}},$$

$$\text{s.t. } (\tilde{\beta} - \beta)\left[E_{\mathsf{S}} + \frac{\partial}{\partial\tilde{\beta}} \log \tilde{Z}_{\mathsf{S}}\right] - \log \tilde{Z}_{\mathsf{S}} - \tilde{\beta} E_{\mathsf{S}} + S(\rho_{\mathsf{S}}) \leq 0,$$

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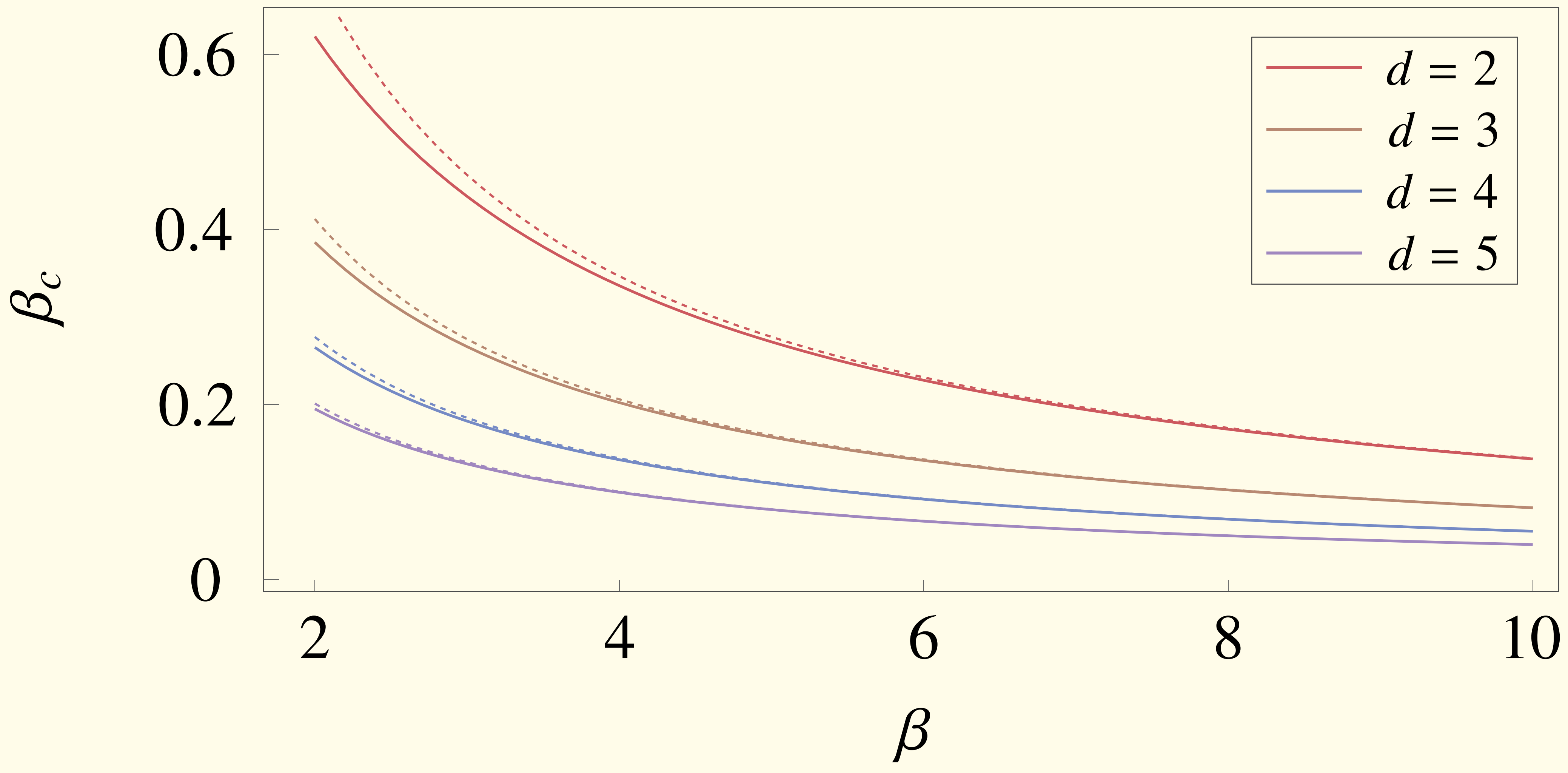
- **Asymptotic expansion**

$$\beta_c \approx -\frac{6 \log d}{\beta (d^2 - 1)}$$



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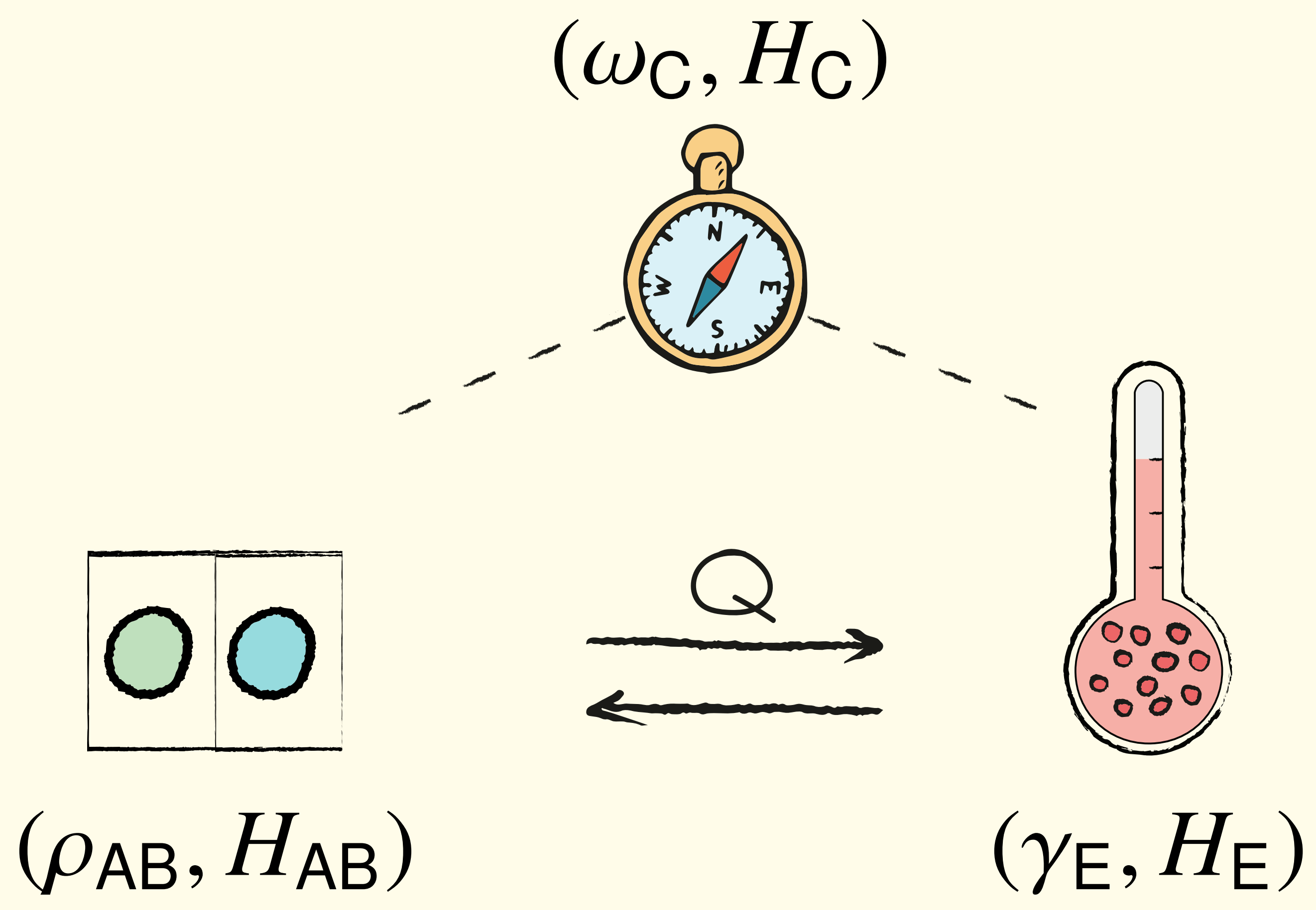
Optimal heat exchange

- **Composite system:** AB, C and E



N. J. Cerf and C. Adami,
Negative entropy in quantum information theory (Springer, 1997) pp. 77–84

Optimal heat exchange

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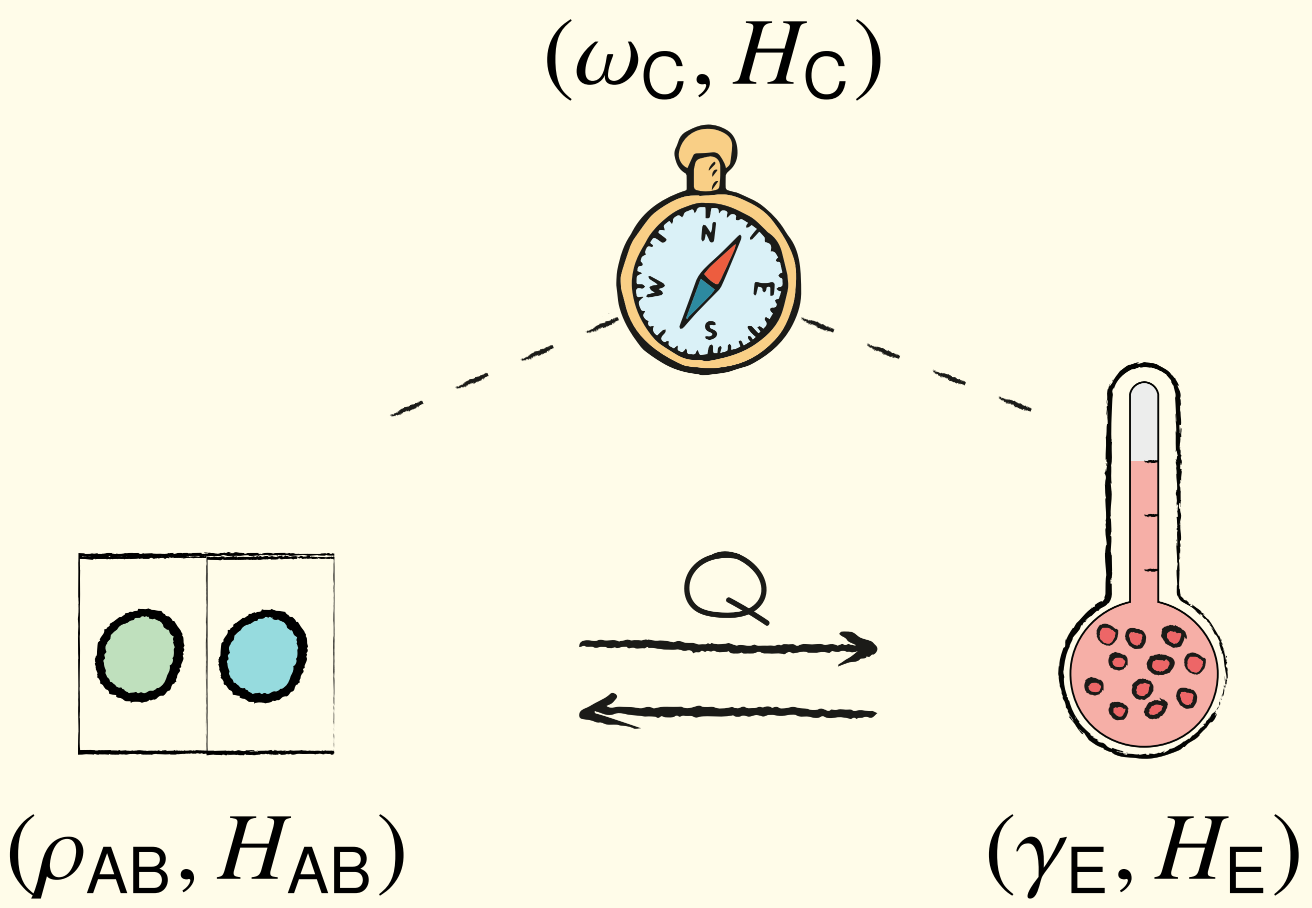

conditional entropy

$$Q_{c/h}(\rho_{\mathsf{S}}) = \frac{1}{\beta}\left[S(\rho_{\mathsf{B}}) - S(\gamma_{\mathsf{AB}}^{(c/h)}) + S(\mathsf{A}|\mathsf{B})_{\rho}\right]$$

Conditional entropy in a nutshel

- $S(\mathsf{A}|\mathsf{B})_{\rho} := S(\rho_{\mathsf{AB}}) - S(\rho_{\mathsf{B}})$ can be negative!

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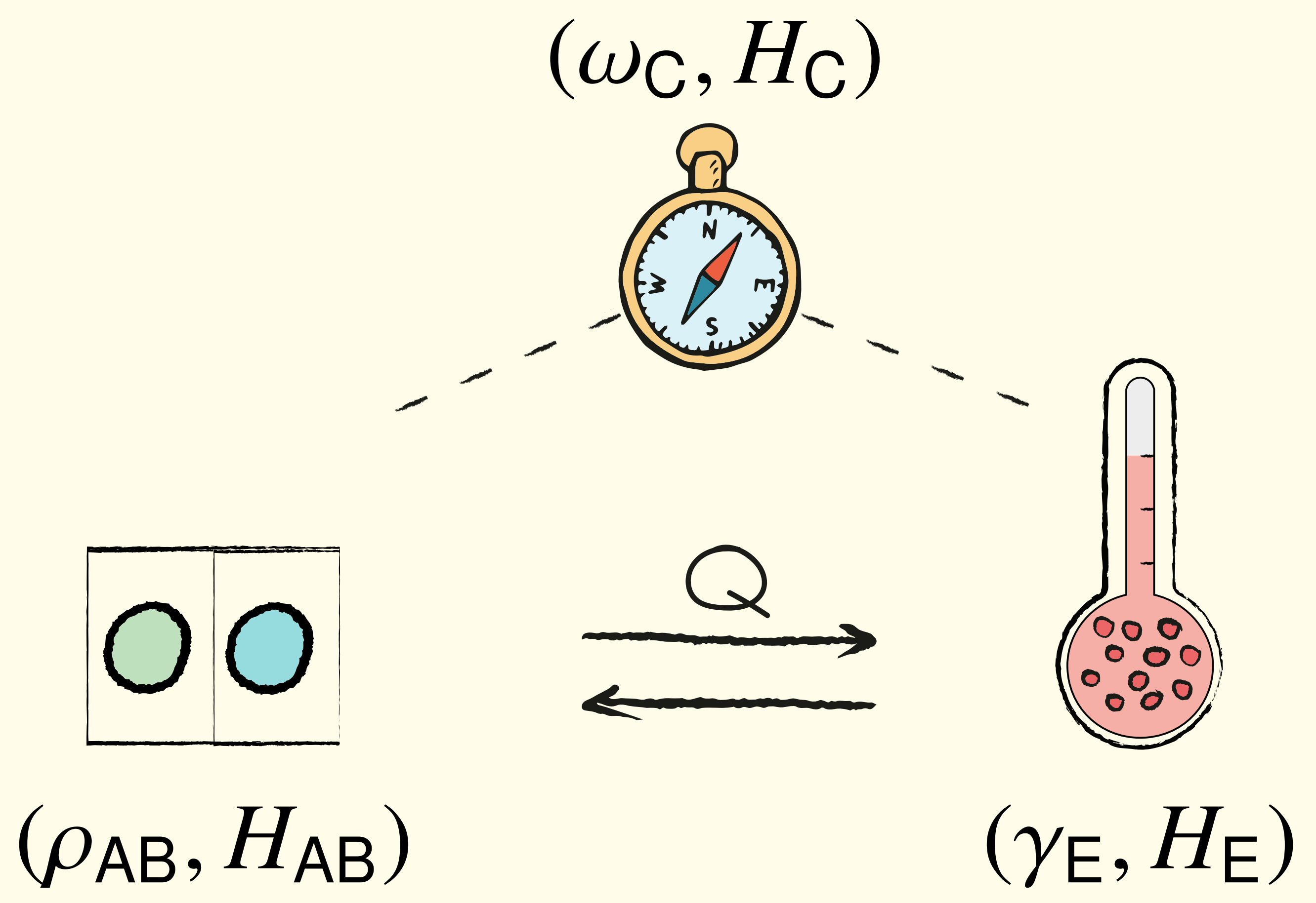

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Conditional entropy in a nutshel

- $S(\mathsf{A}|\mathsf{B})_\rho := S(\rho_{\mathsf{AB}}) - S(\rho_{\mathsf{B}})$ can be negative!
- If $S(\mathsf{A}|\mathsf{B})_\rho \leq 0 \implies \rho_{\mathsf{AB}}$ is entangled!

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Optimal heat exchange

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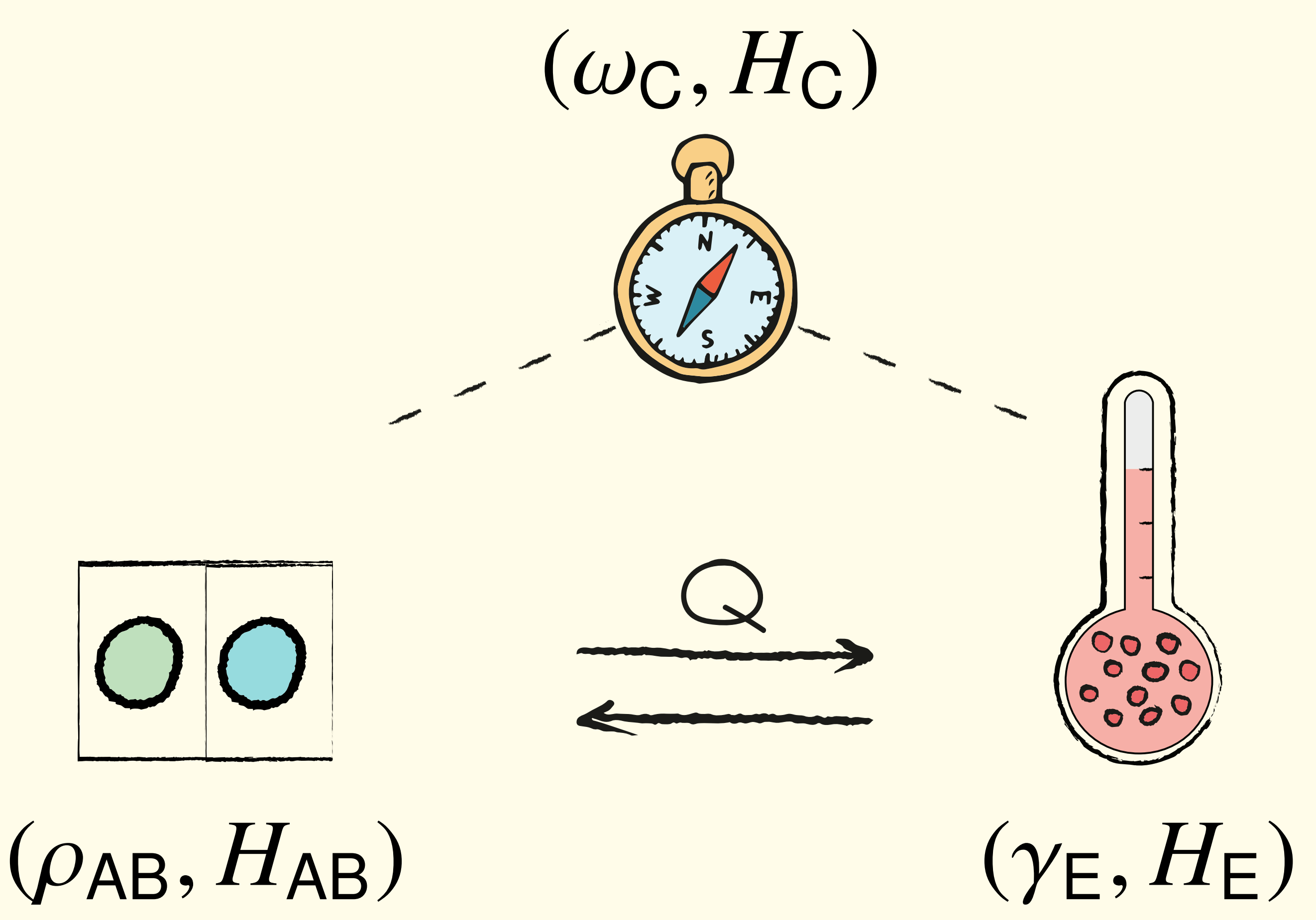

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Conditional entropy in a nutshel

- $S(\mathsf{A}|\mathsf{B})_{\rho} := S(\rho_{\mathsf{AB}}) - S(\rho_{\mathsf{B}})$ can be negative!
- If $S(\mathsf{A}|\mathsf{B})_{\rho} \leq 0 \implies \rho_{\mathsf{AB}}$ is entangled!
- Nice applications in quantum info!

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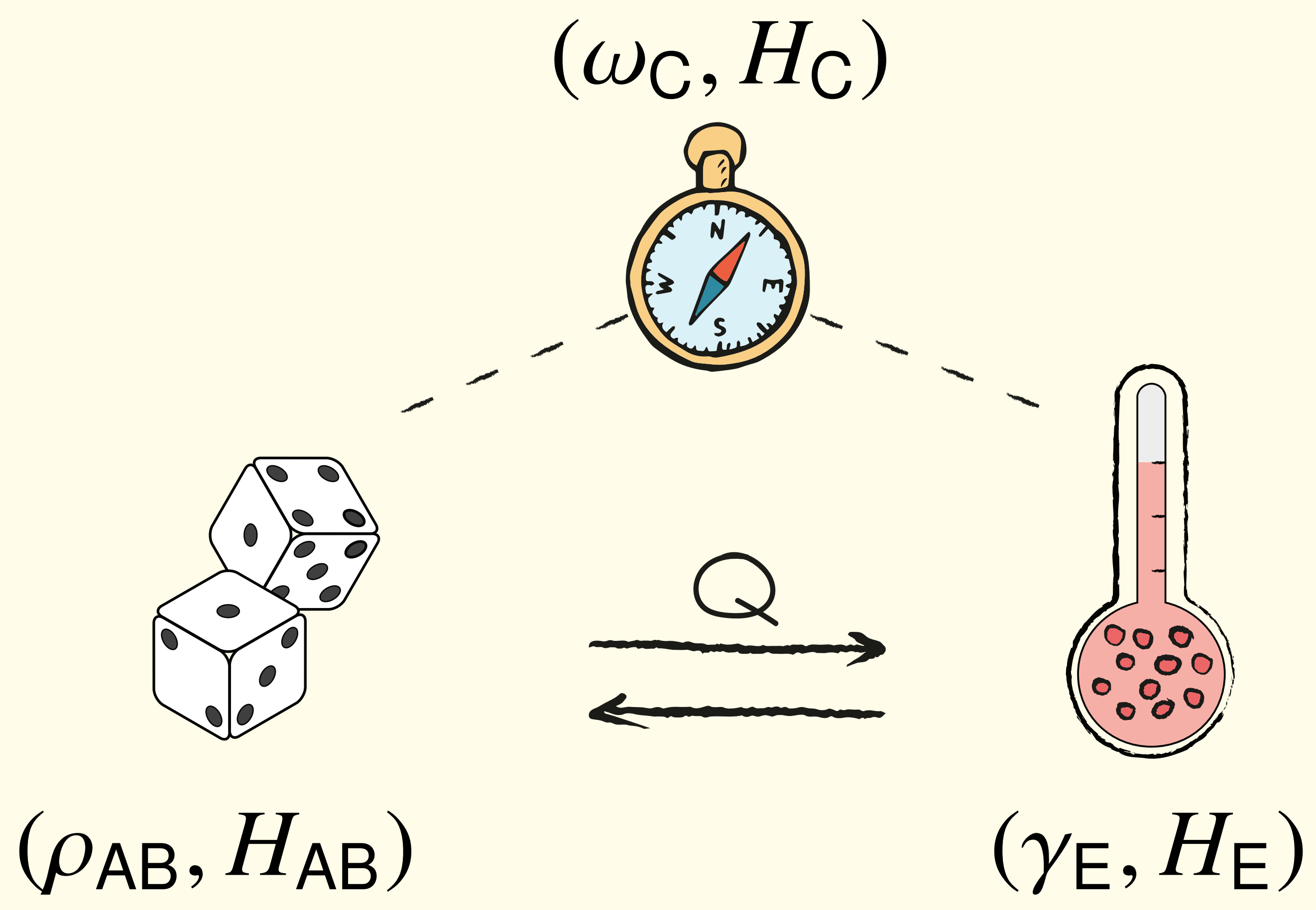
Classical heat exchange

- **Composite system:** AB, C and E



$$Q_{\text{class}} := \min_{\rho_{\mathsf{AB}} \in \mathsf{SEP}} / \max \quad Q_{c/h}(\rho_{\mathsf{AB}})$$

Classical heat exchange

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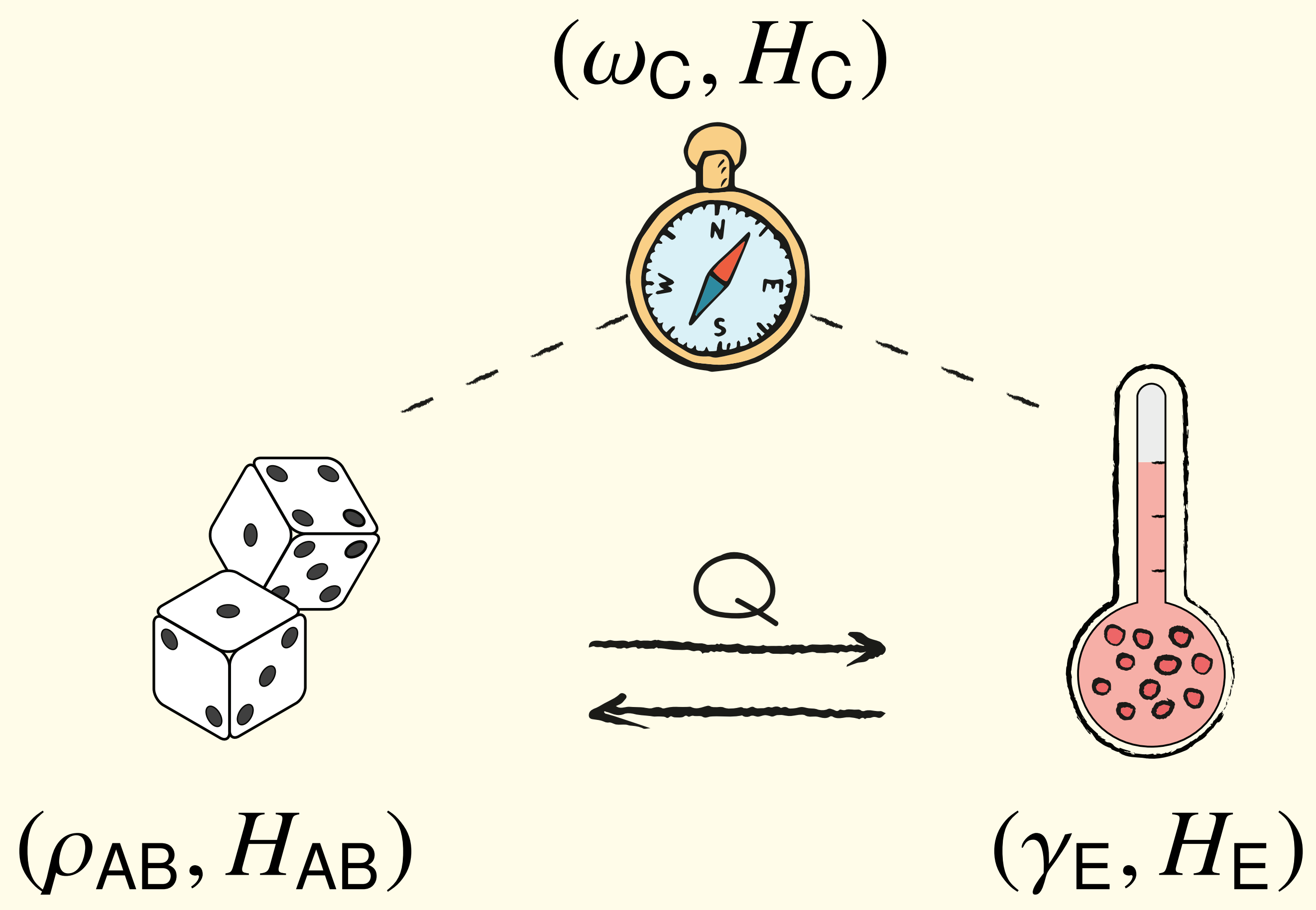

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$$Q^{\text{class}}_{c/h}(\rho_\mathsf{S}) := -\max_{\sigma_\mathsf{S}} / \min \ \operatorname{tr}[H_\mathsf{S}(\sigma_\mathsf{S} - \rho_\mathsf{S})], \quad \text{s.t.} \quad F^{\text{class}}_\beta(\rho_\mathsf{S}) \geq F_\beta(\sigma_\mathsf{S})$$

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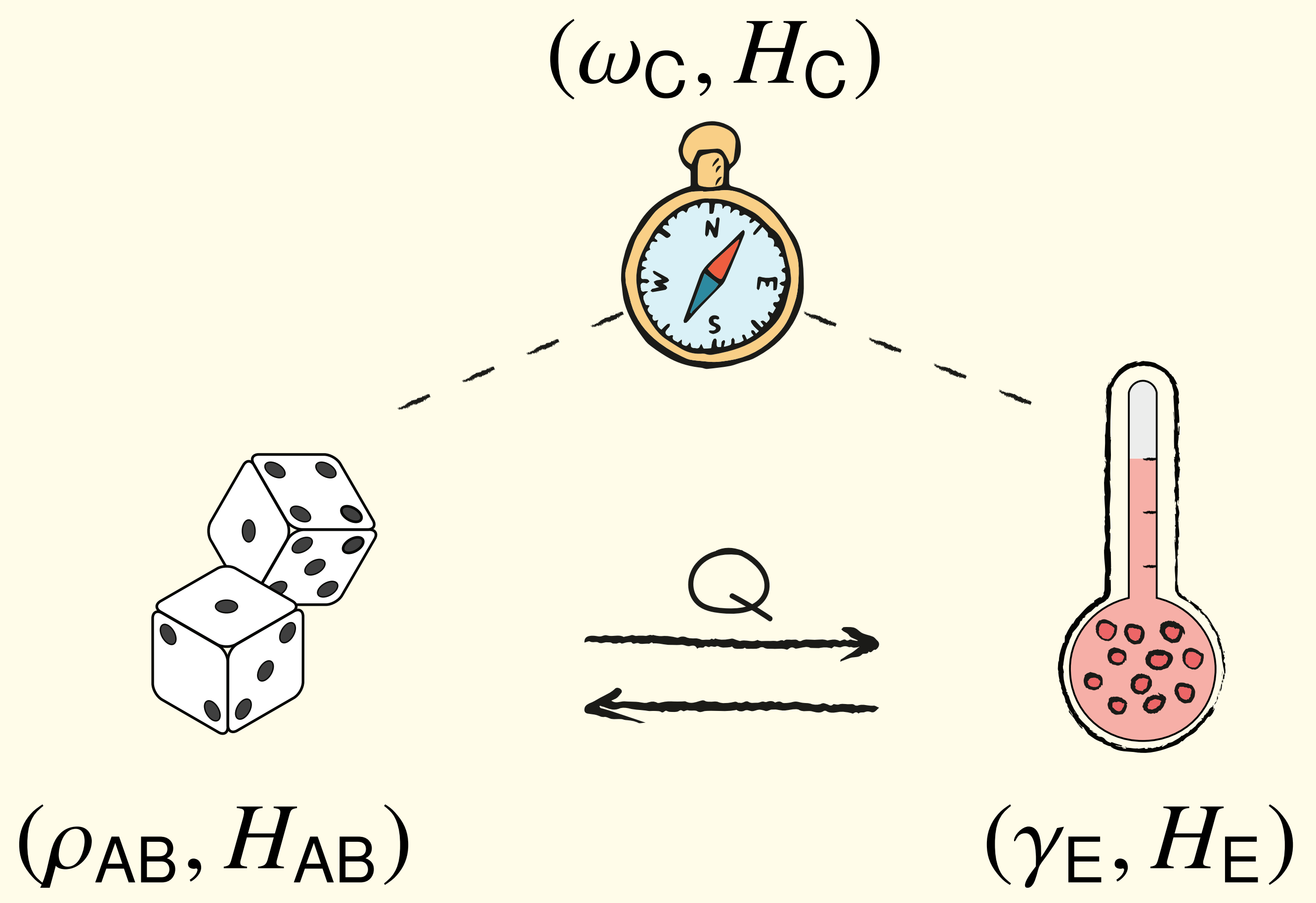

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- Min/max Q_{class} cannot be lower/greater than $Q_{c/h}$ for when $S(\mathsf{A}|\mathsf{B})_{\rho} = 0$
- The solution for Q_{class} will be bounded by $Q_{c/h}$

tight!

Classical heat exchange

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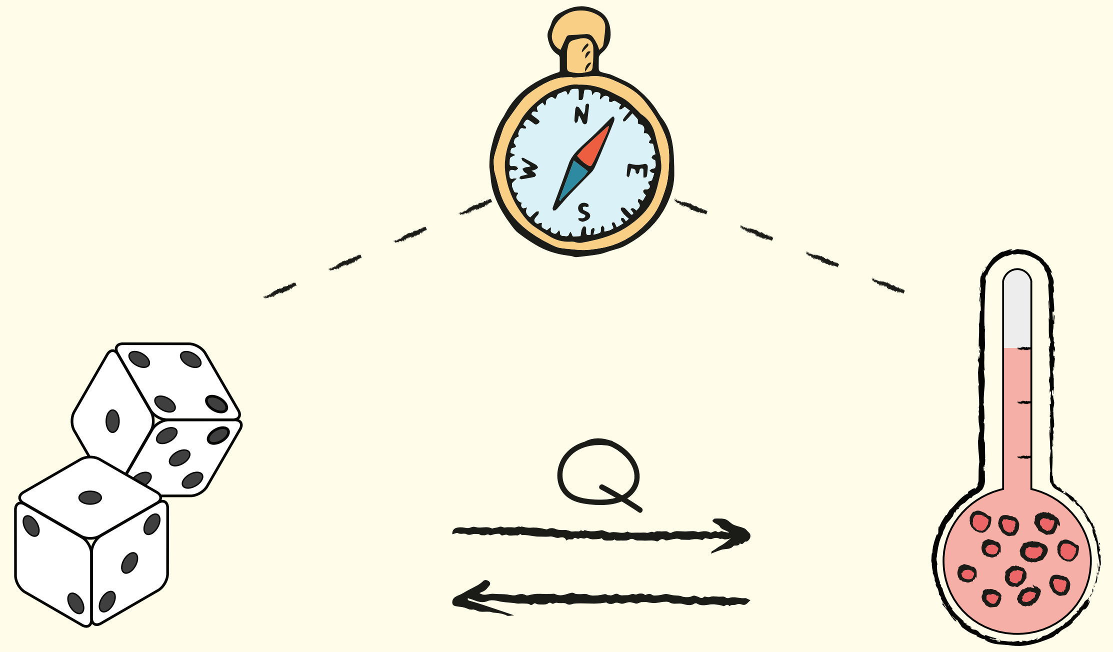
$(\omega_{\mathsf{C}}, H_{\mathsf{C}})$



$(\rho_{\mathsf{AB}}, H_{\mathsf{AB}})$ $(\gamma_{\mathsf{E}}, H_{\mathsf{E}})$

$$Q_{\text{class}} := \min_{\rho_{\mathsf{AB}} \in \mathsf{SEP}} / \max \quad Q_{c/h}(\rho_{\mathsf{AB}}) \quad \Longleftrightarrow \quad Q^{\text{class}}_{c/h}(\rho_{\mathsf{S}}) := -\max_{\sigma_{\mathsf{S}}} / \min \ \operatorname{tr}[H_{\mathsf{S}}(\sigma_{\mathsf{S}} - \rho_{\mathsf{S}})], \quad \text{s.t.} \ \ F^{\text{class}}_{\beta}(\rho_{\mathsf{S}}) \geq F_{\beta}(\sigma_{\mathsf{S}})$$

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$$\frac{1}{\beta}[S(\rho_{\mathsf{B}}) - S(\gamma^{(c)}_{\mathsf{AB}})] \leq Q_{\text{class}}$$

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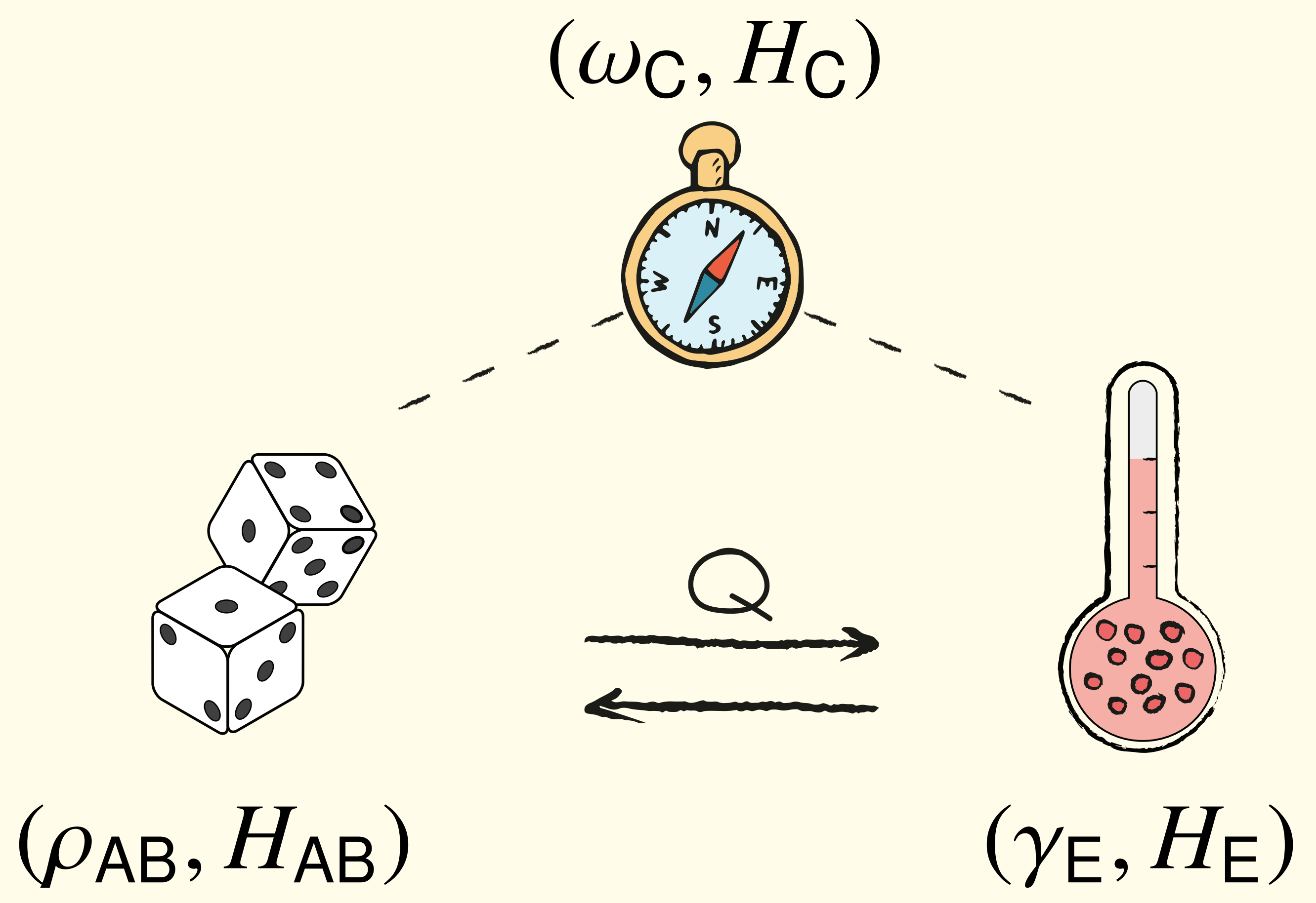

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tight!

Heaviside function

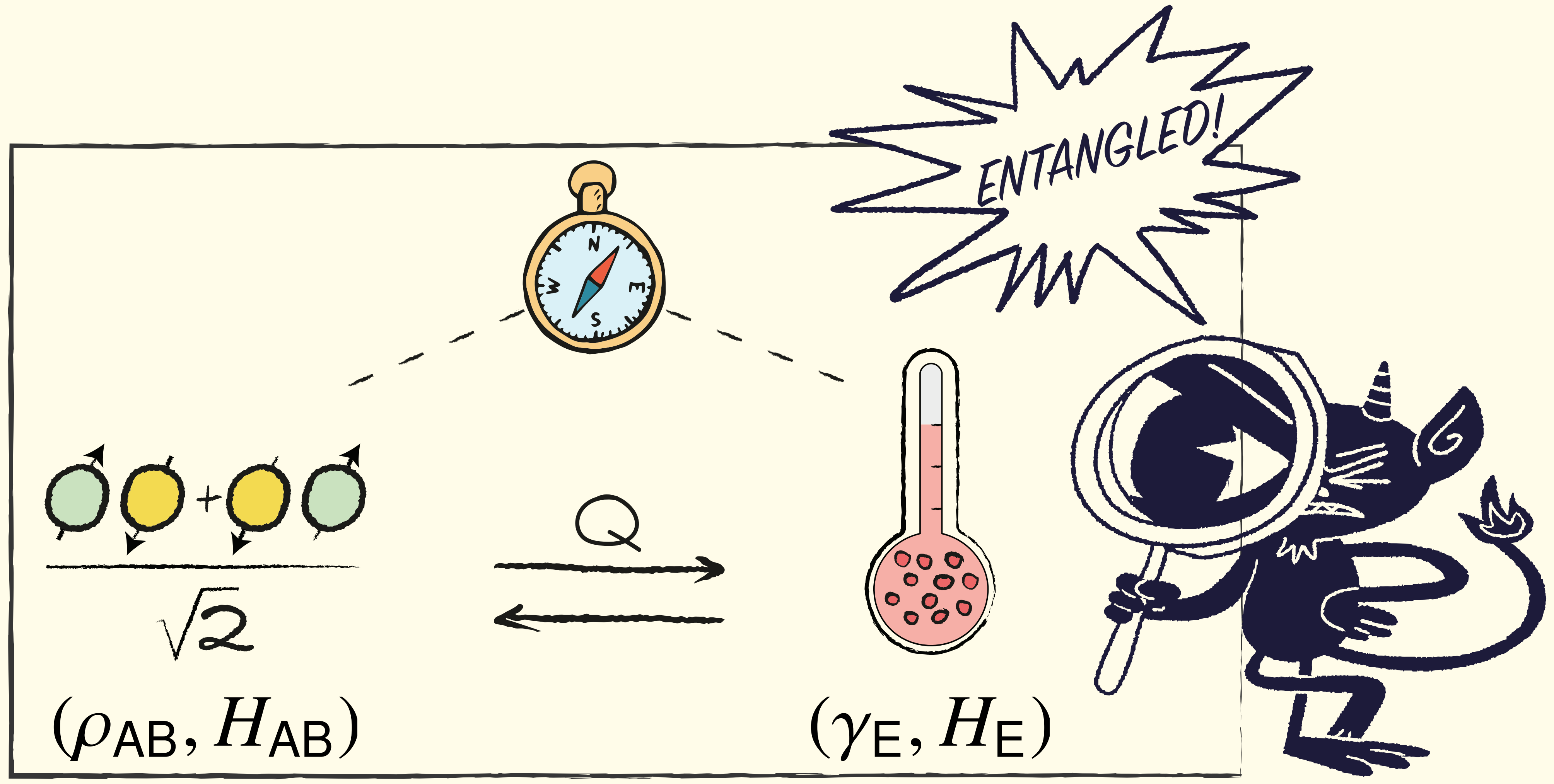
critical inverse temperature

$$\frac{1}{\beta}[S(\rho_{\mathsf{B}}) - S(\gamma^{(c)}_{\mathsf{AB}})] \leq Q_{\text{class}} \leq \frac{1}{\beta}[S(\rho_{\mathsf{B}}) - S(\gamma^{(h)}_{\mathsf{AB}})]\Theta(\beta^{\star} - \beta) + [1 - \Theta(\beta^{\star} - \beta)]E_{\mathsf{AB}}$$

iii. Entanglement thermometer

Entanglement witness

- **Composite system:** AB, C and E



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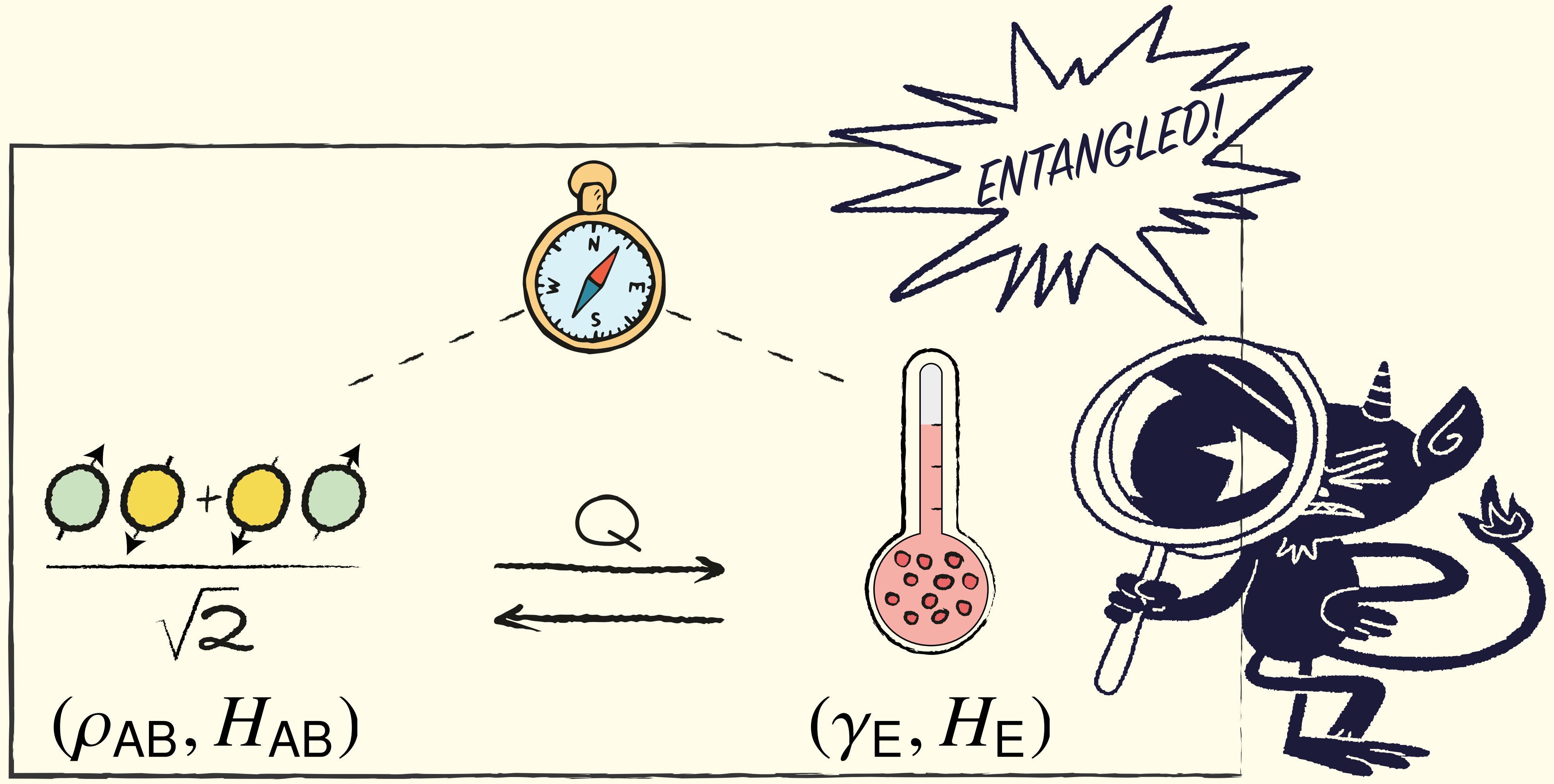

Entanglement thermometer

★ Any bipartite pure entangled state can be detected via heat exchange!

$$\boxed{Q_{c/h}(\rho_{\mathsf{AB}}) = Q^{(c/h)}_{\text{class}} \pm \frac{1}{\beta} S\,(\mathsf{A}|\mathsf{B})_{\rho}}$$

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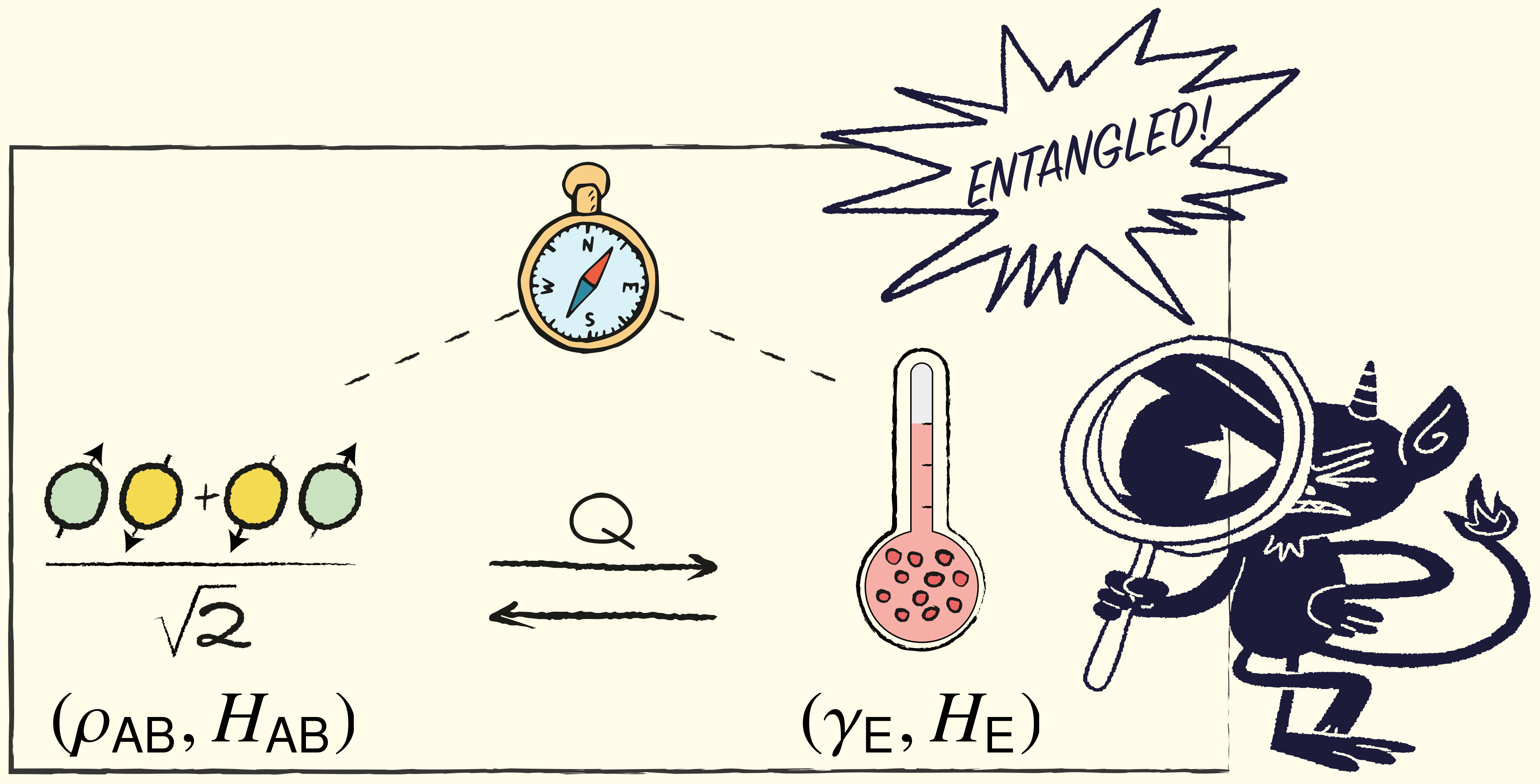

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How about for mixed states?

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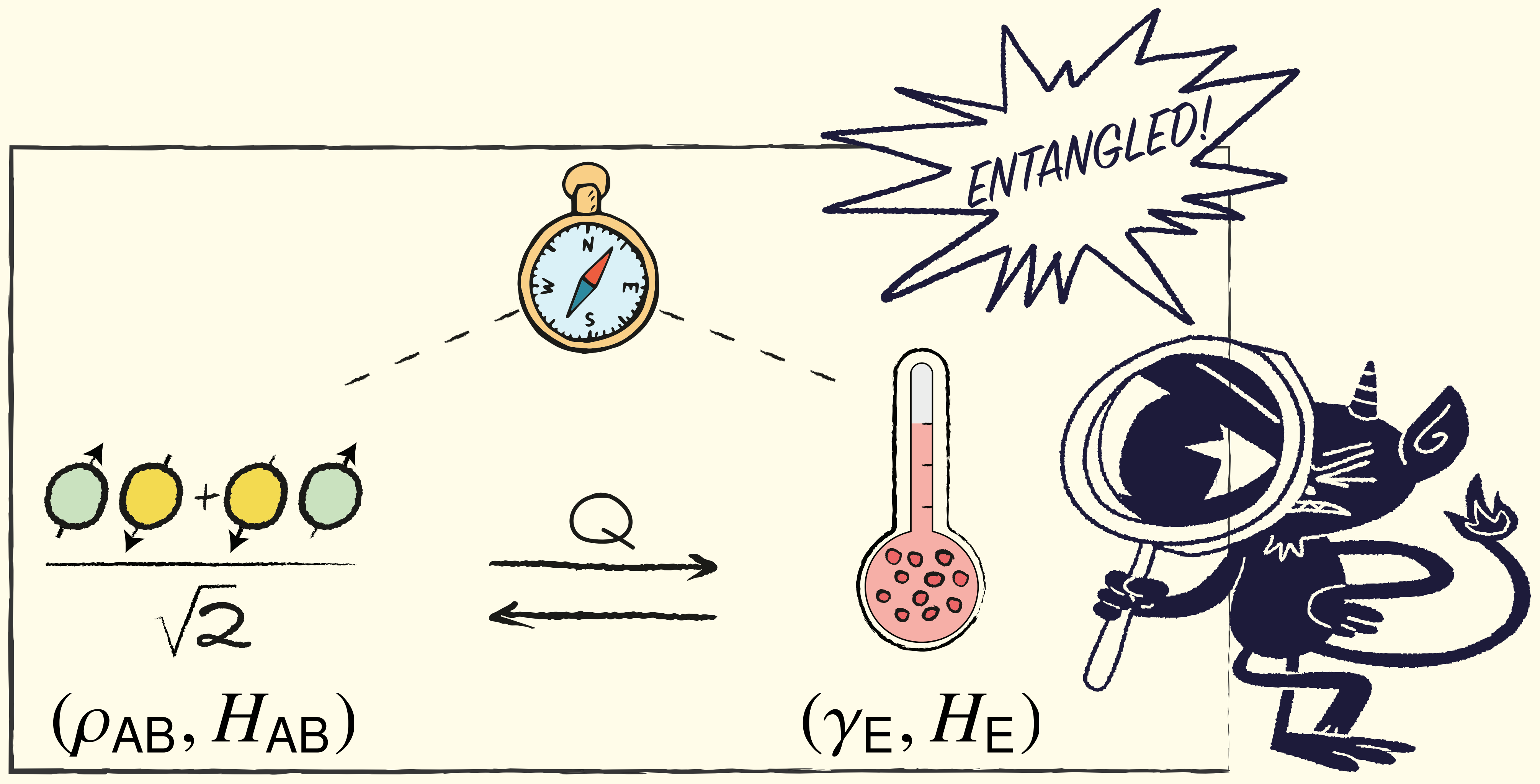

Entanglement thermometer

- **Isotropic state:** $\rho_\lambda = (1-\lambda)\left|\psi^+\middle\rangle\!\middle\langle\psi^+\right| + \lambda\frac{\mathbb{1}}{d^2}$

$$\left|\psi^+\right\rangle = \frac{1}{\sqrt{d}}\sum_{i=1}^{d}|ii\rangle$$

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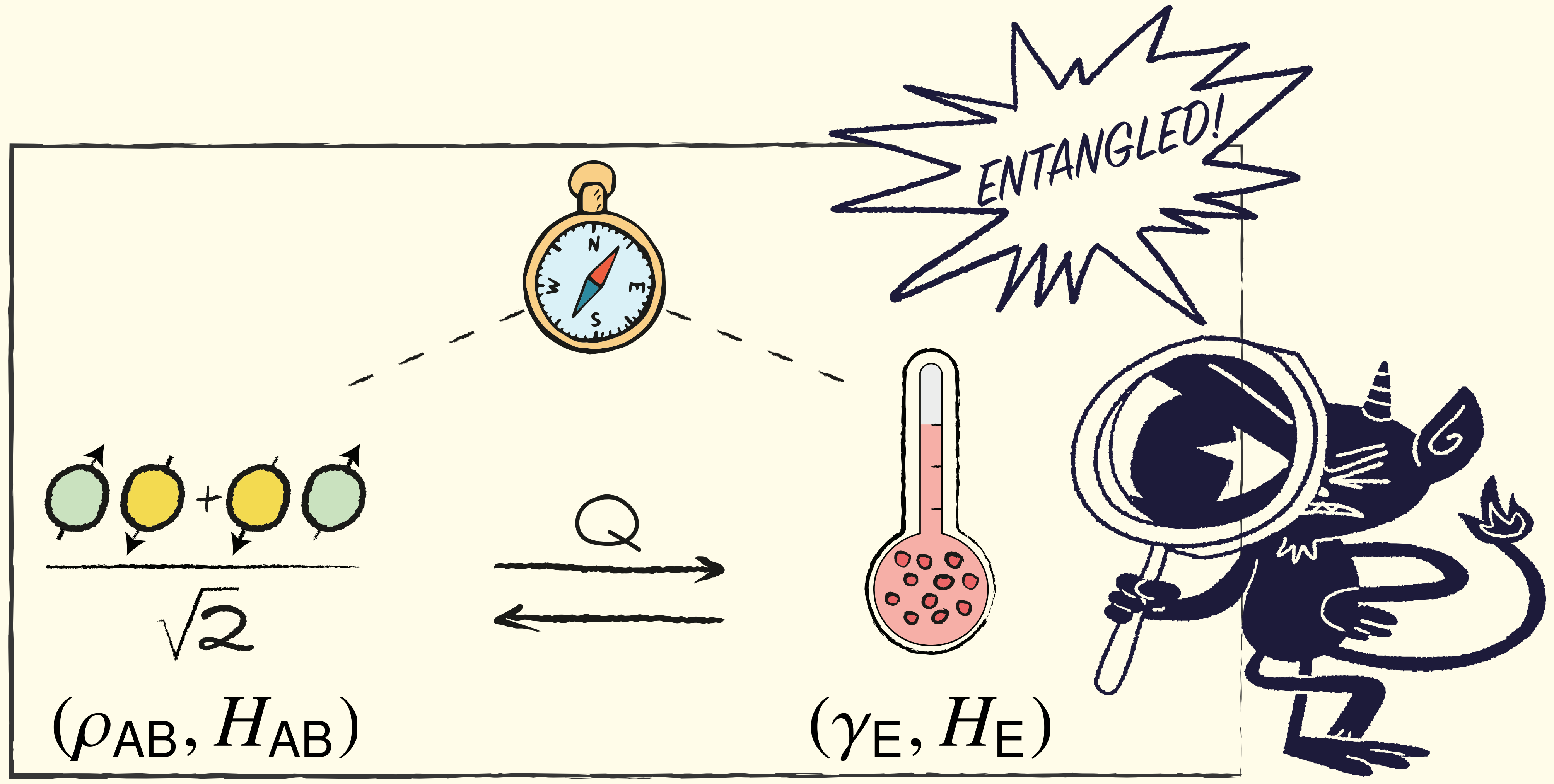

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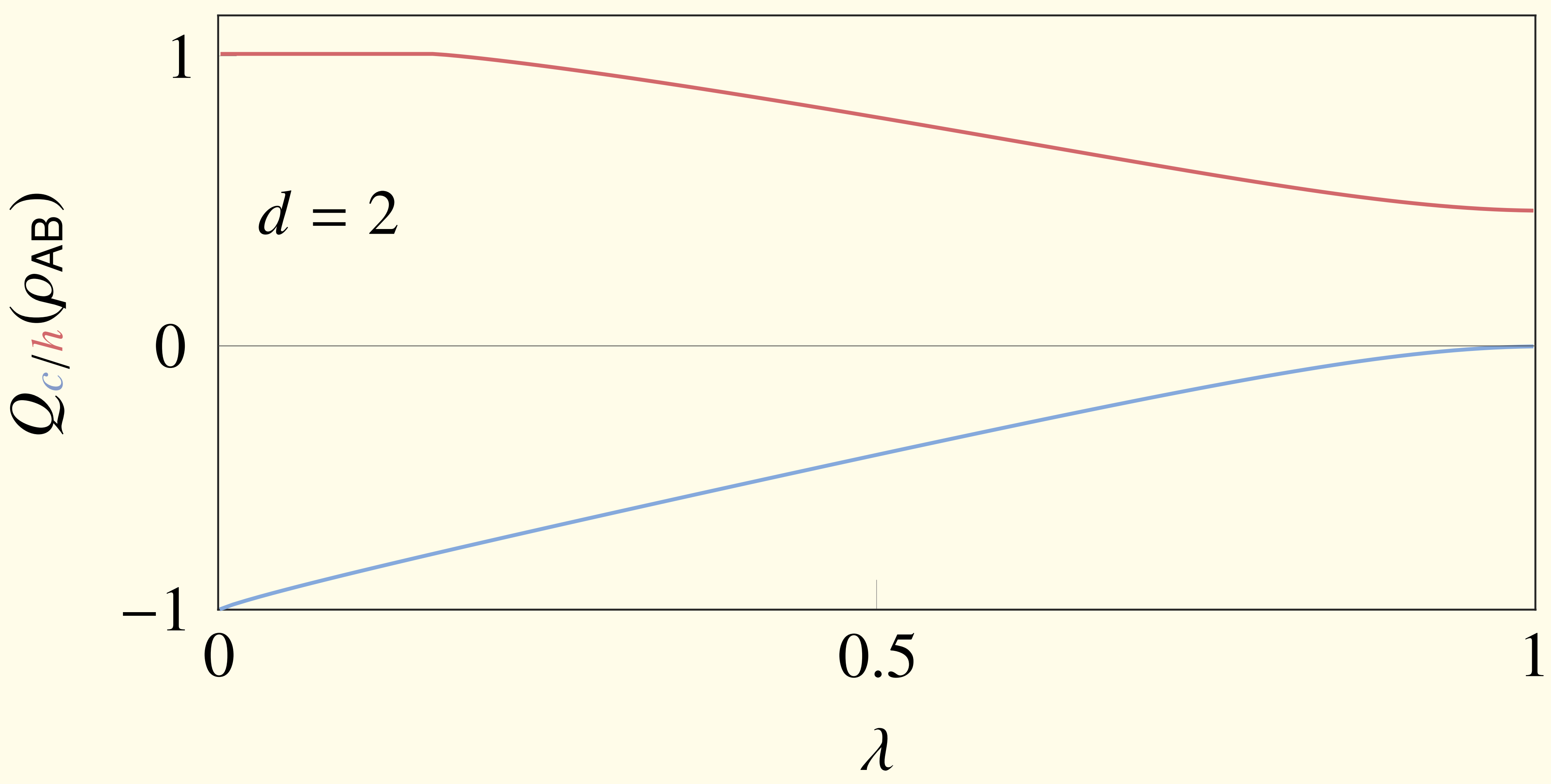

A. de Oliveira Junior

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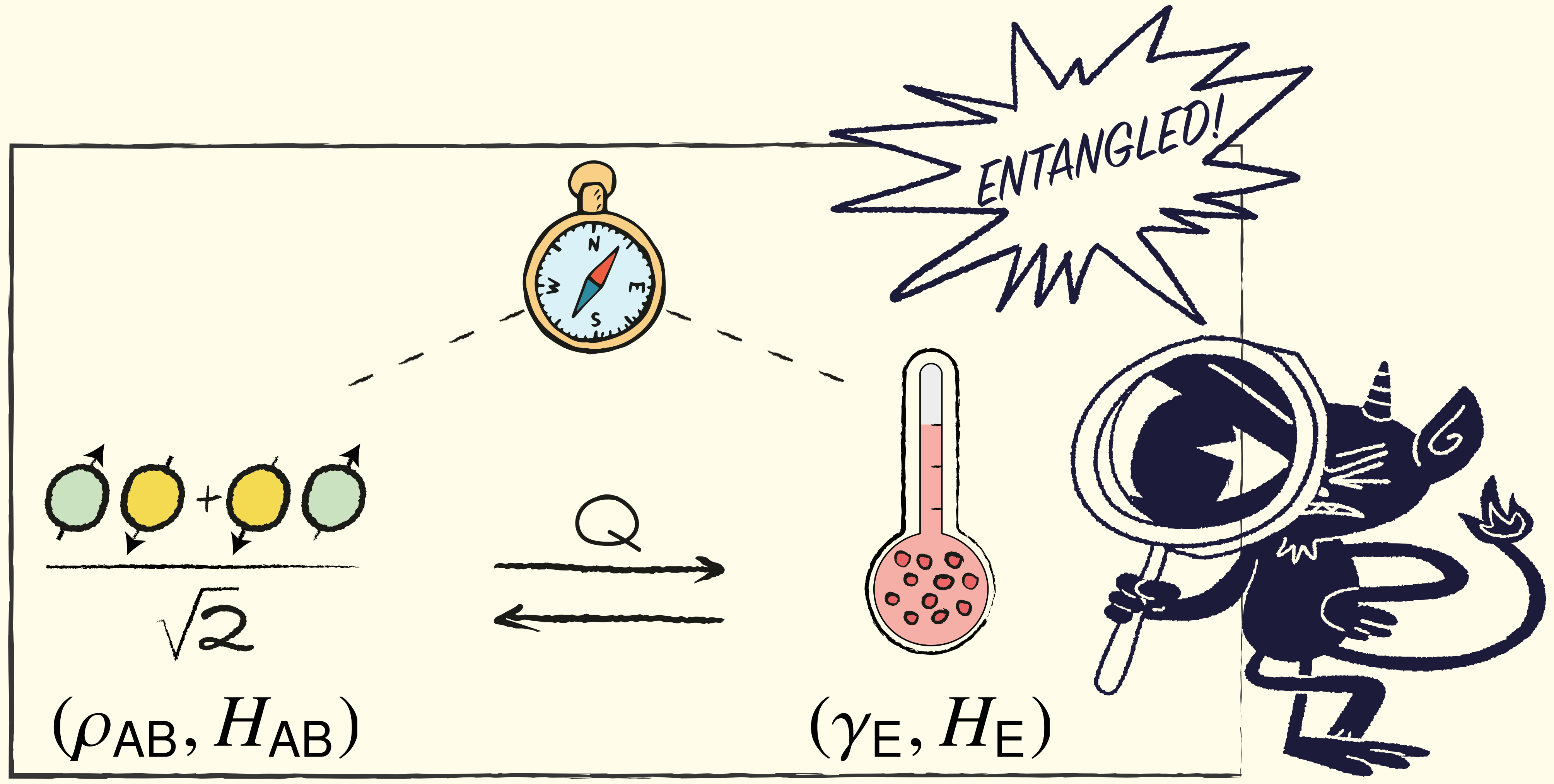

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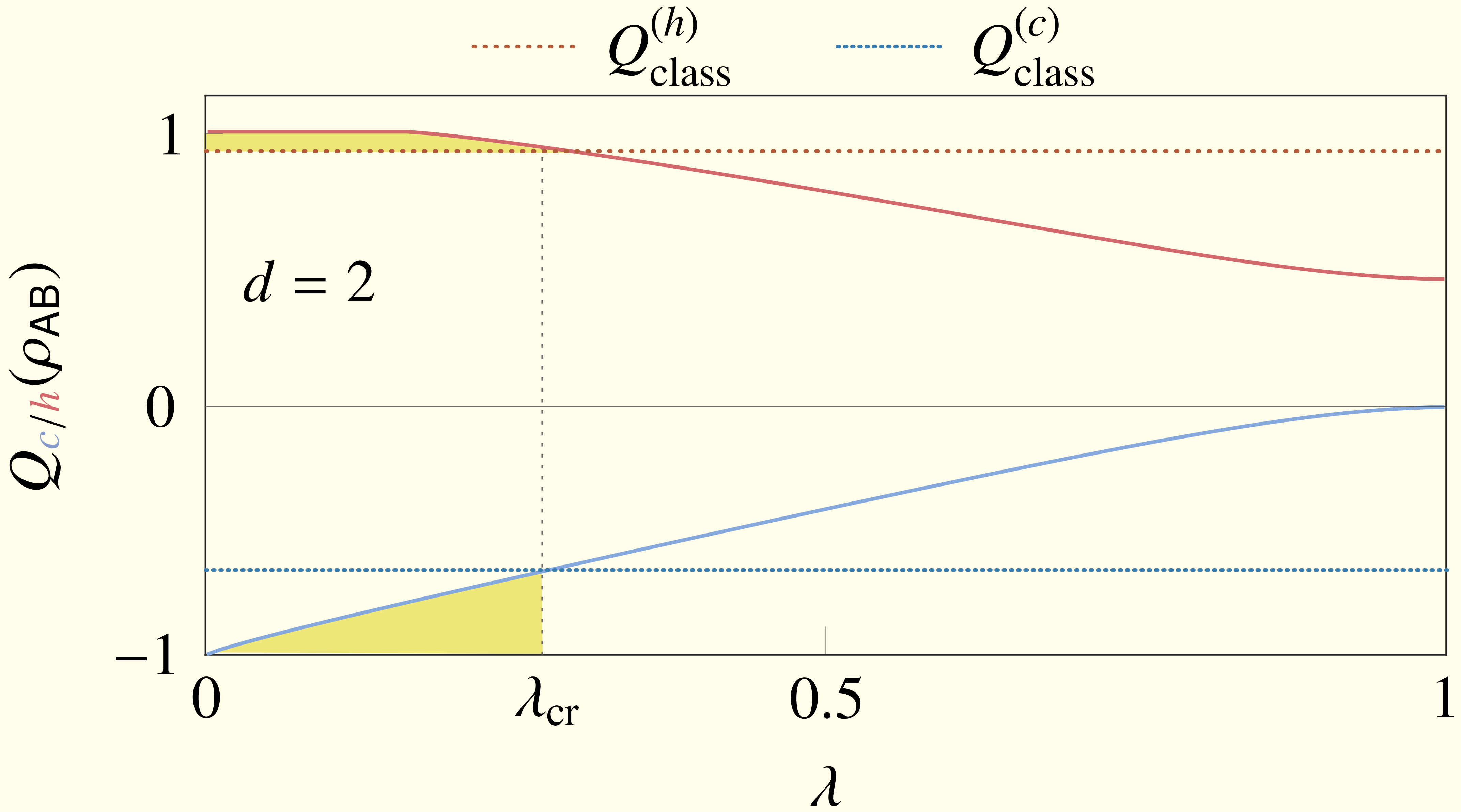

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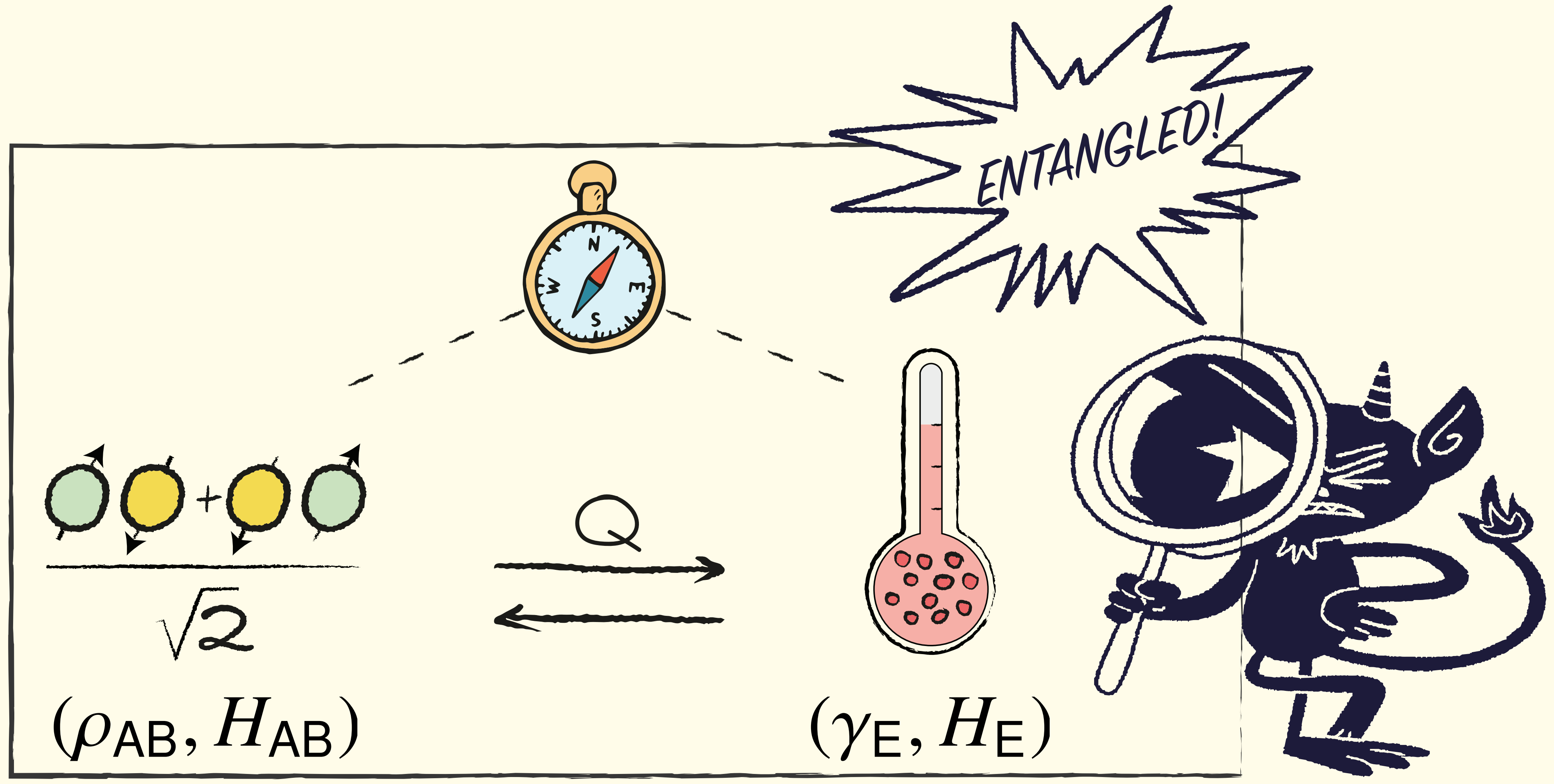

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What's the limit of our detection?

&

How about for higher d?

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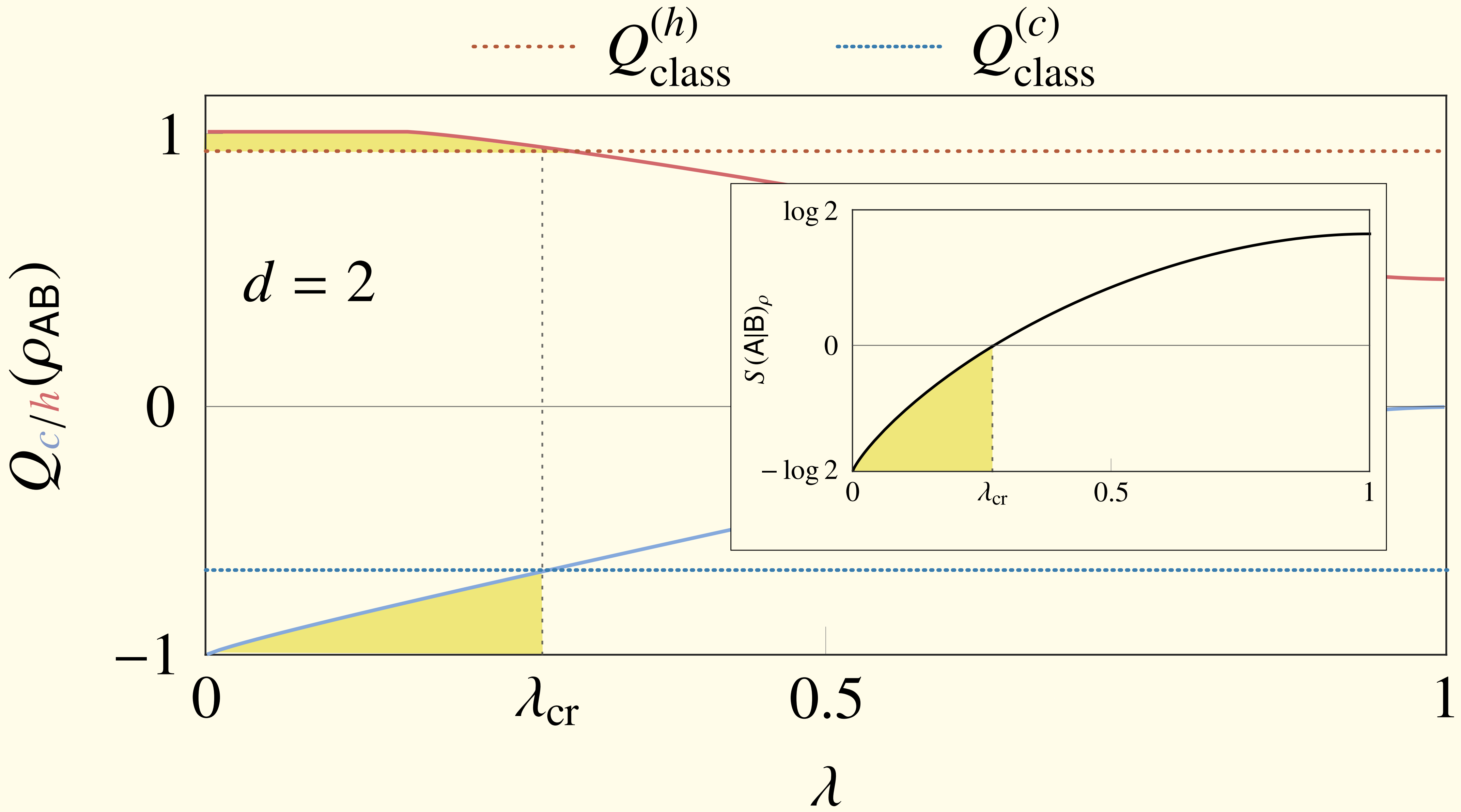

Entanglement witness

- **For arbitrarily dimensions our witness can detect up to**

$$\boxed{\begin{aligned}&\left(\lambda_{\text{crt}} - d^2\lambda_{\text{crt}}\right)\log\left(\frac{\lambda_{\text{crt}}}{d^2}\right) + \left[d^2(\lambda_{\text{crt}} - 1) - \lambda_{\text{crt}}\right]\times \\ &\qquad \times \log\left[\left(\frac{1}{d^2} - 1\right)\lambda_{\text{crt}} + 1\right] = d^2 \log d\end{aligned}}$$

only looks ugly
(but can be easily solved)

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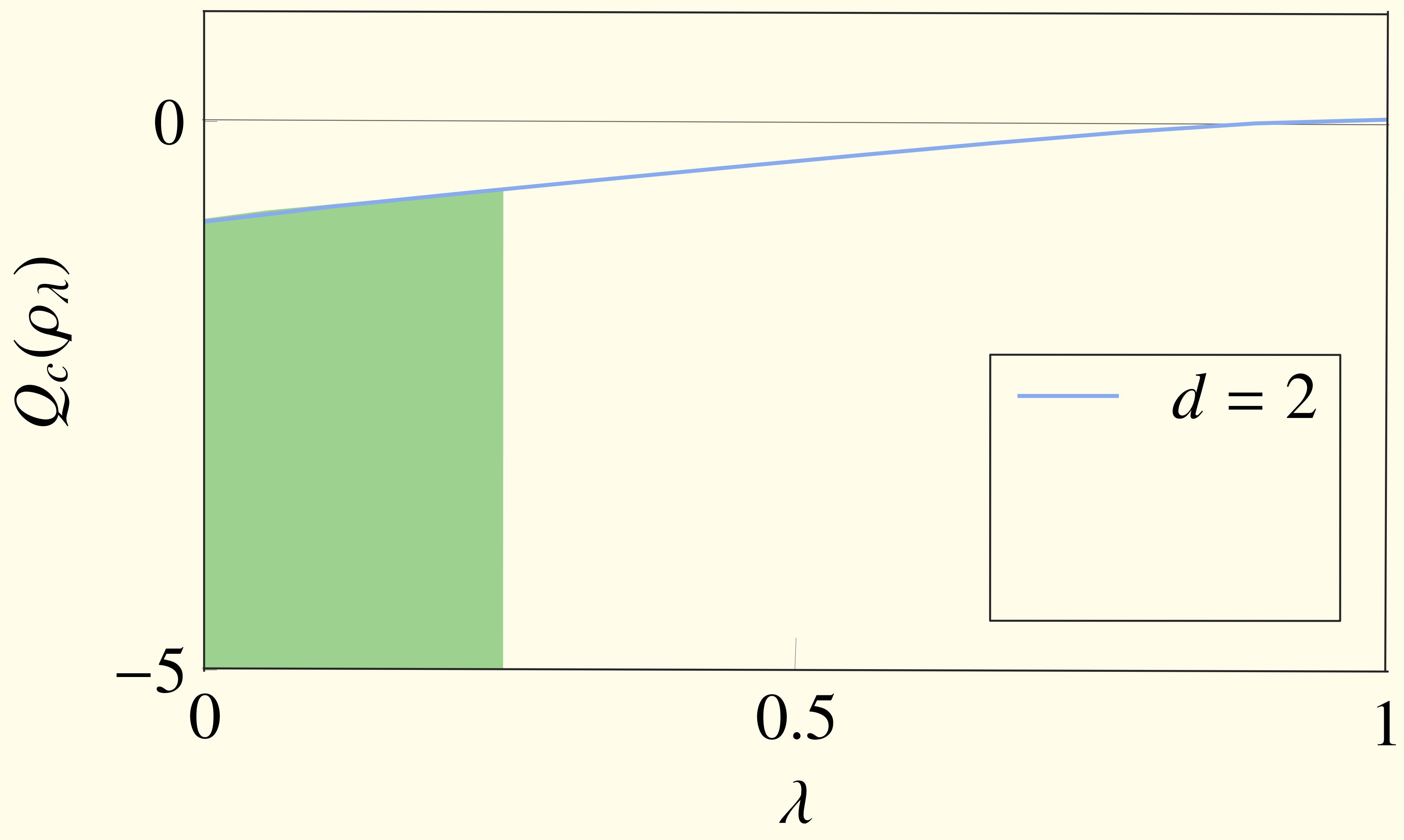

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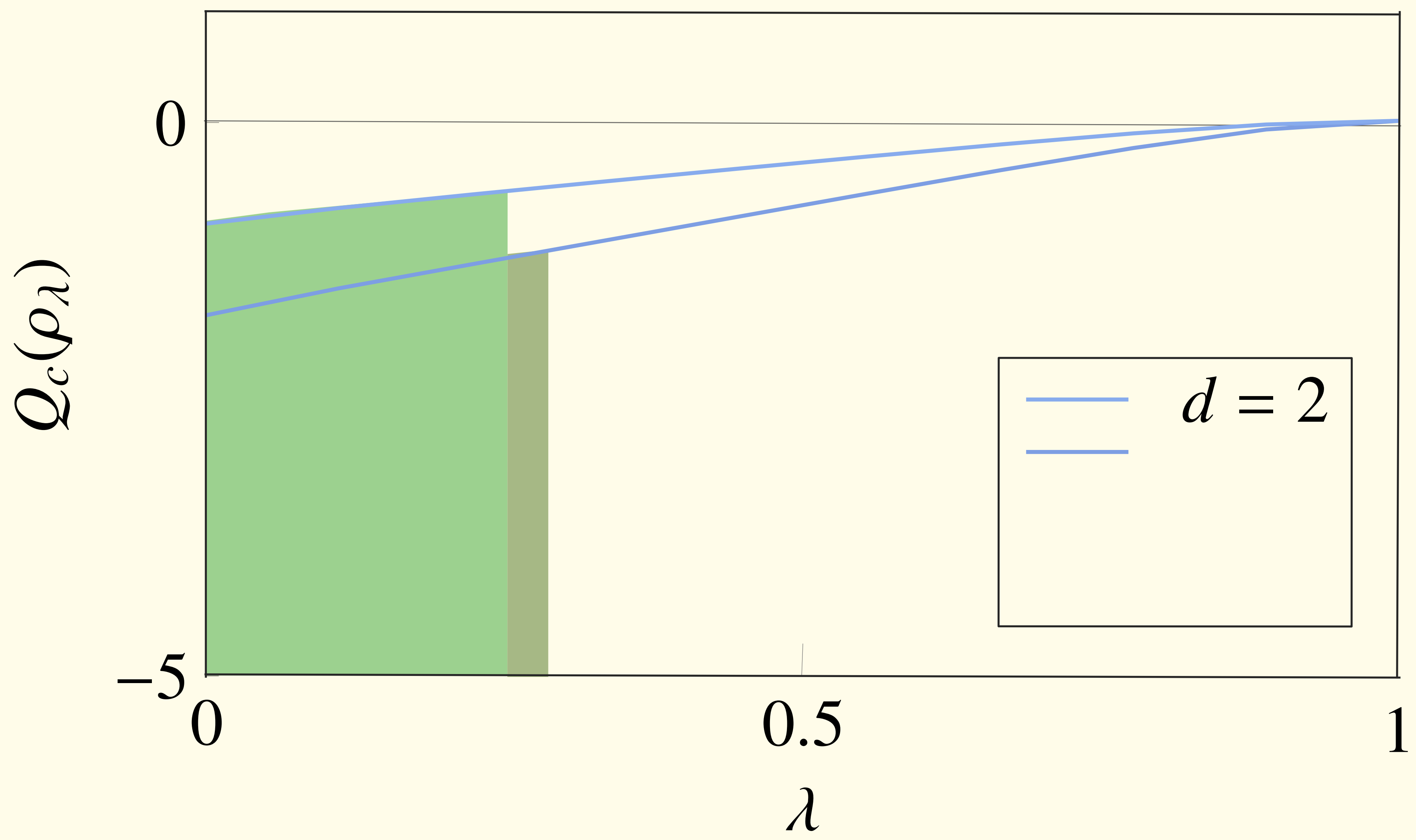

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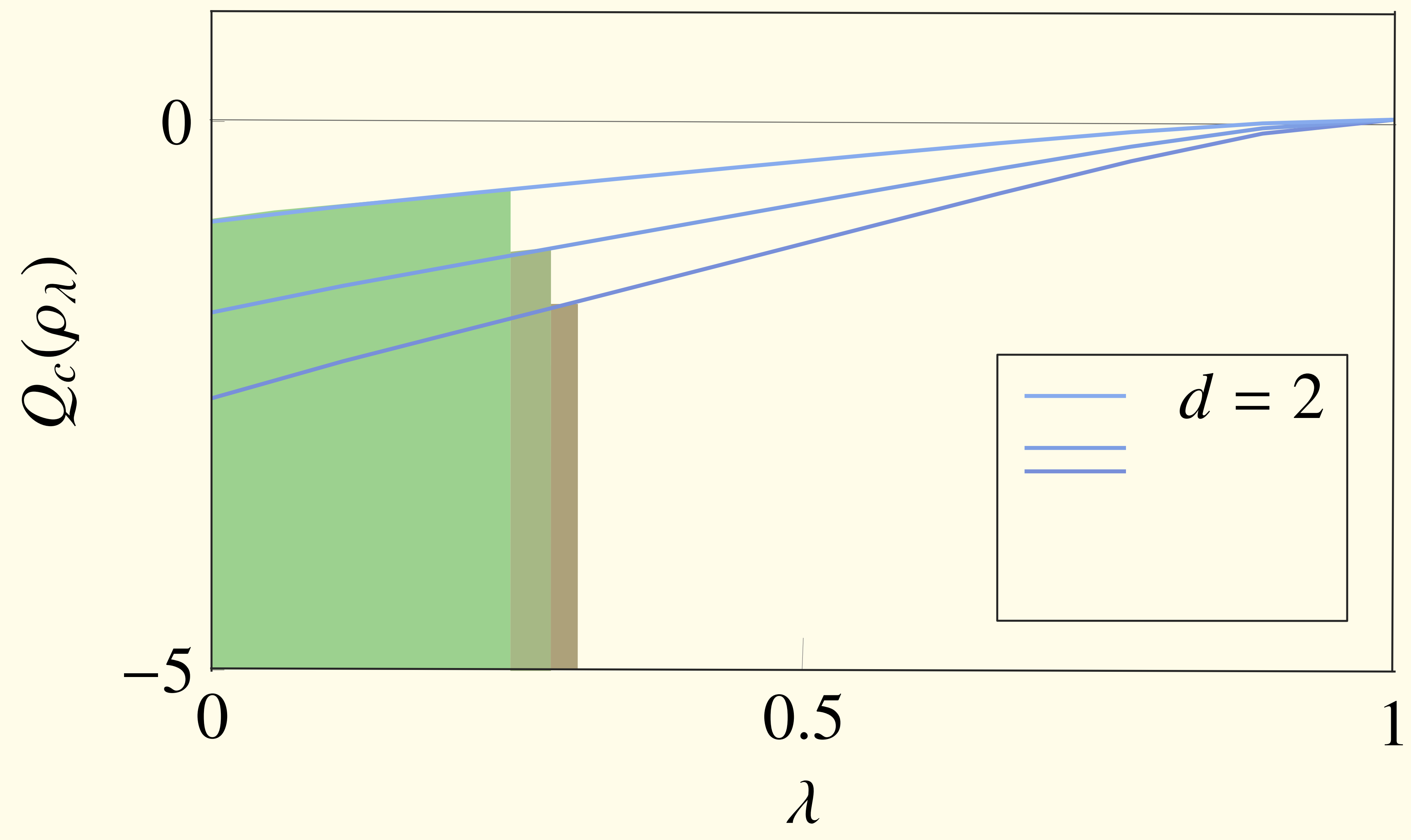

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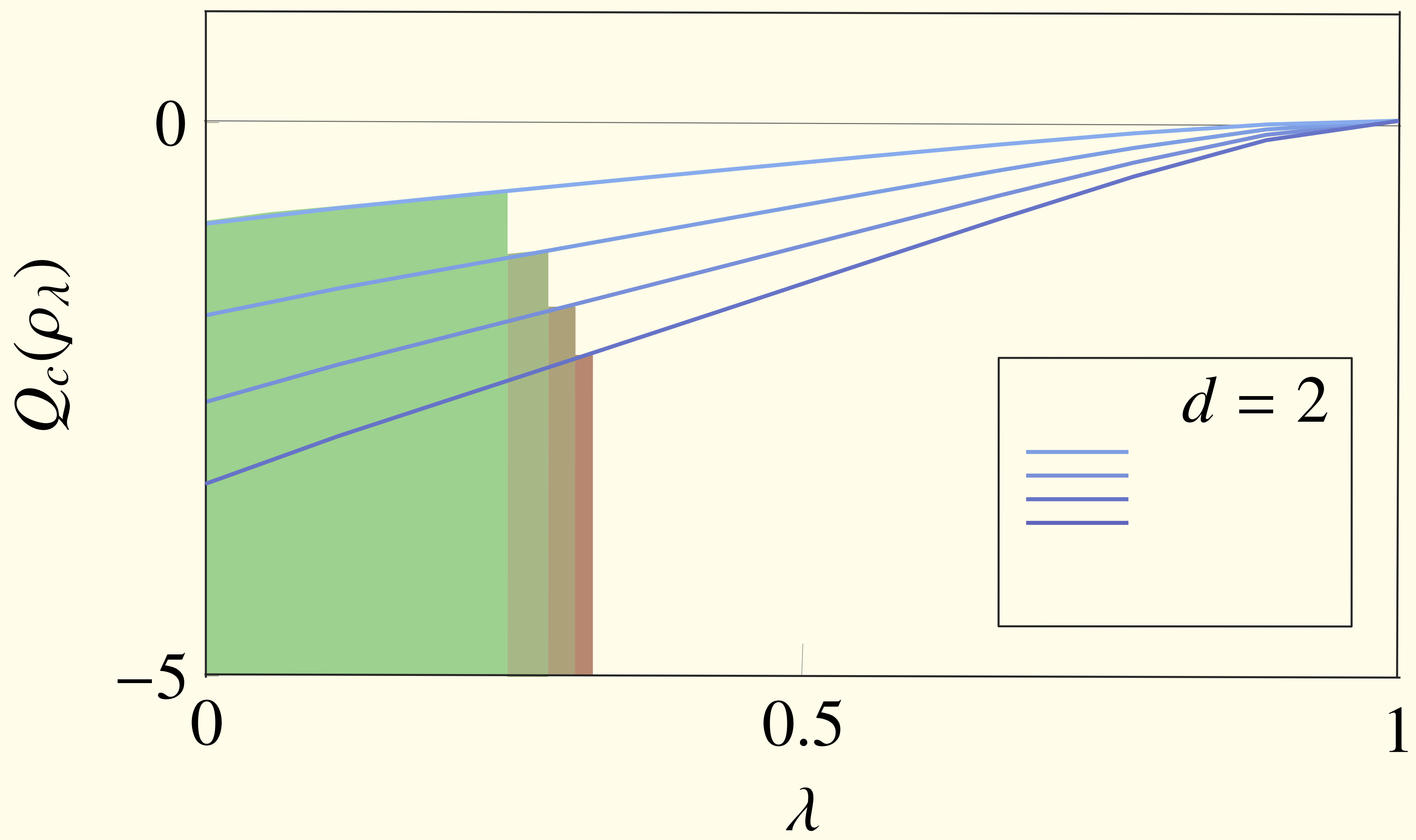

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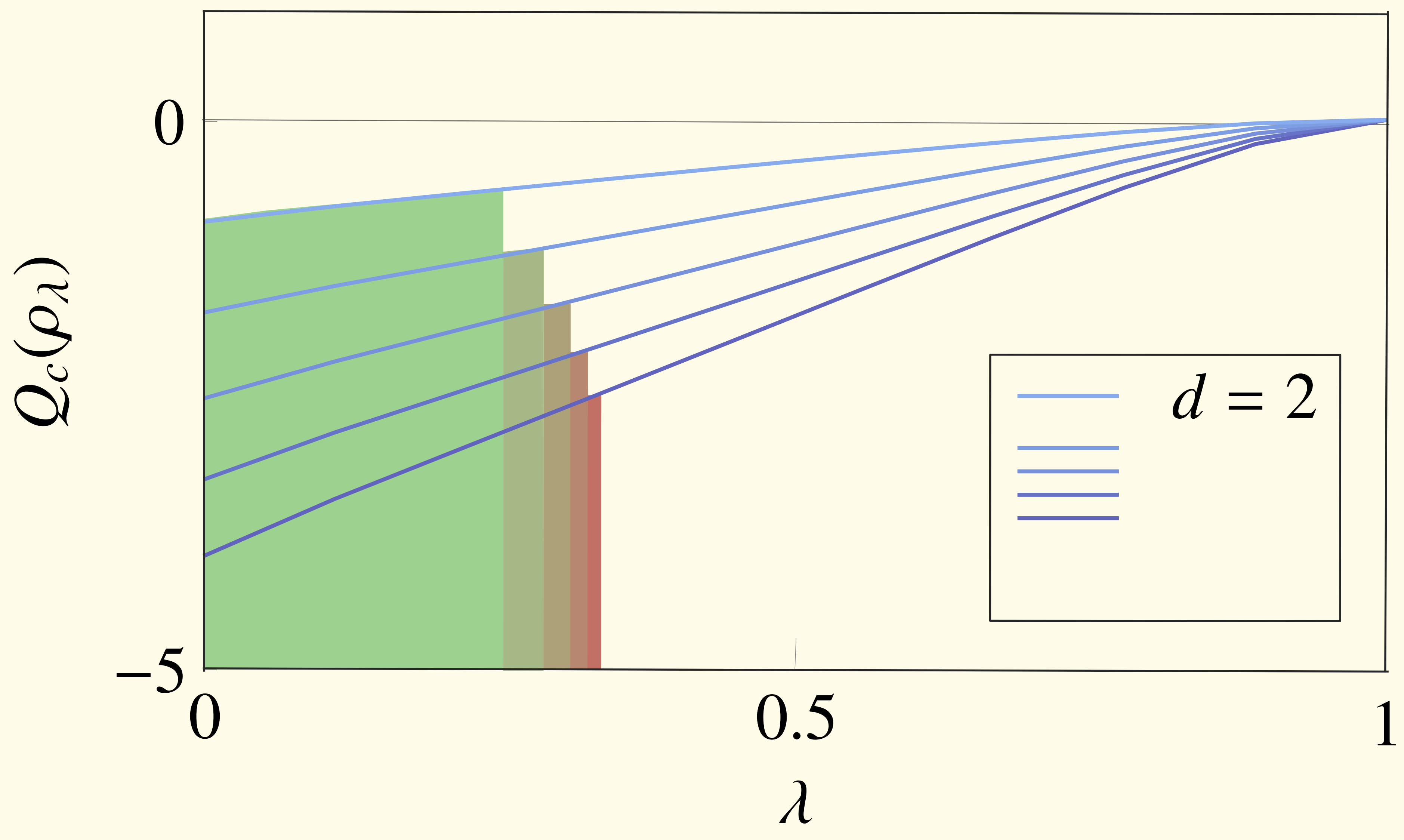

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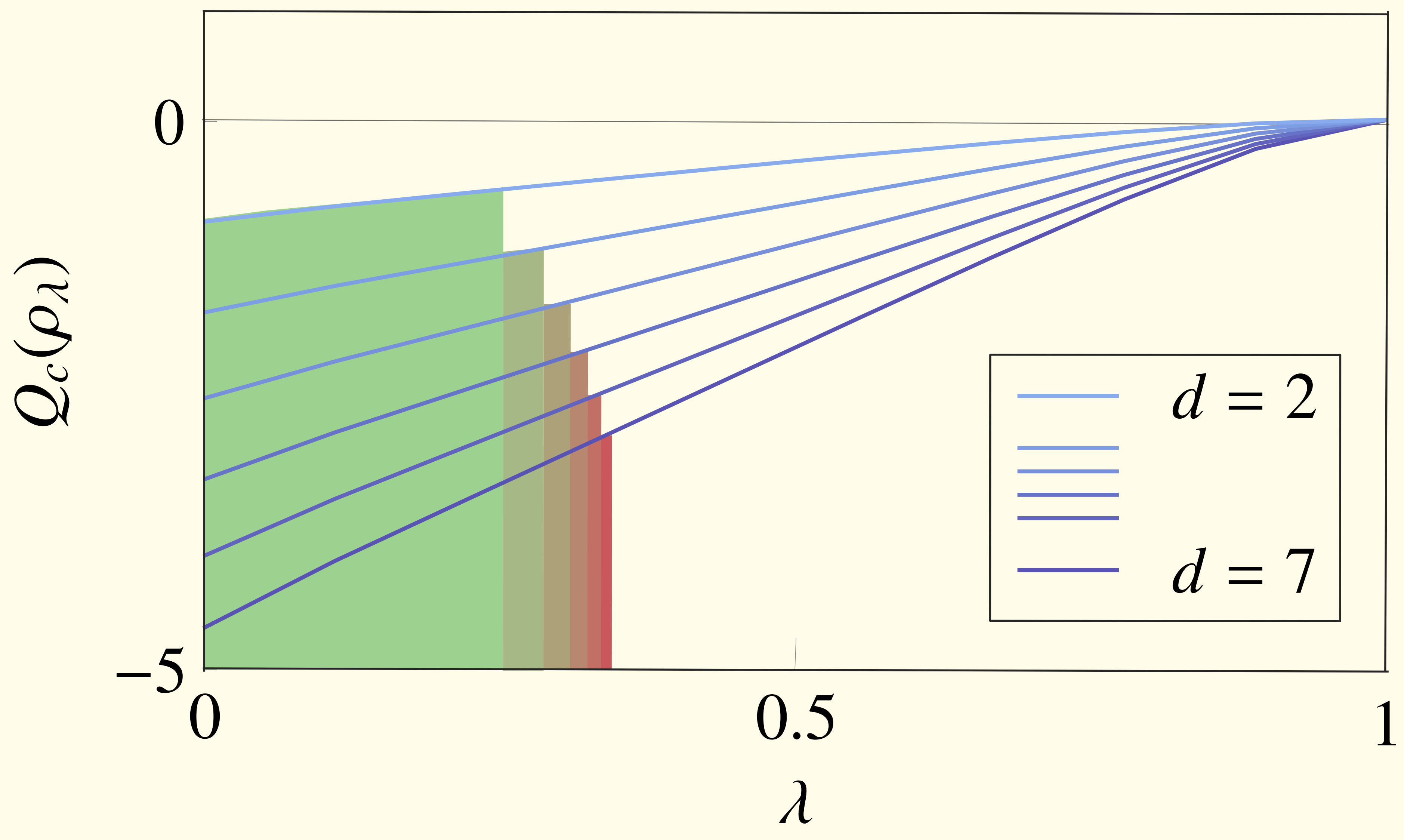

Outlook

What I talked about

- **Optimal heat** exchange in a **thermodynamic process**
- **Thermodynamic process** as a new type of **entanglement witness**

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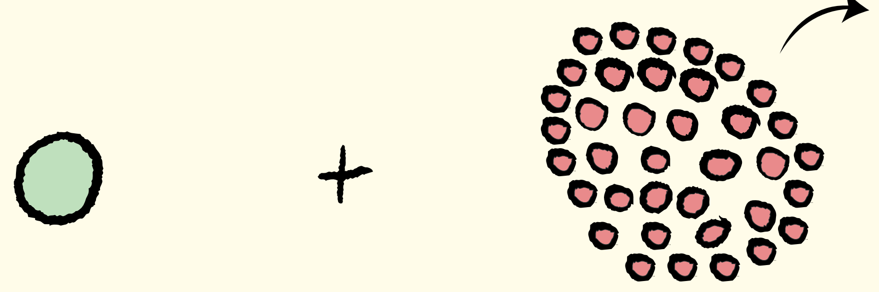
Construct your own witness:

https://github.com/AdeOliveiraJunior/heatW

Framework



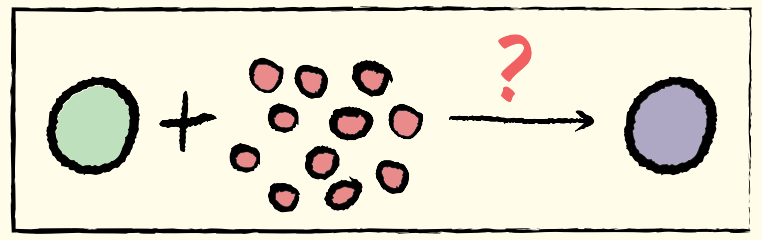
(ρ, H) + $\beta = 1/k_B T$

$\gamma_E = e^{-\beta H_\mathsf{E}}/Z_\mathsf{E}$

$\xrightarrow[\text{operations}]{\text{thermal}}$ $\mathcal{E}(\rho) = \mathrm{tr}_\mathsf{E}[U(\rho \otimes \gamma_\mathsf{E})U^\dagger]$

$[U, H + H_\mathsf{E}] = 0$

Fundamental question: Given ρ and σ does there exist a thermal operation $\mathcal{E}$: $\mathcal{E}(\rho) = \sigma$?

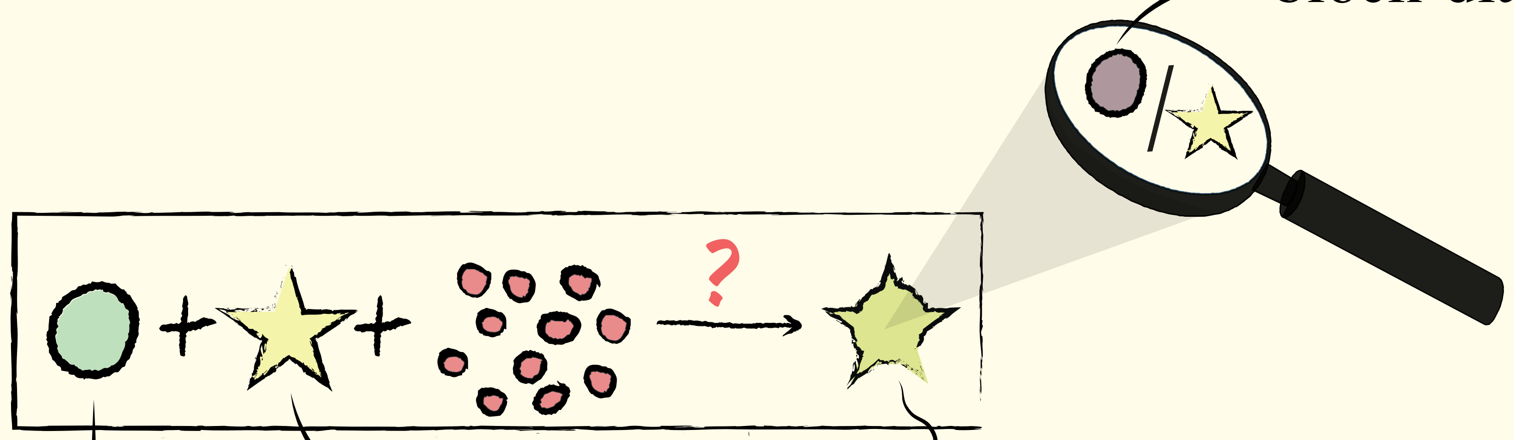


— $[\rho, H] = 0$ & $[\sigma, H] = 0 \rightarrow$ $p \succ_\beta q$ $\longrightarrow$ $F_\alpha(\rho) \geq F_\alpha(\sigma)$ (family of second laws)

$(\alpha = 1)$

$$F_\beta(\rho) := \left[\mathrm{tr}(H\rho) - \frac{1}{\beta} S(\rho)\right] - \frac{1}{\beta}\log Z$$

Nonequilibrium free-energy



$\longrightarrow$ $F_\beta(\rho) \geq F_\beta(\sigma)$

$\| \cdot - \cdot \|_1 \leq \epsilon$ $\rightarrow \mathcal{O}(e^{-an})$

D. Janzing, P. Wocjan, R. Zeier, R. Geiss, and T. Beth, *Int. J. Theor. Phys.* **39**, 2717 (2000)

M. Horodecki and J. Oppenheim, *Nat. Commun.* **4**, 2059 (2013)

F. G. S. L. Brandao, M. Horodecki, N. H. Y. Ng, J . Oppenheim & S. Wehner, *Proc. Natl. Acad. Sci. U.S.A.* **112**, 3275 (2015)