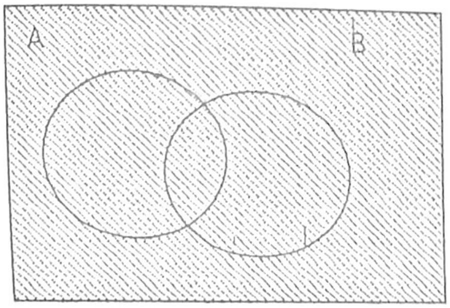
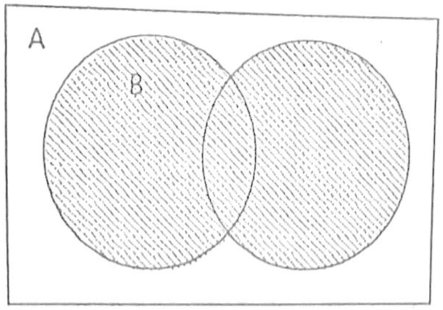
VENN DIAGRAMS FOR VARIOUS SET NOTATIONS

2-LOOPS

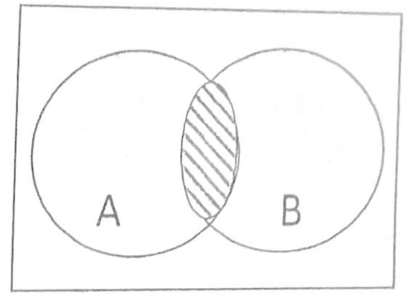
μ



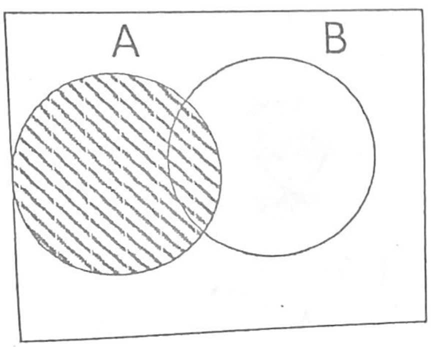
$A \cup B$



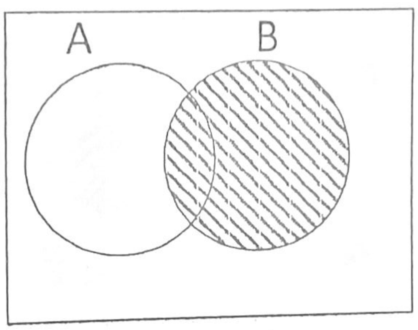
$A \cap B$



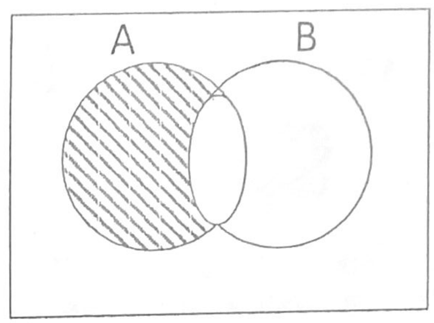
A



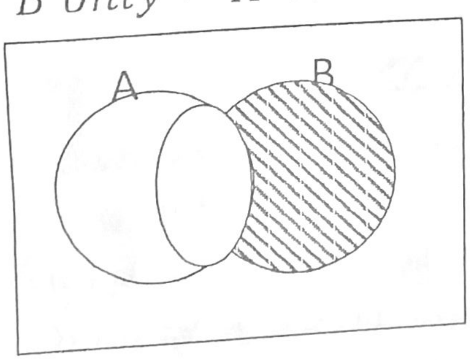
B



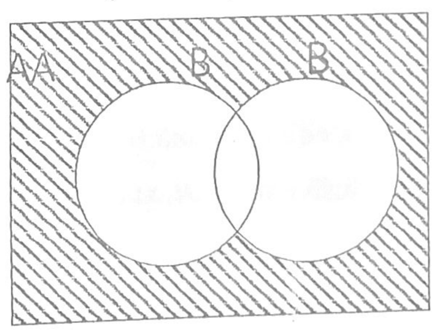
$A\ only = A \cap B' = A - B$



$B\ only = A' \cap B = B - A$

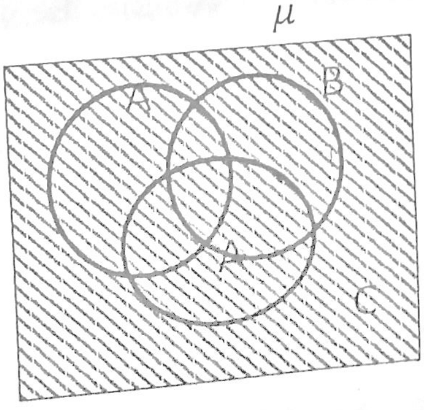


$(A \cup B)'$

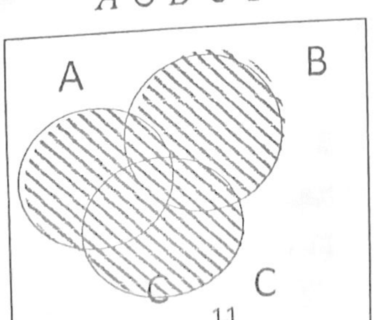


3 – LOOPS

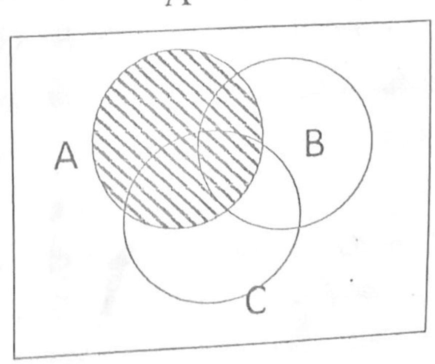
μ



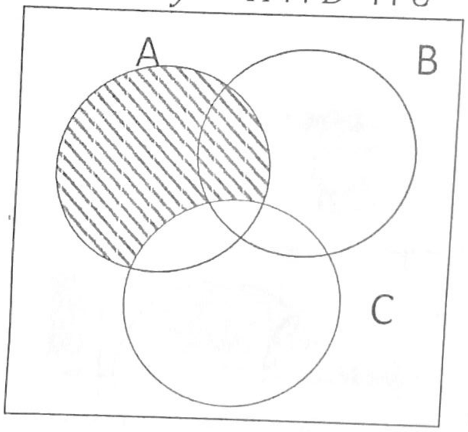
$A \cup B \cup C$



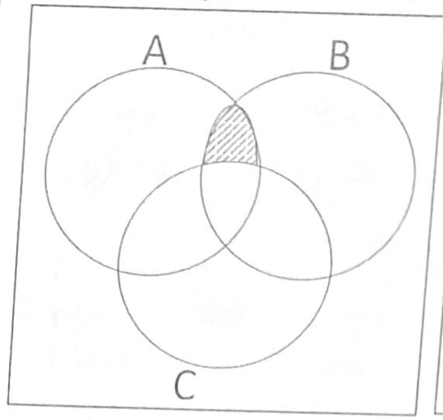
A



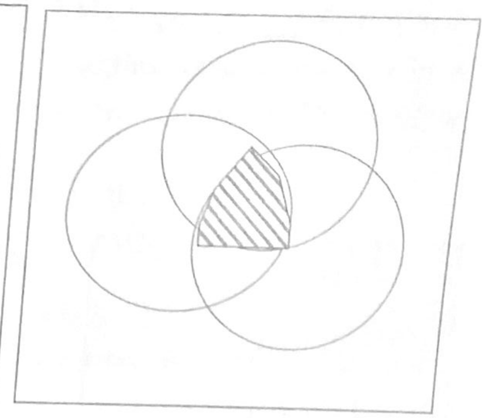
$A\ only = A \cap B' \cap C'$



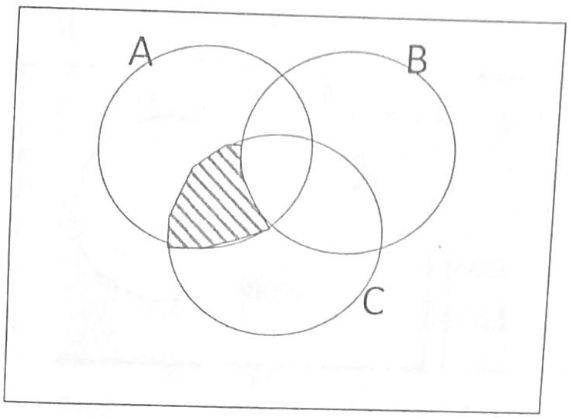
$A\ \&\ B\ only = A \cap B \cap C'$



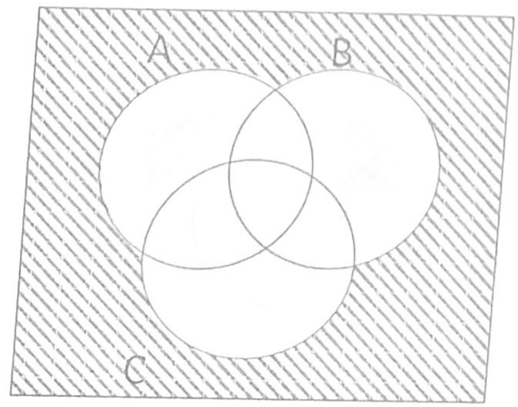
$A \cap B \cap C$



N $A \cap B' \cap C = A\&C\ only$



$A' \cap B' \cap C'$



MULTIPLE CHOICE QUESTIONS

1. Suppose D is divided into A and B (i.e A $\subset$ D and B $\subset$ D) for A $\neq$B , then we say A and B are mutually exclusive if (A) A $\cup$ B = ϕ (B) A $\cap$ B = A' (C) A $\cup$ B = D (D) $A \cap B = \phi$
2. If S and T are the only partitions (subsets) of A, we then say that the partitions S and T are (A) collectively exhaustive with respect to A (B) selectively exhaustive with respect to A (C) collectively exhaustive with respect to T (D) selectively exhaustive with respect to S
3. E = {natural numbers}; $A = \{2, 4, 6, 8, 10\}$; $B = \{1, 3, 6, 7, 8\}$. Which of the following is false: (A) $2 \in A$ (B) $11 \in B$ (C) $4 \notin B$ (D) $A \in$ U
4. $(X \cap Y)' = X' \cup Y'$ represents (A) Idempotent law (B) Domination law (C) De Morgan's law (D) Compliment law
5. Let P={x,2,3,y, 4,5}. The Cardinal number of P is (A) 2 (B) 3 (C) 4 (D) 6
6. Given that W={a, b, c, d}. The Cardinality of W is (A) 5 (B) 16 (C) 8 (D) 7

7. The power set of $X=\{x,z\}$ is (A). $\{\{x\},\{z\},\{x,z\}\}$ (B) $\{X,\phi\}$ (C) $\{\{x\},\{z\},X,\phi\}$ (D)$\{\{x\},\{z\},X,\{x,z\}\}$

8. $X \cap Y =$ (A) $\{ x\in \mu : x\in A$ and $x\in B \}$ (B) $\{ x\in \mu : x\in A$ or $x\in B \}$ (C) $\{ x\in \mu : x\notin A$ but $x\in B \}$ (D) $\{ x\in \mu : x\in A$ and $x\notin B \}$

9. If V and W are sets and $V \cup W = V \cap W$, then (A) $V = \phi$ (B) $W = \phi$ (C) $V = W$ (D) NOTA

10. If A and B are two sets, $A \cap (B \cup A)^c$ equals (A) A (B) ϕ (C) B (D) Universal set

11. Given $A = \{1, 3, 6, 8, 9, 12, 15\}$ and $B = \{6, 9, 12\}$, which is TRUE? (A) B is the complement of A (B) $A \cap B = \phi$ (C.) A and B are disjoint sets (D) $B \subset A$

12. The order of a Power set of a set of order n is (A). 2n (B). n (C) 2^n (D) n^2

13. All the followings are void sets except (A) { } (B) ϕ (C) $B \cap B'$ (D) $B \cup B'$

14. The number of all possible subsets of a set of order 5 is A. 6 B. 64 C. 32 D. 10

15. A – B equals (A) $A \cap B'$ (B) $A' \cap B'$ (C) $A' \cap B$ (D) $A \cup B'$

16. Which of the following statements is FALSE? (A) $2 \in A \cup B$ implies that if $2 \in/ A$ then $2 \in B$. (B) $\{2, 3\} \subseteq A$ implies that $2 \in A$ and $3 \in A$. (C) $A \cap B \supseteq \{2, 3\}$ implies that $\{2, 3\} \subseteq A$ and $\{2, 3\} \subseteq B$. (D) $A - B \supseteq \{3\}$ and $\{2\} \subseteq B$ implies that $\{2, 3\} \subseteq A \cup B$. (e) $\{2\} \in A$ and $\{3\} \in A$ implies that $\{2, 3\} \subseteq A$. 2. Let $A = \{0, 1\} \times \{0, 1\}$ and $B = \{a, b, c\}$.

17. Suppose A is listed in lexicographic order based on $0 < 1$ and B is in alphabetic order. If $A \times B \times A$ is listed in lexicographic order, then the next element after ((1, 0), c,(1, 1)) is (A) ((1, 0), a,(0, 0)) (B) ((1, 1), c,(0, 0)) (C) ((1, 1), a,(0, 0)) (D) ((1, 1), a,(1, 1)) (e) ((1, 1), b,(1, 1))

18.. Which of the following statements is TRUE? (A) For all sets A, B, and C, $A - (B - C) = (A - B) - C$. (B) For all sets A, B, and C, $(A - B) \cap (C - B) = (A \cap C) - B$. (C) For all sets A, B, and C, $(A - B) \cap (C - B) = A - (B \cup C)$. (D) For all sets A, B, and C, if $A \cap C = B \cap C$ then $A = B$. (E) For all sets A, B, and C, if $A \cup C = B \cup C$ then $A = B$.

19.. Which of the following statements is FALSE? (A) $C - (B \cup A) = (C - B) - A$ (B) $A - (C \cup B) = (A - B) - C$ (C) $B - (A \cup C) = (B - C) - A$ (D) $A - (B \cup C) = (B - C) - A$ (E) $A - (B \cup C) = (A - C) - B$

20. Consider the true theorem, "For all sets A and B, if $A \subseteq B$ then $A \cap B^c = \emptyset$." Which of the following statements is **NOT** equivalent to this statement: (A) For all sets A^c and B, if $A \subseteq B$ then $A^c \cap B^c = \emptyset$. (B) For all sets A and B, if $A^c \subseteq B$ then $A^c \cap B^c = \emptyset$. (C) For all sets A^c and B^c, if $A \subseteq B^c$ then $A \cap B = \emptyset$. (D) For all sets Ac and B^c, if $A^c \subseteq B^c$ then $A^c \cap B = \emptyset$. (E) For all sets A and B, if $A^c \supseteq B$ then $A \cap B = \emptyset$.

21. The power set P((A × B) ∪ (B × A)) has the same number of elements as the power set P((A × B) ∪ (A × B)) if and only if (A) A = B (B) A = ∅ or B = ∅ (C) B = ∅ or A = B (D) A = ∅ or B = ∅ or A = B (E) A = ∅ or B = ∅ or A ∩ B = ∅

Questions 16 – 21 is a puzzle !

SET-BUILDER NOTATIONS AND APPLICATIONS TO SET OF REAL NUMBERS

For better illustration, we shall introduce you to the set of real numbers and relevant examples. The basic classes of the sets on the real line are sets of (1) Natural numbers (2) Integers (3) rational numbers (4) Irrational numbers and (5) real numbers.

1. The set of **Natural numbers** is a set containing counting numbers 0,1, 2, 3,…… with the symbol ℕ described as ℕ ={ 0,1,2,3,………}. It is an infinite set.

The set ℕ can only allow for **addition** (+) and **multiplication** (×) since if any two elements in the set are added or multiplied also gives an element in the set. We then say "the set of Natural numbers is closed under the additive and multiplicative operations". The following set-builder notations can be used to describe various subsets of ℕ.

1. $A = \{x \in \mathbb{N} : 3 < x < 8\} = \{4, 5, 6, 7\}$
2. B= $\{x \in \mathbb{N} : x \leq 12 \text{ } and \text{ } x \text{ } is \text{ } a \text{ } multiple \text{ } of \text{ } 4\} = \{4, 8, 12\}$
3. C = $\{x \in \mathbb{N} : x \text{ } is \text{ } a \text{ } divisor \text{ } of \text{ } 8\} = \{4, 8\}$
4. D = $\{x \in \mathbb{N} : x \text{ } is \text{ } odd \text{ } and \text{ } x < -12\} = \phi$

2. The set of **Integers** is a set containing the positive and negative counting numbers 0, ±1, ±2, ±3,…… with the symbol ℤ described as ℤ ={ 0, ±1, ±2, ±3,……….} or ℤ={ …….,-3,-2,-1,0,1,2,3………}. It is an infinite set.

The set ℤ can only allow for **addition** (+), **multiplication** (×) **and subtraction** (-) since if any two elements in the set are added, multiplied or subtracted also gives an element in the set. We then say

3. The set of **rational numbers** (also called **Quotients**) is the set of the numbers expressed as a quotient of two integers. The second integer (the denominator) cannot be zero (0) with the symbol ℚ described as $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$. It is an infinite set.

"the set of Integers is closed under the additive, multiplicative and subtractive operations". The following set-builder notations can be used to describe various subsets of $\mathbb{Z}$

5. $A = \{x \in \mathbb{Z} : -3 < x < 8\}$ = {-2,-1,0,1,2,3,4,5,6,7}
6. $A = \{x \in \mathbb{Z} : x \text{ is less than } 10 \text{ and } x \text{ is prime}\}$ = {1,3,5,7}

The set $\mathbb{Q}$ allows for **addition, multiplication, subtraction and division** since if any two elements in the set are added , multiplied ,subtracted or divided also gives an element in the set. We then say "the set of Rational numbers is closed under the additive, multiplicative, subtractive and division operations". The set of rational numbers when converted into a decimal form either terminates (such as $\frac{1}{5} = 0.2$) or gives a non-terminating re-occurring (periodic) decimals (such as $\frac{1}{3} = 0.3333333 \ldots\ldots$).

4. The set of **irrational numbers** $\mathbb{Q}^*$ is the set of the numbers expressed as a quotient of two integers expressed as decimals that are non-terminating and non-recurring (non-periodic) and represented in the form $\mathbb{Q}^* = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \text{ and decimal value non} - \text{periodic}\}$. It is

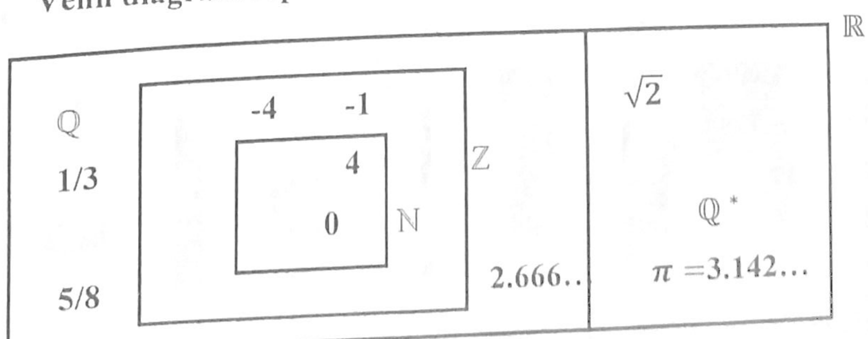
The set of Irrational numbers is also closed under the additive, multiplicative, subtractive and division operations". The set of irrational numbers when converted into a decimal form gives a non-terminating non-recurring (non-periodic) decimals (such as $\frac{22}{7} = 3.142857.. or \sqrt{2} = 0.414213 \ldots\ldots..$).

6. The set of **Real numbers** $\mathbb{R}$ is the set of numbers that are rational or irrational. $\mathbb{R} = \{x : x \in \mathbb{Q} \; or \; x \in \mathbb{Q}^*\}$. It is an infinite set.

Summary:

1. Any number that belongs to $\mathbb{N}$ also belongs to $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$ (i.e $x \in \mathbb{N} \Rightarrow x \in \mathbb{Z}, \mathbb{Q} \; and \; \mathbb{R}$)
2. Any number that belongs to $\mathbb{Z}$ also belongs $\mathbb{Q}$ and $\mathbb{R}$ (i.e $x \in \mathbb{Z} \Rightarrow x \in \mathbb{Q} \; and \; \mathbb{R}$)
3. Any number that belongs to $\mathbb{Q}$ also belongs $\mathbb{R}$ (i.e. $x \in \mathbb{Q} \Rightarrow x \in \mathbb{R}$)
4. Any number that belongs to $\mathbb{Q}^*$also belongs $\mathbb{R}$ (i.e. $x \in \mathbb{Q}^* \Rightarrow x \in \mathbb{R}$)

Venn diagram representation of the sets of Real numbers



NOTE : $[(\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}) \cup \mathbb{Q}^*] = \mathbb{R}$

Classwork:- tick (√) the boxes appropriately.

$\in$	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}^*$	$\mathbb{R}$
-5/2					
π					
$\sqrt{3}$					
-7					
12					
3.142					
$\sqrt{25}$					
0					

(Students practice Exercises!)

1. In a club of 27 readers 12 read Guardian, 16 read Punch and 20 read Vanguard. 6 read the Guardian and the Punch, 9 read the Guardian and Vanguard, and 12 read the vanguard and the Punch. If 1 read neither of the newspaper. Find the number of reader that read
   (i) All the three newspaper (ii) only two newspapers (iii).The Guardian and the punch only
   (iv). The Guardian and the Vanguard only (v). The punch and the Vanguard
   (vi). The Guardian only (vii). The Punch only (viii). The vanguard only
2. In a music academy of 48 instruments 30 played a guitar 22 played the piano, if 9 played both the guitar and the piano, how many played neither of the instrument?
3. In a group of Student's 1,200 take and 200 take neither of the courses, how many student are in the group?.
4. In a Home Economics club of 160 members, 75 bake, 85 sow, and 110 weave. 40 bake and sow, 60 bake and weave and 55 sow and weave. If 35 do the three craft, how many do neither of the craft?
5. Twenty-four dogs are in a kennel. Twelve of the dogs are black, six of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Two of the dogs are black with short tails and do not have long hair. Two of the dogs have short tails and long hair but are not black. If all of the dogs in the kennel have at least one of the mentioned characteristics, how many dogs are black with long hair?