Plan

Cartesian definion of a general plan :

At least one value of a, b, c must be different from 0, equivalent to mean than the normal vector to surface $\vec{n}$ cannot be null.

$$ax + by + cz = d$$

A normal vector to the surface is $\vec{n}$:

$$\vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

To find the following parametric definition:

$$\vec{X}(u, v) = \vec{D} + u\vec{U} + v\vec{V}$$

We need to find $\vec{D}, \vec{U}$ and $\vec{V}$ as:

$$\begin{cases} \vec{U} \wedge \vec{V} = \vec{n} \\ \vec{X} \cdot \vec{n} = d \end{cases}$$

Find orthogonal vector to $\vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$:

- https://math.stackexchange.com/questions/137362/how-to-find-perpendicular-vector-to-another-vector

If $a = 0$:

$$\vec{n}^{\perp} = \begin{bmatrix} 0 \\ c \\ -b \end{bmatrix}$$

Else:

$$\vec{n}^{\perp} = \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix}$$

We define $\vec{U} = \vec{n}^{\perp}$ and $\vec{V}$ as $\vec{V} = \vec{n} \wedge \vec{U}$

We can normalize both vectors:

$$\overrightarrow{U_n} = \frac{\vec{U}}{\|\vec{U}\|}, \overrightarrow{V_n} = \frac{\vec{V}}{\|\vec{V}\|}$$

Finally the shift is

$$\vec{D} = d\frac{\vec{n}}{\|\vec{n}\|^2}$$