IS3301 COMPLEX ANALYSIS & MATHEMATICAL TRANSFORMATION

ASSIGNMENT 03

NAME	: THENNAKOON T.M.R.G.
REG. NO	: EG/2022/5369
SEMESTER	: 03
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Q1)

the complen emponential series of fourier series can be written as,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} \quad \text{——— (1)}$$

where, C_n - complen emponential fourier coefficient

$$C_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t) e^{-jn\omega t} \, dt \quad \text{——— (2)}$$

$$\omega = 2\pi / T$$

let, $\Delta f = \frac{1}{T}$ Δf - change in frequency

$$\therefore \omega = 2\pi \Delta f \quad \text{——— (3)}$$

(1), (3) $\Rightarrow$ $$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn \times 2\pi \Delta f t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n \Delta f t} \quad \text{——— (4)}$$

②, ③ ⇒

$$C_n = \Delta f \int_{t_0}^{t_0+T} x(t) e^{-j2\pi n \Delta f t} dt \quad \text{——— ⑤}$$

④, ⑤ ⇒

$$x(t) = \sum_{n=-\infty}^{\infty} \left\{ \left[\Delta f \int_{t_0}^{t_0+T} x(t) e^{-j2\pi n \Delta f t} dt \right] \cdot e^{j2\pi n \Delta f t} \right\}$$

let $t_0 = -T/2$

$$x(t) = \sum_{n=-\infty}^{\infty} \left\{ \left[\Delta f \int_{-T/2}^{T/2} x(t) e^{-j2\pi n \Delta f t} \cdot dt \right] \cdot e^{j2\pi n \Delta f t} \right\}$$

when $T \to \infty$

$$\Sigma \to \int, \quad \Delta f \to df, \quad n\Delta f \to f$$

therefore,

$$x(t) = \int_{-\infty}^{\infty} df \cdot \underbrace{\int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt}_{X(\omega)} \cdot e^{j2\pi f t}$$

$$x(t) = \int_{-\infty}^{\infty} \frac{X(\omega) e^{j\omega t}}{2\pi} d\omega \quad \text{— (A)} \quad ; \omega = 2\pi f$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

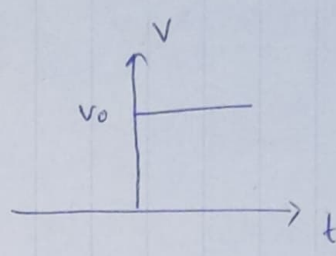
as well as, from (A) inverse fourier transform is,

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Q2)

$$R\frac{dq}{dt} + \frac{1}{C}q = V_o$$

also $q(0) = 0$



here V_o is causal

therefore, $V_o = V_o\, u(t)$

$$R\frac{dq}{dt} + \frac{1}{C}q = V_o\, u(t)$$

let take the laplance transform of the equation,

$$L\left[R\frac{dq}{dt} + \frac{q}{C}\right] = L\left[V_o\, u(t)\right]$$

$$R\, L\left[\frac{dq}{dt}\right] + \frac{1}{C}L\left[q(t)\right] = V_o\, L\left[u(t)\right]$$

$$RSQ(s) - R\cancel{q(0)}^{\,0} + \frac{1}{C}Q(s) = \frac{V_o}{s}$$ where $L[q(t)] = Q(s)$

$$\left(RS + \frac{1}{C}\right)Q(s) = \frac{V_o}{s}$$

$$Q(s) = \frac{V_o C}{s(RSC + 1)}$$

$$\frac{V_0 C}{S(RSC+1)} = \frac{A}{S} + \frac{B}{RSC+1}$$

$$V_0 C = ARC \cdot S + A + B \cdot S$$

considering constant values,

$$V_0 C = A$$

$$A = V_0 C$$

considering coefficients of S,

$$0 = ARC + B$$

$$0 = V_0 RC^2 + B$$

$$B = -V_0 RC^2$$

$$\therefore Q(S) = \frac{V_0 C}{S(RSC+1)} = \frac{V_0 C}{S} - \frac{V_0 RC^2}{RSC+1}$$

$$Q(S) = \frac{V_0 C}{S} - \frac{V_0 RC^2}{RSC+1}$$

let take the inverse laplance transform of both sides,

$$L^{-1}\left[Q(S)\right] = L^{-1}\left[\frac{V_0 C}{S} - \frac{V_0 RC^2}{RSC+1}\right]$$

$$q(t) = L^{-1}\left[\frac{V_0 C}{S}\right] - L^{-1}\left[\frac{V_0 RC^2}{RSC+1}\right]$$

$$q(t) = V_0 C \, L^{-1}\left[\frac{1}{S}\right] - \frac{V_0 RC^2}{RC} L^{-1}\left[\frac{1}{S + \frac{1}{RC}}\right]$$

$$q(t) = V_0 C u(t) - V_0 C \times e^{-\frac{1}{RC} \times t} u(t)$$

$$q(t) = V_0 C \left(1 - e^{\frac{-t}{RC}}\right) u(t)$$