1. $\hat{y}^{(i)} = \sigma(z^{(i)}) \qquad z^{(i)} = \underline{x}^{(i)T}\underline{w}$

i labels example

$$J(\underline{w}) = \frac{1}{N}\sum_{i=1}^{N} L(\hat{y}^{(i)}(\underline{w}), y^{(i)})$$

$$L(\hat{y}^{(i)}, y^{(i)}) = -\left[y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)}) \right]$$

$$\frac{\partial J}{\partial \underline{w}} = \frac{1}{N}\sum_{i=1}^{N} \frac{\partial L}{\partial \underline{w}}$$

$$= \frac{1}{N}\sum_{i=1}^{N} \frac{\partial L}{\partial \hat{y}^{(i)}} \frac{\partial \hat{y}^{(i)}}{\partial z^{(i)}} \frac{\partial z^{(i)}}{\partial \underline{w}}$$

$$\frac{\partial L}{\partial \hat{y}^{(i)}} = -\frac{y^{(i)}}{\hat{y}^{(i)}} + \frac{1-y^{(i)}}{1-\hat{y}^{(i)}}$$

$$\hat{y}^{(i)} = \sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$$

$$\frac{\partial \hat{y}^{(i)}}{\partial z^{(i)}} = \frac{e^{-z^{(i)}}}{(1+e^{-z^{(i)}})^2}$$

$$1 - \hat{y}^{(i)} = \frac{e^{-z^{(i)}}}{1 + e^{-z^{(i)}}}$$

$$\frac{\partial \hat{y}^{(i)}}{\partial z^{(i)}} = \hat{y}^{(i)} \left(1 - \hat{y}^{(i)} \right)$$

$$\frac{\partial L}{\partial \hat{y}^{(i)}} \frac{\partial \hat{y}^{(i)}}{\partial z^{(i)}} = -y^{(i)} \left(1 - \hat{y}^{(i)} \right) + \hat{y}^{(i)} \left(1 - y^{(i)} \right)$$

$$= \hat{y}^{(i)} - y^{(i)}$$

$$\frac{\partial z^{(i)}}{\partial \underline{w}} = \underline{x}^{(i)}$$

$$\frac{\partial J}{\partial \underline{w}} = \frac{1}{N} \sum_{i=1}^{N} \underline{x}^{(i)} \left(\hat{y}^{(i)} - \hat{y}^{(i)} \right)$$

$$= \frac{1}{N} \begin{pmatrix} \vdots & \vdots & \\ \underline{x}^{(1)} & \underline{x}_2^{(2)} & \cdots \\ \vdots & \vdots & \end{pmatrix} \begin{pmatrix} \hat{y}^{(1)} - y^{(1)} \\ \hat{y}^{(2)} - y^{(2)} \\ \vdots \end{pmatrix}$$

$$= \frac{1}{N} \underline{X}^T (\hat{y} - y)$$

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(2) Need $\Delta^{(2)} = \frac{\partial J}{\partial z_1^{(2)}}$

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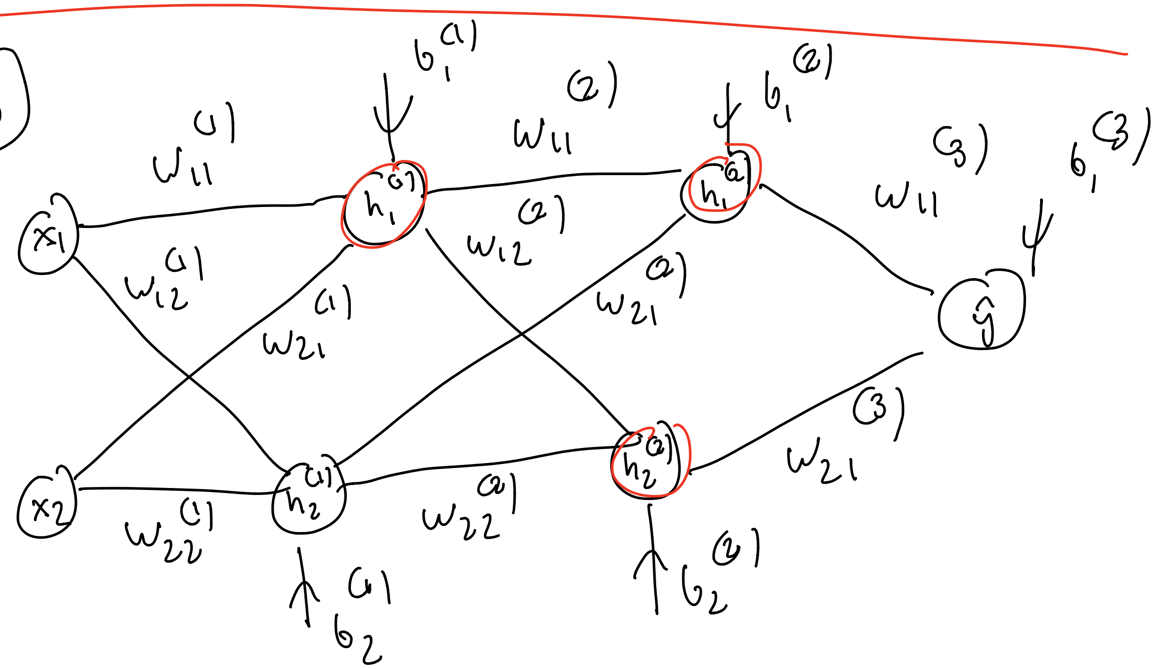
(3)



Error at output neuron

$$\Delta_1^{(3)} = \frac{\partial J}{\partial z_1^{(3)}}$$

$$\Delta_j^{(l)} = \left(\sum_k \Delta_k^{(l+1)} w_{jk}^{(l+1)} \right) \Phi'(z_j^{(l)})$$

$$\Delta_1^{(2)} = \Delta_1^{(3)} w_{11}^{(3)} \Phi'(z_1^{(2)})$$

$$\Delta_2^{(2)} = \Delta_1^{(3)} w_{21}^{(3)} \Phi'(z_2^{(2)})$$

$$\Delta_1^{(1)} = \Delta_1^{(2)} w_{11}^{(2)} \Phi'(z_1^{(1)})$$

$$+ \Delta_2^{(2)} w_{12}^{(2)} \Phi'(z_1^{(1)})$$

$$\Delta_2^{(1)} = \Delta_1^{(2)} w_{21}^{(2)} \Phi'(z_2^{(1)})$$

$$+ \Delta_2^{(2)} w_{22}^{(2)} \Phi'(z_2^{(1)})$$

$$\frac{\partial J}{\partial w_{kj}^{(l)}} = \Delta_j^{(l)} a_k^{(l-1)}$$

$$\frac{\partial J}{\partial w_{11}^{(3)}} = \Delta_1^{(3)} a_1^{(2)}$$

$$\frac{\partial J}{\partial w_{21}^{(3)}} = \Delta_1^{(3)} a_2^{(2)}$$

$$\frac{\partial J}{\partial w_{11}^{(2)}} = \Delta_1^{(2)} a_1^{(1)}$$

$$\frac{\partial J}{\partial w_{12}^{(2)}} = \Delta_2^{(2)} a_1^{(1)}$$

$$\frac{\partial J}{\partial w_{21}^{(1)}} = \Delta_1^{(2)} a_2^{(1)}$$

$$\frac{\partial J}{\partial w_{22}^{(1)}} = \Delta_2^{(2)} a_2^{(2)}$$

$$\frac{\partial J}{\partial w_{11}^{(1)}} = \Delta_1^{(1)} X_1$$

$$\frac{\partial J}{\partial w_{12}^{(1)}} = \Delta_2^{(1)} x_1$$

$$\frac{\partial J}{\partial w_{21}^{(1)}} = \Delta_1^{(1)} x_2$$

$$\frac{\partial J}{\partial w_{22}^{(1)}} = \Delta_2^{(1)} x_2$$