① $\hat{y}^{(i)} = \sigma(z^{(i)}) \qquad z^{(i)} = \underline{x}^{(i)T}\underline{w}$

$$J(\underline{w}) = \frac{1}{N}\sum_{i=1}^{N} L(\hat{y}^{(i)}(\underline{w}), y^{(i)})$$

$$\frac{\partial J}{\partial \underline{w}} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial L}{\partial \underline{w}} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial L}{\partial \hat{y}^{(i)}}\frac{\partial \hat{y}^{(i)}}{\partial z^{(i)}}\frac{\partial z^{(i)}}{\partial \underline{w}}$$

$$L(\hat{y}^{(i)}, y^{(i)}) = -\left[y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$$

$$\frac{\partial L}{\partial \hat{y}^{(i)}} = -\frac{y^{(i)}}{\hat{y}^{(i)}} + \frac{1-y^{(i)}}{1-\hat{y}^{(i)}}$$

$$\frac{\partial \hat{y}^{(i)}}{\partial z^{(i)}} = \hat{y}^{(i)}(1-\hat{y}^{(i)})$$

$$\frac{\partial L}{\partial \hat{y}^{(i)}}\frac{\partial \hat{y}^{(i)}}{\partial z^{(i)}} = -y^{(i)}(1-\hat{y}^{(i)}) + \hat{y}^{(i)}(1-y^{(i)})$$

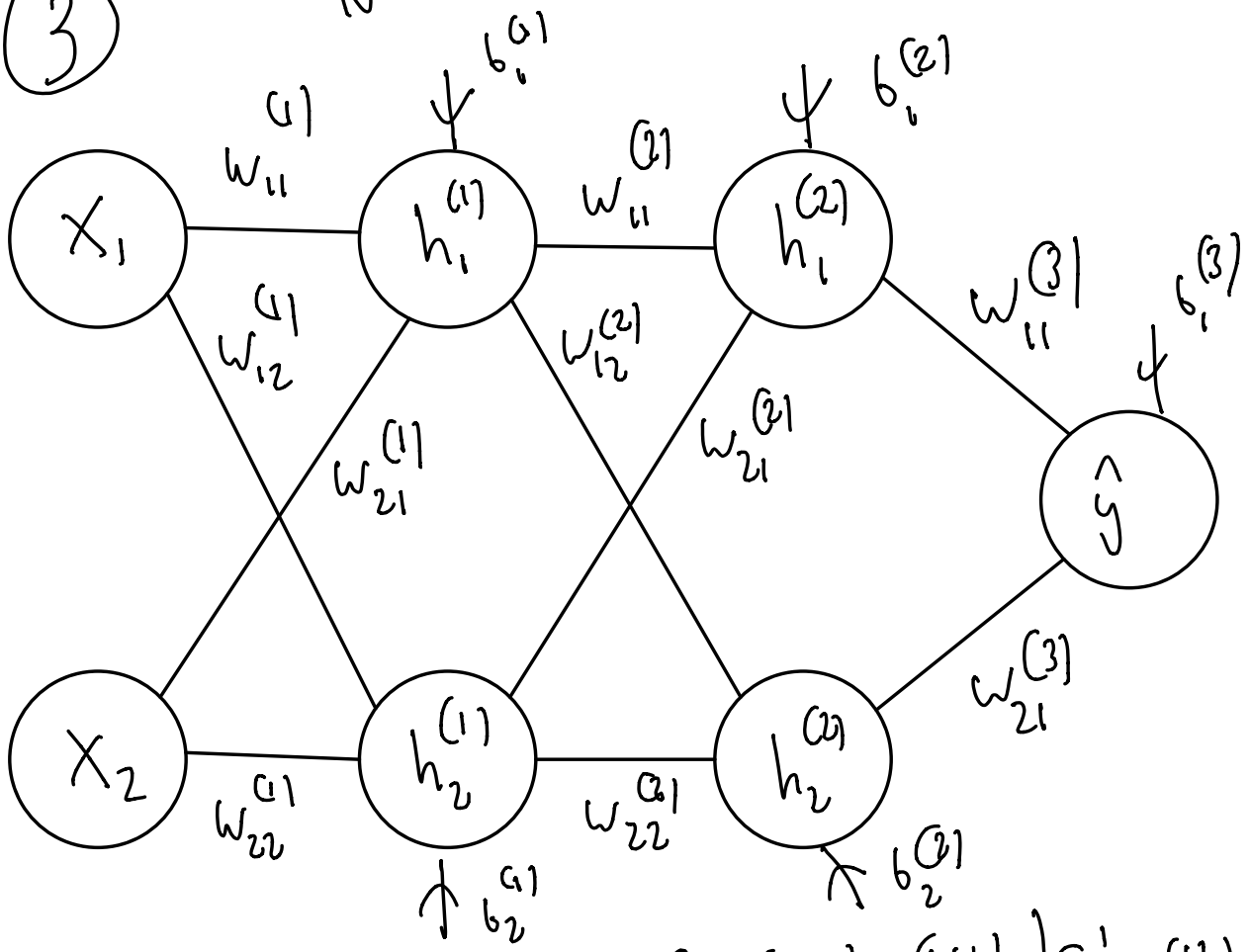
$$= \hat{y}^{(i)} - y^{(i)}$$

$$\frac{\partial J}{\partial \underline{w}} = \frac{1}{N}\sum_{i=1}^{N}\underline{x}^{(i)}(\hat{y}^{(i)} - y^{(i)})$$

$$= \frac{1}{N}\begin{pmatrix} | & | & \\ \underline{x}^{(1)} & \underline{x}^{(2)} & \cdots \\ | & | & \end{pmatrix}\begin{pmatrix} \hat{y}^{(1)} - y^{(1)} \\ \hat{y}^{(2)} - y^{(2)} \\ \vdots \end{pmatrix}$$

$$= \frac{1}{N} \underline{X}^T (\underline{\hat{y}} - \underline{y})$$

(3)



$$\Delta_1^{(3)} = \frac{\partial J}{\partial a_1^{(3)}} \qquad \Delta_j^{(l)} = \left(\sum_k \Delta_k^{(l+1)} w_{jk}^{(l+1)} \right) \Phi'(z_j^{(l)})$$

$$\Delta_1^{(2)} = \Delta_1^{(3)} \; w_{11}^{(3)} \; \Theta(z_1^{(2)})$$

$$\Delta_2^{(2)} = \Delta_1^{(3)} \; w_{21}^{(3)} \; \Theta(z_2^{(2)})$$

$$\Delta_1^{(1)} = \Delta_1^{(2)} w_{11}^{(2)} \theta(z_1^{(1)}) + \Delta_2^{(2)} w_{12}^{(2)} \theta(z_1^{(1)})$$

$$\Delta_2^{(1)} = \Delta_1^{(2)} w_{21}^{(2)} \theta(z_2^{(1)}) + \Delta_2^{(2)} w_{22}^{(2)} \theta(z_2^{(1)})$$

$$\frac{\partial J}{\partial w_{kj}^{(1)}} = \Delta_j^{(1)} a_k^{(l-1)}$$

$$\frac{\partial J}{\partial w_{11}^{(3)}} = \Delta_1^{(3)} a_1^{(2)}$$

$$\frac{\partial J}{\partial w_{21}^{(3)}} = \Delta_1^{(3)} a_2^{(2)}$$

$$\frac{\partial J}{\partial w_{11}^{(2)}} = \Delta_1^{(2)} a_1^{(1)}$$

$$\frac{\partial J}{\partial W_{12}^{(2)}} = \Delta_2^{(2)} a_1^{(1)}$$

$$\frac{\partial J}{\partial W_{21}^{(1)}} = \Delta_1^{(2)} a_2^{(1)}$$

$$\frac{\partial J}{\partial W_{22}^{(1)}} = \Delta_2^{(2)} a_2^{(2)}$$

$$\frac{\partial J}{\partial W_{11}^{(1)}} = \Delta_1^{(1)} x_1$$

$$\frac{\partial J}{\partial W_{12}^{(1)}} = \Delta_2^{(1)} x_1$$

$$\frac{\partial J}{\partial W_{21}^{(1)}} = \Delta_1^{(1)} x_2$$

$$\frac{\partial J}{\partial W_{22}^{(1)}} = \Delta_2^{(1)} x_2$$