The Shannon entropy and the rate-distortion function characterize the theoretical performance limits in lossless and lossy data compression, respectively. The rate redundancy refers to how quickly the performance of practical compression algorithms approaches the theoretical limit as the number n of symbols encoded increases. For decades, from the 1980s through the early 2000s, the $\Theta\left(\frac{\log n}{n}\right)$ rate redundancy result was well-established in both lossless and lossy compression, including in the universal setting where the source distribution is unknown; "the question was essentially settled," Kontoyiannis [1].

However, the *minimax* universal setting is the gold standard because it accounts for the worst-case performance of universal compression schemes over all source distributions, thus giving uniform convergence guarantees under potentially adversarial conditions. In the *minimax* universal setting, the same $\Theta\left(\frac{\log n}{n}\right)$ rate redundancy result has been proven for lossless codes since 1981. However, minimax universal results for lossy compression remained elusive until 2023, when our work [2] broke new ground. Our work in [2] shattered the long-standing $\Theta\left(\frac{\log n}{n}\right)$ paradigm by proving that the optimal rate redundancy for lossy compression under the universal *minimax* framework is actually $\tilde{\Theta}\left(\frac{1}{\sqrt{n}}\right)$. This result stands in stark contrast to the prior $\Theta\left(\frac{\log n}{n}\right)$ results in lossy compression which only gave pointwise convergence guarantees. Even more importantly, we showed that $\tilde{\Theta}\left(\frac{1}{\sqrt{n}}\right)$ rate redundancy persists even in the non-universal lossy setting. We gave a detailed study in [2] on how regularity conditions imposed in prior works led to a faster $\Theta\left(\frac{\log n}{n}\right)$ convergence by not accounting for all i.i.d. sources and distortion measures. This fundamentally redefines the landscape of lossy compression theory, upending the $\Theta\left(\frac{\log n}{n}\right)$ standard across the board.

In a related work [3], we pioneered a novel variant of lossy compression called *universal distortion* in which the distortion measure - traditionally fixed - is now an input to the encoder along with the source data to be compressed. The *universal distortion* framework affords greater versatility in compression systems and is an especially useful model for modern compression algorithms, where nonlinear transforms make compression in the transform domain sensitive to the input source data. We proved rate redundancy results under the combined framework of *minimax* and *universal distortion*, providing uniform convergence guarantees over both all i.i.d. sources and all distortion measures.

In our work on channel coding [4, 5], we introduced two significant innovations. First, we developed a more practical cost model that superseded existing ones. Our novel cost formulation constrains the cost (or power) of the transmission in expectation as well as in its variance. This *mean and variance (m.v.) cost constraint* provides an alternative to the two standard cost constraints in the literature: the strict peak-power constraint and the weaker expected cost constraint. Our m.v. cost formulation with a variance parameter V generalizes these existing frameworks in the sense that letting $V \to 0$ recovers the peak-power constraint and letting $V \to \infty$ recovers the expected cost constraint. Beyond generalization, we showed that the *m.v. cost constraint* for $0 < V < \infty$ offers practical benefits over both prior cost models. Unlike the peak-power constraint, it allows for an improved coding performance with feedback; even without feedback, the coding performance under the m.v. cost constraint is superior. Unlike the expected cost constraint, it enforces an ergodic and controlled use of transmission power, which is desirable for several practical reasons detailed in the introduction section of [5]. The benefits of the new cost constraint come about by merely allowing the cost to fluctuate above the threshold in a manner consistent with a noise process, thus making it a more realistic and natural cost model in practice than the restrictive peak-power constraint.

Our second innovation was in feedback communication where we unveiled new ways in which feedback can improve communication performance. For *any* $V > 0$, we showed that feedback improvement is possible for a significantly larger class of channels than in the prior study on unconstrained channels [6]. Additionally, we proved that for a broad class of channels, feedback improvement is possible *if and only if*

$V > 0$. This finding highlights the important role of cost variability V in enabling feedback mechanisms to improve coding performance. These are also the first results to establish second-order feedback improvement for discrete memoryless channels *with* cost constraints, thus giving a broader understanding of how and when feedback improves the coding performance.

The two pillars of information theory, data compression and channel coding, are often considered as having reached maturity a long time ago. Our research brings fresh perspectives to classical information theory, challenging long-standing assumptions and introducing new frameworks in both these areas. By reformulating foundational problems and upending established results, we have expanded the theoretical boundaries of the field, offering more robust, practical and adaptable models for modern applications.

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