Introduction: The two pillars of information theory, data compression and channel coding, are often considered as having reached maturity a long time ago. Our research brings fresh perspectives to both these areas by challenging long-standing assumptions and introducing new frameworks. By reformulating foundational problems and upending established results, we have expanded the theoretical boundaries of the field, offering more robust, practical, and adaptable models for modern applications.

Data Compression: The Shannon entropy and rate-distortion function characterize the theoretical performance limits in lossless and lossy data compression, respectively. Rate redundancy measures how quickly practical compression algorithms converge to these limits as the number n of encoded symbols increases. Since the 1980s and 1990s, the optimal rate redundancy has been firmly established as $\Theta\left(\frac{\log n}{n}\right)$ in both lossless and lossy compression, specifically in the *universal* setting where the source distribution is unknown. As Kontoyiannis [1] stated, "the question was essentially settled."

However, the *minimax* universal setting, which accounts for the worst-case performance of universal compression schemes over all source distributions, is the gold standard because it gives uniform convergence guarantees, even under adversarial conditions. In the minimax universal setting, the same $\Theta\left(\frac{\log n}{n}\right)$ rate redundancy result has been proven for lossless codes since 1981. However, minimax universal results for lossy compression remained elusive until 2023, when our work [2] broke new ground. Our work shattered the long-standing $\Theta\left(\frac{\log n}{n}\right)$ paradigm by proving that the optimal rate redundancy for lossy compression under the minimax universal framework is actually $\tilde{\Theta}\left(\frac{1}{\sqrt{n}}\right)$. This contrasts sharply with the prior $\Theta\left(\frac{\log n}{n}\right)$ results in lossy compression, which only gave pointwise convergence guarantees. Even more importantly, we showed that the $\tilde{\Theta}\left(\frac{1}{\sqrt{n}}\right)$ rate redundancy result holds even when the source distribution is known (the non-universal setting). We provided a detailed study in [2] showing how regularity conditions imposed in prior works led to the faster $\Theta\left(\frac{\log n}{n}\right)$ convergence because they did not account for all i.i.d. sources and distortion measures. This fundamentally redefines the landscape of lossy compression theory, upending the $\Theta\left(\frac{\log n}{n}\right)$ standard across the board.

In another work [3], we introduced a novel paradigm in lossy compression called *universal distortion*, in which the distortion measure—traditionally fixed—is now a runtime input only to the encoder. The universal distortion framework affords greater adaptability and is an especially useful model for modern compression algorithms based on nonlinear transforms, where optimal distortion in the transform domain must dynamically adjust to the source data characteristics. We proved rate redundancy results under the combined framework of *minimax* and *universal distortion*, providing uniform convergence guarantees over all i.i.d. sources and all distortion measures.

Channel Coding: In our work on channel coding [4], [5], we introduced two significant innovations. First, we developed an advanced cost model that supersedes the two cost constraints that have dominated the literature: the strict peak-power constraint and the weaker expected cost constraint. Our novel cost formulation constrains the cost (or power) of the transmission both in expectation and variance. With a variance parameter V, our *mean and variance (m.v.) cost constraint* generalizes the existing frameworks, with $V \to 0$ recovering the peak-power constraint and $V \to \infty$ recovering the expected cost constraint. Beyond generalization, we showed that the m.v. cost constraint for $0 < V < \infty$ has practical advantages over both prior cost models. Unlike the peak-power constraint, it allows for an improved coding performance with feedback; even without feedback, the coding performance under the m.v. cost constraint is superior. Unlike the expected cost constraint, it enforces a controlled, ergodic use of transmission power, which is desirable for several practical reasons as discussed in [5]. The new cost constraint achieves its benefits by allowing the cost to fluctuate above the threshold in a manner consistent with a noise process, thus making it a more realistic and natural cost model in practice than the restrictive peak-power constraint.

Our second innovation was in feedback communication, where we unveiled new ways in which feedback can improve communication performance. For *any* $V > 0$, we showed that feedback improvement is possible for a significantly larger class of channels than in prior studies on unconstrained channels [6]. Additionally, we proved that for a broad class of channels, feedback improvement is possible *if and only if* $V > 0$. Thus, our results reveal the critical role of cost variability V in enabling feedback mechanisms to improve coding performance. These are also the first results to establish second-order feedback improvement for discrete memoryless channels *with* cost constraints, thus providing a broader understanding of how and when feedback improves coding performance.

References

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