R: Spatial regression

Roger Bivand

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Required current contributed CRAN packages:

```
I am running R 3.6.1, with recent update.packages().
needed <- c("MatrixModels", "lme4", "spatialreg", "spdep", "sf", "sp", "HSAR")</pre>
```

Beijing data set

library(sf)

Lower level point support data and upper level district boundaries (polygon support)

```
library(HSAR)
library(sp)
data(landSPDF)
data(landprice)
data(Beijingdistricts)
```

Convert to \mathbf{sf} class and merge data with point geometries

Simple feature collection with 1117 features and 12 fields

```
## Linking to GEOS 3.7.2, GDAL 3.0.1, PROJ 6.2.0
land_sf <- st_as_sf(landSPDF)
landprice_sf <- merge(land_sf, landprice, by="obs")
(landprice_sf <- landprice_sf[order(landprice_sf$district.id.x),])</pre>
```

```
## geometry type: POINT
## dimension:
                  XΥ
                  xmin: 428553.1 ymin: 4406815 xmax: 463693.2 ymax: 4440423
## bbox:
## epsg (SRID):
                  +proj=tmerc +lat_0=0 +lon_0=117 +k=1 +x_0=500000 +y_0=0 +ellps=krass +units=m +no_de
## proj4string:
## First 10 features:
        obs district.id.x lnprice
                                   lnarea lndcbd dsubway
                                                            dpark
## 187
       189
                       3 5.57430 10.27820 9.94866 6.83023 7.06579 6.81916
## 188
                       3 7.16382 11.58780 9.93534 7.14334 6.78243 6.67827
       190
## 700
       968
                       3 7.61282 8.94551 9.91779 7.64360 6.84364 4.60356
## 701
       969
                       3 6.81564 5.81928 9.91940 7.64640 6.88254 4.10025
## 702
       970
                       3 6.93528 7.71869 9.91752 7.65810 6.86760 4.53460
## 709
       992
                       3 7.45757 9.20029 9.84785 7.78904 6.95662 7.05138
```

```
## 710
                       3 7.12569 7.97788 9.84388 7.81991 7.00792 7.11267
       993
## 711
       994
                       3 7.48522 7.78634 9.84203 7.83398 7.03089 7.13981
## 717 1001
                       3 5.87349 10.70910 9.95534 7.89121 7.11019 5.67984
                       5 6.79302 6.39403 9.92025 6.76006 6.23524 6.10494
## 181 183
        popden crimerate district.id.y year
                                                            geometry
## 187 0.548966 10.75110
                                     3
                                          1
                                              POINT (430237 4422804)
## 188 0.548966 10.75110
                                     3
                                          1 POINT (430547.1 4423001)
## 700 0.548966 10.75110
                                    3
                                          0 POINT (431029.4 4423667)
## 701 0.548966 10.75110
                                     3
                                          0 POINT (431001.8 4423695)
```

```
## 702 0.548966 10.75110
                                      3
                                           0 POINT (431040.9 4423697)
## 709 0.548966 10.75110
                                      3
                                               POINT (432164 4422080)
                                           0 POINT (432239.7 4422081)
## 710 0.548966 10.75110
                                      3
## 711 0.548966 10.75110
                                      3
                                               POINT (432275 4422082)
## 717 0.548966
                 10.75110
                                      3
                                           0 POINT (430436.8 4424601)
## 181 1.407250
                  2.25832
                                      5
                                           1 POINT (430606.5 4420186)
```

Check that the input IDs match and that the data are correctly ordered

```
all.equal(landprice_sf$district.id.x, landprice_sf$district.id.y)
```

[1] TRUE

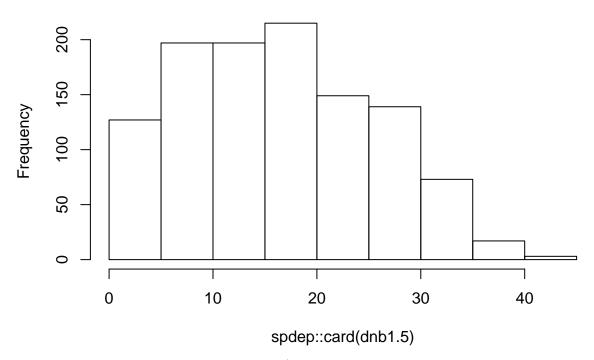
Create the original 1.5 km distance threshold spatial weights, with a few no-neighbour observations (so set zero policy option)

library(spatialreg)

```
## Loading required package: spData
## Loading required package: Matrix
## Registered S3 methods overwritten by 'spatialreg':
##
     method
                               from
     residuals.stsls
##
                               spdep
##
     deviance.stsls
                               spdep
##
     coef.stsls
                               spdep
##
     print.stsls
                               spdep
##
     summary.stsls
                               spdep
##
     print.summary.stsls
                               spdep
##
     residuals.gmsar
                               spdep
##
     deviance.gmsar
                               spdep
##
     coef.gmsar
                               spdep
##
     fitted.gmsar
                               spdep
##
     print.gmsar
                               spdep
     summary.gmsar
##
                               spdep
     print.summary.gmsar
##
                               spdep
##
     print.lagmess
                               spdep
##
     summary.lagmess
                               spdep
##
     print.summary.lagmess
                               spdep
##
     residuals.lagmess
                               spdep
##
     deviance.lagmess
                               spdep
##
     coef.lagmess
                               spdep
##
     fitted.lagmess
                               spdep
##
     logLik.lagmess
                               spdep
     fitted.SFResult
##
                               spdep
##
     print.SFResult
                               spdep
##
     fitted.ME res
                               spdep
##
     print.ME_res
                               spdep
##
     print.lagImpact
                               spdep
##
     plot.lagImpact
                               spdep
##
     summary.lagImpact
                               spdep
##
     HPDinterval.lagImpact
                               spdep
     print.summary.lagImpact
##
                               spdep
##
     print.sarlm
                               spdep
##
     summary.sarlm
                               spdep
     residuals.sarlm
##
                               spdep
```

```
##
     deviance.sarlm
                               spdep
     coef.sarlm
##
                               spdep
     vcov.sarlm
##
                               spdep
##
     fitted.sarlm
                               spdep
##
     logLik.sarlm
                               spdep
##
     anova.sarlm
                               spdep
     predict.sarlm
##
                               spdep
     print.summary.sarlm
##
                               spdep
##
     print.sarlm.pred
                               spdep
##
     as.data.frame.sarlm.pred spdep
##
     residuals.spautolm
                               spdep
##
     deviance.spautolm
                               spdep
##
     coef.spautolm
                               spdep
##
     fitted.spautolm
                               spdep
                               spdep
##
     print.spautolm
##
     summary.spautolm
                               spdep
##
     logLik.spautolm
                               spdep
##
     print.summary.spautolm
                               spdep
                               spdep
##
    print.WXImpact
##
     summary.WXImpact
                               spdep
##
     print.summary.WXImpact
                               spdep
##
    predict.SLX
                               spdep
dnb1.5 <- spdep::dnearneigh(landprice_sf, 0, 1500, row.names=as.character(landprice_sf$obs))</pre>
dnb1.5
## Neighbour list object:
## Number of regions: 1117
## Number of nonzero links: 18798
## Percentage nonzero weights: 1.506625
## Average number of links: 16.82901
## 7 regions with no links:
## 517 53 292 1764 33 1785 1126
dists <- spdep::nbdists(dnb1.5, st_geometry(landprice_sf))</pre>
edists <- lapply(dists, function(x) exp((-((x/1000)^2))/(1.5^2)))
ozpo <- spdep::set.ZeroPolicyOption(TRUE)</pre>
oo <- set.ZeroPolicyOption(TRUE)</pre>
lw <- spdep::nb2listw(dnb1.5, glist=edists, style="W")</pre>
hist(spdep::card(dnb1.5))
```

Histogram of spdep::card(dnb1.5)



Reconstruct the input data for R formula use (do not log in advance, do use factors for categorical variables to permit automatic generation of dummies)

```
landprice_sf$fyear <- factor(landprice_sf$year + 2003)
landprice_sf$price <- exp(landprice_sf$lnprice)
landprice_sf$area <- exp(landprice_sf$lnarea)
landprice_sf$Dcbd <- exp(landprice_sf$lndcbd)
landprice_sf$Dsubway <- exp(landprice_sf$dsubway)
landprice_sf$Dpark <- exp(landprice_sf$dpark)
landprice_sf$Dele <- exp(landprice_sf$dele)
landprice_sf$f_district.id <- factor(landprice_sf$district.id.x)
(t1 <- table(table(landprice_sf$f_district.id)))</pre>
```

```
## 1 2 3 4 5 6 7 8 9 10 11 12 14 15 16 17 18 19 20 21 23 25 26 27 28 ## 7 8 10 4 5 10 8 10 8 4 6 2 4 3 1 2 5 2 1 1 1 1 1 1 1 2 ## 31 32 33 52 ## 1 1 1 1 1
```

Some covariates are observed at the district level rather than the land parcel level

```
sapply(as.data.frame(landprice_sf[, c("price", "area", "Dcbd", "Dele", "Dpark", "Dsubway", "crimerate",
##
       price
                   area
                             Dcbd
                                        Dele
                                                 Dpark
                                                          Dsubway crimerate
                                                  1114
##
        1062
                   1098
                             1114
                                        1116
                                                             1117
                                                                         105
##
      popden geometry
##
         111
                   1117
```

Check the matching of district IDs and counts of land parcels in districts

```
Beijingdistricts$id1 <- Beijingdistricts$id+1
all.equal(unique(landprice_sf$district.id.x), Beijingdistricts$id1)
```

```
## [1] TRUE
(Beijingdistricts_sf <- st_as_sf(Beijingdistricts))
## Simple feature collection with 111 features and 2 fields
## geometry type:
                   MULTIPOLYGON
## dimension:
                   XΥ
## bbox:
                   xmin: 426987.3 ymin: 4403559 xmax: 467920.9 ymax: 4443287
## epsg (SRID):
                   +proj=tmerc +lat_0=0 +lon_0=117 +k=1 +x_0=500000 +y_0=0 +ellps=krass +units=m +no_de
## proj4string:
## First 10 features:
##
     id id1
                                   geometry
## 0 2
         3 MULTIPOLYGON (((428183.1 44...
         5 MULTIPOLYGON (((432472.2 44...
## 1 4
         7 MULTIPOLYGON (((432446.1 44...
## 3 7
          8 MULTIPOLYGON (((433534.5 44...
         9 MULTIPOLYGON (((443807.9 44...
## 5 9 10 MULTIPOLYGON (((444461 4420...
## 6 10 11 MULTIPOLYGON (((447530.6 44...
        12 MULTIPOLYGON (((443849.6 44...
## 7 11
## 8 12 13 MULTIPOLYGON (((446810 4417...
## 9 13 14 MULTIPOLYGON (((445954.8 44...
Beijingdistricts_sf$counts <- sapply(st_contains(Beijingdistricts_sf, landprice_sf), length)
Check point counts by district from input data and topological points in polygon counts
t2 <- table(Beijingdistricts_sf$counts)
all.equal(t1, t2)
## [1] TRUE
Basic formula object from the original paper and examples in package, fit and display OLS model (note that
fyear is split into dummies with 2003 in the intercept)
form <- log(price) ~ log(area) + log(Dcbd) + log(Dele) + log(Dpark) + log(Dsubway) +
  crimerate + popden + fyear
OLS <- lm(form, data=landprice_sf)
summary(OLS)
##
## lm(formula = form, data = landprice_sf)
##
## Residuals:
##
       Min
                10 Median
                                 3Q
                                        Max
## -2.5915 -0.5752 -0.0496 0.5206
                                    3.4042
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
                            0.370103 38.243 < 2e-16 ***
## (Intercept) 14.153917
## log(area)
                -0.008253
                            0.018675 -0.442 0.65863
## log(Dcbd)
                -0.250601
                            0.047752 -5.248 1.84e-07 ***
## log(Dele)
                -0.085528
                            0.032308 -2.647 0.00823 **
## log(Dpark)
                -0.284372
                            0.046046 -6.176 9.24e-10 ***
## log(Dsubway) -0.245748
                            0.034755 -7.071 2.73e-12 ***
## crimerate
                 0.007668
                            0.004458
                                        1.720 0.08575 .
```

```
## popden
               0.032827
                          0.010304 3.186 0.00148 **
               ## fyear2004
## fyear2005
               0.017635 0.124986
                                     0.141 0.88782
## fyear2006
               -0.120314
                          0.107209 -1.122 0.26201
## fyear2007
                0.551384
                          0.117431
                                     4.695 3.00e-06 ***
                                      3.058 0.00229 **
## fyear2008
                0.396172 0.129571
                2.113691
                           0.228688
                                      9.243 < 2e-16 ***
## fyear2009
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.8335 on 1103 degrees of freedom
## Multiple R-squared: 0.3524, Adjusted R-squared: 0.3448
## F-statistic: 46.17 on 13 and 1103 DF, p-value: < 2.2e-16
Are the residuals spatially autocorrelated?
spdep::lm.morantest(OLS, listw=lw)
##
##
  Global Moran I for regression residuals
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
## Moran I statistic standard deviate = 15.1, p-value < 2.2e-16
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I
                        Expectation
                                            Variance
##
      0.1944768641
                      -0.0054494313
                                        0.0001753028
What do the robust LM tests say?
spdep::lm.LMtests(OLS, listw=lw, test=c("RLMerr", "RLMlag"))
##
   Lagrange multiplier diagnostics for spatial dependence
##
##
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## RLMerr = 118.22, df = 1, p-value < 2.2e-16
##
##
##
  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
## RLMlag = 3.7725, df = 1, p-value = 0.0521
How dow we do with a linear model including selected spatially lagged covariates?
SLX <- lmSLX(form, data=landprice_sf, listw=lw, Durbin= ~ log(area) + log(Dcbd) + log(Dele) + log(Dpark
summary(impacts(SLX))
```

```
## Impact measures (SLX, estimable, n-k):
##
                     Direct
                               Indirect
                                               Total
## log(area)
               -0.028250900 0.15061221 0.122361308
## log(Dcbd)
               -0.575325229   0.34199016   -0.233335072
## log(Dele)
                0.001927612 -0.15153784 -0.149610227
## log(Dpark) -0.066376027 -0.29670998 -0.363086011
## log(Dsubway) -0.180677830 -0.09171154 -0.272389368
                0.002228943 0.00697053 0.009199473
## crimerate
## popden
                0.003782010 0.04501340 0.048795406
## fyear2004
               -0.163634606
                                    NA -0.163634606
## fyear2005
                0.006112745
                                   NA 0.006112745
                                   NA -0.120821815
## fyear2006
               -0.120821815
## fyear2007
                0.619330756
                                   NA 0.619330756
## fyear2008
                0.444819013
                                   NA 0.444819013
## fyear2009
                2.186988035
                                   NA 2.186988035
## Standard errors:
##
                             Indirect
                    Direct
                                            Total
## log(area)
               0.019588590 0.03882315 0.037437930
## log(Dcbd)
               0.121875390 0.13150000 0.052242219
## log(Dele)
               0.051081439 0.06620277 0.043070631
## log(Dpark)
               0.107848818 0.12908527 0.057968077
## log(Dsubway) 0.059325583 0.07722793 0.046759734
## crimerate
               0.009157083 0.01134039 0.005658334
               0.017573848 0.02455712 0.014701241
## popden
## fyear2004
               0.058433277
                                  NA 0.058433277
## fyear2005
                                   NA 0.124199473
               0.124199473
## fyear2006
               0.106956715
                                   NA 0.106956715
## fyear2007
               0.117305346
                                   NA 0.117305346
## fyear2008
               0.128990353
                                   NA 0.128990353
## fyear2009
               0.227449165
                                   NA 0.227449165
## Z-values:
##
                           Indirect
                    Direct
                                            Total
## log(area)
               -1.44221201 3.8794431 3.26837801
## log(Dcbd)
              -4.72060215 2.6006857 -4.46640817
## log(Dele)
                0.03773605 -2.2889955 -3.47360195
## log(Dpark)
              -0.61545437 -2.2985581 -6.26355107
## log(Dsubway) -3.04552976 -1.1875436 -5.82529759
## crimerate
                ## popden
                0.21520671 1.8330078 3.31913527
## fyear2004
                                  NA -2.80036674
               -2.80036674
## fyear2005
                0.04921716
                                  NA 0.04921716
## fyear2006
                                 NA -1.12963282
               -1.12963282
## fyear2007
                5.27964648
                                  NA 5.27964648
## fyear2008
                                  NA 3.44846729
                3.44846729
## fyear2009
                9.61528276
                                   NA 9.61528276
##
## p-values:
##
               Direct
                          Indirect Total
## log(area)
               0.14924257 0.0001047 0.00108166
## log(Dcbd)
               2.3515e-06 0.0093038 7.9544e-06
## log(Dele)
               0.96989813 0.0220796 0.00051352
## log(Dpark)
               0.53825469 0.0215300 3.7631e-10
```

```
## log(Dsubway) 0.00232271 0.2350133 5.7011e-09
## crimerate 0.80768630 0.5387765 0.10398645
## popden
              0.82960616 0.0668014 0.00090297
## fyear2004 0.00510446 NA
                                    0.00510446
## fyear2005
               0.96074624 NA
                                     0.96074624
## fyear2006
              0.25863098 NA
                                    0.25863098
## fyear2007
              1.2943e-07 NA
                                    1.2943e-07
## fyear2008
                0.00056378 NA
                                     0.00056378
## fyear2009
                < 2.22e-16 NA
                                     < 2.22e-16
And the spatial residual autocorrelation?
spdep::lm.morantest(SLX, listw=lw)
##
   Global Moran I for regression residuals
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## Moran I statistic standard deviate = 14.802, p-value < 2.2e-16
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I
                         Expectation
                                             Variance
       0.1885391149
                       -0.0069886190
                                         0.0001745023
And robust LM tests?
spdep::lm.LMtests(SLX, listw=lw, test=c("RLMerr", "RLMlag"))
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## RLMerr = 39.985, df = 1, p-value = 2.56e-10
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## RLMlag = 0.17014, df = 1, p-value = 0.68
So let's fit a spatial Durbin error model, with the same selection of spatially lagged covariates
e <- eigenw(lw)
SDEM <- errorsarlm(form, data=landprice_sf, listw=lw, Durbin= ~ log(area) + log(Dcbd) + log(Dele) + log
summary(impacts(SDEM))
```

```
## Impact measures (SDEM, estimable, n):
##
                     Direct
                               Indirect
                                               Total
               -0.021085385 0.133094132 0.112008747
## log(area)
## log(Dcbd)
               ## log(Dele)
               -0.005661913 -0.114782232 -0.120444145
## log(Dpark) -0.074378191 -0.273739346 -0.348117537
## log(Dsubway) -0.164798714 -0.090024131 -0.254822845
               0.006411748 -0.001121658 0.005290091
## crimerate
## popden
                ## fyear2004
               -0.213875338
                                     NA -0.213875338
## fyear2005
               -0.074548268
                                     NA -0.074548268
## fyear2006
               -0.136022976
                                     NA -0.136022976
## fyear2007
                0.713777760
                                     NA 0.713777760
## fyear2008
                0.488689750
                                     NA 0.488689750
## fyear2009
                2.167398316
                                     NA 2.167398316
## Standard errors:
##
                            Indirect
                    Direct
## log(area)
               0.018070856 0.05086004 0.055146985
## log(Dcbd)
              0.124751995 0.14355527 0.091948225
## log(Dele)
              0.045634066 0.08014074 0.072364276
## log(Dpark)
               0.101149512 0.14212885 0.094724658
## log(Dsubway) 0.054362580 0.09395633 0.078343698
## crimerate
               0.008305157 0.01274632 0.009474872
               0.015899732 0.02876860 0.023482266
## popden
## fyear2004
               0.055676729
                                  NA 0.055676729
## fyear2005
                                  NA 0.116369514
               0.116369514
## fyear2006
               0.100092117
                                  NA 0.100092117
## fyear2007
               0.115930158
                                  NA 0.115930158
## fyear2008
               0.122087311
                                  NA 0.122087311
## fyear2009
               0.205699716
                                  NA 0.205699716
## Z-values:
##
                            Indirect
                                          Total
                   Direct
## log(area)
              -1.1668171 2.61687048 2.0310947
              -4.5879242 2.21018802 -2.7740454
## log(Dcbd)
## log(Dele)
               -0.1240721 -1.43225824 -1.6644144
## log(Dpark)
              -0.7353292 -1.92599427 -3.6750467
## log(Dsubway) -3.0314734 -0.95814867 -3.2526272
## crimerate
               0.7720201 -0.08799852 0.5583285
## popden
                0.3134164 1.57734403 2.1446488
## fyear2004
                                 NA -3.8413776
               -3.8413776
## fyear2005
               -0.6406168
                                  NA -0.6406168
## fyear2006
                                 NA -1.3589779
               -1.3589779
## fyear2007
               6.1569636
                                 NA 6.1569636
## fyear2008
                                  NA 4.0027890
               4.0027890
## fyear2009
               10.5367103
                                  NA 10.5367103
##
## p-values:
##
               Direct
                         Indirect Total
## log(area)
               0.24328423 0.008874 0.04224539
## log(Dcbd)
               4.4768e-06 0.027092 0.00553640
## log(Dele)
               0.90125821 0.152070 0.09602964
## log(Dpark)
               0.46213902 0.054105 0.00023781
```

```
## log(Dsubway) 0.00243363 0.337988 0.00114343
## crimerate 0.44010252 0.929878 0.57662011
## popden
               0.75396431 0.114716 0.03198094
## fyear2004
               0.00012235 NA
                                    0.00012235
## fyear2005
               0.52177167 NA
                                    0.52177167
## fyear2006
               0.17415359 NA
                                    0.17415359
## fyear2007
               7.4153e-10 NA
                                    7.4153e-10
## fyear2008
               6.2600e-05 NA
                                    6.2600e-05
## fyear2009
                < 2.22e-16 NA
                                    < 2.22e-16
```

The likelihood ratio test shows that the SDEM model fits much better than the SLX model

```
LR1.sarlm(SDEM)
```

The Hausman test is perhaps significant, suggestion that the non-spatial coefficients shift somewhat between the SLX model and the SDEM model

```
Hausman.test(SDEM)
```

```
##
## Spatial Hausman test (asymptotic)
##
## data: NULL
## Hausman test = 41.781, df = 21, p-value = 0.004482
But ...
```

is this the end of the story? Reach out to very general mixed model IID random effects at the district level (fixed effects would give 111 dummies); here without spatially lagged covariates

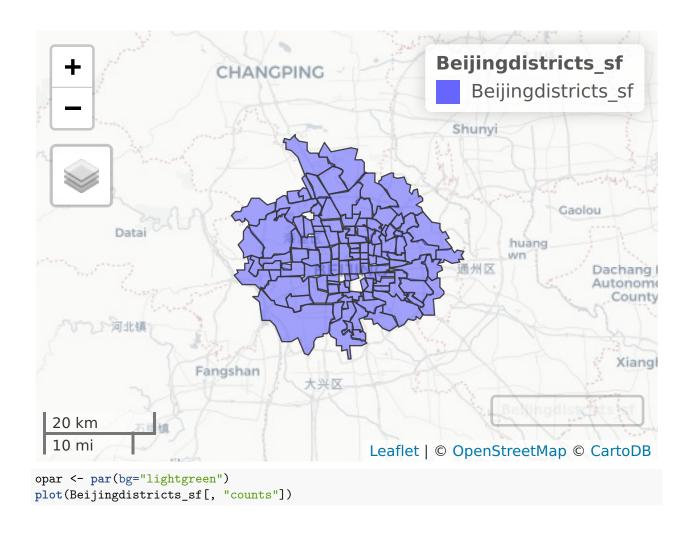
```
library(lme4)
mlm_1 <- lmer(update(form, . ~ . + (1 | f_district.id)), data=landprice_sf, REML=FALSE)
Beijingdistricts_sf$mlm_re <- ranef(mlm_1)[[1]][,1]</pre>
```

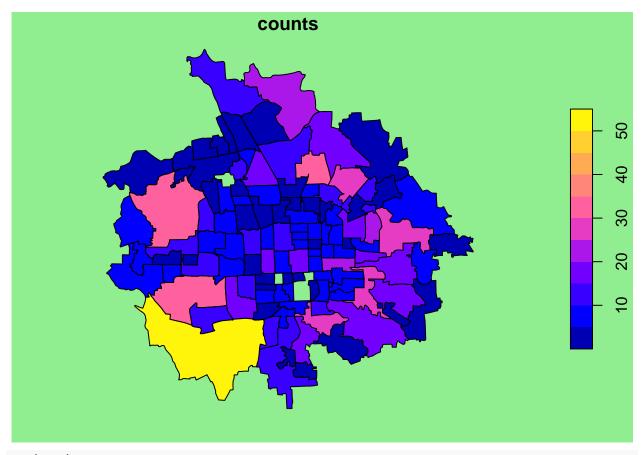
The **HSAR** model gives a spatial error model at the district level, defining a sparse matrix **Delta** assigning parcels to districts

```
library(Matrix)
suppressMessages(library(MatrixModels))
Delta <- as(model.Matrix(~ -1 + f_district.id, data=landprice_sf, sparse=TRUE), "dgCMatrix")</pre>
```

There are gaps in the land parcel and district coverage

```
library(mapview)
mapview(Beijingdistricts_sf)
```





par(opar)

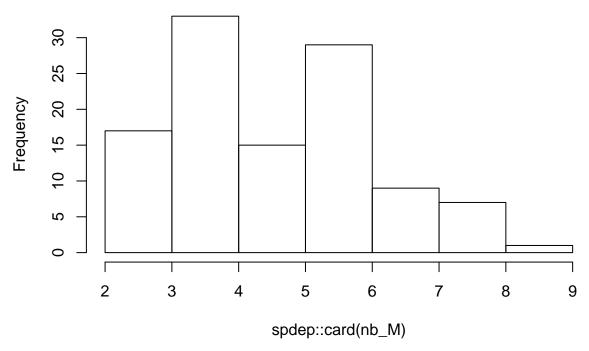
Construct the spatial weights for the disticts

```
nb_M <- spdep::poly2nb(Beijingdistricts, queen=FALSE, row.names=as.character(Beijingdistricts$id1))
M <- as(spdep::nb2listw(nb_M, style="B"), "CsparseMatrix")
dim(M)</pre>
```

[1] 111 111

hist(spdep::card(nb_M))

Histogram of spdep::card(nb_M)

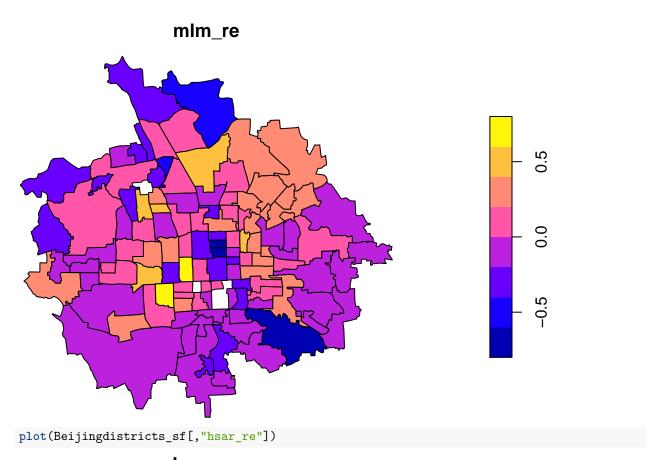


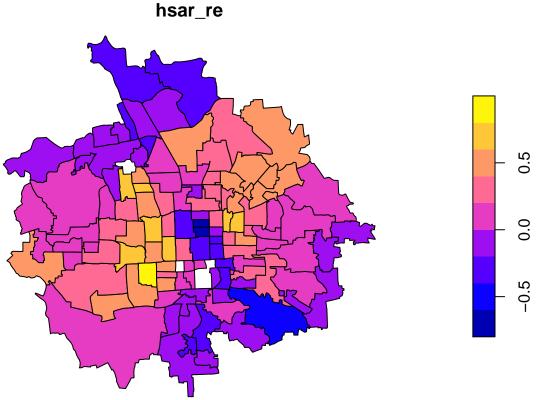
Using the M sparse spatial weights matrix, fit a model with district level simultaneous error autoregression; without spatially lagged covariates

```
m_hsar <- hsar(form, data=landprice_sf, W=NULL, M=M, Delta=Delta, burnin=500, Nsim=5000, thinning=1)
Beijingdistricts_sf$hsar_re <- m_hsar$Mus[1,]</pre>
```

The IID and SAR random effects are rather similar

```
plot(Beijingdistricts_sf[,"mlm_re"])
```





We do not have tests for residual autocorrelation for these fitted multilevel models, so (speculatively) let's

```
copy out the district level random effects to the parcels, checking first for matching
```

```
o <- match(landprice_sf$district.id.x, Beijingdistricts_sf$id1)
landprice_sf$id1 <- Beijingdistricts_sf$id1[o]</pre>
all.equal(landprice_sf$district.id.x, landprice_sf$id1)
## [1] TRUE
landprice_sf$mlm_re <- Beijingdistricts_sf$mlm_re[o]</pre>
landprice_sf$hsar_re <- Beijingdistricts_sf$hsar_re[o]</pre>
Now we can refit the SLX model but including the district level IID random effect, and test the residual
autocorrelation
spdep::lm.morantest(lmSLX(update(form, . ~ . + mlm_re), data=landprice_sf, listw=lw, Durbin= ~ log(area
##
##
   Global Moran I for regression residuals
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## Moran I statistic standard deviate = 2.7727, p-value = 0.002779
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I
                          Expectation
                                               Variance
                         -0.007654903
##
        0.028860257
                                            0.000173432
and for the spatially structured random effect
spdep::lm.morantest(lmSLX(update(form, . ~ . + hsar_re), data=landprice_sf, listw=lw, Durbin= ~ log(are
##
##
   Global Moran I for regression residuals
##
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
## Moran I statistic standard deviate = 3.3167, p-value = 0.0004554
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I
                          Expectation
                                               Variance
       0.0359635684
                        -0.0077079227
##
                                           0.0001733743
Next fit SDEM models with the IID random effect, and it turns out that the SLX model does almost as well,
so maybe most of the residual autocorrelation was at the district level rather than the parcel level?
SDEM1 <- errorsarlm(update(form, . ~ . + mlm_re), data=landprice_sf, listw=lw, Durbin= ~ log(area) + log
LR1.sarlm(SDEM1)
##
   Likelihood Ratio diagnostics for spatial dependence
##
##
## data:
```

```
## Likelihood ratio = 4.3635, df = 1, p-value = 0.03672
## sample estimates:
## Log likelihood of spatial error model
##
                                -1224.315
##
             Log likelihood of OLS fit y
##
                                -1226.496
The SSRE doesn't do as well (perhaps because it oversmooths the districts)
SDEM2 <- errorsarlm(update(form, . ~ . + hsar_re), data=landprice_sf, listw=lw, Durbin= ~ log(area) + 1
LR1.sarlm(SDEM2)
##
## Likelihood Ratio diagnostics for spatial dependence
##
## Likelihood ratio = 6.6411, df = 1, p-value = 0.009965
## sample estimates:
## Log likelihood of spatial error model
##
                                -1229.587
             Log likelihood of OLS fit y
##
##
                                -1232.908
```

Isn't spatial regression fun!!