

# R: Spatial regression

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## Required current contributed CRAN packages:

I am running R 3.6.1, with recent `update.packages()`.

```
needed <- c("MatrixModels", "lme4", "spatialreg", "spdep", "sf", "sp", "HSAR")
```

## Beijing data set

```
library(HSAR)
library(sp)
data(landSPDF)
data(landprice)
data(Beijingdistricts)
```

```
library(sf)
```

```
## Linking to GEOS 3.7.2, GDAL 3.0.1, PROJ 6.2.0
```

```
land_sf <- st_as_sf(landSPDF)
landprice_sf <- merge(land_sf, landprice, by="obs")
(landprice_sf <- landprice_sf[order(landprice_sf$district.id.x),])
```

```
## Simple feature collection with 1117 features and 12 fields
```

```
## geometry type: POINT
```

```
## dimension: XY
```

```
## bbox: xmin: 428553.1 ymin: 4406815 xmax: 463693.2 ymax: 4440423
```

```
## epsg (SRID): NA
```

```
## proj4string: +proj=tmerc +lat_0=0 +lon_0=117 +k=1 +x_0=500000 +y_0=0 +ellps=krass +units=m +no_de
```

```
## First 10 features:
```

```
##      obs district.id.x lnprice  lnarea  lndcbd dsubway  dpark    dele
## 187 189              3 5.57430 10.27820 9.94866 6.83023 7.06579 6.81916
## 188 190              3 7.16382 11.58780 9.93534 7.14334 6.78243 6.67827
## 700 968              3 7.61282 8.94551 9.91779 7.64360 6.84364 4.60356
## 701 969              3 6.81564 5.81928 9.91940 7.64640 6.88254 4.10025
## 702 970              3 6.93528 7.71869 9.91752 7.65810 6.86760 4.53460
## 709 992              3 7.45757 9.20029 9.84785 7.78904 6.95662 7.05138
## 710 993              3 7.12569 7.97788 9.84388 7.81991 7.00792 7.11267
## 711 994              3 7.48522 7.78634 9.84203 7.83398 7.03089 7.13981
## 717 1001             3 5.87349 10.70910 9.95534 7.89121 7.11019 5.67984
## 181 183              5 6.79302 6.39403 9.92025 6.76006 6.23524 6.10494
```

```
##      popden crimerate district.id.y year      geometry
```

```
## 187 0.548966 10.75110          3    1 POINT (430237 4422804)
## 188 0.548966 10.75110          3    1 POINT (430547.1 4423001)
## 700 0.548966 10.75110          3    0 POINT (431029.4 4423667)
## 701 0.548966 10.75110          3    0 POINT (431001.8 4423695)
## 702 0.548966 10.75110          3    0 POINT (431040.9 4423697)
## 709 0.548966 10.75110          3    0 POINT (432164 4422080)
```

```
## 710 0.548966 10.75110      3  0 POINT (432239.7 4422081)
## 711 0.548966 10.75110      3  0 POINT (432275 4422082)
## 717 0.548966 10.75110      3  0 POINT (430436.8 4424601)
## 181 1.407250  2.25832      5  1 POINT (430606.5 4420186)
```

```
all.equal(landprice_sf$district.id.x, landprice_sf$district.id.y)
```

```
## [1] TRUE
```

```
library(spatialreg)
```

```
## Loading required package: spData
```

```
## Loading required package: Matrix
```

```
## Registered S3 methods overwritten by 'spatialreg':
```

## method	from
## residuals.stsls	spdep
## deviance.stsls	spdep
## coef.stsls	spdep
## print.stsls	spdep
## summary.stsls	spdep
## print.summary.stsls	spdep
## residuals.gmsar	spdep
## deviance.gmsar	spdep
## coef.gmsar	spdep
## fitted.gmsar	spdep
## print.gmsar	spdep
## summary.gmsar	spdep
## print.summary.gmsar	spdep
## print.lagmess	spdep
## summary.lagmess	spdep
## print.summary.lagmess	spdep
## residuals.lagmess	spdep
## deviance.lagmess	spdep
## coef.lagmess	spdep
## fitted.lagmess	spdep
## logLik.lagmess	spdep
## fitted.SFResult	spdep
## print.SFResult	spdep
## fitted.ME_res	spdep
## print.ME_res	spdep
## print.lagImpact	spdep
## plot.lagImpact	spdep
## summary.lagImpact	spdep
## HPDinterval.lagImpact	spdep
## print.summary.lagImpact	spdep
## print.sarlm	spdep
## summary.sarlm	spdep
## residuals.sarlm	spdep
## deviance.sarlm	spdep
## coef.sarlm	spdep
## vcov.sarlm	spdep
## fitted.sarlm	spdep
## logLik.sarlm	spdep
## anova.sarlm	spdep

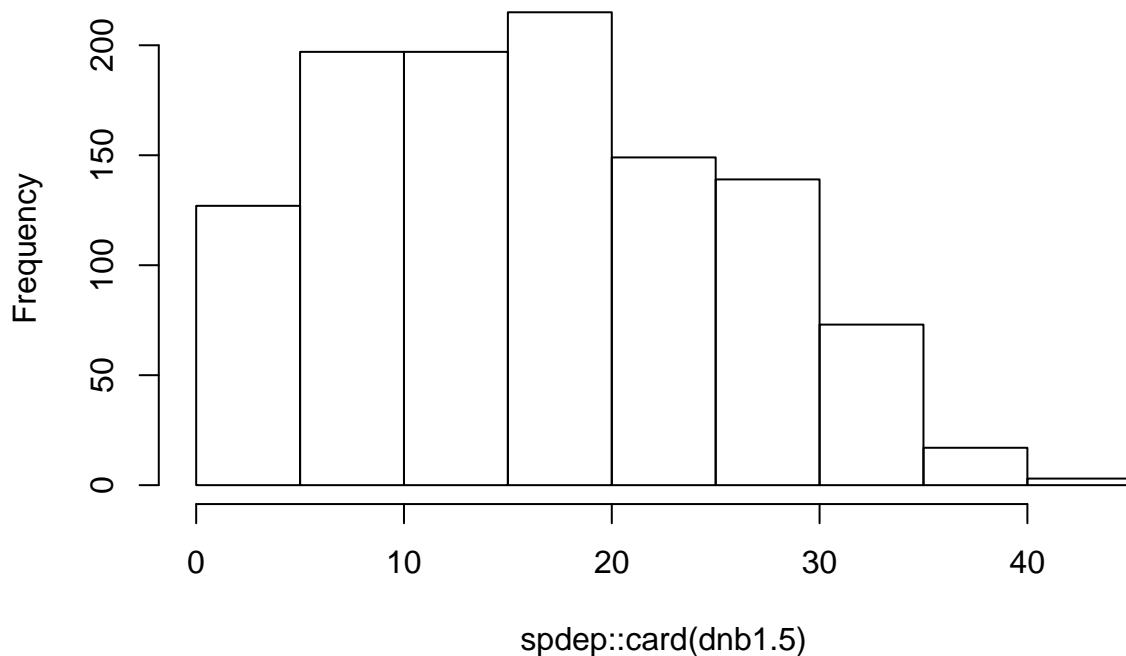
```
## predict.sarlm          spdep
## print.summary.sarlm    spdep
## print.sarlm.pred       spdep
## as.data.frame.sarlm.pred spdep
## residuals.spautolm     spdep
## deviance.spautolm      spdep
## coef.spautolm          spdep
## fitted.spautolm        spdep
## print.spautolm         spdep
## summary.spautolm       spdep
## logLik.spautolm        spdep
## print.summary.spautolm spdep
## print.WXImpact         spdep
## summary.WXImpact       spdep
## print.summary.WXImpact spdep
## predict.SLX            spdep

dnb1.5 <- spdep::dnearneigh(landprice_sf, 0, 1500, row.names=as.character(landprice_sf$obs))
dists <- spdep::nbdists(dnb1.5, st_geometry(landprice_sf))
edists <- lapply(dists, function(x) exp(-(x/1000)^2)/(1.5^2)))
ozpo <- spdep::set.ZeroPolicyOption(TRUE)
set.ZeroPolicyOption(TRUE)

## [1] FALSE

lw <- spdep::nb2listw(dnb1.5, glist=edists, style="W")
hist(spdep::card(dnb1.5))
```

**Histogram of `spdep::card(dnb1.5)`**



```
landprice_sf$year <- factor(landprice_sf$year + 2003)
landprice_sf$price <- exp(landprice_sf$lnprice)
landprice_sf$area <- exp(landprice_sf$lnarea)
```

```

landprice_sf$Dcbd <- exp(landprice_sf$lndcbd)
landprice_sf$Dsubway <- exp(landprice_sf$dsubway)
landprice_sf$Dpark <- exp(landprice_sf$dpark)
landprice_sf$Dele <- exp(landprice_sf$dele)
landprice_sf$f_district.id <- factor(landprice_sf$district.id.x)
(t1 <- table(table(landprice_sf$f_district.id)))

##
##  1  2  3  4  5  6  7  8  9 10 11 12 14 15 16 17 18 19 20 21 23 25 26 27 28
##  7  8 10  4  5 10  8 10  8  4  6  2  4  3  1  2  5  2  1  1  1  1  1  1  2
## 31 32 33 52
##  1  1  1  1

Beijingdistricts$id1 <- Beijingdistricts$id+1
all.equal(unique(landprice_sf$district.id.x), Beijingdistricts$id1)

## [1] TRUE

(Beijingdistricts_sf <- st_as_sf(Beijingdistricts))

## Simple feature collection with 111 features and 2 fields
## geometry type:  MULTIPOLYGON
## dimension:      XY
## bbox:           xmin: 426987.3 ymin: 4403559 xmax: 467920.9 ymax: 4443287
## epsg (SRID):    NA
## proj4string:     +proj=tmerc +lat_0=0 +lon_0=117 +k=1 +x_0=500000 +y_0=0 +ellps=krass +units=m +no_de
## First 10 features:
##   id id1 geometry
## 0  2   3 MULTIPOLYGON (((428183.1 44...
## 1  4   5 MULTIPOLYGON (((432472.2 44...
## 2  6   7 MULTIPOLYGON (((432446.1 44...
## 3  7   8 MULTIPOLYGON (((433534.5 44...
## 4  8   9 MULTIPOLYGON (((443807.9 44...
## 5  9  10 MULTIPOLYGON (((444461 4420...
## 6 10  11 MULTIPOLYGON (((447530.6 44...
## 7 11  12 MULTIPOLYGON (((443849.6 44...
## 8 12  13 MULTIPOLYGON (((446810 4417...
## 9 13  14 MULTIPOLYGON (((445954.8 44...

t2 <- table(sapply(st_contains(Beijingdistricts_sf, landprice_sf), length))
all.equal(t1, t2)

## [1] TRUE

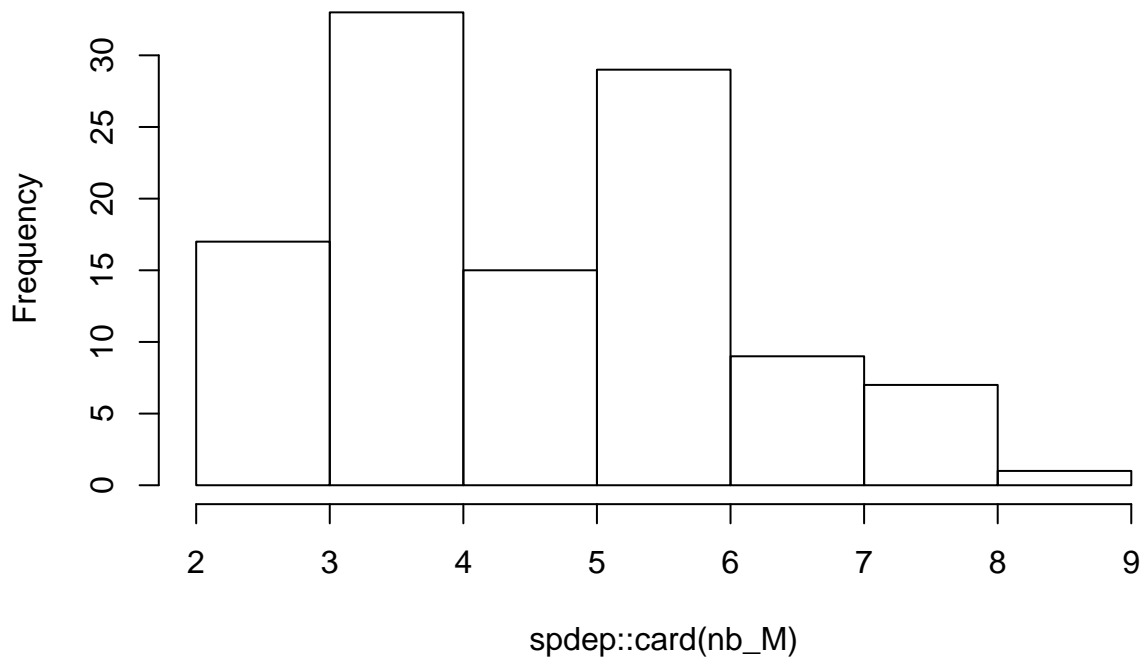
nb_M <- spdep::poly2nb(Beijingdistricts, queen=FALSE, row.names=as.character(Beijingdistricts$id1))
M <- as(spdep::nb2listw(nb_M, style="B"), "CsparseMatrix")
dim(M)

## [1] 111 111

hist(spdep::card(nb_M))

```

## Histogram of spdep::card(nb\_M)



```
form <- log(price) ~ log(area) + log(Dcbd) + log(Dele) + log(Dpark) + log(Dsubway) +
  crimerate + popden + fyear
OLS <- lm(form, data=landprice_sf)
summary(OLS)
```

```
##
## Call:
## lm(formula = form, data = landprice_sf)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.5915 -0.5752 -0.0496  0.5206  3.4042
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  14.153917   0.370103  38.243  < 2e-16 ***
## log(area)    -0.008253   0.018675  -0.442  0.65863
## log(Dcbd)    -0.250601   0.047752  -5.248  1.84e-07 ***
## log(Dele)    -0.085528   0.032308  -2.647  0.00823 **
## log(Dpark)   -0.284372   0.046046  -6.176  9.24e-10 ***
## log(Dsubway) -0.245748   0.034755  -7.071  2.73e-12 ***
## crimerate     0.007668   0.004458   1.720  0.08575 .
## popden        0.032827   0.010304   3.186  0.00148 **
## fyear2004    -0.164503   0.058380  -2.818  0.00492 **
## fyear2005     0.017635   0.124986   0.141  0.88782
## fyear2006    -0.120314   0.107209  -1.122  0.26201
## fyear2007     0.551384   0.117431   4.695  3.00e-06 ***
## fyear2008     0.396172   0.129571   3.058  0.00229 **
## fyear2009     2.113691   0.228688   9.243  < 2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8335 on 1103 degrees of freedom
## Multiple R-squared:  0.3524, Adjusted R-squared:  0.3448
## F-statistic: 46.17 on 13 and 1103 DF,  p-value: < 2.2e-16
```

```
spdep::lm.morantest(OLS, listw=lw)
```

```
##
## Global Moran I for regression residuals
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## Moran I statistic standard deviate = 15.1, p-value < 2.2e-16
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I      Expectation      Variance
##    0.1944768641    -0.0054494313    0.0001753028
```

```
spdep::lm.LMtests(OLS, listw=lw, test="all")
```

```
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## LMerr = 207.15, df = 1, p-value < 2.2e-16
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## LMlag = 92.712, df = 1, p-value < 2.2e-16
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## RLMerr = 118.22, df = 1, p-value < 2.2e-16
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
```

```
## weights: lw
##
## RLMlag = 3.7725, df = 1, p-value = 0.0521
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## SARMA = 210.93, df = 2, p-value < 2.2e-16
SLX <- lmSLX(form, data=landprice_sf, listw=lw)
summary(impacts(SLX))
```

```
## Impact measures (SLX, estimable, n-k):
##
```

	Direct	Indirect	Total
## log(area)	-0.027393865	0.090812258	0.063418393
## log(Dcbd)	-0.586485128	0.362485133	-0.223999995
## log(Dele)	0.017324132	-0.159256358	-0.141932226
## log(Dpark)	-0.091259953	-0.212539298	-0.303799251
## log(Dsubway)	-0.176849400	-0.146595779	-0.323445180
## crimerate	0.003700684	0.006267203	0.009967887
## popden	0.003018936	0.038261481	0.041280417
## fyear2004	-0.203603540	0.448210479	0.244606939
## fyear2005	-0.037845735	0.874443554	0.836597820
## fyear2006	-0.117303538	0.212143967	0.094840429
## fyear2007	0.689975245	-0.246715186	0.443260059
## fyear2008	0.465854623	-0.266940282	0.198914342
## fyear2009	2.203749162	0.406971450	2.610720612

```
## =====
## Standard errors:
##
```

	Direct	Indirect	Total
## log(area)	0.019571325	0.04274094	0.041806133
## log(Dcbd)	0.123554643	0.13231893	0.053287909
## log(Dele)	0.051056912	0.06662771	0.043850829
## log(Dpark)	0.107566463	0.13104491	0.061869504
## log(Dsubway)	0.059404607	0.07922483	0.049145364
## crimerate	0.009139954	0.01132696	0.005692364
## popden	0.017518547	0.02466541	0.014932683
## fyear2004	0.059967444	0.14620810	0.144181383
## fyear2005	0.125200756	0.34549622	0.348471521
## fyear2006	0.108193628	0.31019156	0.314906166
## fyear2007	0.126343166	0.24786610	0.234084184
## fyear2008	0.131769313	0.36565659	0.364477241
## fyear2009	0.226910003	0.50284041	0.549874172

```
## =====
## Z-values:
##
```

	Direct	Indirect	Total
## log(area)	-1.3996939	2.1247136	1.5169639
## log(Dcbd)	-4.7467672	2.7394806	-4.2035802
## log(Dele)	0.3393102	-2.3902422	-3.2367056
## log(Dpark)	-0.8484053	-1.6218813	-4.9103231
## log(Dsubway)	-2.9770318	-1.8503767	-6.5813976

```
## crimerate      0.4048909  0.5532996  1.7510981
## popden         0.1723280  1.5512203  2.7644339
## fyear2004     -3.3952346  3.0655654  1.6965224
## fyear2005     -0.3022804  2.5309786  2.4007638
## fyear2006     -1.0842001  0.6839128  0.3011704
## fyear2007      5.4611204 -0.9953567  1.8935925
## fyear2008      3.5353802 -0.7300300  0.5457524
## fyear2009      9.7119965  0.8093452  4.7478509
##
## p-values:
##               Direct      Indirect  Total
## log(area)      0.16160499  0.0336105  0.1292758
## log(Dcbd)      2.0669e-06  0.0061536  2.6273e-05
## log(Dele)      0.73437605  0.0168373  0.0012092
## log(Dpark)     0.39621231  0.1048288  9.0926e-07
## log(Dsubway)   0.00291054  0.0642593  4.6605e-11
## crimerate      0.68555771  0.5800583  0.0799290
## popden         0.86317968  0.1208489  0.0057022
## fyear2004      0.00068570  0.0021726  0.0897870
## fyear2005      0.76243832  0.0113745  0.0163609
## fyear2006      0.27827611  0.4940303  0.7632845
## fyear2007      4.7314e-08  0.3195628  0.0582791
## fyear2008      0.00040719  0.4653718  0.5852361
## fyear2009      < 2.22e-16  0.4183166  2.0559e-06
```

```
spdep::lm.morantest(SLX, listw=lw)
```

```
##
## Global Moran I for regression residuals
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## Moran I statistic standard deviate = 14.294, p-value < 2.2e-16
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I      Expectation      Variance
##      0.1768367391     -0.0095750057      0.0001700666
```

```
spdep::lm.LMtests(SLX, listw=lw, test="all")
```

```
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## LMerr = 171.28, df = 1, p-value < 2.2e-16
##
##
## Lagrange multiplier diagnostics for spatial dependence
```



```

##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## LMlag = 166.26, df = 1, p-value < 2.2e-16
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## RLMerr = 7.3124, df = 1, p-value = 0.006848
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## RLMlag = 2.2935, df = 1, p-value = 0.1299
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## SARMA = 173.57, df = 2, p-value < 2.2e-16
e <- eigenw(lw)
SDEM <- errorsarlm(form, data=landprice_sf, listw=lw, Durbin=TRUE, control=list(pre_eig=e))
summary(impacts(SDEM))

## Impact measures (SDEM, estimable, n):
##
##           Direct      Indirect      Total
## log(area)   -0.025312192  0.0899318870  0.064619695
## log(Dcbd)   -0.588066020  0.3297094158 -0.258356604
## log(Dele)   -0.002311498 -0.1215960956 -0.123907593
## log(Dpark)  -0.081889383 -0.2187482895 -0.300637673
## log(Dsubway) -0.168100990 -0.1136152278 -0.281716218
## crimerate    0.006368132 -0.0007289184  0.005639214
## popden       0.004766271  0.0394667820  0.044233053
## fyear2004   -0.206852047  0.2244675077  0.017615461
## fyear2005   -0.047901527  0.5532037864  0.505302260
## fyear2006   -0.132401052  0.0290395422 -0.103361510
## fyear2007    0.693289805 -0.4672464525  0.226043352

```

```

## fyear2008      0.485618731 -0.1354646418  0.350154090
## fyear2009      2.299169487  0.7608233880  3.059992875
## =====
## Standard errors:
##               Direct      Indirect      Total
## log(area)      0.018213195  0.05439058  0.059111166
## log(Dcbd)      0.125909677  0.14337589  0.091299942
## log(Dele)      0.045766222  0.08001448  0.071996590
## log(Dpark)     0.101522411  0.14359787  0.096960618
## log(Dsubway)   0.054624512  0.09528292  0.079291144
## crimerate      0.008310533  0.01266441  0.009369951
## popden         0.015882383  0.02871415  0.023371534
## fyear2004      0.055951753  0.19532568  0.209065378
## fyear2005      0.118378556  0.44432426  0.480862483
## fyear2006      0.102489167  0.35908167  0.392488897
## fyear2007      0.116560989  0.32719106  0.350864647
## fyear2008      0.123098193  0.45396729  0.483946755
## fyear2009      0.222961274  0.56093975  0.677756394
## =====
## Z-values:
##               Direct      Indirect      Total
## log(area)     -1.38977220   1.65344597   1.09318930
## log(Dcbd)     -4.67053869   2.29961551  -2.82975649
## log(Dele)     -0.05050663  -1.51967605  -1.72102031
## log(Dpark)    -0.80661386  -1.52333933  -3.10061630
## log(Dsubway)  -3.07739115  -1.19239863  -3.55293420
## crimerate      0.76627244  -0.05755644   0.60184029
## popden         0.30009801   1.37447145   1.89260378
## fyear2004     -3.69697167   1.14919608   0.08425815
## fyear2005     -0.40464699   1.24504519   1.05082488
## fyear2006     -1.29185411   0.08087169  -0.26334888
## fyear2007      5.94787165  -1.42805385   0.64424659
## fyear2008      3.94497044  -0.29840177   0.72353846
## fyear2009     10.31196782   1.35633709   4.51488603
##
## p-values:
##               Direct      Indirect Total
## log(area)      0.16459806  0.09824  0.27431069
## log(Dcbd)      3.0041e-06  0.02147  0.00465834
## log(Dele)      0.95971867  0.12859  0.08524714
## log(Dpark)     0.41988899  0.12767  0.00193118
## log(Dsubway)   0.00208821  0.23310  0.00038096
## crimerate      0.44351421  0.95410  0.54728045
## popden         0.76410240  0.16930  0.05841058
## fyear2004      0.00021819  0.25048  0.93285119
## fyear2005      0.68573701  0.21312  0.29333903
## fyear2006      0.19640767  0.93554  0.79228169
## fyear2007      2.7165e-09  0.15328  0.51941554
## fyear2008      7.9810e-05  0.76540  0.46934913
## fyear2009      < 2.22e-16  0.17499  6.3351e-06

```

```
LR1.sarlm(SDEM)
```

```

##
## Likelihood Ratio diagnostics for spatial dependence

```

```
##
## data:
## Likelihood ratio = 106.85, df = 1, p-value < 2.2e-16
## sample estimates:
## Log likelihood of spatial error model
##                               -1296.305
##       Log likelihood of OLS fit y
##                               -1349.730

Hausman.test(SDEM)

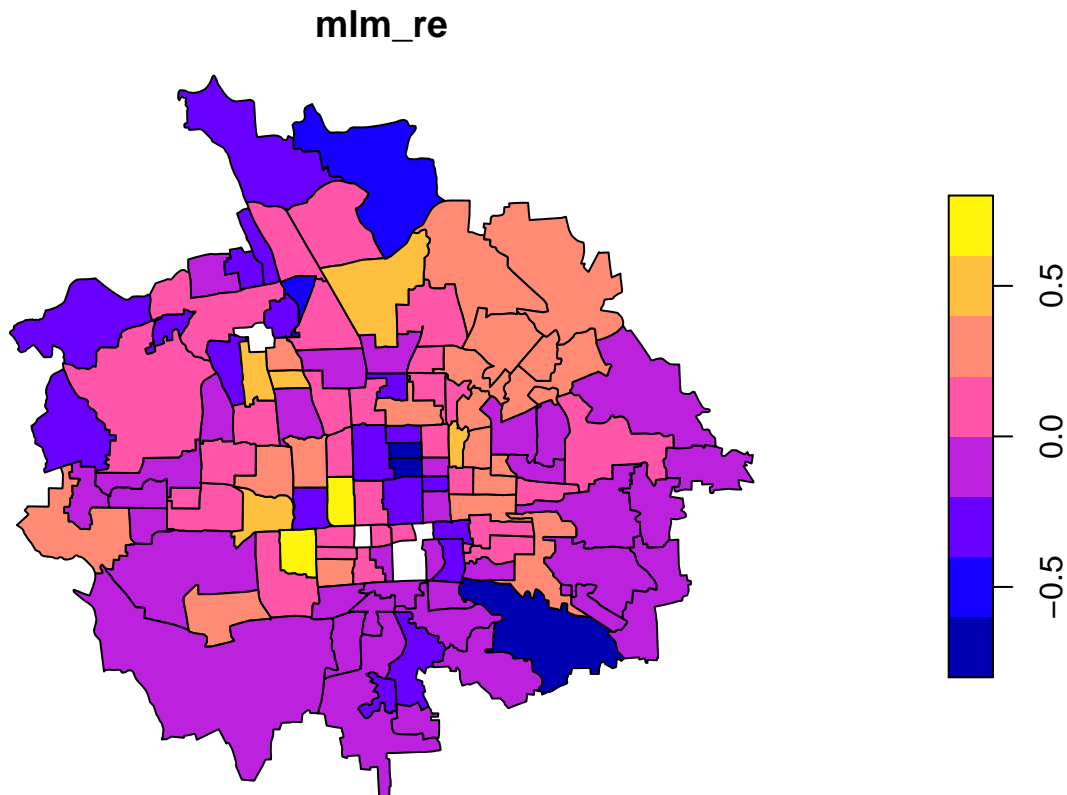
##
## Spatial Hausman test (asymptotic)
##
## data:  NULL
## Hausman test = 12.872, df = 27, p-value = 0.99

library(lme4)
mlm_1 <- lmer(update(form, . ~ . + (1 | f_district.id)), data=landprice_sf, REML=FALSE)
Beijingdistricts_sf$mlm_re <- ranef(mlm_1)[[1]][,1]

library(Matrix)
suppressMessages(library(MatrixModels))
Delta <- as(model.Matrix(~ -1 + f_district.id, data=landprice_sf, sparse=TRUE), "dgCMatrix")

m_hsar <- hsar(form, data=landprice_sf, W=NULL, M=M, Delta=Delta, burnin=500, Nsim=5000, thinning=1)
Beijingdistricts_sf$hsar_re <- m_hsar$Mus[1,]

plot(Beijingdistricts_sf[, "mlm_re"])
```



```
plot(Beijingdistricts_sf[, "hsar_re"])
```

