# R: Spatial regression

### Roger Bivand

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#### Required current contributed CRAN packages:

I am running R 3.6.1, with recent update.packages().

**##** 187 0.548966 10.75110

## 188 0.548966 10.75110

## 700 0.548966 10.75110

## 701 0.548966 10.75110

## 702 0.548966 10.75110

## 709 0.548966 10.75110

```
needed <- c("sf", "stars", "sp", "classInt", "raster", "colorspace", "RColorBrewer", "ggplot2", "cartog</pre>
Beijing data set
library(HSAR)
library(sp)
data(landSPDF)
data(landprice)
data(Beijingdistricts)
library(sf)
## Linking to GEOS 3.7.2, GDAL 3.0.1, PROJ 6.2.0
land_sf <- st_as_sf(landSPDF)</pre>
landprice_sf <- merge(land_sf, landprice, by="obs")</pre>
(landprice_sf <- landprice_sf[order(landprice_sf$district.id.x),])</pre>
## Simple feature collection with 1117 features and 12 fields
## geometry type: POINT
## dimension:
                   XΥ
                   xmin: 428553.1 ymin: 4406815 xmax: 463693.2 ymax: 4440423
## bbox:
## epsg (SRID):
## proj4string:
                   +proj=tmerc +lat_0=0 +lon_0=117 +k=1 +x_0=500000 +y_0=0 +ellps=krass +units=m +no_de
## First 10 features:
        obs district.id.x lnprice
                                    lnarea lndcbd dsubway
                                                              dpark
## 187 189
                        3 5.57430 10.27820 9.94866 6.83023 7.06579 6.81916
## 188 190
                        3 7.16382 11.58780 9.93534 7.14334 6.78243 6.67827
## 700
       968
                        3 7.61282 8.94551 9.91779 7.64360 6.84364 4.60356
## 701
       969
                        3 6.81564 5.81928 9.91940 7.64640 6.88254 4.10025
## 702
       970
                        3 6.93528 7.71869 9.91752 7.65810 6.86760 4.53460
                        3 7.45757 9.20029 9.84785 7.78904 6.95662 7.05138
## 709
       992
                        3 7.12569 7.97788 9.84388 7.81991 7.00792 7.11267
## 710
       993
## 711
       994
                        3 7.48522 7.78634 9.84203 7.83398 7.03089 7.13981
## 717 1001
                        3 5.87349 10.70910 9.95534 7.89121 7.11019 5.67984
## 181 183
                        5 6.79302 6.39403 9.92025 6.76006 6.23524 6.10494
##
         popden crimerate district.id.y year
                                                              geometry
```

POINT (430237 4422804)

POINT (432164 4422080)

1 POINT (430547.1 4423001)

0 POINT (431029.4 4423667)

0 POINT (431001.8 4423695)

0 POINT (431040.9 4423697)

3

3

3

3

3

3

1

```
## 710 0.548966 10.75110
                                       3
                                            0 POINT (432239.7 4422081)
                                                POINT (432275 4422082)
## 711 0.548966 10.75110
                                       3
## 717 0.548966 10.75110
                                            0 POINT (430436.8 4424601)
                                       3
## 181 1.407250
                  2.25832
                                            1 POINT (430606.5 4420186)
all.equal(landprice_sf$district.id.x, landprice_sf$district.id.y)
## [1] TRUE
library(spatialreg)
## Loading required package: spData
## Loading required package: Matrix
## Registered S3 methods overwritten by 'spatialreg':
##
     method
                               from
     residuals.stsls
##
                               spdep
     deviance.stsls
##
                               spdep
##
     coef.stsls
                               spdep
##
     print.stsls
                               spdep
##
     summary.stsls
                               spdep
##
     print.summary.stsls
                               spdep
##
     residuals.gmsar
                               spdep
##
     deviance.gmsar
                               spdep
##
     coef.gmsar
                               spdep
##
     fitted.gmsar
                               spdep
##
     print.gmsar
                               spdep
##
     summary.gmsar
                               spdep
##
     print.summary.gmsar
                               spdep
##
     print.lagmess
                               spdep
##
     summary.lagmess
                               spdep
##
     print.summary.lagmess
                               spdep
##
     residuals.lagmess
                               spdep
##
     deviance.lagmess
                               spdep
##
     coef.lagmess
                               spdep
##
     fitted.lagmess
                               spdep
##
     logLik.lagmess
                               spdep
##
     fitted.SFResult
                               spdep
##
     print.SFResult
                               spdep
##
     fitted.ME_res
                               spdep
##
     print.ME_res
                               spdep
##
     print.lagImpact
                               spdep
##
     plot.lagImpact
                               spdep
##
     summary.lagImpact
                               spdep
##
     HPDinterval.lagImpact
                               spdep
##
     print.summary.lagImpact
                               spdep
##
     print.sarlm
                               spdep
##
     summary.sarlm
                               spdep
##
     residuals.sarlm
                               spdep
##
     deviance.sarlm
                               spdep
##
     coef.sarlm
                               spdep
```

spdep

spdep

spdep

spdep

##

##

##

##

vcov.sarlm

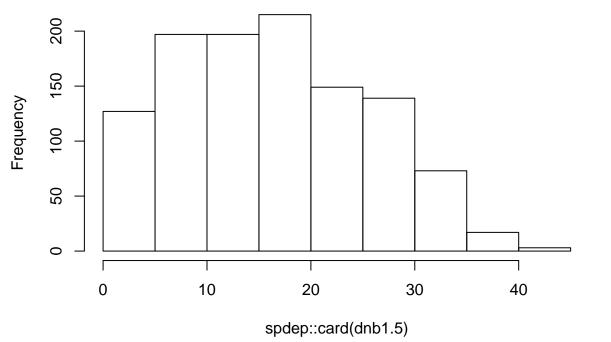
fitted.sarlm

logLik.sarlm

anova.sarlm

```
predict.sarlm
##
                                spdep
##
     print.summary.sarlm
                                spdep
##
     print.sarlm.pred
                                spdep
##
     as.data.frame.sarlm.pred spdep
##
     residuals.spautolm
                                spdep
##
     deviance.spautolm
                                spdep
##
     coef.spautolm
                                spdep
##
     fitted.spautolm
                                spdep
##
     print.spautolm
                                spdep
##
     summary.spautolm
                                spdep
##
     logLik.spautolm
                                spdep
##
     print.summary.spautolm
                                spdep
##
     print.WXImpact
                                spdep
##
     summary.WXImpact
                                spdep
##
     print.summary.WXImpact
                                spdep
##
     predict.SLX
                                spdep
dnb1.5 <- spdep::dnearneigh(landprice_sf, 0, 1500, row.names=as.character(landprice_sf$obs))</pre>
dists <- spdep::nbdists(dnb1.5, st_geometry(landprice_sf))</pre>
edists \leftarrow lapply(dists, function(x) exp((-((x/1000)^2))/(1.5^2)))
ozpo <- spdep::set.ZeroPolicyOption(TRUE)</pre>
set.ZeroPolicyOption(TRUE)
## [1] FALSE
lw <- spdep::nb2listw(dnb1.5, glist=edists, style="W")</pre>
hist(spdep::card(dnb1.5))
```

# Histogram of spdep::card(dnb1.5)



```
landprice_sf$fyear <- factor(landprice_sf$year + 2003)
landprice_sf$price <- exp(landprice_sf$lnprice)
landprice_sf$area <- exp(landprice_sf$lnarea)</pre>
```

```
landprice_sf$Dcbd <- exp(landprice_sf$Indcbd)</pre>
landprice_sf$Dsubway <- exp(landprice_sf$dsubway)</pre>
landprice_sf$Dpark <- exp(landprice_sf$dpark)</pre>
landprice_sf$Dele <- exp(landprice_sf$dele)</pre>
landprice_sf$f_district.id <- factor(landprice_sf$district.id.x)</pre>
(t1 <- table(table(landprice_sf$f_district.id)))</pre>
##
## 1 2 3 4 5 6 7 8 9 10 11 12 14 15 16 17 18 19 20 21 23 25 26 27 28
## 7 8 10 4 5 10 8 10 8 4 6 2 4 3 1 2 5 2 1 1 1 1 1 1 2
## 31 32 33 52
## 1 1 1 1
Beijingdistricts$id1 <- Beijingdistricts$id+1
all.equal(unique(landprice_sf$district.id.x), Beijingdistricts$id1)
## [1] TRUE
(Beijingdistricts_sf <- st_as_sf(Beijingdistricts))
## Simple feature collection with 111 features and 2 fields
## geometry type: MULTIPOLYGON
                  XY
## dimension:
## bbox:
                   xmin: 426987.3 ymin: 4403559 xmax: 467920.9 ymax: 4443287
## epsg (SRID):
## proj4string:
                   +proj=tmerc +lat_0=0 +lon_0=117 +k=1 +x_0=500000 +y_0=0 +ellps=krass +units=m +no_de
## First 10 features:
##
    id id1
                                  geometry
## 0 2 3 MULTIPOLYGON (((428183.1 44...
## 1 4 5 MULTIPOLYGON (((432472.2 44...
## 2 6 7 MULTIPOLYGON (((432446.1 44...
## 3 7 8 MULTIPOLYGON (((433534.5 44...
## 4 8 9 MULTIPOLYGON (((443807.9 44...
## 5 9 10 MULTIPOLYGON (((444461 4420...
## 6 10 11 MULTIPOLYGON (((447530.6 44...
## 7 11 12 MULTIPOLYGON (((443849.6 44...
## 8 12 13 MULTIPOLYGON (((446810 4417...
## 9 13 14 MULTIPOLYGON (((445954.8 44...
t2 <- table(sapply(st_contains(Beijingdistricts_sf, landprice_sf), length))
all.equal(t1, t2)
## [1] TRUE
nb_M <- spdep::poly2nb(Beijingdistricts, queen=FALSE, row.names=as.character(Beijingdistricts$id1))
M <- as(spdep::nb2listw(nb_M, style="B"), "CsparseMatrix")</pre>
dim(M)
## [1] 111 111
hist(spdep::card(nb_M))
```

# Histogram of spdep::card(nb\_M)

```
25
Frequency
     20
     15
     10
     2
     0
             2
                      3
                                4
                                          5
                                                    6
                                                              7
                                                                        8
                                                                                  9
                                      spdep::card(nb_M)
form <- log(price) ~ log(area) + log(Dcbd) + log(Dele) + log(Dpark) + log(Dsubway) +
  crimerate + popden + fyear
OLS <- lm(form, data=landprice_sf)
summary(OLS)
##
## Call:
## lm(formula = form, data = landprice_sf)
##
## Residuals:
       Min
                1Q Median
                                 3Q
                                        Max
## -2.5915 -0.5752 -0.0496 0.5206 3.4042
##
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                14.153917
                            0.370103 38.243 < 2e-16 ***
## log(area)
                -0.008253
                            0.018675
                                       -0.442
                                               0.65863
## log(Dcbd)
                -0.250601
                            0.047752
                                       -5.248 1.84e-07 ***
## log(Dele)
                -0.085528
                            0.032308
                                       -2.647 0.00823 **
## log(Dpark)
                -0.284372
                            0.046046
                                       -6.176 9.24e-10 ***
## log(Dsubway) -0.245748
                            0.034755
                                       -7.071 2.73e-12 ***
## crimerate
                                              0.08575 .
                 0.007668
                            0.004458
                                        1.720
## popden
                 0.032827
                            0.010304
                                        3.186
                                               0.00148 **
## fyear2004
                                       -2.818
                                               0.00492 **
                -0.164503
                            0.058380
## fyear2005
                 0.017635
                            0.124986
                                               0.88782
                                        0.141
## fyear2006
                -0.120314
                            0.107209
                                       -1.122 0.26201
## fyear2007
                 0.551384
                            0.117431
                                        4.695 3.00e-06 ***
## fyear2008
                                        3.058 0.00229 **
                 0.396172
                            0.129571
## fyear2009
                 2.113691
                            0.228688
                                        9.243 < 2e-16 ***
## ---
```

```
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.8335 on 1103 degrees of freedom
## Multiple R-squared: 0.3524, Adjusted R-squared: 0.3448
## F-statistic: 46.17 on 13 and 1103 DF, p-value: < 2.2e-16
spdep::lm.morantest(OLS, listw=lw)
##
##
   Global Moran I for regression residuals
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## Moran I statistic standard deviate = 15.1, p-value < 2.2e-16
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I
                         Expectation
                                             Variance
      0.1944768641
                       -0.0054494313
                                         0.0001753028
##
spdep::lm.LMtests(OLS, listw=lw, test="all")
## Lagrange multiplier diagnostics for spatial dependence
##
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## LMerr = 207.15, df = 1, p-value < 2.2e-16
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## LMlag = 92.712, df = 1, p-value < 2.2e-16
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
## RLMerr = 118.22, df = 1, p-value < 2.2e-16
##
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = form, data = landprice_sf)
```

```
## weights: lw
##
## RLMlag = 3.7725, df = 1, p-value = 0.0521
##
##
  Lagrange multiplier diagnostics for spatial dependence
##
##
## data:
## model: lm(formula = form, data = landprice_sf)
## weights: lw
##
## SARMA = 210.93, df = 2, p-value < 2.2e-16
SLX <- lmSLX(form, data=landprice_sf, listw=lw)</pre>
summary(impacts(SLX))
## Impact measures (SLX, estimable, n-k):
                              Indirect
                    Direct
                                             Total
## log(area)
               -0.027393865 0.090812258 0.063418393
## log(Dcbd)
              ## log(Dele)
               0.017324132 -0.159256358 -0.141932226
## log(Dpark)
              -0.091259953 -0.212539298 -0.303799251
## log(Dsubway) -0.176849400 -0.146595779 -0.323445180
## crimerate
               0.003700684 0.006267203 0.009967887
## popden
               ## fyear2004
               -0.203603540 0.448210479 0.244606939
## fyear2005
              ## fyear2006
              ## fyear2007
               0.689975245 -0.246715186 0.443260059
## fyear2008
               0.465854623 -0.266940282
                                        0.198914342
## fyear2009
               2.203749162 0.406971450
                                        2.610720612
## Standard errors:
                   Direct
                            Indirect
                                          Total
## log(area)
              0.019571325 0.04274094 0.041806133
## log(Dcbd)
              0.123554643 0.13231893 0.053287909
## log(Dele)
              0.051056912 0.06662771 0.043850829
## log(Dpark)
              0.107566463 0.13104491 0.061869504
## log(Dsubway) 0.059404607 0.07922483 0.049145364
## crimerate
              0.009139954 0.01132696 0.005692364
## popden
              0.017518547 0.02466541 0.014932683
## fyear2004
              0.059967444 \ 0.14620810 \ 0.144181383
## fyear2005
              0.125200756 0.34549622 0.348471521
## fyear2006
              0.108193628 0.31019156 0.314906166
## fyear2007
              0.126343166 0.24786610 0.234084184
## fyear2008
              0.131769313 0.36565659 0.364477241
## fyear2009
              0.226910003 0.50284041 0.549874172
## Z-values:
##
                         Indirect
                  Direct
                                        Total
## log(area)
              -1.3996939 2.1247136 1.5169639
## log(Dcbd)
              -4.7467672 2.7394806 -4.2035802
## log(Dele)
               0.3393102 -2.3902422 -3.2367056
## log(Dpark)
              -0.8484053 -1.6218813 -4.9103231
## log(Dsubway) -2.9770318 -1.8503767 -6.5813976
```

```
## crimerate
                0.4048909 0.5532996 1.7510981
## popden
                0.1723280 1.5512203 2.7644339
## fyear2004
               -3.3952346 3.0655654 1.6965224
## fyear2005
               -0.3022804 2.5309786 2.4007638
## fyear2006
               -1.0842001 0.6839128 0.3011704
## fyear2007
                5.4611204 -0.9953567 1.8935925
## fyear2008
                3.5353802 -0.7300300 0.5457524
                9.7119965 0.8093452 4.7478509
## fyear2009
##
## p-values:
               Direct
                           Indirect Total
## log(area)
               0.16160499 0.0336105 0.1292758
## log(Dcbd)
               2.0669e-06 0.0061536 2.6273e-05
## log(Dele)
               0.73437605 0.0168373 0.0012092
## log(Dpark)
               0.39621231 0.1048288 9.0926e-07
## log(Dsubway) 0.00291054 0.0642593 4.6605e-11
## crimerate
               0.68555771 0.5800583 0.0799290
## popden
               0.86317968 0.1208489 0.0057022
## fyear2004
               0.00068570 0.0021726 0.0897870
## fyear2005
               0.76243832 0.0113745 0.0163609
## fyear2006
               0.27827611 0.4940303 0.7632845
## fyear2007
               4.7314e-08 0.3195628 0.0582791
## fyear2008
               0.00040719 0.4653718 0.5852361
## fyear2009
               < 2.22e-16 0.4183166 2.0559e-06
spdep::lm.morantest(SLX, listw=lw)
##
  Global Moran I for regression residuals
##
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## Moran I statistic standard deviate = 14.294, p-value < 2.2e-16
## alternative hypothesis: greater
## sample estimates:
## Observed Moran I
                         Expectation
                                             Variance
##
       0.1768367391
                       -0.0095750057
                                         0.0001700666
spdep::lm.LMtests(SLX, listw=lw, test="all")
##
## Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## LMerr = 171.28, df = 1, p-value < 2.2e-16
##
##
##
  Lagrange multiplier diagnostics for spatial dependence
```

```
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## LMlag = 166.26, df = 1, p-value < 2.2e-16
##
##
##
  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## RLMerr = 7.3124, df = 1, p-value = 0.006848
##
##
##
   Lagrange multiplier diagnostics for spatial dependence
##
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
## RLMlag = 2.2935, df = 1, p-value = 0.1299
##
##
  Lagrange multiplier diagnostics for spatial dependence
##
## data:
## model: lm(formula = formula(paste("y ~ ", paste(colnames(x)[-1],
## collapse = "+"))), data = as.data.frame(x), weights = weights)
## weights: lw
##
## SARMA = 173.57, df = 2, p-value < 2.2e-16
e <- eigenw(lw)
SDEM <- errorsarlm(form, data=landprice_sf, listw=lw, Durbin=TRUE, control=list(pre_eig=e))
summary(impacts(SDEM))
## Impact measures (SDEM, estimable, n):
##
                    Direct
                                Indirect
              -0.025312192  0.0899318870  0.064619695
## log(area)
## log(Dcbd)
              ## log(Dele)
              -0.002311498 -0.1215960956 -0.123907593
              -0.081889383 -0.2187482895 -0.300637673
## log(Dpark)
## log(Dsubway) -0.168100990 -0.1136152278 -0.281716218
## crimerate 0.006368132 -0.0007289184 0.005639214
## popden
              0.004766271 0.0394667820 0.044233053
            -0.206852047 0.2244675077 0.017615461
## fyear2004
## fyear2005
              ## fyear2006
              ## fyear2007
               0.693289805 -0.4672464525 0.226043352
```

```
0.485618731 -0.1354646418 0.350154090
## fyear2008
## fyear2009
                2.299169487 0.7608233880 3.059992875
## -----
## Standard errors:
                    Direct
                            Indirect
                                           Total
              0.018213195 0.05439058 0.059111166
## log(area)
## log(Dcbd)
               0.125909677 0.14337589 0.091299942
## log(Dele)
               0.045766222 0.08001448 0.071996590
## log(Dpark)
               0.101522411 0.14359787 0.096960618
## log(Dsubway) 0.054624512 0.09528292 0.079291144
## crimerate
               0.008310533 0.01266441 0.009369951
## popden
               0.015882383 0.02871415 0.023371534
## fyear2004
               0.055951753 0.19532568 0.209065378
## fyear2005
               0.118378556 0.44432426 0.480862483
## fyear2006
               0.102489167 0.35908167 0.392488897
## fyear2007
               0.116560989 0.32719106 0.350864647
               0.123098193 0.45396729 0.483946755
## fyear2008
               0.222961274 0.56093975 0.677756394
## fyear2009
## -----
## Z-values:
##
                             Indirect
                    Direct
                                            Total
## log(area)
               -1.38977220 1.65344597 1.09318930
## log(Dcbd)
              -4.67053869
                           2.29961551 -2.82975649
## log(Dele)
               -0.05050663 -1.51967605 -1.72102031
## log(Dpark)
               -0.80661386 -1.52333933 -3.10061630
## log(Dsubway) -3.07739115 -1.19239863 -3.55293420
## crimerate
                0.76627244 -0.05755644 0.60184029
## popden
                0.30009801 1.37447145 1.89260378
## fyear2004
               -3.69697167
                           1.14919608 0.08425815
## fyear2005
               -0.40464699 1.24504519
                                       1.05082488
## fyear2006
               -1.29185411
                           0.08087169 -0.26334888
## fyear2007
                5.94787165 -1.42805385
                                      0.64424659
## fyear2008
                3.94497044 -0.29840177
                                       0.72353846
## fyear2009
               10.31196782 1.35633709 4.51488603
## p-values:
##
               Direct
                          Indirect Total
## log(area)
               0.16459806 0.09824 0.27431069
## log(Dcbd)
               3.0041e-06 0.02147
                                  0.00465834
## log(Dele)
               0.95971867 0.12859 0.08524714
## log(Dpark)
               0.41988899 0.12767
                                  0.00193118
## log(Dsubway) 0.00208821 0.23310 0.00038096
## crimerate
               0.44351421 0.95410
                                  0.54728045
## popden
               0.76410240 0.16930 0.05841058
## fyear2004
               0.00021819 0.25048
                                  0.93285119
## fyear2005
               0.68573701 0.21312
                                  0.29333903
## fyear2006
               0.19640767 0.93554
                                  0.79228169
## fyear2007
               2.7165e-09 0.15328
                                  0.51941554
## fyear2008
               7.9810e-05 0.76540
                                  0.46934913
## fyear2009
               < 2.22e-16 0.17499
                                  6.3351e-06
LR1.sarlm(SDEM)
##
```

Likelihood Ratio diagnostics for spatial dependence

```
##
## data:
## Likelihood ratio = 106.85, df = 1, p-value < 2.2e-16
## sample estimates:
## Log likelihood of spatial error model
##
                                -1296.305
             Log likelihood of OLS fit y
##
                                -1349.730
##
Hausman.test(SDEM)
##
## Spatial Hausman test (asymptotic)
## data: NULL
## Hausman test = 12.872, df = 27, p-value = 0.99
library(lme4)
mlm_1 <- lmer(update(form, . ~ . + (1 | f_district.id)), data=landprice_sf, REML=FALSE)</pre>
Beijingdistricts_sf$mlm_re <- ranef(mlm_1)[[1]][,1]</pre>
library(Matrix)
suppressMessages(library(MatrixModels))
Delta <- as(model.Matrix(~ -1 + f_district.id, data=landprice_sf, sparse=TRUE), "dgCMatrix")
m_hsar <- hsar(form, data=landprice_sf, W=NULL, M=M, Delta=Delta, burnin=500, Nsim=5000, thinning=1)</pre>
Beijingdistricts_sf$hsar_re <- m_hsar$Mus[1,]</pre>
plot(Beijingdistricts_sf[,"mlm_re"])
```

# mlm\_re

