UNIVERSITY OF GHANA

BSc. FIRST SEMESTER EXAMINATIONS: 2017/2018

PHYS 443: PHYSICS OF LARGE SYSTEMS II (3 Credits)

ANSWER ***QUESTION 1*** *AND* ***TWO OTHER*** *QUESTIONS*

.

TIME ALLOWED: ***TWO AND A HALF (2½) HOURS***

VALUES OF CONSTANTS

Boltzmann's Constant	$k = 1.38 \times 10^{-23}$ J/K
Speed of light	$c = 3.00 \text{ x } 10^{8}$ m s^{-1}
Planck's constant	$h = 6.63 \text{ x } 10^{-34}$ J s

1.

a) Define entropy and briefly discuss its physical meaning.

[2 marks]

b) Explain the *micro-canonical*, *canonical* and *grand-canonical* ensembles.

[6 marks]

c)
i) Write down the Maxwell-Boltzmann, Fermi-Dirac and Bose-Einstein distribution functions.
ii) What are the properties of particles which obey the different distributions?
iii) Under what conditions do the three statistics tend to be the same?

[8 marks]

d) A two-level system of $N = n_1 + n_2$ particles is distributed among two eigen-states 1 and 2 with eigenenergies E_1 and E_2 respectively. The system is in contact with a heat reservoir at temperature T. If a single quantum emission into the reservoir occurs, population changes $n_2 \longrightarrow n_2 - 1$ and $n_1 \rightarrow n_1 + 1$ take place in the system. (Assume n_1 >> 1 and n_2 >> 1).

i) Obtain the expression for entropy change of the two-level system and the reservoir.

ii) From i) derive the Boltzmann relation for the ratio $\frac{n_1}{n_2}$.

[6 marks]

e) Three identical, indistinguishable particles are placed into a system consisting of four energy levels with energies 1.0, 2.0, 3.0, and 4.0 eV, respectively. The total energy of the three particles is 6.0 eV. What is the average number of particles occupying each energy level, if the particles are

i) bosons

ii) fermions?

[4 marks]

2. The partition function for a single particle is given by

$$z = \left(\frac{2\pi mkT}{h^2}\right)^{\frac{3}{2}} V$$

where the symbols have their usual meanings.

a) Use z to obtain an expression for the partition function Z of a system of N ideal gas particles in the canonical ensemble using the Boltzmann approximation.

[3 marks]

b) With the result for Z, derive the following relations for a system of ideal gas particles:

i) Energy, E.

ii) Entropy, S

iii) Pressure, P

iv) Heat capacity, C_V

v) Chemical potential, μ

[10 marks]

c) Comment on the result for the energy E and the heat capacity C_V

[4 marks]

3. The energy of a particle can be expressed as $E(x) = \alpha x^2$ where x is a coordinate or momentum and can take on values from $-\infty$ to $+\infty$.

a) Show that the average energy per particle for a system of such particles subject to Maxwell-Boltzmann statistics is $\bar{E} = \frac{kT}{2}$.

[10 marks]

b) State the principle of equipartition of energy and discuss briefly its relation to the results in a) above.

[7 marks]

4. For the Fermi-Dirac distribution function F_{FD}:

a) Show that as $T \longrightarrow 0$ K

i) $F_{FD} = 1$ for $E < \varepsilon_F$
ii) $F_{FD} = 0$ for $E > \varepsilon_F$

[4 marks]

b) Sketch f, the Fermi-Dirac probability, vs E, energy, at $T = 0\ K$.

[3 marks]

c) On the same plot sketch $F_{FD}\ vs. E$ two temperatures $T_2 > T_1 > 0\ K$.
Comment on the shape of the electron distribution as T increases?

[4 marks]

d) For a given semiconductor at $E = E_F + 0.5$ eV. Calculate the temperature required to have 10% probability of finding an electron at that energy?

[6 marks]

5. Consider a quantum-mechanical gas of non-interacting spin zero bosons, each of mass m which are free to move within a volume V.

a) Determine the energy and heat capacity in the very low temperature region. Discuss why it is appropriate at low temperatures to put the chemical potential equal to zero.

[9 marks]

b) Show how the calculation is modified for a photon (mass = 0) gas. Prove that the energy is proportional to T^4.

[8 marks]

(Note: Put all integrals in dimensional form, but do not evaluate.)

Useful Integrals

$$\int_{-\infty}^{\infty} dz\, e^{-\alpha z^2} = 2\int_{0}^{\infty} dz\, e^{-\alpha z^2} = 2\int_{-\infty}^{0} dz\, e^{-\alpha z^2} = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}}$$

$$\int_{-\infty}^{\infty} dz\, z^2 e^{-\alpha z^2} = 2\int_{0}^{\infty} dz\, z^2 e^{-\alpha z^2} = 2\int_{-\infty}^{0} dz\, z^2 e^{-\alpha z^2} = \frac{\pi^{\frac{1}{2}}}{2\alpha^{\frac{3}{2}}}$$

Useful Formulae

$$F = -kTlnZ$$

$$dF = -SdT - pdV + \mu dN$$

$$U = kT^2\left(\frac{\partial lnZ}{\partial T}\right)_{V,N}$$

$$\Omega_{MB} = \frac{N!}{n_1!n_2!\ldots}(g_1)^{n_1}\cdot(g_2)^{n_2}\ldots = N!\prod_i\left(\frac{\{g_i\}^{n_i}}{n_i!}\right)$$

$$\Omega_{BE} = \prod_i \Omega_i = \prod_i \frac{(n_i+g_i-1)!}{n_i!(g_i-1)!}$$

$$\Omega_{FD} = \prod_i \frac{g_i!}{n_i!(g_i-n_i)!}$$

$$\mathrm{g}(E) = \frac{2\pi V}{h^3}(2m)^{\frac{3}{2}}E^{\frac{1}{2}}$$