UNIVERSITY OF GHANA

BSc/BA, FIRST SEMESTER EXAMINATIONS: 2019/2020

DEPARTMENT OF MATHEMATICS

MATH 121: Algebra and Trigonometry (3 credits)

INSTRUCTION:

ANSWER ANY FOUR OUT OF THE FOLLOWING SIX QUESTIONS

TIME ALLOWED:

TWO HOURS AND THIRTY MINUTES $\left(2\frac{1}{2} \text{ hours}\right)$

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1. (a) Let P, Q and R be simple statements.

   i. Construct a truth table for the compound statement $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$.

   ii. From the truth table, determine if the compound statement is a tautology or not.

   [20 marks]

   (b) If $\log_2 3 = a$, find the value of $\log_{\sqrt{2}} 54$ in terms of a. *[15 marks]*

   (c) Suppose that for the given quadratic equation $ax^2 + bx + c = 0$, one root is twice the other. Show that $2b^2 = 9ac$.

   [15 marks]

2. (a) Suppose $(x-2)$ and $(x+1)$ are factors of the polynomial $f(x) = 2x^3 + ax^2 - 3x + b$, where $a, b \in \mathbb{Z}$. Find the values of a and b and factorize the polynomial completely. *[15 marks]*

   (b) By writing $3x$ as $2x + x$, establish the *triple-angle* formula

   $$\cos(3x) = 4\cos^3 x - 3\cos x.$$

   Hence, solve the equation $8\cos^3 x - 6\cos x - 1 = 0$. *(You only need to write down the general solution).* *[15 marks]*

   (c) Solve the inequality $\dfrac{5}{x-6} > \dfrac{3}{x+2}$. *[20 marks]*

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3. (a) Prove by using mathematical induction, that

$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^2+4\left(\frac{1}{2}\right)^3+\cdots+n\left(\frac{1}{2}\right)^{n-1}=4-\frac{n+2}{2^{n-1}}.$$

[20 marks]

(b) Let $f(x)=\dfrac{x}{x+1}$.

i. State the domain of f.

ii. Find the composite functions $f^2=f\circ f$ and $f^3=f\circ f\circ f$, and simplify the expressions as far as possible.

iii. From f^2 and f^3, deduce an expression for $f^5(x)$.

[15 marks]

(c) Solve the exponential equation $2^{2x+1}-9(2^x)+4=0$. [15 marks]

4. (a) Prove the following statement by contradiction:

$$\forall x\in\mathbb{R},\ \exists y\in\mathbb{R}\text{ such that } x\leq y.$$

[15 marks]

(b) Establish the following trigonometric identity:

$$\frac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta}=\cos(2\theta).$$

[15 marks]

(c) Solve the radical equation $\sqrt{x+2}+\sqrt{x+1}=3$. [20 marks]

5. (a) Solve the simultaneous equations for exact values of x and y:

$$\begin{cases} 2^{x+y}=6^y \\ 3^{x-1}=2^{y+1} \end{cases}.$$

[20 marks]

(b) Consider the function $f(x)=\dfrac{x-5}{2x+3}$.

i. Show that $f(x)$ is an *injective function.*

ii. Find the inverse function of f. [15 marks]

(c) Express the rational function $R(x)=\dfrac{3x^2-2x+3}{(x-1)(x^2+x+2)}$ in partial fractions. [15 marks]

6. (a) Let A, B and C be subsets of a universal set. Prove that $(A\setminus C)\cap B=(A\cap B)\setminus C$.

[15 marks]

(b) Find the square root of the radical expression $17+12\sqrt{2}$ in the form $a+b\sqrt{2}$, where a,b are integers. [20 marks]

(c) Find the values of k such that the equation

$$\log_2(x^2+2kx)=2$$

has real roots. [15 marks]

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