

## Effective Field Theories and Inflation

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**ABSTRACT:** We investigate the possible influence of very-high-energy physics on inflationary predictions focussing on whether effective field theories can allow effects which are parametrically larger than order  $H^2/M^2$ , where  $M$  is the scale of heavy physics and  $H$  is the Hubble scale at horizon exit. By investigating supersymmetric hybrid inflation models, we show that decoupling does not preclude heavy-physics having effects for the CMB with observable size even if  $H^2/M^2 \ll O(1\%)$ , although their presence can only be inferred from observations given some *a priori* assumptions about the inflationary mechanism. Our analysis differs from the results of hep-th/0210233, in which other kinds of heavy-physics effects were found which could alter inflationary predictions for CMB fluctuations, inasmuch as the heavy-physics *can* be integrated out here to produce an effective field theory description of low-energy physics. We argue, as in hep-th/0210233, that the potential presence of heavy-physics effects in the CMB does *not* alter the predictions of inflation for generic models, but *does* make the search for deviations from standard predictions worthwhile.

**KEYWORDS:** Cosmology; Inflation.

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## 1. Introduction and Discussion

The recent inflationary literature contains considerable discussion of whether or not detailed observations of fluctuations in the temperature of the Cosmic Microwave Background (CMB) can be used to infer the properties of extremely high energies — usually assumed to be above the Planck scale. This discussion is particularly timely due to the arrival of ever-more-accurate measurements of these fluctuations [1, 2], most recently by the Wilkinson Microwave Anisotropy Probe (WMAP) collaboration [3]. There is likely to be even further improvement in the not-too-distant future [4].

Broadly speaking, two points of view have emerged from this discussion.

1. CMB fluctuations can depend on the details of high-energy (trans-Planckian) physics, and so represents an opportunity to probe these otherwise inaccessible scales [5, 6, 7].
2. General decoupling arguments require the influence of physics at scale  $M \gg H$  to contribute at most of order  $H^2/M^2$  to observable late-time effects, where  $H$  is the Hubble scale at horizon exit. This precludes the intrusion of higher-energy physics into CMB fluctuations [8].

Clearly much is at stake. On the one hand, given specific guesses for what trans-Planckian physics might be, observable effects for the CMB have been calculated. Although some of these calculations remain controversial [9] – in particular the choice of nonstandard vacua in de Sitter space – it is hard to argue trans-Planckian physics cannot interfere with inflationary predictions without better understanding what trans-Planckian physics is.

On the other hand, if general decoupling arguments do not apply in an inflationary context then the very *predictability* of inflationary models is lost. Indeed, since the nature

of the higher-energy physics is at present unknown any of its implications must be included under the general heading of theoretical uncertainty when comparing with experiments. In this sense decoupling is a prerequisite for using the properties of the CMB as evidence for an earlier inflationary period [10] in the first place.

To the extent that the central issue is the validity and implications of decoupling during inflation, it may be addressed without invoking unknown trans-Planckian physics. The conclusions drawn might then also be applicable to trans-Planckian physics to the extent that it also satisfies the assumptions of the analysis. Here (and in [11]) we use simple sub-Planckian field theories to explore these issues. Within this context two separate questions may be addressed.

1. Is it required that higher-energy physics decouple at all, in the sense that its low-energy effects must be described in terms of a low-energy effective field theory?
2. Given that the low-energy implications of heavy physics *can* be described by an effective field theory during the epoch of horizon exit, need its influence for the CMB be limited to effects which are of order  $H^2/M^2$ ?

Ref. [11] addresses the first of these questions, and shows that oscillating background fields before horizon exit can invalidate an effective-field-theory description. They can do so by preventing the time evolution of the relevant modes of the inflaton from being adiabatic, which is also an obstacle to using an effective-lagrangian description outside of a cosmological context.

In this paper we address the second question of whether an effective field theory can produce detectable effects even if those of order  $H^2/M^2$  are too small to be observable. We here show that for some models heavy physics generates contributions to the inflaton potential which depend logarithmically on the heavy mass  $M$ , and so contribute to the slow-roll parameters an amount of order  $M_0^2/M^2$ , with  $M_0 \gg H$ . In so doing we also show that since the CMB measurements in themselves only probe the inflaton potential in a limited way, at present the influence of heavy physics can only be inferred using CMB measurements given some sort of *a priori* assumptions about the nature of the physics which is responsible for making the inflaton potential flat in the first place.

Although the thrust of both of these conclusions is that high-energy physics *can* intrude into CMB fluctuations, we argue that they do so in a way which does *not* introduce uncontrollable theoretical errors into the predictions of inflationary models, and so do not undermine the successful comparison of these predictions with observations. They do not do so because our basic conclusion is that the criteria for decoupling in inflation are precisely the same criteria which apply in other non-cosmological contexts. In particular, the vast majority of high-energy effects *do* decouple and so cannot alter standard predictions. But just as in other areas of physics, some specific kinds of effective interactions can be sensitive to higher-energy details and these are worth scrutinizing for the information they may contain about the microscopic physics of very small distances. In this sense our results, like those of [11], represent in some ways the best of all possible worlds.

Our presentation is organized as follows. Section 2 starts with a toy model of hybrid inflation consisting of two scalar fields having very different masses. We use this model to explicitly integrate out the heavier of the two (the heavy physics) at one loop to see its effects on the lighter scalar (the inflaton). This example illustrates first that, in the generic case heavy-field loop contributions to the inflaton potential are large and dangerous, since they tend to ruin inflation by destroying the flatness of the inflaton potential. This is one of the well-known naturalness problems of inflation, and any careful treatment of the effects for inflation of higher-energy physics must properly address this.

We can also use this example to show that even if the above naturalness problems are addressed, there is a further obstruction to identifying heavy-physics effects within the observed fluctuations of the CMB if the inflaton is really rolling slowly at the epoch of horizon exit. This is because the inflationary predictions in this case only sample the first few derivatives of the inflaton potential, and these can generically be adjusted by small changes in the renormalizable inflaton couplings. Because of this a detection of heavy-physics effects in inflation is only possible within the context of a specific inflationary model, for which the inflaton couplings can be subject to *a priori* conditions (such as those due to symmetries).

Section 3 sharpens the analysis by addressing these two issues within a supersymmetric model. Supersymmetry provides an attractive framework for addressing the above questions because supersymmetry both controls the naturalness issue and provides *a priori* constraints on the form of the low-energy inflaton potential. Section 3.1 examines these issues within a standard globally supersymmetric hybrid-inflation model. The light scalar for this model parameterizes a precisely flat direction of the classical potential, whose degeneracy is lifted purely by the virtual effects of heavy fields. In this model it is therefore purely heavy-physics effects which make the difference between the success and failure of the theory as a model of inflation, since it is their small size which explains why the inflaton potential is very shallow (but not exactly flat). Simple modifications of this higher-energy physics produce sizeable changes in the predictions for the CMB precisely because in these models it is the higher-energy physics which controls the entire effect.

Section 3.2 generalizes the model of section 3.1 to supergravity. The main message of this section is that such an extension is possible despite the well-known  $\eta$  problem of supergravity models. We illustrate this using the example of D-term inflation.

We emphasize that all of the examples we use are extremely orthodox, and have been considered in various contexts in the literature. Our purpose in bringing them all together here is to first show that existing models already provide explicit examples of how higher-energy physics can have important effects during inflation, and so to illustrate that such effects can happen in ordinary theories which decouple, without relying on other exotic properties (like violations of Lorentz invariance, for example). A secondary goal is to raise the bar for putative models of trans-Planckian physics, by arguing that any serious candidate for this must: (i) establish that the proposed trans-Planckian physics does decouple from lower-energy phenomena so as to not completely sacrifice the predictivity of lower-energy physics, and (ii) contain a precise model of the low-energy inflaton dynamics as a benchmark against which the high-energy physics effects can be compared.

## 2. Inflation in a Toy Effective Theory

In this section we accomplish two things using a simple toy model involving an inflaton  $\phi$  and some heavy fields  $\chi_i$ . First, we explicitly integrate out the heavy fields to derive the dominant terms in the resulting low-energy effective field theory for  $\phi$ . Then we apply this result to an inflationary model and show how these heavy-field contributions are poison to inflationary models, since they destroy the flatness of the inflaton potential. This well-known naturalness problem shows that the virtual effects of high-energy fields for inflation are generically too *large* rather than too small. This calculation is standard [12] and is mainly used to motivate the calculations of the next section, where we repeat the analysis using supersymmetric examples. A reader familiar with these issues, and who is in a hurry, can skip directly to section 3.

### 2.1 The Model

Consider the following toy model of very-high-energy physics, which is a very minor generalization of a standard hybrid-inflation model [13]:

$$-\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \chi_i \partial^\mu \chi_i + V(\phi, \chi), \quad (2.1)$$

with  $V(\phi, \chi) = V_{\text{inf}}(\phi) - \frac{1}{2} m^2 \chi_i^2 + \frac{1}{2} g \chi_i^2 \phi^2.$

We assume the  $N$  massive scalars,<sup>1</sup>  $\chi_i, i = 1, \dots, N$  — which represent the heavy physics whose lower-energy influence we wish to determine — satisfy  $\langle \chi_i \rangle = 0$ , thereby excluding the non-adiabatic effects discussed in ref. [11]. For the simplicity of later formulae we choose the couplings to be invariant under the  $O(N)$  which rotates the  $\chi_i$ 's amongst themselves. In eq. (2.1)  $\phi$  is the inflaton, whose potential

$$V_{\text{inf}}(\phi) = \rho + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \quad (2.2)$$

is chosen to ensure sufficient  $e$ -foldings of expansion before an eventual inflationary exit and reheating. This is ensured if the two slow-roll parameters [14] satisfy<sup>2</sup>

$$\epsilon_0(\phi) = \frac{1}{2} \left( \frac{M_p V'_{\text{inf}}}{V_{\text{inf}}} \right)^2 = \left( \frac{M_p \phi (m_\phi^2 + \lambda \phi^2)}{\rho + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda \phi^4} \right)^2 \ll 1$$

$$\eta_0(\phi) = \left( \frac{M_p^2 V''_{\text{inf}}}{V_{\text{inf}}} \right) = \left( \frac{M_p^2 (m_\phi^2 + 3\lambda \phi^2)}{\rho + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda \phi^4} \right) \ll 1. \quad (2.3)$$

For instance, these conditions are satisfied if  $m_\phi^2 M_p^2 \ll \rho$  and if we choose initial conditions for which  $\chi_i = 0$  and  $\rho/m_\phi^2 \gg \phi^2 \gg m^2/g$  (provided  $\lambda$  is small enough also to ensure  $\lambda \phi^2 \ll m_\phi^2$  throughout this range of  $\phi$ ). Inflation then occurs with  $H^2 = V_{\text{inf}}(\phi)/(3M_p^2) \approx$

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<sup>1</sup>We introduce  $N$  heavy fields simply to amplify the effects of the  $\chi$  fields, and much of our discussion applies equally well if  $N = 1$ . Perturbative quartic self-interactions for  $\chi$  can also be included without materially affecting our discussion.

<sup>2</sup>We use here the rationalized Planck mass,  $M_p^{-2} = 8\pi G$ .

$\rho/(3M_p^2)$  while  $\phi$  rolls slowly down to  $\phi_{\text{end}}$ , where inflation ends. Inflation ends either when the slow-roll parameters become of order unity, or when  $\phi^2 \sim m^2/g$ , at which point  $\chi_i$  moves quickly away from zero, and  $\phi_{\text{end}}$  is the field determined by whichever of these occurs first.

In order to study the effects of high energy physics we assume the masses,

$$M^2(\phi) = -m^2 + g\phi^2, \quad (2.4)$$

to be much larger than the Hubble scale,  $H^2$ , during horizon exit. In order to keep the analysis under control we also assume the  $\chi_i$  fields to be sub-Planckian throughout the inflationary slow roll of  $\phi$ :  $M^2(\phi) \ll M_p^2$ .

## 2.2 Integrating out the Heavy Fields

Under the above assumptions the heavy fields may be explicitly integrated out and an effective-field-theory analysis applies. Since our main interest is in effects which do not decouple we focus on those terms in the effective theory which are unsuppressed by powers of  $1/M$ . These come in two types: (i) relevant interactions, which are proportional to positive powers of  $M$ ; or (ii) marginal interactions, which grow logarithmically with  $M$ .

A simple calculation of the virtual effects of the heavy scalars on the light scalar potential is obtained by matching the one-loop corrected effective potential for the full theory with the one-loop effective potential in the theory involving only  $\phi$ , but including effective interactions. This gives the following result:

$$\begin{aligned} V_{\text{eff}}(\phi) &= V_{\text{inf}}(\phi) + \Delta V(\phi), \\ \text{with} \quad \Delta V(\phi) &= V_{\text{ct}}(\phi) + \frac{N}{64\pi^2} M^4(\phi) \ln \left( \frac{M^2(\phi)}{\mu^2} \right), \end{aligned} \quad (2.5)$$

where  $V_{\text{inf}}$  now denotes the renormalized inflaton potential, eq. (2.2), with constants  $\rho(\mu)$ ,  $m_\phi^2(\mu)$  and  $\lambda(\mu)$  chosen according to a renormalization prescription which is described in detail below.  $V_{\text{ct}}$  contains the counter-terms,  $\delta\rho$ ,  $\delta m_\phi^2$  and  $\delta\lambda$ , which enforce the renormalization condition, and  $\mu$  is a floating scale which also depends on the renormalization scheme. As usual, the implicit dependence of  $V_{\text{inf}}$  on  $\mu$  is just such as to ensure that  $\mu$  cancels in physical observables.

For inflationary purposes it is convenient to cast the renormalization conditions in terms of the slow-roll parameters since we must really demand that  $V_{\text{eff}}$ , rather than  $V_{\text{inf}}$ , produces sufficient inflation. For our later purposes, a convenient way to do so is to require that  $V_{\text{eff}}$  and  $V_{\text{inf}}$  share the same value for the Hubble constant and the two slow-roll parameters at a particular point in field space, which we can choose to be the point when a specific mode  $k_*$  leaves the horizon. We denote the time of horizon exit for this mode by  $t_*$ , where  $H(t_*) = k_{*\text{phys}} = k_*/a(t_*)$ , and we further denote the value of the inflaton field at this time by  $\phi(t_*) = \phi_*$ . Then this renormalization condition states that  $V_{\text{eff}}(\phi_*)$ ,  $\epsilon(\phi_*)$  and  $\eta(\phi_*)$  are given by their tree-level expressions in terms of  $\rho$ ,  $m_\phi$  and  $\lambda$ :

$$V_* = V_{\text{eff}}(\phi_*) = \rho + \frac{1}{2} m_\phi \phi_*^2 + \frac{1}{4} \lambda \phi_*^4 = 3 M_p^2 H_*^2$$

$$\begin{aligned}\epsilon_* &= \epsilon_0(\phi_*) = \frac{1}{2} \left[ \frac{M_p \phi_* (m_\phi^2 + \lambda \phi_*^2)}{V_{\text{eff}}(\phi_*)} \right]^2 = \frac{1}{2} \left[ \frac{\phi_* (m_\phi^2 + \lambda \phi_*^2)}{3 M_p H_*^2} \right]^2 \\ \eta_* &= \eta_0(\phi_*) = \frac{M_p^2 (m_\phi^2 + 3 \lambda \phi_*^2)}{V_{\text{eff}}(\phi_*)} = \frac{(m_\phi^2 + 3 \lambda \phi_*^2)}{3 H_*^2}.\end{aligned}\quad (2.6)$$

The subscript ‘0’ on the quantities  $\epsilon_0(\phi)$  and  $\eta_0(\phi)$  indicates that their functional dependence is as calculated using the potential  $V_{\text{inf}}$ , as in eq. (2.3).

These conditions amount to the following requirements for  $\Delta V$ :  $\Delta V(\phi_*) = \Delta V'(\phi_*) = \Delta V''(\phi_*) = 0$ , and so

$$\Delta V(\phi) = \frac{N}{64\pi^2} \left\{ M^4(\phi) \left[ \ln \left( \frac{M^2(\phi)}{M_*^2} \right) - \frac{3}{2} \right] + \frac{1}{2} M_*^2 [4 M^2(\phi) - M_*^2] \right\}, \quad (2.7)$$

where  $M_* = M(\phi_*)$ .

Using the following derivatives:

$$\begin{aligned}\Delta V'(\phi) &= \frac{g N \phi}{16\pi^2} \left[ (m^2 + g \phi^2) \ln \left( \frac{M^2}{M_*^2} \right) - g(\phi^2 - \phi_*^2) \right], \\ \Delta V''(\phi) &= \frac{g N}{16\pi^2} \left[ (m^2 + 3g \phi^2) \ln \left( \frac{M^2}{M_*^2} \right) - g(\phi^2 - \phi_*^2) \right], \\ \Delta V'''(\phi) &= \frac{g^2 N \phi}{8\pi^2} \left[ 3 \ln \left( \frac{M^2}{M_*^2} \right) + \frac{2g \phi^2}{M^2} \right],\end{aligned}\quad (2.8)$$

and taking  $M^2 \approx g \phi^2 \gg m^2$  and  $\Delta V \ll V_{\text{inf}} \approx \rho$ , we find  $H^2 \approx \rho/(3M_p^2)$  and the following expressions for the slow-roll parameters:

$$\begin{aligned}(2\epsilon)^{1/2} &\approx (2\epsilon_0)^{1/2} + \frac{g^2 N \phi}{48\pi^2 M_p H^2} \left[ \phi^2 \ln \left( \frac{\phi^2}{\phi_*^2} \right) - (\phi^2 - \phi_*^2) \right] \\ \eta &\approx \eta_0 + \frac{g^2 N}{48\pi^2 H^2} \left[ 3 \phi^2 \ln \left( \frac{\phi^2}{\phi_*^2} \right) - (\phi^2 - \phi_*^2) \right],\end{aligned}\quad (2.9)$$

as well as the second-order slow-roll quantity:

$$\xi^2 = (2\epsilon)^{1/2} \left( \frac{M_p^3 V'''}{V} \right) \approx (2\epsilon)^{1/2} \frac{M_p \phi}{H^2} \left\{ 2\lambda + \frac{g^2 N}{24\pi^2} \left[ 3 \ln \left( \frac{\phi^2}{\phi_*^2} \right) + 2 \right] \right\}. \quad (2.10)$$

For our purposes, there are two lessons to be learned from these expressions: the naturalness problems they imply, and the obstruction they raise to the inference of heavy-physics properties purely using measurements of the CMB.

### Naturalness Problems

The biggest difficulty with these expressions is in maintaining inflation itself. Although we’ve ensured (by construction) that the  $\phi$  motion is sufficiently slow near  $\phi = \phi_*$ , this is not in itself sufficient to obtain the more than 50  $e$ -foldings of inflation which are required for successful cosmology. In particular, this requires  $\eta$  to remain small over a significant range of  $\phi$ , which requires

$$|\phi^2 - \phi_*^2| \ll \frac{48\pi^2 H^2}{g^2 N}. \quad (2.11)$$

This is typically difficult to satisfy unless  $g$  is extremely small. For instance, if we take  $H$  to be approximately constant over the  $N_e$   $e$ -foldings of inflation between horizon exit and inflation's end, and if we take  $dV_{\text{inf}}/d\phi \approx m_\phi^2 \phi$  over this region, we have  $\phi_*^2 \approx (m^2/g) \exp \left[ (2m_\phi^2 N_e)/(3H^2) \right]$ . In this case we see eq. (2.11) implies

$$\left| \exp \left[ \frac{2m_\phi^2 N_e}{3H^2} \right] - 1 \right| \ll \frac{48\pi^2}{gN} \left( \frac{H^2}{m^2} \right), \quad (2.12)$$

and so even if we have already assumed  $H^2/m_\phi^2 \sim N_e \gtrsim 50$ , we must now in addition tune  $m$  to satisfy  $Nm^2 \ll 48\pi^2 H^2/g$ .

We see that heavy loops can have big effects because they compete with unusually small low-energy interactions. The low-energy interactions are small precisely because inflation requires the low-energy inflaton potential must be chosen to be so very flat. In these circumstance generic kinds of heavy physics not only do not decouple, they can completely ruin inflation.

### Detecting Heavy Physics Using the CMB

Eq. (2.9) implies another obstruction to learning about higher-energy physics using inflationary predictions for the CMB, independent of the naturalness issues associated with obtaining the slow roll itself. This obstruction arises because if the inflaton describes a sufficiently slow roll, its effects for the CMB are completely determined by the quantities  $H$ ,  $\epsilon$  and  $\eta$  evaluated at horizon exit. For instance standard expressions [14] for the corrections to the CMB fluctuation spectrum are<sup>3</sup>

$$\begin{aligned} \delta_H^2(k_*) &\approx \frac{V_*}{150\pi^2 M_p^4 \epsilon_*} = \frac{H_*^2}{50\pi^2 M_p^2 \epsilon_*} \\ n(k_*) &\approx 1 + 2\eta_* - 6\epsilon_* \\ \frac{dn}{d\ln k}(k_*) &\approx 16\epsilon_* \eta_* - 24\epsilon_*^2 - 2\xi_*^2. \end{aligned} \quad (2.13)$$

Since the above renormalization scheme is designed not to change the slow-roll parameters at horizon exit, it ensures that the heavy fields cannot alter the inflationary predictions for modes near  $k = k_*$ , to leading order in the slow-roll parameters. To the extent that this is true, and that observations are only sensitive to fluctuation properties near  $k_*$ , any detection of heavy physics as a distortion of the CMB spectrum must rely on the breakdown of the slow-roll conditions near horizon exit (such as might be true if the preliminary WMAP indications for nonzero  $dn/d\ln k$  [15] should prove to be significant).

Until more detailed observational information is available, we conclude that the effects of very heavy physics for the CMB can at present only be inferred relative to some *a priori* information about the nature of inflationary physics. If, for instance, inflation were believed to be due to a particular mechanism — such as perhaps the string-motivated brane-inflation proposals of [16] — then detailed knowledge of the form for  $V_{\text{inf}}$  can allow sufficiently large corrections to this form to be inferred from observations.

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<sup>3</sup>Note that there is a sign error in the formula for  $\xi$  in ref. [14]



An extreme example of *a priori* constraints is the case where  $V_{\text{inf}}$  is precisely constant, in which case the inflaton potential is *entirely* due to loop-generated heavy-physics effects. This situation is actually fairly common for supersymmetric models, which are typically rife with classically-flat directions. In these models the inflaton can be a modulus parameterizing one of these directions, and so the very flatness of the inflaton potential is then partially explained by its origin as a loop-generated effect.

### 3. Flat Directions and Supersymmetric Models

In this section we repeat the calculation of the previous section for a supersymmetric example, for which the generic naturalness issues raised above can be controlled. This allows us to more precisely compute the size of high-energy loop effects on CMB fluctuations, and so to illustrate how these effects need not be unobservably small even if their scale is much higher than  $H$ . They also provide examples wherein the entire inflaton potential arises as such a heavy-physics loop effect, and so for which the very detection of inflationary effects in the CMB is necessarily also a detection of heavy-physics effects. We first describe models in global supersymmetry in section 3.1, for which the calculations are simple. We shall find self-consistency forces us then to generalize these to supergravity, and this is done in section 3.2.

The results we obtain in this section are simple to state. In the models we consider here, the tree-level inflaton potential is exactly flat, but this flat direction is lifted by virtual loops of heavy particles yielding nonzero slow-roll parameters  $\eta$  and  $\epsilon$ . In particular, integrating out the heavy particles produces slow-roll parameters which are suppressed by factors of order  $M_0^2/M^2$ , where  $M$  is the relevant heavy mass scale, and it is this decoupling which makes the slow-roll parameters small. But, and this is the crux of the matter, the reference scale  $M_0$  is much bigger than  $H$ , and so these models may be taken as an existence proof that heavy physics can decouple and yet still alter inflationary predictions for the CMB since the figure of merit for deciding the observability of the heavy-physics effects can be larger than  $H^2/M^2$ .

#### 3.1 Globally Supersymmetric Models

Part of the attraction of supersymmetric models for hybrid inflation is the ubiquity with which their scalar potentials have flat directions. Two kinds of models have been proposed, which differ according to whether the inflationary potential arises as an  $F$ -term [18] or a  $D$ -term [19]. We focus here on  $D$ -term models in order to avoid the usual  $\eta$  problem when we generalize to supergravity.

Consider, then, a model containing the chiral multiplets,  $\Phi = \{\phi, \psi\}$  and  $H_{\pm} = \{h_{\pm}, \chi_{\pm}\}$ , coupled to a  $U(1)$  gauge multiplet,  $V = \{A_{\mu}, \lambda\}$ . We take  $H_+$  and  $H_-$  to carry opposite  $U(1)$  charge  $\pm e$ , and the multiplet  $\Phi$  to be neutral. The model's superpotential and Kähler potential are

$$K = H_+^* H_+ + H_-^* H_- + \Phi^* \Phi \quad \text{and} \quad W = g\Phi(H_+ H_- - v^2), \quad (3.1)$$

where  $g$  and  $v$  are real constants. The associated scalar potential is  $V = V_F + V_D$  where

$$\begin{aligned} V_F &= g^2 \left( |h_+ h_- - v^2|^2 + |\phi h_-|^2 + |\phi h_+|^2 \right), \\ V_D &= \frac{e^2}{2} \left( |h_+|^2 - |h_-|^2 + \xi \right)^2, \end{aligned} \quad (3.2)$$

where  $\xi > 0$  is the Fayet-Iliopoulos term. Notice that  $V_D = 0$  implies  $|h_-|^2 = |h_+|^2 + \xi$  for any  $\phi$  while  $V_F = 0$  implies  $\phi = 0$  and  $h_+ h_- = v^2$ , so the global minimum is supersymmetric and has

$$\phi = 0, \quad |h_{\pm}|^2 = \frac{1}{2} \left( \mp \xi + \sqrt{\xi^2 + 4v^4} \right). \quad (3.3)$$

The feature of most interest for the present purposes is the potential's trough at  $h_{\pm} = 0$  and large  $|\phi|$ , for which  $V_{\text{trough}}(\phi) = V(h_{\pm} = 0, \phi) = g^2 v^4 + \frac{1}{2} e^2 \xi^2$ . For sufficiently large  $|\phi|$  this is a local minimum in the  $h_{\pm}$  directions, with scalar excitations in these directions having masses

$$M_{\pm}^2(\phi) = g^2 |\phi|^2 \pm \sqrt{g^4 v^4 + e^4 \xi^2}. \quad (3.4)$$

Everywhere along this trough the scalar  $\phi$  is precisely massless, while linear combinations of the complex scalars  $h_{\pm}$  and  $h_{\mp}^*$  are massive, with masses,  $M_{\pm}^2(\phi)$ , which are large for large  $|\phi|$ . Since the  $U(1)$  gauge invariance is not broken along the trough, the gauge bosons are massless. The fermion masses along the trough, on the other hand, are zero for the gaugino,  $\lambda$ , and the chiral-multiplet fermion  $\psi$ . They are nonzero for the fermions  $\chi_{\pm}$ , with mass eigenvalues

$$m_{\pm}^2(\phi) = g^2 |\phi|^2. \quad (3.5)$$

Along the trough's bottom the tree-level spectrum therefore breaks up into a sector of massless particles,  $\{A_{\mu}, \lambda, \phi, \psi\}$ , which do not classically directly couple among themselves, but which do couple to a massive sector,  $\{h_+, \chi_+, h_-, \chi_-\}$ . In this section we integrate out this massive sector to determine the effective interactions which are generated in this way amongst the light fields, with the goal of using this as a candidate hybrid inflation model [20].

At the classical level the model does *not* describe viable hybrid inflation, with  $\phi$  interpreted as the inflaton rolling along the trough's bottom. This is because at the classical level the  $\phi$  potential is precisely flat, and so there is nothing to drive the inflaton's slow roll. This is no longer true once loop corrections are included since the virtual heavy fields will generate an inflaton potential.

Following the same procedure as for the previous section we obtain the heavy-field contribution to the inflaton potential by matching the one-loop results. It is useful to extend the model to include  $N$  charged chiral multiplets,  $H_{\pm}^i$ , in an  $O(N)$ -invariant way, in which case we find the result

$$\begin{aligned} V_{\text{eff}}(\phi) &= \rho + \Delta V(\phi), \\ \text{with} \quad \Delta V(\phi) &= \delta\rho + \frac{2N}{64\pi^2} \sum_{i=\pm} \left[ M_i^4(\phi) \ln \left( \frac{M_i^2(\phi)}{\mu^2} \right) - m_i^4(\phi) \ln \left( \frac{m_i^2(\phi)}{\mu^2} \right) \right], \end{aligned} \quad (3.6)$$

where  $\rho$  is the renormalized (constant) classical potential along the trough (which classically is  $\rho = g^2 v^4 + \frac{1}{2} e^2 \xi^2$ ), and  $\delta\rho$  is the corresponding counter-term.

It is convenient to evaluate this result using the previous expressions  $M_{\pm}^2(\phi) = m^2(\phi) \pm \Delta$  and  $m_{\pm}^2(\phi) = m^2(\phi)$ , where  $m(\phi) = g|\phi|$  and  $\Delta = \sqrt{g^4 v^4 + e^4 \xi^2}$ . In terms of these the potential becomes

$$\begin{aligned} \Delta V_{\text{eff}}(\phi) = & \delta\rho + \frac{N}{32\pi^2} \left[ m^4(\phi) \ln \left( \frac{m^4(\phi) - \Delta^2}{m^4(\phi)} \right) + \Delta^2 \ln \left( \frac{m^4(\phi) - \Delta^2}{\mu^4} \right) \right. \\ & \left. + 2m^2(\phi)\Delta \ln \left( \frac{m^2(\phi) + \Delta}{m^2(\phi) - \Delta} \right) \right], \end{aligned}$$

which for  $m^2(\phi) \gg \Delta$  becomes

$$\Delta V_{\text{eff}}(\phi) \approx \frac{N \Delta^2}{16\pi^2} \left[ \ln \left( \frac{m^2(\phi)}{m_*^2} \right) + \mathcal{O} \left( \frac{\Delta^2}{m^4} \right) \right], \quad (3.7)$$

where  $m_*^2 = m^2(\phi_*) = g^2 |\phi_*|^2$  and we adopt the renormalization condition that  $\Delta V$  must vanish when  $\phi = \phi_*$ .

This induced potential causes the inflaton  $\varphi = |\phi|$  to roll, and this roll can be slow if  $|\phi|$  is sufficiently large, since

$$\begin{aligned} (2\epsilon)^{1/2} &= \frac{M_p}{V_{\text{eff}}} \left( \frac{\partial V_{\text{eff}}}{\partial \varphi} \right) \approx \frac{M_p N \Delta^2}{8\pi^2 \rho \varphi}, \\ \eta &= \frac{M_p^2}{V_{\text{eff}}} \left( \frac{\partial^2 V_{\text{eff}}}{\partial \varphi^2} \right) \approx - \frac{M_p^2 N \Delta^2}{8\pi^2 \rho \varphi^2}. \end{aligned} \quad (3.8)$$

With these expressions we can examine more quantitatively the conditions for inflaton. For simplicity we take for these purposes  $e \sim g$  and  $\xi \sim v^2$ , which implies  $\rho \sim g^2 v^4$  and  $\Delta \sim g^2 v^2$  and so  $\Delta^2/\rho \sim g^2$ . We also choose  $N = 1$ , although we return to other choices for  $N$  at the end. Suppose now  $N_e$   $e$ -foldings of inflation occur between horizon exit and the end of inflation. During this time the field  $\phi$  evolves from  $\phi_*$  to  $\phi_{\text{end}}$ , with

$$\phi_*^2 \approx \phi_{\text{end}}^2 + \frac{N N_e \Delta^2}{12\pi^2 H^2} \approx \phi_{\text{end}}^2 + \frac{g^2 N N_e M_p^2}{4\pi^2}, \quad (3.9)$$

where  $\phi_{\text{end}}^2$  is defined by the condition that the fields  $h_{\pm}$  start to roll ( $\phi_{\text{end}}^2 \sim \Delta/g^2$ ) or that the slow-roll parameter  $\eta$  becomes order unity ( $\phi_{\text{end}}^2 \sim g^2 N M_p^2/(8\pi^2)$ ), whichever is reached first. It turns out that  $\Delta/g^2$  is reached first if  $g \gtrsim 10^{-3}$ , and it is to this case we now specialize for concreteness' sake.

With the above choices, and taking  $\phi_{\text{end}}^2 \ll \phi_*^2$ , the amplitude of scalar fluctuations becomes

$$\delta_s^2 = \frac{H^2}{50\pi^2 M_p^2 \epsilon_*} \sim \left( \frac{16 N_e v^4}{75 N M_p^4} \right), \quad (3.10)$$

which for 60  $e$ -foldings agrees with the CMB value,  $\delta_s \sim 10^{-5}$  if  $v/M_p \sim g \sim 10^{-3}$ . The slow-roll parameters at horizon exit similarly become

$$\epsilon_* \approx \frac{g^2 N}{32\pi^2 N_e}, \quad \eta_* \approx - \frac{1}{2N_e}. \quad (3.11)$$

As is easily checked,  $\phi_*$  (and so also  $m_*$ ) is less than  $M_p$  provided  $g^2 N N_e / (4\pi^2) < O(1)$ . This is important because so long as  $\phi \ll M_p$  we are justified to restrict our analysis to global supersymmetry, instead of supergravity.

In this model  $\epsilon_* \ll \eta_*$ , so  $n_s - 1 \approx 2\eta \sim 10^{-2}$  (if  $N_e \sim 50$ ). This is perfectly compatible with the WMAP data; for example with the choice of cosmological parameters  $\Omega_b = 0.046$ ,  $\Omega_{cdm} = 0.178$ ,  $\Omega_\Lambda = 1 - \Omega_b - \Omega_{cdm}$ ,  $h = 0.72$ ,  $\tau = 0.17$  and  $n_s = 0.99$  in CMBFAST [21], we obtain  $\chi^2 = 1430$  using the WMAP likelihood code [22]. This is as good a fit to the data as the WMAP preferred parameter values [23]. Moreover since  $\epsilon \ll \eta$  the model is not constrained by gravitational wave production, with the WMAP gravity-wave constraint giving  $16\epsilon = r < 1.3$  [24], implying the weak constraint  $\epsilon < 0.08$ . It is interesting to note that other simple models of inflation, such as  $\lambda\phi^4$  chaotic inflation, are close to being ruled out by the CMB data due to having too large a tilt [24, 25].

Having established that the model is experimentally viable, let us return to the issue of decoupling. The implications of decoupling are most easily seen by writing the slow-roll parameters as

$$(2\epsilon_*)^{1/2} \approx \frac{g^2 N \Delta^2}{8\pi^2 \rho} \left( \frac{\varphi_* M_p}{m_*^2} \right) \quad \text{and} \quad \eta_* \approx -\frac{g^2 N \Delta^2}{8\pi^2 \rho} \left( \frac{M_p^2}{m_*^2} \right), \quad (3.12)$$

which shows that both are suppressed by two powers of the heavy mass  $m_*$ . What is important is that the scale against which  $m_*^2$  is compared is not  $H^2$ , but is instead  $(g^2 N \Delta^2 / 8\pi^2 \rho) M_p^2 \sim (g^4 N / 8\pi^2) M_p^2$ . As we dial the parameters of the model so that  $m_*$  becomes larger, the slow roll parameters get closer to the scale-invariant point  $\epsilon_* = \eta_* = 0$ . This is a consequence of decoupling, since the entire inflaton potential for the model is generated by virtual effects of the heavy physics. But because the benchmark for observability is *not*  $H^2/m_*^2$ , the difference from scale invariance can be kept observable even if  $H^2/m_*^2$  is much smaller than a few percent.

In particular, we can consider two models which differ only in their spectrum of particles at scale  $m_*$ , by comparing the effects two theories which differ only in the number of heavy multiplets. We take  $N = 1$  for model 1 and  $N = 2$  for model 2, and we suppose both models to undergo the same number of  $e$ -foldings of inflation,  $N_e$ . With these choices these models agree on their predictions for  $\eta_*$ . They also predict identical fluctuation amplitudes provided  $v_1^4 = v_2^4/2$ , and so if they further share the same couplings,  $g_1 \sim g_2$ , then we have  $\epsilon_{*2} = 2\epsilon_{*1}$ . Clearly the two models therefore can have detectable differences in their predictions for CMB observables, provided only that  $\epsilon_*$  is larger than  $O(0.01)$ .

### 3.2 Supergravity Models

This section makes a technical point by showing that a model similar to the previous hybrid inflation model can be embedded into supergravity without losing its main features. We do so because we have seen that inflationary applications can (but need not, for  $g \ll 1$ ) require fields  $|\phi| \gg M_p$ , for which supergravity corrections to the action are generally not negligible.

The ability to make the extension from global to local supersymmetry is not trivial, and would have been much more difficult if we had adopted an example relying purely on

the scalar potential's  $F$ -terms. In the  $F$ -term case the supergravity corrections typically introduce new terms which are of order  $H^2|\phi|^2$ , and so which contribute an  $\mathcal{O}(1)$  amount to the slow-roll parameter  $\eta$  — a result known as the supersymmetric  $\eta$ -problem [26]. Our main purpose here is to show how this problem is evaded in the case of  $D$ -term inflation [19].

Recall for these purposes the form of the supergravity scalar sector [27]. Given the Kähler function,  $K(z, z^*)$ , and superpotential,  $W(z)$ , the kinetic and potential terms for a collection of chiral scalars,  $z^i$ , are:

$$\mathcal{L} = -\sqrt{-g} \left[ \frac{1}{2} K_i^{j*}(z, z^*) \partial_\mu z^i \partial^\mu z_j^* + V(z, z^*) \right], \quad (3.13)$$

and  $V = V_F + V_D$  with

$$V_F = e^{K/M_p^2} \left[ \hat{K}_{j*}^i (D_i W) (D^j W)^* - \frac{3|W|^2}{M_p^2} \right], \quad (3.14)$$

and

$$V_D = \frac{1}{2} f^{ab} \left[ K_i (T_a z)^i + \xi_a \right] \left[ K_j (T_b z)^j + \xi_b \right]. \quad (3.15)$$

Here  $K_i^{j*} = \partial^2 K / \partial z^i \partial z_j^*$  and  $\hat{K}_{j*}^i$  is its inverse matrix. Similarly  $K_i = \partial K / \partial z^i$ ,  $T_b$  is a gauge-group generator,  $\xi_a$  is a Fayet-Iliopoulos term (and so is only present for  $U(1)$  gauge-group factors) and  $f^{ab}$  is the inverse matrix of  $\text{Re } F_{ab}(z)$ , where  $F_{ab}$  is the holomorphic gauge-kinetic function. In these expressions the Kähler derivative is

$$D_i W = \frac{\partial W}{\partial z^i} + \frac{W}{M_p^2} \frac{\partial K}{\partial z^i} = W_i + \frac{K_i W}{M_p^2}. \quad (3.16)$$

We now specialize as before to an inflaton multiplet,  $\Phi$ , plus two electrically charged multiplets,  $H_\pm$ . Write  $\varphi = (\Phi + \Phi^*)/M_p$  and take the no-scale ansatz [28]

$$K(\varphi, H_+^* H_+, H_-^* H_-) = -3 M_p^2 \log \varphi + k(H_+^* H_+) + \bar{k}(H_-^* H_-), \quad (3.17)$$

$F_{ab} = \delta_{ab}$  and  $W = W(H_+, H_-)$ . With these choices we have  $W_\Phi = \partial W / \partial \Phi = 0$  and the Kähler derivatives are:

$$D_\Phi W = -\frac{3W}{M_p \varphi} \quad (3.18)$$

$$D_{H_+} W = W_{H_+} + \frac{k' H_+^* W}{M_p^2}, \quad (3.18)$$

$$D_{H_-} W = W_{H_-} + \frac{\bar{k}' H_-^* W}{M_p^2}, \quad (3.19)$$

where  $k' = \partial k / \partial x$ , for  $x = H_+^* H_+$ , and so on. For instance, if  $k = H_+^* H_+$  then  $k' = 1$  etc.

The Kähler metric is

$$K_i^{j*} = \begin{pmatrix} 3/\varphi^2 & 0 & 0 \\ 0 & k' + k'' H_+^* H_+ & 0 \\ 0 & 0 & \bar{k}' + \bar{k}'' H_-^* H_- \end{pmatrix} \quad (3.20)$$

and so its inverse matrix becomes

$$\hat{K}_{j*}^i = \begin{pmatrix} \varphi^2/3 & 0 & 0 \\ 0 & 1/(k' + k''H_+^*H_+) & 0 \\ 0 & 0 & 1/(\bar{k}' + \bar{k}''H_-^*H_-) \end{pmatrix}. \quad (3.21)$$

Positivity of the kinetic energies requires  $k' + k''H_+^*H_+ > 0$  and  $\bar{k}' + \bar{k}''H_-^*H_- > 0$ .

With these choices the scalar potential,  $V = V_F + V_D$ , becomes

$$V_F = \frac{e^{(k+\bar{k})/M_p^2}}{\varphi^3} \left[ \frac{|D_{H_+}W|^2}{k' + k''H_+^*H_+} + \frac{|D_{H_-}W|^2}{\bar{k}' + \bar{k}''H_-^*H_-} \right], \quad (3.22)$$

and

$$V_D = \frac{e^2}{2} \left[ k'|H_+|^2 - \bar{k}'|H_-|^2 + \xi \right]^2, \quad (3.23)$$

where we take a  $U(1)$  gauge group with Fayet-Iliopoulos term  $\xi$ .

For inflationary purposes, we may further specialize to

$$W = M(H_+H_- - v^2), \quad (3.24)$$

and so  $W_{H_+} = MH_-$  and  $W_{H_-} = MH_+$ . Also suppose both  $k'$  and  $\bar{k}'$  are finite and nonzero as  $H_{\pm} \rightarrow 0$ , and that both  $k''$  and  $\bar{k}''$  are also finite (but possibly zero) in this limit. In this case we have  $D_{H_+}W \rightarrow 0$  and  $D_{H_-}W \rightarrow 0$  as  $H_{\pm} \rightarrow 0$ , and so also  $V_F \rightarrow 0$  in this limit.

Any zero of both  $V_F$  and  $V_D$  is a global minimum for  $V$ , which in our case is obtained by choosing  $k'|H_+|^2 + \xi = \bar{k}'|H_-|^2$  to ensure  $V_D = 0$ , and then choosing  $\varphi \rightarrow \infty$  to make  $V_F = 0$ . (This solution exists, for instance, for the minimal case  $k' = \bar{k}' = 1$ .) In this limit supersymmetry is broken (since  $D_iW \neq 0$ ) but with  $V = 0$  classically in the limit of large  $\varphi$ .

There is also a trough with nonzero  $V$  which is independent of  $\phi$ , corresponding to the case  $H_{\pm} = 0$ . In this limit we have  $D_{H_{\pm}}W = 0$ , and so  $V_F = 0$ , and so  $V = V_D = e^2\xi^2/2$ . The mass of the  $H_{\pm}$  scalar fields about this trough are both positive and of order  $M$ , provided that  $v \ll M_p$  and  $M \gg e\sqrt{\xi}$ . This trough is the direct analogue of the trough considered above in the globally-supersymmetric case, and we may apply a similar analysis again here to the same effect. Just as for the globally-supersymmetric case we have a classically flat potential, whose degeneracy gets lifted by virtual loops of heavy particles, and so for which the relative effect of a heavy multiplet for the slow-roll parameters need not be small.

The effective potential for this SUGRA model is similar to that of the previous SUSY model, with an important qualitative difference. Here the field-dependent scalar field masses for  $H_{\pm}$  take the form

$$M^2_{\pm} = \frac{M^2}{\phi^3} \mp e^2\xi \quad (3.25)$$

while their superpartners have masses  $m^2_{\pm} = \frac{M^2}{\phi^3}$ . In the limit where  $\frac{M^2}{\phi^3} \gg e^2\xi$ , the effective potential is approximately

$$V_{\text{eff}} = \frac{1}{2}e^2\xi^2 \left( 1 - \frac{3e^2}{8\pi^2} \ln \frac{\phi}{\phi_+} \right) \quad (3.26)$$

Therefore  $\phi$  rolls to larger instead of smaller values, which is expected since the global minimum is at  $\phi \rightarrow \infty$  in this model. Unlike the SUSY model, the slow roll approximation only gets better as the inflaton rolls, so the end of inflation is definitely triggered by the instability of  $H_+$ , at a field value given by

$$\phi_e^3 = \frac{M^2}{e^2 \xi} \quad (3.27)$$

The smallest allowable initial value of  $\phi$  is determined by the observational constraint on  $n_s - 1 \cong 2\eta \lesssim 0.2$ ,

$$\phi_0^2 = \frac{3e^2}{8\pi^2 \eta} \quad (3.28)$$

and the relation between  $\phi$  and the number of e-foldings,  $\phi^2 = \phi_0^2 + \frac{3e^2}{4\pi^2} N_e$ , gives the constraint that

$$\phi_e^2 - \phi_0^2 = \frac{3e^2}{4\pi^2} N_e \quad \rightarrow \quad e^5 = \frac{M^2}{\xi} \left( \frac{1}{4\pi^2} \left( N_e + \frac{3}{2\eta} \right) \right)^{-3/2} \quad (3.29)$$

The COBE normalization implies

$$\xi = 5 \times 10^{-4} \frac{\sqrt{3\eta}}{4\pi} M_p^2 \quad (3.30)$$

independently of the coupling  $e^2$ ; thus  $\xi$  is safely below the Planck scale. In fact, this model has the possibility of choosing  $M$  and  $\phi_e$  to be subPlanckian if desired, since  $M^2$  is freely adjustable. If we choose  $M$  such that  $\phi_e < 1$ , then (3.29) implies that  $e^3 < (4\pi^2/(N_e + 3/2\eta))^{3/2}$ , which is a weak constraint on the coupling. Even though the global minimum is at the superPlanckian value  $\phi \rightarrow \infty$ , all the inflationary dynamics can be comfortably accomplished in the regime where  $\phi < 1$ .

## 4. Conclusions

What we hope to have made clear is that some care must be taken when trying to use decoupling to limit the influence of heavy physics on low energy observables. We have argued that for a class of inflationary models, the inflaton potential is *solely* due to the effects of integrating out the heavy particles of the model. This allows for changes in the CMB parameters of order unity, instead of the  $H^2/M^2$  effects that might naïvely be expected. Together with the non-adiabatic mechanism of ref. [11] this gives two ways for heavy physics to be relevant to the physics of horizon exit, using only garden-variety hybrid inflation models.

Although our calculations are purely sub-Planckian, our conclusions do allow some inferences concerning trans-Planckian physics. If the trans-Planckian physics decouples, it may be able to take advantage of the same kinds of mechanisms we have found to leave some residue of itself in the CMB temperature anisotropies. If, on the other hand, the trans-Planckian physics does not decouple (as the  $\alpha$ -vacua proposals may not do) then its low-energy implications are likely to be everywhere, and the first problem is really to understand why low-energy predictions have been possible at all to this point, before worrying specifically about its implications for the CMB.

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