

A Brief Introduction into Quantum Gravity and Quantum Cosmology

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Why quantum gravity?

- ▶ Unification of all interactions
- ▶ Singularity theorems
 - ▶ Black holes
 - ▶ 'Big Bang'
- ▶ Problem of time
- ▶ Absence of viable alternatives

Wolfgang Pauli (1955):

Es scheint mir . . . , daß nicht so sehr die Linearität oder Nichtlinearität Kern der Sache ist, sondern eben der Umstand, daß hier eine allgemeinere Gruppe als die Lorentzgruppe vorhanden ist

Matvei Bronstein (1936):

The elimination of the logical inconsistencies connected with this requires a radical reconstruction of the theory, and in particular, the rejection of a Riemannian geometry dealing, as we see here, with values unobservable in principle, and perhaps also the rejection of our ordinary concepts of space and time, modifying them by some much deeper and nonevident concepts. *Wer's nicht glaubt, bezahlt einen Taler.*

The problem of time

- **Absolute time** in quantum theory:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$$

- **Dynamical time** in general relativity:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

QUANTUM GRAVITY?

Planck units

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33} \text{ cm}$$

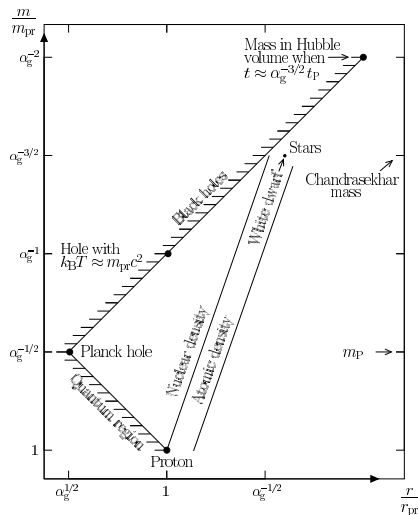
$$t_P = \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44} \text{ s}$$

$$m_P = \frac{\hbar}{l_P c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5} \text{ g} \approx 1.22 \times 10^{19} \text{ GeV}/c^2$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Structures in the Universe



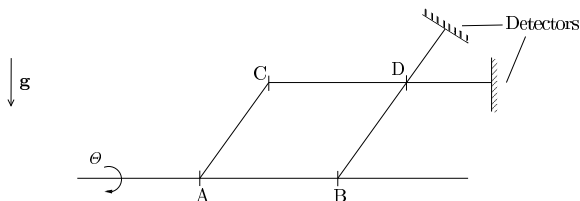
$$\alpha_g = \frac{G m_{\text{pr}}^2}{\hbar c} = \left(\frac{m_{\text{pr}}}{m_{\text{P}}} \right)^2 \approx 5.91 \times 10^{-39}$$

Steps towards quantum gravity

- ▶ Interaction of micro- and macroscopic systems with an external gravitational field
- ▶ Quantum field theory on curved backgrounds (or in flat background, but in non-inertial systems)
- ▶ Full quantum gravity

Quantum systems in external gravitational fields

Neutron and atom interferometry



Experimente:

- ▶ Neutron interferometry in the field of the Earth (Colella, Overhauser, and Werner ("COW") 1975)
- ▶ Neutron interferometry in accelerated systems (Bonse and Wroblewski 1983)
- ▶ Discrete neutron states in the field of the Earth (Nesvizhevsky *et al.* 2002)
- ▶ Atom interferometry (z. B. Peters, Chung, Chu 2001: measurement of g with accuracy $\Delta g/g \sim 10^{-10}$)

Non-relativistic expansion of the Dirac equation yields

$$i\hbar \frac{\partial \psi}{\partial t} \approx H_{\text{FW}} \psi$$

mit

$$\begin{aligned} H_{\text{FW}} = & \underbrace{\beta m c^2}_{\text{rest mass}} + \underbrace{\frac{\beta}{2m} \mathbf{p}^2}_{\text{kinetic energy}} - \underbrace{\frac{\beta}{8m^3 c^2} \mathbf{p}^4}_{\text{SR correction}} + \underbrace{\beta m (\mathbf{a} \cdot \mathbf{x})}_{\text{COW}} \\ & - \underbrace{\frac{\omega \mathbf{L}}{2}}_{\text{Sagnac effect}} - \underbrace{\frac{\omega \mathbf{S}}{2}}_{\text{Mashhoon effect}} \\ & + \frac{\beta}{2m} \mathbf{p} \frac{\mathbf{a} \cdot \mathbf{x}}{c^2} \mathbf{p} + \frac{\beta \hbar}{4mc^2} \vec{\Sigma} (\mathbf{a} \times \mathbf{p}) + \mathcal{O} \left(\frac{1}{c^3} \right) \end{aligned}$$

Black-hole radiation

Black holes radiate with a **temperature** proportional to \hbar :

$$T_{\text{BH}} = \frac{\hbar \kappa}{2\pi k_{\text{B}} c}$$

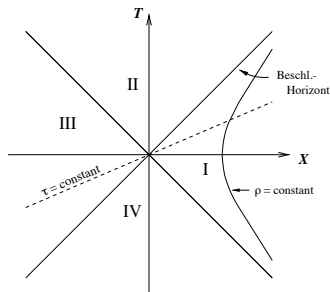
Schwarzschild case:

$$\begin{aligned} T_{\text{BH}} &= \frac{\hbar c^3}{8\pi k_{\text{B}} G M} \\ &\approx 6.17 \times 10^{-8} \left(\frac{M_{\odot}}{M} \right) \text{ K} \end{aligned}$$

Black holes also have an **entropy**:

$$S_{\text{BH}} = k_{\text{B}} \frac{A}{4l_{\text{P}}^2} \stackrel{\text{Schwarzschild}}{\approx} 1.07 \times 10^{77} k_{\text{B}} \left(\frac{M}{M_{\odot}} \right)^2$$

Analogous effect in flat spacetime



Accelerated observer in the Minkowski vacuum experiences thermal radiation with temperature

$$T_{\text{DU}} = \frac{\hbar a}{2\pi k_{\text{B}} c} \approx 4.05 \times 10^{-23} a \left[\frac{\text{cm}}{\text{s}^2} \right] \text{ K} .$$

(Davies–Unruh temperature)

Main Approaches to Quantum Gravity

No question about quantum gravity is more difficult than the question, “What is the question?” (John Wheeler 1984)

- ▶ Quantum general relativity
 - ▶ Covariant approaches (perturbation theory, path integrals, ...)
 - ▶ Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, ...)
- ▶ String theory
- ▶ Other approaches
(Quantization of topology, causal sets ...)

Covariant quantum gravity

Perturbation theory:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \sqrt{\frac{32\pi G}{c^4}} f_{\mu\nu}$$

- ▶ $\bar{g}_{\mu\nu}$: classical background
- ▶ Perturbation theory with respect to $f_{\mu\nu}$
(Feynman rules)
- ▶ “Particle” of quantum gravity: **graviton**
(massless spin-2 particle)

Perturbative non-renormalizability

Effective field theory

Concrete predictions possible at low energies
(even in non-renormalizable theory)

Example:

Quantum gravitational correction to the Newtonian potential

$$V(r) = -\frac{Gm_1m_2}{r} \left(1 + \underbrace{3\frac{G(m_1+m_2)}{rc^2}}_{\text{GR-correction}} + \underbrace{\frac{41}{10\pi} \frac{G\hbar}{r^2c^3}}_{\text{QG-correction}} \right)$$

(Bjerrum–Bohr *et al.* 2003)

Analogy: Chiral perturbation theory (small pion mass)

Beyond perturbation theory?

Example: self-energy of a thin charged shell

Energy of the shell using the bare mass m_0 is

$$m(\epsilon) = m_0 + \frac{Q^2}{2\epsilon} ,$$

which diverges for $\epsilon \rightarrow 0$. But the inclusion of gravity leads to

$$m(\epsilon) = m_0 + \frac{Q^2}{2\epsilon} - \frac{Gm^2(\epsilon)}{2\epsilon} ,$$

which leads for $\epsilon \rightarrow 0$ to a finite result,

$$m(\epsilon) \xrightarrow{\epsilon \rightarrow 0} \frac{|Q|}{\sqrt{G}} .$$

The sigma model

Non-linear σ model: N -component field ϕ_a satisfying $\sum_a \phi_a^2 = 1$

- ▶ is non-renormalizable for $D > 2$
- ▶ exhibits a non-trivial UV fixed point at some coupling g_c ('phase transition')
- ▶ an expansion in $D - 2$ and use of renormalization-group (RG) techniques gives information about the behaviour in the vicinity of the non-trivial fixed point

Example: superfluid Helium

The specific heat exponent α was measured in a space shuttle experiment (Lipa *et al.* 2003): $\alpha = -0.0127(3)$, which is in excellent agreement with three calculations in the $N = 2$ non-linear σ -model:

- ▶ $\alpha = -0.01126(10)$ (4-loop result; Kleinert 2000);
- ▶ $\alpha = -0.0146(8)$ (lattice Monte Carlo estimate; Campostrini *et al.* 2001);
- ▶ $\alpha = -0.0125(39)$ (lattice variational RG prediction; cited in Hamber 2009)

Asymptotic Safety

Weinberg (1979): A theory is called **asymptotically safe** if all essential coupling parameters g_i of the theory approach for $k \rightarrow \infty$ a non-trivial fix point

Preliminary results:

- ▶ Effective gravitational constant vanishes for $k \rightarrow \infty$
- ▶ Effective gravitational constant increases with distance (simulation of Dark Matter?)
- ▶ Small positive cosmological constant as an infrared effect (Dark Energy?)
- ▶ Spacetime appears two-dimensional on smallest scales

(H. Hamber *et al.*, M. Reuter *et al.*)

Path integrals

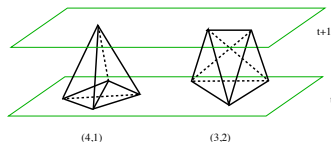
$$Z[g] = \int \mathcal{D}g_{\mu\nu}(x) \, e^{iS[g_{\mu\nu}(x)]/\hbar}$$

In addition: sum over all topologies?

- ▶ Euclidean path integrals
(e.g. for Hartle–Hawking proposal [see quantum cosmology] or Regge calculus)
- ▶ Lorentzian path integrals
(e.g. for dynamical triangulation)

Dynamical triangulation

- ▶ makes use of *Lorentzian* path integrals
- ▶ edge lengths of simplices remain fixed; sum is performed over all possible combinations with equilateral simplices
- ▶ Monte-Carlo simulations



Preliminary results:

- ▶ Hausdorff dimension $H = 3.10 \pm 0.15$
- ▶ Spacetime two-dimensional on smallest scales (cf. asymptotic-safety approach)
- ▶ positive cosmological constant needed
- ▶ continuum limit?

A brief history of early covariant quantum gravity

- ▶ L. Rosenfeld, Über die Gravitationswirkungen des Lichtes, *Annalen der Physik* (1930)
- ▶ M. P. Bronstein, Quantentheorie schwacher Gravitationsfelder, *Physikalische Zeitschrift der Sowjetunion* (1936)
- ▶ S. Gupta, Quantization of Einstein's Gravitational Field: Linear Approximation, *Proceedings of the Royal Society* (1952)
- ▶ C. Misner, Feynman quantization of general relativity, *Reviews of Modern Physics* (1957)
- ▶ R. P. Feynman, Quantum theory of gravitation, *Acta Physica Polonica* (1963)
- ▶ B. S. DeWitt, Quantum theory of gravity II, III, *Physical Review* (1967)

Canonical quantum gravity

Central equations are **constraints**:

$$\hat{H}\Psi = 0$$

Different canonical approaches

- ▶ **Geometrodynamics** –
metric and extrinsic curvature
- ▶ **Connection dynamics** –
connection (A^i_a) and coloured electric field (E^a_i)
- ▶ **Loop dynamics** –
flux of E^a_i and holonomy

Erwin Schrödinger 1926:

We know today, in fact, that our classical mechanics fails for very small dimensions of the path and for very great curvatures. Perhaps this failure is in strict analogy with the failure of geometrical optics . . . that becomes evident as soon as the obstacles or apertures are no longer great compared with the real, finite, wavelength. . . . Then it becomes a question of searching for an undulatory mechanics, and the most obvious way is by an elaboration of the Hamiltonian analogy on the lines of undulatory optics.¹

¹ *wir wissen doch heute, daß unsere klassische Mechanik bei sehr kleinen Bahndimensionen und sehr starken Bahnkrümmungen versagt.* Vielleicht ist dieses Versagen eine volle Analogie zum Versagen der geometrischen Optik . . . , das bekanntlich eintritt, sobald die ‘Hindernisse’ oder ‘Öffnungen’ nicht mehr groß sind gegen die wirkliche, endliche Wellenlänge. . . . Dann gilt es, eine ‘undulatorische Mechanik’ zu suchen – und der nächstliegende Weg dazu ist wohl die wellentheoretische Ausgestaltung des Hamiltonschen Bildes.

Hamilton–Jacobi equation

Hamilton–Jacobi equation \longrightarrow guess a wave equation

In the vacuum case, one has

$$16\pi G G_{abcd} \frac{\delta S}{\delta h_{ab}} \frac{\delta S}{\delta h_{cd}} - \frac{\sqrt{h}}{16\pi G} ({}^{(3)}R - 2\Lambda) = 0 ,$$
$$D_a \frac{\delta S}{\delta h_{ab}} = 0$$

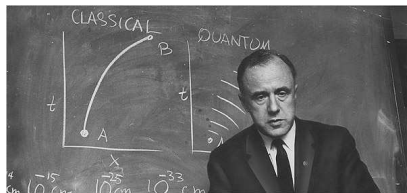
(Peres 1962)

Find wave equation which yields the Hamilton–Jacobi equation in the semiclassical limit:

$$\text{Ansatz : } \Psi[h_{ab}] = C[h_{ab}] \exp \left(\frac{i}{\hbar} S[h_{ab}] \right)$$

The dynamical gravitational variable is the three-metric h_{ab} ! It is the argument of the wave functional.

Quantum geometrodynamics



In the vacuum case, one has

$$\hat{H}\Psi \equiv \left(-2\kappa\hbar^2 G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - (2\kappa)^{-1} \sqrt{h} ({}^{(3)}R - 2\Lambda) \right) \Psi = 0,$$
$$\kappa = 8\pi G$$

Wheeler–DeWitt equation

$$\hat{D}^a \Psi \equiv -2\nabla_b \frac{\hbar}{i} \frac{\delta \Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraint

Problem of time

- ▶ no external time present; spacetime has *disappeared!*
- ▶ local **intrinsic time** can be defined through local hyperbolic structure of Wheeler–DeWitt equation (‘wave equation’)
- ▶ related problem: *Hilbert-space problem* – which inner product, if any, to choose between wave functionals?
 - ▶ Schrödinger inner product?
 - ▶ Klein–Gordon inner product?
- ▶ Problem of *observables*

The semiclassical approximation

Wheeler–DeWitt equation and momentum constraints in the presence of matter (e.g. a scalar field):

$$\left\{ -\frac{1}{2m_{\text{P}}^2} G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} - 2m_{\text{P}}^2 \sqrt{h} {}^{(3)}R + \hat{\mathcal{H}}_{\perp}^{\text{m}} \right\} |\Psi[h_{ab}]\rangle = 0 ,$$
$$\left\{ -\frac{2}{i} h_{ab} D_c \frac{\delta}{\delta h_{bc}} + \hat{\mathcal{H}}_a^{\text{m}} \right\} |\Psi[h_{ab}]\rangle = 0$$

(bra and ket notation refers to non-gravitational fields)

Make comparison with a quantum-mechanical model:

$$\begin{aligned} -\frac{1}{2M} \frac{\partial^2}{\partial Q^2} &\leftrightarrow -\frac{1}{2m_{\text{P}}^2} G_{abcd} \frac{\delta^2}{\delta h_{ab} \delta h_{cd}} , \\ V(Q) &\leftrightarrow -2m_{\text{P}}^2 \sqrt{h} {}^{(3)}R , \\ h(q, Q) &\leftrightarrow \hat{\mathcal{H}}_{\perp}^{\text{m}} , \\ \Psi(q, Q) &\leftrightarrow |\Psi[h_{ab}]\rangle . \end{aligned}$$

A quantum-mechanical model

Divide the total system into a 'heavy part' described by the variable Q and a 'light part' described by the variable q ;
full system be described by a stationary Schrödinger equation:

$$\boxed{H\Psi(q, Q) = E\Psi(q, Q)}$$

with

$$H = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial Q^2} + V(Q) + h(q, Q)$$

$$\text{Ansatz : } \Psi(q, Q) = \sum_n \chi_n(Q) \psi_n(q, Q)$$

(assume that $\langle \psi_n | \psi_m \rangle = \delta_{nm}$ for each Q)

Get an effective (exact) equation for the “heavy” part:

$$\sum_n \left(\frac{\mathcal{P}_{mn}^2}{2M} + \epsilon_{mn}(Q) \right) \chi_n(Q) + V(Q) \chi_m(Q) = E \chi_m(Q) ,$$

$\epsilon_{mn}(Q) \equiv \langle \psi_m | h | \psi_n \rangle$: “Born–Oppenheimer potential”

$$\mathcal{P}_{mn} \equiv \frac{\hbar}{i} \left(\delta_{mn} \frac{\partial}{\partial Q} - \frac{i}{\hbar} A_{mn} \right) ,$$

$$A_{mn}(Q) \equiv i\hbar \left\langle \psi_m \left| \frac{\partial \psi_n}{\partial Q} \right. \right\rangle: \text{“connection”}$$

Born–Oppenheimer approximation

First approximation: neglect off-diagonal terms in the effective equation

$$\left[\frac{1}{2M} \left(\frac{\hbar}{i} \frac{\partial}{\partial Q} - A_{nn}(Q) \right)^2 + V(Q) + E_n(Q) \right] \chi_n(Q) = E \chi_n(Q),$$

$$E_n(Q) \equiv \epsilon_{nn}(Q) = \langle \psi_n | h | \psi_n \rangle$$

Second approximation: WKB-ansatz for the “heavy” part

$$\chi_n(Q) = C_n(Q) e^{iMS_n(Q)/\hbar}$$

Neglecting the connection, the above equation then becomes the Hamilton–Jacobi equation:

$$H_{\text{cl}} \equiv \frac{P_n^2}{2M} + V(Q) + E_n(Q) = E$$

(in gravity: semiclassical Einstein equations)

One can now introduce a time coordinate t_n (“WKB time”) via the Hamilton equations of motion for the “heavy” part,

$$\begin{aligned}\frac{d}{dt_n}P_n &= -\frac{\partial}{\partial Q}H_{\text{cl}} = -\frac{\partial}{\partial Q}(V(Q) + E_n(Q)) , \\ \frac{d}{dt_n}Q &= \frac{\partial}{\partial P_n}H_{\text{cl}} = \frac{P_n}{M}\end{aligned}$$

Use the WKB time in the effective equation for the “light” part

Get an effective (exact) equation for the “light” part:

$$\sum_n \chi_n(Q) \left[h(q, Q) - \left(E - V(Q) + \frac{\hbar^2}{2M\chi_n} \frac{\partial^2 \chi_n}{\partial Q^2} \right) - \frac{\hbar^2}{2M} \frac{\partial^2}{\partial Q^2} - \frac{\hbar^2}{M\chi_n} \frac{\partial \chi_n}{\partial Q} \frac{\partial}{\partial Q} \right] \psi_n(q, Q) = 0$$

- ▶ Neglect $\frac{\partial^2 \chi_n}{\partial Q^2}$
(assume slow variation of ψ_n with respect to Q);
- ▶ use the definition of WKB time in the last term:

$$-\frac{\hbar^2}{M\chi_n} \frac{\partial \chi_n}{\partial Q} \frac{\partial \psi_n}{\partial Q} \approx -i\hbar \frac{\partial S_n}{\partial Q} \frac{\partial \psi_n}{\partial Q} \equiv -i\hbar \frac{\partial \psi_n}{\partial t_n}$$

ψ_n is thus evaluated along a particular classical trajectory of the “heavy” variable, $\psi_n(Q(t_n), q) \equiv \psi_n(t_n, q)$.

Further algebra leads to

$$\sum_n \chi_n \left[h(q, t_n) - E_n(t_n) - i\hbar \frac{\partial}{\partial t_n} \right] \psi_n(t_n, q) = 0$$

Restriction to one component and absorption of $E_n(t)$ into ψ would yield

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = h\psi}$$

cf. Mott (1931)

Back to quantum gravity

Ansatz:

$$|\Psi[h_{ab}]\rangle = C[h_{ab}]e^{im_{\text{P}}^2 S[h_{ab}]}|\psi[h_{ab}]\rangle$$

One evaluates $|\psi[h_{ab}]\rangle$ along a solution of the classical Einstein equations, $h_{ab}(\mathbf{x}, t)$, corresponding to a solution, $S[h_{ab}]$, of the Hamilton–Jacobi equations; this solution is obtained from

$$\dot{h}_{ab} = N G_{abcd} \frac{\delta S}{\delta h_{cd}} + 2D_{(a} N_{b)}$$

$$\frac{\partial}{\partial t} |\psi(t)\rangle = \int d^3x \dot{h}_{ab}(\mathbf{x}, t) \frac{\delta}{\delta h_{ab}(\mathbf{x})} |\psi[h_{ab}]\rangle$$

→ functional Schrödinger equation for quantized matter fields in the chosen external classical gravitational field:

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle &= \hat{H}^m |\psi(t)\rangle \\ \hat{H}^m &\equiv \int d^3x \left\{ N(\mathbf{x}) \hat{\mathcal{H}}_{\perp}^m(\mathbf{x}) + N^a(\mathbf{x}) \hat{\mathcal{H}}_a^m(\mathbf{x}) \right\} \end{aligned}$$

\hat{H}^m : matter-field Hamiltonian in the Schrödinger picture, parametrically depending on (generally non-static) metric coefficients of the curved space–time background.

WKB time t controls the dynamics in this approximation

Quantum gravitational corrections

Next order in the Born–Oppenheimer approximation gives

$$\hat{H}^m \rightarrow \hat{H}^m + \frac{1}{m_{\text{P}}^2} (\text{various terms})$$

(C. K. and T. P. Singh (1991); A. O. Barvinsky and C. K. (1998))

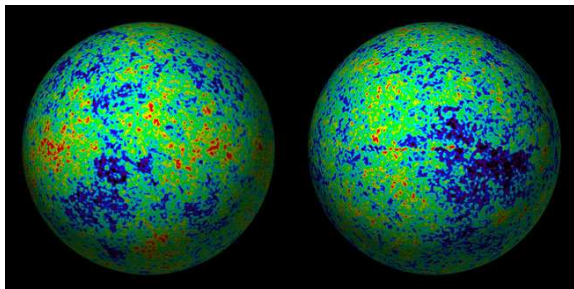
Simple example: Quantum gravitational correction to the trace anomaly in de Sitter space:

$$\delta\epsilon \approx -\frac{2G\hbar^2 H_{\text{dS}}^6}{3(1440)^2\pi^3 c^8}$$

(C. K. 1996)

Observations

Does the anisotropy spectrum of the cosmic background radiation contain information about quantum gravity?



Eagerly awaited: Results of the PLANCK satellite (Launch: May 2009)

A brief history of early quantum geometrodynamics

- ▶ F. Klein, *Nachrichten von der Königlischen Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-physikalische Klasse*, **1918**, 171–189:
first four Einstein equations are “Hamiltonian” and “momentum density” equations
- ▶ L. Rosenfeld, *Annalen der Physik*, 5. Folge, **5**, 113–152 (1930):
general constraint formalism; first four Einstein equations are constraints; consistency conditions in the quantum theory (“Dirac consistency”)

- ▶ P. Bergmann and collaborators (from 1949 on): general formalism (mostly classical); notion of observables
Bergmann (1966): $H\psi = 0$, $\partial\psi/\partial t = 0$
("To this extent the Heisenberg and Schrödinger pictures are indistinguishable in any theory whose Hamiltonian is a constraint.")
- ▶ P. Dirac (1951): general formalism; Dirac brackets
- ▶ P. Dirac (1958/59): application to the gravitational field; reduced quantization
("I am inclined to believe from this that four-dimensional symmetry is not a fundamental property of the physical world.")
- ▶ ADM (1959–1962): lapse and shift; rigorous definition of gravitational energy and radiation by canonical methods

- ▶ B. S. DeWitt, Quantum theory of gravity. I. The canonical theory. *Phys. Rev.*, **160**, 1113–48 (1967):
general Wheeler–DeWitt equation; configuration space;
quantum cosmology; semiclassical limit; conceptual issues, . . .
- ▶ J. A. Wheeler, Superspace and the nature of quantum
geometrodynamics. In *Battelle rencontres* (ed. C. M. DeWitt and
J. A. Wheeler), pp. 242–307 (1968):
general Wheeler–DeWitt equation; superspace; semiclassical
limit; conceptual issues; . . .

Path Integral satisfies Constraints

- ▶ Quantum mechanics: path integral satisfies Schrödinger equation
- ▶ Quantum gravity: path integral satisfies Wheeler–DeWitt equation and diffeomorphism constraints

A. O. Barvinsky (1998): direct check in the **one-loop approximation** that the quantum-gravitational path integral satisfies the constraints

→ **connection between covariant and canonical approach**
application in quantum cosmology: **no-boundary condition**

Ashtekar's new variables

- ▶ new momentum variable: densitized version of triad,

$$E_i^a(x) := \sqrt{h(x)} e_i^a(x) ;$$

- ▶ new configuration variable: 'connection' ,

$$GA_a^i(x) := \Gamma_a^i(x) + \beta K_a^i(x)$$

$$\boxed{\{A_a^i(x), E_j^b(y)\} = 8\pi\beta\delta_j^i\delta_a^b\delta(x,y)}$$

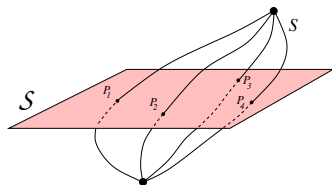
Loop quantum gravity

- ▶ new configuration variable: holonomy,

$$U[A, \alpha] := \mathcal{P} \exp \left(G \int_{\alpha} A \right) ;$$

- ▶ new momentum variable: densitized triad flux

$$E_i[\mathcal{S}] := \int_{\mathcal{S}} d\sigma_a E_i^a$$



Quantization of area:

$$\hat{A}(\mathcal{S})\Psi_S[A] = 8\pi\beta l_P^2 \sum_{P \in \mathcal{S} \cap \mathcal{S}} \sqrt{j_P(j_P + 1)} \Psi_S[A]$$

String theory

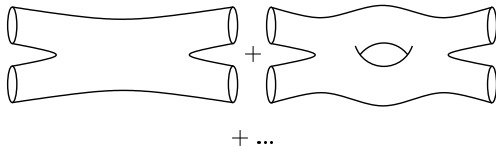
Important properties:

- ▶ Inclusion of gravity unavoidable
- ▶ Gauge invariance, supersymmetry, higher dimensions
- ▶ Unification of all interactions
- ▶ Perturbation theory probably finite at all orders, but sum diverges
- ▶ Only **three** fundamental constants: \hbar, c, l_s
- ▶ Branes as central objects
- ▶ Dualities connect different string theories

Space and time in string theory

$$Z = \int \mathcal{D}X \mathcal{D}h e^{-S/\hbar}$$

(X : Embedding; h : Metric on worldsheet)



Absence of quantum anomalies \longrightarrow

- ▶ Background fields obey Einstein equations up to $\mathcal{O}(l_s^2)$; can be derived from an effective action
- ▶ Constraint on the number of spacetime dimensions: 10 resp. 11

Generalized uncertainty relation:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \frac{l_s^2}{\hbar} \Delta p$$

Problems

- ▶ Too many “string vacua” (problem of landscape)
- ▶ No background independence?
- ▶ Standard model of particle physics?
- ▶ What is the role of the 11th dimension? What is M-theory?
- ▶ Experiments?

Black holes

Microscopic explanation of entropy?

$$S_{\text{BH}} = k_{\text{B}} \frac{A}{4l_{\text{P}}^2}$$

- ▶ *Loop quantum gravity*: microscopic degrees of freedom are the spin networks; S_{BH} only follows for a specific choice of β : $\beta = 0.237532 \dots$
- ▶ *String theory*: microscopic degrees of freedom are the “D-branes”; S_{BH} only follows for special (extremal or near-extremal) black holes
- ▶ *Quantum geometrodynamics*: e.g. $S \propto A$ in the LTB model

Problem of information loss

- ▶ Final phase of evaporation?
- ▶ Fate of information is a consequence of the fundamental theory (unitary or non-unitary)
- ▶ Problem does not arise in the semiclassical approximation (thermal character of Hawking radiation follows from decoherence)
- ▶ Empirical problems:
 - ▶ Are there primordial black holes?
 - ▶ Can black holes be generated in accelerators?

Primordial black holes

Primordial Black Holes could form from density fluctuations in the early Universe (with masses from 1 g on); black holes with an initial mass of $M_0 \approx 5 \times 10^{14}$ Gramm would evaporate “today” → typical spectrum of Gamma rays



Fermi Gamma-ray Space Telescope; Launch: June 2008

Generation of mini black holes at the LHC?



CMS detector

Only possible if space has more than three dimensions

My own research on quantum black holes

- ▶ Primordial black holes from density fluctuations in inflationary models
- ▶ Quasi-normal modes and the Hawking temperature
- ▶ Decoherence of quantum black holes and its relevance for the problem of information loss
- ▶ Hawking temperature from solutions to the Wheeler–DeWitt equation (for the LTB model) as well as quantum gravitational corrections
- ▶ Area law for the entropy from solutions to the Wheeler–DeWitt equation (for the 2+1-dimensional LTB model)
- ▶ Origin of corrections to the area law
- ▶ Model for black-hole evaporation

Why Quantum Cosmology?

Gell-Mann and Hartle 1990:

Quantum mechanics is best and most fundamentally understood in the framework of quantum cosmology.

- ▶ Quantum theory is universally valid:
Application to the Universe as a whole as the only closed quantum system in the strict sense
- ▶ Need quantum theory of **gravity**, since gravity dominates on large scales

Quantization of a Friedmann Universe

Closed Friedmann–Lemaître universe with scale factor a ,
containing a homogeneous massive scalar field ϕ
(two-dimensional *minisuperspace*)

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega_3^2$$

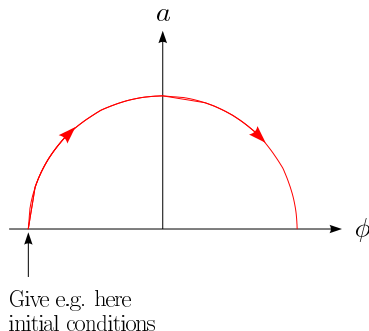
The **Wheeler–DeWitt equation** reads (with units $2G/3\pi = 1$)

$$\frac{1}{2} \left(\frac{\hbar^2}{a^2} \frac{\partial}{\partial a} \left(a \frac{\partial}{\partial a} \right) - \frac{\hbar^2}{a^3} \frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2 \right) \psi(a, \phi) = 0$$

Factor ordering chosen in order to achieve covariance in
minisuperspace

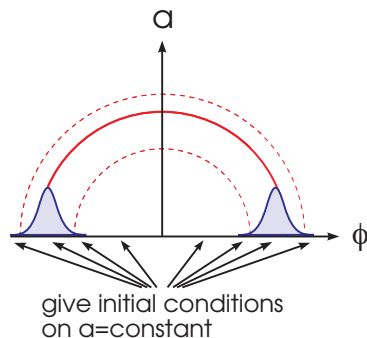
Determinism in classical and quantum theory

Classical theory



Recollapsing part is
deterministic successor of
expanding part

Quantum theory

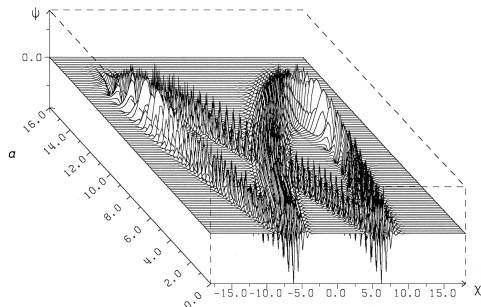


“Recollapsing” wave packet
must be present “initially”

Example

Indefinite Oscillator

$$\hat{H}\psi(a, \chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2 \right) \psi = 0$$



Validity of Semiclassical Approximation?

Closed universe: 'Final condition' $\psi \xrightarrow{a \rightarrow \infty} 0$



wave packets in general **disperse**



WKB approximation not always valid

Solution: **Decoherence** (see below)

Introduction of inhomogeneities

Describe small inhomogeneities by **multipoles** $\{x_n\}$ around the minisuperspace variables (e.g. a and ϕ)

$$\left(H_0 + \sum_n H_n(a, \phi, x_n) \right) \Psi(\alpha, \phi, \{x_n\}) = 0$$

(Halliwell and Hawking 1985)

If ψ_0 is of WKB form, $\psi_0 \approx C \exp(iS_0/\hbar)$ (with a slowly varying prefactor C), one will get with $\Psi = \psi_0 \prod_n \psi_n$,

$$i\hbar \frac{\partial \psi_n}{\partial t} \approx H_n \psi_n$$

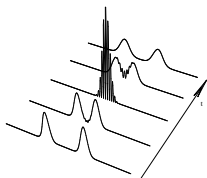
with

$$\frac{\partial}{\partial t} \equiv \nabla S_0 \cdot \nabla$$

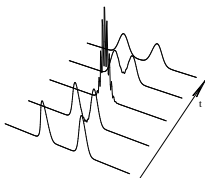
t : ‘**WKB time**’ – controls the dynamics in this approximation

Decoherence

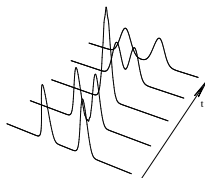
Irreversible emergence of classical properties through the unavoidable interaction with the environment (irrelevant degrees of freedom)



without decoherence



medium decoherence



strong decoherence

Decoherence in quantum cosmology

Quantum gravity \Rightarrow superposition of different metrics

Decoherence?

- ▶ ‘System’: Global degrees of freedom (radius of Universe, inflaton field, ...)
- ▶ ‘Environment’: Density fluctuations, gravitational waves, other fields

(Zeh 1986, C.K. 1987)

Example: Scale factor a of de Sitter space ($a \propto e^{H_I t}$) (‘system’) is decohered by gravitons (‘environment’) according to

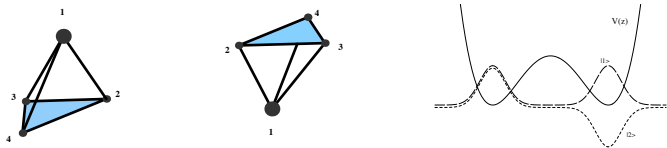
$$\rho_0(a, a') \rightarrow \rho_0(a, a') \exp \left(-CH_I^3 a(a - a')^2 \right) , \quad C > 0$$

The **Universe assumes classical properties** at the ‘beginning’ of the inflationary phase

(Barvinsky, Kamenshchik, C.K. 1999)

Time from Symmetry Breaking

Analogy from molecular physics: emergence of chirality



dynamical origin: decoherence due to scattering with light or air molecules

quantum cosmology: decoherence between $\exp(iS_0/\hbar)$ - and $\exp(-iS_0/\hbar)$ -part of wave function through interaction with multipoles

one example for decoherence factor:

$$\exp\left(-\frac{\pi m H_0^2 a^3}{128 \hbar}\right) \sim \exp(-10^{43}) \text{ (C. K. 1992)}$$

Decoherence of primordial fluctuations

The modes for the inflaton field and the gravitons evolve into a 'squeezed quantum state' during inflation ($r > 100$)

They decohere through coupling to other fields
(pointer basis = field basis)

Decoherence time is given by

$$t_d \sim H_I^{-1} \sim 10^{-34} \text{ s}$$

(C.K., Lohmar, Polarski, Starobinsky 1998, 2007)

Fluctuations assume classical properties during inflation

Interpretation of quantum cosmology

Both quantum general relativity and string theory preserve the linear structure for the quantum states

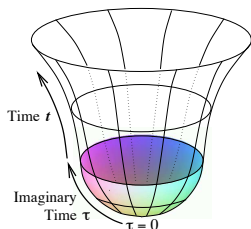
⇒ strict validity of the **superposition principle**

only interpretation so far: *Everett interpretation*
(with decoherence as an essential part)

B. S. DeWitt 1967:

Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarrassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

No-boundary proposal



S. W. Hawking, Vatican conference 1982:

There ought to be something very special about the boundary conditions of the universe and what can be more special than the condition that there is no boundary.

$$\Psi[h_{ab}, \Phi, \Sigma] = \sum_{\mathcal{M}} \nu(\mathcal{M}) \int_{\mathcal{M}} \mathcal{D}g \mathcal{D}\Phi \, e^{-S_E[g_{\mu\nu}, \Phi]}$$

Problems with the no-boundary proposal

- ▶ Four-manifolds not classifiable
- ▶ Problems with Euclidean gravitational action
→ evaluation for general complex metrics
- ▶ Many solutions in minisuperspace
- ▶ Solutions do in general not correspond to classical solutions
(e.g. increase exponentially for large a)

main merit perhaps in the semiclassical approximation
(selection of extrema for the classical action); e.g.

$$\psi_{\text{NB}} \propto (a^2 V(\phi) - 1)^{-1/4} \exp\left(\frac{1}{3V(\phi)}\right) \cos\left(\frac{(a^2 V(\phi) - 1)^{3/2}}{3V(\phi)} - \frac{\pi}{4}\right)$$

Other boundary conditions

- ▶ The wave function should obey $\Psi [^{(3)}\mathcal{G}] = 0$ for all singular three-geometries $^{(3)}\mathcal{G}$ (DeWitt 1967)
- ▶ **Tunnelling Condition**: Only *outgoing* modes near singular boundaries of superspace (Vilenkin 1982); e.g.

$$\psi_T \propto (a^2 V(\phi) - 1)^{-1/4} \exp\left(-\frac{1}{3V(\phi)}\right) \exp\left(-\frac{i}{3V(\phi)}(a^2 V(\phi) - 1)^{3/2}\right)$$

- ▶ **SIC!**: Demand normalizability for $a \rightarrow 0$ through introduction of a ‘Planck potential’ (Conradi and Zeh 1991); can be justified e.g. from loop quantum cosmology

prediction of inflation?

Criteria for quantum avoidance of singularities

No general agreement!

Sufficient criteria in quantum geometrodynamics:

- ▶ Vanishing of the wave function at the point of the classical singularity (dating back to DeWitt 1967)
- ▶ Spreading of wave packets when approaching the region of the classical singularity

concerning the second criterium:

only in the semiclassical regime (narrow wave packets following the classical trajectories) do we have an approximate notion of geodesics → only in this regime can we apply the classical singularity theorems

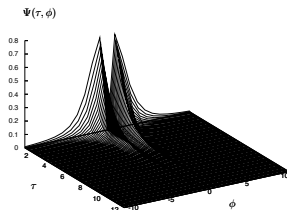
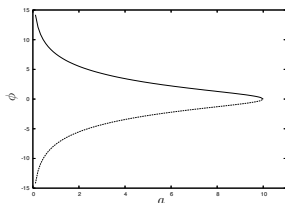
Quantum cosmology with big brake

Classical model: Equation of state $p = A/\rho$, $A > 0$, for a Friedmann universe with scale factor $a(t)$ and scalar field $\phi(t)$ with potential ($24\pi G = 1$)

$$V(\phi) = V_0 \left(\sinh(|\phi|) - \frac{1}{\sinh(|\phi|)} \right) ;$$

develops pressure singularity (only $\ddot{a}(t)$ becomes singular)

Quantum model: Normalizable solutions of the Wheeler–DeWitt equation vanish at the classical singularity



(Kamenshchik, C. K., Sandhöfer 2007)

$D = 11$ supergravity: near spacelike singularity
cosmological billiard description based on the
Kac–Moody group $E_{10} \longrightarrow$ discussion of
Wheeler–DeWitt equation

- ▶ $\Psi \rightarrow 0$ near the singularity
- ▶ Ψ is generically complex and oscillating

(Kleinschmidt, Koehn, Nicolai 2009)

Quantum phantom cosmology

Classical model: Friedmann universe with scale factor $a(t)$ containing a scalar field with negative kinetic term ('phantom')
→ develops a **big-rip singularity**
(ρ and p diverge as a goes to infinity at a *finite time*)

Quantum model: Wave-packet solutions of the Wheeler–DeWitt equation disperse in the region of the classical big-rip singularity
→ time and the classical evolution come to an end;
only a stationary quantum state is left

Exhibition of quantum effects at large scales!

(Dąbrowski, C. K., Sandhöfer 2006)

Loop Quantum Cosmology

- ▶ Difference equation instead of Wheeler–DeWitt equation; the latter emerges as an effective description away from the Planck scale
- ▶ Singularity avoidance (from difference equation or from effective Friedmann equation via a *bounce*)
- ▶ Prediction of inflation (?)
- ▶ Observable effect in the CMB spectrum (?)
- ▶ **but:** not yet derived from full loop quantum gravity

(cf. M. Bojowald, C.K., P. Vargas Moniz, arXiv:1005.2471v1 [gr-qc])

Effective equations in loop quantum cosmology

Effective Hamiltonian constraint reads

$$H_{\text{eff}} = -\frac{3}{8\pi G\beta^2} \frac{\sin^2(\lambda p)}{\lambda^2} a^3 + H_m ,$$

where $\lambda = 2(\sqrt{3}\pi\beta)^{1/2}l_P$

This leads to a modified Hubble rate:

$$H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_c} \right) ,$$

where $\rho_c = 3/(8\pi G\beta^2\lambda^2) \approx 0.41\rho_P$

→ **bounces** which may prevent singularities

(P. Singh, arXiv:0901.2750: “*All strong singularities are generically resolved in loop quantum cosmology.*”)

Corresponds to the second of the criteria above (breakdown of semiclassical approximation near the classical singularity)

How special is the Universe

Penrose (1981):

Entropy of the observed part of the Universe is maximal if all its mass is in one black hole; the probability for our Universe would then be (updated version from C.K. arXiv:0910.5836)

$$\frac{\exp\left(\frac{S}{k_B}\right)}{\exp\left(\frac{S_{\max}}{k_B}\right)} \sim \frac{\exp(3.1 \times 10^{104})}{\exp(1.8 \times 10^{121})} \approx \exp(-1.8 \times 10^{121})$$

Arrow of time from quantum cosmology

Fundamental asymmetry with respect to “intrinsic time”:

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial \alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\rightarrow 0 \text{ for } \alpha \rightarrow -\infty} \right] \right) \Psi = 0$$

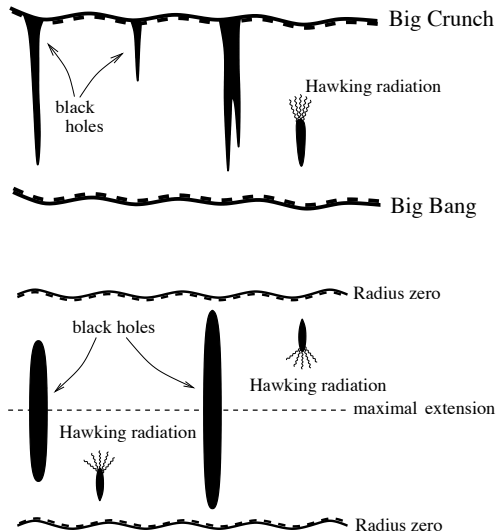
Is compatible with simple boundary condition:

$$\Psi \xrightarrow{\alpha \rightarrow -\infty} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

Entropy increases with increasing α , since entanglement with other degrees of freedom increases

→ **defines** time direction

Is the expansion of the Universe a tautology?



(C. K. and Zeh 1995)

Observations and experiments

Up to now only expectations!

- ▶ Evaporation of black holes (but need primordial black holes or big extra dimensions)
- ▶ Origin of masses and coupling constants (Λ !)
- ▶ Quantum gravitational corrections observable in the anisotropy spectrum of the cosmic background radiation?
- ▶ Time-dependent coupling constants, violation of the equivalence principle, . . .
- ▶ Signature of a discrete structure of spacetime (γ -ray bursts?)
- ▶ Signature of extra dimensions (LHC)? Supersymmetry?

Einstein (according to Heisenberg) : *Erst die Theorie entscheidet darüber, was man beobachten kann.*

More details in

C. K., *Quantum Gravity*, second edition
(Oxford 2007).