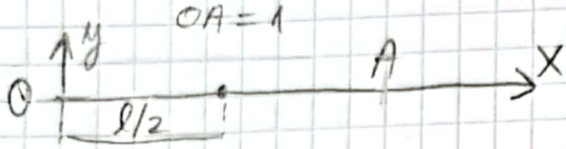
27.10.2021

LABORATOR 3

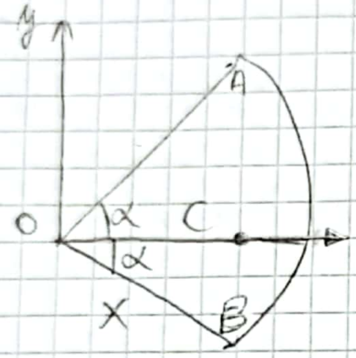
Centre de greutate bare și plăci omogene

Tipuri de bare – drepte (trebuie să știm lungimea)



$OA = l \quad x_C = \frac{l}{2} \quad y_C = 0$

– arc circular

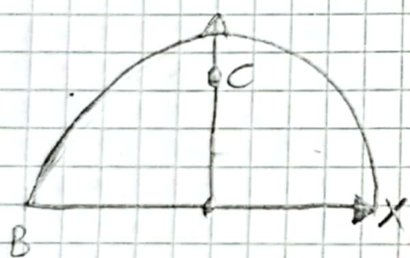


$$AB = 2r\alpha$$

$$x_C = OC = r\frac{\sin\alpha}{\alpha}$$

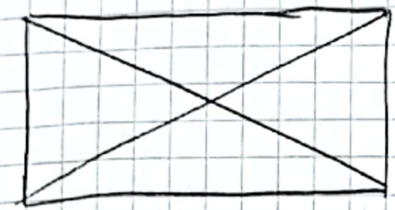
$$y_C = 0$$

– semicirculară



Plăci

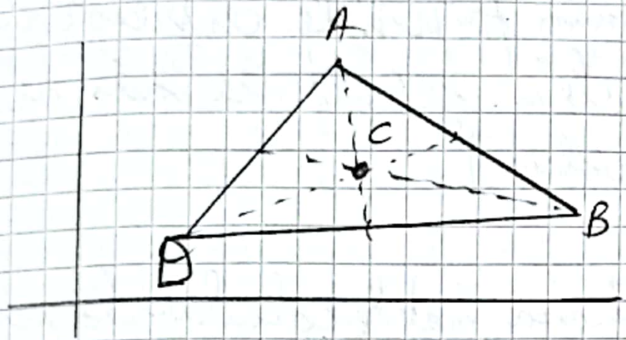
Dreptunghi



$$A = l \cdot L$$

$$x_c = \frac{l}{2} \qquad y_c = \frac{L}{2}$$

Triunghi:

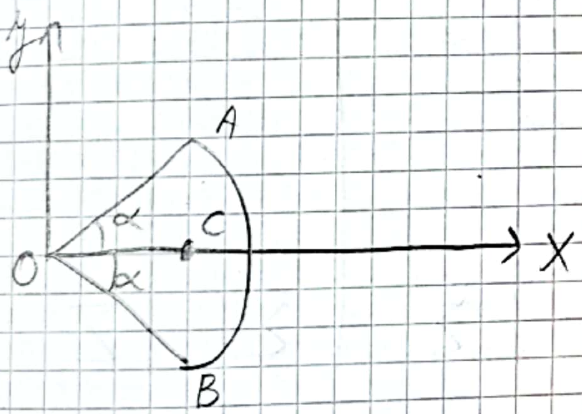


$$x_c = \frac{x_A + x_B + x_D}{3}$$

$$y_c = \frac{y_A + y_B + y_D}{3}$$

$$A = \frac{b \cdot h}{2}$$

Sector circular

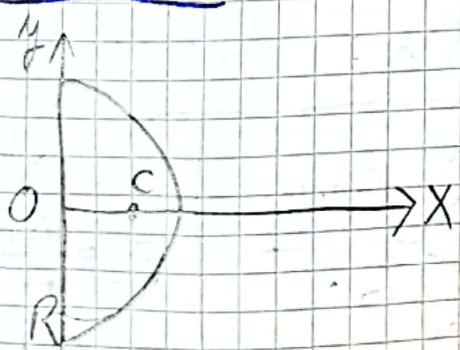


$$A = \alpha \cdot R^2$$

$$x_c = OC = \frac{2}{3} R \cdot \frac{\sin\alpha}{\alpha}$$

$$y_c = 0$$

Semicerc:

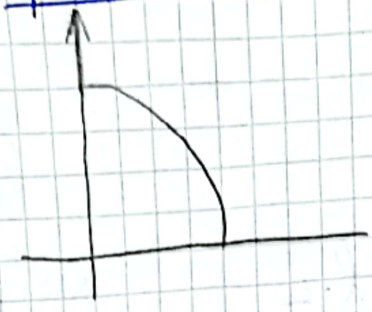


$$A = \frac{\pi R^2}{2}$$

$$x_c = OC = \frac{4R}{3\pi}$$

$$y_c = 0$$

Sfert de cerc:



Pasul 2 – alegere sistem de axe

Pasul 3 – determinăm pozițiile centrelor de greutate și se calculează elem. geometrice ale acestora (lungimi, arii, volume)

Pasul 4 – Se completează datele calculate la pasul 3 într-un tabel de următ formă:

Nr crt	G_i	x_i	y_i	z_i	$G_i \cdot x_i$	$G_i \cdot y_i$	$G_i \cdot z_i$
1							
2							
⋮							
n							
Σ	Σ	---	---	---	Σ	Σ	Σ

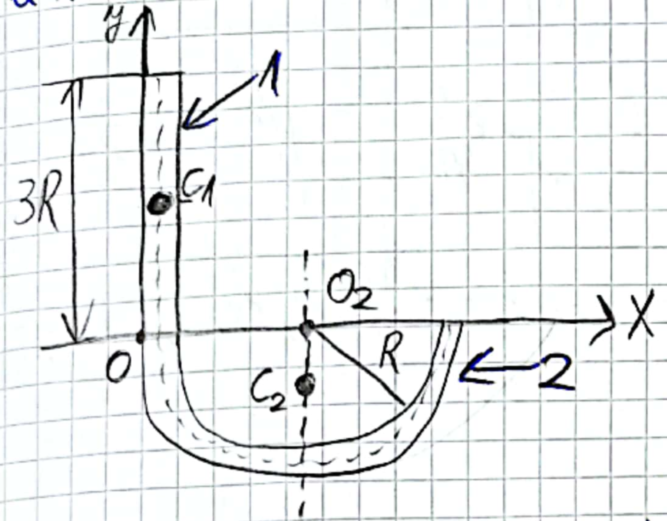
Pasul 5. Se determină coordonatele centrului de greutate ale corpului complex:

$$x_c = \frac{\sum_{i=1}^{n} G_i \cdot x_i}{\sum_{i=1}^{n} G_i} \qquad y_c = \frac{\sum_{i=1}^{n} G_i \cdot y_i}{\sum_{i=1}^{n} G_i} \qquad z_c = \frac{\sum_{i=1}^{n} G_i \cdot z_i}{\sum_{i=1}^{n} G_i}$$

unde G_i = lungimea / aria / volumul, după caz.

Problema 1

Să se det coordonatele centrului de greutate a sistemului de bare din figura, cunoscând raza R



Bara 1: Lungimea barei 1 este 3R: $l_1 = 3R$

$$x_1 = 0$$

$$y_1 = \frac{3R}{2}$$

Bara 2: $l_2 = \pi \cdot R$

$$O_2C_2 = R\,\frac{\sin\alpha}{\alpha}$$

$$\alpha = \frac{\pi}{2}\ \text{rad}$$

$$\Rightarrow O_2C_2 = R\,\frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2R}{\pi} \Rightarrow \boxed{O_2C_2 = \frac{2R}{\pi}}$$

$$x_2 = R$$

$$y_2 = -O_2C_2 = -\frac{2R}{\pi}$$

Tabel

Nr.crt	l_i	x_i	y_i	$l_i \cdot x_i$	$l_i \cdot y_i$
Bara 1	$3R$	0	$\frac{3R}{2}$	0	$\frac{9R^2}{2}$
Bara 2	πR	R	$-\frac{2R}{\pi}$	πR^2	$-2R^2$
	$\sum l_i$	---	---	$\sum l_i \cdot x_i$	$\sum l_i \cdot y_i$

$$\sum l_i = 3R + \pi R$$

$$\sum l_i \cdot x_i = 0 + \pi R^2 = \pi R^2$$

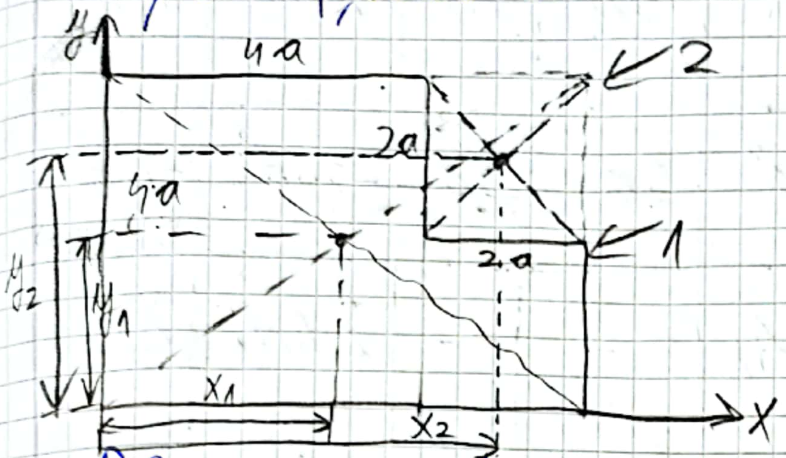
$$\sum l_i \cdot y_i = \frac{9R^2}{2} - 2R^2 = \frac{5R^2}{2}$$

$$x_c = \frac{\sum l_i \cdot x_i}{\sum l_i} = \frac{\pi R^2}{3R + \pi R} = \frac{\pi R}{3 + \pi} = 0{,}51R$$

$$y_c = \frac{\sum l_i \cdot y_i}{\sum l_i} = \frac{\frac{5R^2}{2}}{3R+}$$

Problema 2

Det. coordonatele centrului de greutate $C(x_c, y_c)$ al plăcii, știind laturile



Placa se poate împărți astfel

Corp 1 – dreptunghi de lungime 6a și lățime 4a, considerată plină din care decupăm corpul 2

Corp 2 – pătrat cu $l = 2a$

Corp 1: $A_1 = L \cdot l = 6a \cdot 4a = 24a^2$

$$x_1 = \frac{L}{2} = \frac{6a}{2} = 3a$$

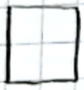
$$y_1 = \frac{l}{2} = \frac{4a}{2} = 2a$$

Corp 2: $A_2 = l^2 = (2a)^2 = 4a^2$

$$x_2 = 6a - \frac{2a}{2} = 6a - a = 5a$$

$$y_2 = 4a - \frac{2a}{2} = 4a - a = 3a$$

Tabel

Nr	Ord	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
1.		$24a^2$	$3a$	$2a$	$72a^3$	$48a^3$
2.		$-4a^2$	$5a$	$3a$	$-20a^3$	$-12a^3$
3.		$\sum A_i$	— —	— —	$\sum A_i x_i$	$\sum A_i y_i$

$$\sum A_i = 24a^2 - 4a^2 = 20a^2$$

$$\sum A_i x_i = 72a^3 - 20a^3 = 52a^3$$

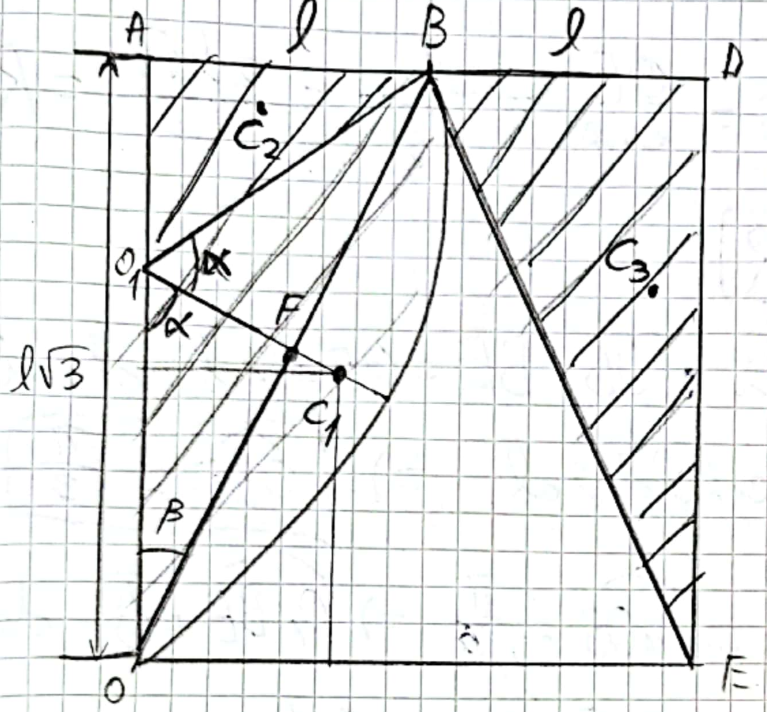
$$\sum A_i y_i = 48a^3 - 12a^3 = 36a^3$$

$$x_c = \frac{\sum A_i x_i}{\sum A_i} = \frac{52a^3}{20a^2} = 2{,}6\,a$$

$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{36a^3}{20a^2} = 1{,}8\,a$$

Problema 3

Să se det coordonatele centrului de greutate $C(x_c, y_c)$
Știm că arcul de cerc este tangent la latura OE.



- ΔOAB - dreptunghic

$$OB^2 = (l\sqrt{3})^2 + l^2 \Rightarrow OB = 2l$$

- Punctul F se află pe mediatoarea coardei deci este la mijlocul lui OB

$$OF = \frac{OB}{2} = l$$

- Unghiul format de OB cu axa y

$$tg\beta = \frac{AB}{OA} = \frac{l}{l\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \beta = \frac{\pi}{6}$$

• În ΔOO_1F - dreptunghic:

$$\alpha = \pi - \frac{\pi}{2} - \beta = \pi - \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \Rightarrow \boxed{\alpha = \frac{\pi}{3}}$$

• Deci $\widehat{OO_1B} = 2\alpha = \frac{2\pi}{3}$

$$OO_1 = O_1B = \frac{OF}{\cos\beta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2l\sqrt{3}}{3} = R$$

$$\Rightarrow \boxed{OO_1 = R}$$

• Observăm că: $OB = BE = OE = 2l$ deci

ΔOBE este echilateral $\Rightarrow \boxed{\widehat{OBE} = \frac{\pi}{3}}$

Cum $\widehat{O\cancel{B}O_1B} = \widehat{O_1OB} = \frac{\pi}{6} \Rightarrow \widehat{O_1BE} = \frac{\pi}{2}$ deci

BE este tangentă în B la O_1B

$$O_1C_1 = \frac{2}{3}R\frac{\sin\alpha}{\alpha} = \frac{2}{3}\,\frac{2l\sqrt{3}}{3}\cdot\frac{\frac{\sqrt{3}}{2}}{}$$

Corpul 1

$$A_1 = \frac{\pi}{3}R^2 = \frac{\pi}{3}\left(\frac{2l\sqrt{3}}{3}\right)^2 = \frac{4\pi l^2}{9}$$

$$X_1 = O_1C_1 \sin\frac{\pi}{3} = \frac{2l}{\pi}\,\frac{\sqrt{3}}{2} = \frac{l\sqrt{3}}{\pi}$$

$$Y_1 = R - O_1C_1\cos\frac{\pi}{3} = \frac{2l\sqrt{3}}{3} - \frac{2l}{\pi}\cdot\frac{1}{2} = l\left(\frac{2\sqrt{3}}{3} - \frac{1}{\pi}\right.$$

Corp 2

$$A_2 = \frac{O_1A \cdot AB}{2} = \frac{1}{2}\frac{l\sqrt{3}}{3} l = \frac{l^2\sqrt{3}}{6}$$

$$x_2 = \frac{l+0+0}{3} = \frac{l}{3}$$

$$y_2 = \frac{1}{3}\left(\frac{2l\sqrt{3}}{3} + l\sqrt{3} + l\sqrt{3}\right) = \frac{8l\sqrt{3}}{9}$$

$$y_2 = \frac{y_{O_1} + y_A + y_B}{3}$$

Corp 3

$$A_3 = \frac{DE \cdot BB}{2} = \frac{c_1 \cdot c_2}{2} = \frac{1}{2} l\sqrt{3}\, l = \frac{l^2\sqrt{3}}{2}$$

$$x_3 = \frac{1}{3}(l + 2l + 2l) = \frac{5l}{3}$$

$$y_3 = \frac{1}{3}(0 + l\sqrt{3} + l\sqrt{3}) = \frac{2l\sqrt{3}}{3}$$

Nr corp	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
1	$\frac{4\pi l^2}{9}$	$\frac{l\sqrt{3}}{\pi}$	$l\left(\frac{2\sqrt{3}}{3} - \frac{1}{\pi}\right)$	$\frac{4\sqrt{3}l^3}{9}$	$l^3\left(\frac{8\pi\sqrt{3}}{27} - \frac{4}{9}\right)$
2	$\frac{l^2\sqrt{3}}{6}$	$\frac{l}{3}$	$\frac{8l\sqrt{3}}{9}$	$\frac{l^3\sqrt{3}}{18}$	$\frac{4l^3}{9}$
3	$\frac{l^2\sqrt{3}}{2}$	$\frac{5l}{3}$	$\frac{2l\sqrt{3}}{3}$	$\frac{5l^3\sqrt{3}}{6}$	l^3
Σ	$l^2\frac{6\sqrt{3}+4\pi}{9}$	— —	— —	$\frac{4l^3\sqrt{3}}{3}$	$l^3\left(1 + \frac{8\pi\sqrt{3}}{27}\right)$

$$x_C = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{\frac{4l^3\sqrt{3}}{3}}{l^2\frac{6\sqrt{3}+4\pi}{9}} = \frac{12\sqrt{3}}{6\sqrt{3}+4\pi} l = 0{,}9l$$

$$y_C = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{l^3\left(1 + \frac{8\pi\sqrt{3}}{27}\right)}{l^2\frac{6\sqrt{3}+4\pi}{9}} = \frac{27 + 8\pi\sqrt{3}}{3(6\sqrt{3}+4\pi)} l = 1{,}02l$$