Computer Exercise 3
EL2520 Control Theory and Practice

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Suppression of disturbances

Since y = Sd, the disturbance at $\omega = 100\pi\ \frac{rad}{sec}$, can be suppressed by ensuring $|S(j \cdot 100\pi)| < 1$. Using the hint, a complex-conjugate pole is placed such that a peak is obtained at the frequency $\omega = 100\pi\ \frac{rad}{sec}$.

The weight is

$$\begin{aligned} W_S(s) &= \frac{1}{\left(s + 0.01 - i\sqrt{(100\pi)^2 - 0.01^2}\right)\left(s + 0.01 + i\sqrt{(100\pi)^2 - 0.01^2}\right)} \\ &= \frac{1}{s^2 + 0.02s + (100\pi)^2} \end{aligned}$$

How much is the disturbance damped on the output? What amplification is required for a P-controller to get the same performance, and what are the disadvantages of such a controller?

From Figure 1 it can be seen that the disturbance is damped approximately to a factor of $8 \cdot 10^{-4}$. Hence, to obtain the same amplification using a P-controller, the gain of the controller must be

$$P \approx \frac{8 \cdot 10^{-4}}{|G(i100\pi)|} \approx 0.0087. \tag{1}$$

The P-controller has the advantages of being of lower-order and hence easier to implement. Additionally, a larger bandwidth of disturbances is attenuated not only at $\omega = 100\pi\ \frac{rad}{sec}$. However, due to its simplicity the P-controller could result in steady-state errors in the output if the model is uncertain.

Robustness

What is the condition on T to guarantee stability according to the small gain theorem, and how can it be used to choose the weight W_T?

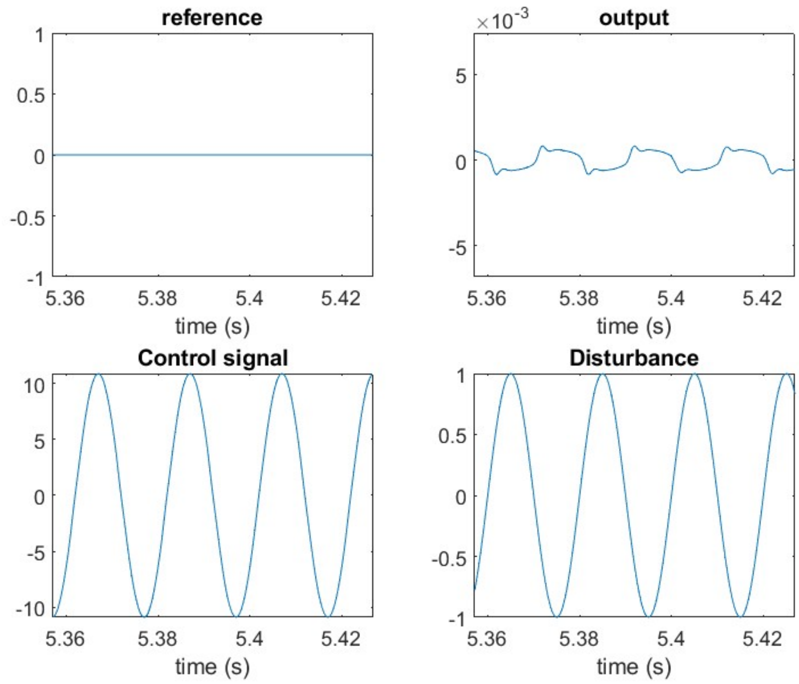

Figure 1: Simulation results with system G, using W_S.

The uncertainty $\Delta_G(s)$ is pulled out and the transfer function from w to ξ is computed to be $\xi = -Tw$. The small gain theorem guarantees the robust stability of the closed-loop of the perturbed system $G_0(s)$ if Δ_G and T are both stable and the condition

$$|T(i\omega)| < |\Delta_G^{-1}(i\omega)|, \qquad \forall\omega \tag{2}$$

.

The exact uncertainty transfer function can be computed as

$$\Delta_G(s) = \frac{G_0(s)}{G(s)} - 1 = -\frac{3}{s+2}. \tag{3}$$

Hence, the robustness condition translates into

$$\big|T(i\omega)\big| < \Big|\frac{i\omega + 2}{3}\Big| = \frac{\sqrt{\omega^2+4}}{3}, \qquad \forall\omega. \tag{4}$$

One of the requirements that the stacked should have, is a H_∞-norm less than γ is

$$|T(i\omega)| \leq \gamma|W_T^{-1}(i\omega)|, \qquad \forall\omega. \tag{5}$$

The robust stability condition in Eq. 4 and the above condition on the complementary sensitivty function can be combined to obtain the relation

$$|W_T(i\omega)| > \gamma |\Delta_G(i\omega)| = \gamma \left| \frac{3}{iw + 2} \right|. \tag{6}$$

$$(1/\gamma)|\Delta_G^{-1}(i\omega)| > |W_T^{-1}(i\omega)| \tag{7}$$

Here we choose $\gamma = 10^{-3}$.

The weights are

$$W_S(s) = \frac{1}{s^2 + 0.02s + (100\pi)^2}$$
$$W_T(s) = 10^{-3} \cdot \frac{3}{s+2}$$

Is the small gain theorem fulfilled?
Yes, since Δ_G and T are stable, from Figure 2 it is clear that the condition on Eq. 4 is satisfied and hence the small gain theorem is fulfilled.

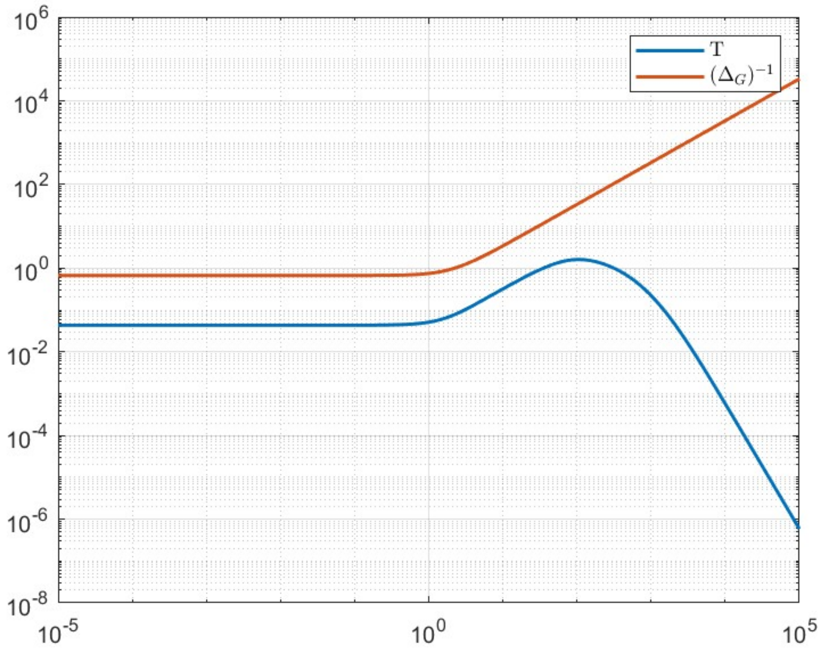

Figure 2: Bode diagram showing that the small gain theorem is satisfied.

Compare the results to the previous simulation
The summarized results of the simulation in exercise 4.1.iv is compared with the simulation results in exercise 4.1.ii in Table 1. In conclusion, the disturbance attentuation is about a factor of 0.27 worse for the second simulation, while the magnitude of the used control signal is the same.

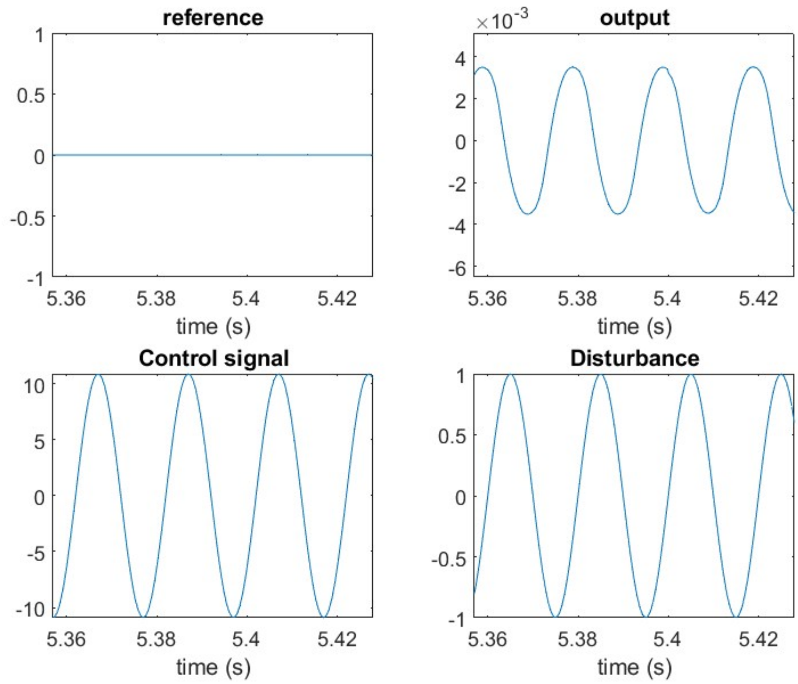

Figure 3: Simulation results with system G_0, using W_S and W_T.

Table 1: Summary of simulation results in exercise 4.1.ii and 4.2.iv.

	Disturbance (|w|)	Output (|y|)	Control Signal (|u|)
4.1.(ii)	≤ 1	$\leq 8 \cdot 10^{-4}$	≤ 11
4.2.(iv)	≤ 1	$\leq 3.5 \cdot 10^{-3}$	≤ 11

Control signal

The weights are

$$W_S(s) = \frac{1}{s^2 + 0.02s + (100\pi)^2}$$
$$W_T(s) = 10^{-3} \cdot \frac{3}{s+2}$$
$$W_U(s) = \frac{5}{s+0.1}$$

Compare the results to the previous simulations
The results of the simulation is summarized in Table 2. The control signal is reduced to almost half, however, the disturbance attenuation at the output is worse. In general, the more the control signal is reduced the more disturbances pass through to the output.

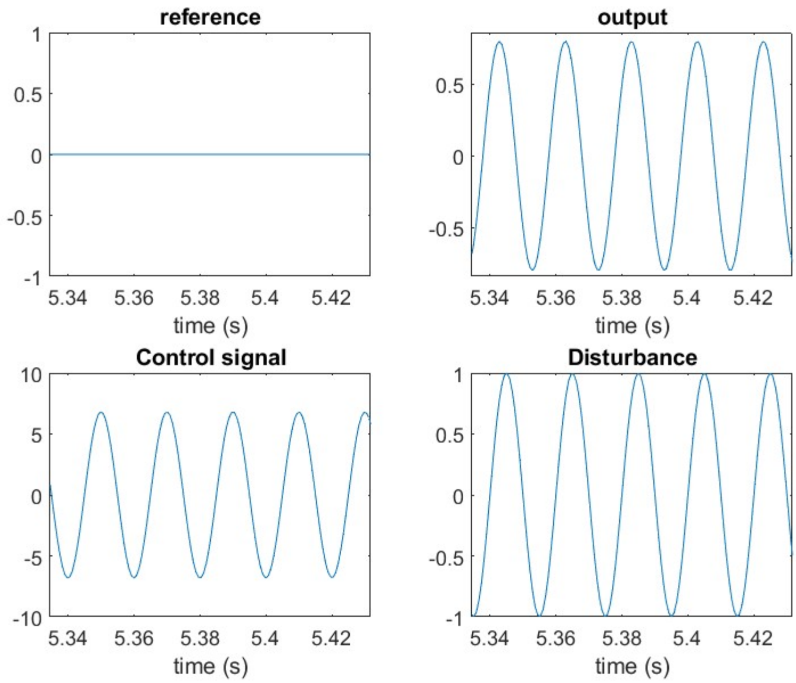

Figure 4: Simulation results with system G_0, using W_S, W_T and W_U.

Table 2: Summary of simulation results in exercise 4.1.ii, 4.2.iv. and 4.3.i

	Disturbance (|w|)	Output (|y|)	Control Signal (|u|)
4.1.(ii)	≤ 1	$\leq 8 \cdot 10^{-4}$	≤ 11
4.2.(iv)	≤ 1	$\leq 3.5 \cdot 10^{-3}$	≤ 11
4.3.(i)	≤ 1	≤ 0.8	≤ 6