Computer Exercise 4
EL2520 Control Theory and Practice

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Minimum phase case

Dynamic decoupling

The RGA-matrix at steady state is computed as

$$RGA(G(0)) = \begin{pmatrix} 1.5625 & -0.5625 \\ -0.5625 & 1.5625 \end{pmatrix}. \tag{1}$$

Hence, the pairings (y_1-u_1 and y_2-u_2) were used for the imminent decoupling process. Following the hint in the instructions, $w_{11}(s) = w_{22}(s) = 1$ and the off-diagonal terms of W were chosen such that $\tilde{G}(s) = G(s)W(s)$ is diagonal. The dynamic decoupling in exercise 3.2.1 is

$$W(s) = \begin{pmatrix} 1 & \frac{-0.01336}{s+0.02572} \\ \frac{-0.01476}{s+0.0213} & 1 \end{pmatrix}$$

The Bode diagram of $\tilde{G}(s)$ is shown in Figure 1. The off-diagonal terms can be seen to be almost zero (due to the numerical errors mentioned in the instructions) and hence decoupling is successful.

Is the controller good?

The H_∞-norm of the sensitivity function is $||S||_\infty = 1.1318 < 2$ and of the complementary sensitivity $||T||_\infty = 1.1308 < 1.25$ and hence the performance should be good. Regarding the time domain specifications, the step response for each output exhibits an overshoot of M= 12%, a settling time of T_s =85.063 sec and a rise time of $T_r = 14.629$ sec, as can be seen in Figure 2.

Hence, the control performance is good.

Are the output signals coupled?

No, the outputs are completely decoupled, i.e. there is no interaction between the pairings y_1-u_2 and y_2-u_1 as shown in the simulation. Hence, the decoupling is successful.

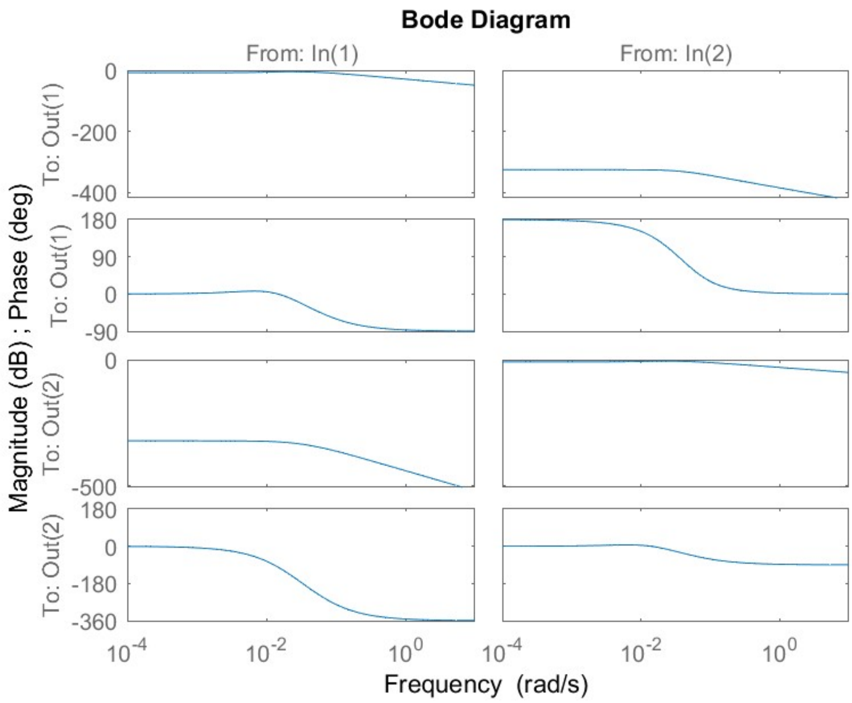

Figure 1: Bode diagram of $\tilde{G}(s)$ derived in exercise 3.2.1

Glover-MacFarlane robust loop-shaping

What are the similarities and differences compared to the nominal design? The H_∞-norm of the sensitivity function is $||S||_\infty = 1.1314$ and of the complementary sensitivity is $||T||_\infty = 1$. Both are lower than in the nominal design. However, since the robust stability condition depends on the complementary sensitivity function, the robustified design decreases the $||T||_\infty$ more than the sensitivty function. The robustified sensitivity function and the nominal ones are shown in Figure 4.

Regarding the time-domain performance, the robustified system is only slightly slower compared to the nominal design. This is reflected in the increased settling time $T_s = 94.2266$ sec and rise time $T_r = 14.629$ sec.

The outputs of the system still remain completely decoupled as in the nominal design. Hence, performance is not affected much after robustification.

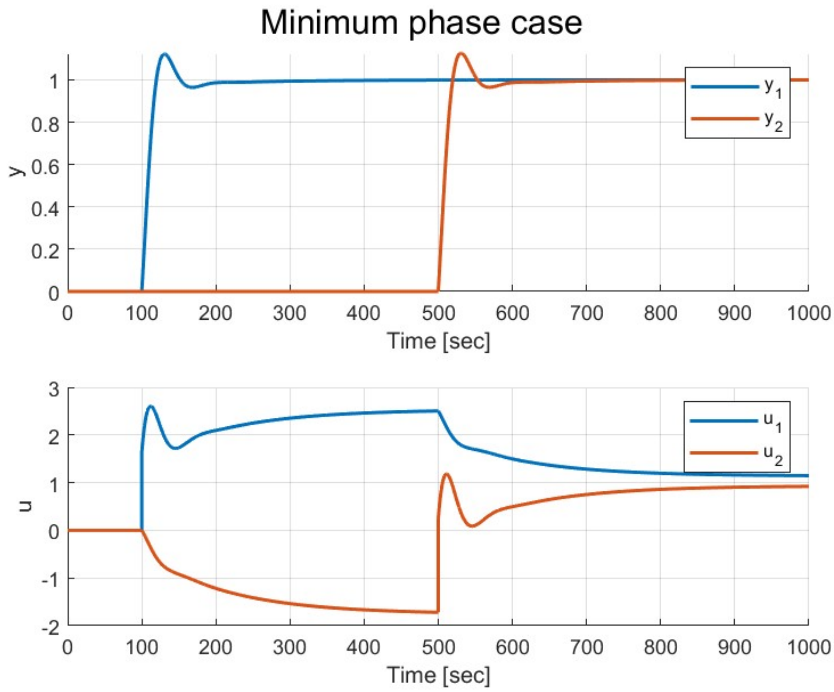

Figure 2: Simulink plots from exercise 3.2.4

Non-minimum phase case

Dynamic decoupling

The RGA-matrix at steady state is computed as

$$RGA(G(0)) = \begin{pmatrix} -0.5625 & 1.5625 \\ 1.5625 & -0.5625 \end{pmatrix}. \tag{2}$$

Hence, the pairings (y_1-u_2 and y_2-u_1) were used for the imminent decoupling process. Following the hint in the instructions, $w_{12}(s) = w_{21}(s) = 1$ and the diagonal terms of W were chosen such that $\tilde{G}(s) = G(s)W(s)$ is diagonal. Additionally, to obtain a semi-proper decoupling compensator W, the hint in the instruction is used and all elements in W are multiplied with $\frac{10\omega_c}{s+10\omega_c}$. The dynamic decoupling in exercise 3.2.1 is

$$W(s) = \begin{pmatrix} \frac{-1.143s-0.1039}{s+0.2} & \frac{0.2}{s+0.2} \\ \frac{0.2}{s+0.2} & \frac{-1.615s-0.1386}{s+0.2} \end{pmatrix}$$

The Bode diagram of $\tilde{G}(s)$ is shown in Figure 5. Again, the off-diagonal terms can be seen to be almost zero (same numerical errors discussed in the instructions) and hence decoupling is successful.

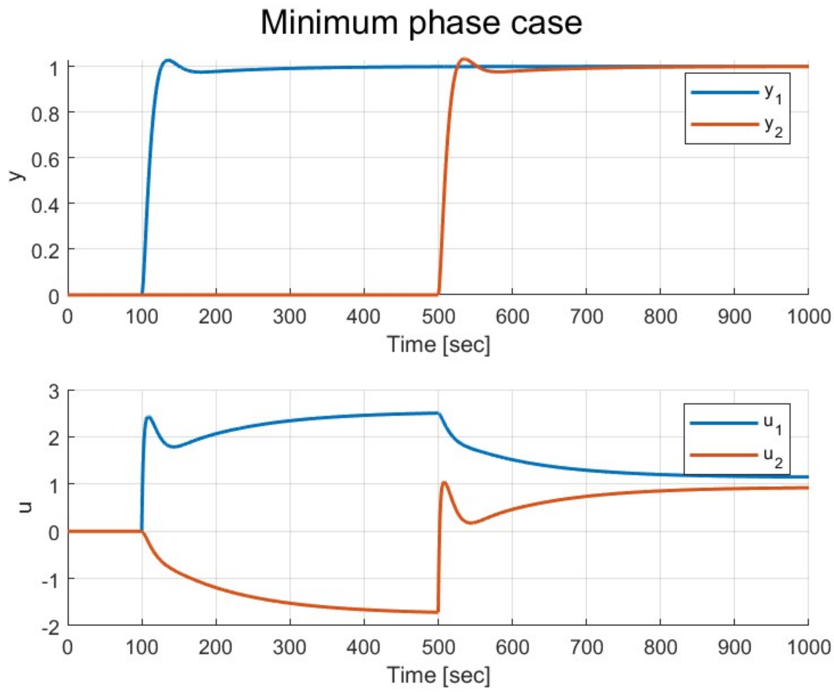

Figure 3: Simulink plots from exercise 3.3.4

Is the controller good?
The H_∞-norm of the sensitivity function is $||S||_\infty = 1.6819 < 2$ and of the complementary sensitivity $||T||_\infty = 1.0019 < 1.25$ and hence the performance should be good. Regarding the time domain specifications, the step response for each output exhibits an overshoot of M= 14.805%, a settling time of T_s =132.9362 sec and a rise time of T_r = 46.0182 sec, as can be seen in Figure 6. Furthermore, we note that due to the RHP-zero an inverse-response is justified.

Hence, the control performance is good given the inherent system limitations.
Are the output signals coupled?
No, the outputs are completely decoupled, i.e. there is no interaction between the pairings y_1-u_1 and y_2-u_2 as shown in the simulation. Hence, the decoupling is successful.

Glover-MacFarlane robust loop-shaping

What are the similarities and differences compared to the nominal design?
The H_∞-norm of the sensitivity function is $||S||_\infty = 1.13583$ and of the complementary sensitivity is $||T||_\infty = 1$. Both are lower than in the nominal design. However, since the robust stability condition depends on the com-

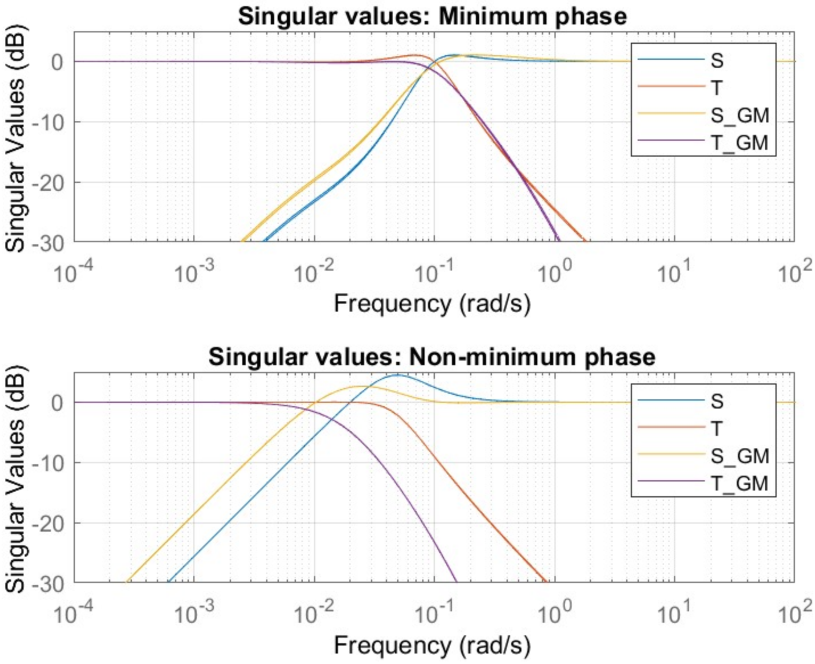

Figure 4: Comparison of the sensitivity functions between robustitified and nominal design

plementary sensitivity function, the robustified design decreases the $||T||_\infty$ more than the sensitivity function. The robustified sensitivity function and the nominal ones are also shown in Figure 4.

Regarding the time-domain performance, the robustified system is significantly slower compared to the nominal design. This is reflected in the increased settling time $T_s = 259.1832$ sec and rise time $T_r = 144.125$ sec.

The outputs of the system still remain completely decoupled as in the nominal design and the inverse-response-behaviour can still be seen in simulation. However, robustification comes at a cost; a much slower system.

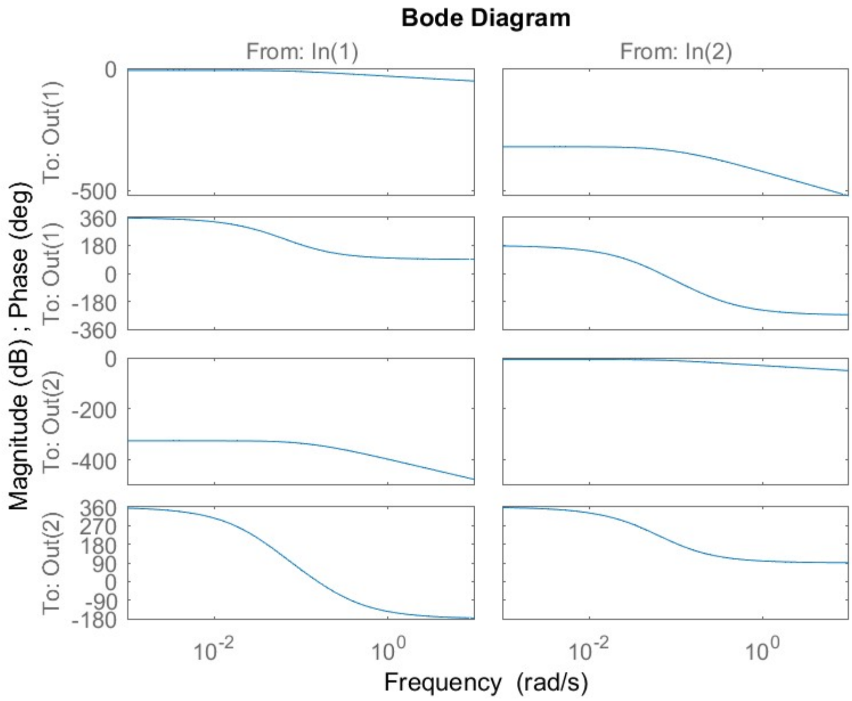

Figure 5: Bode diagram of $\tilde{G}(s)$ derived in exercise 3.2.1

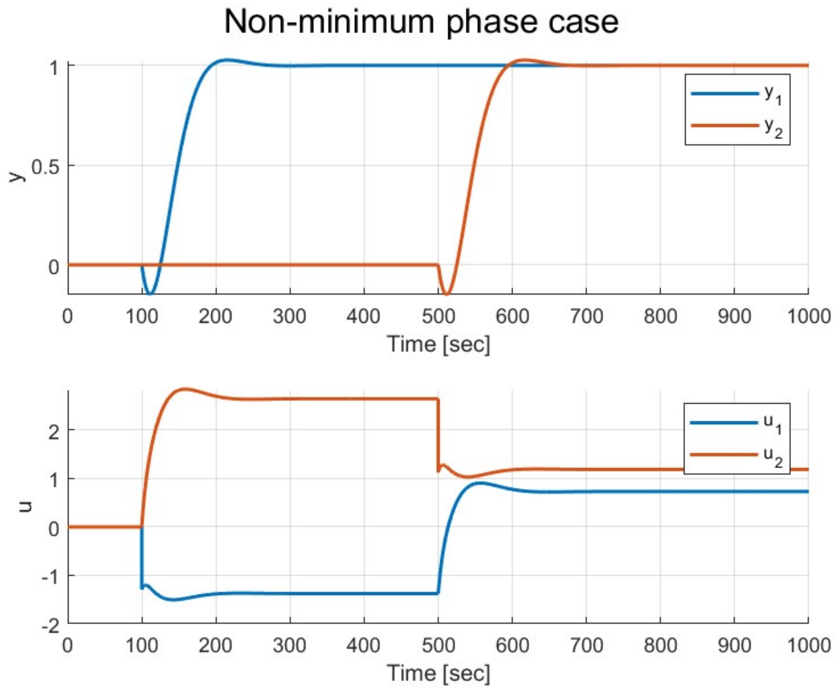

Figure 6: Simulink plots from exercise 3.2.4

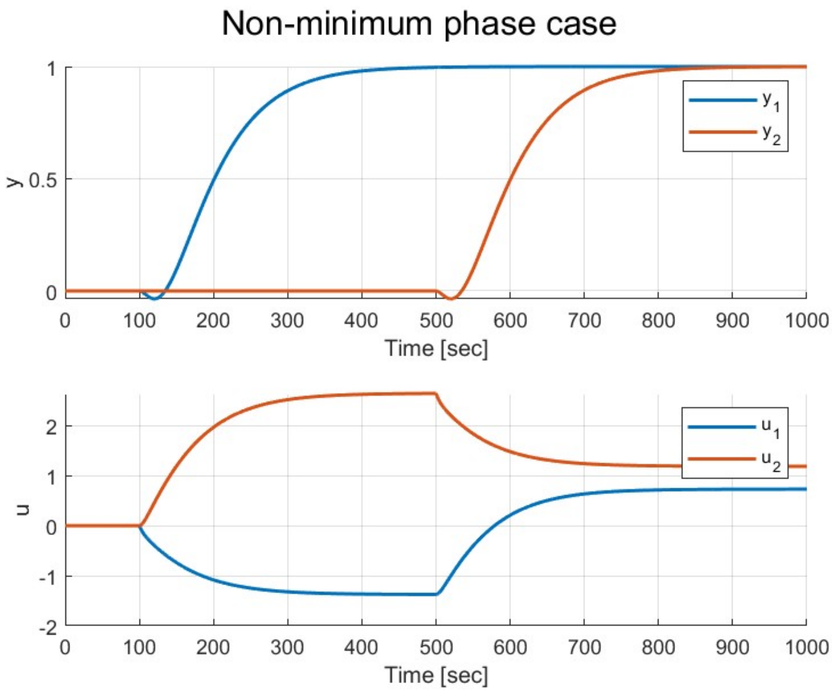

Figure 7: Simulink plots from exercise 3.3.4