Project Lab
EL2520 Control Theory and Practice

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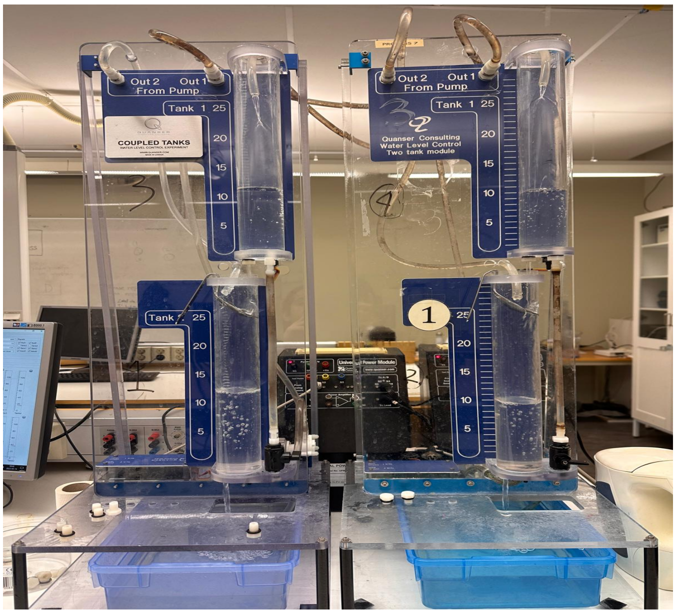

Exercise 2.1.1

The following relation applies for all tanks

$$A\frac{dh}{dt} = q_{\text{in}} - q_{\text{out}}. \tag{1}$$

The inflow of tank 1 is generated by pump 1, described as $q_{L,1} = \gamma_1 k_1 u_1$, and the outflow of water from tank 3, which is described as $q_{\text{out},3} = a_3\sqrt{2gh_3}$. The outflow of tank 1 is $q_{\text{out},1} = a_1\sqrt{2gh_1}$. Thus, using the relation in Eq. (1) we obtain

$$\begin{aligned}\frac{dh_1}{dt} &= \frac{1}{A_1}\big(q_{\text{in}} - q_{\text{out},1}\big) = \frac{1}{A_1}\big(q_{L,1} + q_{\text{out},3} - q_{\text{out},1}\big)\\ &= \frac{1}{A_1}\big(\gamma_1 k_1 u_1 + a_3\sqrt{2gh_3} - a_1\sqrt{2gh_1}\big).\end{aligned} \tag{2}$$

On the other hand, the inflow of tank 3 is generated by pump 2, i.e. $q_{U,2} = (1-\gamma_2)k_2u_2$ and the outflow is $q_{\text{out},3} = a_1\sqrt{2gh_3}$. Thus, using the relation in Eq. (1) we obtain

$$\begin{aligned}\frac{dh_3}{dt} &= \frac{1}{A_3}\big(q_{\text{in},1} - q_{\text{out},3}\big) = \frac{1}{A_3}\big(q_{U,2} - q_{\text{out},3}\big)\\ &= \frac{1}{A_3}\big((1-\gamma_2)k_2u_2 - a_3\sqrt{2gh_3}\big).\end{aligned} \tag{3}$$

The same logic can be followed to obtain the following equations

$$\begin{aligned}\frac{dh_2}{dt} &= \frac{1}{A_2}\big(\gamma_2 k_2 u_2 + a_4\sqrt{2gh_4} - a_1\sqrt{2gh_2}\big)\\ \frac{dh_4}{dt} &= \frac{1}{A_4}\big((1-\gamma_1)k_1u_1 - a_4\sqrt{2gh_4}\big)\end{aligned} \tag{4}$$

In order to simplify notation we introduce the state variable z(t)

$$z(t) = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \end{pmatrix}^T. \tag{5}$$

Furthermore, we define the vector fields $f : \mathbb{R}^4 \times \mathbb{R}^2 \to \mathbb{R}^4$

$$\dot{z}(t) = f(z,u) = \begin{pmatrix} -\frac{a_1}{A_1}\sqrt{2gz_1} + \frac{a_3}{A_1}\sqrt{2gz_3} + \frac{\gamma_1 k_1}{A_1}u_1 \\ -\frac{a_2}{A_2}\sqrt{2gz_2} + \frac{a_4}{A_2}\sqrt{2gz_4} + \frac{\gamma_2 k_2}{A_2}u_2 \\ -\frac{a_3}{A_3}\sqrt{2gz_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \\ -\frac{a_4}{A_4}\sqrt{2gz_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \end{pmatrix}, \tag{6}$$

and $h : \mathbb{R}^4 \times \mathbb{R}^2 \to \mathbb{R}^2$

$$y(t) = h(z,u) = \begin{pmatrix} k_c z_1 \\ k_c z_2 \end{pmatrix}. \tag{7}$$

Exercise 2.1.2

Given the differential equations in Eq. (6) describing the water levels in the tanks and the output voltages in Eq. (7) the equilibrium states u_1^0, u_2^0, h_1^0, h_2^0, h_3^0, h_4^0, y_1^0, y_2^0 are calculated by setting $f(z^0, u^0) \stackrel{!}{=} 0$ and $y^0 = h(z^0, u^0)$. The result is validated with MATLAB's symbolic toolbox and yields the following steady-state equations

$$\begin{aligned}
h_1^0 &= \frac{\left(2 \cdot a_3 \cdot \sqrt{\frac{k_2^2 \cdot u_2^2 \cdot (\gamma_2 - 1)^2}{2 \cdot a_3^2}} + \sqrt{2} \cdot \gamma_1 \cdot k_1 \cdot u_1\right)^2}{4 \cdot a_1^2 \cdot g}, \\
h_2^0 &= \frac{\left(2 \cdot a_4 \cdot \sqrt{\frac{k_1^2 \cdot u_1^2 \cdot (\gamma_1 - 1)^2}{2 \cdot a_4^2}} + \sqrt{2} \cdot \gamma_2 \cdot k_2 \cdot u_2\right)^2}{4 \cdot a_2^2 \cdot g}, \\
h_3^0 &= \frac{k_2^2 \cdot u_2^2 \cdot (\gamma_2 - 1)^2}{2 \cdot a_3^2 \cdot g}, \\
h_4^0 &= \frac{k_1^2 \cdot u_1^2 \cdot (\gamma_1 - 1)^2}{2 \cdot a_4^2 \cdot g}, \\
\begin{pmatrix} y_1^0 \\ y_2^0 \end{pmatrix} &= \begin{pmatrix} k_c \cdot h_1^0 \\ k_c \cdot h_2^0 \end{pmatrix}.
\end{aligned} \tag{8}$$

Evidently, there is only 6 equations to describe the 8 steady states. The conclusion we can draw from this is that there is an infinite amount of steady states for the water levels and signal voltages. However, most of these are unachievable since there is a limit on how big the input voltage can be and the height of the water should not exceed the height of the tank.

Exercise 2.1.3

After finding the equations for the equilibria and by introducing the deviation variables

$$u = \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \end{pmatrix}, x = \begin{pmatrix} \Delta h_1 \\ \Delta h_2 \\ \Delta h_3 \\ \Delta h_4 \end{pmatrix}, y = \begin{pmatrix} \Delta y_1 \\ \Delta y_2 \end{pmatrix}, \tag{9}$$

where $\Delta u_i = u_i - u_i^0, \Delta h_i = h_i - h_i^0$ and $\Delta y_i = y_i - y_i^0$. It is noted that here we abuse notation and define u again. Whenever, the linear sytsem is used, then u denotes deviation variables. The system is linearized about a given equilibrium point resulting in the system

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t).
\end{aligned} \tag{10}$$

The matrices of the linearized system are then obtained through

$$
\begin{aligned}
A &= \left.\frac{\partial f(z,u)}{\partial z}\right|_{z=z^0,u=u^0} \\
&= \begin{pmatrix}
-\frac{a_1}{A_1}\frac{g}{\sqrt{2gh_1^0}} & 0 & \frac{a_3}{A_1}\frac{g}{\sqrt{2gh_3^0}} & 0 \\
0 & -\frac{a_2}{A_2}\frac{g}{\sqrt{2gh_2^0}} & 0 & \frac{a_4}{A_2}\frac{g}{\sqrt{2gh_4^0}} \\
0 & 0 & -\frac{a_3}{A_3}\frac{g}{\sqrt{2gh_3^0}} & 0 \\
0 & 0 & 0 & -\frac{a_4}{A_4}\frac{g}{\sqrt{2gh_4^0}}
\end{pmatrix} \\
&= \begin{pmatrix}
-\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\
0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\
0 & 0 & -\frac{1}{T_3} & 0 \\
0 & 0 & 0 & -\frac{1}{T_4}
\end{pmatrix},
\end{aligned}
\tag{11}
$$

where the last equality follows from $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$. Subsequently, the remaining matrices are computed

$$
\begin{aligned}
B &= \left.\frac{\partial f(z,u)}{\partial u}\right|_{z=z^0,u=u^0} &&= \begin{pmatrix}
\frac{\gamma_1 k_1}{A_1} & 0 \\
0 & \frac{\gamma_2 k_2}{A_2} \\
0 & \frac{(1-\gamma_2)k_2}{A_3} \\
\frac{(1-\gamma_1)k_1}{A_4} & 0
\end{pmatrix} \\
C &= \left.\frac{\partial h(z,u)}{\partial x}\right|_{z=z^0,u=u^0} &&= \begin{pmatrix}
k_c & 0 & 0 & 0 \\
0 & k_c & 0 & 0
\end{pmatrix} \\
D &= \left.\frac{\partial h(z,u)}{\partial u}\right|_{z=z^0,u=u^0} &&= 0.
\end{aligned}
\tag{12}
$$

Exercise 2.1.4

In order to compute the transfer matrix, the following formula is used

$$
G(s) = C \cdot (sI - A)^{-1} \cdot B + D = C \cdot \frac{adj(sI-A)}{det(sI-A)} \cdot B, \tag{13}
$$

where the fact that the system is proper (D = 0) is utilized to obtain the second equality. Furthermore, we utilize the structure of the C-matrix and only compute the first and second rows of the adjoint matrix yielding

$$G(s) = \begin{pmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{T_1}{T_1 s+1} & 0 & \frac{k_c A_3 T_1}{A_1 (T_1 s+1)(T_3 s+1)} & 0 \\ 0 & \frac{T_2}{T_2 s+1} & 0 & \frac{k_c A_4 T_2}{A_2 (T_2 s+1)(T_4 s+1)} \\ \star & \star & \star & \star \\ \star & \star & \star & \star \end{pmatrix}$$

$$\cdot \begin{pmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2) k_2}{A_3} \\ \frac{(1-\gamma_1) k_1}{A_4} & 0 \end{pmatrix}.$$

By multiplying out the above matrices and using $c_i = \frac{T_i k_c}{A_i}$ the following transfer matrix is obtained

$$G(s) = \begin{pmatrix} \frac{\gamma_1 k_1 c_1}{1+sT_1} & \frac{(1-\gamma_2) k_2 c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1 k_1 c_2)}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 k_2 c_2}{1+sT_2} \end{pmatrix}. \tag{14}$$

Exercise 2.1.5

The zeros of the system are obtained by setting $det\ G(s) \overset{!}{=} 0$. This yields the second-order equation

$$s^2 \cdot T_3 T_4 + s \cdot (T_3 + T_4) - \frac{\gamma_1 + \gamma_2 - 1}{\gamma_1 \gamma_2} \overset{!}{=} 0. \tag{15}$$

The zeros are hence placed at

$$s_{z_{1/2}} = -\frac{T_3 + T_4}{2T_3 T_4} \pm \frac{\sqrt{\left(T_3 + T_4\right)^2 - 4T_3 T_4 \left(\frac{\gamma_1+\gamma_2-1}{\gamma_1 \gamma_2}\right)}}{2T_3 T_4}. \tag{16}$$

Two cases are distinguished

- **Case (i)**: $\gamma_1 + \gamma_2 \in (0, 1]$

  This leads to $\gamma_1 + \gamma_2 - 1 \in (-1, 0]$ and since $T_i > 0$, the radicand is positive. It is then easy to see that this results in a RHP zero; and hence implies a non-minimum phase system.

- **Case (ii)**: $\gamma_1 + \gamma_2 \in (1, 2]$

  This leads to $\gamma_1 + \gamma_2 - 1 \in (0, 1]$ and since $T_i > 0$ the second term under the square root is negative. Here we distinguish again two cases.

  - First, if $\left(T_3 + T_4\right)^2 - 4T_3 T_4 \left(\frac{\gamma_1+\gamma_2-1}{\gamma_1 \gamma_2}\right) < 0$, this results in a complex-conjugate zero located at

$$s_{z_{1/2}} = -\frac{T_3 + T_4}{2T_3 T_4} \pm j \cdot \frac{\sqrt{\left(T_3 + T_4\right)^2 - 4T_3 T_4 \left(\frac{\gamma_1+\gamma_2-1}{\gamma_1 \gamma_2}\right)}}{2T_3 T_4}. \tag{17}$$

Since $Re\{s_{z_{1/2}}\} < 0$, the system is minimum phase in this case.

- Second, if $\big(T_3 + T_4\big)^2 - 4T_3T_4\Big(\frac{\gamma_1+\gamma_2-1}{\gamma_1\gamma_2}\Big) \geq 0$, it is straightforward to see that the zeros are placed in the LHP and thus the system is minimum phase.

Hence, in case (ii) the system is always minimum phase.

Exercise 2.1.6

The steady-state RGA is computed using

$$\begin{aligned} RGA\big(G(0)\big) &= G(0) \times \big[G(0)^{-1}\big]^T \\ &= \frac{1}{\gamma_1+\gamma_2-1}\begin{pmatrix} \gamma_1\gamma_2 & -1+\gamma_1+\gamma_2-\gamma_1\gamma_2 \\ -1+\gamma_1+\gamma_2-\gamma_1\gamma_2 & \gamma_1\gamma_2 \end{pmatrix} \\ &= \begin{pmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{pmatrix}, \end{aligned} \tag{18}$$

where the last equality follows from $\lambda = \gamma_1\gamma_2/(\gamma_1+\gamma_2-1)$. By substituting the values for both cases, we get in the

- **Minimum phase case:** $(\lambda_1 = \lambda_2 = 0.625)$

$$RGA\big(G(0)\big) = \begin{pmatrix} 1.5625 & -0.5625 \\ -0.5625 & 1.5625 \end{pmatrix}. \tag{19}$$

  Since the diagonal elements are positive and close to one, output 1 should be controlled with input 1 and output 2 with input 2. Furthermore, the off-diagonal elements are not significant and hence minimal coupling should be expected.

- **Non-minimum phase case:** $(\lambda_1 = \lambda_2 = 0.375)$

$$RGA\big(G(0)\big) = \begin{pmatrix} -0.5625 & 1.5625 \\ 1.5625 & -0.5625 \end{pmatrix}. \tag{20}$$

  Since the off-diagonal elements are positive and close to one, output 1 should be controlled with input 2 and vice versa. This indicates that the coupling here is significant.

Exercise 2.1.7

To determine the proportionality coefficients of the pumps k_1 and k_2, we propose two experiments; one for each coefficient. First, the outlet hole of tank 3 that allows water to flow to tank 1, is closed. The voltage of pump 2 was set to $u_2 = 50\%$ (7.5 V). Tank 3 is filled up with water until a certain height (20 cm) and the time for the water to reach the targeted height is measured.

The third equation is manipulated to compute k_2. Since the outlet hole of tank 3 is closed, no water flows out of tank 3, such that the equation is simplified to

$$\frac{dh_3(t)}{dt} = \frac{(1-\gamma_2)k_2}{A_3}u_2. \tag{21}$$

The above equation integrated with respect to time yields

$$\begin{aligned} \int_0^T \dot{h}_3(\tau)d\tau &= \int_0^T \frac{(1-\gamma_2)k_2}{A_3}u_2 d\tau. \\ h_3(T) - h_3(0) = h_3(T) &= T\frac{(1-\gamma_2)k_2}{A_3}u_2 \end{aligned} \tag{22}$$

Here we use the fact that the tank is empty at the start of the experiment ($h_3(0) = 0$). Thus, k_2 can be determined directly as

$$k_2 = \frac{h_3(T)A_3}{T(1-\gamma_2)u_2}. \tag{23}$$

Using $u_2 = 50\%$ (7.5 V) the tank reached 20 cm after $T = 43.5$ s and hence we obtain $k_2 = 2.54\ \frac{\text{cm}^3}{\text{s}\cdot\text{V}}$.

The same experiment is done with tank 4, where a time of $T = 42.5$ s was measured. Hence, $k_1 = 2.59\ \frac{\text{cm}^3}{\text{s}\cdot\text{V}}$.

Exercise 2.1.8

To determine the four effective outlet hole areas, the following simple experiment was conducted once for the minimum phase case and once for the non-minimum phase case. The inputs u_1, u_2 were set to 50% (7.5 V). Then, we waited long enough for the system to reach a steady-state and read off the values of the four tanks directly.

The outlet hole areas a_i can be obtained by substituting the measured steady-state values as well as the determined k_1 and k_2 and the constant inputs in the steady-state equations in Eq. (8). The following linear system of equations is obtained

$$\begin{pmatrix} -\frac{\sqrt{2\cdot g\cdot h_1^0}}{A_1} & 0 & \frac{\sqrt{2\cdot g\cdot h_3^0}}{A_1} & 0 \\ 0 & -\frac{\sqrt{2\cdot g\cdot h_2^0}}{A_2} & 0 & \frac{\sqrt{2\cdot g\cdot h_4^0}}{A_2} \\ 0 & 0 & -\frac{\sqrt{2\cdot g\cdot h_3^0}}{A_3} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2\cdot g\cdot h_4^0}}{A_4} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} -\frac{k_1\cdot\gamma_1\cdot u_{10}}{A_1} \\ -\frac{k_2\cdot\gamma_2\cdot u_{20}}{A_2} \\ -\frac{k_2\cdot(1-\gamma_2)\cdot u_{20}}{A_3} \\ -\frac{k_1\cdot(1-\gamma_1)\cdot u_{10}}{A_4} \end{pmatrix}. \tag{24}$$

Table 1: Summary of the steady-state values for the minimum phase case (i) and non-minimum phase case (ii).

Case	h_1^0 [cm]	h_2^0 [cm]	h_3^0 [cm]	h_4^0 [cm]
(i)	18.5	19	8.5	10
(ii)	17	20	8.5	11

- **Minimum phase case:** The steady-state values of the water heights for the minimum phase case are shown in Table 1. By solving Eq. (24) for the outlet hole areas we obtain for the minimum phase case

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 0.1012\,\mathrm{cm}^2 \\ 0.1026\,\mathrm{cm}^2 \\ 0.0553\,\mathrm{cm}^2 \\ 0.0564\,\mathrm{cm}^2 \end{pmatrix}. \tag{25}$$

- **Non-minimum phase case:** The steady-state values of the water heights for the non-minimum phase case are shown in Table 1. By solving Eq. (24) for the outlet hole areas we obtain for the minimum phase case

$$\begin{pmatrix} a_1 \\ a_2 \\ a_{3,\mathrm{nonmin}} \\ a_{4,\mathrm{nonmin}} \end{pmatrix} = \begin{pmatrix} 0.1051\,\mathrm{cm}^2 \\ 0.1058\,\mathrm{cm}^2 \\ 0.0922\,\mathrm{cm}^2 \\ 0.0940\,\mathrm{cm}^2 \end{pmatrix}. \tag{26}$$

For each phase-case we compute the four outlet hole areas independently. Hence, although the nozzles of tank 1 and tank 2 are not changed when we change the phase-case, the corresponding outlet hole areas are not constrained. However, the difference between the phase cases in a_1 and a_2 (as seen in Eq. (25) and Eq. (26)) is minimal. For later use, we take the average of the two values.

Exercise 2.2.1

The steady-state values are in accordance with the computed outlet hole areas, since the steady-state equations are used to obtain the outlet hole areas in Exercise 2.1.8. This applies both for the minimum and non-minimum phase case.

Note: In hindsight, a different steady-state value other than the proposed $u_1 = u_2 = 50\%$ could've been tested to increase the confidence in the computed values.

Exercise 2.2.2

Minimum phase case: The step responses of the two inputs are shown in Figure 1. From the step responses it is clear that the system is minimally coupled. A step in input 1 almost only affects tank 1 (and input 2 similarly for tank 2). This is in accordance with the result of the RGA in Eq. (19) and the conclusions drawn there.

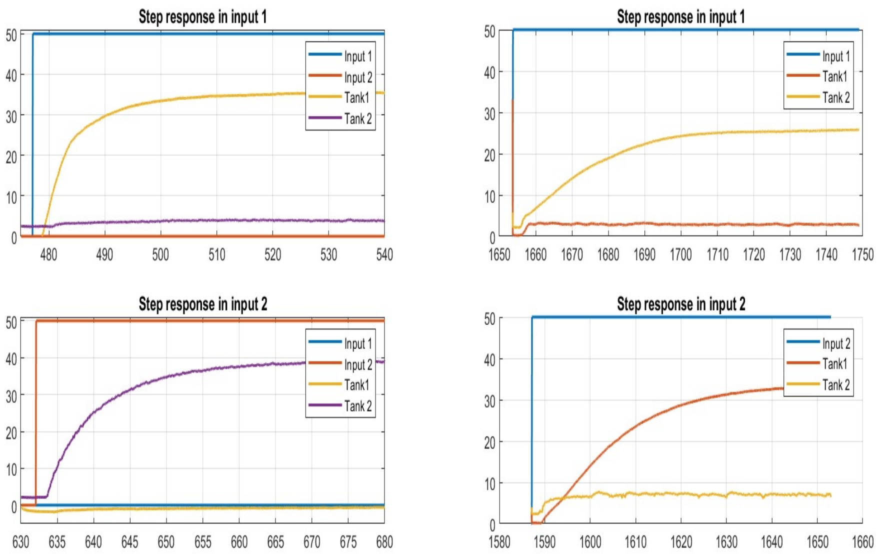

Figure 1: Step responses from one input at a time for the minimum phase case (left) and the non-minimum phase case (right).

Non-minimum phase case: The step responses of the two inputs are shown in Figure 1. From the figure it is clear that a step in input 1 affects tank 2 and vice versa. Hence, the interaction is stronger and the effect of the input on the same output is much weaker. This again is in accordance with the result of the RGA in Eq. (19) and the conclusions drawn there.

Exercise 2.2.3

Minimum phase case: The proposed reference value (60% of full tank) was chosen for both tanks. Using manual control the inputs were varied until the reference values are reached. The results are shown in Figure 2. The first input was set to $u_1 = 43\%$ and the second input to $u_2 = 48\%$. The transient time was estimated to be approximately 85.5 sec.

Non-minimum phase case: Unfortunately, the pressure sensor measuring the height of tank 1 stopped working and hence manual control could

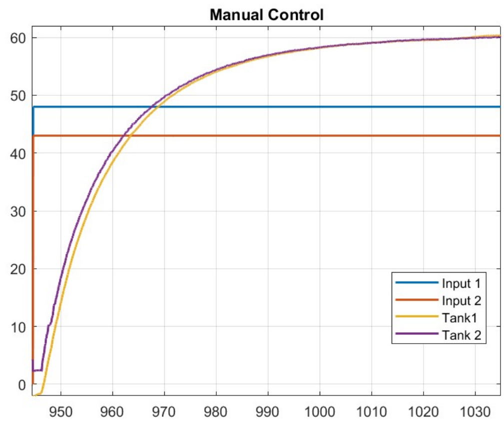

Figure 2: Manual control: Reaching a reference value of 60% of full tank in both tanks.

not be performed for this case. This was also communicated to the attending supervisor.

Exercise 2.2.4

The main difference between both cases can be seen studying the step responses in Figure 1. In the non-minimum phase case a strong interaction in the system is very visible. This was also expected by calculating the steady-state RGA-matrix. However, in the minimum phase case the interaction can be deemed insignificant.

Since non-minimum phase systems are much more difficult to control, we expect a larger transient time during manual control. However, this could not be validated as mentioned in the exercise before.

Exercise 3.1.1

First, in each phase-case a dynamic decoupler followed by decentralized PI-control was favoured over the static decoupler. The choice was based on the simulation performance (shown in Figure 3). The dynamic decoupler exhibited a faster response in both cases and less cross-coupling of the outputs.

In a second step, the above controllers were used as a nominal design and were robustified using Glover-McFarlane.

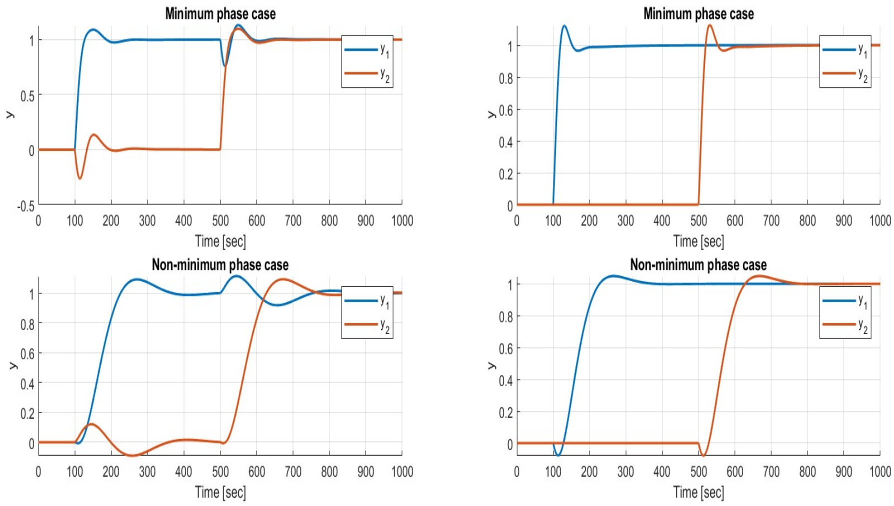

Figure 3: Comparison between static (left) and dynamic decoupling (right) approach in simulation.

Exercise 4.1.1

- **Minimum phase case**

  The system response to a step input as well as the disturbance rejection test are shown in Figure 4. Regarding the step response, the system exhibits no overshoot contrary to the simulation in Exercise 3.1.1. Furthermore, the rise time is 5.8 sec.

  In the right plot, a disturbance was introduced by pouring water in tank 2. It takes the controller approximately 16 sec to reject the disturbance.

- **Non-minimum phase case**

  For the non-minimum phase case, the system response to a step input as well as the disturbance rejection test are shown in Figure 5. Regarding the step response, the system exhibits an overshoot of 1.1011%. Furthermore, the rise time is 44.5 sec.

  In the right plot, a disturbance was introduced by opening the valve of tank 3 so the input to tank 1 dropped. It takes the controller approximately 110 sec to reject the disturbance.

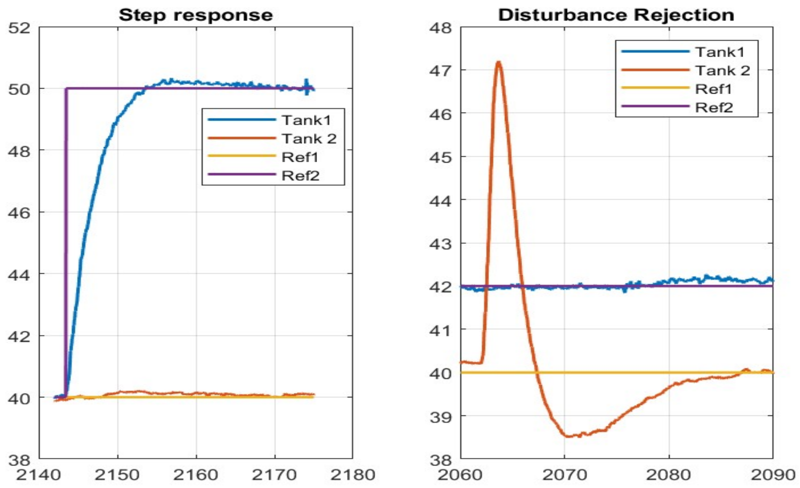

Figure 4: Nominal decentralized controller for minimum phase. Step response (left) and a disturbance (right).

Exercise 4.2.1

- **Minimum phase case**
  For the case of the minimum phase system the robust Glover-McFarlane controller was tested both in a step response and with a disturbance caused by adding water to the tank 1. The result of this experiment on the controller can be viewed in Figure 6. From this figure we can deduce the rise time of the step response to be $t_r \approx 6s$. Likewise, we observe no overshoot. Additionally, the tank has a static error (or stationarity was not reached). From the introduced disturbance we can observe the time to eliminate the load disturbance to be 6 sec.

- **Non-minimum phase case**
  The system response was changed when using the Glover-McFarlane controller for the non-minimum phase case. In Figure 7 the step response and effect of disturbances are shown. The rise time for the step response for this configuration is $t_r \approx 125s$. Unfortunately, the overshoot couldn't correctly be measured, since one of the water magazines was very close to overflow. This problem persisted albeit multiple tries. However, a small overshoot of 2% was noticed.

  The time it took to eliminate disturbances was tested by opening the valve on one of the upper tanks. The result can be seen in the right plot in figure 7. The time it took for the controller to eliminate the disturbance was $\approx 150s$.

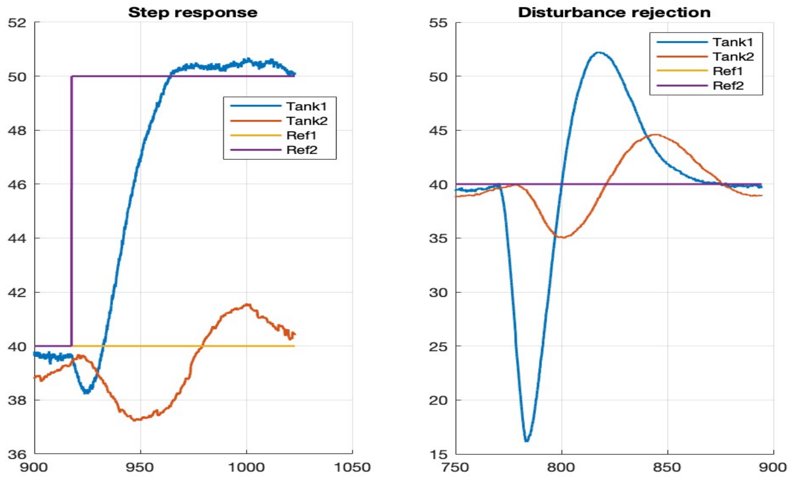

Figure 5: Nominal decentralized controller for non-minimum phase. Step response (left) and a disturbance (right).

Exercise 4.2.2

When comparing the different controllers, the key differences are the rise time and overshoot. The Glover-McFarlane controller has a significantly bigger rise time compared to the decoupled controller for the non-minimum phase case. However, the disturbance rejection results in no overshoot contrary to the nominal decentralized controller. For the minimal phase case, the rise time and disturbance rejection was similar for both controllers. However, the Glover-McFarlane controller had a steady state error (we suspect that stationarity was not completely reached).

Exercise 4.2.3

The most important differences between the minimum phase and non-minimum phase case for the Glover-McFarlane controller is the time it took to eliminate disturbance and the rise time.

For the minimal phase case the time to eliminate the disturbances was 6 sec seconds. For the non-minimal phase case the time was approximately 150 sec. Regarding the rise times it was approximately 6 sec for the minimal phase and 125 sec for the non-minimal phase case.

Hence, robustifying the controllers comes at the cost of the performance.

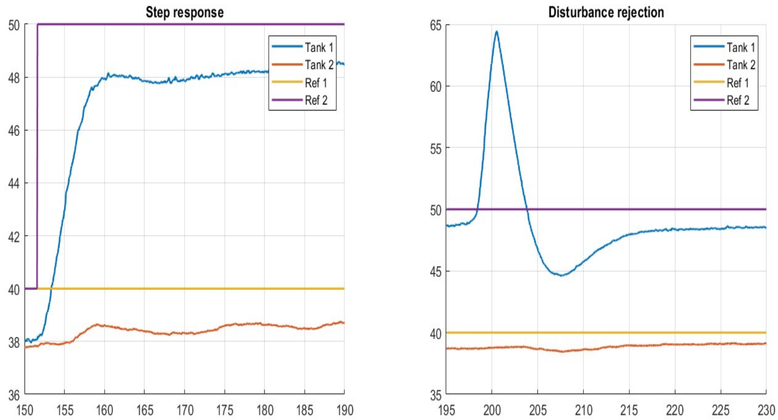

Figure 6: Glover-McFarlane controller for minimum phase. Step response (left) and a disturbance (right)

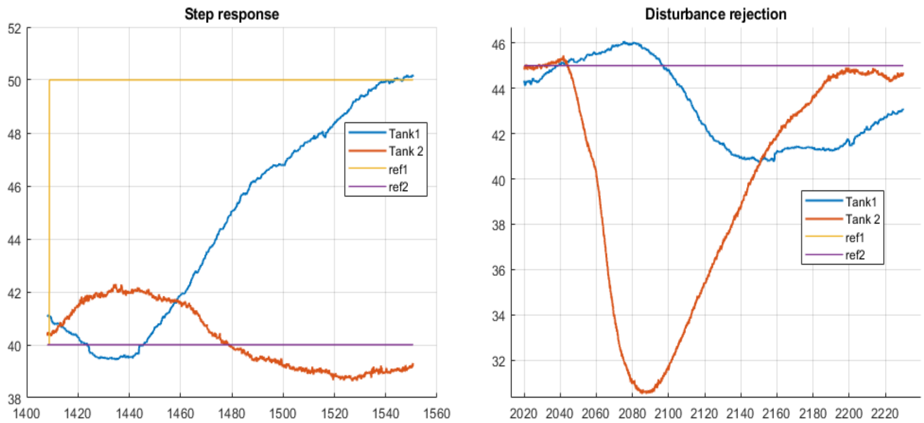

Figure 7: Glover-McFarlane controller for non-minimum phase. Step response (left) and a disturbance (right)