Graph Theory in Lambda Calculus

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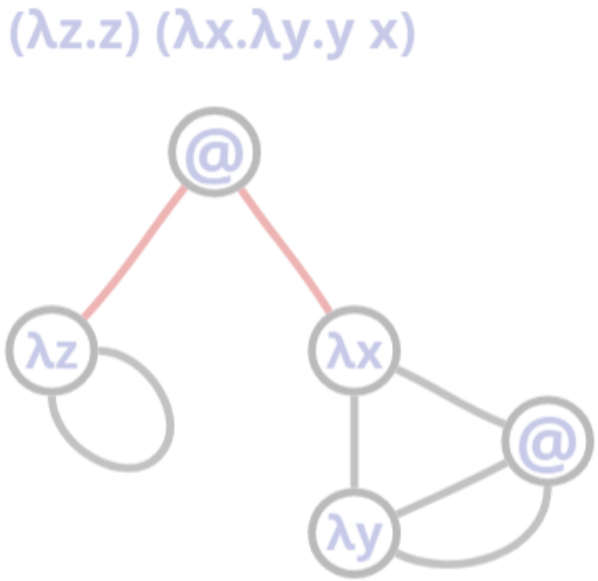
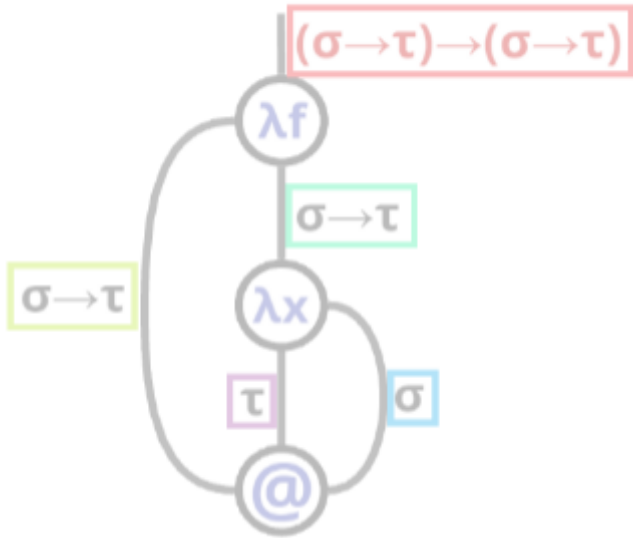


July 2024



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Alan Turing
(1912-1954)

Alonzo Church
(1903-1995)

Lambda Calculus in Functional Programming

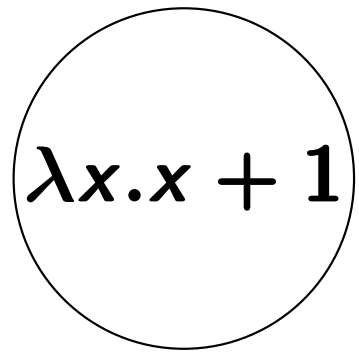
```
\x -> x + 1
```

```
(lambda (x) (+ x 1))
```


```
(x) => x + 1
```

```
lambda x:  x + 1
```

```
fun x -> x + 1
```

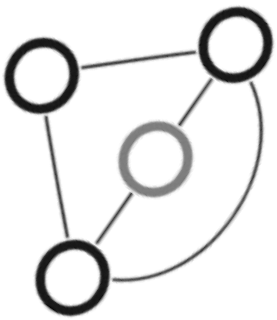


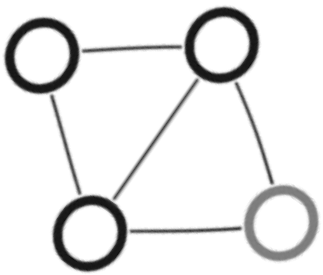
Lambda Terms - definitions

- `lambda term = variable | application | abstraction`
  x $M\ N$ $\lambda x.M$
- **Example:** $\lambda x.x\ y \rightarrow$ function that takes x and returns x applied to y
- $M\ N\ P \equiv (M\ N)\ P$
- $\lambda x.M\ N \equiv \lambda x.(M\ N) \not\equiv (\lambda x.M)\ N$

What is a graph?

- ... $G = (V, E)$ ...
- visual representation!





Map

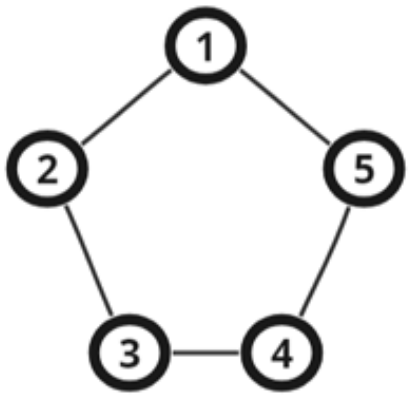
Map Graph

...

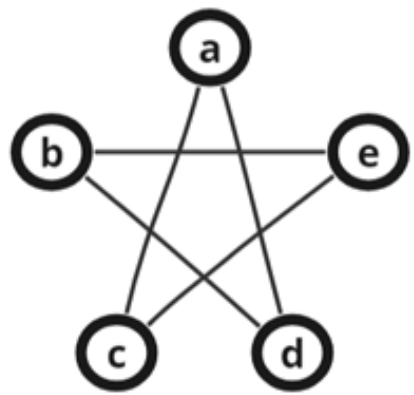
Plane Graph

Planar Map

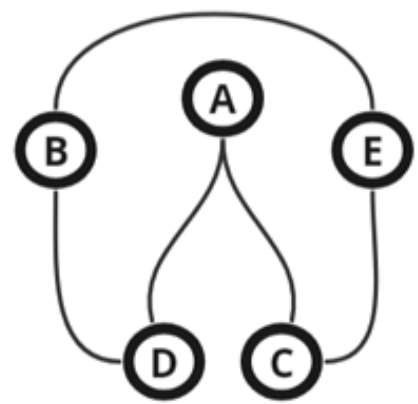
Some graph theory concepts



G1



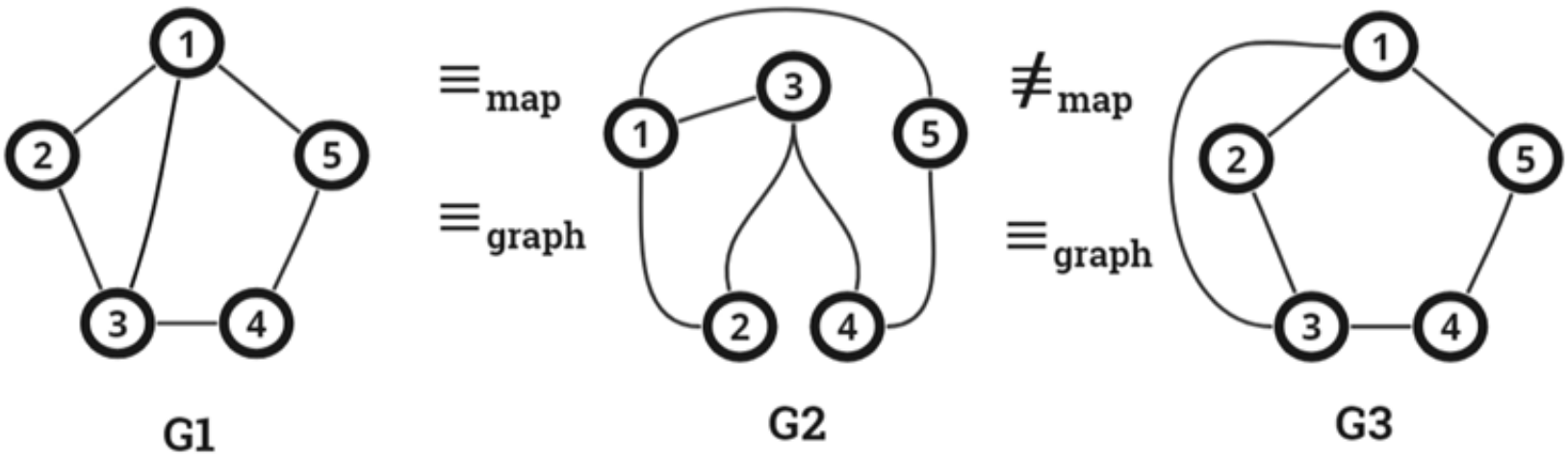
G2



G3

- graph isomorphism
- embeddings onto plane
- planarity

Maps



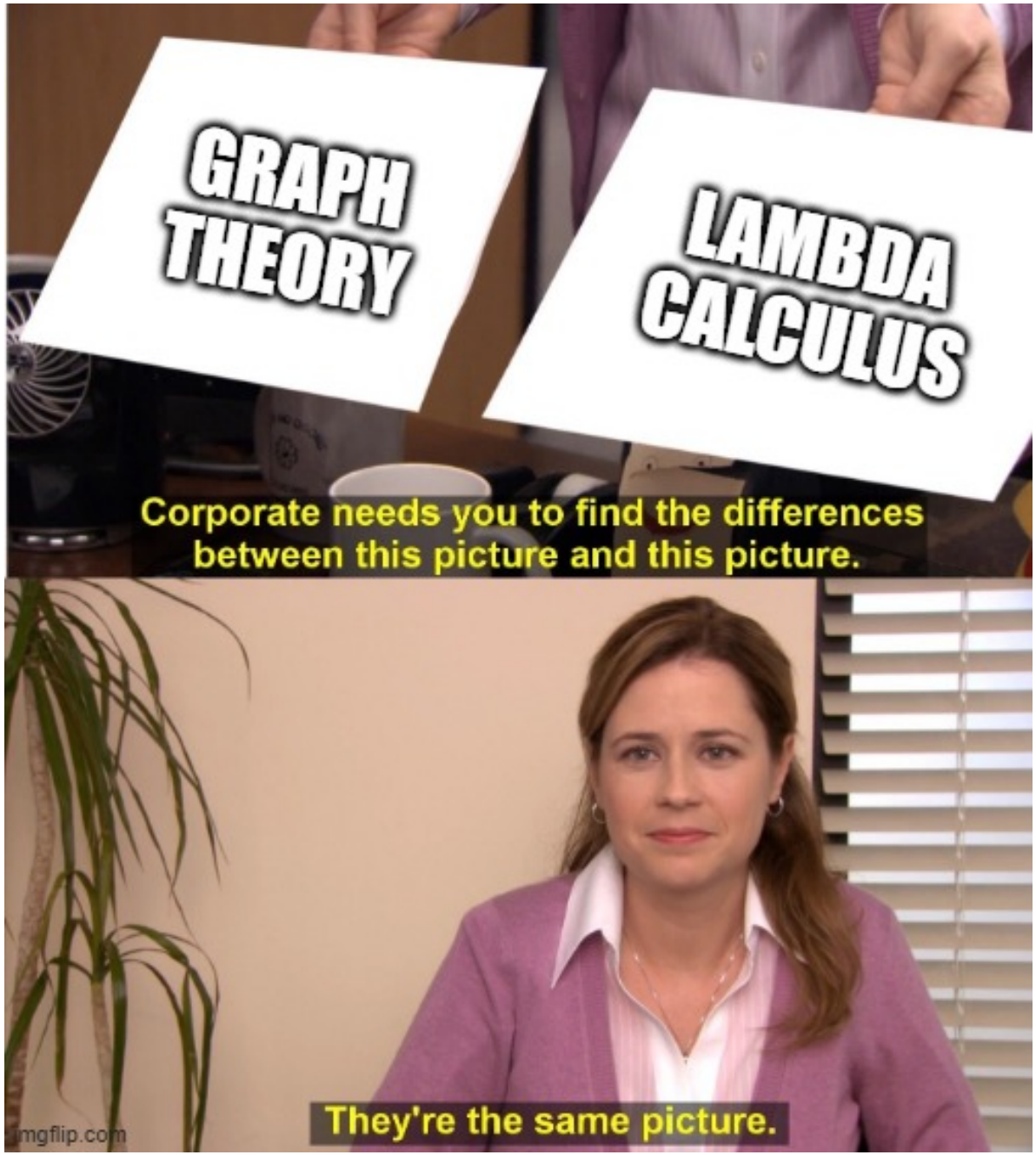
Three isomorphic *graphs*, but **not** three isomorphic *maps*

β-normal ordered linear lambda terms

- **not** ordered: $\lambda x.\lambda y.y\ x$
- ordered: $\lambda x.\lambda y.x\ y$

- **not** linear: $\lambda x.x\ x$
- linear: $\lambda x.\lambda y.x\ y$

- **not** β-normal: $(\lambda x.x\ y)\ z$
- β-normal: $z\ y$, $\lambda x.x\ y$


GRAPH THEORY
LAMBDA CALCULUS
Corporate needs you to find the differences between this picture and this picture.
They're the same picture.
imgflip.com

Counting β-normal ordered linear lambda terms

One variable:

1. $\lambda x.x$

Two variables:

1. $\lambda x.x\ (\lambda y.y)$
2. $\lambda x.\lambda y.x\ y$

Three variables:

1. $\lambda x.x\ (\lambda y.y\ (\lambda z.z))$
2. $\lambda x.x\ (\lambda y.\lambda z.y\ z)$
3. $\lambda x.x\ (\lambda y.y)\ (\lambda z.z)$
4. $\lambda x.\lambda y.x\ (y\ (\lambda z.z))$
5. $\lambda x.\lambda y.x\ (\lambda z.y\ z)$
6. $\lambda x.\lambda y.x\ (\lambda z.z)\ y$
7. $\lambda x.\lambda y.x\ y\ (\lambda z.z)$
8. $\lambda x.\lambda y.\lambda z.x\ (y\ z)$
9. $\lambda x.\lambda y.\lambda z.x\ y\ z$

1, 2, 9, 54...

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

1,2,9,54 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **seq:1,2,9,54**

Displaying 1-10 of 12 results found. page 1 2

Sort: relevance | references | number | modified | created Format: long | short | data

A000168 a(n) = 2*3^n*(2*n)!/(n!*(n+2)!). +30 35

(Formerly M1940 N0768)

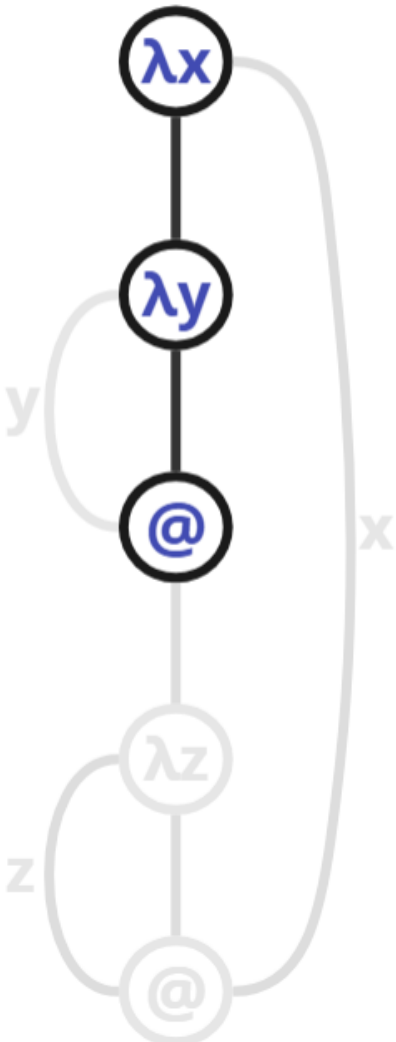
1, 2, 9, 54, 378, 2916, 24057, 208494, 1876446, 17399772, 165297834, 1602117468, 15792300756, 157923007560, 1598970451545, 16365932856990, 169114639522230, 1762352559231660, 18504701871932430, 195621134074714260, 2080697516976506220, 22254416920705240440, 239234981897581334730, 2583737804493878415084 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,2

COMMENTS Number of rooted planar maps with n edges. - Don Knuth, Nov 24 2013

From Lambda Terms to Maps

λx.λy.y (λz.x z)



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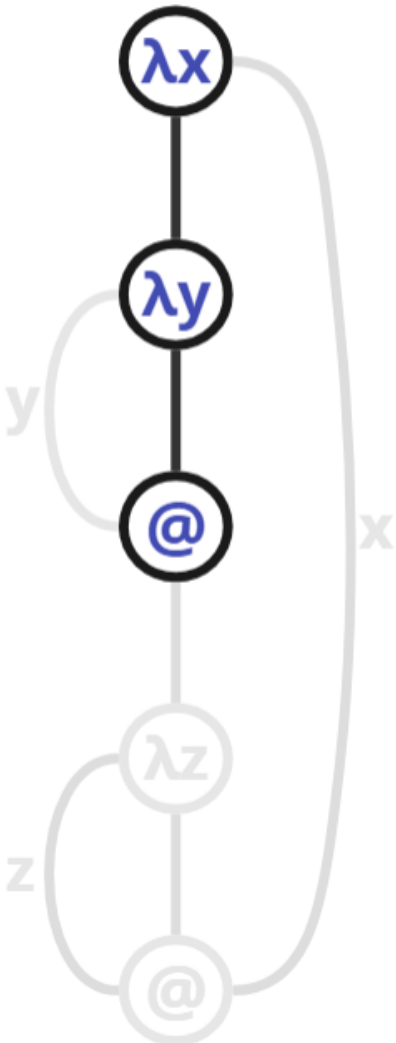

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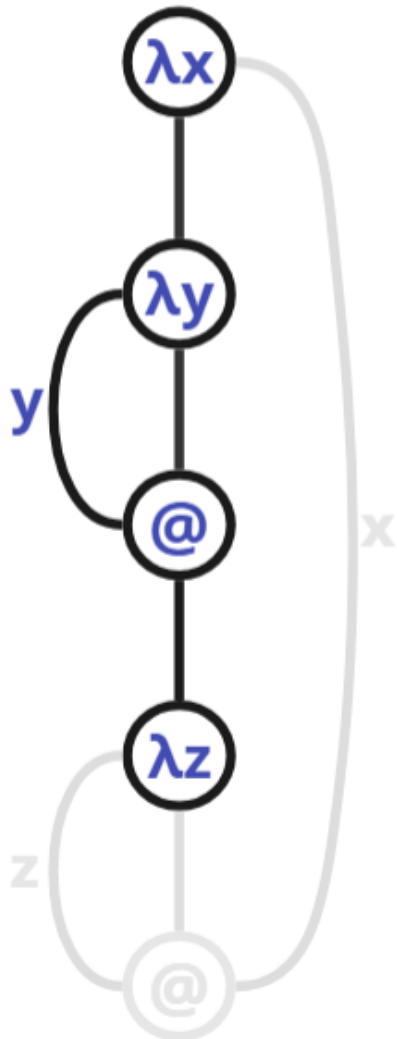

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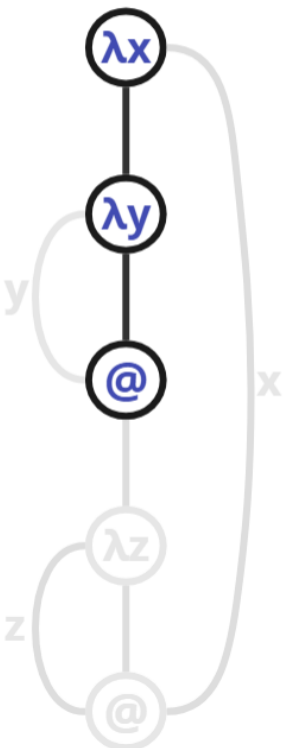

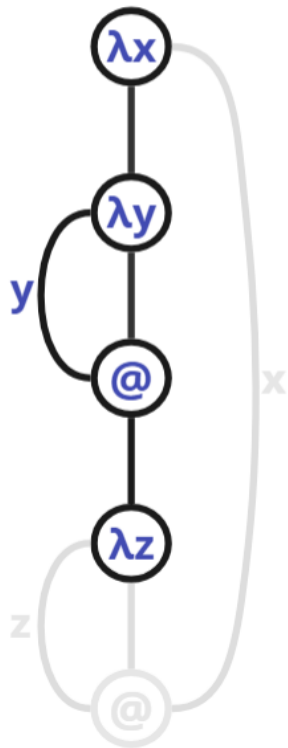

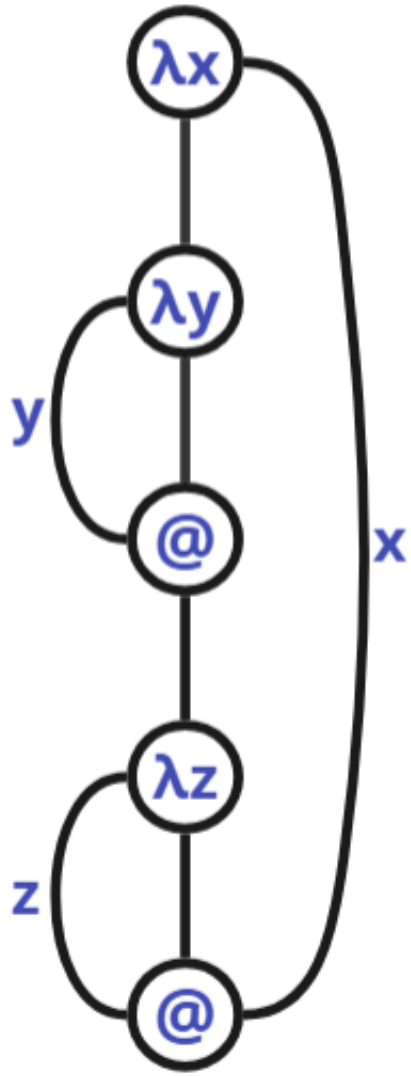

Some correspondences

- linear lambda terms $\equiv$ trivalent maps
- typing of lambda terms $\equiv$ Four Color Theorem
- ... unitless lambda terms $\equiv$ bridgeless maps, etc. ...

Future perspectives

- a complete bilingual dictionary between lambda calculus and graph theory
- an enumerative perspective of lambda calculus

> *"From time to time in a graph-theoretical career, one's thoughts turn to the Four Colour Problem."*
>
> — W. T. Tutte, *Graph Theory as I Have Known It*

Thank you!

Discussion time!