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Faculty of Civil Engineering
Department of Mechanics

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Reliability-Based Design Optimization using Approximation Techniques

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Abstract

Reliability-based design optimization (RBDO) searches for a trade-off between costs and safety under the assumption of uncertainties. Classical multi-objective RBDO formulates the task with several objective functions but keeps the reliability in the inequality constraints. This thesis defines the task, such that the resulting Pareto front provides a dataset of compromising solutions concerning both costs and reliability answering the question of what is the price of reliability. The proposed multi-objective evolutionary double-loop RBDO method utilizes a meta-model-based Monte Carlo type approach which is enhanced by a repetitively optimized adaptive Design of (Computer) Experiments (DoE). The outer loop solves the design process while an advanced simulation technique based on Monte Carlo principles assesses the reliability for each individual in the inner loop evaluating the meta-model instead of the original model. The DoE update uses suitable sampling points from the simulation technique optimized by space-filling and limit-state-proximity objectives, subsequently evaluated using the original model, to enrich the DoE within each generation of the evolutionary algorithm. Since the design space is wide, in addition to a classic global meta-model (GMM) with a dense Gram matrix assembled for the entire design space, we use sparse global meta-models (SGMM) assembled for each generation in the entire design space and local meta-models (LMM) constructed for each individual from each generation individually.

The presented thesis compares the results obtained from RBDO with different degrees of approximation – only at the level of the meta-model using quasi-Monte Carlo simulation, at the level of reliability using the original model, or a combination of both. If the approximation is at the level of reliability, we compare seven different reliability assessment methods, namely an Asymptotic sampling, classical Monte Carlo simulation with a low number of samples, Enhanced Monte Carlo simulation, First-order reliability method (FORM), Importance sampling, Scaled sigma sampling, and Subset simulation. The presented results show that the GMM and SGMM are a suitable choice to replace the original model, that is relatively smooth. The global meta-model is also less sensitive to the choice of the simulation method. However, if the design space is large, more design points or failure modes exist, and the limit state is highly nonlinear, then LMM even with a smaller number of points in DoE serve better as a replacement for the original model. The proposed method using adaptively updated meta-models is more accurate and computationally less demanding than double-loop RBDO using Reliability index approach, i.e. the use of FORM to evaluate the reliability. FORM needs at least two orders of magnitude more original model evaluations than our method for less accurate results. Advanced simulation techniques, in addition to better accuracy and precision in estimating structural reliability with a lower number of model evaluations, also offer more suitable points for DoE update, as they better cover the space around the limit state than the classical Monte Carlo method.

Abstrakt

Spolehlivostní optimalizace konstrukcí (RBDO, z anglického Reliability-based design optimization) hledá kompromisní řešení mezi cenou a spolehlivostí systému. Klasická vícekriteriální RBDO rozšiřuje jednokriteriální formulaci více účelovými funkcemi se zachováním omezujících spolehlivostních podmínek. Tato práce definuje úlohu tak, aby byla výsledná řešení z Paretovy množiny indiferentním kompromisem mezi cenou a spolehlivostí, a tedy poskytla odpověď na otázku, jaká je cena spolehlivosti. Navrhovaná vícekriteriální evoluční RBDO metoda se dvěma cykly využívá pokročilé simulační techniky založené na principu Monte Carlo s použitím meta-modelů vylepšených o optimalizovaný adaptivní návrh experimentu (DoE, z anglického Design of Experiments). Vnější cyklus obstarává optimalizační část, zatímco spolehlivost návrhu je vyhodnocena pokročilou simulační technikou, která evaluuje meta-model místo původního modelu. Vhodné vzorky z této simulační techniky, které jsou optimalizovány pomocí kritéria rovnoměrného rozprostření návrhových bodů a kritéria vzdálenosti od hranice poruchy, se následně využijí pro aktualizaci DoE během každé generace evolučního algoritmu. Jelikož je návrhový prostor široký, v předkládané práci jsou využity kromě klasického meta-modelu s plnou Gramovou maticí sestaveného na celém návrhovém prostoru (GMM, z anglického global meta-model) i řídké globální meta-modely (SGMM, z anglického sparse global meta-model), sestavené na celém návrhovém prostoru pro každou generaci evolučního algoritmu, a lokální meta-modely (LMM), vytvořené pro každého řešení z každé generace evolučního algoritmu zvlášť.

V rámci celé práce jsou porovnávány Pareto-fronty jako výsledná řešení z RBDO ze třech skupin aproximace – pouze na úrovni meta-modelu s využitím metody Monte Carlo, na úrovni spolehlivosti s evaluací původního modelu, nebo kombinací obojího. Pokud je aproximace provedena na úrovni spolehlivosti, je v práci porovnáno sedm různých metod – Asymptotic sampling, klasická metoda Monte Carlo s nízkým počtem vzorků, vylepšená metoda Monte Carlo (Enhanced Monte Carlo simulation), spolehlivostní metoda prvního řádu FORM, Importance sampling, Scaled sigma sampling a Subset simulation. Z prezentovaných výsledků vyplývá, že pokud je funkce relativně hladká, pak jsou globální meta-modely s plnou, respektive i řídkou Gramovou maticí, vhodnou volbou náhrady původního modelu. Tento globální meta-model je i méně citlivý na volbu simulační metody. Pokud je ale návrhový prostor velký, existuje v něm více návrhových bodů nebo módů porušení a hranice poruchy je vysoce nelineární, pak jako náhrada originálního modelu lépe poslouží lokální meta-modely i s menším počtem bodů v DoE, tzn. nižším počtem evaluací původního modelu, v porovnání s modely globálními. Navrhovaná metoda v této práci využívající průběžně aktualizované meta-modely je přesnější a výpočetně méně náročná než RBDO se dvěma cykly využívající formulaci metody spolehlivostního indexu (Reliability index approach), tedy využití FORM pro výpočet spolehlivosti, který i pro méně přesné výsledky v porovnání s naší metodou potřebuje minimálně o dva řády více evaluací původního modelu. Pokročilé simulační metody kromě vyšší přesnosti při odhadu spolehlivosti s nižším počtem evaluací modelu také nabízí vhodnější body pro aktualizaci DoE, neboť lépe pokrývají oblast poruchy v porovnání s klasickou metodou Monte Carlo.

Contents

1 Research Aims and Objectives

This thesis aims to develop a methodology that provides fast and computationally simple solution of multi-objective reliability-based design optimization with cost and reliability defined in the objectives. The main objective of this thesis is to formulate, implement, optimize, and validate a methodology for multi-objective reliability-based design optimization. The most computationally expensive part is evaluating the reliability of the system whilst utilizing crude Monte Carlo simulation and the original complicated model (e.g. a finite element model). Therefore, the partial objectives are in a reduction of computational costs in these three significant areas:

1. a multi-objective optimization algorithm should converge with the fewest solutions as possible;
2. a fast evaluation of the probability of failure by advanced simulation techniques while maintaining acceptable result accuracy;
3. and effective and suitable meta-models mimicking the behaviour of the original (full) model in the critical regions.

2 Literature Review

Optimization and search methodologies have become very popular for making products more desirable. A shape of the structure [48], topology [67], reinforcement of concrete structures [41], cross-sections [15], design of the concrete mix [5], and many other properties can be optimized. Structural optimization [26] is a process that seeks the best design under some predefined constraints. A deterministic model, i.e. model, which has identical outputs for a given input, is usually unrealistic due to uncertain inputs such as material properties, a structural topology, and loadings. Moreover, the deterministic optimization techniques can often lead to unacceptable results [29] since an optimal design with deterministic variables often terminates at a boundary between the failure domain and the safe domain. Even a small perturbation in inputs can lead to a fatal failure. For that reason, the model uncertainties are introduced; parameter uncertainties are associated with input data, whereas structural uncertainties express that a model need not clearly describe the physics of the problem [63]. Yao et al. [59] define uncertainty as an "incompleteness in knowledge and the inherent variability of the system and its environment". Forrester [23] mentioned that computational errors are mainly human-made (e.g. incorrectly entered boundary conditions) and systematic errors (e.g. a wrong model). Since computational experiments are deterministic, a random error is more common in experimental data. Another possible categorization of the sources of uncertainty is to consider *aleatoric uncertainty* (from the Latin *alea* with meaning rolling of dice) that is not influenceable by the experiment repetition and *epistemic uncertainty* (from Greek *episteme* that means knowledge) that disappears with better information of the problem [33]. Uncertainties can be represented through interval bounds that is the vaguest definition; by membership functions which are used in fuzzy logic approaches; or by probability density functions that provide the best description of uncertainty [63].

An optimal design provides a small probability of failure assuming structural economy and reduction of the system variability to unexpected variations. These requirements categorize the optimization under uncertainty into two groups [29] as depicted in Figure 1. Economical design with high reliability is provided by *reliability-based design optimization* (sometimes referred just as *reliability-based optimization*) concentrating on worse-case scenarios that occur only in extreme events. *Robust design optimization* covers daily fluctuations minimizing the price as well as sensitivity to small changes in model inputs such as loading, structural parameters, and geometry. Schuëller and Jensen [52] include the third group of optimization under uncertainty - *model updating and system identification*; the goal is to reduce discrepancies which arise when the model prediction is compared with the test data [52].

Reliability-based design optimization (RBDO) minimizes an objective function (e.g. a structural weight, maximal displacement, benefits, construction costs or expected life-time costs) with respect to deterministic constraints as well as probabilistic constraints evaluating a probability of failure. Optimization, as well as reliability assessment, require repetitive computation of a structural response with different settings of uncertain parameters and design variables; this is the most computationally expensive part of the problem. It is possible to evaluate a probability of failure analytically only for a particular type of problems (for instance, by Gauss quadrature approaches, Laplace Approximation approaches) which is limiting for RBDO applications. Exact computation is impractical [2]. *Approximation techniques* such as a First-order reliability method (FORM) or Second-order reliability method (SORM) and *simulation techniques* such as a crude Monte Carlo (MC) and variance reduction techniques are commonly used. FORM [27] is frequently preferred for its speed and only a few necessary evaluations of the performance measure, see e.g. [16]. The drawback is that the obtained reliability assessment is accurate only if the limit state function of the problem is a hyperplane and the variables are independent and normally distributed – the more nonlinear the problem, the more significant the error. A nonlinearity can also be hidden in the transformation from the non-normal space into the Gaussian space. If the problem is nonlinear, SORM [46] utilizing second-order derivatives is more precise since SORM respects the curvature around the most probable point. However, SORM is also computationally more expensive.

The methods for solving RBDO problems can be classified into three groups, namely *double-loop approaches* (also referred to as *two-level approaches*), *single-loop approaches* (also referred to as *mono-level approaches*), and *decoupling approaches* [2]. A *double-loop approach* is the most direct approach and the most accurate method to solve RBDO problems [36]. It consists of two merged loops, where the outer loop solves the designing process, and the inner loop appraises the reliability evaluations. The double-loop approach may be divided into four main classes: (i) double-loop utilizing sampling techniques;

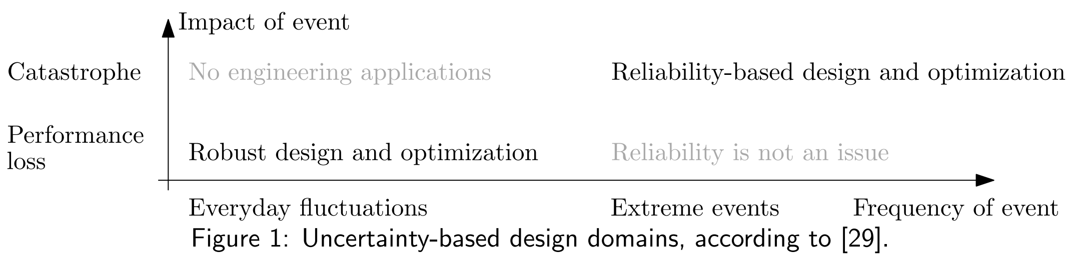

Figure 1: Uncertainty-based design domains, according to [29].

(ii) Reliability index approach (RIA) that is similar to class (i) – the main difference is that RIA uses FORM; (iii) Performance measure approach which may be understood as an inverse task to RIA; and (iv) our proposed approach converting single-objective optimization into the multi-objective formulation. In a sampling-based approach, crude Monte Carlo simulation is the most robust tool for all reliability assessment tasks, but it is also the most expensive one. Nevertheless, replacing the true function with a meta-model can reduce the computational time, e.g. by Monte Carlo utilizing Neural Networks [47]. Advanced simulation techniques in RBDO such as Importance sampling [4], Subset simulation [57] or Asymptotic sampling [49] require less computational effort than crude Monte Carlo simulation. FORM is a very common method for solving this reliability task; Aoues and Chateauneuf [2] and other authors as well (e.g. [56]) call the application of FORM the *Reliability index approach* (RIA). A lot of general-purpose optimization software makes use of the implementation of this approach [2]. Tu et al. [56] formulated an inverse reliability assessment called a *Performance measure approach* (PMA) (also referred to as *inverse FORM* or *inverse reliability approach*) which seeks the threshold value for the given probability of failure. This method is announced to be more robust and efficient than RIA [56] because the search direction is simply determined by exploring the spherical equality constraint [2]. A single-objective problem can be reformulated into a multi-objective form[1] [49]. In a *single-loop approach*, some or all probabilistic constraints are replaced with the approximate deterministic constraints. For example, Li et al. [36] transform the limit state function into the standard normal space, and then they add to it a constant, which is equal to the minimum allowable reliability index and is just a distance which abridges the feasible region. This constraint is then wholly deterministic. *A decoupled approach* separates a reliability assessment and an optimization task. Du and Chen [17] formulated a *Sequential optimization and reliability assessment method* (SORA); they shift boundaries of violated deterministic constraints to the feasible direction based on the reliability information obtained in the previous cycle [58].

Simulation techniques have better accuracy in predicting a probability of failure. However, they are much more time demanding due to repetitive evaluations of the virtual simulation, such as a finite element model. Monte Carlo simulation [42] itself is the most universal and robust method for the assessment of reliability [47]. However, it requires millions of samples for low probabilities of failure. Advanced simulation techniques, such as an Asymptotic sampling [6], Enhanced Monte Carlo simulation [44], Importance sampling [4, 30], Subset simulation [3, 57], or Scaled Sigma sampling demand [55] less computational effort while evaluating the performance function, which can still last quite a long time. Since the accuracy of the probability of failure is needed, reducing demands of simulation techniques is essential.

Some model of the original model with a very similar behaviour can replace the exact model to speed up the design process; those models of models are called *meta-models* or *surrogate models*. The original function is evaluated only in some so-called support points;

[1]RBDO considers structural reliability to be an inequality constraint because the limits on which it is proposed are known. The basic recommended minimum value of the reliability index is 3.8 for reliability class RC2 and 4.3 for RC3 at 50 years reference period for the ultimate limit states [21, 50]. Outside the structural engineering area, such as industrial electronics [61, 62], or wind energy engineering [12], these limits do not have to be defined. Such type of optimization is called Reliability and Cost Optimization. The task is multi-objective in the basic definition but can be converted to single-objective, e.g. by using the weighted sum approach [22] or the epsilon-constraint method [51].

any support point is a combination of input variables of the model. These support points form a *Design of the experiments* (DoE) [43]. Scientific experiments, as well as industrial, very often use a *uniform* Design of experiments [31] since it is robust and only a small number of support points are necessary to get a large amount of information about the relationship between the function inputs and outputs [37]. For some specific tasks, it is advantageous to use information concentrated in an important subdomain of the task. An initial small Design of experiments can detect this significant region. Subsequently, an *updating* procedure adds more support points there to improve the meta-model behaviour mimicking the original model. Initial meta-models do not have the same accuracy as the original models particularly in locations that are the most interesting for engineering design, e.g. a vicinity of the border between the safe and the failure domain called a *limit state*. Adaptive updating of meta-models can make a meta-model more accurate in those significant locations. Many types of meta-models are used in RBDO, namely Response surfaces [66], Support vector machines [57], Kriging [18], and Polynomial chaos expansion [9].

If the optimization problem is complex, and the exact methods are not applicable, the meta-heuristic strategies [25] are convenient to use [64]. Simulated annealing [32], TABU search [24], Evolutionary algorithms [20], and Swarm intelligence algorithms [19] are just a few groups of algorithms belonging to these strategies. Evolutionary algorithms are prevalent algorithms that are inspired by evolutionary theory. They are well-liked for their simplicity, robustness, no need of the good initial estimation of a solution nor the derivatives or for a capability of finding the global optimum in the presence of several local optima. There exist several multi-objective versions of evolutionary algorithms, to name a few Pareto-archived evolution strategy (PAES) [34], Strength-Pareto evolutionary algorithm (SPEA) [69], improved SPEA (SPEA2) [68], Pareto envelope-based selection algorithm (PESA) [11], improved PESA (PESA-II) [10], Multi-objective evolutionary algorithm based on decomposition (MOEA/D) [65], Dynamic multi-objective evolutionary algorithm (DMOEA) [60], or Non-dominated sorting genetic algorithm II (NSGA-II) [14]. For our purposes, we need a solver that does not use an archive. Since we will use an updated Design of Experiments and therefore, have an updated meta-model for each generation of the algorithm, the values of the objective functions will differ for identical individuals in different generations. The values of the objective functions obtained from meta-models will be more accurate with each next generation. When using the archive, we would have to recalculate the entire archive with every new generation, and that would be very computationally demanding. Unfortunately, PAES, SPEA, SPEA2, PESA, and PESA-II use an external population [35] that would have to be re-evaluated each time when DoE and subsequently the meta-model is updated. NSGA-II uses a different approach for keeping the elitist solution in the population instead of storing an external list of solutions. It is also well tested, efficient, popular, widely used, and it uses only a few parameters to be easily tuned [1]. According to Google Scholar[2], a group of NSGA algorithms is the most commonly used algorithm of the algorithms mentioned above. As of April 2020, NSGA, NSGA-II, and NSGA-III have 28,700 occurrences, SPEA, and SPEA 2 have 11,800 occurrences, PAES has 4,090 occurrences, MOEA/D has 3,480 occurrences, PESA, and PESA-II have 1,430 occurrences, and DMOEA has 274 occurrences.

[2]Google Scholar is a freely available web search engine, indexing full text and metadata of academic publishing including results from databases Scopus and Web of Science.

3 Research Methods Used

Reliability-based design optimization provides a design that is economical as well as reliable in the presence of uncertainties. Multi-objective formulation of the reliability-based design optimization task provides larger room for a decision-maker, whereas single-objective formulation forces a researcher to constrain the search space before an optimization procedure is started. The multi-objective formulation is as follows:

$$\min_{\mathbf{d}\in\mathbb{D}} \quad C(\mathbf{d}) \tag{1}$$

$$\max_{\mathbf{d}\in\mathbb{D}} \quad \beta(\mathbf{x},\mathbf{d}) \tag{2}$$

$$\text{subject to} \quad h_i(\mathbf{d}) \leq 0,\ i = 1,\dots,n_I, \tag{3}$$

$$\beta^{\text{min,tol}} \leq \beta(\mathbf{x},\mathbf{d}) \leq \beta^{\text{max,tol}}. \tag{4}$$

The objective function $C(\cdot)$ is to be minimized with optimal values of design variables arranged in a vector $\mathbf{d}$ that contains deterministic variables or probability distribution parameters (e.g. the mean of random variables). Design variables belong to the design space $\mathbb{D}$. A generalized reliability index $\beta(\cdot)$ is to be minimized; generally, β is defined as an inverse cumulative distribution function of the standard normal distribution $\beta = \Phi^{-1}(1-p_F)$, where p_F is a probability of failure. A vector $\mathbf{x}$ contains uncertain parameters. Hard constraints $h_i(\mathbf{d})$ specify the description of the design space; n_I is their total number. Reliability index $\beta(\mathbf{x},\mathbf{d})$ beta can be limited from the bottom by $\beta^{\text{min,tol}}$ and from above by $\beta^{\text{max,tol}}$ value.

3.1 Multi-Objective Optimization Solver: Non-Dominated Sorting Genetic Algorithm II

NSGA-II [14] is an elitist multi-objective $(\mu+\lambda)$ evolutionary algorithm without any external archive. It creates the next generation using a tournament selection according to two fitness functions (a non-dominated sorting approach, and crowding distance), Simulated binary crossover, and mutation operator. Non-dominated sorting approach assigns an ordinal rank to each individual; this rank is equal to its non-domination level – the smaller, the better. A crowding distance is a metric evaluating the density of solutions surrounding a solution in the population – the greater, the better. Since NSGA-II is a $(\mu+\lambda)$ algorithm, where μ and λ are the sizes of the parental and offspring population, the algorithm selects next parental generation from the unification of both populations using non-dominated sorting ranks, so that the next population remains quality solutions from previous generations. Individuals with the lowest possible ranks fill free slots in the offspring population. If there are more solutions with the same rank in the last set than free slots, the algorithm selects the individuals based on the crowding distance. The non-dominated sorting approach solves constraint handling implicitly without having any penalty parameters and penalty constraints. If both solutions are feasible, the non-domination rank is better for the solution that dominates the other solution. If both solutions are infeasible, the non-domination rank is better for the solution with a smaller constraint violation.

3.2 A Failure Probability Assessment

A probability of failure in an n-dimensional space of random variables $X_1 \dots X_n$ can be computed as

$$p_F = \text{Prob}[g(\mathbf{X}) \leq 0] = \int \cdots \int_{g(\mathbf{X}) \leq 0} f_X(\mathbf{x}) \mathrm{d}\mathbf{x}, \tag{5}$$

where $f_X(\mathbf{x})$ is a joint probability distribution function, $g(\mathbf{X})$ stands for a *limit state function* and $g(\mathbf{X}) \leq 0$ denotes the failure domain. A set of several realizations of the limit state function $y_i = g(x_i)$ is a random variable Y.

Analytical evaluation of the failure probability is possible only for a particular type of problems (e.g. by Gauss quadrature approaches, Laplace Approximation approaches) which is limiting for RBDO applications. The most famous approximation technique is the First-Order Reliability Method [7, 27, 28, 46], which approximates the limit state function by a hyperplane in a design point in the standard normal space. Simulation techniques are numerical methods that solve mathematical problems through random sampling. Unlike approximation methods, the sampling methods require only the information about the value of the model without knowledge of additional information, such as the first or second derivative. In case that a number of samples in a crude Monte Carlo simulation, i.e. the most direct simulation method, is not sufficient, several techniques are available for the improvement of the probability of failure estimate. An Asymptotic sampling predicts a reliability index from an asymptotic behaviour of the probability of failure in an n-dimensional independent and identically distributed normal space. An Enhanced Monte Carlo simulation sequentially shifts the limit state function to obtain more responses in the failure domain. An Importance sampling samples in the important region to gain more knowledge about the area of interest. A Subset simulation reformulates the rare event problem into a series of more frequent events that are easier to solve. A Scaled sigma sampling fits the relation of traditional numerical integration for failure probability evaluation by sequential importance sampling with increasing standard deviations. The principle is similar to an Asymptotic sampling, the difference is in the model of extrapolation and its fitting.

3.3 A Meta-Model: A Radial Basis Function Model

A Radial Basis Functions model (RBF) [8] approximates a complicated but relatively smooth true function or produces an estimate to an unknown function from a set of input data. The approximation is made by evaluation of easier basis functions using input dataset, multiplying these results by weight coefficients and summing them together. For noise-free data, the model is typically of the form

$$\hat{y}(\mathbf{x}) = \mathbf{w}^T \boldsymbol{\psi} = \sum_{i=1}^{n_c} w_i \psi(||\mathbf{x} - \mathbf{c}^{(i)}||), \tag{6}$$

where $\mathbf{w}$ is a weighting vector, $\boldsymbol{\psi}$ is a vector of length n_c holding evaluated basis functions on Euclidean distances between the prediction $\mathbf{x}$ and centres of basis functions $\mathbf{c}$. The

Euclidean norm is evaluated as

$$||r|| = \sqrt{(\sum_{i=1}^{n_e} r_i^2)} \tag{7}$$

for a vector r with n_e elements. Other metrics are possible; however, the Euclidean metric is quite often used. The basis functions are symmetrical due to the Euclidean norm [8] and centred on a set of support points [23]. As the basis, we work with a Gaussian radial basis function

$$\psi(r) = \exp\,[\frac{-r^2}{\sigma^2}]. \tag{8}$$

According to [45], the approximation of parameter σ can be computed as

$$\sigma = \sqrt{\frac{d_{\max}}{\sqrt[n_d]{n_d \cdot n_c}}} \tag{9}$$

where $d_{\max}$ is a maximum distance among the data and n_d is the number of dimensions equivalent to the number of variables. This formula represents how large would be a space around each point in DoE if the points in DoE had been optimally uniformly distributed.

Since the design space is wide, in addition to a classic global meta-model with a dense Gram matrix assembled for the entire design space, we use our novel sparse global meta-models (SGMM) assembled for each generation in the entire design space and local meta-models (LMM) constructed for each individual from each generation individually. The SGMM utilizes the fact that the mutual influence of the DoE points diminishes with their increasing distance, and thus the distant samples have a little impact on each other. Therefore, instead of dense Gram matrix, including all links between all support points, each support point has its influence domain in the closest neighbourhood, and other points outside the influence domain are omitted. Local meta-models are constructed only in the influence domain having dense Gram matrix since this domain is smaller than the design space. Figure 2 shows a DoE with 20 support points on the left, and the other images visualise the sparsity pattern of Gram matrices.

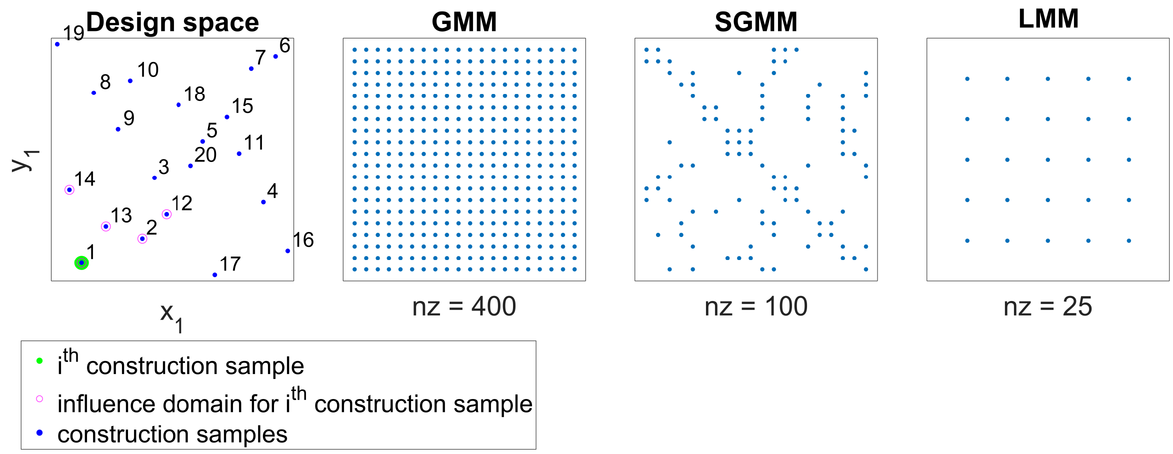

Figure 2: Influence domain for meta-models in design space and sparsity pattern visualization of Gram matrices for a global meta-model (GMM), sparse global meta-model (SGMM), and local meta-model (LMM). Abbreviations: nz - nonzero elements.

3.4 Multi-Objective RBDO using DoE update

Figure 3 shows a simplified flow-chart of a double-loop RBDO, including DoE updates and meta-models within optimization. DoE update uses samples from a simulation method, from which it selects the most suitable points for updating DoE. Only samples from DoE are evaluated using the original model, the simulation method uses the meta-model to evaluate the limit state function. If local meta-models are used for the limit state function evaluation, the meta-model is assembled both for each individual in each generation, but also for each candidate sample to be added to the DoE. Otherwise, the method constructs a global meta-model for each generation throughout the design space. Figure 4 depicts an RBDO application with an update on one evolutionary generation with four individuals.

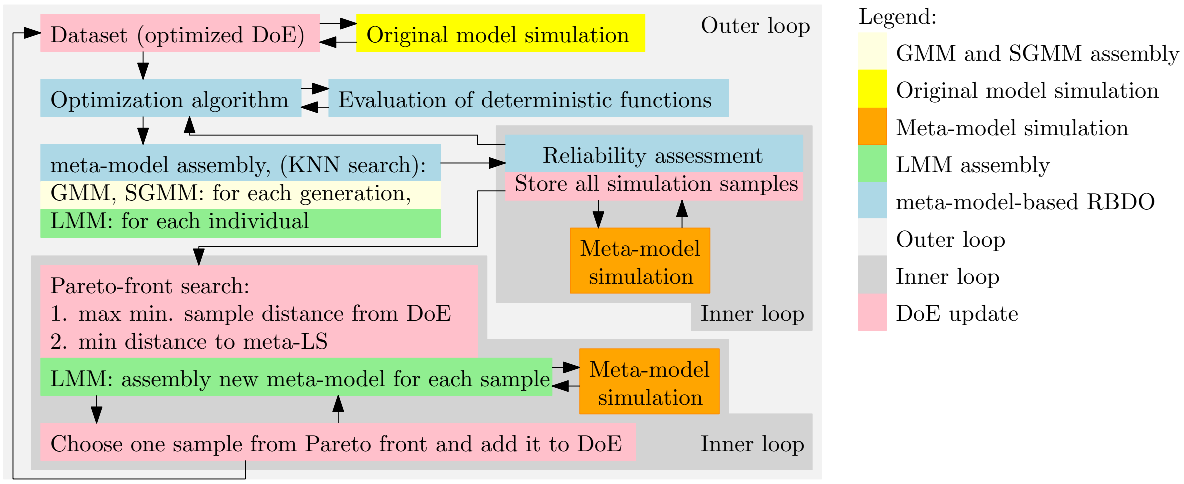

Figure 3: A flow-chart of a double-loop RBDO problem extended to the global and local meta-models update.

4 Results

The presented thesis compares Pareto-fronts as RBDO final solutions from three groups of approximations on three mathematical examples and the design of two simple structures. The examples differ in their complexity; they include different degrees of nonlinearity either in the model or in the necessary transformation info normal space, reliability of components and series systems with a different number of limit state functions, they also differ in a number of stochastic variables or design constraints. The first group of approximations uses the preconditioned quasi-Monte Carlo method to assess the reliability, where the preconditioning guarantees an approximately constant coefficient of variation of the failure probability estimate. The approximation is included only in the meta-model. The second group approximates reliability using six simulation techniques, namely an Asymptotic sampling, Importance sampling, Subset simulation, classical Monte Carlo simulation, Enhanced Monte Carlo simulation, and Scaled sigma sampling, and one approximation technique, namely First-order reliability method. If we evaluate the accuracy and precision of the methods used for individual examples using Pareto-efficiency conditions with criteria of mean value and standard deviation of the reliability index error, then we obtain the classification according to ranks – the lower the rank, the better the method. Table 1 presents the results. The third group combines the three best simulation

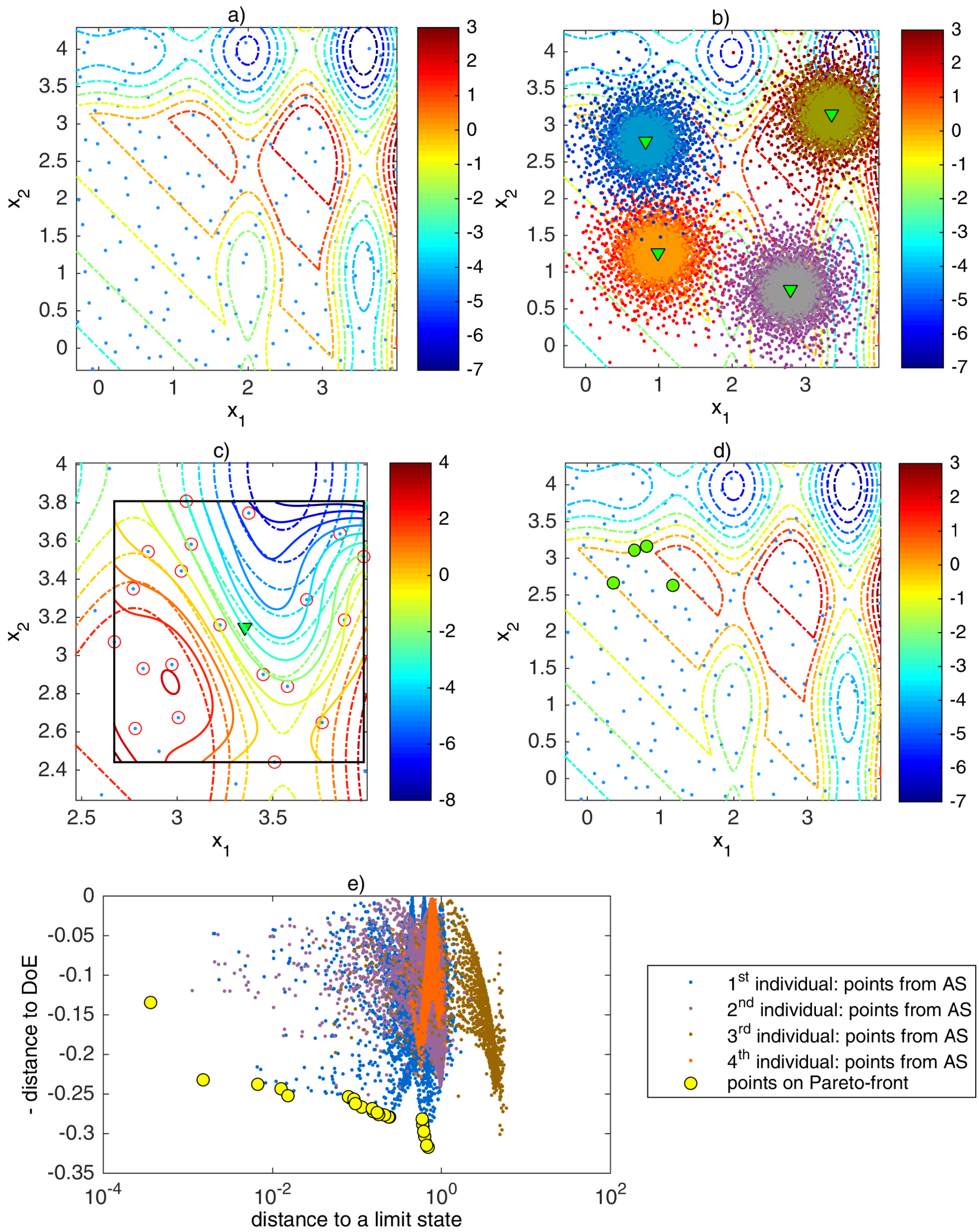

Figure 4: An illustrative example of the updating procedure using local meta-models. Image a) represents original LSF (dash-dotted contours) and initial DoE (blue dots). Image b) shows four individuals (green triangles) of an evolutionary algorithm with Asymptotic sampling samples (dots). Image c) is devoted to one local meta-model (solid contours) with support points (red circles) in the influence domain assembled for one individual of an evolutionary algorithm (green triangle), the black box represents the meta-model interpolation domain. Image d) shows new support points (green circles) in DoE (blue dots). Image e) depicts two criteria (a distance to a limit state and a distance to the nearest DoE support point) for decision making of DoE potential candidates.

Example	AS	eMC	FORM	IS	MC	SS	SSS	GMM	LMM	SGMM
1	4	5	2	1	2	3	6	2	3	1
2	1	3	5	4	2	1	4	1	3	2
3	1	5	4	2	3	3	3	1	2	3
4	2	4	3	1	3	2	3	2	1	1
5	3	4	3	2	4	1	3	3	1	2
Sum	**11**	**21**	**17**	**10**	**14**	**10**	**19**	**9**	**10**	**9**
Placings	**3**	**7**	**5**	**1-2**	**4**	**1-2**	**6**	**1-2**	**3**	**1-2**

Table 1: Sorting to fronts according to Pareto efficiency conditions with criteria a mean and standard deviation of the error. Reliability assessment techniques and meta-models are compared independently. The green tinge represents the best results, while the red colouring is for the worst results.

methods, namely an Asymptotic sampling, Importance sampling, and Subset simulation, with three meta-models, i.e. GMM, SGMM, and LMM. Several performance measures and error indicators evaluate all resulting Pareto-optimal fronts; a recurrent Monte Carlo simulation with a prescribed coefficient of variation of the failure probability estimate maps the Pareto-optimal sets into the objective space to evaluate the reliability index error.

The following three figures illustrate an example of some results. Figure 5 shows the output of one RBDO using an Asymptotic sampling and sparse global meta-model for Example 1. The Pareto front is almost identical to the superior Pareto front, which is the most accurate solution we can get with a very accurate Monte Carlo simulation and an analytical model. Due to the shape of the limit state function, the reliability space is separated from the design space. Only the reliability assessment uses the meta-model; therefore, for four different combinations of design variables and thus four different levels of reliability, the analytical model is plotted via the meta-model (dashed lines) and analytical model (solid lines). The bold contour shows a limit state as the most important part of the model for simulation techniques. Figure 6 presents the output of one RBDO simulation using a preconditioned quasi-Monte Carlo simulation and SGMM for Example 3. The points added to the DoE are mainly concentrated on the limit state around the place of population convergence. The meta-model is accurate mainly at the interface between reliable and unreliable space because it is the most important part of the space of interest for reliability calculation. The Pareto front is again almost identical to the superior Pareto front. Figure 7 provides a comparison of 10 independent outputs from RBDO using an Importance sampling and local meta-models with the superior Pareto set (left) and front (right) for Example 5. Meta-models needed an average of 334 points in DoE, i.e. 334 analytical function evaluations; the mean and standard deviation of the reliability index error are 0.10 and 0.09, respectively.

Accuracy and precision are the most important indicators for Pareto fronts obtained by MO-RBDO using approximations at different levels. Figure 8 shows a plot of these indicators. Another important indicator is the number of evaluations of the analytical model, Figure 9 shows the scatter plot of this indicator against the accuracy of the methods.

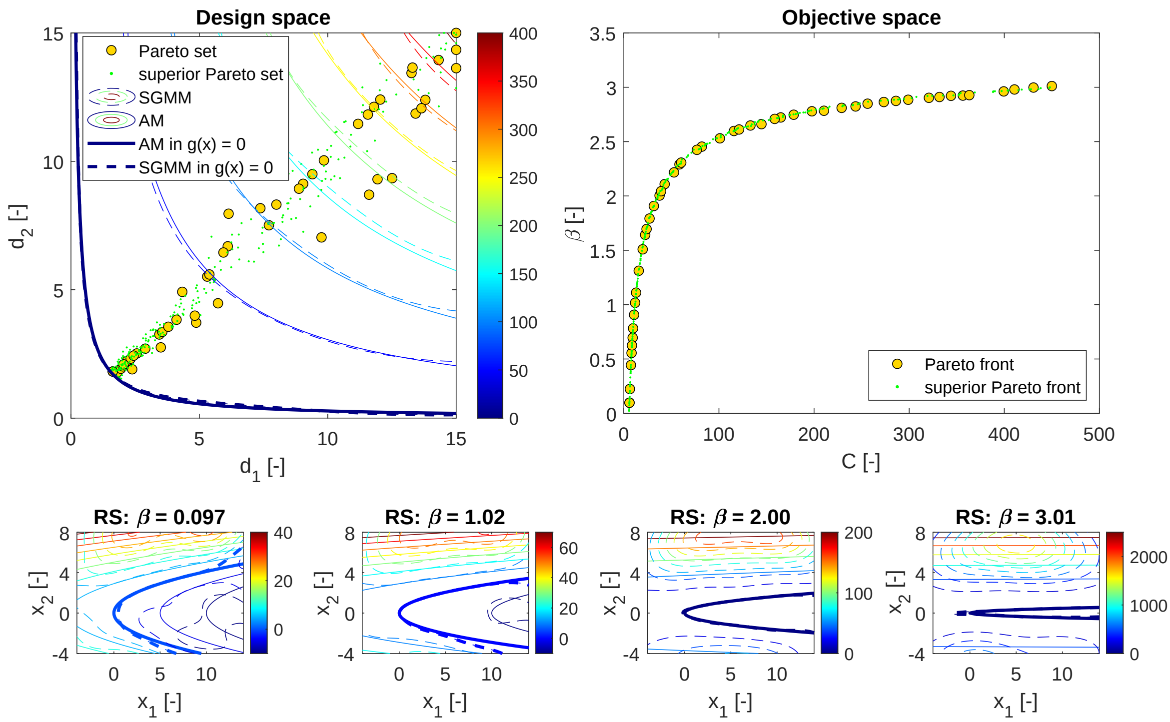

Figure 5: Example 1: Pareto sets and Pareto fronts from one MO-RBDO run utilizing a sparse global meta-model (upper row of images) and Asyptotic sampling. The reliability assessment uses SGMM and the reliability space (RS) changes for each set of design variables. The bottom row of images represents four randomly selected RSs from the Pareto set by analytical models (solid lines) and SGMM (dashed lines). Bold contour shows the limit state $g(x) = 0$.

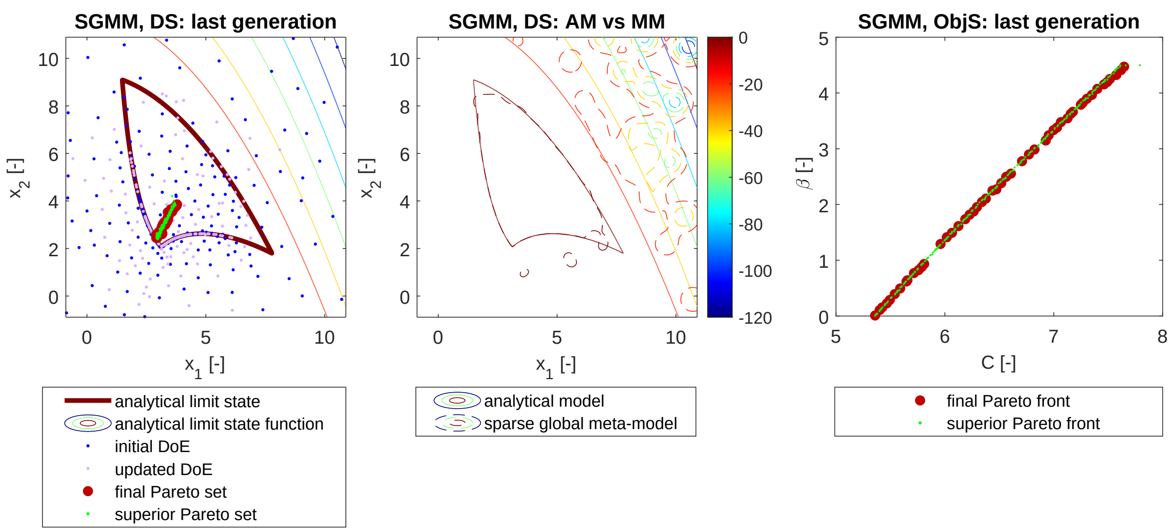

Figure 6: Example 3: Pareto sets and Pareto fronts from one MO-RBDO run utilizing a preconditioned quasi-Monte Carlo simulation and sparse global meta-model (SGMM). Legend: DS - design space, ObjS - Objective space, AM - analytical model, MM - meta-model.

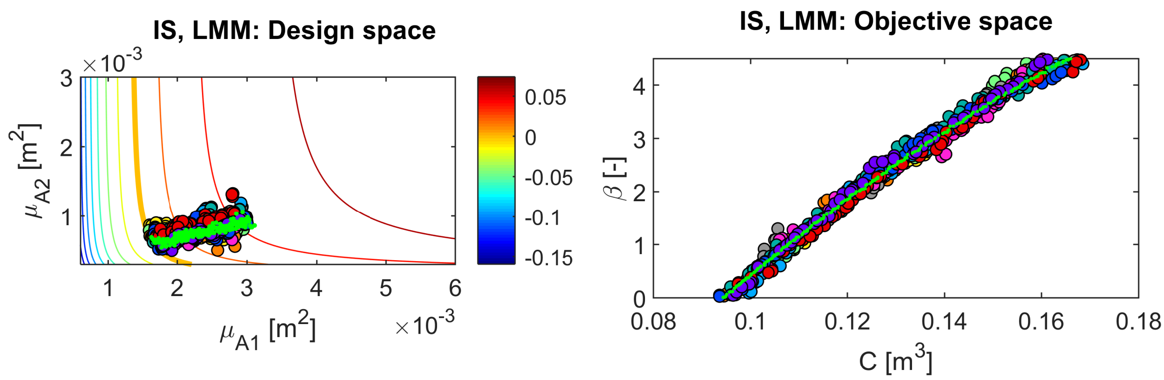

Figure 7: Example 5: Ten independent Pareto set and fronts (large circles) obtained from MO-RBDO using an Importance sampling and local meta-models. Different colours represent different runs. Green dot sets depict the superior Pareto set and front.

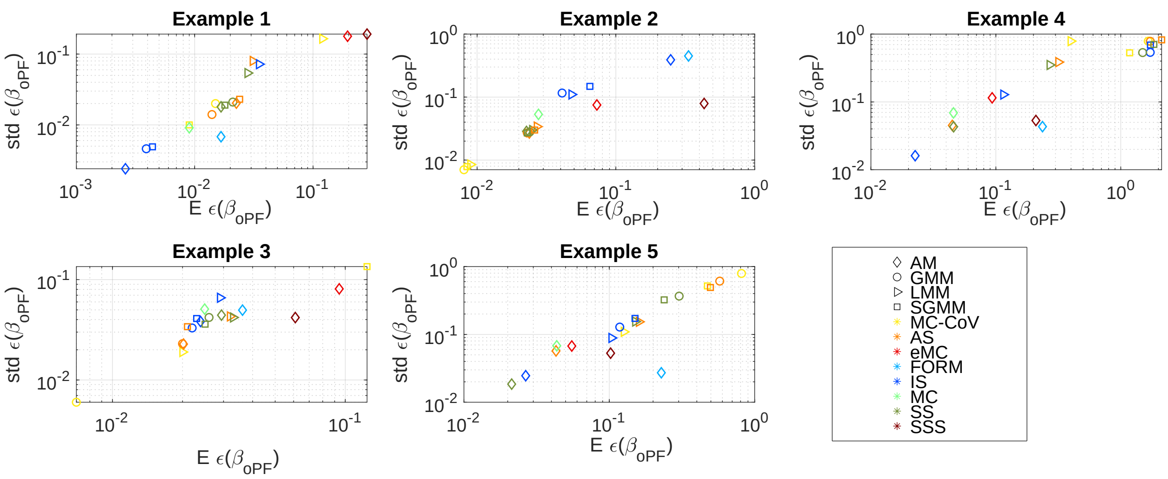

Figure 8: Accuracy ($\mathrm{E}\ \epsilon(\beta_{\mathrm{oPF}})$) and precision ($\mathrm{std}\ \epsilon(\beta_{\mathrm{oPF}})$) for all methods used and all examples.

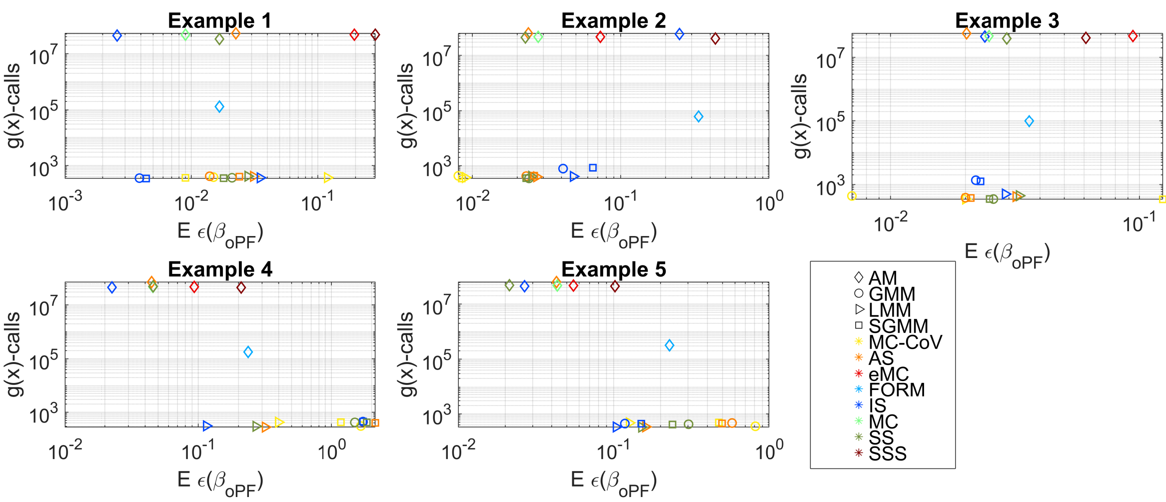

Figure 9: A number of analytical limit state function evaluations over accuracy for all methods used and all examples.

5 Conclusion

This thesis aimed to develop a methodology that provides fast and computationally simple solution of multi-objective reliability-based design optimization. However, our formulation of MO-RBDO is different from the formulation in the literature. Several papers [13, 38–40, 53, 54] define multi-objective RBDO essentially as an extension of single-objective RBDO so that there are several objective functions, but the reliability stays in the constraints inequalities. We formulate the reliability as an objective function because we want to answer the question: "How much does the reliability cost?" with the Pareto fronts as an answer. That is why we formulated the assignment of the reliability problem as one of the objective functions. Since this formulation is, to the author's knowledge, unique in the structural optimization field, we looked for the least computationally demanding implementation, optimized its implementation and validated the methodology on selected examples. In terms of computational complexity, the greatest demands appeared in the area of selection of (i) a multi-objective algorithm, (ii) fast and reliable evaluation of reliability, (iii) and effective replacement of the original model with a meta-model. We solved the (i) objective problem of computational complexity by studying various literature and found the most used and most recommended algorithm, which does not use the archive or the external population to preserve elitist solutions. Because our methodology consists of updated DoE in each generation, recalculating the external archive for each new generation would dramatically increase the computational demands. We solved the (ii) objective problem by finding seven different methods for evaluating the reliability in the available literature and using these methods together with the original model in multi-objective RBDO to compare their qualities for our problem. We then mapped the resulting Pareto sets to recalculated "Pareto" fronts using a quasi-Monte Carlo simulation with a guaranteed coefficient of variation below 5%. We used quotation marks because these "Pareto" fronts may not be Pareto-optimal, as the Pareto-efficiency conditions are not used in the mapping. We used these recalculated fronts only to evaluate the errors of the reliability methods. However, for the computation of other metrics evaluating the quality of the Pareto fronts in general, we selected only the Pareto-optimal solutions from these fronts. We solved the (iii) objective problem by formulating two special forms of the meta-model, namely local and sparse global meta-modes, and we compare these meta-models with the classical global meta-model. We also compared their error first separately with the preconditioned quasi-Monte Carlo simulation, and then we applied them with selected best reliability methods. On all these results, we again evaluated the errors of the reliability index.

We showed that our proposed methodology utilizing updated meta-models is better than MO-RBDO utilizing FORM and an original model since FORM needs at least two orders of magnitude more evaluations of the original model than our proposed method as evident from Figure 9. We have always been able to achieve better accuracy in the reliability index with at least one meta-model. This discovery is an interesting entry step for further research into how to fill the Pareto front individuals if we add or remove the number of support points in the updated DoE. There are three solutions to this problem. First, it is possible that adding more support points into DoE will greatly improve the accuracy of the reliability indices, as more information about the real problem gets to DoE. Second, DoE may be adequately filled, and the addition of support points do not

	GMM			SGMM			LMM		
Example	pqMC	$\bar{\text{SM}}$	diff.	pqMC	$\bar{\text{SM}}$	diff.	pqMC	$\bar{\text{SM}}$	diff.
1	13	5.67	7.33	5	7	2.00	7	11.67	4.67
2	3	6.33	3.33	2	7.33	5.33	1	8	7.00
3	2	4.33	2.33	10	4.67	5.33	1	7	6.00
4	8	10	2.00	8	12	4.00	10	6	4.00
5	6	8.67	2.67	10	8.33	1.67	12	6.33	5.67
	32	35	**3.00**	35	39.33	**4.33**	31	39	**8.00**

Table 2: Change in the behaviour of the meta-model depending on the simulation method. Ranks evaluate the Pareto-efficiency conditions for a mean and standard deviation of the error. Legend: pqMC - ranks for a preconditioned quasi-Monte Carlo simulation, $\bar{\text{SM}}$ - averaged ranks for an Asymptotic sampling, Importance sampling, and Subset simulation, diff. - difference between ranks for pqMC and $\bar{\text{SM}}$, GMM - a global meta-model, LMM - local meta-models, SGMM - a sparse global meta-model. The green tinge represents an average improvement of the meta-model behaviour with the advanced simulation methods in comparison with a preconditioned quasi-Monte Carlo simulation, the red tinge worsening of the same behaviour.

improve the accuracy of the reliability index. Third, with the addition of data to DoE, the meta-model will be subsequently overfitted and be prone to numerical noise. However, our acquired solutions using meta-models always dominate solutions obtained by using FORM calling the analytical model both in the accuracy of the solution and in the number of simulations of the analytical function.

The global meta-model is the least sensitive to the choice of a simulation method. Conversely, the local meta-models are most sensitive to its choice. If we evaluate the ranks for all used methods concerning the mean and standard deviation of the error, and compare these ranks for a preconditioned quasi-Monte Carlo simulation and averaged values via advanced simulation techniques (AST) as shown in Table 2 (or Figure 8), then the sum of the differences across all examples is the smallest just for the global meta-model and largest for local meta-models. If we evaluate the influence of a preconditioned quasi-MC and AST for each example, as shown in Table 2, then the behaviour improved ones, twice, and twice by using advanced simulation techniques for the GMM, LMM, and SGMM. For real applications, it will be necessary to test the meta-model accuracy and precision as an intermediate step for selecting a meta-model type. In case of inappropriate behaviour of the meta-model for the given example, the update procedure cannot improve it to be a credible replacement for the original model.

If we examine the influence of the meta-model on the change of the result of the simulation method using the analytical model, Table 3a) shows the trends of accuracy and precision for Example 1 – 3. These examples use mathematical functions as a limit state function that are relatively smooth if we omit the intersections of functions. The global meta-models for these three examples dominate, as both accuracy and precision have improved or only slightly deteriorated for these models regardless of the reliability assessment method compared to the analytical model and the simulation method. The table already contains subtracted values, where a negative number means improvement and a positive number means deterioration. Conversely, local meta-models dominate for

	GMM			SGMM			LMM		
Example	AS	IS	SS	AS	IS	SS	AS	IS	SS
accuracy change									
1	-0.008	0.001	0.004	0.001	0.002	0.001	0.008	**0.032**	0.011
2	$-4{\cdot}10^{-4}$	**-0.207**	0.001	0.002	-0.183	$\mathbf{2{\cdot}10^{-4}}$	0.003	-0.200	0.002
3	$-3{\cdot}10^{-5}$	-0.002	-0.003	0.001	-0.001	-0.004	0.012	0.005	0.004
precision change									
1	-0.006	0.002	0.003	0.003	0.003	0.001	0.059	**0.070**	0.036
2	$\mathbf{1{\cdot}10^{-4}}$	-0.275	-0.001	0.003	-0.242	-0.001	0.007	**-0.280**	0.002
3	$5{\cdot}10^{-4}$	-0.006	-0.002	0.011	0.002	-0.009	0.020	0.028	-0.002

(a) Example 1 – 3

	GMM			SGMM			LMM		
Example	AS	IS	SS	AS	IS	SS	AS	IS	SS
accuracy change									
4	1.679	1.699	1.451	**2.080**	1.706	1.794	0.272	0.092	0.225
5	0.532	0.091	0.281	0.453	0.124	0.217	0.117	**0.077**	0.130
precision change									
4	0.737	0.522	0.491	**0.777**	0.674	0.662	0.341	0.112	0.308
5	0.553	0.104	0.349	0.435	0.148	0.305	0.097	**0.064**	0.136

(b) Example 4 – 5

Table 3: Accuracy and precision change in the behaviour of the simulation method depending on the meta-model in comparison with the analytical model for a) Example 1 – 3 and b) Example 4 – 5. Legend: GMM - a global meta-model, LMM - local meta-models, SGMM - a sparse global meta-model, IS - an Importance sampling, AS - an Asymptotic sampling, SS - a Subset simulation. The green-yellow-red colour scale represents a deterioration of the method using a meta-model, where the green colour means the least deterioration, while the red colour shows the worst. The blue colour represents an improvement.

application on simple structures as evident from Table 3b) for Examples 4 – 5. Limit state functions are no longer as smooth as in Examples 1 – 3, but they have more failure modes and therefore more design points. The design space is larger, and the limit state functions are more nonlinear as in Examples 1 – 3. Global meta-models then fail to capture this complicated trend, and conversely, local meta-models work very well here due to their smaller influence domain. Also, local meta-models have better accuracy and precision in comparison with both types of meta-models, although the updating procedure ended up with fewer points in DoE in total; they are therefore better copying the original model, and at the same time they need less evaluation of the original model for accuracy and precision.

6 List of publications by the dissertant related to the dissertation

Peer-reviewed articles

1. Pospíšilová, A. – Lepš, M.: *Global Optima for Size Optimization Benchmarks by Branch and Bound Principles.* In: Acta Polytechnica, 52(6), 2012, pp. 74–81. 80% participation.

Conference papers

1. Hlobilová, A. – Lepš, M.: *Parallelization of Reliability-based Design Optimization using Surrogates.* In: Proceedings of the Fifth International Conference on Parallel, Distributed, Grid and Cloud Computing for Engineering. Stirling: Civil-Comp Press Ltd, 2017, pp. 1–16. 80% participation.
2. Hlobilová, A.; Lepš, M.: *Parameter Study on Subset Simulation for Reliability Assessment.* In: PADEVĚT, P., ed. Modern Methods of Experimental and Computational Investigations in Area of Construction II. Nano a Makro Mechanika 2016 – 7. ročník konference. Praha, 22.09.2016. Pfaffikon: Trans Tech Publications Inc. 2016, Advanced Materials Research 1144, pp. 128–135. 70% participation.
3. Pospíšilová, A.; Lepš, M.: *Multi-Objective Reliability-Based Design Optimization using Subset Simulation Enhanced by Meta-Models.* In: REC 2016. Bochum: Ruhr Universitat Bochum, 2016, pp. 427–440. 75% participation.
4. Pospíšilová, A.; Lepš, M.: *Comparison of different simulation techniques for reliability-based design optimization.* In: Engineering Mechanics 2016 – Book of full texts. Prague: Institute of Thermomechanics, AS CR, v.v.i., 2016, pp. 474–477. 80% participation.
5. Pospíšilová, A. – Lepš, M.: *Multi-Objective Reliability-Based Design Optimization utilizing an Adaptively Updated Surrogate Model.* In: Proceedings of the Fourth International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering. Stirling: Civil-Comp Press Ltd, 2015, art. no. 8. 60% participation.
6. Pospíšilová, A., Lepš, M.: *Comparison of Advanced Simulation Techniques for Reliability Assessment.* In: Proceedings of the 5th Conference Nano and Macro Mechanics NMM 2014. Praha: České vysoké učení technické v Praze, Fakulta stavební, 2014, pp. 121–128. 80% participation.
7. Pospíšilová, A.: *On Inverse Formulation of Reliability-Based Design Optimization.* In: Proceedings of the 5rd Conference Nano and Macro Mechanics NMM 2014. Praha: České vysoké učení technické v Praze, Fakulta stavební, 2014, pp. 129–138.
8. Pospíšilová, A. – Lepš, M.: *Parameter Study of Asymptotic Sampling for Truss Structures Reliability.* In: Advanced Materials Research (AMR) volume 59. 2014,

vol. 969, pp. 288–293. 80% participation.

9. Myšáková, E. – Pospíšilová, A. – Lepš, M.: *Optimized Design of Computer Experiments: A Review.* In: Computational Methods for Engineering Technology. Stirling: Saxe-Coburg Publications, 2014, pp. 325–344. 33% participation.
10. Pospíšilová, A. – Lepš, M.: *Adaptive update of surrogate models for reliability-based design optimization: A review.* In: Engineering Mechanics 2014. Brno: Institute of Solid Mechanics, Mechatronics and Biomechanics, Faculty of Mechanical Engineering, Brno University of Technology, 2014, p.. 508–511. 60% participation.
11. Pospíšilová, A. – Lepš, M.: *Parameter study of Asymptotic Sampling for Truss Structures Reliability.* In: Structural and Physical Aspects of Civil Engineering. Košice: Technical University of Košice, 2013. 80% participation.
12. Pospíšilová, A. – Myšáková, E. – Lepš, M.: *Multi-objective Adaptive DoE for RBDO.* In: Proceedings of the 11th International Probabilistic Workshop. Brno: Ing. Vladislav Pokorný – LITERA, 2013, pp. 325–336. 50% participation.
13. Pospíšilová, A. – Lepš, M.: *Multi-Objective Optimization with Asymptotic Sampling for RBDO.* In: Proceedings of the Third International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering. Stirling: Civil-Comp Press Ltd, 2013, Paper 2. 50% participation.
14. Myšáková, E. – Pospíšilová, A. – Lepš, M.: *Multiobjective Adaptive Updating of Surrogate Models.* In: Engineering Mechanics 2013 [CD-ROM]. Prague: Institute of Thermomechanics, Academy of Sciences of the Czech Republic, 2013, p. 453–460. 40% participation.

Research reports

1. Hlobilová, A.; Myšáková, E.; Kučerová, A.; Lepš, M.: *TC6: Report: Strategy for probabilistic life prediction.* [Research report] OTTOBRUNN: AIRBUS DEFENCE AND SPACE GMBH, 2016. FLC-TN-11000-H-0007-CTU. 40% participation.

2. Lepš, M. – Pospíšilová, A.: *Geometry optimization.* [Research Report]. 2014. FLP-TN-22180-H-0005-CTU. 50% participation.

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