What is a (Mathematical) Convolution

- The definition

    - Continuous (https://mathworld.wolfram.com/Convolution.html)

    $$[f * g](t) = \int_0^t f(\tau)g(t - \tau)d\tau$$

    - Discrete

    $$[x * w](n) = \sum_{m=-\infty}^{\infty} x[n - m]w[m]$$

    - What to do with finite sequences

        - pad with zeros and suffer edge effects
        - circular convolution (assumed periodicity)
    - 2D and more (can be generalised to images/matrices, tensors, groups ++)

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- Another Mathsy View

    - Convolution is a linear transform by a Toeplitz matrix

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- The Signal Processing View

    - Think of x as a signal, and w as a *kernel*

    - A convolution is a Linear Time-Invariant (LTI) filter

        - Keywords: Low-pass, High-pass, Band-pass, ARMA, IIR, FIR, ...
    - Implemented as taps, block diagrams, ...

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- EEEngineers view

    - Convulition is a LTI *system*, e.g., represents dynamics

- The Frequency View

    - $x * w = F^{-1}\left(F(x)F(w)\right)$

    - Fourier transform is just a set of convolutions (1 for each frequency band)

- The Statisticians View

    - A convolution is just a correlation coefficient between x and lagged versions of $\bar{w}$ (time reversed w)

    - In time series, a convolution is used to create an ARMA system

    - A convolution is used in kernel smoothing/density approaches

- The probabilists View

    - A convolution is how you compute the distribution function of the sum of two independent RVs

- The physicist View

    - Doppler blurring
    - Out-of-focus optics

- Machine Learning

    - CNNS

    - blurring, smoothing, edge detection, ...

These views are all the same.