List of acronyms

A/D Analogue to Digital Convertor
AR Auto-Regressive
ARMA Auto-Regressive Moving Average
ARIMA Auto-Regressive Integrated Moving Average
CD Compact Disc
CFT Continuous Fourier Transform
CGI Computer Generated Imagery
CWT Continuous Wavelet Transform
DAC Digital to Analogue Convertor
DCT Discrete Cosine Transform
DFT Discrete Fourier Transform
DWF Discrete Wavelet Filters
DWT Discrete Wavelet Transform
EWMA Exponentially Weighted Moving Average
FFT Fast Fourier Transform
FIR Finite Impulse Reponse
FS Fourier Series
FT Fourier Transform
IFFT Inverse Fast Fourier Transform
IFT Inverse Fourier Transform
IIR Infinite Impulse Reponse
JPEG Joint Photographics Experts Group
LTI Linear Time-Invariant (filter/system)
MRA Multi-Resolution Analysis (or Approximation)
MA Moving Average
RMS Root Mean Square
STFT Short Time Fourier Transform
WFT Windowed Fourier Transform

Units

Some frequently used terminology (see notes for details)

- **dB:** Decibels (defined WRT a reference power level p_{ref}) by
$$dB = 10 \log_{10} \frac{p}{p_{\text{ref}}}.$$
Note that $p = m^2$, where m is the RMS magnitude of a signal, so we may write $dB = 20 \log_{10} \frac{m}{m_{\text{ref}}}$
- **Dynamic range:** expresses the range of values we can represent in our digital format expressed in dB. In a representation with b bits (ignoring the sign bit), the dynamic range is roughly 6dB per bit.
- **Hz** Unit of frequency. 1 Hz = 1 cycle per second

Common Mathematical Notation

$\mathbb{R}$	the real numbers
$\mathbb{C}$	the complex numbers
$\mathbb{R}^n$	length n vectors of real numbers
$\mathbb{R}^{n\times m}$	$n \times m$ matrices of real numbers
$\mathbb{R}^{n\times m\times k}$	$n \times m \times k$ tensors of real numbers
$\|x\|$	absolute value of x
$\|S\|$	cardinality of set S
$\\|\cdot\\|$	a norm
$\\|\cdot\\|_p$	the L_p norm
$\\|\cdot\\|_2$	the L_2 norm
$\\|\cdot\\|_\infty$	the L_∞ norm
$\\|\cdot\\|_F$	the Frobenius norm
$\langle f, g\rangle$	the inner product of f and g
$\text{argmin}_x f(x)$	the value (argument) of x that minimises f
$\text{argmax}_x f(x)$	the value (argument) of x that maximises f
$[a, b]$	the closed interval $\{x \mid a \le x \le b\}$
(a, b)	the open interval $\{x \mid a < x < b\}$

Complex Number Notation and Terminology

For any $x \in \mathbb{C}$ we can write
$$x = a + ib, \text{ where } i = \sqrt{-1}.$$

- real part of x is $\Re(x) = a$
- imaginary part of x is $\Im(x) = b$
- complex conjugate $\overline{x} = x^* = a - ib$
- commonly used identities
  - $e^{ix} = \cos(x) + i\sin(x)$
  - $\cos(x) = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$
  - $\sin(x) = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$
- Hermitian of a complex matrix $A = [a_{ij}]$ is $A^H = [a^*_{ji}]$.

Matrix and Vector Notation and Terminology

The standard convention is that

- Lower case letters denote scalars, *e.g.,* $x = 6$, though some are used to denote functions, *e.g.,* f and g.
- Boldface letters denote column vectors, *e.g.,* $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, but note that it is becoming more common to omit the distinction between scalars and vectors.
- Upper case letters denote matrices (but may also denote random variables in a probabilistic context).
- Tensors are higher-dimensional analogues of vectors and matrices. These are typically also represented using upper case letters but usually with subscripts to indicate the order, *e.g.,* $T_{\mu\nu\sigma}$.
- We indicate elements of an array with square brackets, *e.g.,* $[A]_{ij}$ is the (i, j)th element of the matrix A. When its obvious the brackets may be omitted, *e.g.,* A_{ij}, but this can get confusing with tensors so be aware of context.

Vector, Matrix and Tensor notation

$[A]_{ij}$	the element of A from the ith row and jth column
$\mathbf{e}^{(i)}$	ith standard basis vector $(0, 0, \ldots, 0, 1, 0, \ldots, 0)$, with 1 in the ith position
A^T	the transpose of A
A^H	the Hermitian transpose (the transpose and complex conjugate)
$\det A$	the determinant of A
$\text{tr}(A)$	the trace of A
A^{-1}	the inverse of A, *i.e.,* $AA^{-1} = I$
$N(A)$	the null-space of A, *i.e.,* .....
$\text{rank}\,(A)$	the rank of A
$\text{vect}\,(A)$	the vectorised matrix (converted to a vector by stacking)

$$\text{vect}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a \\ c \\ b \\ d \end{bmatrix}$$

Matrix types

I	the identity matrix
symmetric	$A = A^T$
skew-symmetric	$A = -A^T$
Hermitian	$A = A^H$
skew-Hermitian	$A = -A^H$
diagonal	all elements off the (principle) diagonal are 0
lower triangular	all elements above the (principle) diagonal are 0
upper triangular	all elements below the (principle) diagonal are 0
square	a matrix with the same number of rows and columns
permutation	a square binary matrix with exaclty one '1' in each row and column
Toeplitz	a matrix whose diagonals are constant
circulant	a Toeplitz matrix where each row is the previous one shifted one element to the right (and wrapped back around)
orthogonal	a real, square matrix Q such that $QQ^T = Q^TQ = I$
unitary	a complex, square matrix U such that $UU^H = U^HU = I$
normal	a complex, square matrix A such that $AA^H = A^HA$
projection	a square matrix P such that $P^2 = P$, also called *idempotent*

https://en.wikipedia.org/wiki/List_of_named_matrices

Probability Notation and Terminology

$P(A)$	probaility of event A
$P(X = x)$	probaility of random variable x takes the value x
$E[X]$	the expectation of random variable X
$\mathrm{Var}(X)$	the variance of random variable X

Function Notation and Terminology

Standard functions

- unit step: $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$
- rectangular pulse: $r(t) = u(t + 1/2) - u(t - 1/2)$.
- sign (signum) function: $\mathrm{sgn}(t) = \begin{cases} -1, & t < 0 \\ 1, & t > 0 \end{cases}$
- sinc function: $\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$
- Delta "function" $\delta(t)$ definition

$$\begin{aligned} \delta(-t) &= \delta(t) \\ \int_{-\infty}^{t} \delta(s)\, ds &= u(t) \\ \int_{-\infty}^{\infty} f(t)\delta(t - t_0)\, dt &= f(t_0) \end{aligned}$$

Function characteristics

- even: $f(-t) = f(t)$
- odd: $f(-t) = -f(t)$
- any signal $x(t) = x_{\text{even}}(t) + x_{\text{odd}}(t)$ where

  $x_{\text{even}}(t) = \dfrac{1}{2}[x(t) + x(-t)]$ and $x_{\text{odd}}(t) = \dfrac{1}{2}[x(t) - x(-t)]$
- Hermitian: $x(-t) = x^*(t)$
- periodic: $x(t + nT) = x(t)$ for any $n = 1, 2, \ldots$, and some $T > 0$.

  The minimal value $T = T_0 > 0$ for which periodic signal $x(t + nT) = x(t)$ for any $n = 1, 2, \ldots$, and some $T > 0$ is called the fundamental period, and has units of seconds.

$$\begin{aligned} T &= \text{ period (maybe measured in seconds)} \\ f &= 1/T = \text{ frequency (measured in Hz)} \\ \omega &= 2\pi f, \text{ frequency (measured in radians per second)} \end{aligned}$$

Useful Results

Inequalities

- Hölder: Take $p, q \in [1, \infty]$ with $1/p + 1/q = 1$ then

  $$\|fg\|_1 \leq \|f\|_p \|g\|_q,$$

  for all (suitable constrained) functions f and g.

- Cauchy-Schwarz: For all vectors $\mathbf{u}$ and $\mathbf{v}$

  $$|\langle \mathbf{u}, \mathbf{v} \rangle| \leq \|\mathbf{u}\| \|\mathbf{v}\|,$$

  where we use the norm defined by $\|\mathbf{u}\|^2 = \langle \mathbf{u}, \mathbf{u} \rangle$

- Cauchy-Schwarz, common case of vectors in $\mathbb{R}^n$

  $$\left(\sum_{i=1}^n u_i v_i \right)^2 \leq \left(\sum_{i=1}^n u_i^2 \right) \left(\sum_{i=1}^n v_i^2 \right).$$

- Jensen: For any convex function $\phi(\cdot)$ and random variable X

  $$\phi\left(E\left[X\right]\right) \leq E\left[\phi(X)\right].$$

- Log-sum: a_i and b_i are non-negative numbers, and $a = \sum_i a_i$ and $b = \sum_i b_i$, then

  $$\sum_i a_i \log \frac{a_i}{b_i} \geq a \log \frac{a}{b},$$

  with convention that $0 \log 0 = 0$ and equality if and only if $a_i = c b_i$ for some constant c.

- Gibbs: For discrete probability distributions $P = \{p_1, \ldots, p_n\}$ and $Q = \{q_1, \ldots, q_n\}$ then

  $$-\sum_{i=1}^n p_i \log p_i \leq -\sum_{i=1}^n p_i \log q_i,$$

  with equality if and only if $p_i = q_i$.

- Nyquist: Perfect reconstruction of a signal (from discrete samples) is possible if the sampling frequency f_s is

  $$f_s > 2B,$$

  where B is the bandwidth of the signal.

Fourier Cheatsheet

Fourier series: We can write a periodic function as an (infinite) discrete sum of trigonometric terms, *e.g.*, period 2π

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\cos(nx)\,dx \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\sin(nx)\,dx \end{aligned}$$

Fourier transform: $F(s) = \int_{-\infty}^{\infty} f(t) e^{-i2\pi st}\,dt$

Inverse Fourier transform: $f(t) = \int_{-\infty}^{\infty} F(s) e^{i2\pi st}\,ds$

Example FTs

Function	**Transform**
$\delta(t)$	1
$\delta(t - t_0)$	$e^{-i2\pi t_0 s}$
$r(t)$	$\mathrm{sinc}(s)$
$e^{-\lvert t\rvert}$	$\frac{2}{4\pi^2 s^2 + 1}$
$e^{-\pi t^2}$	$e^{-\pi s^2}$
1	$\delta(t)$
$e^{i2\pi s_0 t}$	$\delta(s - s_0)$
$\mathrm{sinc}(t)$	$r(s)$

Discrete Fourier Transformation

A signal $x(n)$, with N data points, has DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-i2\pi kn/N},$$

Inverse DFT

$$x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(k) e^{i2\pi kn/N},$$