# What is a (Mathematical) Convolution

- The definition
  - Continuous (https://mathworld.wolfram.com/Convolution.html)

$$[f * g](t) = \int_0^t f(\tau)g(t-\tau)d\tau$$

  - Discrete

$$[x * w](n) = \sum_{m=-\infty}^{\infty} x[n-m]w[m]$$

  - What to do with finite sequences
    - pad with zeros and suffer edge effects
    - circular convolution (assumed periodicity)
  - 2D and more (can be generalised to images/matrices, tensors, groups ++ )

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- Another Mathsy View
  - Convolution is a linear transform by a Toeplitz matrix

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- The Signal Processing View
  - Think of $x$ as a signal, and $w$ as a *kernel*
  - A convolution is a Linear Time-Invariant (LTI) filter
    - Keywords: Low-pass, High-pass, Band-pass, ARMA, IIR, FIR, ...
  - Implemented as taps, block diagrams, ...

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- EEEngineers view
  - Convolution is a LTI *system*, e.g., represents dynamics

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- The Frequency View
  - $x * w = F^{-1}\Big(F(x)F(w)\Big)$
  - Fourier transform is just a set of convolutions (1 for each frequency band)

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- The Statisticians View
  - A convolution is just a correlation coefficient between $x$ and lagged versions of $\bar{w}$ (time reversed $w$)
  - In time series, a convolution is used to create an ARMA system
  - A convolution is used in kernel smoothing/density approaches

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- The probabilists View
  - A convolution is how you compute the distribution function of the sum of two independent RVs

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- The physicist View
  - Doppler blurring
  - Out-of-focus optics

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- Machine Learning
  - CNNS
  - blurring, smoothing, edge detection, ...

These views are all the same.