QSM Quantization

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April 22, 2021

1. The capacitance matrix C of the system is simulated by Ansys
2. Circuit simulations are run with the qubit inductance set as L_i (user-defined). These simulations yield the lumped resonator L_ℓ, C_ℓ.
3. The resonator capacitance to ground is corrected in C via the lumped model.
4. Gaussian elimination is performed on C to get the C_{GE} for quantization. This is then inverted to get C_{GE}^{-1} (the "GE" is dropped moving forward).
5. Qubit definitions (From Solgun "Simple Impedance Response Formulas..."):

$$E_{c,i} = \frac{e^2}{2}[C^{-1}]_i \tag{1}$$

$$L_i = \frac{L_J}{1 - \frac{2E_c}{\hbar\omega_i}} = \frac{1}{\omega_i^2}[C^{-1}]_i \tag{2}$$

$$\omega_i = \omega_J - \frac{E_c}{\hbar\left(1 - \frac{E_c}{\hbar\omega_J}\right)} \tag{3}$$

$$\omega_J = \frac{1}{\sqrt{L_J \frac{e^2}{2E_c}}} \tag{4}$$

$$L_J = 11.5\text{nH} \quad \text{(User defined in componentParameters)} \tag{5}$$

6. There exists a cyclic definition, namely that L_i is a function of E_C, and E_C depends on a circuit simulation requiring L_i. This is managed by comparing the "calculated" L_i (eq.2) to the user-defined L_i. When they agree everything is consistent.
7. The hamiltonian is constructed via

$$H = \frac{1}{2}Q^T C^{-1} Q - \sum_i E_{J,i}\cos\left(\frac{2\pi\Phi_i}{\Phi_0}\right) + \sum_j \frac{\Phi_j^2}{2L_{\ell,j}} \tag{6}$$

with (Junling thesis equation 2.48)

$$E_{J,i} = \frac{(\Phi_0/2\pi)^2}{L_{J,i}} \tag{7}$$

and $\Phi_0 \equiv \frac{h}{2e}$.

8. We then make the second quantization substitutions (Junling thesis equations 2.22/2.23)

$$Q_k = i\sqrt{\frac{\hbar}{2Z_k}}\left(a_k^\dagger - a_k\right) \tag{8}$$

$$\Phi_k = \sqrt{\frac{\hbar Z_k}{2}}\left(a_k^\dagger + a_k\right) \tag{9}$$

$$Z_k = L_k\omega_k \tag{10}$$

$$\omega_k = \sqrt{C^{-1}[k,k]/L_k} \tag{11}$$

Where for qubits $L_k = L_{J,k}$ and for resonators $L_k = L_{\ell,k}$.

9. In the quantize simulation parameters file there is a parameter called "numPhotons". The QSM currently uses fock states of dimension numPhotons+2 (due to the truncation error of ladder operators), substitutes these matrices into the hamiltonian and simplifies, then removes all states in which any qubit/resonator has more than numPhotons excitations. So, for example, in a 2 qubit/resonator system with numPhotons=6, it will keep all states up to [6,6,6,6] when assembling the Hamiltonian, which cause it to have 4096 rows/columns. Now that the Hamiltonian is fully assembled, it gets truncated to 2401 (numPhotons+1, including 0) rows/columns.

10. Now that the Hamiltonian is truncated, we want to prepare H_output, which is the subset of the Hamiltonian that will be written to the CSV file. Currently, it includes everything up to the two-photon manifold. Instead of leaving it in the fock basis, however, we shift to a basis in which the individual components are diagonal. In this basis $\langle m0| H |n0\rangle = 0$ for $m \neq n$ (and for all components, not just the first). This is achieved as such. First, the "subspace Hamiltonians" for each component are constructed by extracting from H $|00\rangle, |10\rangle, |20\rangle$ etc. Then these are all diagonalized. Their eigenvectors then correspond to $|\tilde{0}\rangle, |\tilde{1}\rangle, |\tilde{2}\rangle$ etc. Finally, the ouptut matrix is assembled by transforming to the $|\tilde{0}\tilde{0}\rangle, |\tilde{1}\tilde{0}\rangle, |\tilde{0}\tilde{1}\rangle$ etc. basis and truncating up to the two-photon manifold.

11. Finally, the hamiltonian is diagonalized, and ZZ is calculated via

$$ZZ = E11 - E10 - E01$$