Redundancy under Discussion[1]

Adèle HÉNOT-MORTIER — *Massachusetts Institute of Technology*

Abstract. This paper presents novel data taking the form of a family of sentences, all derived from a redundant structure ($p \vee p \vee q$) *via* the *or*-to-*if* tautology and core properties of disjunction (commutativity, associativity). In a way similar to Hurford Disjunctions vs. Conditionals, the sentences at stake exhibit differing degrees of pragmatic oddness, which represents a challenge for standard theories of redundancy. Building on a recent QuD-informed model of pragmatic oddness, we propose a solution covering almost all the cases at stake, and discuss how the remaining problematic cases might be solved assuming an incremental view on Redundancy.

Keywords: redundancy, relevance, question under discussion

1. Introduction

Disjunctive sentences featuring a repeated disjunct, such as (1a), (1b) and (1c), appear sharply infelicitous. From a logical point of view, these three sentences are related to each other *via* applications of $\vee$-commutativity and $\vee$-associativity.[2] From a pragmatic point of view, these sentences can be argued to be odd because they are all contextually equivalent to their complex disjunct, $p \vee q$ or $q \vee p$.

(1) a. # Either Ido is at SuB, or else he is at SuB or in Cambridge. $p \vee (p \vee q)$
b. # Either Ido is at SuB, or else he is in Cambridge or at SuB. $p \vee (q \vee p)$
c. # Either Ido is at SuB or in Cambridge, or else he is at SuB. $(p \vee q) \vee p$
d. # Either Ido is in Cambridge or at SuB, or else he is at SuB. $(q \vee p) \vee p$

(2-5) below show variants of (1a-1d) obtained *via* the *or*-to-*if* tautology. In each pair of sentences, the a. instances are derived by applying the tautology to the outer disjunction, while the the b. instances are derived by applying the tautology to the inner disjunction.[3]. Surprisingly, those variants exhibit different degrees of oddness: (2b) and (4b) are the only two variants which seem to escape infelicity. This is unexpected given that all the sentences in (2-5) have same logical structure as the infelicitous sentences in (1a-1d), assuming implications are material.

(2) Derived from (1a):
a. # If Ido is not at SuB then he is at SuB or in Cambridge. $\neg p \rightarrow (p \vee q)$
b. Either Ido is at SuB or if he is not at SuB then he is in Cambridge. $p \vee (\neg p \rightarrow q)$

(3) Derived from (1b):
a. # If Ido is not at SuB then he is in Cambridge or at SuB. $\neg p \rightarrow (q \vee p)$
b. # Either Ido is at SuB or if he is not in Cambridge then he is at SuB. $p \vee (\neg q \rightarrow p)$

[1] Many thanks to Amir Anvari, Athulya Aravind, Danny Fox, Nina Haslinger, and Viola Schmitt for their precious input on that project. All mistakes are my own.

[2] More variants could be derived, for instance $q \vee (p \vee p)$. Here, we focus on the less obvious variants where two instances of p do not directly combine together.

[3] One could also apply the *or*-to-*if* tautology to *both* the inner and the outer disjunction in the sentences in (1). Nested conditionals however, are hard to judge. That is why we omit them in this introduction.

(4) Derived from (1c):

a. # If it's not true that Ido is at SuB or in Cambridge, then he is at SuB. $\neg(p \vee q) \to p$

b. ? Either Ido is in Cambridge if not at SuB, or he is at SuB. $(\neg p \to q) \vee p$

(5) Derived from (1d):

a. # If it's not true that Ido is in Cambridge or at SuB, then he is at SuB. $\neg(q \vee p) \to p$

b. # Either Ido is at SuB if not in Cambridge, or he is at SuB. $(\neg q \to p) \vee p$

The intuitive generalization seems to be the following: the sentences in (2-5) that retain an outer disjunction, and whose complex (conditional) disjunct has the negation of their simple disjunct as antecedent, are rescued. Building on the machinery laid out in Hénot-Mortier (2024), we propose that this descriptive generalization follows from the idea that oddness arises when sentences cannot evoke any well-formed accommodated Questions under Discussion (Van Kuppevelt, 1995; Roberts, 1996); and that disjunctions and conditionals have different inquisitive contributions, in the sense that disjunctions force their disjuncts to raise *parallel* QuDs, while conditionals "stack" the QuDs of their antecedent and consequent. More generally, this predicts "connectivity effects" in disjunctions-of-conditionals, in that the antecedents and consequents respectively have to address similar QuDs; and no such effect in conditionals-of-disjunctions, in that disjuncts coming from the antecedent and consequent may be inquisitively unrelated. Assuming that $\vee$-commutativity does not affect oddness (in line with the data presented here), we now focus on sentences (1a), (2a), (2b), (3b), and (4a), repeated in (6) below.

(6) a. # Either Ido is at SuB, or else he is at SuB or in Cambridge. $p \vee (p \vee q)$

b. # If Ido is not at SuB then he is at SuB or in Cambridge. $\neg p \to (p \vee q)$

c. Either Ido is at SuB or if he is not at SuB then he is in Cambridge. $p \vee (\neg p \to q)$

d. # Either Ido is at SuB or if he is not in Cambridge then he is at SuB. $p \vee (\neg q \to p)$

e. # If it's not true that Ido is at SuB or in Cambridge, then he is at SuB. $\neg(p \vee q) \to p$

The rest of this paper is structured as follows. In the next Section, we briefly review why some of the sentences in (6) are problematic for existing accounts of redundancy. In Section 3 we show how the model of accommodated QuDs laid out in Hénot-Mortier (2024) captures the target asymmetries. In Section 4 we explore further predictions of he model in elaborations of the sentences in (6). Section 5 concludes.

2. Previous accounts

In this section we briefly present three existing accounts of redundant structures: Local Redundancy Checking, Non-triviality, and Super-Redundancy. We show how the they straightforwardly account for the double disjunction case (6a), but fall short in explaining the contrast between the felicitous (6c) vs. (6b), (6d), and (6e).

2.1. Local Redundancy Checking

Katzir and Singh (2014) propose that the semantic computation evaluates, at certain nodes, whether the semantic composition principle that applies there is non-vacuous. This gives rise to the principle in (7).

(7) *Local Redundancy Checking.* S is deviant if S contains γ s.t. $[\![\gamma]\!] = [\![O(\alpha,\beta)]\!] \equiv_c [\![\zeta]\!]$, $\zeta \in \{\alpha,\beta\}$.

This predicts the double disjunction (6a) to be deviant, because it is contextually equivalent to its complex disjunct ($p \vee q$). But, assuming conditionals denote material implications, this also predicts (6b-6e) to be deviant, meaning, the felicity of (6c) is not derived.

The issue persists if we adopt a non-material analysis of conditionals. Under this assumption, the whole conditional will never be contextually equivalent to its antecedent or consequent regardless of what they denote. Candidate simplifications for (6b-6c) are then $\neg p \rightarrow q$, and for (6d-6e), $\neg q \rightarrow p$. Under a strict analysis of conditionals, (6b-6e) can be shown to be equivalent to their respective simplifications and thus devious, while (6c-6d) are not. But this is not the expected contrast. Under a variably strict analysis, all the cases but (6b) are predicted into felicity. Again, it is not the expected contrast.

2.2. Non-triviality

Another line of work, building on the notion of local contexts (Schlenker, 2009), associates redundancy with triviality (Stalnaker, 1999): a sentence should not contain a part that is trivially true or false when evaluated against its local context (Mayr and Romoli, 2016).

(8) *Non-triviality.* A sentence S cannot be used in a context c if some part π of S is entailed or contradicted by the local context of π in c.

Assuming the local context of the second disjunct is the negation of the first, and the local context of a consequent is the antecedent, (6a-6e) are all predicted to be deviant under a material implication analysis: in all cases, the second occurrence of p gets interpreted in a local context entailing $\neg p$.

nonmaterial conditionals TODO

2.3. Super-Redundancy

Kalomoiros (2024) proposes an adaptation of the REDUNDANCY view, based on the novel notion of SUPER-REDUNDANCY. Roughly, a sentence is super-redundant if there is no way of strengthening one of its subconstituents that that would make the resulting sentence non-redundant.

(9) *Super-redundancy.* A sentence S is infelicitous if it contains a subconstituent C s.t. $(S)^-_C$ is defined and for all D, $(S)^-_C \equiv S_{Str(C,D)}$.

Roughly, $(S)^-_C$ in the above definition designates S where C got deleted, while $Str(C,D)$ refers to a strengthening of C with D, defined inductively and whose key property is that it commutes with negation: $Str(\neg\alpha, D) = \neg(Str(\alpha, D))$ – as well as with binary operators $Str(O(\alpha,\beta),D) = O(Str(\alpha,D), Str(\beta,D))$. $S_{Str(C,D)}$ designates S where C is replaced by $Str(C,D)$

This account elegantly solved the puzzle posed by Hurford Disjunctions (Hurford, 1974) and Conditionals (Mandelkern and Romoli, 2018), by predicting that, when deriving a logical expression from another, double negation introduction influences redundancy. However, this account fails to predict any contrast for (6a-6e), precisely because those sentences are isomorphic *without* any appeal to double negation introduction. More specifically, if we set $C = p$ for

$S \in \{(6a)...(6e)\}$, it can be shown that, assuming material implication, $S_{Str(C,D)}$ will always be $(p \wedge D) \vee (p \vee q)$, which turns out to be equivalent to $p \vee q \equiv \neg p \rightarrow q \equiv \neg q \rightarrow p \equiv (S)^{-}_{C}$. In other words, (6a-6e) are all super-redundant.

3. Capturing the contrast

Building on the model proposed for Hurford Sentences by Hénot-Mortier (2024), we assume that Logical Forms evoke accommodated QuDs in the form of parse trees of the Context Set (Qtrees); and that the Qtrees evoked by a complex LF are derived from the Qtrees evoked by the LF's constitutive parts in a compositional way. This compositional machinery is supplemented by Qtree-LF well-formedness constraints (RELEVANCE, REDUNDANCY), which rule-out certain derived Qtrees. As a result, specific LFs can end up with no accommodated Qtrees, and are therefore deemed odd.

For the present case study, we will need two key ingredients borrowed from Hénot-Mortier (2024): that disjunctions and conditionals give rise to distinct kinds of Qtrees, and that derived Qtrees-LFs pairs are subject to a REDUNDANCY constraint, stating that a Qtree evoked by a LF is suboptimal if if it evoked by a simplification of this LF. The interaction between these two ingredients predicts that the Qtrees evoked by (6a) are redundant given $p \vee q$, those evoked by (6b) are redundant given $p \vee q/\neg p \rightarrow q$, the one evoked by (6d) is redundant given p, and the one evoked by (6e) is redundant given $\perp$. In the following two sections, we sketched the account presented in more detail in Hénot-Mortier (2024).

3.1. Compositional QuDs

We model QuDs as parse trees of the Context Set (Stalnaker, 1974), which can also be seen as nested partitions. The definition of such trees ("Qtrees") is given in (10).

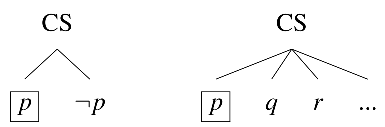
(10) *Structure of Question-trees (Qtrees)*
Qtrees are trees whose nodes are all subsets of the CS and s.t.:
- Their root denotes the CS;
- Any intermediate node is partitioned by the set of its children.

The nodes of such trees can be assigned the following interpretation. The root denotes a tautology over the CS, and any other node, a possible answer to the global question denoted by the tree. Intermediate nodes can generally be seen as non-maximal answers, while leaves can generally be seen as maximal answers. Any subtree rooted in a node N can be understood as conditional question taking for granted the proposition denoted by N. Finally, a path from the root to any node N can be seen as a strategy of inquiry (or a sequence of conditional questions) leading to the answer denoted by N.

Additionally, we assume that QuDs *evoked* by LFs receive some extra information from the LF, in the form of the maximal answers the LF provides to the QuD. More specifically, we assume that, when evoking a Qtree, a given LFs "flags" specific nodes on the Qtrees as maximal true answers. These nodes, that we dub verifying nodes, are typically the leaves of the Qtree which are subsets (i.e. entail) the proposition denoted by the LF. Those verifying nodes, just like Qtree structure, are compositionally derived.

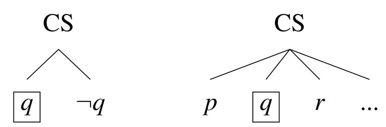
We assume that a simplex LF denoting a tautology is compatible with only one Qtree, namely the Qtree whose root and unique (verifying) leaf is the whole CS. Besides, we assume a contradiction evokes all possible well-formed Qtrees, with an empty set of verifying nodes. Now turning to the other simplex cases, we assume that a simplex LF denoting a proposition p can give rise to two types of Qtree:[4] a “polar-question” depth-1 Qtree whose leaves are the p and $\neg p$ worlds respectively; and a “*wh*-question” depth-1 Qtree whose leaves are p and relevant, mutually exclusive alternatives to p. Moreover, verifying nodes are defined on such trees as simply the p-leaf.

Looking back at (6a-6e), where S_p = *Ido is at SuB* denotes p and S_q = *Ido is in Cambridge* denotes q, it is reasonable to think S_p and S_q are exclusive mutual alternatives. Other similar alternatives may be S_r = *Ido is in Paris*, S_s = *Ido is in Chicago* etc. As a result, the Qtrees compatible with S_p and S_q are given in Figures 1 and 2. From those Figures it can be noticed that the “*wh*” Qtrees raised by S_p and S_q have similar structures (ignoring verifying nodes) – while the corresponding “polar” Qtrees do not.



(a) “Polar” (b) “*Wh*”

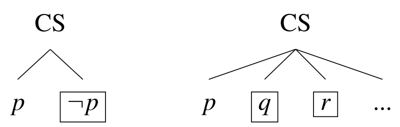
Figure 1: Qtrees for S_p = *Ido is at SuB*. $\boxed{\text{Boxed}}$ nodes are verifying.



(a) “Polar” (b) “*Wh*”

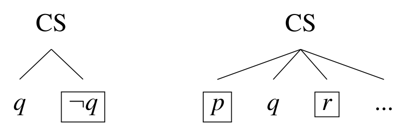
Figure 2: Qtrees for S_q = *Ido is in Cambridge*. $\boxed{\text{Boxed}}$ nodes are verifying.

We now proceed to define Qtrees raised by complex LFs, in particular, negated LFs, disjunctive LFs, and conditional LFs. The negative case is straightforward: simply change the verifying nodes to their same level non-verifying siblings/cousins. This is done for S_p and S_q in Figures 3 and 4



(a) “Polar” (b) “*Wh*”

Figure 3: Qtrees for $\neg S_p$ = *Ido is not at SuB*. $\boxed{\text{Boxed}}$ nodes are verifying.



(a) “Polar” (b) “*Wh*”

Figure 4: Qtrees for $\neg S_q$ = *Ido is not in Cambridge*. $\boxed{\text{Boxed}}$ nodes are verifying.

Disjunctive and conditional Qtree involve a heavier machinery, whose complete definitions and predictions can be found in Hénot-Mortier (2024). Here it is enough to say that disjunction returns all the well-formed unions of Qtrees evoked by its individual disjuncts. The set of verifying nodes attached to two disjoined Qtrees, are also unioned. In other words, a disjunctive Qtree is a Qtree adressing the questions evoked by each disjunct *in parallel*, making both

[4] This is a simplification; Hénot-Mortier (2024) assumes that even simplex LFs can give rise to layered Qtrees, whose layers are ordered by some notion of granularity. But this assumption is not relevant here, because we assume p and q are same-granularity alternatives.

disjuncts at issue. It is a symmetric operation: the order of the disjuncts does not influence the output. The only possible Qtree for $S_p \vee S_q$ / $S_q \vee S_p$ is given in Figure 5. It is obtained from Qtrees 1b and 2b, which have similar structures. Other possible unions of Qtrees are shown in Figure 6 but appear ill-formed, because the leaves of such Qtrees do not properly partition the CS. Following a similar line of reasoning, one can use the Qtrees in Figures 1b (for p) and 5 (for $p \vee q$), to derive the only possible Qtree for (6a) = $p \vee (p \vee q)$. Because Qtrees 1b and 5 have same structure, and are s.t. the former Qtree's set of verifying nodes is a subset of the latter Qtree's set of verifying nodes, Qtree union simply returns Qtree 5 as output. So Qtree 5 is compatible with both $p \vee q$ and (6a).

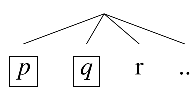
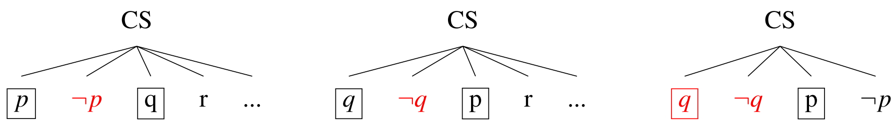


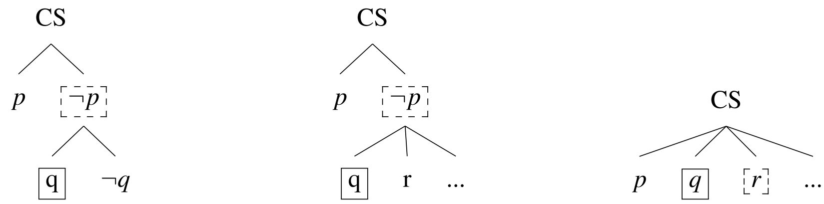
Figure 5: Only well-formed Qtree evoked by $S_p \vee S_q =$ *Ido is at SuB or in Cambridge*, obtained from 1b $\vee$ 2b. This Qtree is also the only Qtree compatible with (6a).



(a) 1a $\vee$ 2b (b) 1b $\vee$ 2a (c) 1a $\vee$ 2a

Figure 6: Ill-formed Qtrees resulting from the union of the Qtrees in Figures 1 and 2. Red nodes are nodes that should be removed for the leaves to form a proper partition of the CS.

Let us now turn to the conditional case. Following Hénot-Mortier (2024), we assume that the "inquisitive" contribution of $\rightarrow$ is not material, i.e. a conditional Qtree is not derived by disjoining the negations of its antecedent Qtrees, with its consequent Qtreess. Rather, we propose that conditional Qtrees are derived by "plugging" consequent Qtree into the verifying nodes of antecedent Qtrees – where "plugging" technically refers to Qtree-node intersection. The verifying nodes for the output conditional Qtrees are inherited from the consequent Qtree (meaning, verifying nodes from the antecedent Qtree are disregarded). The core idea behind this operation is that conditionals do not make antecedent and consequent QuDs at issue at the same time, rather, they introduce a hierarchy between these two objects, by raising the consequent QuD only in the cells of the CS (as defined by the antecedent QuD), where the antecedent holds. Yet another way to phrase this is by saying that, through the process of Qtree-conditionalization, the consequent Qtree gets *restricted* by the antecedent Qtree. This is done for $\neg S_p \rightarrow S_q$ in Figure 7, using Qtrees for $\neg S_p$ from Figure 3 and Qtrees for S_q from Figure 2.



(a) “Polar” antecedent (3a) and “polar” consequent (2a).

(b) “Polar” antecedent (3a) and “*wh*” consequent (2b).

(c) “*Wh*” antecedent (3b) and “polar”/“*wh*” consequent (2a)/(2b).[5]

Figure 7: Qtrees for $\neg S_p \rightarrow S_q =$ *If Ido is not at SuB then he is in Cambridge*. Nodes in dashed boxes refer to the nodes that were verifying in the input antecedent Qtree, but are no longer verifying in the output conditional Qtree. Nodes in $\boxed{\text{solid boxes}}$ refer to the nodes that were verifying in the input consequent Qtree, and are thus still verifying in the output conditional Qtree.

4. Exploring extensions and elaborations of the target sentences

5. Conclusion and outlook

(11) Double *or*-to-*if*

a. # If Ido is not at SuB then, if he is not at SuB then he is in Cambridge. $\neg p \rightarrow (\neg p \rightarrow q)$

b. # If Ido is not at SuB then, if he is not in Cambridge then he is at SuB. $\neg p \rightarrow (\neg q \rightarrow p)$

c. If it’s not that Ido is in Cambridge if not at SuB, then Ido is not at SuB. $\neg(\neg p \rightarrow q) \rightarrow p$

d. # If it’s not that Ido is at SuB if not in Cambridge, then Ido is at SuB. $\neg(\neg q \rightarrow p) \rightarrow p$

6. Parallel with Hurford Sentences

$p \vee q$ is like r and p like r^+. The only diff is that $p \vee q$, unlike r has internal structure.

ido had dessert or cheese and dessert if ido did not have dessert he had cheese and dessert if ido did not have cheese and dessert he had dessert

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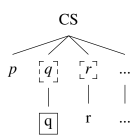
[5]Note that, technically, this Qtree is derived *via* the “plugging in” operation, plus a Qtree reduction step recursively collapsing single children with their mother and percolating the “verifying” property if needed. The Qtree obtained from (3b) and (2a)/(2b) *before* the reduction step is given in (i). Reduction collapses the two q-nodes and makes the resulting node verifying; collapses the two r-nodes and makes the resulting node non-verifying; and so on for all other nodes different from the p-node.

(i)



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