Contents

Abstract

At a broad level, this dissertation's main claim is that many cases of pragmatic oddness, do not stem from assertions alone, but rather from their interaction with the questions they implicitely evoke. Felicitous assertions, must evoke felicitous questions. To operationalize this claim, a model of compositionally derived implicit question is devised, along with conditions of their well-formedness, drawing from familiar concepts in pragmatics, such as REDUNDANCY and RELEVANCE. This model assigns a central role to the degree of specificity, or granularity, conveyed by assertions.

At a more narrow level, this dissertation argues that disjunctions and conditionals fundamentally differ in terms of the questions they evoke, and that this difference has direct consequences on the oddness/felicity profiles of sentences involving these operators. Disjunctions are shown to be prone to REDUNDANCY issues, while conditionals are shown to be prone to RELEVANCE issues. In other words, disjunctions and conditionals typically display distinct flavors of oddness. This is supported by three main classes of sentences. First, sentences that can be seen as equivalent, but which combine conditionals and disjunctions in distinct ways, display varying felicity profiles. Second, "pure" disjunctions and conditionals that can be seen as isomorphic, if not equivalent, display varying felicity profiles. Third, some differences between these disjunctions and conditionals remain when additional pragmatic phenomena, in particular scalar implicatures, are at play, and such differences shift in a way predicted by our approach.

This dissertation therefore justifies the appeal to a more elaborate model of (implicit) questions, which, when fed to the pragmatic module, is characterized by a better empirical accuracy on challenging data, than previous model solely based on assertive content.

Chapter 1

Assertions and Questions

Assertions and questions can be seen as the two sides of the same coin, as they form the two core building blocks of any given conversation. Question typically request information, while assertions typically provide information. (1a) for instance, is a question that requests information about the country where Jo grew up (presupposing there is one such country). (1b) can be seen as a good (assertive) answer to this question, providing the piece of information that Jo grew up in France. Semanticists have observed that the pairs formed by questions and answers are restricted: some are obviously good, while some others are (sometimes surprisingly) odd. So, questions and answers have to be somewhat *congruent*. For instance, (1c) cannot be seen as a suitable answer to (1a), even if it seems to indicate something about Jo's nationality.

(1) a. In which country did Jo grow up?

b. –Jo grew up in France.

c. # –Jo speaks French natively.

This Chapter motivates and lays the foundation of the main contribution of this dissertation: a constrained machinery "retro-engineering" questions out of assertions, allowing to capture intricate patterns in the domain of pragmatic oddness, that were not previously seen as an issue of question-answer congruence. This Chapter is organized as follows. Section 1.1 provides a broad overview of the semantics of assertions, and discusses to what extent they can meaningfully contribute to a conversation. Section 1.2 turns to the semantics and pragmatics of questions and highlights how questions relate to alternative assertions, and their possible answers. Section 1.3 bridges Sections 1.1 and 1.2, by discussing how questions further constrain which assertions should matter in a given conversation. It also points out a few cases in which question-answer is (seemingly) unhelpful. Section 1.5 constitutes a more technical appendix sketching how the semantics of questions is standardly derived. This whole Chapter heavily builds on the section of **?** dedicated to Questions.

1.1 Assertions provide information in the form of propositions

1.1.1 Extension and intension of assertions

When studying the semantics of natural language expressions, one usually starts with assertions, because they appear intuitively simpler. We will use the simple assertion in (1b), as a running example. At the most basic level, assertions are truth-conditional, i.e. their meaning corresponds to the set of conditions under which they hold. For instance, *Jo grew up in France* will be true if and only if whoever *Jo* is, grew up in whatever geographical entity *France* is. The *extension* of an assertion is therefore of type $\mathtt{t}$, the type of truth-values.

Additionally, the truth-conditions of a sentence are parametrized by (at least) a world variable.[1]. So, *Jo grew up in France* will be true as evaluated against a world w_0 if and only if whoever *Jo* is in w_0, grew up in w_0 in whatever geographical entity *France* is in w_0. One can then abstract over this world-parameter, and define the *intension* of an assertion as a function from worlds to truth-values. Such functions are called *propositions*, and have type $\langle \mathtt{s}, \mathtt{t} \rangle$, where $\mathtt{s}$ is the type of world-variables. So, the intension, or propositional content of *Jo grew up in France*, will be a function mapping any world variable w, to true if and only if, whoever *Jo* is in w, grew up in w in whatever geographical entity *France* is in w. This is formalized (with some simplifications) in (2).

(2) $$[\![\text{ Jo grew up in France }]\!] = \lambda w.\ \text{Jo grew up in France in } w \quad : \langle \mathtt{s}, \mathtt{t} \rangle$$

Propositions can receive an alternative, equivalent interpretation in terms of sets, based on the idea that any function with domain D and range R is just a (potentially infinite) set of pairs of elements in $D \times R$. A proposition is then simply the set of worlds in which it holds. This interpretation of propositions will be heavily used throughout the dissertation, and is outlined in (3).

(3) $$\begin{aligned}[\![\text{ Jo grew up in France }]\!] &= \lambda w.\ \text{Jo grew up in France in } w \\ &\simeq \{ w \mid \text{ Jo grew up in France in } w \ \}\end{aligned}$$

[1] Other parameters can also be relevant, like times, and assignments. But we choose to keep things simple here.

1.1.2 Assertions in conversation

Propositions either denote functions of type $\langle \mathtt{s}, \mathtt{t} \rangle$, or subsets of the set of elements of type $\mathtt{s}$. Should all elements of type $\mathtt{s}$ be considered when evaluating such functions, or computing such subsets? It is commonly assumed that the worlds under consideration at any point of a conversation, are the ones that are compatible with the premises of the said conversation (**??**). For instance, if two people have a discussion about *France*, it is often reasonable to assume that they agree on what geographical area *France* encompasses, and more generally about the topology of Earth. Moreover, they agree that they agree on this; and agree that they agree that they agree on this; etc. Propositions subject to this recursive, mutual, tacit agreement pattern, form what is called a Common Ground (henceforth **CG**, (**?**)). Each conversation has its own CG, as defined in (4). The set of worlds in which all the propositions of the CG hold, is called the Context Set (henceforth **CS**). The CS associated with a conversation is therefore a subset of the set of all possible worlds; and can also be seen (under the set interpretation of propositions) as the grand intersection of the propositions in the CG. This is defined in (5).

(4) **Common Ground (CG)**. Let $\mathcal{C}$ be a conversation between participants $\{P_1, ..., P_k\}$. Let $K(x, p)$ is a proposition meaning that individual x knows p, and p is a proposition. The Common Ground of $\mathcal{C}$ is the set of propositions that are recursively taken for granted by all the participants in $\mathcal{C}$:
$p \in CG(\mathcal{C}) \iff \forall n \in \mathbb{N}^*. \forall \{k_1, ...k_n\} \in [1; k]^n. K(P_{k_1}, K(P_{k_2}, ...K(P_{k_n}, p)...)$

(5) **Context Set (CS)**. Let $\mathcal{C}$ be a conversation between participants $\{P_1, ..., P_k\}$. Let CG $CG(\mathcal{C})$ be the Common Ground of this conversation. Under a set interpretation of propositions, the resulting Context Set $CS(\mathcal{C})$ is the set of worlds verifying all propositions of the CG, i.e.:
$CS(\mathcal{C}) = \bigcap \{p \mid p \in CG(\mathcal{C})\}$.

The concepts of CG and CS help delineate which worlds to focus on when evaluating an assertion in context, and determining to what extent this assertion is informative. If uttering an assertion is akin to *adding* it to the CG, then, it also amounts to *intersecting* this assertion with the CS.

(6) *Updating the Common Ground.* Let $\mathcal{C}$ be a conversation, and $CG(\mathcal{C})$ its Common Ground. If a sentence S denoting p is uttered, then p is added to $CG(\mathcal{C})$ to form a new Common Ground $CG'(\mathcal{C})$:
$CG'(\mathcal{C}) = CG(\mathcal{C}) \cup \{p\}$

(7) *Updating the Context Set.* Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S denoting p is uttered, then a new Context Set $CS'(\mathcal{C})$ is derived by intersecting $CS(\mathcal{C})$ with p:
$CS'(\mathcal{C}) = CS(\mathcal{C}) \cap p$

(8) *Link between the two updates.* (7) can be derived from (6) and the definition of the CG in (5):

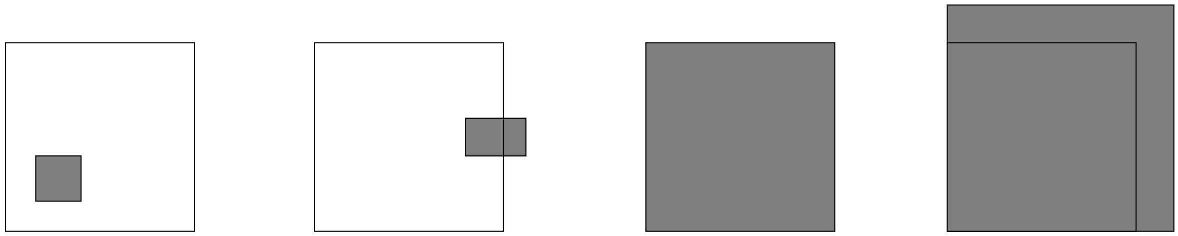
$$\begin{aligned} CS'(\mathcal{C}) &= \bigcap\{q \mid q \in CG'(\mathcal{C})\} \\ &= \bigcap\{q \mid q \in CG(\mathcal{C}) \cup \{p\}\} \\ &= \bigcap\{q \mid q \in CG(\mathcal{C})\} \cap p \\ &= CS(\mathcal{C}) \cap p \end{aligned}$$

Note that updating the CG will always create a bigger set, because the CG is simply a collection of propositions. For instance, if *Jo grew up in Paris* is already in the CG, then, adding the proposition denoted by *Jo grew up in France* to the CG will mechanically expand it. Updating the CS however, does not always lead to a different, smaller CS. For instance, taking for granted that Paris is in France (i.e., all the *Paris*-worlds of the CS are *France*-worlds), and assuming that *Jo grew up in Paris* is already common ground, intersecting the CS with the proposition that *Jo lives in France* will not have any effect. This seems to capture the idea that a proposition like *Jo lives in France* is *uninformative* once it is already known by all participants that *Jo lives in Paris*.

More generally, if it is Common Ground that p, and a sentence S denoting p^- s.t. $p \vDash p^-$ is uttered, then S will feel uninformative. An informative assertion should lead to a non-vacuous update of the CS, i.e. it should properly *shrink* the CS. This is spelled out in (9).

(9) **Informativity** (propositional view). A sentence S denoting a proposition p is informative in a conversation $\mathcal{C}$, iff $CS(\mathcal{C}) \cap p \subset CS(\mathcal{C})$.

In that framework, an assertion provides information in the sense that it reduces the set of live possibilities, and allows to better guess which world is the "real" one. Figure A illustrates how an asserted proposition can be informative or uninformative, depending on its set-theoretic relationship to the CS.



(1) Informative update (p strictly contained in the CS).

(2) Informative update (p strictly overlaps with the CS).

(3) Uninformative update (p equal to the CS).

(4) Uninformative update (p containing the CS).

Figure A: A few examples of informative and uninformative updates of the CS. The big squares represent the CS. The grey shapes refer to p, the proposition added to the CG (and intersected with the CS to update it).

1.1.3 Dynamic Semantics

So far, we have mainly considered "simplex" assertions that did not make use of operators, connectives or quantifiers. But what about sentences like those in (10)? How should they interact with the Context Set?

(10) a. Jo did not grow up in France.
b. Jo grew up in France or Belgium.
c. Jo grew up in France and Ed in Belgium.

The simplest way to deal with these sentences, would be to compute their intension (the proposition they denote) based on the semantics of negation, disjunction, and conjunction, and then, intersect the resulting proposition with the Context Set. We will call this approach the naive "bulk" CS update. There is evidence, coming from the behavior of presuppositions, that this might not be the way to go, and that complex assertions should be added to the Context Set "bit by bit" (**???**).

To see this, let us consider the pair in (11). The sentences in (11) are conjunctive and only vary in the order of their conjuncts. Additionally, one of their conjuncts contains the presupposition trigger *too*, associated with the predicate *grew up in France*. In the felicitous variant (11a), *too* occurs in the second conjunct; in the the infelicitous variant (11a), *too* occurs in the first conjunct. Intuitively, *X too VP* imposes that whatever predicate *VP* denotes be true of at least one individual different from the one *X* denotes. This presupposition can be seen as a precondition on the Context Set (as defined prior to the update step). In the case of (11a) and (11a), *Ed too grew up in France* then imposes that the Context Set at the time of the update entail that somebody other than Ed (e.g., Jo) grew up in France.

(11) a. Jo grew up in France, and Ed too grew up in France.

b. # Ed too grew up in France, and Jo grew up in France.

Let us attempt a naive "bulk" CS update with sentences (11a)/(11b). The first step is to compute (11a)/(11b)'s presuppositions and (propositional) assertions. The CS, as defined prior to the utterance of (11a)/(11b), then gets updated, provided that it verifies (11a)/(11b)'s presupposition. Let us start with (11a) and (11b)'s presuppositional component. We can assume that the presupposition that somebody other than Ed grew up in France projects from inside the conjunctive operator. Under this assumption, both (11a) and (11b) end up imposing that the CS prior to their utterance entail that somebody other than Ed grew up in France. This will in principle *not* be verified. So, the naive "bulk" Context Set update correctly predicts the infelicity of (11b), but, also, incorrectly predicts (11a) to be odd. Assuming the presupposition does not project does not address the issue. Under this assumption, both (11a) and (11b) end up being presuppositionless, and the naive "bulk" CS update correctly predicts (11a)'s felicity, but also incorrectly predicts (11b) to be just as felicitous. So, regardless of how presupposition should exactly behave in complex sentences, the asymmetry between (11a) and (11b) does not seem to be captured by the naive "bulk" CS update.

The linear asymmetry in (11) in fact suggests an alternative, "bit by bit" update strategy for complex sentences like conjunctions. If each conjunct were to update the CS one at a time, following the linear order of the sentence, then, the first conjunct of (11a) would create an updated CS that would incorporate the information that *Jo grew up in France*, and as such verify the presupposition of (11a)'s second conjunct (that somebody other than Ed grew up in France). This would allow (11a)'s second conjunct to be subsequently intersected to the CS, and would predict the whole conjunction in (11a) to be felicitous. By contrast, (11b)'s first conjunct would still be problematic in this framework, because its presupposition would not be satisfied by the original CS.

In this toy example, a presupposition was used as a diagnostic to better determine the nature of the CS update triggered by a conjunctive sentence. The conclusion is that the update should be dynamic: the two conjuncts should be intersected with the CS one by one, in the order in which they appear. This should apply to presuppositionless sentences as well; and is summarized in (12).

(12) *Conjunctive update of the CS.* Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S of the form $X \wedge Y$, with $[\![X]\!] = p$ and $[\![Y]\!] = q$ is uttered, then a new Context Set $CS''(\mathcal{C})$ is derived by, first intersecting $CS(\mathcal{C})$ with p to create $CS'(\mathcal{C})$, and second, intersecting $CS'(\mathcal{C})$ with q to create $CS''(\mathcal{C})$:
$CS''(\mathcal{C}) = (CS(\mathcal{C}) \cap p) \cap q = CS'(\mathcal{C}) \cap q$

The potential presuppositions of X and Y are tested on the CS at the time of their respective update, i.e. on $CS(\mathcal{C})$ and $CS'(\mathcal{C})$ respectively.

Dynamic Semantics is a framework that proposes to extend this view to other kinds of complex sentences, e.g. disjunctive and conditional sentences. In Dynamic Semantics, sentences give rise to different kinds of CS updates, depending on how they are constructed. More fundamentally, Dynamic Semantics proposes a shift of perspective when it comes to the meaning of assertions: assertions no longer denote propositions, instead they denote proposals to update the CS in specific ways. In that sense, assertions can be seen as functions from an input CS, to an output CS–sometimes called Context-Change Potentials (**CCP**). CCPs for disjunctive and conditional sentences are spelled out in (13) and (14) respectively.

(13) *Disjunctive update of the* CS. Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S of the form $X \vee Y$, with $[\![X]\!] = p$ and $[\![Y]\!] = q$ is uttered, then a new Context Set $CS'(\mathcal{C})$ is derived by intersecting $CS(\mathcal{C})$ with $p \cup q$:
$CS'(\mathcal{C}) = CS(\mathcal{C}) \cap (p \cup q)$
The potential presuppositions of X and Y are tested on, respectively, $CS(\mathcal{C})$ and $CS(\mathcal{C}) \cap \neg p$.[2]

(14) *Conditional update of the* CS. Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S of the form *if* X *then* Y, with $[\![X]\!] = p$ and $[\![Y]\!] = q$ is uttered, then a new Context Set $CS''(\mathcal{C})$ is derived by, first intersecting $CS(\mathcal{C})$ with p to create $CS'(\mathcal{C})$, and second, intersecting $CS'(\mathcal{C})$ with q to create $CS''(\mathcal{C})$:
$CS''(\mathcal{C}) = (CS(\mathcal{C}) \cap p) \cap q = CS'(\mathcal{C}) \cap q$
The potential presuppositions of X and Y are tested on the CS at the time of their respective update, i.e. on $CS(\mathcal{C})$ and $CS'(\mathcal{C})$ respectively.

This incremental view of assertions leads to a revised, incremental definition of informativity, given in (15).

(15) **Informativity** (CCP view). A sentence S is informative in a conversation $\mathcal{C}$, iff all the updates of $CS(\mathcal{C})$ it gives rise to are non-vacuous.

In sum, assertions can be seen as proposals to update (shrink) the CS. The specific update they give rise to is compositionally derived, and incrementally performed, following the structure of the sentence. We will use a similar approach in Chapter **??** when defining

[2] There is a debate on whether or not disjunctions should behave symmetrically w.r.t. the presupposition(s) carried by their disjuncts. An alternative, symmetric way to evaluate X and Y's potential presuppositions, would be to test them against $CS(\mathcal{C}) \cap \neg q$ and $CS(\mathcal{C}) \cap \neg p$ respectively.

questions *evoked* by assertions. But this first requires to define what questions mean. This is what we do in the next section, in which we show that questions influence, not the size, but rather, the topology of the CS.

1.2 Questions indicate which kind of information is worth providing

1.2.1 Questions as answerhood conditions

Participants in a conversation utter assertions to shrink the CS, and hopefully, jointly figure out which world they are in. But this allows for very unnatural interactions like (16), taking the forms of sequences of intuitively unrelated sentences–as long as each of them denotes propositions shrinking the CS!

(16) –Jo grew up in France.
–I like cheese.
–Al is arriving tomorrow.

This is where questions enter the game. Intuitively, a question indicates an interest in *which* proposition(s) hold, among a restricted set. The proposition at stake are typically possible answers to the question **??**. Questions therefore denote sets of sets of worlds (equivalent to a type $\langle\langle \mathtt{s}, \mathtt{t}\rangle, \mathtt{t}\rangle$), and constrain which kind of (informative) propositions can be uttered as a follow-up. For instance, a polar question such as *Is it raining?* will typically request information of the form *It is raining*, or *It is not raining*, see (17).

(17) –Is it raining?
–Yes, it is raining. / No, it is not raining.

The question *Is it raining?* can thus be represented as a set made of two propositions, namely, the proposition that *it is raining*, and the proposition that *it is not raining*.

(18)
$$\begin{aligned} [\![\text{ Is it raining? }]\!] &= \{\, [\![\text{ It is raining }]\!], [\![\text{ It is not raining }]\!] \,\} \\ &= \{\lambda w.\ \text{it is raining in } w,\ \lambda w.\ \text{it is not raining in } w\} \\ &: \langle\langle \mathtt{s}, \mathtt{t}\rangle, \mathtt{t}\rangle \end{aligned}$$

In the case of the question *is it raining?*, the set of possible answers is fairly simple: it only contains two elements. These two elements cover the space of all possibilities,[3] and

[3] This is the case assuming there is no vagueness-induced "grey area", i.e. any salient situation is either a *raining*-situation, or a *not raining*-situation

are *exclusive*: if it's the case that it's raining (at a salient place, at a salient time) in w, then, it's not the case that it is not raining (at the same place, at the same time), in w. We will see in the next section that this configuration amounts to a partition of the CS. A definition of exclusivity under the set interpretation of propositions is given in (19).

(19) *Exclusive propositions.* $p : \langle \texttt{s}, \texttt{t} \rangle$ and $q : \langle \texttt{s}, \texttt{t} \rangle$ are exclusive if $p \cap q = \emptyset$.

But questions may not always intuitively request information about exclusive propositions. For instance, a *wh*-question like *Which students passed the class?* expects answers that convey a subset of students who passed the class, see (20). But there are many possible, overlapping subsets of students, so, the corresponding propositions will be overlapping as well. For instance, the proposition that *Jo passed the class*, denotes the set of worlds in which Jo passed the class, and this set happens to contain the set of worlds where both Jo and Al passed the class. It also overlaps with the set of worlds in which Al passed the class.

(20) Which students passed the class?
–Jo did.
–Al did.
–Jo and Al did.

We will call propositions like *Jo passed the class*, and *Jo and Al passed the class*, alternatives associated to the question *Which students passed the class?* Alternatives may be overlapping; and, as we will see, can be obtained from the original question by substituting its *wh*-component (e.g., *which students*), with relevant, same-type material (e.g., students or groups of students).[4]

(21) Question : $[\![$ Which students passed the class? $]\!]$
Alternatives: $\{[\![$ Jo passed $]\!]$, $[\![$ Al passed $]\!]$, $[\![$ Jo and Al passed $]\!]$ ... $\}$

Why would this overlap between alternative answers be an issue in modeling the meaning of questions? The fact that entailing or merely overlapping propositions should be considered equally good answers does not capture the idea that more specific propositions constitute more exhaustive answers than less specific ones. For instance, answering that *Jo passed*, in theory leaves the fate of the other students undecided–for instance, it does

[4]It is worth mentioning that the set $\{\lambda w.$ it is raining in $w,\ \lambda w.$ it is not raining in $w\}$ does not strictly speaking correspond to the set of alternatives raised by *Is it raining?* Section 1.5 further specifies how alternatives get compositionally derived, and predicts that *Is it raining?* should only give rise to one alternative: $\lambda w.$ it is raining in w. The set $\{\lambda w.$ it is raining in $w,\ \lambda w.$ it is not raining in $w\}$ is derived from this singleton alternative *via* the "pragmatic" process presented in (24), in the next Section.

not settle if *Al passed*, or not. Answering that *Jo and Al passed* by contrast, settles Al's fate, in addition to Jo's. Ideally, an answer to *Which students passed?* should explicitly address whether *each* student of the class passed, or not. That would be an exhaustive answer.

1.2.2 Questions as partitions of the Context Set

We have just discussed that, at the semantic level, questions characterize the conditions under which they are answered, i.e. denote a set of potentially overlapping propositions. But, just like we did with assertions, the effect of this semantics on the Context Set has to be defined. There is in fact a deterministic way to change a set of overlapping propositions P (i.e. a set of subsets of the CS), into a set of exclusive subsets of the CS (called *cells*, for reasons made clear in (24)). To do so, one can group in the same cell the worlds of the Context Set that all "agree" on all propositions in P. This "agreement" property amounts to the same-cell relation in (22). This relation is reflexive, symmetric and transitive, i.e. is an equivalence relation (see proof in (23)). From this, we can conclude that the set of subsets of the CS induced by P, obtained by grouping worlds of the CS according to the same-cell relation, forms a partition of the Context Set (see proof in (24)).[5] So, on top of being exclusive, cells are non-empty and together cover the CS. We assume that the process changing the set of alternative propositions raised by a question, to a partition of the CS, belongs to pragmatics. So, questions *denote* sets of alternative propositions, and this set *pragmatically induces* a partition structure on the CS.

(22) *Same-cell relation* $\equiv_P$. Let P be a set of propositions, i.e. a set of subsets of the Context Set ($P \in \mathcal{P}(\mathcal{P}(CS))$, with $\mathcal{P}$ the powerset operation). Let w and w' be two worlds of the Context Set. $w \equiv_P w'$ iff, $\forall p \in P.\ p(w) = p(w')$.

(23) $\equiv_P$ is an equivalence relation, no matter what P is. Let $\forall P \in \mathcal{P}(\mathcal{P}(CS))$.

- $\equiv_P$ is reflexive: $\forall w \in CS.\ \forall p \in P.\ p(w) = p(w)$.
- $\equiv_P$ is symmetric. Let $\forall (w, w') \in CS^2$.
  $\forall p \in P.\ p(w) = p(w')$ iff $\forall p \in P.\ p(w') = p(w)$.
- $\equiv_P$ is transitive. Let $\forall (w, w', w'') \in CS^3$.
  We assume $\forall p \in P.\ p(w) = p(w')$ and $\forall p \in P.\ p(w') = p(w'')$.
  Let $\forall p \in P$. We have $p(w) = p(w')$ and $p(w') = p(w'')$, so $p(w') = p(w'')$.
  So, $\forall p \in P.\ p(w) = p(w'')$

[5] Cells as we defined them are also called equivalence classes. It's a general property that equivalence classes induced by an equivalence relation on a certain set on which this relation is defined, will create a partition of the set.

(24) *Partition of the CS induced by P.*[6] Let P be a set of propositions. The partition induced by P in the Context Set is the set of subsets of the CS (cells): $\mathfrak{P}_{P,CS} = \{\{w' \mid w' \in CS \wedge w' \equiv_P w\} \mid w \in CS\}$. This set partitions the CS.

- No cell c of $\mathfrak{P}_{P,CS}$ is empty. Let $c \in \mathfrak{P}_{P,CS}$. There is a $w \in CS$ s.t. $c = \{w' \mid w' \in CS \wedge w' \equiv_P w\}$. Then at least $w \in c$, because $w \equiv_P w$.
- Cells cover the CS. Let $w \in CS$. $\mathfrak{P}_{P,CS}$ contains a cell $c = \{w' \mid w' \in CS \wedge w' \equiv_P w\}$. Then $w \in c$ because $w \equiv_P w$.
- Cells are disjoint. Let $(c, c') \in \mathfrak{P}_{P,CS}$, s.t. $c \cap c' \neq \emptyset$. We show $c = c'$. c and c' have resp. the form $c = \{w'' \mid w'' \in CS \wedge w'' \equiv_P w\}$ and $c = \{w'' \mid w'' \in CS \wedge w'' \equiv_P w'\}$, for $(w, w') \in CS^2$. Let $w''' \in c \cap c'$. Then $w''' \equiv_P w$ and $w''' \equiv_P w'$, and so by symmetry and transitivity, $w \equiv_P w'$, and $c = c'$.

It is easy to show that, in the polar example (17), the subsets of the CS defined by *It is raining* and *It is not raining*, which we said were intuitive answers to the question, form a partition of the CS. Section 1.5 will in fact show that polar questions of the form p? *denote* the singleton set formed by p, and *induce* a 2-cell partition of the form $\{p, \neg p\}$.

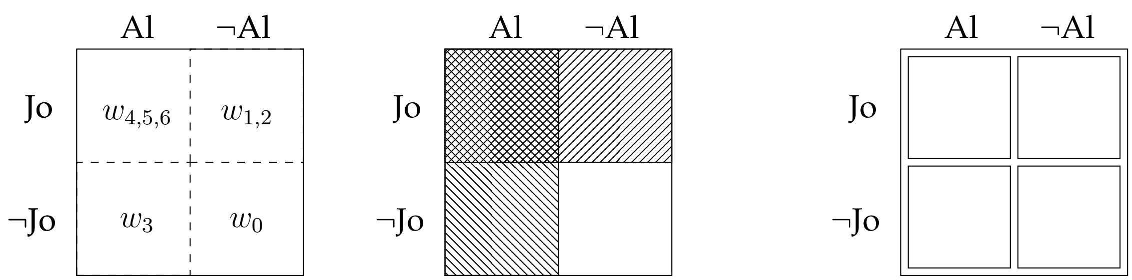
Let us now see how the above definitions apply to a *wh*-question like *Which students passed?* in (20). Let's assume there are only two salient students, Jo and Al. We assume that the alternatives the question raises (labeled P), are the proposition that *Jo passed*, and the proposition that *Al passed*. We assume that the CS contains six possible worlds, which vary according to whether Jo, Al, both, or none passed the class. The worlds may vary in other respects, that are not relevant to us here. The alternatives and cells associated with this question are given in (25). The alternative set P then corresponds to two subsets of the CS, which do not cover it. In particular, the world in which nobody passed (w_0) is included in none of the two alternatives. Moreover, the two subsets are overlapping: both *Jo passed* and *Al passed* contain w_4, w_5, and w_6. Now turning to the cells induced by P on the CS, we notice that there are four of them, which correspond to worlds where nobody, only Jo, only Al, or both Jo and Al passed the class. Such cells cover the CS, are disjoint, and non-empty, so correctly form a partition of the CS. They also fully specify, for *both* Jo and Al, if they passed the class; and as such constitute exhaustive answers to the original question.

(25) Question : *Which students passed the class?*
Context Set: $\{w_0, w_1, w_2, w_3, w_4, w_5, w_6\}$, s.t.:

[6] ***?** proposes an alternative way to derive a partition of the CS from a set of alternative propositions, leveraging the covert operator exh*

- Nobody passed in w_0;
- Only Jo passed in w_1 and w_2;
- Only Al passed in w_3;
- Both Jo and Al passed in w_4, w_5, and w_6.

Alternatives (P): $\{[\![\text{Jo passed}]\!], [\![\text{Al passed}]\!]\} = \{\{w_1, w_2, w_4, w_5, w_6\}, \{w_3, w_4, w_5, w_6\}\}$
Cells induced by $\equiv_P$: $\{\{w_0\}, \{w_1, w_2\}, \{w_3\}, \{w_4, w_5, w_6\}\}$



(1) Distribution of $w_0 ... w_6$ in the CS.

(2) Alternatives: defines *Al passed* and defines *Jo passed*.

(3) Partition induced by and . Cells correspond to the subsets of Figure B2 featuring the same overall pattern.

Figure B: Partitioning of the CS defined in (25) according to the alternatives *Jo passed* and *Al passed*. The CS is organized as follows: counter-clockwise, quadrant I is made of *Jo but not Al*-worlds; quadrant II, *Jo and Al*, quadrant III, *Al but not Jo*, and quadrant IV, *neither Jo nor Al*.

To summarize, at the pragmatic level questions are partitions of the Context Set, as formalized in (26).[7] The cells of such partitions constitute maximal answers to the questions. Unions of two or more cells constitute non-maximal answers, as defined in (27).

(26) *Standard semantics for questions* (**????**). Given a conversation $\mathcal{C}$ and a Context Set $CS(\mathcal{C})$, a question on $CS(\mathcal{C})$ is a partition of $CS(\mathcal{C})$, i.e. a set of subsets of $CS(\mathcal{C})$ ("cells") $\{c_1, ..., c_k\}$ s.t.:

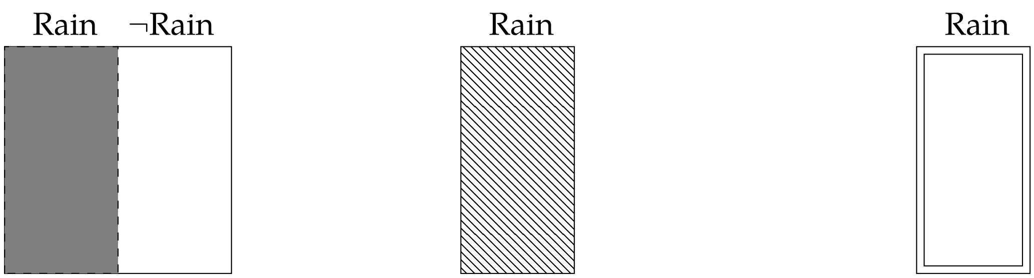
- "No empty cell": $\forall i \in [1;k].\ c_i \neq \emptyset$
- "Full cover": $\bigcup_{i \in [1;k]} c_i = CS(\mathcal{C})$
- "Pairwise disjointness": $\forall (i,j) \in [1;k]^2.\ i \neq j \Rightarrow c_i \cap c_j = \emptyset$

[7]It is important to note that questions may be taken to have a partition *semantics*. But we do not cover this here.

(27) *Maximal and non-maximal answers to a question.* Given a conversation $\mathcal{C}$, a Context Set $CS(\mathcal{C})$, and a question Q forming a partition $\{c_1, ..., c_k\}$ of $CS(\mathcal{C})$:

- Any $c \in \{c_1, ..., c_k\}$ constitutes a maximal answer to Q;
- Any c' s.t. $\exists C \subseteq \{c_1, ..., c_k\}.\ |C| > 1 \wedge c' = \bigcup C$ is a non-maximal answer to Q.

Just like we did with assertions, let us clarify further what it means to be a good question. We have established that the idea of a partition is a good candidate to model the effect of questions on a given CS. But what if the CS is already such that the partition induced by the question's alternatives is just made of one big cell? Such a configuration suggests that the question is already *settled*, meaning, the CS already makes one maximal answer trivial. For instance, if it is already common ground between the conversation's participants that *it is raining* (at the salient place and time) in (17), then, the question *Is it raining?* appears completely trivial. This is illustrated in Figure C and generalized in (28).



(1) Interecting the proposition that *It's raining* ( ) with a CS that is agnostic about the weather.

(2) Alternatives on the restricted CS: defines *It's raining*.

(3) Partition induced by on the CS. Cells correspond to the subsets of Figure C2 featuring the same overall pattern (1 such subset).

Figure C: Updating the CS with the proposition that *It's raining*, and then computing the partition induced by *Is it raining* on the resulting "shrunk" CS. The outcome is a single-cell partition, i.e., the question has a trivial pragmatics.

(28) **Trivial Question**. Let $\mathcal{C}$ be a conversation, $CS(\mathcal{C})$ its associated Context Set, and Q a question. Q is trivial given $CS(\mathcal{C})$ iff the partition induced by Q on $CS(\mathcal{C})$ is made of a singleton cell, i.e. has cardinal 1.

We now have a basic notion of what it mean to be a good assertion, given a CS, and a good question, given a CS. A good assertion has to be informative, i.e. properly shrink the CS (as per (9)/(15)). A good question has to induce a non-trivial, multiple-cell partition on the CS (as per (28)). But being a good question or a good assertion, does not *only* depend on the state of the CS! In particular, good assertions also have to be good answers to good questions. This principle, dubbed *Question-Answer Congruence*, is given in (29).

(29) **Question-Answer Congruence** (**?**). A felicitous assertion has to be a good answer to a good question.

The next Section presents what can be seen as a partial implementation of this principle, in the form of a general principle dubbed Relevance. It also points out the limitations of this principle.

1.3 Assertions as good answers to questions

1.3.1 Relevance mediates questions and assertions

Now that we precisified what assertions and questions are, it becomes possible to (at least partially) define what a good assertion should be, given a question. The principles we introduce in this Section are based on the general concept of Relevance. They will eventually rule out informative but "unnatural" sequences of assertions like (16), but also, more generally, a wide range of odd question-answer pairs.

Following much previous literature (**??????**), we call the question against which assertions are evaluated, *Question under Discussion* (henceforth **QuD**). QuDs are typically seen as partitions of the CS. In (27), we defined cells an unions of cells as respectively maximal and non-maximal answers to a question. Very broadly, Relevance constrains what a proposition should do to the cells of the QuD. Let us now unpack this with an example.

If for instance the QuD is about which country Jo grew up in (as in (30)), the CS will be partitioned according to propositions of the form *Jo grew up in c*, with *c* a country. Utterances such as (30a) or (30b), both seem relevant to that kind of QuD, and both constitute answers to the QuD–maximal, or not. By contrast, utterances such as (30c), (30d) or (30e), do not appear relevant, and do *not* constitute answers to the QuD: there are native and non-native French speakers in virtually all countries; same holds for wine-lovers and wine-haters; as for (30e) it seems completely independent from the subject matter.[8] These various configurations are sketched in Figure D.

(30) QuD: In which country did Jo grow up?

a. Jo grew up in France.

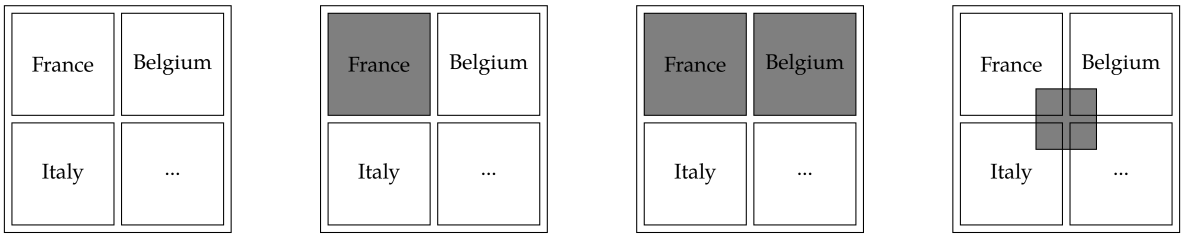
b. Jo grew up in France or Belgium.

[8] It is interesting to note that (30c) and (30d) can be more easily coerced into relevance than (30e). For instance with (30c), one might consider that France is the country which, in proportion, comprises the most native French speakers, and so (30c) may be understood as *It is likely that Jo grew up in France*–which constitutes a modalized answer to the QuD. This kind of reasoning is harder (if not impossible) to perform when facing an utterance like (30e).

c. ?? Jo speaks French natively.

d. ?? Jo enjoys wine.

e. # The cat went outside.



(1) QuD for *In which city did Jo grow up?* (2) Utterance: *Jo grew up in France.* (3) Utterance: *Jo grew up in France or Belgium.* (4) Utterance: (30c), (30d) or (30e).

Figure D: QuD-utterance configurations for a QuD like *In which country did Jo grow up*, and possible follow-up utterance.

From this, we can conclude that a proposition is "relevant" to a question, if it constitutes a maximal or a non-maximal answer to the question. This is similar in spirit to the notion of *Aboutness* developed **?**, according to which a proposition p is about a subject matter (in modern terms, a QuD), if and only if the truth value of that proposition supervenes on that subject matter (i.e. p should not introduce truth-conditional distinctions between cellmates, i.e. p does not "cut across" cells). This is rephrased in (31).

(31) **?'s Relevance** (rephrased in the QuD framework). Let $\mathcal{C}$ be a conversation, Q a QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is **?**-Relevant to Q, iff $\exists C \subseteq Q.\ p \cap CS(\mathcal{C}) = C$

A typical **?**-Relevant configuration is exemplified in Figure F1. Note however two edge cases. The first, is that of a proposition whose intersection with the CS is empty (a contextual contradiction). This kind of proposition verifies (31), because the empty set is a subset of any set, including the set of propositions defined by the QuD–whatever it is. Figure F2 exemplifies this kind of configuration. The second edge case, is that of a proposition whose intersection with the CS is the entire CS (a contextual tautology, uninformative as per (9)). This kind of proposition also verifies (31), because the entire CS corresponds to the unions of all cells of any given QuD defined on that CS. Figure F3 exemplifies this kind of configuration.

But, coming back to the QuD *In which country did Jo grow up?*, what about an utterance of the form *Jo grew up in Paris?* Although overinformative (the QuD was only asking about countries, not cities!), this utterance appears relevant, because it allows to infer that Jo grew up in France, and not, say, Belgium. This kind of configuration is sketched in Figure E.

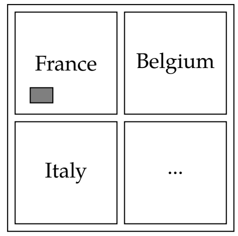
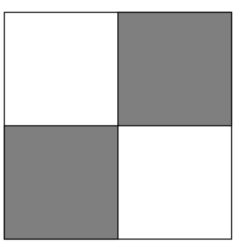


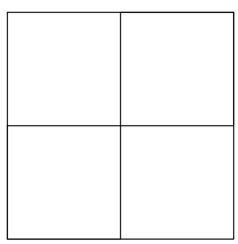
Figure E: QuD-utterance configuration for a QuD like *In which country did Jo grow up?* and an utterance like *Jo grew up in Paris*.

The view of relevance, developed by **?**, captures this intuition, by stating that a relevant proposition has to rule out at least one maximal answer conveyed by the QuD. In other words, a relevant proposition has to be incompatible with at least one cell of the QuD. This is summarized in (32). This definition makes uninformative propositions irrelevant (see Figure F3), but allows certain propositions that do not coincide with the grand union of a subset of the QuD's cells, to be relevant (see Figures F4 and F5). In other words, relevant propositions in the sense of **?** may introduce truth-conditional distinctions between cellmates–as long as they rule out a cell. A particular case is that of propositions like *Jo grew up in Paris*, when the QuD is about countries, which strictly entail a specific cell of the QuD, i.e. are strictly contained in one single cell (see Figure F4).

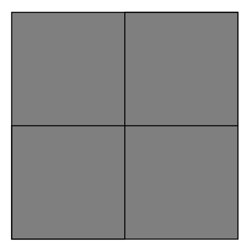
(32) **?'s Relevance** (**?**). Let $\mathcal{C}$ be a conversation, Q a (non-trivial) QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is **?**-Relevant to Q, if $\exists c \in Q.\, p \cap c = \emptyset$.



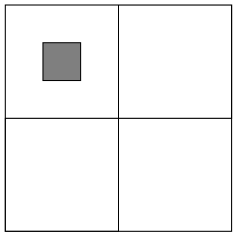
(1) Informative
?-Relevant
?-Relevant.



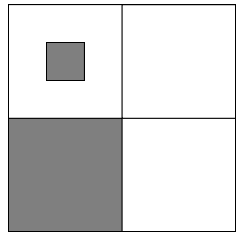
(2) Informative
?-Relevant
?-Relevant.



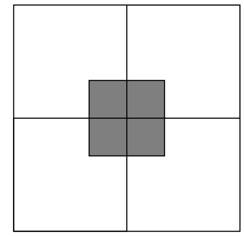
(3) Uninformative
?-Relevant
not **?**-Relevant.



(4) Informative
not ?-Relevant
?-Relevant.



(5) Informative
not ?-Relevant
?-Relevant.



(6) Informative
not ?-Relevant
not ?-Relevant.

In sum, the concept of Relevance (whether it follows ?'s or ?'s implementation) allows to rule-out a wide range of QuD-utterance pairs, by stating that propositions should properly relate to an existing question. We will not discuss which approach between ?'s and ?'s is best here, and will propose an incremental variant of this core concept in Chapter **??**, to deal with certain complex, out-of-the blue sentences. The next two section outline a few limitations of relevance.

1.3.2 A few conceptual shortcomings of Relevance

Regardless on which view of Relevance is adopted, relevant propositions can be added to the Common Ground, and as such, trigger an update of the CS. This, in turn, updates the QuD, which must remain a partition of the CS. It is easy to show, given how partition are "induced" on a set (see definition (24)), that the updated QuD on the smaller CS corresponds to the previous QuD, whose cells are pointwise intersected with the newly added proposition, and such that empty cells are filtered. This is formalized in (33).

(33) *Updating the partitioned Context Set.* Let $\mathcal{C}$ be a conversation, $CS(\mathcal{C})$ its Context Set, and let Q be a partition of $CS(\mathcal{C})$. If a sentence S denoting p is uttered and relevant given Q (as per (31) or (32)), then a new Context Set $CS'(\mathcal{C})$ is derived by intersecting $CS(\mathcal{C})$ with p, and this new context set is partitioned by Q', s.t.:
$Q' = \{c' \mid \exists c \in Q.\ c' = c \cap p \wedge c' \neq \emptyset\}$

There are two shortcomings to the current framework. First, adding a proposition to the CG "mechanically" leads to an update of the CS and of the QuD, but does not directly affect the *structure* of this QuD: even if some cells should shrink, the *limits* of each cell remain the same. This goes against the intuition that sometimes, sentences give rise to brand new QuDs, as exemplified by the exchange in (34).

(34) –Is it raining?
–Yes, I think so. I just so Ed come in with this very pretty umbrella.

(Likely follow-up: Where did Ed find this umbrella?)

Second, and relatedly, one can wonder what is supposed to happen in the case of out-of-the-blue sentences, i.e. sentences for which there is no explicit QuD. In such cases, it is generally assumed that a reasonable QuD is somehow inferred. But, given the fact that a QuD is merely a partition of the current CS, there exists many options. This dissertation will focus on how exactly QuDs are inferred, what additional constraints hold between an assertion and a QuD, and what the consequences are for pragmatic theory.

1.4 Conclusion and roadmap of the dissertation

In this Chapter, we have introduced the dominant view of the semantics of questions and assertions, and of their interplay. In particular, we have seen that assertions should better be informative and relevant to the QuD raised by the conversation. In the rest of this dissertation, we will show that the interplay between questions and assertions may have implications beyond Relevance, and as such explain more cases of oddness that previously assumed. Specifically, we will claim that instead of being a "good" answer to *some* QuD, an out-of-the-blue sentence must be a good answer to a *good* QuD, following insights by **?**.

Chapter 2 will continue the discussion on the pragmatics of questions, and argue that "good" implicit QuDs are determined from the shape of the assertive sentence itself. This is pushing the idea that assertions evoke alternatives one step further, in the sense that sentences will be taken to evoke questions (themselves derived from alternatives). These implicit questions will have a structure that consists in a generalization of the partition structure, namely, they will take the form of nested partitions of the CS.

In Chapters **??**, we will claim that the process deriving questions from assertions as defined in Chapter 2, is subject to constraints that go beyond Relevance, in particular Redundancy. A new concept of Redundancy will be used to explain when, and how, structurally and logically similar sentences involving disjunctions and conditionals, display distinct oddness/felicity profiles. More broadly, the introduction of constraints on QuD derivation will make way for a "lifted" view of pragmatic oddness, under which an assertion is not odd *per se*, but rather, is odd due to its interaction with the QuDs it evokes.

Chapter **??** will further generalize the view of Redundancy introduced in **??**, in order to cover a wider variety of disjunctive sentences related to Hurford Disjunctions (**?**).

Chapter 3 will turn to conditional variants of Hurford Disjunctions (**?**), and introduce a second constraint on QuD derivation, drawing from **?**'s and **?**'s Relevance.

Lastly, Chapter **??** will explore Hurford Disjunctions involving logically entailing scalar items, showing experimental evidence supporting the existence of a pragmatic contrast between the two possible orderings of these disjunctions. The contrast will then be explained by appealing to (incrementally) derived implicit questions, and independently motivated principles constraining question answering.

1.5 Appendix: computing questions from propositions

So far, we have described what could be a reasonable model for questions, in the form of partitions of the CS. But this was done without explaining how exactly such partitions are derived from the Logical Form of questions. This sketches how this is done, while further clarifying the distinction between propositions, alternatives, and questions. We will show that questions are standardly derived from closely related propositions, by abstracting over specific variables.

We will use the question *In which country did Jo grow up?* as an example. The LF associated with this question is given in Figure G.

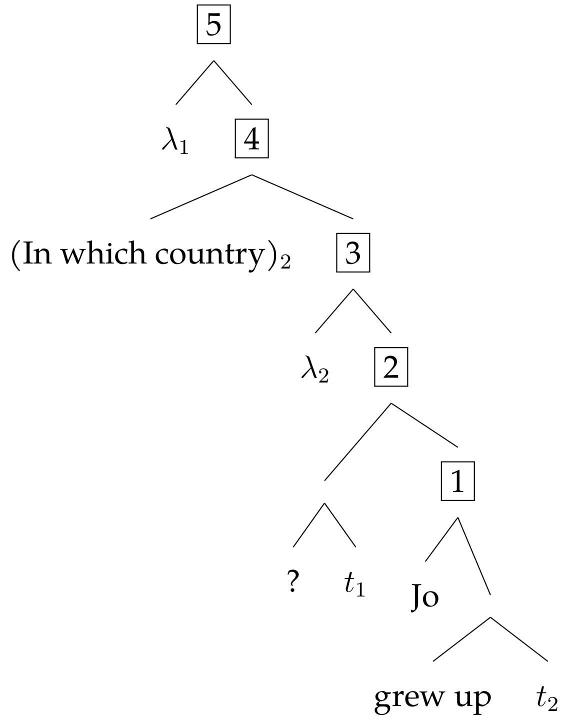


Figure G: LF of the question *In which country did Jo grow up?*

This question involves a *wh*-phrase (*in which country*), which syntactically originates in an adjunct of *grow up*. It is assumed that the *wh*-phrase leaves a trace t_2 in this position. The semantics assigned to the *wh*-phrase is existential, and akin to *some country*. Specifically, *in which country* takes a predicate of type $\langle \mathtt{e}, \mathtt{t} \rangle$ as argument, and returns the quantified statement that *some country* verifies the predicate.

(35) $[\![\text{In which country}]\!]^w = \lambda P.\ \exists l.\ l \text{ is a country in } w \wedge P(l) = 1$

The *wh*-phrase outscopes another "proto-question" operator (**?**). This operator takes two propositions (here, the trace t_1 and the proposition that *Jo grew up in* t_2), and simply equates them.

(36) $[\![?]\!]^w = \lambda p.\ \lambda q.\ p = q$

Applying this operator successively to t_1 and the intension of $\boxed{1}$, yields the following.

(37) $\boxed{1} = [\![\text{Jo grew up } t_2]\!]^w = 1 \text{ iff Jo grew up in } t_2 \text{ in } w$

(38) $\boxed{2} = [\![?\ t_1 \text{ Jo grew up } t_2]\!]^w = 1 \text{ iff } t_1 = \lambda w'.\ \text{Jo grew up in } t_2 \text{ in } w'$

Abstraction then applies to $\boxed{2}$, binds t_2 and yields a predicate that can then serve as an argument of the *wh*-phrase. The *wh*-phrase then turns this predicate into an existentially quantified expression targeting the element being questioned (here, a country).

(39) $\boxed{3} = [\![\lambda_2 \ ? \ t_1 \text{ Jo grew up } t_2]\!]^w = \lambda l.\ t_1 = \lambda w'.$ Jo grew up in l in w'

(40) $\boxed{4} = [\![$ In which country ... Jo grew up $t_2]\!]^w$
$= \exists l.\ l$ is a country in $w \wedge t_1 = \lambda w'.$ Jo grew up in l in w'

Lastly, a t_1 gets bound to produce a set of propositions, namely, the set of propositions that coincide with the proposition that *Jo grew up in l*, for some country l.

(41) $\boxed{5} = [\![\lambda_1$ In which country ... Jo grew up $t_2]\!]^w$
$= \lambda p.\ \exists l.\ l$ is a country in $w \wedge p = \lambda w'.$ Jo grew up in l in w'
$\simeq \{p \mid \exists l.\ l$ is a country in $w \wedge p = \lambda w'.$ Jo grew up in l in $w'\}$

This example showed that the semantics of a question is derived from that of its "assertive counterpart", where the *wh*-phrase is replaced by a quantified variable. Combined with the proto-question operator and λ-abstraction, this allows to generate a set of propositions, which only vary in terms of the variable being questioned. This set of propositions (alternatives) can then be used to induce a partition of the CS, as per (**??**).

Chapter 2

Accommodating QuDs: Qtrees

This Chapter introduces a model of questions that is more sophisticated than standardly assumed (cf. Chapter 1). The goal is two fold: first, capture the intuition that questions may be ordered in terms of specificity; second, relate assertions to implicit questions matching their degree of specificity. To this aim, questions are defined as recursive partitions, or parse trees of the Context Set. The Chapter then describes how such questions can be "retro-engineered" from assertions, in a compositional way. The Chapter additionally hints at ways in which this new model may better predict pragmatic oddness – further explored in the following Chapters.

2.1 Making sense

2.1.1 Oddness despite relevance and informativeness

In Chapter 1, we have seen that assertive sentences should be informative, i.e. lead to an incremental shrinkage of the Context Set (**CS**) (**??**). We have also seen that they should be relevant, i.e. shrink the CS in a way consistent with the Question under Discussion (**QuD**) (**??**). But sometimes, it is unclear what the QuD should be. For instance, consider the exchange in (42).

(42) Al: Have you seen Jo today?
Ed: No I haven't...

In this exchange, Ed's reply fully settles the overt QuD (*Have you seen Jo today?*), by intersecting the CS with the set of worlds in which Ed has not seen Jo on the day that *today* refers to. But one could imagine many possible continuations to Ed's utterance. Any such continuation should be informative and relevant to *some* QuD, but it is unclear how this

QuD should be determined. In principle, it could be any non-vacuous partition of the newly updated CS. But there are many such partitions. How to know which one to pick?

Let us consider the following felicitous follow-up to (42). This continuation is felicitous, so, should be both informative and relevant.

(43) –Have you seen Jo today?
–No I haven't... Either she is sick, or if she's not sick, she is at a conference.

To be relevant, the sentence has to relate to a QuD. But, as mentioned earlier, the overt QuD *Have you seen Jo?* is at that point already settled. This suggests that, when no overt QuD is on the table, a "reasonable" QuD is chosen among all the possible non-vacuous partitions of the CS, and is such that the sentence under consideration properly answers it. More generally, sentences are never uttered in and of themselves; their purpose is to answer a question, overt or not, and to induce further questions **?**. A pragmatic model of assertion therefore needs to integrate what sentences mean, but also what kind of question they attempt to answer – implicitly or explicitly. Back to (43), a "reasonable" QuD is probably along the lines of *Where is Jo?*. Then, the continuation in (43) is predicted to be both informative (it says that Jo is sick or at a conference), and relevant.

But even if some implicit "reasonable" QuD can be inferred in the absence of an overt one, some cases of oddness do not seem to from a lack of informativity or relevance. The follow-up sentence in (44) for instance, is equivalent to the one in (43) assuming implication is material, and so should in principle evoke the same QuD. (43) is thus predicted to be both informative and relevant, just like (43). Yet, this follow-up is sharply odd. Chapter **??** will further detail how this particular contrast is challenging for existing theories of oddness.

(44) –Have you seen Jo today?
–No I haven't... # Either she is sick, or if she's not at a conference, she is sick.

On way to understand the contrast between (43) and (44) is to submit that the two follow-ups at stake do not exactly answer the same kind of QuD. Whatever QuD (43) answers, is "reasonable" given what precedes it; whatever QuD (44) answers, is not "reasonable". If this is indeed the root of the observed contrast, then one must devise a way to systematically derive QuDs from out-of-the-blue assertions, in such a way that semantically similar, yet structurally distinct assertions, sometimes give rise to distinct QuDs.

This Chapter will address this desideratum and introduce a pragmatic model of these sentences (along with many others), in which they end up packaging information differently in terms of their evoked QuDs. This novel difference at the "inquisitive" level will be exploited in all the following Chapters, to explain when, and how, structurally and logically similar sentences, package information in distinct way, some ways being more optimal than others. This will be applied to the follow-ups in (43) and (44) in Chapter **??**.

2.1.2 Overview and motivation of the Chapter

The machinery we introduce in this Chapter aims to account for the above datapoints (among others), by relating their felicity or oddness to the QuD(s) inferred from them.[1] The fundamental principle we want to operationalize is *Question-Answer Congruence* (henceforth **QAC**), as formalized by **?**,[2] and given in (45).

(45) *Question-Answer Congruence* (***QAC***). A felicitous assertion has to be a good answer to a good question.

This take on QAC is interesting because it roots this principle in pragmatics, and is broad enough to encompass a variety of constraints that were previously not grouped under the same umbrella. Chapter 1 for instance, showed that RELEVANCE could rule out a wide range of question-answer pairs, and as such could constitute a partial implementation of QAC. But QAC may in principle involve other constraints applying to question-answer pairs. This dissertation will show that, under a certain interpretation of "good answer" and "good question", many more cases of pragmatic oddness can be understood as an accross-the-board failure of QAC.

In this Chapter, we will lay out the groundwork for this more general pragmatic theory of question-answer well-formedness. We begin by introducing a more new model of questions, based on nested partitions, instead of mere partitions of the CS (as discussed in Chapter 1). This model is building on **???**, among many others. Next, equipped with this model of questions, we will show that questions can be evoked by assertions in a compositional way. As a result, sentences involving different operators (specifically, disjunctions and conditionals), give rise to different kinds of questions. Crucially in this model, each sentence may be associated with multiple potential questions. Finally, we will sketch what a pragmatics for question-answer pairs should look like in that framework. In line with QAC, a sentence which cannot be felicitously paired with *any* question will be deemed

[1] We will not talk extensively about cases in which an assertive sentence constitutes a direct answer to an *overt* QuD. There is in fact an interesting line of work showing that overt QuDs can influence pragmatic oddness, especially when it comes to matters of redundancy (**?**).

[2] This principle has been discussed in several forms for many years, within and outside the field of generative linguistics. See for instance **?** for a discussion on how focused assertions and questions can be systematically related in terms of their *semantics*.

odd. This can happen if *all* the pairs formed by a sentence and a question it evokes, are themselves ill-formed.

We now proceed to define questions, not just as partition, but rather, as parse trees of the Context Set, that we will call Qtrees.

2.2 Structure of Question Trees

2.2.1 From partitions to recursive partitions, to parse trees

Building on the standard model presented in Chapter 1, we introduce a more elaborate view of the pragmatics of questions. This model will incorporate the idea that questions have internal structure, and specifically, are hierarchically organized. This hierarchical organization is meant to capture the intuition that a question such as (46a) for instance, appears more *fine-grained*, than a question like (46b). Alternatively, whatever proposition identifies a cell in (46a), also identifies a cell in (46b). Crucially, this intuition will be incorporated in the pragmatics of questions, and so will be made directly accessible to the grammar.

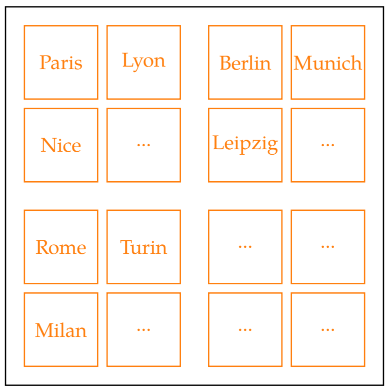
(46) a. In which city did Jo grow up?
b. In which country did Jo grow up?

First, let us observe that these intuitions about question-specificity are *not* readily cashed out by standard partitions or alternative sets associated with questions. (46a)'s and (46b)'s sets of alternatives, given in (47a) and (47b) respectively, are made of disjoint, non empty propositions which, at a certain level of approximation, cover the space of all possibilities.[3] In other words, these alternatives already partition the set of *all* worlds. The partition that (47a) (resp. (47b)) induces on the CS is therefore obtained from (47a) (resp. (47b)) by simply intersecting each of its elements (a proposition/cell) with the CS – discarding empty sets.

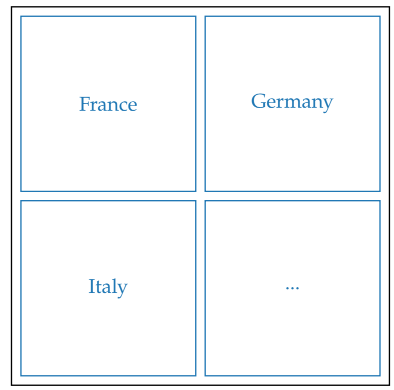
(47) a. $[\![$ In which city did Jo grow up?$]\!]^w$ =
$\{p \mid \exists l.\ l$ is a city $\wedge\ p = \lambda w'.$ Jo grew up in l in $w'\}$
b. $[\![$ In which country did Jo grow up?$]\!]^w$ =
$\{p \mid \exists l.\ l$ is a country $\wedge\ p = \lambda w'.$ Jo grew up in l in $w'\}$

[3]We will assume here, that any point on Earth is associated with one single country, and one single city, in a Voronoi fashion. At this level of approximation, there is no countryless or cityless area. Alternatively, one could assume that there are cityless areas, but that the possibility of Jo growing up in such areas is ruled-out by the presupposition carried by *which*-questions like (46a). Under this assumption, *where*-questions may require more work.

(46a) therefore induces a by-city partition of the CS (see Figure A1), while (46b) induces a by-country partition (see Figure A2). But nothing in (46a)'s partition signals that each of its cells is properly contained in a cell of (46b)'s partition. This property can de derived from the two structures, but is not readily *encoded* by them.



(1) By-city partition associated with (47a). Cells are ordered on a grid for clarity only.



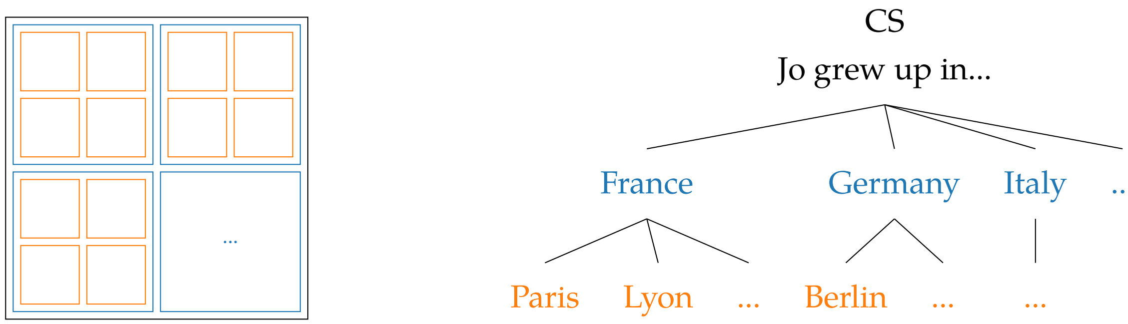
(2) By-country partition associated with (47b). Cells are ordered on a grid for clarity only.

Figure A: Standard partitions induced by a fine-grained (47a) and a coarser-grained question (47b).

Intuitively, grouping together the propositions listed in (47a) talking about cities belonging to the same country, would help capture the desired property. This is done in (48). (48) then defines a set of sets of propositions.

(48) $[\![$ In which city did Jo grow up?$]\!]^w =$
$\{\{p \mid \exists l.\ l$ is a city in $l' \wedge\ p = \lambda w'.$ Jo grew up in l in $w'\} \mid l'$ is a country$\}$

Grouping together cells within bigger sets (which are cells themselves), amounts to building a *nested* partition of the CS. In our example, the "outer" partition is by-country, and the "inner" partition, is by-city. Graphically, this is equivalent to adding the "blue rectangles" from Figure A2, to Figure A1. This operation is performed in Figure B1. The tree in Figure B2 is yet another, more readable way to represent the same thing. In this tree, each node refers to a proposition of the form *Jo grew up in l*, *l* denoting a city or a country. Each node is understood as intersected with the CS, which corresponds to the root of the tree. Therefore, each node forms a proper subset of the CS. Nodes appearing at the same level (forming a "layer"), partition the CS. Deeper layers, correspond to finer-grained partitions. Tree like Figure B2 will be used throughout the dissertation to represent nested partitions like Figure B1. One must always keep in mind that the two representations are equivalent. (49) formally defines the bijective mapping between nested sets of propositions (dubbed *inductive propositions*) like (48), and tree structures like Figure B2.



(1) Recursive partition view

(2) Tree view

Figure B: Alternative representations of the CS corresponding to the nested sets of (48).

(49) *Set-to-tree bijection.* To define this bijection, we first define inductive propositions, and their propositional content. S is an inductive proposition if either:

- S is a set of worlds (i.e. a proposition);
- S is a set of inductive propositions.

The propositional content of an inductive proposition is then defined as:

- If S is a proposition: S;
- If S is a set of inductive propositions: the grand union of the propositional contents of S's elements.

Any inductive proposition S is in a bijection with a tree structure whose nodes are propositions, and defined as:

- If S is a proposition: the tree node denoting S;
- If S is a set of inductive propositions: the tree whose root denotes S's propositional content, and whose children are the tree structures induced by each of S's elements.

So far, we have shown that the standard view linking questions to partitions, fails to account for the intuition that questions differing in terms of specificity, stand in some kind of inclusion relation encoded in their structure. We proposed a way to cash out this intuition, by appealing to recursive partitions, that we represent as trees for clarity.

We now proceed to generalize these observations about the structure of questions. Building on **?????** (among others), we take questions to denote *parse trees* of the CS, i.e. structures that hierarchically organize the worlds of the CS. Such trees (abbreviated **Qtrees**) are defined in (50).

(50) *Structure of Question-trees* (***Qtrees***). Qtrees are rooted trees whose nodes are all subsets of the CS and s.t.:

- Their root generally[4] refers to the CS;
- Any intermediate node is a proposition, which is partitioned by the set of its children.

A Qtree can be bijectively mapped to a nested partition of the CS as defined in (51). Due to this equivalence, we will mostly use Qtrees in the rest of this dissertation.

(51) *Nested partition.* A nested partition P of a set S is a kind of inductive proposition, s.t.:

- If P is a set of inductive propositions, then the propositional contents of P's elements partition P's propositional content. Additionally, P's elements are nested partitions of their own propositional content.

Before investigating the interpretation and the structural properties of model of questions, the next Section covers a few core concepts from graph theory that will be useful in the rest of the Chapter and beyond.

2.2.2 A brief refresher on graph theory (and a few useful concepts for Qtrees)

(50) defines Qtres as rooted trees. Linguists typically understand trees as relations between parent nodes and their children, along the lines of (63).

(52) *Rooted tree* (*inductive version*). A tree rooted in N is either:

- N (single, childless node);
- N, along with N's children, which are all rooted trees.

But we will see throughout this dissertation that it is also useful to see a tree as a specific kind of graph. We will first define graphs, then define trees as a subkind of graph, and lastly, show the importance of defining a root in such trees. The definition of a graph is given in (53). A graph is a way to represent a binary relation, which by default will be

[4] In the case of sentences carrying presuppositions, the root will be assumed to correspond to the intersection between the CS and the sentence's presupposition. In fact, the whole Qtree will be intersected with the presupposition. This will be put to use in Chapters **??** and **??**. But the examples we will see before this, will all involve Qtree rooted in the CS.

symmetric[5]. Elements in the domain of the relation are modeled as nodes, and unordered pairs of nodes are connected with an edge, iff they verify the relation. A graph therefore amounts to a set of nodes, and a set of edges between these nodes. This is illustrated in Figure C.

(53) *Graph.* A graph is defined by a set of nodes $\mathcal{N}$ and by a set of edges $\mathcal{E}$ between elements of $\mathcal{N}$. Edges are defined as unordered pairs of nodes: $\mathcal{E} \subseteq \{\{N_1, N_2\} \mid (N_1, N_2) \in \mathcal{N}^2\}$

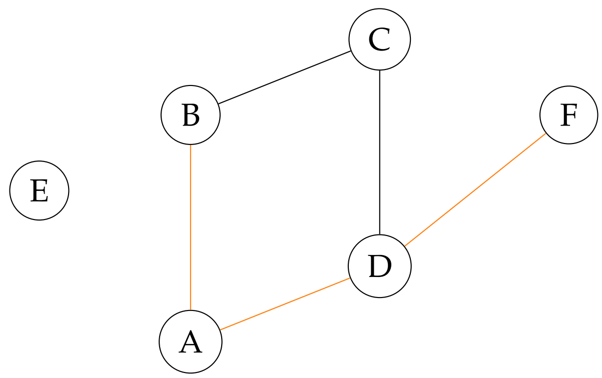


Figure C: A graph $G = (\mathcal{N}, \mathcal{E})$, with $\mathcal{N} = \{A, B, C, D, E, F\}$ and $\mathcal{E} = \{\{A, B\}, \{A, D\}, \{B, C\}, \{C, D\}, \{D, F\}\}$.

This definition allows to define rooted trees as a kind of graph with a few extra properties: connectivity, acyclicity, and rootedness; see (54). We now unpack what these three extra properties mean for graphs. This will lead us to define a few useful concepts applying to trees, namely paths, ancestry, and depth.

(54) *Rooted tree* (*graph version*). A rooted tree is a graph that is connected and acyclic, and features a distinguished node called root.

In graphs, sequences of adjacent edges form paths. For instance, in Figure C, the ordered sequence $[\{A, B\}, \{A, D\}, \{D, F\}]$ forms a path, between node A and node F. This is generalized in (55).

(55) *Path.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. Let $(N_1, N_2) \in \mathcal{N}^2$ be two nodes of G. There is a path in G between N_1 and N_2 (abbreviated $N_1 \overset{G}{\rightsquigarrow} N_2$) iff N_1 and N_2 can be connected by a series of edges in G, i.e. $\exists (e_1, ...e_k) \in \mathcal{E}^k.\ N_1 \in e_1 \wedge N_2 \in e_k \wedge \forall i \in [1; k-1].\ |e_i \cap e_{i+1}| = 1$, where $|.|$ is the cardinality operator.

[5] *Undirected* graphs, that we will simply call graphs, implement symmetric relations, while *directed* graphs implement asymmetric relations.

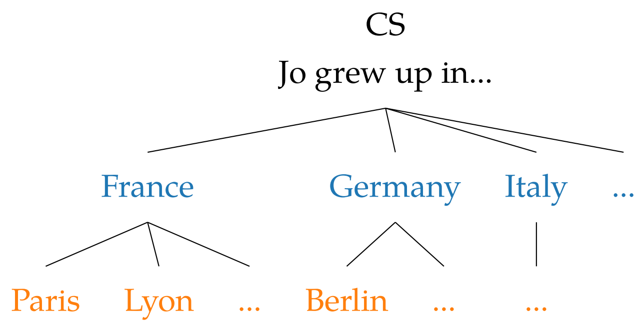
In Figure C, it is easy to see that nodes A, B, C, D and F are all connected to each other by at least one path (in fact, infinitely many of them that cycle through these nodes). Node E on the other hand, is isolated. So, Figure C represents a graph that is *not* connected. If E were removed from the set of nodes, and the edges remained the same, the resulting graph would be connected. This concept of connectivity is generalized in (56). If a graph is a tree, then, it is connected.

(56) *Connectivity.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. G is connected, iff there is a path in G between any pair of nodes in $\mathcal{N}$, i.e. $\forall (N_1, N_2) \in \mathcal{N}^2.\ N_1 \overset{G}{\rightsquigarrow} N_2$.

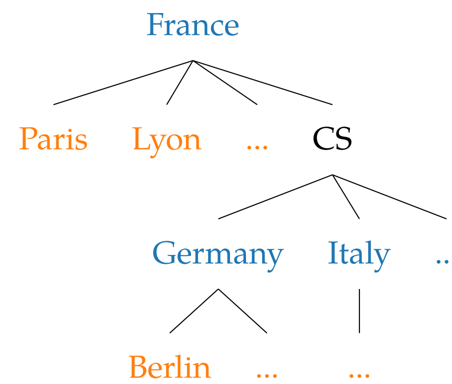
Another thing to note about Figure C, is that nodes A, B, C, and D form a "cycle", there is a path that starts at one of these nodes (e.g., C), and ends at this very same node, *via* B, A, and D. Because of this cycle, there are infinitely many paths between A, B, C, and D, and also between each of these nodes, and F. Removing the edge between, say, A and B, would break the cycle (yet, interestingly, maintain connectivity between A, B, C, and D). The resulting graph would be acyclic. The general definition of an acyclic graph, is given in (57). If a graph is a tree, then, it is acyclic. Moreover, connectivity and acyclicity, are necessary and sufficient for a graph to be a tree.

(57) *Acyclicity.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. G is acyclic, iff no node N of $\mathcal{N}$ is s.t. there is a path starting and ending at N in G, i.e. $\neg\exists N \in \mathcal{N}.\ N \overset{G}{\rightsquigarrow} N$.

We now have a definition of what kind of data structure a tree is. But why do we need Qtrees to be "rooted"? To understand why, let us go back to the tree in Figure B2, repeated in Figure D1 below. The way this tree is represented on paper, is somehow misleading. Recall that, from the point of view of graph theory, a tree is just an undirected graph, with a few extra properties constraining its edges. If the tree represented in Figure D1 were not "rooted", nothing would prevent us from representing it in the form of Figure D2: the nodes and edges are strictly the same, but in Figure D2, *France* "appears" to be the root of the tree, because visually, it is represented at the top. To avoid this confusion, the fact that the CS node should be "at the top" is made part of the representation of the tree – which then becomes a *rooted* tree. So, a rooted tree is just a tree, plus one distinguished node that serves as root.



(1) The "intuitive" view, in which the CS appears at the top.



(2) An alternative "counter-intuitive" view, in which the CS is *not* at the top, yet all edges and nodes are the same.

Figure D: Two equivalent ways to represent the tree corresponding to the question in (46a); assuming trees were connected, acyclic graphs, but not rooted.

The notion of a distinguished root in fact allows to define a few interesting properties on trees that linguist may be more familiar with, and that will be used throughout this dissertation. First, once a tree is rooted, it is possible to define a measure of distance between each node of the tree, and the root. This corresponds to the concept of depth, defined in (58a). In Figure D1 for instance, the CS has depth 0, *Germany* depth 1, and *Lyon* depth 2. This also allows to define the global "size" of the tree, in the form of its maximal depth; see (58b). Figure D1 for instance, is a tree of depth 2.

(58) a. *Depth of a node in a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree, with root R. Let $N \in \mathcal{N}$. The depth of N in T ($d(N, T)$) corresponds to the length of the minimal path between R and N if $N \neq R$,[6] and is set to 0 if $N = R$.

b. *Depth of a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree, with root R. The depth of T ($d(T)$) is the maximal depth of a node in T: $d(T) = max_{N \in \mathcal{N}}(d(N, T))$.

Having a distinguished root, and the derived concepts of depth, gives us the parent-child relation between nodes for free.[7] This relation is defined based on depth and edges in (59), and its transitive closure (the ancestor relation) is defined in (60), in two possible ways.

(59) *Parent-child relation in a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is the parent of N_2 (and N_2 is the child of N_1), iff $\{N_1, N_2\} \in \mathcal{E}$ and $d(N_1, T) < d(N_2, T)$.

[6] This path can be determined using a simple Depth-First Search algorithm starting from the root.
[7] in the next Section, we will introduce another definition of tree, that takes this relation as a primitive

(60) a. *Ancestor relation* (*recursive version*). Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is an ancestor of N_2 iff either:

- N_1 is the parent of N_2;
- or N_1 is the parent of an ancestor of N_2.

b. *Ancestor relation* (*path version*). Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is an ancestor of N_2 iff $N_1 \overset{T}{\rightsquigarrow} N_2$ and $d(N_1, T) < d(N_2, T)$.

Lastly, in the rest of this dissertation, we will extensively use the concept of *layer*, that we define as a the maximal set of same-depth nodes in a rooted tree; see (61). Figure D1 features a country-layer at depth 1, and a city-layer at depth 2. Layers therefore reflect an intuitive notion of granularity.

(61) *Depth-k layer of a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree, with root R. Let k be an integer s.t. $0 \geq k < d(T)$. The depth-k layer of T is the set of nodes in $\mathcal{N}$ whose depth is k, i.e. $\{N \in \mathcal{N} \mid d(N, T) = k\}$.

Now that we have defined the core structure of Qtrees along with a few related properties and metrics, we proceed to assign an interpretation to this kind of structure.

2.2.3 Interpreting Qtrees

At the end of Section 2.2.1, we showed that question should better be represented as nested partition, in order to encode their degree of specificity, and the grammar sensitive to how more or less fine-grained questions relate to each other. We also discussed how nested partitions could be unequivocally represented as Qtrees. It is easy to see that the tree in Figure B2/D1, repeated again in Figure E, is a Qtree according to (50). We saw that this Qtree intuitively capture the idea that a *Which country?* kind of question, is contained in a *Which city?* kind of question. We now investigate how to exploit this hierarchy in a meaningful way. We will use Figure E as an example, and now assign an interpretation to nodes and paths in such structures. We will focus on three meaningful aspects of Qtrees: answer-granularity (understood as node depth), strategies of inquiry (understood as paths), and question refinement (understood as tree inclusion)

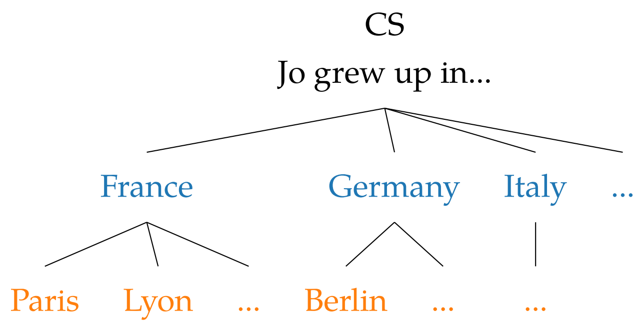


Figure E: "Intuitive" Qtree for *Which city did Jo grow up in?*

We start with the interpretation of nodes as possible answers, with different granularities. The root of Figure E for instance, which corresponds to the entire CS, defines a tautology: it is a proposition which is true of all worlds of the CS, because it simply coincides with it.[8] It can be understood as identifying the unique cell of coarsest-grained partition of the CS, that is, the CS itself. By contrast, leaves like *Paris*, *Lyon*, *Berlin* in Figure E, correspond to the "smallest" cells of the recursive partition that the Qtree defines. They can be seen as maximal answer to the underlying question, e.g., *In which city did Jo grow up?*. Intermediate nodes like *France* or *Germany* in Figure E, form cells of "intermediate" size, and can always be seen as unions of leaves. They appear to correspond to non-maximal answers. Because Qtrees can be made of many layers, they induce a hierarchy between non-maximal answers: an non-maximal answer p is "more maximal" than another non-maximal answer q, iff the node corresponding to p is located deeper in the Qtree than the node corresponding to q. This is formalized in (62).

(62) *Answer granularity.* Let T be a Qtree and (N_1, N_2) be two nodes in T. N_1 constitutes a finer-grained answer than N_2 iff $d(N_1, T) > d(N_2, T)$. This implies that leaves of T correspond to the finest-grained answers (maximal answers) to the question T represents.

Next, we discuss how Qtree encapsulate **?**'s notion of *Strategy of Inquiry*. To this end, we observe that nodes in a tree can receive a "recursive" interpretation, that incorporates everything the node dominates. Under this interpretation, a node N in a Qtree is not only what N denotes; it is the whole subtree ($\sim$subquestion) rooted in N, as defined in (63)

(63) *Recursive interpretation of tree nodes.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $N \in \mathcal{N}$ be a node of T. N's recursive interpretation corresponds to:

[8] Chapter 1 moreover identifies it as an uninformative proposition that is Lewis-relevant but not Roberts-relevant.

- N, if N is a leaf;
- the subtree of T rooted in N, otherwise.

This point of view originates from the inductive definition of a rooted tree given in (52) and repeated below.

(52) *Rooted tree* (*inductive version*). A tree rooted in N is either:

- N (single, childless node);
- N, along with N's children, which are all rooted trees.

If T is a Qtree, then N's recursive interpretation will be the Qtree rooted in N. This Qtree's root can be seen as a "local" CS, which is equal to the global CS, updated with N. For instance, the recursive interpretation of the *France*-node in Figure E, corresponds to the subtree of Figure E rooted in *France*. This subtree, given in Figure F, amounts to the question *In which city did Jo grow up?*, granted that *Jo lives in France*, since its root corresponds to the CS intersected with the proposition that *Jo lives in France*.

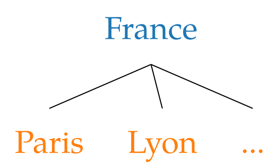


Figure F: "Recursive" interpretation of the *France*-node in Figure E.

In fact, this subtree as a whole, can be understood as the tree-node intersection of Figure E and the proposition that *Jo grew up in France*. Tree-node intersection is defined in (64). This operation takes a Qtree and a proposition p, and creates a Qtree whose nodes are each intersected with p, and resulting empty nodes are removed. Edges from the original Qtree are retained, as long as the nodes they connect are still part of the newly formed Qtree. Note that, because the nodes and edges of a tree form *sets* (and not *multisets*), tree-node intersection automatically collapses nodes from the original tree whose intersections with p yield the same result; and it also collapses the edges between such nodes. Figure G provides a decomposition of this procedure, computing the tree-node intersection between Figure E and the proposition that *Jo grew up in France*, and illustrating how nodes and edges may "collapse". (65) generalizes this point, by stating that the subtree of a Qtree rooted in a node N, can be reconstructed by tree-node intersecting the entire Qtree with the proposition N corresponds to. In other words, the subquestion corresponding to a node N, can be seen as a *restriction* of the entire Qtree, taking N for granted. This is proved in (66).

(64) **Tree-node Intersection.** Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a Qtree. Let p be a proposition. The tree-node intersection between T and p, noted $T \cap p$, is defined iff $R \cap p \neq \emptyset$ and, if so, is the Qtree $T' = (\mathcal{N}', \mathcal{E}', R')$ s.t.:

- $\mathcal{N}' = \{N \cap p \mid N \in \mathcal{N} \wedge N \cap p \neq \emptyset\}$
- $\mathcal{E}' = \{\{N_1 \cap p, N_2 \cap p\} \mid \{N_1, N_2\} \in \mathcal{E} \wedge (N_1 \cap p) \neq (N_2 \cap p) \wedge N_1 \cap p \neq \emptyset \wedge N_2 \cap p \neq \emptyset\}$
- $R' = R \cap p$

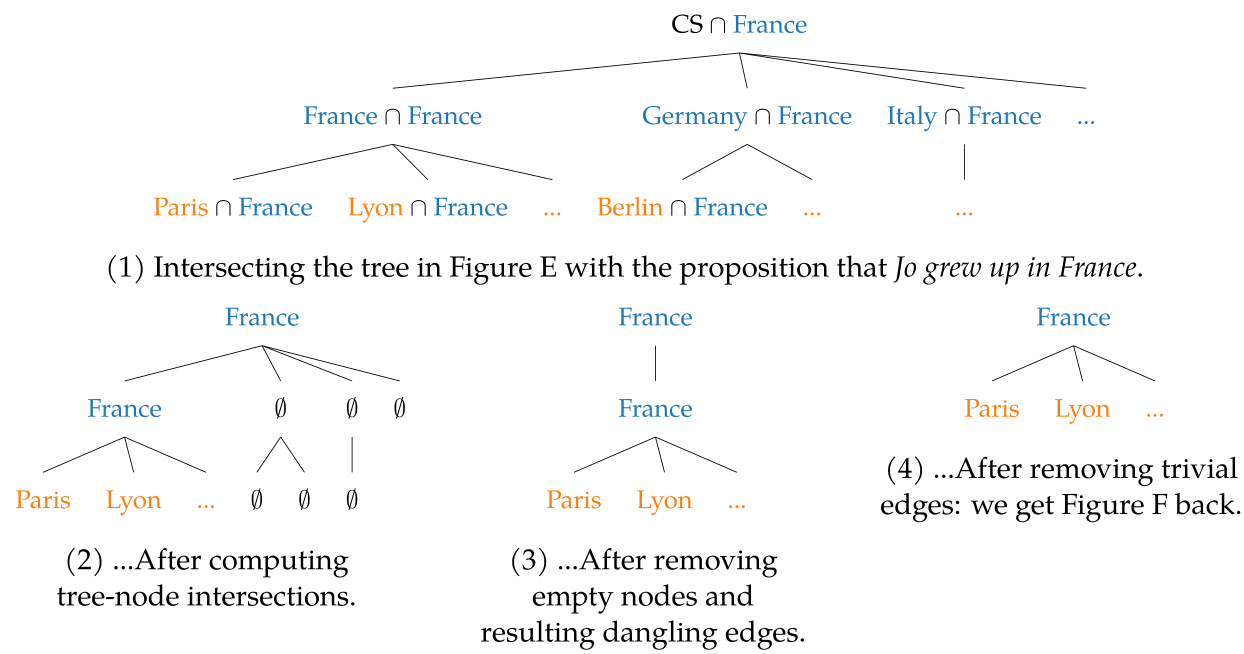


Figure G: The recursive interpretation of a node, can be obtained by intersecting the whole Qtree with that node, removing empty nodes and trivial edges (formed by a parent node and its only child).

(65) *Recursive interpretation and CS update.* Let T be a Qtree. Let N be a node of T. N's recursive interpretation corresponds to the tree-node intersection of T with N.

(66) *Proof of* (65). Let T be a Qtree. Let N be a node of T. Because T is a Qtree, any node N dominates is a subset of N; any node dominating N, is a superset of N, and any node that is neither dominated nor dominating N, is disjoint from N. By definition, N's recursive interpretation is the subtree of T rooted in N, noted T'. We show that T' corresponds to the tree-node intersection between T and N, $T \cap N$. Let N' be N or a node dominated by N. $N' \subseteq N$, so $N' \cap N = N'$. This holds for any N' dominated by N or equal to N. So $T \cap N$ preserves T'. Let N' be an ancestor of N. $N \subseteq N'$ so $N' \cap N = N$. So any ancestor of N in T, is reduced

to N in $T \cap N$. Let N' be a node in T that is neither dominated nor dominating N. $N \cap N' = \emptyset$, and so any sibling/uncle/cousin of N in T is absent in $T \cap N$, along with any incident edges. Therefore, $T \cap N$ ends up being just T'.

We have just seen that under the recursive interpretation of nodes, each node N can be seen as a subquestion of the whole Qtree, which takes N's propositional content for granted. Under this interpretation, a path from the root (CS) to any node N, can then be seen as a series of subquestions, taking for granted increasingly strong propositions. In Figure E for instance, a path of the form [*CS*, *France*, *Paris*], can be interpreted as a series of inquiries of the form: [*In which city did Jo grow up* (*I have no idea*)?, *In which city did Jo grow up* (*given Jo grew up in France*)?, *Jo grew up in Paris*]. If the path terminates on a leaf, then the series of inquiries converges to a maximal answer. We will call such paths complete strategies of inquiry.

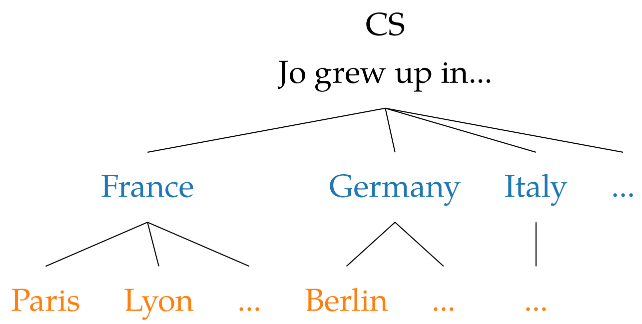
(67) *Complete Strategy of Inquiry.* Let T be a Qtree. A complete strategy of inquiry on T is a path from T's root to one of T's leaves.

This model is very close to what the previous literature had posited at the conversational level, whereby sentences answer questions and sometimes evoke new, finer-grained questions. The key difference here, is that individual questions are assumed to encapsulate the same kind of dynamic, hierarchical information. How does this relate to question granularity? Note that intuitively finer-grained questions yield deeper Qtrees than intuitively coarser-grained ones. Additionally, (58b) defined Qtree depth as the maximal length of a path from the root to a leaf in the tree. This leads to the equivalence in (68).

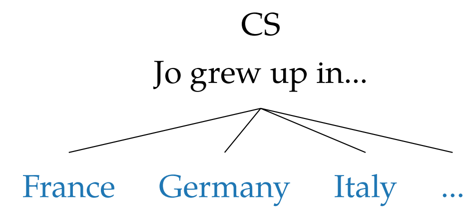
(68) *Depth and Complete Strategies of Inquiry.* Let T be a Qtree. T's depth can be recovered by finding the length of its longest complete strategy of inquiry, dubbed maximal complete strategy of inquiry.

In other words, finer-grained questions are linked to deeper Qtrees, which are characterized by a longer, maximal complete strategy of inquiry. In sum, a fine-grained question is a question for which converging to a maximal answer may require a lot of intermediate steps, or subquestions. This is useful as an absolute measure of question-complexity, but probably not enough to determine if a question is finer-grained than another question. For instance, this incorrectly predicts two completely independent questions to be comparable in terms of granularity, just because they give rise to Qtree of different depths.

There is in fact another way in which the recursive interpretation of nodes can help clarify in what sense a *Which city* kind of question, is more fine-grained than a *Which country* kind of question, in the current framework. Figure H shows what a Qtree for (47a) and a Qtree for (47b) should intuitively look like.



(1) "Intuitive" Qtree for (47a) = *Which city did Jo grow up in?*



(2) "Intuitive" Qtree for (47b) = *Which country did Jo grow up in?*

Figure H: Comparing *Which city* and *Which country* Qtrees.

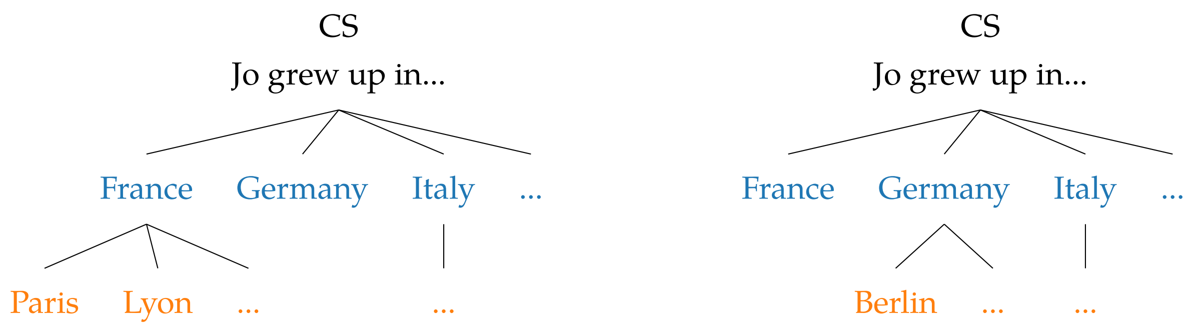
The Qtree for (47a) stops at the city-level, because cities should constitute maximal answer to that kind of question; the Qtree for (47b) on the other hand, stops at the country level, for similar reasons. And it is easy to notice that the Qtree for (47b) somehow forms a "subset" of the Qtree for (47a): it forms a subset of the nodes, and a subsets of the edges, of the Qtree for (47a). Additionally, it is not a random subgraph of Figure H1 (as defined in (69)). It remains a Qtree, that constitues a refinement of Figure H1, as defined in (70). It can also be shown, that all the possible refinements of a Qtree T, correspond to all the possible subgraphs of T that have the Qtree property.[9]

(69) *Subgraph.* Let $G = (\mathcal{N}, \mathcal{E})$ and $G' = (\mathcal{N}', \mathcal{E}')$ be two graphs. $G' \subseteq G$, iff $\mathcal{N}' \subseteq \mathcal{N}$ and $\mathcal{E}' \subseteq \mathcal{E}$.

[9] We identify the refinement operation between T and T', as a set $\mathcal{T}$ of subtrees of T, that is closed under root-sisterhood. We assume T is a refinement of a Qtree T' and show T' is a subgraph of T with the Qtree property. T' is obtained from T by removing the subtrees in $\mathcal{T}$ from T. So it is obviously a subgraph of T. We now show T' is a Qtree. $\mathcal{T}$ cannot contain the tree rooted in T, otherwise T' would be empty. So T' has same root as T, and this root is the CS. Let N be an intermediate node in T'. N has at least one child N', which means that $\mathcal{T}$ cannot contain the subtree of T rooted in N'. To be partitioned by its children, N in T' must have the same children as N in T, i.e. $\mathcal{T}$ should not contain any tree rooted in a child of N. If $\mathcal{T}$ did, then $\mathcal{T}$ would also contain the subtree of T rooted in N', du to its closure property. Contradiction. So N retained all its children from T, and is partitioned by them, given that T is a Qtree.
We assume T' is a subgraph of T with the Qtree property and show T is a refinement of a Qtree T'. T' is a subgraph of T, so we can define S the set of nodes of T not in T'. We then define $\mathcal{T}$ as the set of subtrees of T rooted in a maximal element of S w.r.t. the ancestor relation, as induced by T's edges. We show this set is closed under root-sisterhood. Let $T'' \in \mathcal{T}$. It is subtree of T rooted in some node N, and N is maximal in S w.r.t. the ancestor relation. If N has no sister in T, then the closure property is trivially verified. If N has a sister N' in T, we show the closure property by contradiction. If the subtree of T rooted in N' did not belong to $\mathcal{T}$, then, either N' would be part of a subtree of $\mathcal{T}$ (but not as root), or, N' would be a node in T'. The former option would imply that some common ancestor of N and N' would be the root of a subtree in $\mathcal{T}$. But then both N and some ancestor of N, would be maximal in S w.r.t. the ancestor relation. Contradiction. The former option would mean that N''s parent in T', would have N', but not N has child, and so would not be partitioned by its children. Therefore, T' would not be a Qtree. Contradiction.

(70) *Qtree refinement.* Let T and T' be Qtrees. T is a refinement of T' (or: T is finer-grained than T'), iff T' can be obtained from T by removing a subset $\mathcal{T}$ of T's subtrees, s.t., if $\mathcal{T}$ contains a subtree rooted in N, then, for each node N' that is a sibling of N in T, the subtree of T rooted in N', is also in $\mathcal{T}$.

Two other possible refinements of Figure H1 are given below. It is worth noting that the process deriving a refinement from a Qtree need not remove entire layers.



(1) A refinement where the children of the *Germany*-node are deleted.

(2) A refinement where the children of the *France*-node are deleted.

Figure I: Possible refinements of the Qtree for *Which city did Jo grow up in?* in Figure H1.

2.2.4 Flagging Qtrees

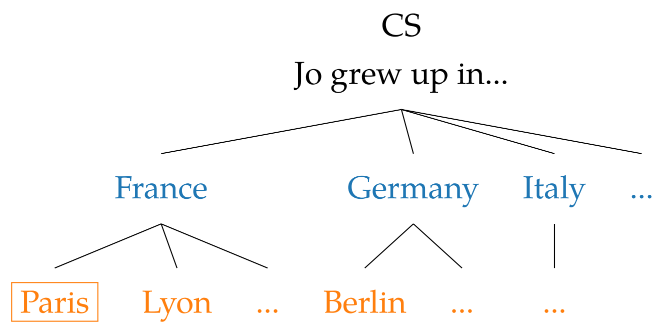
So far, we have considered Qtrees directly associated with questions, like (46a) and (46b). But, as suggested in the introduction to this Chapter, we want to go one step further, and posit that assertive sentences, like (71a) and (71b), also evoke questions in the form of Qtrees. Such questions will correspond to the ones a given assertion could be a good answer to.

(71) a. Jo grew up in Paris.
b. Jo grew up in France.

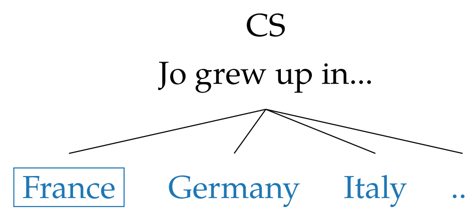
So (71a) and (71b) for instance, should evoke Qtrees associated with questions like (46a) and (46b), respectively. We sketched an intuitive representation of these Qtrees in Figure H. Are these Qtrees representing everything that the assertions in (71a) and (71b) convey though? One major difference between questions and assertions, is that questions are ignorant of the answer, while assertions provide such an answer. So, if (71a) were directly mapped to the Qtree in Figure H1, the information that (71a) actually answers the question by identifying the *Paris*-node, would be lost. Another way to see the issue, is to observe that (71a) and (72) would then be associated with the exact same Qtree.

(72) Jo grew up in Berlin.

To avoid such collisions in the case of assertive sentences, we define an extra piece of machinery on top of the Qtree architecture, that consists in a set of "verifying" nodes keeping track of *how* the assertion answers the question it evokes. I Figures, these nodes will be represented in boxes; given a Qtree T, T's set of verifying nodes will be referred to as $\mathcal{N}^+(T)$. (71a) and (71b) for instance, will intuitively evoke Qtree that *structurally* match those in Figure H, but whose *Paris* and *France* noes respectively, are "boxed", i.e. flagged as verifying.



(1) "Intuitive" Qtree for (47a) = *Which city did Jo grow up in?*



(2) "Intuitive" Qtree for (47b) = *Which country did Jo grow up in?*

Figure J: Comparing *Which city* and *Which country* Qtrees.

In that particular case, the nodes that are flagged as verifying in both Qtrees, strictly coincide with the proposition conveyed by the assertions, namely, that *Jo grew up in Paris*, and that *Jo grew up in France*. For an assertion like (72), the only flagged node would be *Berlin*. But we will not take this strict equivalence between prejacent proposition and verifying nodes to be a generality. In the model laid out in the next Section, we will assume that verifying nodes, just like Qtree structure, are compositionally "retro-engineered" from the structure and meaning of the sentence. As a result, there may be more than on verifying node in a given Qtree, and, the grand union of a Qtree's verifying nodes, may not always coincide with the proposition denoted by the assertion.[10] The next Section therefore introduces a more systematic way to derive Qtrees and their verifying nodes from simplex sentences.

Moreover, an accommodated Qtree should allow the sentence evoking it to properly answer it; that is why we assume that any well-formed Qtree derived from a sentence should come with a non-empty set of verifying nodes.(see (73)). More generally, we as-

[10]This will in particular be true of conditional assertions.

sume that oddness results from the fact that a given sentence, through its LF, cannot give rise to any well-formed Qtree. This is summarized in (74).

(73) *Empty labeling of verifying nodes.* If a sentence S evokes a Qtree T but does not flag any node as verifying on T, then T is deemed odd given S.

(74) *Oddness of a sentence.* A sentence S is odd if any Qtree T it evokes is odd given S.

2.3 Compositional Qtrees: base case

So far, we have established that assertive sentences can evoke the questions they are good answers to, in the form of Qtrees. And we used "intuitive" Qtrees like the ones in Figures J1 and J2 to show how such structures could capture a wide range of properties connecting questions and answers, and questions to each other. We now introduce a principled algorithm to derive Qtrees out of assertive sentences. This will heavily build on the notion of alternatives and how they can be ordered in terms of specificity. We will show that this ordering of alternatives induces a specific "layering" on Qtrees.

2.3.1 Alternatives

It is quite uncontroversial that assertive sentences evoke alternatives (**???**). An alternative is a sentence that is sufficiently "similar" to the sentence it is evoked by, but may have a different meaning. For instance, (75a) is felt to have (75b) as alternative, and vice versa. Pre-theoretically, this is because, in many contexts, (75a) is utterable iff (75b) is, too. The same holds for (76a) and (76b).

(75) a. Jo ate all of the cookies.
b. Jo ate some of the cookies.

(76) a. Jo grew up in Paris.
b. Jo grew up in Lyon.

Moreover, the computation of alternatives is driven by focus. In (75) and (76), it was implicitely assumed that quantifiers and city names respectively, were focused, and so gave rise to alternative varying in terms of the focused element. But note that, if the object of (75a) and the verb of (76a) had been focused instead, the alternatives to these sentences would have been different, along the lines of (77b) and (78b), respectively.

(77) a. Jo ate all of the COOKIES.

b. Jo ate all of the muffins.

(78) a. Jo GREW UP in Paris.

b. Jo resides in Paris.

We define what counts as an alternative to a given sentence (or more generally an LF) in (121), based on **?** and **?**.

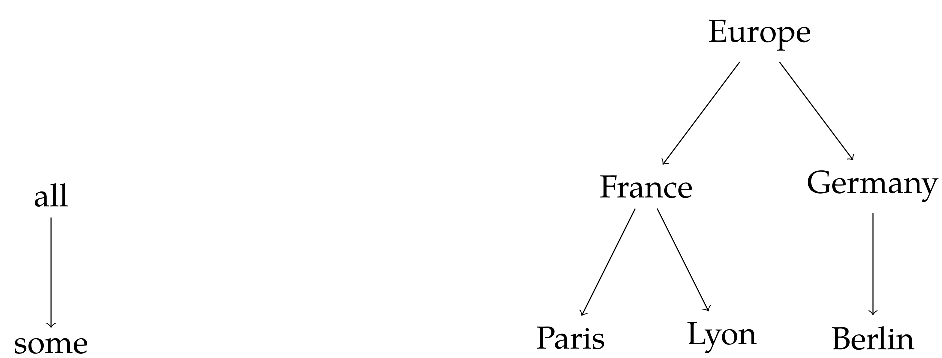
(79) *Structural alternatives.* Let X be an LF containing a focused constituent. The set of X's alternatives is the set $\mathcal{A}_X$ of LFs Y, obtained by substituting X's focused constituent with an element that is at most as complex.

(80) *Structural complexity.* Let X and Y be two LFs. X is at most as complex as Y iff X can be obtained from Y *via* substitutions of lexical items with other lexical items, or constituent-to-subconstituent substitutions.

Structural alternatives to a given LF (which are also LFs), induce a set of propositional alternatives, as defined in (81).

(81) *Propositional alternatives.* Let X be an LF denoting a proposition p. The set of X's propositional alternatives is the set $\mathcal{A}_{p,X}$ of propositions denoted by X's structural alternatives: $\mathcal{A}_{p,X} = \{q \mid \exists Y \in \mathcal{A}_X . [\![Y]\!] = q\}$

Propositional alternatives can be related to each other by entailment – forming a partial order. This partial order between propositional alternatives can be graphically represented in the form of a Hasse diagram. Hasse diagrams for two possible sets of propositional alternatives, roughly corresponding to (77) and (76), are given in Figure K.



(1) Hasse diagram for $\{all, some\}$

(2) Hasse diagram for $\{Europe, France, Germany, Paris, Lyon, Berlin\}$

Figure K: Hasse diagrams generated by $\vDash$ on two possible sets of propositional alternatives.

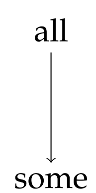
How are these diagrams obtained from propositional alternatives, and the entailment relations between them? Formally, a Hasse diagram is a directed graph, as defined in (82). The only difference between a graph and a directed graph, is that the edges of a directed graph have a direction, i.e. they correspond to ordered pairs instead of sets of cardinality 2. If $[N_1, N_2]$ is a directed edge, $[N_1, N_2]$ is visually represented as $N_1 \to N_2$. Paths are also directed, and so is the ancestor relation (see (83) and (84)). Directed graphs are designed to model *asymmetric* relations, like $\vDash$.

(82) *Directed graph.* A directed graph is defined by a set of nodes $\mathcal{N}$ and by a set of directed edges $\mathcal{E}$ between elements of $\mathcal{N}$. Directed edges are defined as ordered pairs of nodes: $\mathcal{E} \subseteq \{[N_1, N_2] \mid (N_1, N_2) \in \mathcal{N}^2\}$

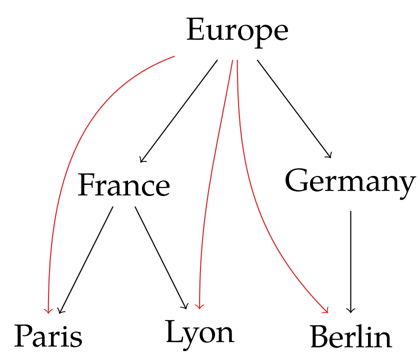
(83) *Directed Path.* Let $G = (\mathcal{N}, \mathcal{E})$ be a directed graph. Let $(N_1, N_2) \in \mathcal{N}^2$ be two nodes of G. There is a path in G between N_1 and N_2 (abbreviated $N_1 \overset{G}{\rightsquigarrow} N_2$) iff N_1 can be connected to N_2 by a series of directed edges in G, i.e. $\exists(e_1, ...e_k) \in \mathcal{E}^k.\ e_1^{(0)} = N_1 \wedge e_k^{(1)} = N_2 \wedge \forall i \in [1; k-1].\ e_i^{(1)} = e_{i+1}^{(0)}$, where, for any edge e, $e = [e^{(0)}, e^{(1)}]$.

(84) *Ancestor relation (directed path version).* Let $G = (\mathcal{N}, \mathcal{E})$ be a directed graph. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is an ancestor of N_2 iff $N_1 \overset{G}{\rightsquigarrow} N_2$.

The directed graphs (not yet the Hasse diagrams) induced by $\vDash$ on the sets of alternatives from Figure K, are given in Figure L. In these graphs, there is a directed edge $[p, q]$ between two nodes corresponding to propositions p and q, iff $q \vDash p$. Figure L1 already looks like the corresponding Hasse diagram in Figure K1, but Figure L2 does not: it features a few more directed edges (in red) than its Hasse counterpart in Figure K2.



(1) Directed graph induced by $\vDash$ on $\{$*all*, *some*$\}$



(2) Directed graph induced by $\vDash$ on $\{$*Europe*, *France*, *Germany*, *Paris*, *Lyon*, *Berlin*$\}$

Figure L: Directed graphs generated by $\vDash$ on two possible sets of propositional alternatives.

How do Hasse diagrams eliminate these few superfluous edges? The Hasse diagrams we are interested in correspond to the transitive reduction of the graphs in Figure L, which were induced by $\vDash$ on sets of propositional alternatives. The transitive reduction operation precisely gets rid of the red edges in Figure K2, based on the idea that such edges correspond to paths formed by the black ones. The formal (though, non constructive) definition of a transitive reduction, is given in (85). This definition maps the graphs in Figure L, to the Hasse diagrams in Figure K.

(85) *Transitive reduction of a graph.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. The transitive reduction G' of G is the graph:

- Whose set of nodes is $\mathcal{N}$;
- Whose edges are the smallest set $\mathcal{E}'$ s.t. $\forall(N_1, N_2) \in \mathcal{N}.\ N_1 \overset{G}{\leadsto} N_2 \iff N_1 \overset{G'}{\leadsto} N_2$

The Hasse diagram in Figure K2 is basically a rooted tree, and may look like a Qtree, but this is not a generality. For instance, the Hasse diagram in Figure K1 is not connected, so is not even a tree. Additionally, not all sets of propositional alternatives, even if their Hasse diagram is tree-like, are guaranteed to verify the partition property of Qtrees. The next Section focuses on how Hasse diagrams can be used to determine how alternatives relate to each other in terms of granularity. This will eventually allow us to encode granularity in the structure of Qtree.

2.3.2 Alternatives and granularity

In this Section, we use a notion of granularity to constrain what kind of Qtree can be evoked by a simplex assertive LFs. We consider simplex LFs to be LFs which do not contain a node of type $\mathtt{t}$ besides their root.[11] The goal is to organize the layers of a Qtree in terms of how specific the nodes in this layer are. Why is an external notion of granularity needed to structure Qtree? In the Qtree sketched in e.g. Figure J1, each layer corresponds to an intuitive degree of specificity: a by-country layer dominates a by-city layer. Though intuitive, this kind of configuration is not the only one to verify the Qtree property. The tree in Figure M, where the *Germany*-node is replaced by its children, is also a Qtree: at our level

[11]Nina Haslinger suggested that this condition *may* be relaxed under certain contexts. For instance, a disjunctive LF denoting p, may sometimes be understood as simplex, and generate Qtrees based on the recipe presented later in this Chapter. The exact circumstances under which this is possible, should be fleshed out, but will not be the focus of this dissertation. One intuition, suggested by Nina, is that an overt QuD may indicate what is relevant, and in turn determine what should be considered "simplex" when computing implicit Qtrees (and comparing such Qtrees to the overt QuD). Another intuition, is that the "simplex" character of an LF, may be partly driven by focus.

of approximation, all countries but Germany, plus all the German cities, partition the set of all possible locations, and, each country represented in this tree is properly partitioned by the set of its cities.

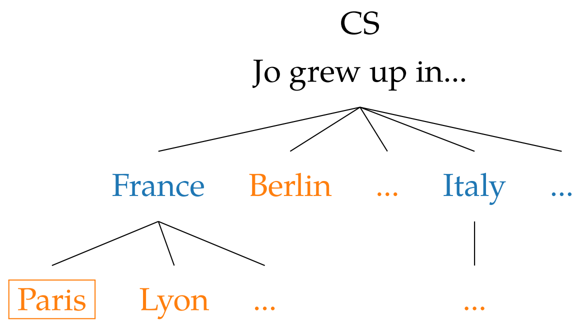


Figure M: "Unintuitive" Qtree for (47a) = *Which city did Jo grow up in?*, where layers exhibit "mixed granularity".

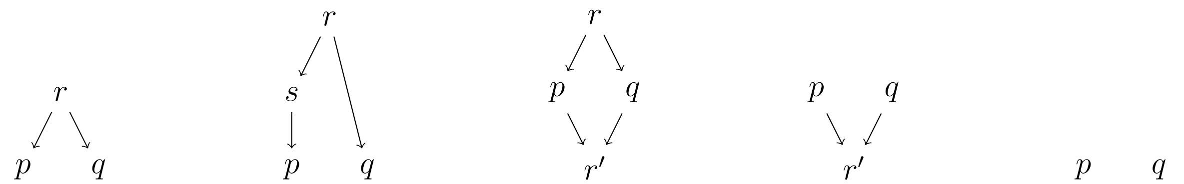
To derive Qtrees like Figure J1, and rule-out Qtrees like Figure M, we need a notion of granularity that can transfer into the Qtree layers. We now show that a relation of same-granularity can be derived from Hasse diagrams induced by $\vDash$ on "complete" sets of propositional alternatives. Considering such diagrams, where *all* relevant alternatives are considered, we take that two nodes (two propositional alternatives) have same granularity if they are equidistant (in terms of path length) to any common ancestor and common descendant they may have. We also take that any node has the same granularity as itself. This relation, defined in (86) is close in spirit to **?**'s *Specificity Condition*.[12] Note that this definition constitutes a universal statement, so nodes that do *not* have any common ancestor or descendant, automatically have same-granularity.[13]

(86) *Same granularity relation* $\sim_g$. Let p and q be two propositions belonging to the same set of propositional alternatives. If $p = q$, then $p \sim_g q$. If not, let H be the Hasse diagram induced by $\vDash$ on the set of propositional alternatives to p and q. If for all common ancestor r of both p and q in H and for all common descendant r' of both p and q in H, the paths from r to p and r to q have same length, and the paths from p to r' and q to r' have same length, then $p \sim_g q$.

[12]However, the structure on which this condition operates in **?**'s model, appears slightly different (*Structured Sets of Alternatives*). Also, the *Specificity Condition* is not taken to be a relation, but a rather, a constraint defining which kind of alternatives can be contrasted in e.g. disjunctive environments.

[13]This will be discussed in more detail when dealing with scalar alternatives such as $\langle$*some*, *all*$\rangle$, in Chapter **??**. It will be crucial that such alternatives, which do *not* have a common ancestor in their Hasse diagram (see Figure K1), *can* be seen as same-granularity.

Figure N show a few simple configurations for which p and q do or do not have same granularity.



(1) $p \sim_g q$ (2) $p \not\sim_g q$ (3) $p \sim_g q$ (4) $p \sim_g q$ (5) $p \sim_g q$

Figure N: Different Hasse diagrams for $\vDash$ influence whether p and q are same-granularity.

We now leverage this relation between propositions to define the layers of a Qtree as partitions induced by sets of same-granularity alternatives. We first observe that the relation $\sim_g$ defined in (86) can be used to divide the set of propositional alternatives to a given LF, into subsets sharing the same level of granularity. This gives rise to a "tiered" set of alternatives, as defined in (87).

(87) *Tiered set of propositional alternatives.* Let X be a sentence denoting a proposition p. A tiered set of propositional alternatives to X, is the set of sets of propositions, whose elements are the maximal sets of propositions related by the same-granularity relation. In other words, $\mathcal{A}^{\sim_g}_{p,X} = \{\{r \in \mathcal{A}_{p,X} \mid r \sim_g q\} \mid q \in \mathcal{A}_{p,X}\}$. If $\sim_g$ is an equivalence relation, $\mathcal{A}^{\sim_g}_{p,X}$ is a partition.

Tiered sets of alternatives are quite close to the *Structured Sets of Alternatives* defined in **?**. One difference however, is that tiered sets of propositional alternatives are *not* assumed to include propositions corresponding to alternatives that are more complex than the original LF. The elements of a tiered set of propositional alternatives are sets of propositions and form same-granularity "tiers", as defined in (88). These tiers will be used to form Qtree layers. If $\sim_g$ is an equivalence relation when restricted to a specific set of propositional alternatives, then the resulting tiered set of alternatives will partition it, i.e. same-granularity tiers will be cells.

(88) *Same-granularity tier.* Let X be a sentence denoting a proposition p, and $\mathcal{A}_{p,X}$ its set of propositional alternatives. Let $q \in \mathcal{A}_{p,X}$. The set of same-granularity alternatives to q (in $\mathcal{A}_{p,X}$), is the set of propositions in $\mathcal{A}_{p,X}$ sharing same-granularity with q. We call this set $\mathcal{A}^q_{p,X}$. $\mathcal{A}^q_{p,X} = \{r \in \mathcal{A}_{p,X} \mid r \sim_g q\}$. $\mathcal{A}^q_{p,X}$ is a subset of the tiered set of propositional alternatives to X, $\mathcal{A}^{\sim_g}_{p,X}$. Moreover, if $\sim_g$ is an equivalence relation, then $\mathcal{A}^q_{p,X}$ constitutes a cells of $\mathcal{A}^{\sim_g}_{p,X}$.

2.3.3 Leveraging alternatives to generate Qtrees

We are now equipped to devise a recipe generating Qtrees out of simplex sentences, based on tiered sets of propositional alternatives. We start by considering the standard constraint on question-answer pairs, given in (89). This constraint establishes a connection between the standard set of alternatives derived from a sentence involving focus, and the kind of question this sentence answers.

(89) *Constraint on question-answer pairs* (**?**, **?**, *to be revised*). A good question-answer pair (Q, A) is s.t. $[\![Q]\!] \subseteq [\![A]\!]^f$, where:

- $[\![Q]\!]$ corresponds to the alternative semantics of the question;
- $[\![A]\!]^f$ corresponds to the focus semantic value of the answer, i.e. the set of propositions denoted by LFs obtained from A *via* the substitution of A's focused material by a same-type element.

Let us show that this constraint is not sufficient (though, a good starter) for a model of questions evoked by assertions. We assume that A corresponds to the sentence *Jo grew up in PARIS*, where *PARIS* is focused. The focus semantic value of A then involves propositions denoted by LFs of the form *Jo grew up in l*, with *l* a location, e.g. *Paris*, *France*, or *Germany*. If the only constraint on the question Q accommodated from A was that $[\![Q]\!]$ should be a subset of $[\![A]\!]^f$, then, in principle, $[\![Q]\!]$ could be made of the three propositions that *Jo grew up in Paris*, *Jo grew up in France*, and *Jo grew up in Germany*. Granted that Paris is in France, and that France and Germany are disjoint, this set of alternatives would induce a partition of the CS of the form $\{\neg\mathit{France} \wedge \neg\mathit{Germany}, \mathit{Germany}, \mathit{France} \wedge \neg\mathit{Paris}, \mathit{Paris}\}$. This appears similar to the mixed-granularity layer that we said was problematic in Figure M. So not all questions allowed by (89), given a fixed assertion, appear to make sense. There are two ways to alter (89) to avoid that kind of configuration: modify the relation between $[\![Q]\!]$ and $[\![A]\!]^f$, and/or, change $[\![A]\!]^f$ into something else.

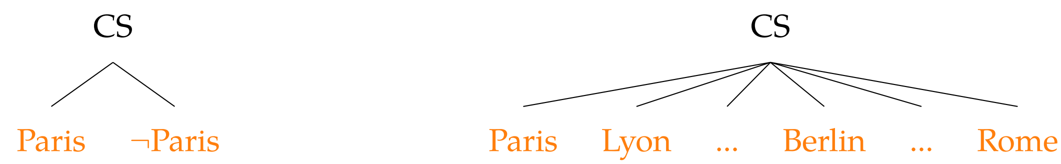
We in fact opt for both options, and reuse the ideas presented in the previous Section. Specifically, we consider two subcases: the case in which $[\![Q]\!]$ simply corresponds to $\{[\![A]\!]\}$, and induces a partition of the CS of the form $\{[\![A]\!], \neg[\![A]\!]\}$; and the case foreshadowed in the previous Section, in which $[\![Q]\!]$ corresponds to same-granularity alternatives to $[\![A]\!]$. These two cases are repeated in (90).

(90) *Constraint on question-answer pairs* (*first revision*). Let X be a LF denoting p. X evokes a question that is either:

(i) $[\![Q]\!] = \{p\}$;

(ii) $[\![Q]\!] = \mathcal{A}^p_{p,X}$, the set of same-granularity alternatives to p.

(90) allows assertions to evoke multiple potential Qtrees. According to (90), an assertion such as *Jo grew up in Paris*, will either evoke the question $[\![Q]\!] = \{Paris\}$, inducing a partition of the CS of the form $\{Paris, \neg Paris\}$, and corresponding to the polar question of whether or not Jo grew up in Paris; or, $[\![Q]\!] = \{Paris, Lyon, Nice, ..., Berlin, ..., Rome, ...\}$, inducing a similar partition of the CS, and corresponding to the *wh*-question *In which city did Jo grow up?*. These partitions are represented in Figure O.



(1) A Qtree for (71a) assuming (90i)

(2) A Qtree for (71a) assuming (90ii)

Figure O: One-layer Qtrees generable from (90) and the sentence (71a)=*Jo grew up in Paris*.

The above partitions seem more in line with intuitions than the pathological ones generable from (89). However, they still do not form layered Qtrees. Therefore, (90) is still not powerful enough to capture the specificity differences sketched in Figure H among others. Going one step further, we can assume that the Qtrees compatible with a sentence, are either generated by the proposition p denoted by the sentence (thus creating a Qtree like Figure O1), or, by the sentence's tiered set of propositional alternatives, as defined in (87). Specifically in the latter case, it will be assumed that each layer of the Qtree corresponds to the partition induced on the CS by a same-granularity tier of propositional alternatives, and that layers are ordered in terms of granularity. Figure O2 constitutes the simplest subcase of this principle, in which only one layer gets generated out of same-granularity alternatives to p. In any case, verifying nodes are defined as the leaves of the tree entailing p (i.e. contained in p). This is formalized in (91). In this definition, (91ii) may be seen as a subcase of (91iii), in which the p-chain set to p only.

(91) *Qtrees for simplex LFs* (*to be further generalized in Chapter* **??**). Let X be a simplex LF denoting p, not settled in the CS. Let $\mathcal{A}_{p,X}$ be the set X's propositional alternatives. For any $q \in \mathcal{A}_{p,X}$, let $\mathcal{A}^q_{p,X} \subseteq \mathcal{A}_{p,X}$ be the set of alternatives from $\mathcal{A}_{p,X}$ sharing same granularity with q. We assume for simplicity that for any q, $\mathcal{A}^q_{p,X}$ partitions the CS. A Qtree for X is either:

(i) A depth-1 Qtree whose leaves denote $\mathfrak{P}_{\{p\},CS} = \{p, \neg p\}$

(ii) A depth-1 Qtree whose leaves denote $\mathfrak{P}_{\mathcal{A}^p_{p,X},CS} = \mathcal{A}^p_{p,X}$.

(iii) A depth-k Qtree ($k > 1$) constructed in the following way:

- Formation of a "p-chain" $p_0 = p \subset p_1 \subset ... \subset p_n$ where $p_0, ..., p_n$ are all in $\mathcal{A}_{p,X}$ but belong to different granularity tiers in $\mathcal{A}^{\sim g}_{p,X}$: $\mathcal{A}^{p_0}_{p,X} \neq \mathcal{A}^{p_1}_{p,X} \neq ... \neq \mathcal{A}^{p_n}_{p,X}$.
- Generation of the "layers" of the Qtree, based on the partitions induced by the granularity tiers corresponding to each element of the p-chain: $\left\{\mathfrak{P}_{\mathcal{A}^{p_i}_{p,X},CS} \mid i \in [0;n]\right\}$.
- Determination of the edges between nodes (cells) of adjacent layers (and between the highest layer and the root), based on the subset relation.[14]

In any case, $\boxed{\text{verifying nodes}}$ are defined as the set of leaves entailing p.

2.3.4 Applying the recipe to two simple sentences

We can now apply (91) to sentences like (71a) and (71b), repeated below.

(71) a. Jo grew up in Paris.
b. Jo grew up in France.

We start with (71a), and assume that its alternatives are of the form *Jo grew up in l*, with *l* a city or a country. Taking for granted that "city" propositions and "country" propositions form two distinct granularity tiers, the tiered set of propositional alternatives to (71a), will be as in (92).

(92)
$$\begin{aligned}\mathcal{A}^{\sim g}_{\mathit{Paris},(71a)} &= \{\{\mathit{Paris}, \mathit{Lyon}, ..., \mathit{Berlin}, ...\}, \{\mathit{France}, \mathit{Germany}, ...\}\} \\ &= \{\{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w.\ \text{Jo grew up in } l \text{ in } w\}, \\ &\qquad \{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w.\ \text{Jo grew up in } l \text{ in } w\}\}\end{aligned}$$

First, we can generate a Qtree for (71a) using principle (91i). This Qtree will have the CS as root, and two leaves corresponding to the propositions that *Jo grew up in Paris*, and *Jo did not grow up in Paris* (assuming this matter is not settled in the CS). This Qtree is depicted in Figure P1. Intuitively, it corresponds to the question of whether or not Jo grew up in Paris.

Second, we can use principle (91ii). To do so, we must determine the set of same-granularity alternatives to the prejacent proposition that *Jo grew up in Paris*. This set, labeled $\mathcal{A}^{\mathit{Paris}}_{\mathit{Paris},(71a)}$, corresponds to the first element of the tiered set of propositional alternatives $\mathcal{A}^{\sim g}_{\mathit{Paris},(71a)}$ in (92). It is repeated in (93). The alternatives contained in $\mathcal{A}^{\mathit{Paris}}_{\mathit{Paris},(71a)}$ are

[14] This may not always create well-formed Qtrees. Chapter **??** will explore such cases update (91) in consequence.

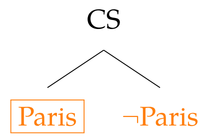
all exclusive (cities are spatially disjoint), and moreover cover the space of possibilities. So, once intersected with the CS, they already form a partition of the CS. According to principle (91ii), this partition correspond to the leaves of the resulting Qtree. This Qtree is depicted in Figure P2. Intuitively, it corresponds to the question of which city Jo grew up in.

(93) $$\begin{aligned}\mathcal{A}^{\textit{Paris}}_{\textit{Paris},(71a)} &= \{\textit{Paris}, \textit{Lyon}, ..., \textit{Berlin}, ...\}\\ &= \{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\}\\ &= \mathfrak{P}_{\{\textit{Paris},\textit{Lyon},...,\textit{Berlin},...\},CS}\end{aligned}$$

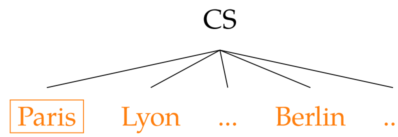
Third and lastly, we can use principle (91iii), which constitutes are multi-layer generalization of principle (91ii). To do so, we need to define a p-chain of propositions entailed $p = \lambda w$. *Jo grew up in Paris in* w. The tiered set of alternatives posited in (92) contains one such proposition, namely $p' = \lambda w$. *Jo grew up in France in* w. The resulting Qtree will therefore be made of three layers: the CS (root), the partition generated by the same granularity alternatives to p', and the partition generated by the same granularity alternatives to p, already defined in (93). The set of same-granularity alternatives to p', labeled $\mathcal{A}^{\textit{France}}_{\textit{Paris},(71a)}$, corresponds to the second element of the tiered set of propositional alternatives $\mathcal{A}^{\sim g}_{\textit{Paris},(71a)}$ in (92). It is repeated in (94). The alternatives contained in $\mathcal{A}^{\textit{France}}_{\textit{Paris},(71a)}$ are all exclusive (country are spatially disjoint), and moreover cover the space of possibilities. So, once intersected with the CS, they already form a partition of the CS.

(94) $$\begin{aligned}\mathcal{A}^{\textit{France}}_{\textit{Paris},(71a)} &= \{\textit{France}, \textit{Germany}, ...\}\\ &= \{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\}\\ &= \mathfrak{P}_{\{\textit{France},\textit{Germany},...\},CS}\end{aligned}$$

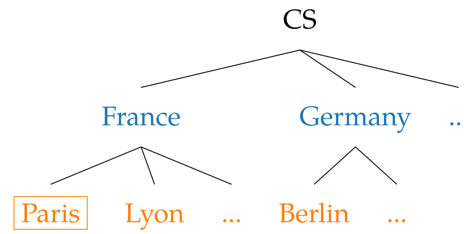
As per principle (91iii), a Qtree evoked by (71a) will then have the CS as top layer, the nodes corresponding to the partition in (94) as middle layer, and the nodes corresponding to the partition in (93) as bottom (leaf) layer. Connectivity between layers is straightforward: it corresponds to the inclusion relation between cities and countries, and between countries and "the whole world" (∼CS). The resulting Qtree is given in Figure P3. Intuitively, it corresponds to the question of which city Jo grew up in, but such that this question is decomposed into two subquestions: first, which country Jo grew up in; then, knowing the country, which city Jo grew up in, in that country.



(1) Following principle (91i).



(2) Following principle (91ii).



(3) Following principle (91iii).

Figure P: Possible Qtrees evoked by the assertion (71a)=*Jo grew up in Paris*.

Of course, if more alternatives to (71a) had been posited in the first place, principle (91iii) would have produced more Qtrees. For instance, if continent alternative had been considered, the tiered set of propositional alternatives to (71a), $\mathcal{A}^{\sim g}_{Paris,(71a)}$, would have been as in (95), and the Qtrees generated by principle (91iii), would have been the one in Figure P3, plus the one in Figure Q.

(95) $$\begin{aligned}\mathcal{A}^{\sim g}_{Paris,(71a)} &= \{\{Paris, Lyon, ..., Berlin, ...\}, \{France, Germany, ...\}, \{Europe, Asia, ...\}\}\\ &= \{\{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\},\\ &\quad\ \{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\}\\ &\quad\ \{p \mid \exists l.\ l \text{ is a continent} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\}\}\end{aligned}$$

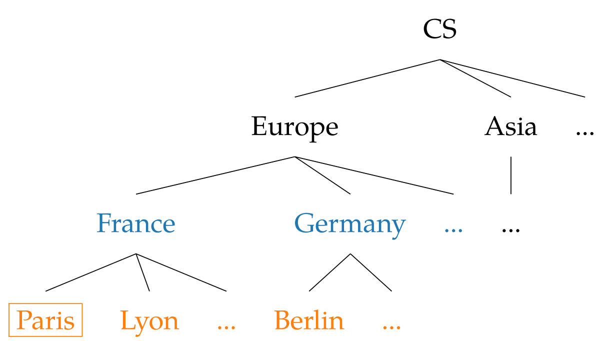


Figure Q: An extra Qtree for (71a), generated by principle (91iii), assuming that (71a)'s tiered set of propositional alternatives is as in (95). .

For simplicity and ease of comparison, we will stick to a tiered set of alternatives involving city- and country-tiers, as defined in (92). Similarly, we can derive Qtrees for (71b)=*Jo grew up in France*. This assertion will in fact require less work, because it appears coarser-grained than (71a). To ensure that (71a) and (71b) are analyzed at the same level of approximation, we assume that (71b)'s alternatives are also of the form *Jo grew up in l*, with *l* a city or a country. The tiered set of propositional alternatives to (71b) is therefore identical to that of (71a), and given in (96).

(96) $\mathcal{A}^{\sim g}_{France,(71b)} = \{\{Paris, Lyon, ..., Berlin, ...\}, \{France, Germany, ...\}\}$
$= \{\{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\},$
$\{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\}\}$
$= \mathcal{A}^{\sim g}_{Paris,(71a)}$

First, we can generate a Qtree for (71b) using principle (91i). This Qtree will have the CS as root, and two leaves corresponding to the propositions that *Jo grew up in France*, and *Jo did not grow up in France* (assuming this matter is not settled in the CS). This Qtree is depicted in Figure R1. Intuitively, it corresponds to the question of whether or not Jo grew up in France.

Second, we can use principle (91ii). To do so, we must determine the set of same-granularity alternatives to the prejacent proposition that *Jo grew up in France*. This set, labeled $\mathcal{A}^{France}_{France,(71b)}$, corresponds to the second element of the tiered set of propositional alternatives $\mathcal{A}^{\sim g}_{France,(71b)}$. It is repeated in (97). This set is also equal to the set of same-granularity alternative to *France*, when the prejacent was *Paris* (see (94)). Thus, the alternatives in this set, once intersected with the CS, already form a partition of the CS. According to principle (91ii), this partition correspond to the leaves of the resulting Qtree. This Qtree is depicted in Figure R2. Intuitively, it corresponds to the question of which country Jo grew up in.

(97) $\mathcal{A}^{France}_{France,(71b)} = \{France, Germany, ...\}$
$= \{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\}$
$= \mathfrak{P}_{\{France, Germany, ...\}, CS}$
$= \mathcal{A}^{France}_{Paris,(71a)}$

Third and lastly, we could use principle (91iii), but this principle would in fact give us nothing more than principle (91ii), given our assumptions about (71b)'s tiered set of propositional alternatives. This is because no proposition in $\mathcal{A}^{\sim g}_{France,(71b)}$ is weaker than p=λw. *Jo grew up in France in* w, and therefore, the only p-chain available in the case of (71b), is made of simply p. This p-chain would generate one single country-layer beyond the CS root, and the resulting Qtree, would simply be the one in Figure R2.[15]

[15]Of course, if we had considered continent-level alternatives as well, principle (91iii) would have generated an extra Qtree for (71b), characterized by a continent-layer on top of a country-layer. But this would have led us to do the same move for (71a), and thus to generate the Qtree in Figure Q for that sentence.

(1) Following principle (91i). (2) Following principle (91ii).

Figure R: Possible Qtrees evoked by the assertion (71b)=*Jo grew up in France*.

Before moving on to "compositional" Qtrees, let us take stock.

First, the recipe in (91) defines way to determine which parse of the CS assertive sentences evoke. We have discussed in Chapter 1 that questions typically correspond to partitions of the CS *in the pragmatic domain*. Semantically, questions are taken to be sets of alternatives. In that sense, Qtrees evoked by sentences should be understood as a form of "inquisitive pragmatics" rather than "inquisitive semantics". Tiered sets of propositional alternatives may be closer to the latter concept.

Second, the recipe in (91) typically generates *multiple* Qtrees out of one assertion. Under our current assumptions, (71a) gives rise to three possible Qtrees, and (71b), to two. So there is some degree of uncertainty about which Qtree any given sentence actually answers. Very roughly, evoked Qtrees can be "polar" (principle (91i)), "*wh*" (principle (91ii)), or "*wh*-articulated" (principle (91iii)). This optionality contrasts with frameworks like inquisitive semantics, in which any given sentence is mapped to a single nonempty downward-closed set of propositions. Given this, our recipe generates more Qtrees than intuitively assumed in the previous Sections. Additionally, this leads us to define the oddness of a sentence as equivalent to the oddness of *all* sentence-Qtree pairs to sentences can generate. This was already defined in (74), repeated below.

(74) *Oddness of a sentence.* A sentence S is odd if any Qtree T it evokes is odd given S.

Third, we mentioned that (71a) and (71b), beyond the fact that they are obviously in a relation of logical entailment, are such that (71a) feels more "fine-grained" than (71b). This is somehow cashed out by the kind of Qtrees these sentences evoke. Specifically, we observe that some Qtrees (71a) evokes (Figure P3) constitute refinements of some Qtree (71b) evokes (Figure R2), where refinement is defined as in (70), repeated below. This implication does not hold in the opposite direction: no Qtree (71b) evokes, constitutes a refinement of a Qtree (71a) evokes. So (in a very weak sense) finer-grained assertions evoke finer-grained Qtrees. This observation will be crucial in Chapter **??**.

(70) *Qtree refinement.* Let T and T' be Qtrees. T is a refinement of T' (or: T is finer-grained than T'), iff T' can be obtained from T by removing a subset $\mathcal{T}$ of T''s subtrees, s.t.:

- if $\mathcal{T}$ contains a subtree rooted in N, then, for each node N' that is a sibling of N in T, the subtree of T rooted in N', is also in $\mathcal{T}$.

We now proceed to define Qtree for complex sentences belonging to the $\{\neg, \vee, \rightarrow\}$-fragment of the language. We will do so inductively, using our recipe for simplex sentences (91) as base case, along with specific combination rules corresponding to the inquisitive effect of each operator.

2.4 Compositional Qtrees: inductive step

In the previous Section, we have seen how to derive Qtrees from simplex sentences, containing no operator or connective. In this Section, we clarify how complex sentences, that may be equally informative, and may even have same propositional meaning, may end up packaging information differently from one another, in terms of their evoked Qtrees. This difference in information packaging, will allow us to derive different felicity profiles for these sentences. We start with Qtrees evoked by negated LFs, before moving on to Qtrees evoked by disjunctions and conditionals.

2.4.1 Questions evoked by negated LFs

We assume negated LFs evoke questions that are structurally similar to those evoked by their non-negated counterpart. The only difference resides in the set of verifying nodes, which is "flipped" by negation. This is formalized in (98).[16]

(98) *Qtrees for negated LFs.* Let T be a Qtree evoked by a LF X. A Qtree $T_\neg$ for $\neg X$ is obtained from T by:

- retaining T's structure; i.e. if $T = (\mathcal{N}, \mathcal{E}, R)$, then $T_\neg = (\mathcal{N}, \mathcal{E}, R)$, too;
- defining $T_\neg$'s set of verifying nodes $\mathcal{N}^+(T_\neg)$ as the set of $T_\neg$'s nodes N that are not verifying in T ($N \notin \mathcal{N}^+(T)$) but belong to a layer containing at least one verifying node N' in T ($N' \in \mathcal{N}^+(T)$). In other words:
$\mathcal{N}^+(T_\neg) = \{N \mid N \notin \mathcal{N}^+(T) \wedge \exists N' \in \mathcal{N}^+(T).\ d(N, T_\neg) = d(N', T_\neg)\}$
With $d(N, T)$ the depth of a node N in a tree T (see (58b)).[17]

[16]This approach is perhaps a bit naive; uttering p vs. $\neg p$, does not seem to preferentially answer the same kind of question. More specifically, it seems that uttering negative statements in general conveys the idea that the original question was more likely to be a polar question of the form *whether p?* – as opposed to a *wh* kind of question. We discuss this more in depth in Chapter **??**.

[17]Because T and $T_\neg$ have same structure, it does not matter which Qtree among T and $T_\neg$ is passed as argument to the depth function; in that particular case, $\forall N \in \mathcal{N}.\ d(N, T) = d(N, T_\neg)$.

The recipe in (98) is exemplified in the abstract Qtrees in Figure S.

(1) An abstract Qtree for X

(2) An abstract Qtree for $\neg X$, derived from Figure S1.

Figure S: An abstract Qtree for X and the abstract Qtree for $\neg X$ derived from it, *via* (98).

It may not seem obvious at this point why and how verifying nodes would occur at intermediate levels in a Qtree; after all, all the Qtrees we have seen so far (derived from simplex sentences) had their verifying nodes at the leaf level. But we will see that Qtrees derived from complex sentences (typically, involving disjunctions and conditionals) can in principle feature intermediate verifying nodes, because such nodes are also derived compositionally. Now, granted that verifying nodes may indeed occur at different levels, the intuition behind the "flipping" algorithm in (98) is the following. If a node N is verifying in a Qtree T corresponding to an LF X, and N is located at depth k in T, then somehow the k-layer of T is "addressed" by X. We aim for a pair $(\neg X, T_{\neg})$ to address the same layers as (X, T), so $T_{\neg}$'s verifying nodes should have similar a similar depth distribution as T's verifying nodes. But of course, the two sets of nodes need to be distinct, because negation standardly flips truth values – hence the by-layer flipping.

It is additionally worth mentioning that, if all verifying nodes in the original Qtree T are leaves, (98) is simplified: $T_{\neg}$'s set of verifying nodes is simply the set of leaves in $T/T_{\neg}$ that are not verifying in T. This is summarized in (99).

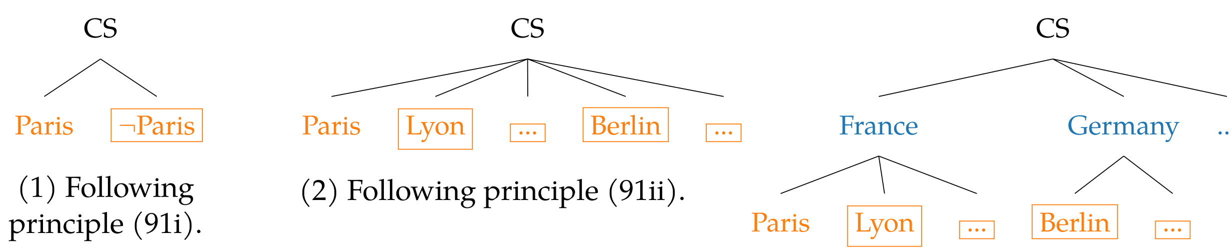
(99) *Qtrees for negated LFs (leaf-only version, subcase of (98)).* Let T be a Qtree evoked by a LF X s.t. $\mathcal{N}^+(T) \subseteq \mathcal{L}(T)$, where $\mathcal{L}(T)$ refers to T's leaves. A Qtree $T_{\neg}$ for $\neg X$ is obtained from T by:

- retaining T's structure;
- defining $T_{\neg}$'s set of verifying nodes as the complement set of $\mathcal{N}^+(T)$ within $\mathcal{L}(T)$: $\mathcal{N}^+(T_{\neg}) = \{N \in \mathcal{L}(T) \mid N \notin \mathcal{N}^+(T)\}$

Following this simplified recipe, Qtrees for (100), which correspond to the negation of (71a), are given below. They are obtained from Figure P, by simply flipping boxed nodes at the leaf level. These new Qtree capture the intuition that (100) can answer three kinds of question: a question about whether or not Jo grew up in Paris; a question about which city Jo grew up in; and a question about which city Jo grew up in question of which city Jo grew up in, but such that this question is decomposed into two subquestions: first, which country Jo grew up in; then, knowing the country, which city Jo grew up in, in that country. The nodes that get flagged as verifying, correspond to sets of worlds disjoint from $\lambda w.$ *Jo grew up in Paris in* w. Interestingly, negation preserves Qtree granularity, simply because it preserves Qtree structure.

(71a) Jo grew up in Paris.

(100) Jo did not grow up in Paris.



(1) Following principle (91i). (2) Following principle (91ii). (3) Following principle (91iii).

Figure T: Possible Qtrees evoked by the assertion (100)=*Jo did not grow up in Paris*.

2.4.2 Questions evoked by disjunctive LFs

Let us consider the disjunction in (101). Intuitively, this sentence is a good, non-maximal answer to a question like (46a), repeated below. It identifies two cities in which Jo could have grown up in, and conveys ignorance about which city Jo actually grew up in. Note that either disjunct taken in isolation, *Jo grew up in Paris*, or *Jo grew up in Lyon*, constitutes a *maximal* answer to (46a).

(46a) In which city did Jo grow up?

(101) Jo grew up in Paris or Lyon.

This observation is consistent with the idea that, in a felicitous disjunction, both disjuncts must answer the same kind of question (**??**). Ou rephrasing of this observation is spelled out in (102).

(102) *Disjunctive answer.* Let $X = Y \vee Z$ be a disjunctive LF. If X is a felicitous assertion, then the set of questions Y answers is equal to the set of questions Z answers. Additionally, if Y/Z answer a question, then X answers it too.

A way to further specify this intuition in our model, is to assume that a Qtree for $X = Y \vee Z$, must contain a Qtree for Y and a Qtree for Z. Containment is understood as the subgraph relation (defined in (69)). This ensures that any node in Y's Qtree is also in X's Qtree, and any node in Z's Qtree, is also in X's Qtree. So, whatever answers Y or Z, also answers X. This is modeled by assuming that the Qtrees evoked by a disjunction are all the possible well-formed unions of Qtrees evoked by each disjunct. This is spelled out in (103). In this definition, Qtree union builds on the notion of graph-union, as formalized in (104).[18] On top of this, Qtree union involves the union of verifying nodes, and the determination of a root node for the output Qtree, defined as the maximum between the two roots of the input Qtrees.

(103) *Qtrees for disjunctive LFs.* A Qtree $T_\vee$ for $X \vee Y$, if defined, is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

- graph-unioning T_X and T_Y;
- defining $T_\vee$'s root as the maximal element (i.e. the weaker proposition) between the root of T_X and the root of T_Y. This will typically be the entire CS. If there is no such maximum, then the output cannot be a Qtree. [19]
- defining $T_\vee$'s verifying nodes as the union of T_X's and T_Y's verifying nodes: $\mathcal{N}^+(T_\vee) = \mathcal{N}^+(T_X) \cup \mathcal{N}^+(T_Y)$.
- returning the output only if it is a Qtree.

In other words, $Qtrees(X \vee Y) = \{T_X \cup T_Y \mid T_X \cup T_Y \text{ verifies (50)} \wedge (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y)\}$

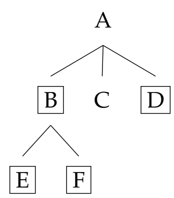
(104) *Graph union.* Let $G = (\mathcal{N}, \mathcal{E})$ and $G' = (\mathcal{N}', \mathcal{E}')$ be two graphs. The union of G and G', noted $G \cup G'$, is the graph $G'' = (\mathcal{N}'', \mathcal{E}'')$ s.t.:

[18] I thank Amir who helped me see this.

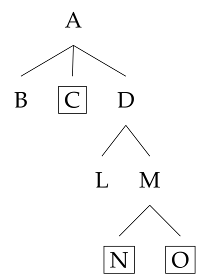
[19] Indeed, suppose R_X and R_Y are the roots of respectively T_X and T_Y, and that R_X and R_Y are not in any kind of inclusion relation. We show by contradiction that $T_X \cup T_Y$ cannot be a Qtree. If $T_X \cup T_Y$ were a Qtree, then, R_X and R_Y would not be in an ancestry relation, meaning, R_X would not be an ancestor of R_Y, and R_Y would not be an ancestor of R_X. So, neither R_X nor R_Y could be the root of $T_X \cup T_Y$, because the root is an ancestor of all the other nodes. Let's call R this root. R is a common ancestor of both R_X and R_Y in $T_X \cup T_Y$. So R must be a strict superset of R_X and R_Y. Also, because $T_X \cup T_Y$ is obtained *via* node- and edge-union, we must have, in the input Qtrees: $R_X \overset{T_X}{\rightsquigarrow} R$ and $R_Y \overset{T_Y}{\rightsquigarrow} R$. In other words, R_X is an ancestor of R in T_X, and R_Y is an ancestor of R in T_Y. Because T_X and T_Y are Qtrees, this implies that R is a strict subset of R_X, and also strict subset of R_Y. Contradiction.

- $\mathcal{N}'' = \mathcal{N} \cup \mathcal{N}'$
- $\mathcal{E}'' = \mathcal{E} \cup \mathcal{E}'$

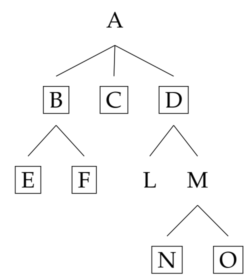
Figure U below exemplifies Qtree union applied to two abstract Qtrees, represented in Figures U1 and U2. In these Qtrees, nodes with different labels are assumed to correspond to a different propositions. By definition, $\{B, C, D\}$ partitions A; $\{E, F\}$ partitions B, $\{L, M\}$ partition D, and $\{N, O\}$ partition M. The disjunction of Figures U1 and U2 is shown in Figure U3. Nodes, edges, and verifying nodes, are unioned, and the output is a Qtree, that contains the two input Qtrees. So, whatever answered either Qtree in Figures U1 and U2, also answers their disjunction in Figure U3.



(1) An abstract Qtree for X



(2) An abstract Qtree for Y.



(3) An abstract Qtree for $X = Y \vee Z$, derived from Figures U1 and U2.

Figure U: Successful attempt at deriving a Qtree from the union of two Qtrees.

It can be shown that if a disjunctive Qtree $T_\vee$ is well-formed and results from the union of two Qtrees T and T' sharing the same root, $T_\vee$ will always constitute a refinement of both T and T'.

What about cases in which the union of two Qtrees, is not a well-formed Qtree? A prediction of (103) is that two Qtrees sharing the same root can be properly disjoined iff they do not involve a common node that gets partitioned in two different ways in the two different input Qtrees.[20] We call this problematic configuration a partition "clash" (or simply a clash). It is formally defined in (105), and related to disjoinability in (106).

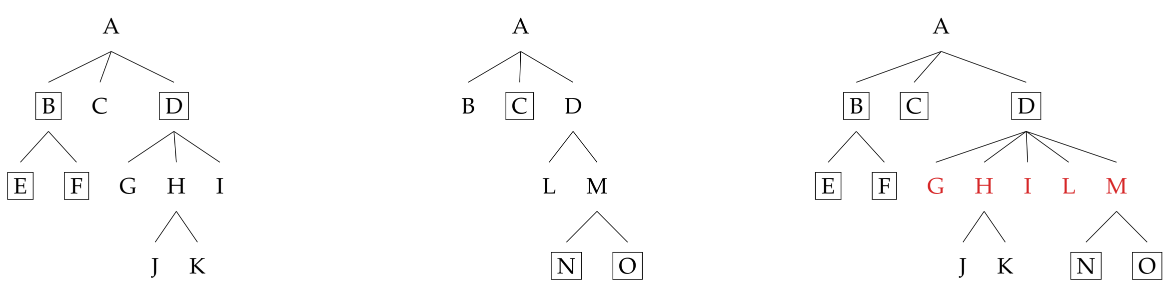
[20]We show that if T and T' exhibit such a clash, their disjunction is not a Q-tree. Let's call C and C' the sets of nodes of resp. T and T' that induce a partition clash; by assumption, C and C' are s.t. $C \neq C'$, and have mothers N and N' s.t. $N = N'$. Because $\vee$ achieves graph-union, $T \vee T'$ will have a node N with $C \cup C'$ as children, and because $C \neq C'$, $C \cup C' \supset C, C'$. Given that both C and C' are partitions of N, $C \cup C'$ cannot be a partition of N. Conversely, if two Q-trees T and T' sharing the same CS as root are s.t. their union $T \cup T'$ is not a Qtree, it must be because T and T' had a partition clash. Indeed, under those assumptions, $T \cup T'$ not being a Qtree means one node N in $T \cup T'$ is not partitioned by its children. Given N is in $T \cup T'$, N is also in T, T', or both. If N was only in, say, T, then it means N's children are also only in T, but then, T itself would have had a node not partitionned by its children, contrary to the assumption T is a Qtree. The same holds *mutatis mutandis* for T', so, N must come from *both* T and T'. Let us call C and C' the partitioning

(105) *Partition clash.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}', R')$ be two Qtrees. T and T' feature a partition clash iff there is $N \in \mathcal{N}$ and $N' \in \mathcal{N}'$ s.t. $N = N'$ but the sets of children of N and N' differ.

(106) *Partition clashes and Qtree disjoinability.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}', R)$ be two Qtrees. T and T' are disjoinable (i.e., their union is a well-formed Qtre) iff T and T' do not exhibit any partition clash.

So, under a recursive interpretation of nodes, two Qtrees with the same root can be disjoined iff, for each node N present in both Qtrees, N's recursive interpretation is the same across Qtrees, or one interpretation constitutes a refinement of the other. This means that, to be disjoinable Qtrees should not introduce different subquestions at the local level.

Figure V illustrates a degenerate case of Qtree union, arising from a partition clash between two abstract input Qtrees. The two input Qtrees, represented in Figures V1 and V2, minimally differ from those in Figures V1 and V2: Figures U2 and V2 are the same, but, in Figure V1, $\{G, H, I\}$ are extra nodes that partition D, and $\{J, K\}$ partitions H. The "clash" between the Qtrees in Figures V1 and V2 comes from the $\{G, H, I\}$ nodes in Figure U1 and the $\{L, M\}$ nodes in Figure U2: these two sets partitions node D in different ways. As a result, the union of these two sets of nodes *cannot* partition D. Figure V3, which represents the disjunction of Figures V1 and V2, thus features nodes $\{G, H, I, L, M\}$ as children of node D, and this configuration violates the partition property of Qtrees. This prevents the tree in Figure V3 from being a well-formed Qtree.



(1) An abstract Qtree for Y

(2) An abstract Qtree for Z.

(3) An abstract Qtree for $X = Y \vee Z$, derived from Figures V1 and V2.

Figure V: Unsuccessful attempt at deriving a Qtree from the union of two Qtrees exhibiting a partition clash.

The badness of this kind of configuration, captures the intuition that two disjoined

introduced by N in resp. T and T'. The fact C, C', but not $C \cup C'$ partition N entails $C \neq C'$, i.e. T and T' feature a partition clash.

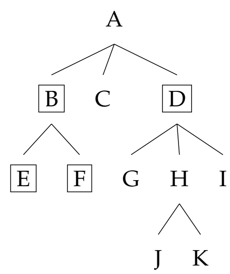
Qtrees should not raise orthogonal issues locally. We call two issues (partitions) orthogonal if they involve two nodes/cells that strictly overlap; see (107). This definition can be shown to be equivalent to that of a partition clash,[21] It is interesting, because it can be more directly related to some concept of RELEVANCE discussed in Chapter 1; see (108).

(107) *Orthogonal partitions.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}, R)$ be two depth-1 Qtrees sharing the same root R (equivalently, two partitions of the same CS). T and T' are orthogonal iff they involves two nodes that are strictly overlapping, i.e. $\exists (N, N') \in \mathcal{N} \times \mathcal{N}'.\ N \cap N' \neq \emptyset \wedge N \neq N'$. T and T' are orthogonal iff T and T' exhibit a partition clash.

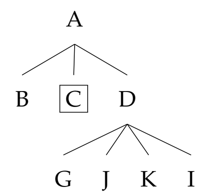
(108) *Orthogonal partitions and relevance.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}, R)$ be two depth-1 Qtrees sharing the same root R. T and T' are orthogonal iff some maximal answer (leaf) of T is not LEWIS-RELEVANT to T'.

Figure W illustrates yet another degenerate case, that may seem more subtle when looking at the two input Qtrees, but with more drastic consequences when looking at the output structure, that is not even a tree. In this example, the two input Qtrees, represented in Figures W1 and W2 clash again at the level of the D node: both $\{G, H, I\}$ $\{G, J, K, I\}$ partition D, but in different ways, since the latter partition is finer grained ($\{J, K\}$ partitions H). This kind of clash, though subtle, generates a disjunctive Qtree that is not even a tree: in Figure W3, J/K is connected to D *via* two distinct paths: directly, and *via* H. So Figure W3 is not acyclic. Zooming out, this degenerate configuration stems from the fact that Qtree union "collapsed" the J and K nodes from the two input Qtrees, and that these nodes, being located at different levels in the two Qtree, were connected differently to the other nodes. This example outlines the idea that, in order to be disjoinable, two Qtrees must match in terms of their layering, i.e. in terms of their degrees of granularity.

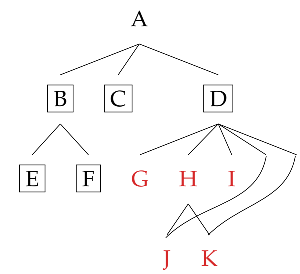
[21] Let us show that if two partitions are different (i.e. involve different cells), then, there is one cell from the former partition and one cell from the latter partition that strictly overlap. Let us assume two partitions P_1 and P_2 are distinct. We show that there is a cell in P_1 and a cell in P_2 that strictly overlap. We consider P_1' and P_2' the partitions obtained from P_1 and P_2 by removing the cells P_1 and P_2 have in common. P_1' and P_2' are not empty, because otherwise P_1 and P_2 would be identical. Moreover, there must be 2 cells c_1 and c_2 in P_1' and P_2' that overlap, because P_1' and P_2' are partitions and as such must be fully covered by their cells. Moreover, c_1 and c_2 cannot be the same, otherwise, they would not be in P_1' and P_2' by construction. So c_1 and c_2 strictly overlap. The other direction of the proof is trivial: if two partitions of the same space P_1 and P_2 involve two strictly overlapping cells, then these two cells must be distinct, and so P_1 and P_2 must be different sets.



(1) An abstract Qtree for Y



(2) An abstract Qtree for Z.



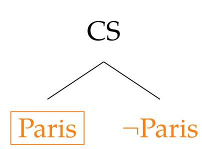
(3) An abstract Qtree for $X = Y \vee Z$, derived from Figures W1 and W2.

Figure W: Yet another unsuccessful attempt at deriving a Qtree from the union of two Qtrees exhibiting a partition clash.

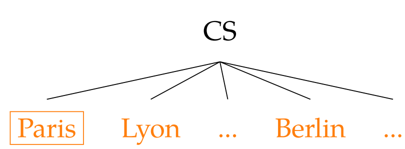
Now that we have defined how disjunctive Qtrees are formed and what the well-formedness conditions for such trees are, we come back to our more concrete disjunctive example (101), repeated below.

(101) Jo grew up in Paris or Lyon.

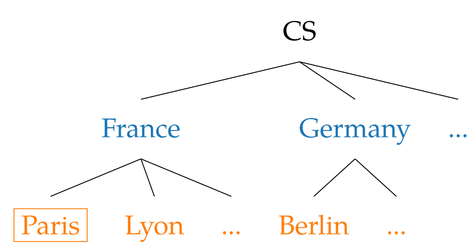
To derive the Qtrees evoked by this disjunctive LF, one must first derive the Qtrees evoked by its two disjuncts, abbreviated *Paris* and *Lyon*. This has been done already in Figure P (repeated in Figure X) for *Paris*. Additionally, *Paris* and *Lyon* have same granularity, and therefore, give rise to the same tiered set of propositional alternatives. This in turn ensures that both *Paris* and *Lyon* give rise to similar Qtrees, that mostly differ in terms of their verifying nodes: *Paris* will flag *Paris*-nodes, and *Lyon*, *Lyon*-nodes. The Qtrees evoked by *Lyon* can be found in Figure Y.



(1) Following principle (91i).

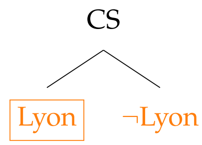


(2) Following principle (91ii).

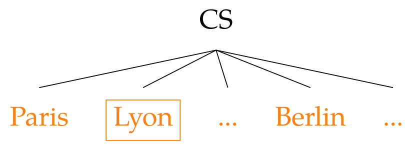


(3) Following principle (91iii).

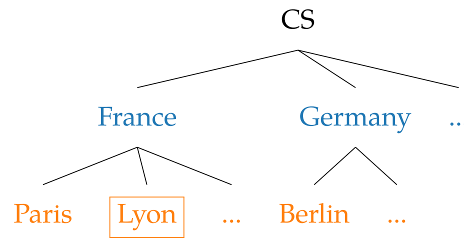
Figure X: Possible Qtrees evoked by the assertion (71a)=*Jo grew up in Paris*.



(1) Following principle (91i).



(2) Following principle (91ii).



(3) Following principle (91iii).

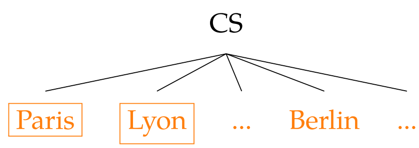
Figure Y: Possible Qtrees evoked by the assertion *Jo grew up in Lyon*.

We could now compute all possible unions of the Qtrees in Figures X and Y, and retain those that are Qtrees. This would effectively yield the Qtree evoked by (101). But instead of computing all these unions, let us use the notion of a partition clash to retain the input Qtrees that will in fact give rise to well-formed disjunctive Qtrees. We now evaluate all pairs of Qtrees from Figures X and Y for partition clashes, and compute unions only if no clash is detected.

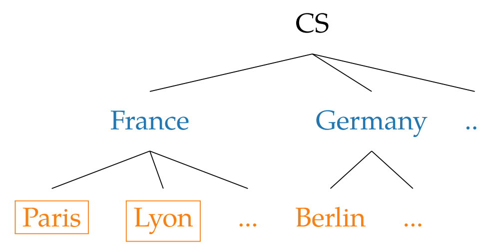
Starting with the two "polar" Qtrees X1 and Y1, we notice an obvious clash between the 2-cells partitions $\{Paris, \neg Paris\}$ and $\{Lyon, \neg Lyon\}$. So we can ignore the union of these two Qtrees. Qtrees X1 and Y2 also clash, because $\{Paris, \neg Paris\}$ and $\{Paris, Lyon...\}$ are different partitions. Again, we ignore this combination. Same holds for Qtrees X1 and Y3, because $\{Paris, \neg Paris\}$ and $\{France, Germany...\}$ are different. We thus once again ignore this combination. From this, we conclude that the "polar" Qtree for *Paris* X1 is not disjoinable with any Qtree *Lyon* evokes. Reciprocally, the "polar" Qtree for *Lyon* Y1 is not disjoinable with any Qtree *Paris* evokes.

Moving on to the "*wh*" Qtree for *Paris* X2, it is structurally identical to the "*wh*" Qtree for *Lyon* Y2. Therefore, these two Qtrees do not clash, and can de disjoined. Their union is given in Figure Z1. Because the two input Qtrees are structurally identical, the structure of the disjunctive output Qtree is also similar. The only difference between inputs and output, is that the nodes flagged as verifying by the output Qtree, are both the *Paris* and the *Lyon* nodes. Considering now the "*wh*" Qtree for *Paris* X2, and the "*wh*-articulated" Qtree for *Lyon* Y3, we notice yet another partition clash: $\{Paris, Lyon, ...\}$ is different from $\{France, Germany, ...\}$. So these two Qtrees cannot be disjoined. Reciprocally, the "*wh*" Qtree for *Lyon* Y2, and the "*wh*-articulated" Qtree for *Paris* X3, will not be disjoinable.

This leaves us with one last pair to evaluate, namely the pair made by the two "*wh*-articulated" Qtrees in Figures X3 and Y3. These two Qtrees are structurally identical, and so can be disjoined. The result of their union is given in Figure Z2. Because the two input Qtrees are structurally identical, the output is also similar. The only difference between inputs and output, is that the nodes flagged as verifying by the output disjunctive Qtree, are both the *Paris* and the *Lyon* node.



(1) Qtree X2 $\vee$ Qtree Y2



(2) Qtree X3 $\vee$ Qtree Y3

Figure Z: Possible Qtrees evoked by the assertion (101)=*Jo grew up in Paris or Lyon*.

Figure Z capture the idea that a disjunction like (101), evokes the same Qtrees as the *wh*-question *In which city did Jo grow up?*: either a simple "*wh*" Qtree partitioning the CS according to cities, or a more complex "*wh*-articulated" Qtree corresponding to the question of which city Jo grew up in, but such that this question is decomposed into two subquestions: first, which country Jo grew up in; then, knowing the country, which city Jo grew up in, in that country.[22] We will see more examples of Qtree disjunctions in the next Chapters, including pathological cases in which the disjuncts may not share the same degree of specificity. We now proceed to define Qtree evoked by conditionals. Crucially, the way such Qtrees will be defined, will not be a function of the "recipes" we just devised for negated and disjunctive LFs. In other words, conditional Qtrees will not be "material".

2.4.3 Questions evoked by conditional LFs

Material implication, defined in (109) is perhaps the simplest way to analyze natural language conditionals.

(109) *Material Implication.* Let X and Y be two LFs denoting p and q respectively. Under the material analysis, $[\![$ *If* X *then* $Y]\!]$ is true iff $\neg p \vee q$ is true. $\rightarrow$ is used as a shorthand for $\lambda p.\ \lambda q.\ \lambda w.\ \neg p \vee q$, s.t. $\neg p \vee q \equiv p \rightarrow q$.

It may be tempting to adapt this definition to the domain of Qtrees evoked by assertions. This tentative translation is given in (110).

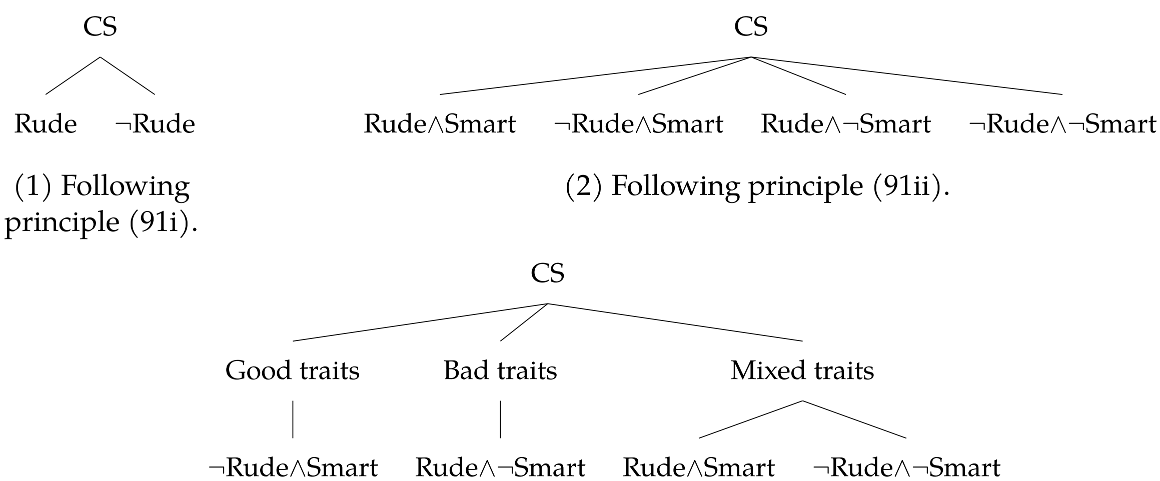
[22] One might wonder at this point why a condition on Qtree disjoinability should not involve structural equality between inputs. After all, the two Qtrees we just derived, depicted in Figure Z, were associated with structurally identical inputs. Chapter **??** will discuss why this identity condition might be too strong, on top of being stipulative.

(110) *"Material" Conditional Qtrees.* Let X be an LF of the form *If* Y *then* Z. A Qtree for X is a Qtree for $\neg Y \vee Z$.

Because we defined Qtrees for negated and disjunctive LFs in the previous Sections, we already have the tools to understand what (110) would predict for Qtrees evoked by natural language conditionals. In particular, we noted that negation preserves Qtree structure, and that disjunction forces the two disjuncts to evoke structurally similar Qtrees. These properties combined, predict that, under (110) the antecedent and consequent of a conditional, should evoke similar Qtrees, devoid of any partition clash. In other words, two Qtrees evoked by X and Y should be "conditionalizable" (in the material sense) iff they are disjoinable. This does not seem to match intuitions about conditionals. (111a) for instance, sounds fine, even if the antecedent *Jo is rude* and the consequent *Jo grew up in Paris*, appear to evoke Qtrees with very different structures. The previous Section already detailed what the latter Qtrees for *Jo grew up in Paris* should looks like, and Figure AA sketches how Qtrees for *Jo is rude* should look like. Clearly, partitions of the CS induced by personality traits, are unlikely to match partitions induced by countries, so under the material analysis, a sentence like (111a) should behave exactly like (111b) at the inquisitive level. Therefore, it should not give rise to any Qtree and should be deemed odd.

(111) a. If Jo is rude, she grew up in Paris.

b. # Jo is not rude, or she grew up in Paris.



(1) Following principle (91i).

(2) Following principle (91ii).

(3) Following principle (91iii).

Figure AA: Possible Qtrees evoked by the assertion *Jo is rude*.

This empirical difference between conditionals and disjunctions regarding the questions they evoke, motivates a non-material model of conditionals at the inquisitive level.

Intuitively, what a conditional statement like (111a) seems to convey, is that figuring out Jo's rudeness may help narrow down where Jo grew up. So, (111a) seems to primarily answer a question about where Jo grew up, taking for granted that she is a rude person. This introduces an asymmetry between antecedent and consequent; it seems that the question evoked by the consequent gets *restricted* to the CS updated with the antecedent. A Qtree for (111a) would then look like the one in Figure AB. In this tree, the Qtree corresponding to the consequent, is "plugged" into the node corresponding to the *Jo is rude* worlds.

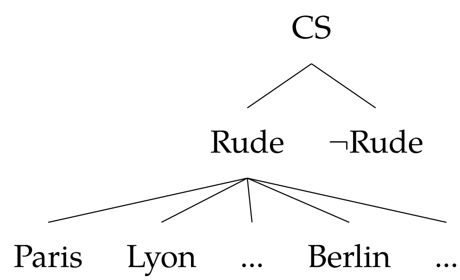


Figure AB: An intuitive Qtree for (111a)= *If Jo is rude, she grew up in Paris*.

We already have the tools to cash out this intuition in a compositional way. Recall that in Section 2.2.3, we discussed how a subtree rooted in N in a given Qtree, could be interpreted as the intersection between the entire Qtree and N, following (64). The definition of this operation is repeated below.

(64) *Tree-node intersection.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a Qtree. Let p be a proposition. The tree-node intersection between T and p, noted $T \cap p$, is defined iff $R \cap p \neq \emptyset$ and, if so, is the Qtree $T' = (\mathcal{N}', \mathcal{E}', R')$ s.t.:

- $\mathcal{N}' = \{N \cap p \mid N \in \mathcal{N} \wedge N \cap p \neq \emptyset\}$
- $\mathcal{E}' = \{\{N_1 \cap p, N_2 \cap p\} \mid \{N_1, N_2\} \in \mathcal{E} \wedge (N_1 \cap p) \neq (N_2 \cap p) \wedge N_1 \cap p \neq \emptyset \wedge N_2 \cap p \neq \emptyset\}$
- $R' = R \cap p$

Tree-node intersection, seen as a form of contextual restriction, in fact allows to "plug" specific Qtrees into the node(s) of another Qtree – producing an output that is still a well-formed Qtree. This gives rise to the definition of conditional Qtrees in (112). This definition defines a Qtree evoked by a conditional *If X then Y*, as a Qtree for X whose verifying nodes get replaced by their intersection with a Qtree evoked by Y.

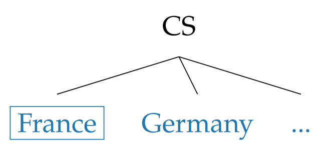
(112) *Qtrees for conditional LFs.* A Qtree T for $X \to Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

- replacing each node N of T_X that is in $\mathcal{N}^+(T_X)$ with $N \cap T_Y$ (see (**??**));
- returning the result only if it is a Qtree.

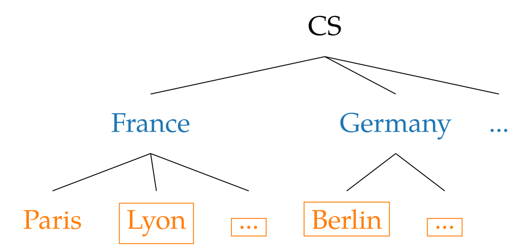
In other words, $Qtrees(X \rightarrow Y) = \{T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)}(N \cap T_Y) | (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y) \wedge T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)}(N \cap T_Y)\text{verifies (50)}\}$, and $\mathcal{N}^+(T_X \rightarrow T_Y) = \{N \cap N' | (N, N') \in \mathcal{N}^+(T_X) \times \mathcal{N}^+(T_Y) \wedge N \cap N' \neq \emptyset\}$.

Let us see what (112) predicts for a conditional Qtree corresponding to (113), assuming the input Qtrees for the antecedent *France*, and the consequent *not Paris*, are those in Figure AC.[23]

(113) If Jo grew up in France, she did not grow up in Paris



(1) A Qtree for(71b)=*Jo grew up in France*, repeated from Figure R2.

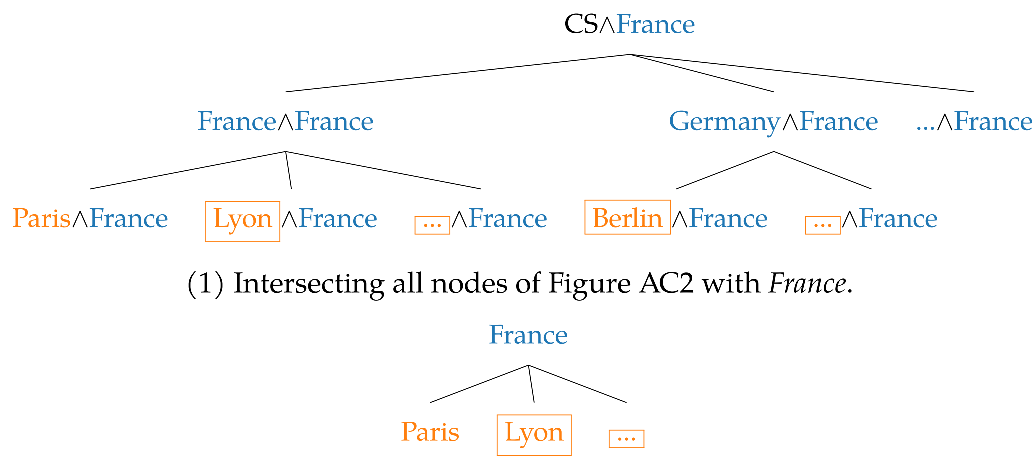


(2) A Qtree for (100)=*Jo did not grow up in Paris*, repeated from Figure T3.

Figure AC: Qtrees corresponding to the antecedent and consequent of (113).

According to definition (112), the conditional Qtree generated from Figure AC, should be Figure AC1, whereby the *France* verifying node gets replaced by its intersection with Figure AC2. This intersection is computed in Figure, following the tree-node intersection intersection principle (64). The result of this operation, is the subtree of Figure AC2 rooted in *France*, in other words, the recursive interpretation of the *France*-node in Figure AC2 – in line with property (65).

[23]Of course, more Qtrees are available for the antecedent and the consequent, and each pairings should be considered when generating Qtrees corresponding to the conditional. We focus on one possible pairing here.



(1) Intersecting all nodes of Figure AC2 with *France*.

(2) Filtering out empty nodes and removing trivial edges.

Figure AD: A Qtree for (100)=*Jo did not grow up in Paris*, intersected with the *France*-node.

The Qtree in Figure AD, can then replace the original *France*-node of Figure AC1 (the antecedent Qtree), to create a Qtree for the conditional assertion (113). This is done in Figure AE.

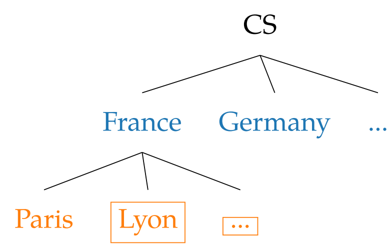


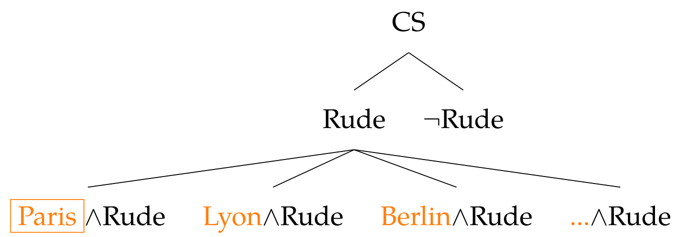
Figure AE: A Qtree for (71b)=*Jo grew up in France*, whose verifying node gets replaced by its intersection with a Qtree for (100)=*Jo did not grow up in Paris*. This creates a Qtree for (113)=*If Jo grew up in France, she did not grow up in Paris*.

It is worth noting that replacing the *France*-node by its intersection with a consequent Qtree in the above example, "erased" the verifying character of the *France*-node. In other words, the verifying nodes of a conditional Qtree, are inherited from its consequent only. The nodes that were verifying in the antecedent Qtree, but were not so in the consequent Qtree, are no longer verifying in the output conditional Qtree.

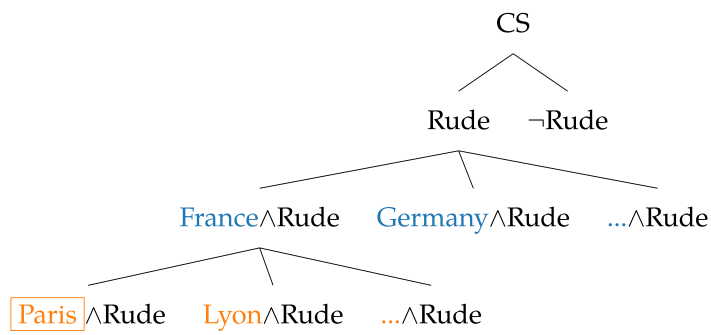
The core idea behind this operation is that conditionals do not make antecedent and consequent QuDs at issue at the same time; rather, they introduce a hierarchy between these two objects, by raising the consequent QuD only in the cells of the CS (as defined by the antecedent QuD), where the antecedent holds. Yet another way to phrase this is by saying that, through the process of Qtree-conditionalization, the consequent Qtree gets *re-*

stricted by the antecedent Qtree. This view is consistent with influential finding in psychology, showing that when asked to verify the truth of a conditional statement, participants tend to massively overlook the eventualities falsifying the antecedent (**?**) It is also consistent with insights from the recent linguistic literature, which argues that zero-models should be disregarding by the semantic module (**?**) – a model where the antecedent of a conditional does not hold is an example of such a zero-model.

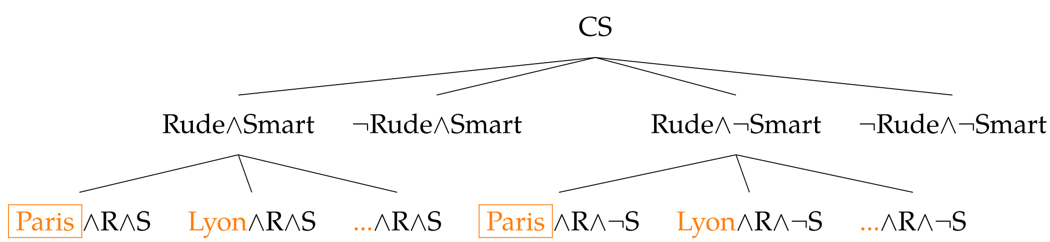
Coming back to the example that motivated this non-material analysis of conditionals at the inquisitive level, a Qtree for (111a), is given in Figure



(1) Conditional Qtree generated from Figure AA1 (antecedent) and P2 (consequent).



(2) Conditional Qtree generated from Figure AA1 (antecedent) and P3 (consequent)



(3) Conditional Qtree generated from Figure AA2 (antecedent) and P2 (consequent)

Figure AF: Possible conditional Qtrees corresponding to (111a)=*If Jo is rude, she grew up in Paris*. Other Qtrees are possible.

Lastly, we observe that node-Qtree intersection between a (non-empty) node N and a tree T, is "vacuous" (i.e., equal to N), iff N entails a specific leaf in T.[24]

[24]Vacuousness is defined structurally only. The verifying status of N will still depends on T's verifying

(114) *Vacuous tree-node intersection.* Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). $T \cap N = N$ iff $\exists N'\mathcal{L}(T).\ N \vDash N'$.

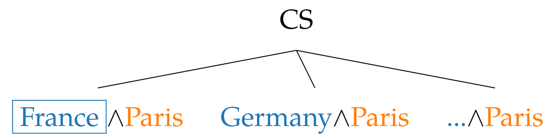
(115) *Proof of (114).* Let T be a Qtree and N a node entailing some leaf in T. T being a Qtree, each node of T is s.t. its ancestors are exactly the nodes entailed by it, its descendants are exactly the nodes entailing it, and other nodes are incompatible with it. If N entailed two or more leaves in T, then N would entail a contradiction, i.e. be empty. So N entails a single leaf L, and all the nodes in T entailed by N must correspond to a path from CS root, to L. Intersecting all such nodes with N, yields N. Intersecting N with any other node, yields the empty set. Therefore, intersecting N with T leads to a single N-root.
Let T be a Qtree and N a node s.t. $T \cap N = N$. We show that N entails some leaf in T by contradiction. If no leaf in T were entailed by N, then at least some leaf in T, when intersected with N, would yield a node different from N. Therefore, $T \cap N$, cannot be equal to N. Contradiction.

The whole conditional Qtree formation process will then be vacuous if each verifying leaf in the antecedent Qtree entails a specific leaf of the consequent Qtree. This configuration is exemplified when considering a sentence like (116).

(116) ?? If Jo grew up in Paris, she grew up in France.

In that sentence, Qtrees corresponding to the antecedent stop at the city-level and mark the *Paris*-node as verifying. Qtrees corresponding to the consequent stop at the country-level, and contain a *France*-leaf. Because *Paris* entails *France*, intersecting a Qtree for the consequent with the "restrictor" *Paris*-node from the antecedent, will simply yield the Paris node. This is shown in Figure AG. In that case, the node resulting from the intersection operation inherits its verifying status from the *France*-node, flagged by the input (consequent) Qtree. Replacing the verifying *Paris*-node in the antecedent Qtree by this node, does not have any effect. Therefore, the output conditional Qtree will be identical to the antecedent Qtree.

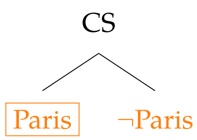
nodes.



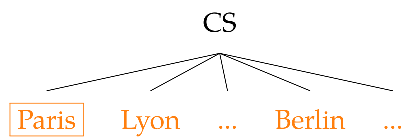
(1) Intersecting all nodes of Figure R2 with *Paris*.

(2) Filtering out empty nodes and removing trivial edges.

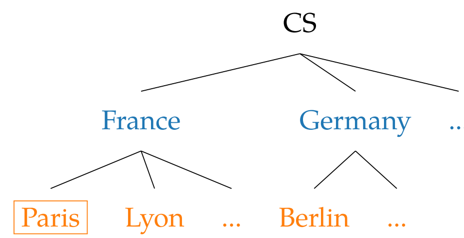
Figure AG: A Qtree for (71b)=*Jo studied in France*, intersected with the *Paris*-node, is just the *Paris*-node itself.



(1) .



(2) .



(3) .

Figure AH: Possible Qtrees evoked by the conditional assertion (116)=*If Jo grew up in Paris, she grew up in France*. Same as those evoked by (71a)=*Jo grew up in Paris*, due to tree-node intersection being vacuous across-the-board.

Moreover, if each verifying leaf in the antecedent Qtree entails a specific *non-verifying* leaf of the consequent Qtree, the output Qtree will be structurally identical to the antecedent Qtree but, will be left with *no* verifying node. Such a tree will be deemed ill-formed as per principle (73) (pertaining to the empty labeling of verifying nodes).

2.5 Conceptual predecessors

This Section compares the model of questions previously introduced, to three earlier approaches exploiting concepts close to Qtrees. It is shown that these earlier models differ from the current framework in three possible ways: (i) question semantics is taken to fully *replace* standard propositional content (the Inquisitive Semantics framework); (ii) the core model is technically very similar, but at the conceptual level assertions are not taken to evoke full-fledged questions (**?**), or (iii) the machinery proposed is based on evoked QuDs, but not fully compositional (**?**).

2.5.1 Inquisitive Semantics

In Inquisitive Semantics (**?**, **?**; **?**, **?**; **?**, **?**; **?**, **?**; **?**, **?**; **?**, to appear), assertive sentences and questions are fundamentally the same kinds of objects: sets of propositions, called "inquisitive" propositions. Inquisitive propositions are assumed to be downward closed, which means that, if p is part of an inquisitive proposition, then any subset of p also belongs to that inquisitive proposition. DAG.

Under the current view, sentences retain a "semantic", truth-conditional component, and evoke Qtrees at a distinct "inquisitive" level. While the semantic module is sensitive to truth conditions, the pragmatic module is assumed to be sensitive to the interaction between form, meaning, and inquisitive content. So our approach may be seen as an "inquisitive pragmatics". The difference between the two frameworks is particularly visible when it comes to negation: in Inquisitive Semantics, negation removes structure by collecting and collapsing all information states incompatible with those of the prejacent. In our framework, negation retains structure, and simply flips verifying nodes.

Additionally, previous accounts were mostly focused on disjunctions, or more generally, configurations where two subconstituents could be taken to answer the same QuD.[25] But it remained unclear how the overarching question was derived in each case, and whether it should be in the first place. Our system also fills this gap, in providing a set of recipes to compositionally derive implicit QuDs, instead of taking them for granted. Together with Q-NON-REDUNDANCY, this machinery captured the target contrasts. The next Chapters will further show how it generalizes beyond the datapoints discussed here.

2.5.2 Ippolito's Structured Sets of Alternatives

? proposes a model of alternatives that is very close in its implementation to the Qtree model proposed in the first half of this Chapter. Under **?**'s view, the way alternatives are structured is seen as a source of oddness. This approach will be shown to differ from ours in two respect: first, sentences are not taken to evoke full-fledged questions (a mainly conceptual difference); second, it leaves unexplained when, and how, sets of alternatives can be combined, cross-sententially and in biclausal sentences.[26]

[25] **?** in fact argues that this comprises Sobel Sequences and sequences of superlatives. We will discuss SObel cases in Chapter **??**.

[26] A third issue will be raised in the upcoming Chapter, namely that under Ippolito's view, oddness arises from a purely structural constraint (the *Specificity Constraint*), that appears independent from familiar competition-based pragmatic principles.

?'s goal was to provide a unified analysis of a number of seemingly independent instances of pragmatic oddness, taking the form of Sobel sequences (117), sequences of superlatives (118), and Hurford Disjunctions (119).

(117) a. If the USA had thrown their nuclear weapons into the sea, there would have been war. But if all the nuclear powers had thrown their weapons into the sea, there would have been peace.

b. # If all the nuclear powers had thrown their nuclear weapons into the sea, there would have been peace. But if the USA had thrown their weapons into the sea, there would have been war.

(118) a. The closest gas stations are crummy; but the closest Shell stations are great.

b. # The closest Shell stations are great; but the closest gas stations are crummy.

(119) a. John ate some of the cookies or all of them.

b. # John ate all of the cookies or some of them.

These three classes of sentences share commonalities. In all three configurations, two sentences or fragments are being contrasted using connectives like *but* and *or*. For instance, in the Sobel case (117a), *If the USA had thrown their nuclear weapons into the sea, there would have been war* gets contrasted with *If all the nuclear powers had thrown their nuclear weapons into the sea, there would have been peace*. Additionally, in all three cases, the two sentences being contrasted exhibit some degree of parallelism, in the sense that they each contain a subconstituent C/C^+, such that $[\![C^+]\!] \vdash [\![C]\!]$. For instance, *all the nuclear powers had thrown their nuclear weapons into the sea*, entails that *the USA had thrown their nuclear weapons into the sea*. Lastly, all configurations are such that the a. examples, which start with the sentences containing the "weaker" C, appear more felicitous than the b. examples, which start with the sentences containing the "stronger" C^+.

To account for these asymmetries, **?** proposes that the alternatives evoked by assertive sentences form "structured sets". Such sets, abbreviated SSAs, are defined in (120). The kind of structures generated by this definition are in essence recursive partitions of the CS, or Qtrees, as defined in (50).[27]

[27] This is what at least is argued in **?**. It is worth mentioning however, that the definition in (120) does not in itself guarantee that any Structured Set of Alternatives should form a tree. Instead, it guarantees that any branching of the form $[_\alpha \beta_1 ... \beta_n]$ is s.t. $(\beta_i)_{i \in [1;n]}$ partitions α. But nothing in principle guarantees the connectedness of the structure: if specific alternatives happen to be "missing" (for relevance/QuD-related reasons, or perhaps due to a missing lexicalization), then, the resulting Structured Set of Alternatives may end up being a forest, instead of a single tree.

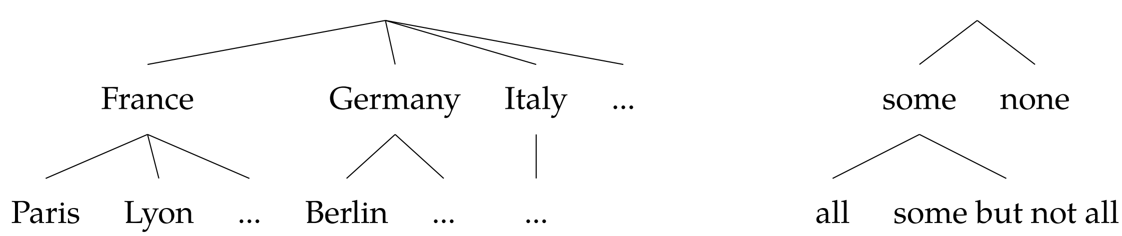
(120) *Structured Set of Alternatives* (*SSA*) (**?**). $T_{\mathcal{A}}$ is a well-formed structured set of alternatives iff the following conditions are met:

- Strength: for any two alternatives α, $\beta \in \mathcal{A}$, β is the daugther of α in $T_{\mathcal{A}}$ just in case $[\![\beta]\!] \subset [\![\alpha]\!]$.
- Disjointness: for any two alternatives β_1, $\beta_2 \in \mathcal{A}$, if β_1 and β_2 are sisters in $T_{\mathcal{A}}$, then $[\![\beta_1]\!] \cap [\![\beta_2]\!] = \emptyset$
- Exhaustivity: for any alternative α with daughters $\beta_1, ...\beta_n$, in $T_{\mathcal{A}}$, $[\![\beta_1]\!] \cup [\![\beta_2]\!] \cup ... \cup [\![\beta_n]\!] = [\![\alpha]\!]$

Alternatives evoked by an assertion are modeled following **?**, i.e. assumed to be obtained by substituting the original sentence's focused material by any expression of the same type. This is spelled out in (121).

(121) *Focus alternatives* (**?**). Let S be a sentence containing a focused element α. The set of focus alternatives to $[\![S]\!]$ is the set of propositions $[\![S']\!]$, where S' is obtained from S by substituting α with any element of the same type as α.

Figure AI illustrates SSAs for simple sentences containing scalar and non-scalar alternatives. It is worth noting that sentences associated with different degrees of granularity (e.g. *Jo grew up in Pairs* vs. *Jo grew up in France*) are not expected to give rise to different SSAs, as shown in Figure AI1. Same holds for scalar sentences in an entailment relation (e.g. *Jo ate some of the cookies* vs. *Jo ate all of the cookies*).



(1) SSA associated with *Jo grew up in Paris$_F$/France$_F$/Italy$_F$* etc.

(2) SSA associated with *Jo ate some$_F$/all$_F$/none$_F$ of the cookies.*

Figure AI: SSAs for simple focused sentences.

Additionally, alternatives are assumed to be constrained by "the" QuD. This constitutes the first, conceptual difference with our account introduced earlier in this Chapter: under **?**'s view, assertions are not assumed to help determine "the" QuD; instead, they are assumed to evoke alternatives, which are themselves constrained by "the" QuD.

In other words, SSAs are not expected to help determine what "the" QuD is–they are partially derived from it. This is far from an esoteric perspective, and appears in line with much past literature. What we want to propose instead, is the reverse perspective: assertions and their alternatives are the primitive, and help *derive* potential QuDs (along with contrasts in pragmatic oddness).

? then proposes that oddness arises from certain SSA configurations. In particular, sequences of sentences belonging to the same SSA are subject to a Specificity Constraint (henceforth **SC**), spelled out in (122). The SC states that the two alternatives in the sequence, should be dominated by the same number of nodes in their common SSA. This is equivalent to saying that two alternatives being contrasted should match in terms of their degree of specificity, or granularity.

(122) *Specificity Condition* (**?**). A sequence $\Sigma = < [_{S_i} ...\alpha_F...], [_{S_j} ...\beta_F...] >$, s.t. both S_i and S_j are answers to the same QuD and β is in the structured set of alternatives evoked by α ($T_{\mathcal{A}_\alpha}$), is felicitous if either:

- α or β is the only node on its branch in $T_{\mathcal{A}_\alpha}$, or
- α and β are dominated by the same number of nodes in $T_{\mathcal{A}_\alpha}$.

A sentence like (119b) then violates the SC, because its "*all*" and its "*some*" disjunct are respectively dominated by 2, and 1 node in the corresponding SSA from Figure AI2. The SC therefore correctly predicts (119b) to be odd. But, because (119a) only differs from (119b) in how the disjuncts are ordered, the SC also incorrectly predicts (119a) to be odd–at least in the absence of any additional assumptions.

The felicity of (119a) is captured in **?**'s framework based on the familiar idea that violations of the SC can be avoided by strengthening the weaker alternative (**?????**). To retain the *contrast* between (119b) and (119a), it is assumed that covert strengthening is governed by an economy condition, which disallows it in (119b). This is shown to generalize to the a. and b. sequences in (117-118).

Even though the SC appears like a reasonable constraint, the deep reason why contrast alternatives with different degrees of specificity should be disallowed, remains relatively mysterious. In particular, the account does not directly relate the SC to general pragmatic principles based on competition *between* sentences: the SC is a constraint that is only sensitive to the SSA associated with the target sentence, independently of the sentence's competitors and their own SSAs. In that respect, it remains close to Hurford's original constraint. Moreover, the constraint amounts to counting the number of parent nodes for each contrasted alternative, and as such is not sensitive to the relative positions of the two alternatives within their common SSA. This perspective might be slightly reductive, and would not capture the observation that oddness gets stronger if the two alternatives are in a dominance relation, as shown by gradience of the judgments in (123).

(123) a. # Jo grew up in Paris or France. — Different specificity, dominance
b. ? Jo grew up in Paris or Germany. — Different specificity,no dominance
c. Jo grew up in France or Germany. — Same specificity,no dominance

In Chapter **??**, we will propose a constraint akin in effect to the SC, but that will constitute a more direct extension of earlier Redundancy-based constraints used to capture Hurford Disjunctions. We will show in Chapter **??** how it applies to basic (non-scalar) Hurford Disjunctions and extends to another challenging family of intuitively redundant sentences. Chapters **??** and **??** will discuss the particular case of scalar Hurford Disjunctions like (119), and extend the account to scalar Sobel sequences.

2.5.3 Buring's and Zhang's (constrained) QuD trees

2.6 Conclusion

In this Chapter, we have build a model of overt and evoked questions that incorporates the concept of specificity *within* the formalism. We have defined how this model derives implicit questions evoked by simple, negated, disjunctive and conditional sentences, in a compositional way. Doing so, we attempted to maintain a few intuitions about the truth-conditional effect of negation, disjunction, and conditionals. For instance, negation "flips" verifying nodes as it would flip truth-values; disjunction forces some parallelism between disjuncts; and conditional act as questions "restrictors". We then discussed how this approach compared to earlier similar approaches: Inquisitive Semantics, Structured Sets of Alternatives, and QuD trees.

From a conceptual perspective, the machinery presented is in fact closer in spirit to Dynamic Semantics (**??**), where different operators give rise to different incremental updates of the Context Set. Under our view, different operators will give rise to different *parses* of the Context Set, at the inquisitive level. This will eventually allow to capture a contrast between (43), (44) (see Chapter **??**), and many other cases, including (113), (116) (see Chapter 3).

Chapter 3

Crossing countries: oddness in non-scalar Hurford Conditionals[1]

This Chapter extends the empirical landscape introduced in Chapter **??**, which was mainly interested in Hurford Disjunctions, of the form $p \vee p^+$ or $p^+ \vee p$, with $p^+ \vDash p$. This Chapter is an investigation of Hurford *Conditionals* (**??**), of the form *If* p *then* $\neg p^+$ or *If* $\neg p^+$ *then* p, with $p^+ \vDash p$. Such conditionals are related to Hurford Disjunctions by the *or*-to-*if* tautology; but (arguably) unlike their disjunctive counterparts, they exhibit a crisp asymmetry: variants in which the negated stronger proposition is in the antecedent (*If* $\neg p^+$ *then* p) are degraded, while variants in which the negated stronger proposition is in the consequent (*If* p *then* $\neg p^+$), are fine. This Chapter explains this asymmetry, by building once again on the implicit QuD framework introduced in Chapter 2, and by proposing a new constraint on Qtree derivation, dubbed Incremental Q-Relevance, building on **?**'s and **?**'s approaches to propositional Relevance. This will predict that the antecedent and consequent of Hurford Conditionals should be ordered in terms of their conveyed degree of granularity; and will be shown to successfully extend to variants of these sentences. More broadly, this Chapter suggests that Hurford Disjunctions and Conditionals, display different "flavors of oddness".

[1]This Chapter builds on **?** (to appear), but develops a different (hopefully clearer and more principled) view on Hurford Conditionals. A number of similar intuitions will be exploited, including intuitions about granularity. I would like to thank the audiences and reviewers of the 2024 BerlinBrnoVienna Workshop, SuB29, and the 2024 Amsterdam Colloquium for relevant questions, datapoints and suggestions regarding this project and adjacent data. I want to give specials thanks to Amir, who first advised me to read **?** almost two years ago, and Viola, who very wisely advised me to take another look at it this Spring.

3.1 Introducing Hurford Conditionals

Let us start by reviewing familiar data. As discussed in Chapter **??**, Hurford Disjunctions (henceforth **HDs**), exemplified in (124), feature contextually entailing disjuncts ($p^+ \vDash_c p$), and are generally odd regardless of the order of their disjuncts (**?**).[2]

(124) a. # SuB29 will take place in Noto or in Italy. $\mathbf{p}^+ \vee \mathbf{p}$

b. # SuB29 will take place in Italy or in Noto. $\mathbf{p} \vee \neg\mathbf{p}^+$

? observed an intriguing, crisp contrast in so-called Hurford *Conditionals* (henceforth **HCs**), exemplified in (125): (125a) is odd while (125b) is fine. As a side note, $\rightarrow$ will be used as a shorthand for *if... then...*, throughout this Chapter (just like we did in Chapter **??**), i.e. $\rightarrow$ will not imply that the conditionals under consideration are necessarily considered material.

(125) a. # If SuB29 will not take place in Noto, it will take place in Italy. $\neg\mathbf{p}^+ \rightarrow \mathbf{p}$

b. If SuB29 will take place in Italy, it will not take place in Noto. $\mathbf{p} \rightarrow \neg\mathbf{p}^+$

Why is the contrast in (125) intriguing? Granted conditionals are material, the HC in (125a) is equivalent to the HD in (124a), and can also be made structurally equivalent to it *modulo* double-$\neg$ elimination. This is shown in (126). Therefore, it is perhaps unsurprising that (125a) appears degraded – it can be understood as an HD "in disguise".

(126) (125a) $= \neg\mathbf{p}^+ \rightarrow \mathbf{p}$

$\equiv \neg(\neg\mathbf{p}^+) \vee \mathbf{p}$ (*or*-to-*if* tautology)

$\equiv \mathbf{p}^+ \vee \mathbf{p}$ (double-$\neg$ elimination)

$=$ (124a)

The surprise comes from (125b). Just like (125a), (125b) can be seen as a conditional of the form $\neg q^+ \rightarrow q$, with $q^+ \vDash q$, by taking q^+ to be the proposition that *SuB29 will not take place in Italy* (so $q^+ = \neg p$), and q to be the proposition that *SuB29 will not take place in Noto* (so $q = \neg p^+$). Again, we have $q^+ \vDash q$, because $p^+ \vDash p$, and negation reverses entailments. This is all detailed in (127).

(127) (125b) $= \mathbf{p} \rightarrow \neg\mathbf{p}^+$

$\equiv \neg(\neg\mathbf{p}) \rightarrow (\neg\mathbf{p}^+)$ (double-$\neg$ introduction)

$\equiv \neg\mathbf{q}^+ \rightarrow \mathbf{q}$ ($\mathbf{q}^+ := \neg\mathbf{p}$; $\mathbf{q} := \neg\mathbf{p}^+$; s.t. $\mathbf{q}^+ \vDash \mathbf{q}$)

$\approxeq$ (125a)

[2] When the two disjuncts are the same modulo scalar expressions (e.g. $\langle$*some*, *all*$\rangle$) HDs *may* be rescued from infelicity **?**, **?**; **?**, **?**; **?**, **?**; **?**, **?** i.a.). Chapter **??** provides an overview of the challenges raised by "scalar" Hurford Sentences, and proposes an account elaborating on the framework introduced here.

(125b) is thus structurally similar to (125a), though not logically equivalent to it. So, one could say (125a) and (125b) are "isomorphic", in the sense that they have same logical structure, and can be derived from each other *via* a variable change preserving logical relations (see (128) for a formal definition).

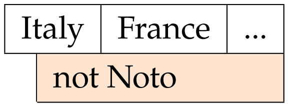
(128) **Isomorphy**. Let X and Y be two LFs. X and Y are isomorphic ($X \approxeq Y$) iff there is a substitution operation $\mathcal{S}$ targeting atomic propositions and preserving the logical relations between the elements in its domain ($aRb \iff \mathcal{S}(a)R\mathcal{S}(b)$, where R denotes entailment, contradiction, or independence) s.t. X and $\mathcal{S}(Y)$ have same parse.

This isomorphy between the infelicitous HC (125a) and the felicitous HC (125b) is problematic, because if (125a) is indeed an HD "in disguise", then so should (125b). Yet, this variant is felicitous. It appears that oddness in HCs is "asymmetric"; descriptively, the weaker item must be the antecedent, while the negated stronger item must be the consequent.

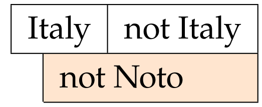
? proposed the first solution to both the HDs (124) and the HCs (125) (as well as other related datapoints). The approach was based on the idea that overt negation has a special status when it comes to evaluating if a sentence is redundant.

This Chapter argues for an alternative view, building on the idea that the questions evoked by an assertion must match the degree of granularity is conveys. The source of pragmatic oddness in HCs will then be tied to "granularity" violations, operationalized in terms of an incremental Relevance constraint. Roughly, this constraint will state that, whenever the question evoked by an assertion gets "restricted" to a certain domain of the Context Set, the domain must be Relevant to the question in a relatively standard sense, blending aspects of **?**'s and **?**'s view on Relevance. The directionality of this constraint will be directly tied to the computation of conditional Qtrees, which Chapter 2 defined as a kind of question-restriction.

According to this constraint, the HC in (125a) will be deemed deviant, essentially because its antecedent (*not Noto*) does not rule out any cell from any question evoked by its consequent (*Italy*). This is sketched in Figure A1.



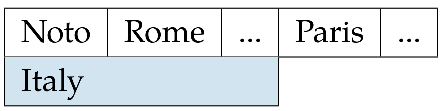
(1) (125a)'s consequent is not Relevant to the "*wh*"-partition evoked by (125a)'s antecedent.



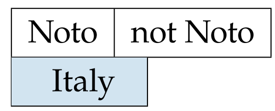
(2) (125a)'s consequent is not Relevant to the "polar"-partition evoked by (125a)'s antecedent.

Figure A: How (125a)'s antecedent interacts with (125a)'s consequent's evoked questions.

The HC in (125b) on the other hand, will be predicted to be fine, because its antecedent (*Italy*), interacts with the by-city partition evoked by its consequent (*not Noto*) in the following way: first, it rules out some cells – namely, all non-Italian city-cells – and second, it rules in some cells – namely, all Italian cities. This is sketched in Figure B1.



(1) (125b)'s antecedent is Relevant to the "*wh*"-partition evoked by (125b)'s consequent.



(2) (125b)'s antecedent is not Relevant to the "polar"-partition evoked by (125b)'s consequent.

Figure B: How (125b)'s antecedent interacts with (125b)'s consequent's evoked questions.

The contrast between the two HCs will therefore arise from an interaction between our novel Relevance constraint, and the maximal level of granularity evoked by propositions, meaning, how fine-grained the leaves of their Qtrees can be.

This Chapter is structured as follows. Section 3.2 provides an overview of **?**'s approach to HCs, and outlines some of its limitations. Section 3.3 uses the machinery introduced in Chapter 2 to derive questions evoked by HCs. Section 3.4 motivates and defines Incremental Q-Relevance on LF-QuD pairs and shows how this constraint captures the HCs in (124) and (125). Section 3.5 explores variants of HCs and shows how Incremental Q-Relevance captures them. Section 3.6 concludes and outlines remaining issues and questions.

3.2 Existing account

? show that HCs are problematic for virtually all accounts of HDs and their variants proposed before **?**. Therefore, we will not review these approaches here, and directly jump to

?'s recent proposal.

3.2.1 Super-Redundancy

?' Super-Redundancy, repeated in (**??**) from Chapter **??**, states that a sentence S is super-redundant if it features a binary operation taking a constituent C as argument, and moreover there is no way of strengthening C to C^+ that would make the resulting sentence S^+ non-redundant (i.e., non-equivalent to its counterpart where C^+ got deleted).

(**??**) **Super-Redundancy** (**?**). A sentence S is infelicitous if it contains $C * C'$ or $C' * C$, with $*$ a binary operation, s.t. $(S)^-_C$ is defined and for all D, $(S)^-_C \equiv S_{Str(C,D)}$. In this definition:

- $(S)^-_C$ refers to S where C got deleted;
- $Str(C,D)$ refers to a strengthening of C with D, defined inductively and whose key property is that it commutes with negation $(Str(\neg\alpha, D) = \neg(Str(\alpha, D)))$, as well as with binary operators $(Str(O(\alpha,\beta), D) = O(Str(\alpha, D), Str(\beta, D)))$;
- $S_{Str(C,D)}$ refers to S where C is replaced by $Str(C,D)$.

As already shown in Chapter **??**, this constraint can capture HDs (124). The proof, adapted from **?**, is repeated in (**??**).

(**??**) HDs are Super Redundant (SR).
We show (**??**)=$\mathbf{p}^+ \vee \mathbf{p}$ and (**??**)=$\mathbf{p} \vee \mathbf{p}^+$ are SR.
In either case, take C = $\mathbf{p}^+$.
We then have (**??**)^-_C = (**??**)^-_C = $\mathbf{p}$

$$
\begin{aligned}
\forall D.\ (\textbf{??})_{Str(C,D)} = (\textbf{??})_{Str(C,D)} &= (\mathbf{p}^+ \wedge D) \vee \mathbf{p} \\
&\equiv (\mathbf{p}^+ \vee \mathbf{p}) \wedge (D \vee \mathbf{p}) \\
&\equiv \mathbf{p} \wedge (D \vee \mathbf{p}) \\
&\equiv (\mathbf{p} \wedge D) \vee \mathbf{p} \\
&\equiv \mathbf{p} = (\textbf{??})^-_C = (\textbf{??})^-_C
\end{aligned}
$$

More interestingly perhaps, (**??**) also captures HCs, whether conditionals are assumed to be material, or strict. The proofs assuming material conditionals, are given in (129) for (125a) and (130) for (125b). In both cases, it is crucial that the local strengthening of $C = p^+$ be conjunctive *under* negation (and thus, disjunctive after applying De Morgan's law). In (129), this allows to remove p^+ from the chain of logical equivalences, and eventually derive Super-Redundancy. In the second part of (130) when $C = \neg p^+$, this ensures that the strengthening D *can* be disregarded, and that the equivalence does *not* obtain – eventually deriving a failure of SUPER-REDUNDANCY. **?** also shows that this account extends to strict (yet not variably strict) conditionals. We omit the proof here for brevity.

(129) Assuming implications are material, "strong-to-weak" HCs like (125a) are Super Redundant (SR).

We show (125a)=$\neg\mathbf{p}^+ \to \mathbf{p}$ is SR.

Take C = $\neg\mathbf{p}^+$.

We then have $(125a)^-_C = \mathbf{p}$.

$$\begin{aligned}\forall D.\ (125a)_{Str(C,D)} &= \neg(\mathbf{p}^+ \wedge D) \to \mathbf{p}\\ &\equiv (\mathbf{p}^+ \wedge D) \vee \mathbf{p}\\ &\equiv (\mathbf{p}^+ \vee \mathbf{p}) \wedge (D \vee \mathbf{p})\\ &\equiv \mathbf{p} \wedge (D \vee \mathbf{p})\\ &\equiv \mathbf{p} \wedge (D \vee \mathbf{p})\\ &\equiv \mathbf{p} = (125a)^-_C\end{aligned}$$

(130) Assuming implications are material, "weak-to-strong" HCs like (125b) are not Super Redundant (SR).

We show (125b)=$\mathbf{p} \to \neg\mathbf{p}^+$ is not SR.

Take C = (125b)'s antecedent = $\mathbf{p}$.

We then have $(125b)^-_C = \neg\mathbf{p}^+$.

Take $D = \bot$.

$$\begin{aligned}(125b)_{Str(C,D)} &= (\mathbf{p} \wedge D) \to (\neg\mathbf{p}^+)\\ &\equiv (\mathbf{p} \wedge \bot) \to (\neg\mathbf{p}^+)\\ &\equiv \bot \to (\neg\mathbf{p}^+)\\ &\equiv \top\\ &\not\equiv \neg\mathbf{p}^+ = (125b)^-_C\end{aligned}$$

Take C = (125b)'s consequent = $\neg\mathbf{p}^+$.

We then have $(125b)^-_C = \mathbf{p}$.

Take $D = \top$.

$$\begin{aligned}(125b)_{Str(C,D)} &= \mathbf{p} \to (\neg(\mathbf{p}^+ \wedge D))\\ &\equiv \mathbf{p} \to (\neg(\mathbf{p}^+ \wedge \top))\\ &\equiv \mathbf{p} \to (\neg\mathbf{p}^+)\\ &\equiv (\neg\mathbf{p}) \vee (\neg\mathbf{p}^+)\\ &\equiv \neg\mathbf{p}^+\\ &\not\equiv \mathbf{p} = (125b)^-_C\end{aligned}$$

This approach is compelling regarding its empirical coverage, but raises one conceptual interrogation. While earlier approaches to Redundancy (**?**, **?**; **?**, **?**; **?**, **?** i.a.) link it to the concept of Brevity in the sense of **?**, it remains unclear, under the Super-Redundancy view, why the notion of local strengthening is defined the way it is (in particular when it comes to its commuting with negation), and why it should be so central in deriving oddness. The next Section adds to this an empirical concern, by presenting data suggesting that overt negation may not be the only source of the contrast in (125).

3.2.2 Is overt negation really the culprit in HCs?

Super-Redundancy was originally motivated by the observation that negated HDs, like (131), appear felicitous. We will dub such sentences "disjunctwise" negated HDs, or **DNHD**s for short, to avoid confusion with expressions of the form $\neg(p^+ \vee p)$, where negation takes wide scope.

(131) Context (taken from **?**, **?**): we go into John's office and see a full pack of Marlboros in the dustbin. We are entertaining hypotheses about what's going on.
John either doesn't smoke or he doesn't smoke Marlboros. $(\neg \mathbf{p}) \vee (\neg \mathbf{p}^+)$

DNHDs are structurally identical to felicitous HCs, granted conditionals are material. This is shown in (132). Super-Redundancy predicts both (131) and (125b) to be fine.

(132) $$\begin{aligned} (125\text{b}) &= \mathbf{p} \rightarrow \neg \mathbf{p}^+ \\ &\equiv (\neg \mathbf{p}) \vee (\neg \mathbf{p}^+) \qquad (\textit{or}\text{-to-}\textit{if}\ \text{tautology}) \\ &= (131) \end{aligned}$$

In this Section, we show that slight variations of (131) display unexpected downgrades in felicity. Moreover, such downgrades can in turn be mitigated by certain operators or expressions. We suggest that the entire paradigm may be better explained by assuming that (131) should in principle be deemed odd, but also that additional pragmatic processes, should be able to rescue it and its variants, only under certain conditions.

First, let us double-check that (**??**) predicts (131) to be fine. This is done in (133).

(133) DNHDs are not Super Redundant (SR).
We show (131)=$(\neg \mathbf{p}) \vee (\neg \mathbf{p}^+)$ is SR.
Take C = $\neg \mathbf{p}$.
We then have $(131)^{-}_{C} = \neg \mathbf{p}^+$.
Take $D = \bot$.

$$
\begin{aligned}
(131)_{Str(C,D)} &= (\neg(\mathbf{p} \wedge D)) \vee (\neg \mathbf{p}^{+}) \\
&\equiv (\neg(\mathbf{p} \wedge \bot)) \vee (\neg \mathbf{p}^{+}) \\
&\equiv (\neg \bot) \vee (\neg \mathbf{p}^{+}) \\
&\equiv \top \vee (\neg \mathbf{p}^{+}) \\
&\equiv \top \\
&\not\equiv \neg \mathbf{p}^{+} = (131)^{-}_{C}
\end{aligned}
$$

Take C = $\neg \mathbf{p}^{+}$.
We then have $(131)^{-}_{C} = \neg \mathbf{p}$.
Take $D = \top$.

$$
\begin{aligned}
(131)_{Str(C,D)} &= (\neg \mathbf{p}) \vee (\neg(\mathbf{p}^{+} \wedge D)) \\
&\equiv (\neg \mathbf{p}) \vee (\neg(\mathbf{p}^{+} \wedge \top)) \\
&\equiv (\neg \mathbf{p}) \vee (\neg \mathbf{p}^{+}) \\
&\not\equiv \neg \mathbf{p} = (131)^{-}_{C}
\end{aligned}
$$

This fact should be unsurprising given that the HD in (131) is equivalent to the HC (125b) granted to *or*-to-*if* tautology, and that we already showed in (130) that (125b) is not Super-Redundant assuming implications are material.

We now show that slight variations of (131) displays felicity downgrades that can be mitigated by certain operators or expressions. First, (131) becomes degraded if its two disjuncts get swapped, as done in (134a). This is *not* expected under Super-Redundancy, whose predictions are insensitive to the order of the disjuncts. Interestingly, adding *at all* to the second disjunct of (134a), recovers felicity, as shown in (134b). *At all* also seems to suggest that the first disjunct, *John doesn't smoke Marlboros*, implied *John smokes cigarettes*.

(134) a. # John either doesn't smoke Marlboros or he doesn't smoke.
b. John either doesn't smoke Marlboros or he doesn't smoke at all.

Although we do intend not provide a full-fledged account of the effect of *at all* here,[3] we believe the paradigm formed by (131) and (134), indicates that some additional incremental pragmatic mechanism is at play in DNHDs – meaning, (131) may be deemed deviant *a priori*, but may end up being rescued by some extra pragmatic mechanism made unavailable in e.g. (134a). This in turn, suggests that the felicity of (131) should not necessarily be accounted for by a Non-Redundancy constraint.

[3]Here is an intuition however. *At all* seems to make the question whether John smokes (p vs. $\neg p$) more salient, and as such may force the $\neg p$ alternative to be considered when attempting exhaustification on the first disjunct $\neg p^{+}$. This would eventually make the two disjuncts contradictory, and rescue (134a) from oddness. See footnote 4 for a more in-depth discussion of the role of covert exhaustification in the sentence at stake, specifically in the subsequent focused variant (135).

Another observation in line with this hypothesis, is that (131) is significantly improved by focus, as shown in (135).

(135) (Either) John doesn't smoke or doesn't smoke MARLBOROS.

Our pre-theoretical understanding of the effect of focus in (135), is that not smoking MALRBOROS implies smoking cigarettes different from Marlboros, i.e. smoking still.[4] If this is indeed the case, (135) would end up meaning $\neg p \vee q$ with $q \equiv \neg p^{+} \wedge p$. Descriptively, this disjunction features incompatible disjuncts, so does not violate **?**'s original condition. It is also predicted by most if not all accounts of oddness to be fine. Assuming that whatever focus achieves in (135), may also be achieved covertly and without explicit focus in (131), would explain (131)'s felicity independently of Super-Redundancy. We believe the pattern described here gets even crisper if disjuncts are picked to be more parallel[5]

Second, we note that (131) is made worse by removing *either*; see (136a). (136b) shows that adding *at all* to the stronger disjunct ($\neg p$) restores felicity.

(136) a. ?? John doesn't smoke or doesn't smoke Marlboros.
b. John doesn't smoke at all or doesn't smoke Marlboros.

Again, we do not wish to propose a full-fledged account of the effect of *either* in (131). But let us just observe that removing *either* in other sentences leads to the same kind of degradation. Such sentences, dubbed *bathroom sentences* (**?**) and attributed to Barbara Partee, are exemplified in (137a).

[4]This may be backed by the theory, as well, assuming that focus forces covert exhaustification *via* the operator *exh* (**??**), and that $\neg p$ (*John does not smoke*) is a salient alternative to $\neg p^{+}$ (*John does not smoke Marlboros*) in (135). In that case, the enriched meaning of $\neg p^{+}$ would end up being $\neg p^{+} \wedge \neg\neg p \equiv \neg p^{+} \wedge p$, i.e. that *John does not smoke Marlboros, but does smoke*. Interestingly, this kind of inference licensed by *exh*, should be unavailable in (134a) – even when *Marlboros* gets focused – in order to capture the infelicity of this sentence. This could be ensured by assuming that relevant alternatives are somehow incrementally computed, and that $\neg p$ is not a Relevant alternative to $\neg p^{+}$ "out-of-the-blue" i.e. if $\neg p^{+}$ is not preceded by $\neg p$ (and, e.g. appears in the first disjunct of a disjunction).

[5]For instance, instead of having V and V+NP as disjuncts, we can have V+NP and V+NP^{+}, with $[\![\text{NP}^{+}]\!] \subset [\![\text{NP}]\!]$).

(i) Analogs of (131) (original sentence), (134a) (swapped disjuncts), and (134b) (swapped disjuncts, plus *at all*), respectively.
a. ? John either doesn't own a dog or he doesn't own a lab.
b. # John either doesn't own a lab or he doesn't own a dog.
c. John either doesn't own a lab or he doesn't own a dog at all.

(ii) Analog of (135) (focused p^{+}).
a. John either doesn't own a dog or doesn't own a LAB.

(137) a. Either there is no bathroom or it's upstairs.

b. ?? There is no bathroom or it's upstairs.

c. Either there is no bathroom or there is a bathroom and it's upstairs.

Roughly, (137a) requires its second disjunct to be interpreted given the negation of its first disjunct to be felicitous. This is because the pronoun *it* in (137a)'s second disjunct, requires an antecedent, which is not overtly introduced in (137a), but could be provided by an existential statement of the form *there is a bathroom*, which correspond to the negation of (137a)'s first disjunct. So, very roughly, (137a) could be felicitous, if understood as (137c), which has the form $\neg p \vee (p \wedge q_p)$, where q_p means that q presupposes p. This is quite similar to the possible pragmatic strengthening of (131)'s second disjunct with p, which we argued made this sentence felicitous. Removing *either* in both DNHDs and bathroom sentences, could be argued to prevent this rescue mechanism[6] – leading to a degradation, as shown in (136a) and (137b) respectively.

Let us now take stock and review the implications for HCs. We have just seen that DNHDs like (131), which constitute the basis of the argument supporting the Super-Redundancy approach, may be felicitous for reasons independent of Non-Redundancy. Specifically, we provided additional data suggesting that independent pragmatic mechanism(s) may force the weaker disjunct of (131) to contradict the stronger one, and that such mechanisms may be blocked or forced, when considering specific variants of (131). In fact, even ignoring such variants, we can observe that the felicity of (131) seems only guaranteed when a precise context is set up; but doing so may force a specific kind of QuD, and such a move was shown to improve other, non-negated HDs just as well (**?**). Besides, we can note that **?**'s Super-Redundancy is challenged by disjunctwise negated *Long-Distance* HDs (see Section **??**), and, outside the domain of Hurford Sentences, by other varieties of redundant sentences obtained from the structure $p \vee p \vee q$ *via* the *or-to-if* tautology (see Chapter **??**).

While the tentative explanations laid out here do not fully explain the complex patterns reported, we believe the overall data to be more in line with an analysis which, unlike Super-Redundancy, would not assign a key role to overt negation in HDs and HCs, but instead, would interact with pragmatic processes themselves influenced by negation and incrementality. In our alternative proposal, we will in fact suggest that *granularity*

[6]This in fact would be in line with the idea that *either* somehow forces exclusivity between disjuncts (**?**, **?** i.a.).

differences (e.g., *Paris* being finer-grained than *France*, *smoking Marlboros* being more fine-grained than *smoking*), drive the contrast in (125).

3.3 QuDs evoked by Hurford Conditionals

To clarify the challenges posed by HCs in our framework, let us first derive Qtrees for these sentences, building on the model presented in Chapter 2. The two sentences under consideration are repeated below.

(125) a. # If SuB29 will not take place in Noto, it will take place in Italy. $\neg \mathbf{p}^+ \rightarrow \mathbf{p}$

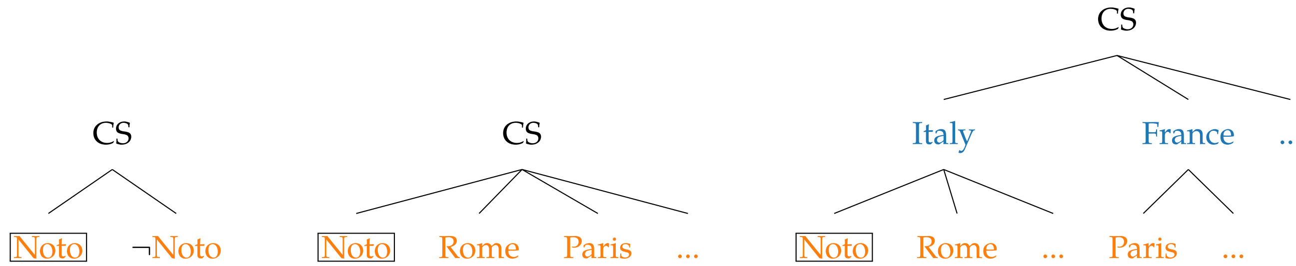
b. If SuB29 will take place in Italy, it will not take place in Noto. $\mathbf{p} \rightarrow \neg \mathbf{p}^+$

Crucial for this Section will be the idea that conditionals evoke Qtrees which assign asymmetric roles to antecedent and consequent. Namely, a conditional Qtree is a Qtree for the antecedent whose verifying nodes are *replaced* by their intersection with a Qtree for the consequent. We will see towards the end of this Chapter that this asymmetry may be exploited to derive the following generalization: a conditional whose consequent evokes at least one Qtree that is finer-grained than some Qtree evoked by its antecedent, is felicitous.

3.3.1 Qtrees for the antecedent and consequent of HCs

We first compute the Qtrees compatible with S_p = *SuB29 will take place in Italy*, and S_{p^+} = *SuB29 will take place in Noto*. The Qtrees for $\neg S_{p^+}$ = *SuB29 will not take place in Noto* will be subsequently derived from those evoked by S_{p^+}.

Chapter 2 extensively discussed how to derive Qtrees from simplex sentences like S_p and S_{p^+}. And Chapter **??** in fact discussed the Qtrees evoked by these exact sentences. Here, it is enough to say that such sentences may evoke three kinds of Qtrees: "polar" ones, splitting the Context Set (henceforth **CS**) into p and $\neg p$ worlds; "*wh*" ones, splitting the CS according to the Hamblin partition generated by same-granularity alternatives to the prejacent; and "*wh*-articulated" ones, whereby each layer corresponds to a Hamblin partition of increasing granularity from the top down, the last layer matching the granularity of the prejacent. In each case, leaves entailed by the prejacent are $\boxed{\text{flagged}}$ as "verifying", and keep track of at-issue content. In this Chapter, and just like in Chapter **??**, we will only consider two levels of granularity for S_p and S_{p^+}: by-city and by-country. This gives rise to the Qtrees in Figure C (for S_{p^+}) and Figure D (for S_p).



(1) "Polar". (2) "*Wh*". (3) "*Wh*-articulated".

Figure C: Qtrees evoked by S_{p^+} = *SuB29 will take place in Noto*.

(1) "Polar". (2) "*Wh*".

Figure D: Qtrees evoked by S_{p} = *SuB29 will take place in Italy*.

We can already note that Figures C3 and D2, introduce consistent partitionings: Figure C3 can be in fact be seen as a refinement of Figure D2, as per (70), repeated below.

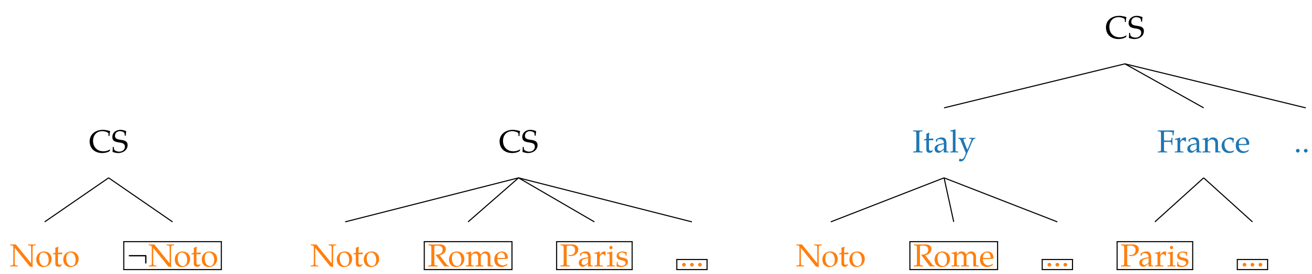
(70) **Qtree refinement.** Let T and T' be Qtrees. T is a refinement of T' (or: T is finer-grained than T'), iff T' can be obtained from T by removing a subset $\mathcal{T}$ of T's subtrees, s.t., if $\mathcal{T}$ contains a subtree rooted in N, then, for each node N' that is a sibling of N in T, the subtree of T rooted in N', is also in $\mathcal{T}$.

More precisely, C3 constitutes a *strict* refinement of Figure D2, as per (138). It may not be obvious at this point, but this stronger characterization will be the most Relevant to our subsequent predictions.

(138) **Strict Qtree refinement.** Let T and T' be Qtrees. T is a strict refinement of T' (or: T is strictly finer-grained than T'), if T is a refinement of T' and $\mathcal{L}(T) \cap \mathcal{L}(T') = \emptyset$.

Figures **??** and **??** thus structurally capture the intuition that S_{p^+} answers a finer-grained question than S_p. More generally, Figures C and D show that some Qtree obtained for S_{p^+} (namely Figure **??**), (strictly) refines some Qtree for S_p (namely, Figure **??**); while no Qtree obtained for S_p refines a Qtree for S_{p^+}.

Before computing the conditional Qtrees corresponding to (125a) and (125b), we need to compute the Qtree corresponding to the negation of S_{p^+}, namely $\neg S_{p^+} =$ *SuB29 will not take place in Noto*, which constitutes the antecedent of (125a) and the consequent of (125b). As discussed in Chapter 2, a negated LF evokes the same kind of question structure as its positive counterpart, but flags a disjoint set of verifying nodes. More specifically, given an LF X, evoking a Qtree T, a Qtree T' for $\neg X$ is obtained by retaining T's structure (nodes and edges), and "swapping" T's verifying nodes, by replacing any set of same-level verifying nodes in T by the set of non-verifying nodes at the same level in T. If the verifying nodes are all leaves, this operation simply corresponds to set complementation in the domain of leaves. This is done for $\neg S_{p^+}$ in Figure E.



(1) "Polar". (2) "*Wh*". (3) "*Wh*-articulated".

Figure E: Qtrees evoked by $\neg S_{\mathrm{p}^+} =$ *SuB29 will not take place in Noto*.

Because negation preserves Qtree structure and only affects verifying nodes, Figure E3, just like Figure C3, constitutes a strict refinement of Figure D2. More broadly, our observation about S_p and S_{p^+} extends to S_p and $\neg S_{p^+}$: some Qtree obtained for $\neg S_{p^+}$ (namely Figure E3), (strictly) refines some Qtree for S_p (namely, Figure D2); while no Qtree obtained for S_p refines a Qtree for $\neg S_{p^+}$.

This double observation will be crucial for our approach to HCs: felicitous HCs like (125b) are the ones whose antecedent evokes a question that is coarser-grained than that of their consequent (i.e. s.t. the antecedent Qtree *can* be strictly refined by a consequent Qtree); odd HCs like (125a) are the ones whose antecedent evokes a question that is finer-grained than that of their consequent (i.e. s.t. the antecedent Qtree *cannot* be strictly refined by any consequent Qtree).

3.3.2 Conditional Qtrees, and one useful result

Let us now turn to the Qtrees evoked by the HCs (125a) = $\neg S_{p^+} \rightarrow S_p$ and (125b) = $S_p \rightarrow \neg S_{p^+}$. Following Chapters 2 and **??**, we assume that the "inquisitive" contribution of *if ... then ...* (glossed $\rightarrow$) is *not* material, meaning, a conditional Qtree is not derived by disjoining the negation of its antecedent Qtrees, with its consequent Qtrees.

Chapter 2 instead proposed that conditionals evoke questions pertaining to their consequent, set in the domain(s) of the CS where the antecedent holds. This was modeled by assuming that conditional Qtrees are derived by "plugging" a consequent Qtree T_C into the verifying nodes of antecedent Qtrees T_A. More concretely, for each verifying node N of T_A, N gets replaced by $T_C \cap N$, where $\cap$ refers to tree-node intersection. This operation is repeated in (64).

(64) **Tree-node intersection**. Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a Qtree. Let p be a proposition. The tree-node intersection between T and p, noted $T \cap p$, is defined iff $R \cap p \neq \emptyset$ and, if so, is the Qtree $T' = (\mathcal{N}', \mathcal{E}', R')$ s.t.:

- $\mathcal{N}' = \{p \cap N \mid N \in \mathcal{N} \wedge p \cap N \neq \emptyset\}$
- $\mathcal{E}' = \{\{N_1 \cap p, N_2 \cap p\} \mid \{N_1, N_2\} \in \mathcal{E} \wedge (N_1 \cap p) \neq (N_2 \cap p) \wedge N_1 \cap p \neq \emptyset \wedge N_2 \cap p \neq \emptyset\}$
- $R' = R \cap p$

From an algorithmic perspective, the formation of a conditional Qtree based on T_A and T_C, amounts to (i) replacing every verifying node of T_A by its intersection with T_C; (ii) removing resulting empty nodes; (iii) removing resulting dangling and unary edges. Additionally, Chapter 2 assumed that only the consequent of a conditional contributes verifying nodes in the resulting conditional Qtree. In particular, nodes falsifying the antecedent are not considered verifying in the resulting conditional Qtree. The core idea behind this operation is that conditionals introduce a hierarchy between antecedent (backgrounded) and consequent (at-issue): the consequent Qtree gets *restricted* by the antecedent Qtree. (112), repeated below, summarizes these assumptions.

(112) *Qtrees for conditional LFs.* A Qtree T for $X \rightarrow Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

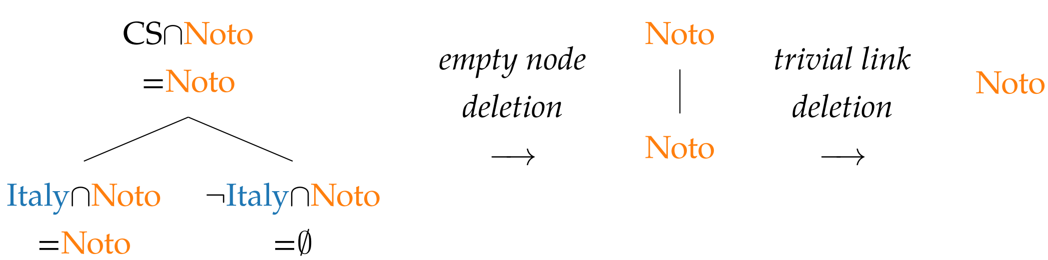
- replacing each node N of T_X that is in $\mathcal{N}^+(T_X)$ with $T_Y \cap N$ (see (**??**));
- returning the result only if it is a Qtree.

In other words, $Qtrees(X \rightarrow Y) = \{T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)} (T_Y \cap N) | (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y) \wedge T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)} (T_Y \cap N)$verifies (50)$\}$, and $\mathcal{N}^+(T_X \rightarrow T_Y) = \{N \cap N' | (N, N') \in \mathcal{N}^+(T_X) \times \mathcal{N}^+(T_Y) \wedge N \cap N' \neq \emptyset\}$.

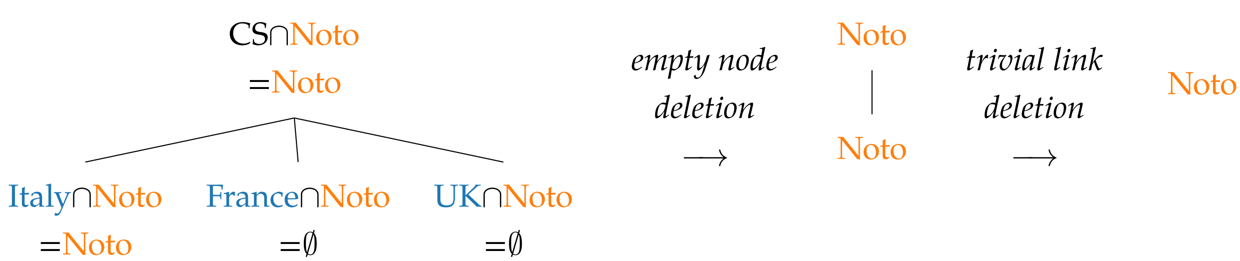
(64) comes with one useful prediction when it comes to HCs, namely that intersecting a city-level node with a country-level Qtree does not have any effect. This is consistent with the intuition that answering a question about cities automatically answers question about countries, and corresponds to the generalization in (114), repeated below.

(114) **Vacuous tree-node intersection**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). $T \cap N = N$ iff $\exists N' \in \mathcal{L}(T).\ N \vDash N'$.

Figure F illustrates this result, considering two possible Qtrees for S_p = *SuB29 will take place in Italy*, and their intersection with a city-level node like *Noto*.



(1) Derivation of *Noto*∩Tree D1=the *Noto*-node.



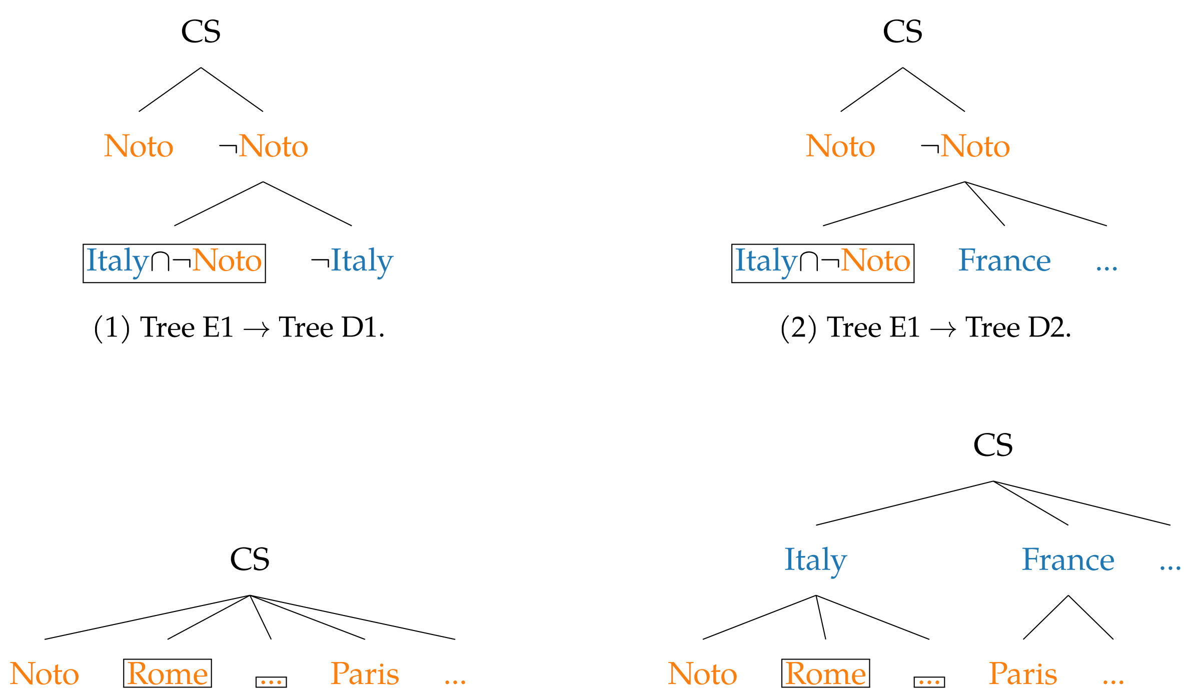
(2) Derivation of *Noto*∩Tree D2=the *Noto*-node.

Figure F: Intersecting a city-level node and a country-level tree yields the input city-level node.

A consequence of this result, given the definition of conditional Qtrees in (112), is the following: if an antecedent Qtree T_X and a consequent Qtree T_Y are such that each verifying node of T_X entails some leaf in T_Y, the conditional Qtree resulting from their composition, will have the same structure as T_X, and its verifying nodes will be exactly the verifying nodes in T_X that entail some verifying node in T_Y. This is the case, if T_X is a strict refinement of T_Y, whose verifying nodes are all leaves. We will use this observation to justify the derivation of conditional Qtrees for HCs in the next two Sections, and later when evaluating their well-formedness.

3.3.3 Qtrees for the HCs in (125)

We can now use the rule (112) to compute Qtrees for HCs. We start with the candidate Qtrees for the infelicitous HC (125a) = $\neg S_{p^+} \rightarrow S_p$. Applying (112) to this LF, using the Qtrees for $\neg S_{p^+}$ from Figure E as antecedent Qtrees, and the Qtrees for S_p from Figure D as consequent Qtrees, leads to the conditional Qtrees in Figure G.



(1) Tree E1 → Tree D1.

(2) Tree E1 → Tree D2.

(3) Tree E2 → Tree D1/D2.

(4) Tree E3 → Tree D1/D2.

Figure G: Qtrees for (125a)=#*If SuB29 will not take place in Noto, it will take place in Italy*.

The Qtrees in Figures G1 and G2 are obtained by replacing the verifying *not Noto* node of the "polar" antecedent Qtree from Figure E1, with the intersection between this node and a Qtree for *Italy* (either from Figure D1, or from Figure D2). Because *not Noto*, does not entail any leaf in the consequent Qtrees for *Italy* (it does not entail any particular city), the whole operation is *not* structurally vacuous, and the output Qtrees are of depth 2. Verifying nodes are inherited from the consequent Qtree after intersection, i.e. correspond to *Italy but not Noto*.

The Qtrees in Figures G3 and Figure G4 are obtained by replacing each leaf different from *Noto* in the non-"polar" antecedent Qtrees (from Figures E2 and E3 respectively), with the intersection between this leaf, and a Qtree for *Italy* (from Figure D1 or D2). Because each node different from *Noto*, is a city-node, it will entail some leaf in the Qtrees evoked by *Italy*. Therefore, the formation of a conditional Qtree based on these inputs, will be structurally vacuous. This explains why the Qtrees in Figures G3 and Figure G4 appear structurally similar to the antecedent Qtrees used to form them, in Figure E2 and Figure E3 respectively. The only difference between these inputs, and the outputs, lies in the verifying nodes, which are inherited from the consequent Qtrees, and correspond to Italian cities different from Noto. Figure H further details the derivation of the Qtree in Figure G3.

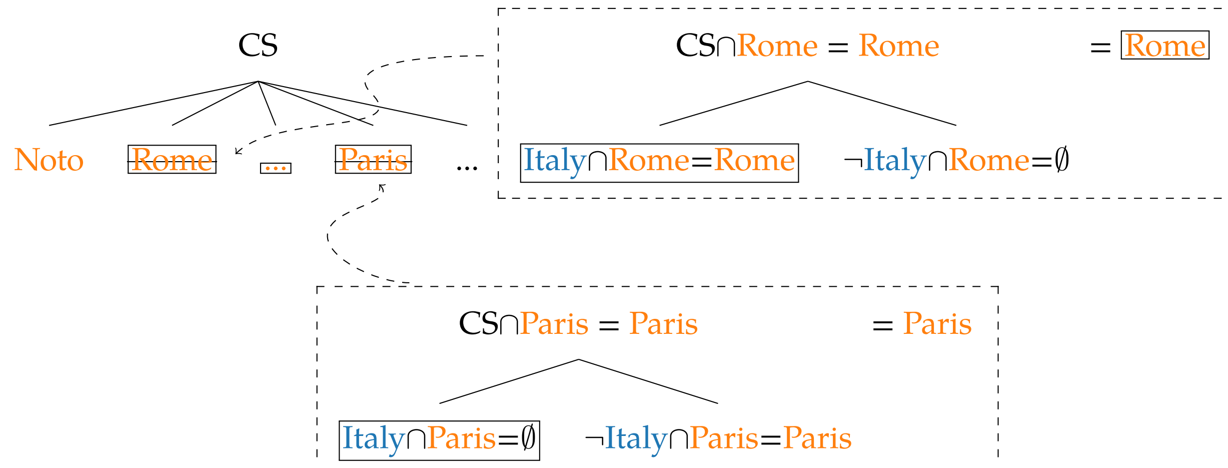


Figure H: Breakdown of the derivation of Figure G3, assuming Figure D1 is the consequent Qtree. The end result is unchanged if Figure D2 is considered instead.

Let us now turn to the candidate Qtrees for the felicitous HC (125b) = $S_p \to \neg S_{p^+}$. Applying (112) to this LF, using now the Qtrees for S_p from Figure D as antecedent Qtrees, and the Qtrees for $\neg S_{p^+}$ from Figure E as consequent Qtrees, leads to the conditional Qtrees in Figure I.

(1) Tree D1 → Tree E1.

(2) Tree D1 → Tree E2/E3.

(3) Tree D2 $\rightarrow$ Tree E1. (4) Tree D2 $\rightarrow$ Tree E2/E3.

Figure I: Qtrees for (125b)=*If SuB29 will take place in Italy, it will not take place in Noto.*

All the Qtrees in Figure I are obtained by replacing the verifying *Italy* node of the antecedent Qtree (from Figure E1 or E2), with the intersection between this node and a Qtree for *not Noto*. Because *Italy*, does not entail any leaf in the consequent Qtrees for *not Noto* (it entails neither *not Noto*, nor any specific city), the whole operation is never structurally vacuous, and the output Qtrees are all of depth 2. Verifying nodes are inherited from the consequent Qtree after intersection, i.e. correspond to *Italy but not Noto*.

At this point, it seems that many Qtrees are available, for both the felicitous variant (125b) and the odd variant (125a). But it appears that the two sets of Qtrees are different from each other, which comes from the fact that Qtrees for S_p and S_{p^+} differ in terms of granularity, and that the recipe for conditional Qtrees in (112), is asymmetric in nature. What is the key difference between these two sets of Qtrees then? It appears that *some* Qtrees compatible with (125b), namely those in Figure I2 and I4, still feature a by-city partition (as conveyed by the consequent) at their lowest level, defined on the *Italy*-subset of the CS.

By contrast, none of the Qtrees evoked by (125a) feature a properly restricted by-country partitions at their lowest level, i.e. a partition which both (i) contains some country nodes (as introduced by the consequent) and (ii) does not contain *all* country nodes. Instead, such Qtrees either feature by-city partitions at the leaf level (Figures G3 and G4), or partitions where no country node is fully missing (Figure G1 and G2).

We will see in the next Section, that these observed differences between the Qtrees evoked by the felicitous HC (125b) and those evoked by the infelicitous HC (125a), translate into the following generalization: the antecedent of (125b), *Italy can* be taken to be "Relevant" to the question evoked by (125b)'s consequent; while the antecedent of (125a), *not Noto*, *cannot* be taken to be "Relevant" to the question evoked by (125a)'s consequent.

We will then further justify this description in the form of a new incremental RELEVANCE constraint targeting Qtree derivation, and specifically tree-node intersection.

3.4 Hurford Conditionals and RELEVANCE

3.4.1 Do we actually need an extra constraint?

We have just seen that both HCs in (125) are in principle compatible with various Qtrees. First, let us double-check that the previous constraints on Qtrees (and LFs) defined in Chapters 2 and **??**, are insufficient to capture the contrast between the two HCs in (125). The first constraint to check is the EMPTY LABELING constraint, which states that well-formed Qtrees should flag at least one node as verifying; see (73).

(73) **EMPTY LABELING**. If a sentence S evokes a Qtree T but does not flag any node as verifying in T, then T is deemed odd given S.

It is easy to see that none of the Qtrees in Figure G (corresponding to the infelicitous HC (125a)) or Figure I (corresponding to the felicitous HC (125b)), violate (73): all these Qtrees flag at least one node. So the EMPTY LABELING does not help at all in deriving the desired contrast.

The second constraint to check, is Q-NON-REDUNDANCY, which states that a Qtree evoked by an LF should not be equivalent to a Qtree evoked by some simplification of that LF (see (**??**) and (**??**)).

(**??**) **Q-NON-REDUNDANCY** (final version). Let X be a LF and let $Qtrees(X)$ be the set of Qtrees evoked by X. For any $T \in Qtrees(X)$, T is deemed Q-REDUNDANT given X, iff there exists a formal simplification of X, X', and $T' \in Qtrees(X')$, such that $T \equiv T'$.

(**??**) **QTREE EQUIVALENCE RELATION**. T and T' are equivalent ($T \equiv T'$) iff T and T' have same structure and same set of minimal verifying paths.

We can show that none of the Qtrees in Figures G and I violate (**??**). To see this, we need to review the Qtrees associated with the simplifications of (125a) and (125b). Let us use p and p^+ as shorthands for S_p = *SuB29 will take place in Italy*, and S_{p^+} = *SuB29 will take place in Noto*. The possible simplifications of (125a) and (125b) are summarized in Table 3.1.

Sentence	Simplifications
(125a)=$\neg\mathbf{p}^+ \to \mathbf{p}$	$\mathbf{p}$, $\mathbf{p}^+$, $\neg\mathbf{p}^+$, $\mathbf{p}^+ \to \mathbf{p}$
(125b)=$\mathbf{p} \to \neg\mathbf{p}^+$	$\mathbf{p}$, $\mathbf{p}^+$,$\neg\mathbf{p}^+$, $\mathbf{p} \to \mathbf{p}^+$

Table 3.1: Gathering the formal simplifications of (125a) and (125b).

Let us start by evaluating the simplifications of the infelicitous variant (125a). The Qtrees for (125a) are given in Figure G. For (125a) to be deemed deviant, all these Qtrees must be equivalent to *some* Qtree evoked by *some* simplification of (125a). For clarity, we proceed simplification-by-simplification.

First, Qtrees for the simplification p, shown in Figure D, have a different structure altogether from all the Qtrees in Figure G. So there is no way Qtree equivalence holds between some Qtree for p and some Qtree for (125a).

Second, among the Qtrees for the simplification p^+ shown in Figure C, two Qtrees are structurally identical to two Qtrees from Figure G. The first pair is made of the Qtree in Figure C2 and the one in Figure G3. These two Qtrees, though structurally identical, are not equivalent, because they each flag different sets of leaves as verifying; so their minimal sets of verifying paths cannot be the same. The second pair is made of the Qtree in Figure C3 and the one in Figure G4. Again, these two Qtrees, though structurally identical, are not equivalent, because they each flag different sets of leaves as verifying.

Third, the reasoning about the simplification p^+, extends to the simplification $\neg p^+$: there are two Qtrees evoked by $\neg p^+$ (in Figures E2 and E3) that are structurally identical to two Qtrees in Figure G, but these pairs, though structurally identical, are not equivalent, because they flag different sets of leaves as verifying.

Lastly, Qtrees for the simplification $p^+ \to p$ are the same as the Qtrees evoked by p^+. This is because, the only verifying node of the antecedent Qtree, *Noto*, always entails a leaf of the consequent Qtree (namely, *Italy*); therefore, the intersection operation performed to create conditional Qtrees is vacuous (as per (114)), and the resulting conditional Qtrees are just the same as the antecedent Qtrees used to form them. Since we have already shown that none of the Qtrees evoked by p^+ make the Qtrees in Figure G Q-Redundant, none of the Qtree evoked by $p^+ \to p$ make the Qtrees in Figure G Q-Redundant, either.

We have just gone through all the possible simplifications of the infelicitous HC (125a), and shown that none of these simplification evoke Qtrees triggering Q-Non-Redundancy. Therefore, Q-Non-Redundancy does not rule out (125a). This already motivates the introduction of a new constraint deriving (125a)'s oddness.

Let us now turn to the simplifications of the felicitous HC (125b). The Qtrees for (125b) are given in Figure I. For (125b) to be fine, some Qtree in Figure I should not be equivalent to *any* Qtree evoked by *any* simplification of (125b). Again, we proceed simplification-by-simplification and show something stronger, namely that no simplification evokes a Qtree equivalent to any Qtree in Figure G.

First, Qtrees for the simplification p, shown in Figure D, have a different structure altogether from all the Qtrees in Figure I. So there is no way Qtree equivalence holds between some Qtree for p and some Qtree for (125b).

Second, Qtrees for the simplification p^+, shown in Figure C, also have a different structure altogether from all the Qtrees in Figure I. So there is no way Qtree equivalence holds between some Qtree for p^+ and some Qtree for (125b). This extends to the simplification $\neg p^+$, whose Qtrees are structurally identical to those evoked by p^+.

Lastly, Qtrees for the simplification $p \rightarrow p^+$ are pairwise structurally identical to the Qtrees in Figure I (evoked by $p \rightarrow \neg p^+$). This is because Qtrees for $p \rightarrow p^+$ and $p \rightarrow \neg p^+$ are built using Qtrees for p as antecedent Qtrees, and Qtrees for $(\neg)p^+$ as consequent Qtrees, and negation does not affect Qtree structure. Still, the Qtrees evoked by the simplification $p \rightarrow p^+$ are *not* pairwise equivalent to the Qtrees in Figure I. This is roughly because, Qtrees for $p \rightarrow p^+$ will flag nodes verifying p^+ (consequent), while Qtrees for $p \rightarrow \neg p^+$ in Figure I, will flag nodes verifying $\neg p^+$. More precisely, the Qtrees evoked by $p \rightarrow p^+$, flag nodes that are Italian cities, and in fact Noto; while the Qtrees in Figure I, flag nodes that are Italian cities different from Noto. Thus, pairs of equivalent Qtrees from these two sets, end up flagging disjoint sets of verifying nodes, which in turn implies that there is no way Qtree equivalence holds between some Qtree for $p \rightarrow p^+$ and some Qtree for (125b).

We have just gone through all the possible simplifications of the felicitous HC (125b), and shown that none of these simplifications evokes Qtrees triggering Q-Non-Redundancy. Therefore, Q-Non-Redundancy does not incorrectly rule out (125b) – which is good news, and means we do not need to amend Q-Non-Redundancy to rule in (125b).

In brief, this Section confirmed that the constraints on Qtrees and LFs posited so far (Empty Labeling, Q-Non-Redundancy), if they do not incorrectly rule out felicitous HCs like (125b), also cannot rule out *in*felicitous HCs like (125a). In other words, both HCs are so far predicted to be felicitous. To account for the infelicity of (125a), while retaining the felicity of (125b), we will appeal to an updated definition of Relevance. The next Section will first motivate the use of a new Relevance constraint, by outlining some limitations of earlier approaches to Relevance.

3.4.2 Can earlier notions of Relevance help?

We have previously suggested that the contrast between felicitous and infelicitous HCs may be a matter of Relevance. Chapter 1 already defined ways in which a proposition could be understood as Relevant to a question, seen as a partition of the CS. Adapting insights from **?** to the QuD framework, we stated that a proposition is **?**-Relevant to a QuD, if it coincides with a (potentially empty) union of cells. This is repeated in (31).

(31) **?'s Relevance** (rephrased in the QuD framework). Let $\mathcal{C}$ be a conversation, Q a QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is **?**-Relevant to Q, iff $\exists C \subseteq Q.\, p \cap CS(\mathcal{C}) = C$.

A corollary of this definition is given in (139). It says that p is **?**-Relevant to a question, iff p does not introduce any truth-conditional distinction in any cell of that question. In other words, all the cells must either entail, or be imcompatible with, p. This view will be useful when we relate our novel approach to Relevance to **?**'s approach.

(139) **?'s Relevance** (corollary). Let $\mathcal{C}$ be a conversation, Q a QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is **?**-Relevant to Q, iff $\forall c \in Q.\, \forall (w, w') \in c.\, p(w) = p(w')$.

We also mentioned the view from **?**, according to which a proposition is Relevant if it rules out a cell; see (32).

(32) **?'s Relevance** (**?**). Let $\mathcal{C}$ be a conversation, Q a (non-trivial) QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is **?**-Relevant to Q, if $\exists c \in Q.\, p \cap c = \emptyset$.

Ideally, we would like to reuse either (31) or (32) in the context of compositional Qtrees, and derive, for instance, that two LFs X and Y can form a conditional $X \to Y$, only if the proposition denoted by Y, is Relevant to a Qtree evoked by X. Note that this would be the most intuitive direction, because X, as antecedent, would be understood as "setting" the QuD, and Y, would be understood as some Relevant answer to it. This (stipulative) idea is summarized in (140).

(140) **Incremental Relevance** (naive version). Let X and Y be two LFs. $X \to Y$ is deviant if none of the questions X evokes (seen as partitions of the CS formed by the leaves of X's Qtrees), make the proposition Y denotes Relevant. Relevance may be understood as (31) or (32).

This however, would not quite work on the HCs at stake. In particular, (140) predicts an infelicitous HC like (125a) to be fine. Indeed, (125a) has its antecedent evoke

Qtrees whose leaves either partition the CS into cities, or partition the CS into *Noto* vs. *not Noto*-worlds. These partitions are represented in Figures J1 and J2. Additionally, (125a)'s consequent denotes *Italy*. Is *Italy* RELEVANT to any of the partitions evoked by (125a)'s antecedent? Figure J1 shows that *Italy* is in fact both **?**- and **?**-RELEVANT to the antecedent's implicit "*wh*" question: it corresponds to a collection of Italian cities (hence **?**-RELEVANT), and rules out the non-Italian cities (hence **?**-RELEVANT). According to (140), this is enough to predict that (125a) should be fine. Note however that, if a polar partition is considered itself for the antecedent, the consequent is neither **?**- not **?**-RELEVANT (see Figure J2).

Noto	Rome	...	Paris	...
Italy				

(1) (125a)'s consequent is both **?**- and **?**-RELEVANT to the "*wh*"-partition evoked by (125a)'s antecedent.

Noto	not Noto
Italy	

(2) (125a)'s consequent is neither **?**- nor **?**-RELEVANT to the "polar"-partition evoked by (125a)'s antecedent.

Figure J: How (125a)'s consequent interacts with (125a)'s antecedent's evoked questions.

Additionally, we can show that (140) predicts the felicitous HC in (125b), to be deviant. Indeed, (125b) has its antecedent evoke Qtrees whose leaves either partition the CS into countries, or partition the CS into *Italy* vs. *not Italy*-worlds. These partitions are represented in Figures K1 and K2. Additionally, (125b)'s consequent denotes *not Noto*. Is *not Noto* RELEVANT to any of the partitions evoked by (125a)'s antecedent? Figure K1 shows that *not Noto* is neither **?**- nor **?**-RELEVANT to the antecedent's implicit "*wh*" question: it does not correspond to a collection of countries (hence not **?**-RELEVANT), and does not rule out any country (hence not **?**-RELEVANT). The same holds for Figure K1.

Italy	France	...
not Noto		

(1) (125b)'s consequent is neither **?**- nor **?**-RELEVANT to the "*wh*"-partition evoked by (125b)'s antecedent.

Italy	not Italy
not Noto	

(2) (125b)'s consequent is neither **?**- nor **?**-RELEVANT to the "polar"-partition evoked by (125b)'s antecedent.

Figure K: How (125b)'s consequent interacts with (125b)'s antecedent's evoked questions.

In fact, reversing the directionality of the principle in (140), i.e. stating that the antecedent should be RELEVANT to one of the consequent's implicit questions (see (141)), would in turn reverse the above predictions, and capture HCs.

(141) **Incremental Relevance** (reversed version). Let X and Y be two LFs. $X \to Y$ is deviant if none of the questions Y evokes (seen as partitions of the CS formed by the leaves of Y's Qtrees), make the proposition X denotes Relevant. Relevance may be understood as (31) or (32).

This principle, though less intuitive in terms of its directionality, is in effect close to a result derived by **?**, who showed that, under a (relatively weak) definition of "inquisitive" Relevance between two *questions*, and assuming conditionals are strict, the antecedent of a conditional *must* be "inquisitively" Relevant to the consequent. However, Appendix 3.7 shows that this result cannot account for the contrast observed in HCs, because the inquisitive take on Relevance assigns symmetric roles to the two questions it evaluates, i.e. cannot distinguish between $p \to q$ and $q \to p$.

In any event, the challenge is now to derive a less stipulative version of (141). In other words, the goal is to state a constraint making the same kind of prediction, but which would avoid stipulating the respective roles of X (antecedent) and Y (consequent) which, as we have seen, can so far be reversed. In the next Section, we propose to build a variant of (141) as a constraint on tree-node intersection, which is already an asymmetric operation, whose directionality was motivated by the data presented in Chapter **??**. Tying Relevance to tree-node intersection (and more generally perhaps, to *restriction* operations), will give us the directionality stipulated in (141) "for free".

3.4.3 Incremental Q-Relevance

We have just seen that HCs could be captured assuming that LFs give rise to Qtrees matching their degree of granularity, and that, in a conditional, the proposition denoted by the antecedent, should be Relevant to some question evoked by the consequent, where a question is defined as the partition formed by the set of leaves of a Qtree (see (141)). Here, we modify this constraint to avoid stipulating the roles of the antecedent (so far assumed to provide a Relevant proposition) and consequent (so far assumed to provide the question). To this end, we rephrase (141) as a constraint on tree-node intersection, which itself assigns asymmetric roles to its "restrictor" node argument (contributed by the consequent) and tree argument (contributed by the consequent). Under that view, Relevance amounts to some notion of non-vacuity constraining tree-node intersection. Specifically, it is assumed that tree-node intersection should eliminate a leaf of the input Qtree, but also, retain at least one other leaf. This is capturing the idea that tree-node intersection should eliminate some relevant information, but not too much of it – crucially, it should retain at least one relevant "distinction" (cell/leaf) established by the input Qtree. This is all summarized in (142).

(142) **Incremental Q-Relevance**. Let N be a node and T be a Qtree. The tree-node intersection of T and N, noted $T \cap N$, is well formed only if Relevant. It is Relevant iff $T \cap N$'s leaves comprise at least one of T's leaves, and exclude at least one of T's leaves. A leaf of T is excluded, if it intersects with no leaf in $T \cap N$.

(143) **Incremental Q-Relevance** (corollary). Let N be a node and T be a Qtree. The tree-node intersection of T and N, noted $T \cap N$, is Relevant iff N is a superset of at least one leaf in T, and is disjoint from at least one leaf in T.

(144) *Proof of corollary (143).* Let N be a node and T be a Qtree. We assume $T \cap N$'s leaves comprise at least one of T's leaves noted L, and excludes at least one of T's leaves, noted L'. By definition, L is in both T and $T \cap N$, so $L = L \cap N$ i.e. $L \subseteq N$. By definition, L' is in T but not $T \cap N$, so $L' \cap N = \emptyset$.
We now assume N is a superset of at least one leaf in T, noted L, and is disjoint from at least one leaf in T, noted L'. Because $N \supseteq L$, $L \cap N = L$ and L is in $T \cap N$ and is a leaf. Because $L' \cap N = \emptyset$, L' cannot be in $T \cap N$. Moreover, given that the leaves of a Qtree are disjoint, no other leaf in $T \cap N$ intersects with L'.

First, let us quickly note that Incremental Q-Relevance does not jeopardize the results we previously established for HDs and their variants back in Chapter **??**. This is simply because Incremental Q-Relevance is checked only when tree-node intersection is performed, i.e. only when conditional Qtrees are computed; and none of the sentences in Chapter **??** involved conditionals. Moreover, Incremental Q-Relevance does not lead to mispredictions in the context of the sentences analyzed in Chapter **??**, even those involving conditionals. To show this, it is enough to focus on the only felicitous variant analyzed in that Chapter, repeated in (**??**), and show that at least one of the Qtrees compatible with that structure after evaluating Q-Non-Redundancy, is ruled in by Incremental Q-Relevance. Figure L below repeats the two Non-Q-Redundant Qtrees evoked by (**??**).

(**??**) Either Jo is at SuB or if he is not at SuB then he is in Cambridge. $\mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$

(1) Tree **??** ∨ Tree **??**
Q-Irrelevant

(2) Tree **??** ∨ Tree **??**
Q-Relevant

Figure L: Non-Q-Redundant Qtrees for (**??**) = $\mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$

These two Qtrees were derived by intersecting "polar" and "*wh*" consequent Qtrees evoked by q, with $\neg p$, and then disjoining the result with a "polar" Qtree for p. The area of interest where tree-node intersection took place, is circled. In both cases, tree-node intersection fully retained the q-leaf from the consequent Qtree. Moreover, in the case of Figure L2, intersection fully ruled out the p leaf from the consequent Qtree, inducing a partition of the $\neg p$-domain of the form $\{q, r, ...\}$. Therefore the Qtree in Figure L2, in addition to being Non-Q-Redundant, is Q-Relevant. And (**??**)'s felicity is preserved, even when assuming Incremental Q-Relevance.

Now that these initial concerns are addressed, let us go back to the definition of Incremental Q-Relevance in (142). The concept of Relevance used in (142) is a hybrid between **?**'s Relevance and **?**'s Relevance. This is perhaps made more obvious by the corollary in (143). The fact that the restrictor node N must comprise at least one cell, relates to the non-pathological instances of **?**'s Relevance;[7] the fact that it must rule out one cell, relates to **?**'s Relevance. Note however, that our principle does not constitute a proper conjunction of **?**'s and **?**'s Relevance. Like **?**'s Relevance, it allows N to be a proper union of cells. Unlike **?**'s Relevance, it disallows this union to be maximal or empty; and allows cells to be partially covered, as soon as one full cell is. Like **?**'s Relevance, it allows N to *not* be a proper union of cells (as soon as one full cell is covered). Unlike **?**'s Relevance, it disallows strictly overinformative configurations whereby no full cell is covered. These properties are illustrated by the configurations in Figure M.

[7]We dub "pathological" the case in which p is a contextual contradiction: in that case, the intersection between p and the CS does not include *any* cell of the question (it is empty!), yet p is still identified as **?**-Relevant.

(1) **?**-Relevant **?**-Relevant Q-Relevant.

(2) **?**-Irrelevant **?**-Relevant Q-Irrelevant.

(3) **?**-Irrelevant **?**-Relevant Q-Irrelevant.

(4) **?**-Irrelevant **?**-Relevant Q-Relevant.

Figure M: Various QuD-proposition configurations (proposition/restrictor node defined by the gray area).

We will see that this "hybrid" definition will be crucial to capture what we will call "Compatible" HCs (see Section 3.5). Before showing how (142) captures HCs, let us establish three useful results that will significantly simplify the argument. The first result, has to do with what we previously called Vacuous tree-node intersection, repeated below.

(114) **Vacuous tree-node intersection**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). $T \cap N = N$ iff $\exists N' \in \mathcal{L}(T).\ N \vDash N'$.

Let us consider a specific subcase of the condition stated in (114), namely, the case in which the restrictor node at stake *strictly* entails some leaf in the Qtree it gets intersected with. In that case, tree-node intersection is vacuous, but also, Irrelevant. This is because it results in a single node, that is a strict subset of the leaf it entails. In other words, the final result does not preserve any leaf from the input Qtree. This is repeated in (145), and will prove very handy when dealing with HCs.

(145) *Irrelevance by* **Single Strict Entailment**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). If $\exists N' \in \mathcal{L}(T).\ N \vDash N' \wedge N \not\equiv N'$, then $T \cap N$ is Irrelevant.

This subcase has an interesting generalization, that will prove useful when analyzing "Compatible" HCs in Section 3.5. Suppose now the restrictor node at stake can be partitioned into a set of propositions, s.t. each of them strictly entails a leaf in the Qtree the node gets intersected with. In that case, tree-node intersection will *not* be vacuous, because the node does not entail a single leaf. It will be Irrelevant however. This is because, if the node N in question, can be partitioned into a set $\{N_1, N_2, ..., N_k\}$ of propositions, each of which strictly entails the leaves $\{L_1, L_2, ..., L_k\}$ in T, respectively, then, the intersection between N and T, will simply be the Qtree whose leaves are $\{N_1, N_2, ..., N_k\}$. Since none of these nodes fully coincides with a leaf in T, due to the assumption of strict entailment, the tree-node intersection operation fails to be Relevant. This is summarized in (146).

(146) *Irrelevance by* **Multiple Strict Entailment**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). If $\exists\{N_1, N_2, ..., N_k\}$ a partition of N and $\exists\{L_1, L_2, ..., L_k\} \subset \mathcal{L}(T)$ s.t. $\forall i \in [1; k]$. $N_i \vDash L_i \wedge N_i \not\equiv L_i$, then $T \cap N$ is Irrelevant.

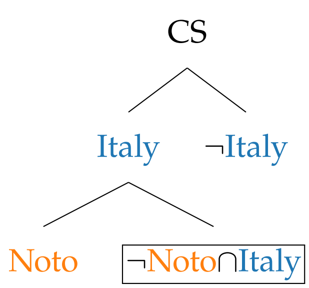
Lastly, let us consider one more case that will turn out useful when analyzing HCs. We now assume that the restrictor node at stake in tree-node intersection, is compatible with all the leaves in the Qtree is gets intersected with. In that case, tree-node intersection may shrink some leaves of the input Qtree, but, all leaves in the original Qtree, will still intersect with some leaf in the output Qtree, due to the assumption of compatibility. In other words, the intersection operation will not exclude any leaf. It will thus be deemed Irrelevant. This is summarized in (147).

(147) *Irrelevance by* **Holistic Compatibility**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). If $\forall L \in \mathcal{L}(T)$. $L \wedge N \nvDash \bot$, then $T \cap N$ is Irrelevant.

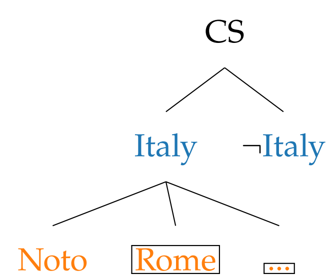
We are now equipped with the tools and definitions to smoothly deal with HCs.

3.4.4 Capturing the contrast in Hurford Conditionals

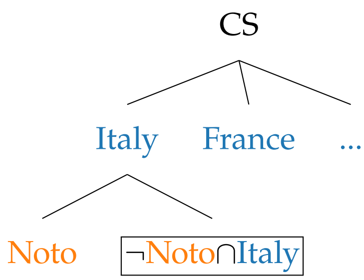
We now explain the oddness pattern of the HCs in (125). We start with the felicitous HC (125b), whose Qtrees are repeated in Figure N below. In such Qtrees, the depth-2 layer, was obtained by intersecting a Qtree for the consequent (*not Noto*), with the *Italy*-node verifying the antecedent.



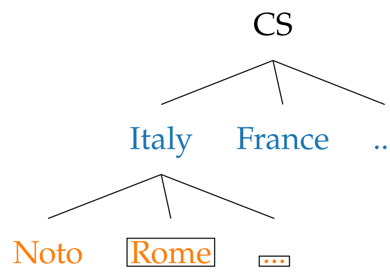
(1) Tree D1 → Tree E1.
Q-Irrelevant.



(2) Tree D1 → Tree E2/E3.
Q-Relevant.



(3) Tree D2 → Tree E1.
Q-Irrelevant.



(4) Tree D2 → Tree E2/E3.
Q-Relevant.

Figure N: Qtrees for (125b)=*If SuB29 will take place in Italy, it will not take place in Noto*.

Let us now review each Qtree and see whether the tree-node intersection operation used to form it, is Relevant. Note that it is enough to find one well-formed Qtree for (125b) to be correctly predicted to be felicitous.

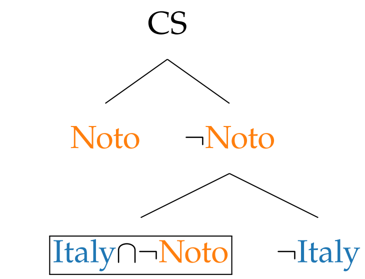
Starting with the Qtree in Figure N1, this Qtree was obtained by intersecting the "polar" Qtree for *not Noto*, with *Italy*. Because *Italy* is compatible with both *Noto* and *not Noto*, the Holistic Compatibility property (147) predicts the intersection operation to be Irrelevant. So this Qtree is odd given (125b).

Now considering the Qtree in Figure N2; this Qtree was obtained by intersecting the "*wh*" (articulated or not) Qtree for *not Noto*, with *Italy*. The resulting set of leaves, is the partition of *Italy* made of Italian cities. This set of leaves, makes the intersection operation Relevant: all Italian cities were leaves in the original "*wh*" (articulated or not) Qtree for *not Noto*, and additionally, the original "*wh*" Qtree for *not Noto*, contained non-Italian city leaves, that were properly excluded by the intersection operation. Therefore, the Qtree in Figure N2, does not violate Incremental Q-Relevance, and (125b) is correctly predicted not to be felicitous.

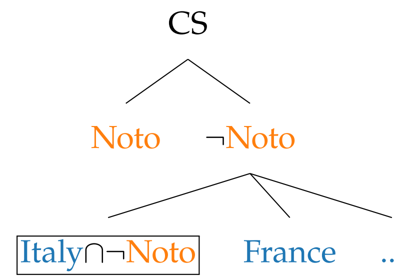
We could stop here, but let us review the Qtrees in Figures N3 and N4 for completeness. Turning to the Qtree in Figure N3; it was obtained by intersecting the "polar" Qtree for *not Noto*, with *Italy* – just like the Qtree in Figure N1. The intersection operation is therefore predicted to be Irrelevant too.

Lastly, the Qtrees in Figure N4 was obtained by intersecting the "*wh*" (articulated or not) Qtree for *not Noto*, with *Italy* – just like the Qtree in Figure N2. The intersection operation is therefore predicted to be Relevant, too.

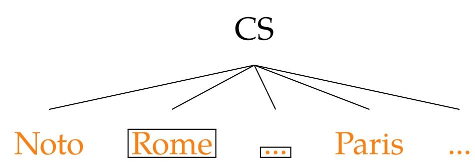
We now proceed to analyzing the infelicitous HC (125a), whose Qtrees are repeated in Figure O below. In the Qtrees in Figure O1 and O2, the depth-2 layer, was obtained by intersecting a Qtree for the consequent (*Italy*), with the *not Noto*-node verifying the antecedent. In the Qtrees in Figures O3 and O4, all layers are contributed by the antecedent Qtree, because the intersection operation between city-level leaves and the country-level Qtree contributed by the consequent, was shown to be vacuous.



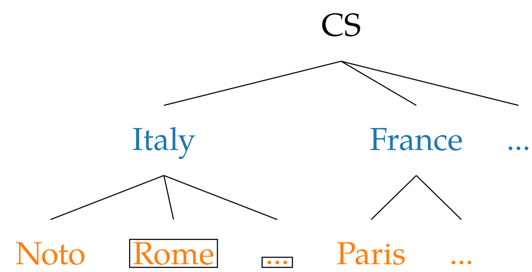
(1) Tree E1 → Tree D1.
Q-Irrelevant.



(2) Tree E1 → Tree D2.
Q-Irrelevant.



(3) Tree E2 → Tree D1/D2.
Q-Irrelevant.



(4) Tree E3 → Tree D1/D2.
Q-Irrelevant.

Figure O: Qtrees for (125a)=#*If SuB29 will not take place in Noto, it will take place in Italy*.

Let us now review each Qtree and show that, in each case, the tree-node intersection operation used to form it, was Irrelevant.

Starting with the Qtree in Figure O1, this Qtree was obtained by intersecting the "polar" Qtree for *Italy*, with *not Noto*. Because *not Noto* is compatible with both *Italy* and *not Italy*, the Holistic Compatibility property (147) predicts the intersection operation to be Irrelevant. So this Qtree is ill-formed.

Now considering the Qtree in Figure O2; this Qtree was obtained by intersecting the "*wh*" Qtree for *Italy*, with *not Noto*. Again, because *not Noto* is compatible with any country-level node, the Holistic Compatibility property (147) predicts the intersection operation to be Irrelevant. So this Qtree is ill-formed as well.

Turning to the Qtree in Figure O3; this Qtree was obtained by intersecting any Qtree for *Italy* (polar or "*wh*"), with city-leaves that are not *not Noto*. Let us start by considering the intersection between a city-leaf, and a polar Qtree for *Italy*, whose leaves are *Italy* and *not Italy*. Because any city, strictly entails *Italy* or strictly entails *not Italy*, the Single Strict Entailment property (145) predicts the intersection operation to be Irrelevant. Now consider the intersection between a city-leaf, and a "*wh*" Qtree for *Italy*, whose leaves are country-level. Because any city, strictly entails some country, the Single Strict Entailment property (145), again predicts the intersection operation to be Irrelevant. So, no matter how it gets derived, the Qtree in Figure O3, is derived *via* an Irrelevant tree-node intersection operation. So this Qtree is ill-formed.

Lastly, the Qtree in Figure O4 was obtained by intersecting any Qtree for *Italy* (polar or "*wh*"), with city-leaves that are not *not Noto*. The intersections operations are thus exactly similar to those performed in Figure O3, and which were just shown to be Irrelevant as per the Single Strict Entailment property (145). So, no matter how it gets derived, the Qtree in Figure O4, is derived *via* an Irrelevant tree-node intersection operation. So this Qtree is ill-formed.

Therefore all Qtrees derived for the infelicitous HC (125a), were derived *via* an Irrelevant tree-node intersection operation. As a result, (125a) is not compatible with any well-formed Qtree, and as such should be deemed odd. The contrast observed in HCs is captured.

3.4.5 Taking stock

In this Section, we have shown that HCs could not be accounted for by previously posited constraints (Empty Labeling; Q-Non-Redundancy). We have then discussed how earlier notions or Relevance could help, *modulo* stipulative assumptions. We then proposed a new notion of Relevance, Incremental Q-Relevance, that we framed as a constraint on the tree-node intersection operation, an operation recruited during the formation of conditional Qtrees. This way, we got the specific directionality of Relevance, for free. We then showed how this new view could capture the contrast in HCs. Specifically, we showed that some Qtrees evoked by the felicitous HC (125b), were derived *via* an intersection operation that was properly "shrinking" the consequent Qtree – retaining at least one leaf; excluding at least one leaf. And we showed that no Qtree evoked by the infelicitous HC (125a), could be derived in a similar fashion.

Zooming out, the asymmetry we derived in HC can be traced back to how Qtrees for *Italy* (p) and (*not*) *Noto* ($(\neg)p^+$) were defined: crucially, we observed beck in Section 3.3.3, that at least one Qtree for *not Noto* formed a strict refinement of a Qtree for *Italy* – eventually leading to a Relevant tree-node intersection operation. This intuition is generalized in (148), which roughly says that two Qtrees can be "conditionalized" if all the verifying nodes of the antecedent Qtree, are further subdivided in the consequent Qtree.

(148) Qtree Refinements and Incremental Q-Relevance. Let T be a Qtree whose verifying leaves are $\mathcal{N}^+(T)$, and all have depth at least 1 (i.e. the root is not verifying). Let T' be a Qtree whose root is the same as T, whose nodes include $\mathcal{N}^+(T)$, and are s.t. $\mathcal{L}(T') \cap \mathcal{N}^+(T) = \emptyset$, i.e. any node in T' that is verifying in T, is further subdivided in T'. Then, a conditional Qtree can be formed out of T (as antecedent) and T' (as consequent).

(149) *Proof of (148).* Let T be a Qtree whose verifying nodes are $\mathcal{N}^+(T)$, and all have depth at least 1. Let T' be a Qtree whose root is the same as T, whose nodes include $\mathcal{N}^+(T)$, and are s.t. $\mathcal{L}(T') \cap \mathcal{N}^+(T) = \emptyset$. To show that a conditional Qtree can be formed out of T (as antecedent) and T' (as consequent), we must show that the intersection between each verifying leaf of T and T', is Relevant. Because all verifying nodes in T have the same characteristics in T', it is enough to show that the intersection between an arbitrary verifying node in T, and T', is Relevant. Let N be such a node. By assumption, N is in T' and further subdivided in T'. So $T' \cap N$ is exactly the subtree of T' rooted in N. Let us call this subtree T''. T'' contains at least a leaf from T' (in fact, all its leaves, are leaves from T'). Additionally, T'' does not contain all leaves from T'. This is because all leaves from T' partition the root of T'. Because N is by assumption different from the root (i.e. a subset of the root), $T'' = T \cap N$'s leaves, partition a subset of the root. So $T'' = T \cap N$'s leaves, form a subset of T''s leaves. Therefore, $T \cap N$ is Relevant, and the conditional Qtree formed out of T (as antecedent) and T' (as consequent), is well-formed.

A special case of this condition, is when the verifying nodes of the antecedent and consequent Qtree are all leaves, and when the consequent strictly refines the antecedent. In that case, the resulting conditional Qtree is defined. This is exactly the kind of configuration that felicitous HCs like (125b) give rise to: we observed that some Qtree for *not Noto* formed a strict refinement of some Qtree for *Italy*, and additionally, both Qtrees only had verifying leaves.

By contrast, when the verifying nodes of the antecedent and consequent Qtree are all leaves, and when the antecedent strictly refines the consequent, the resulting conditional Qtree is *not* defined. This is exactly the kind of configuration that infelicitous HCs like (125a) give rise to: we observed that no Qtree for *Italy* formed a (strict) refinement of a Qtree for *not Noto* – leading to Irrelevant tree-node intersection operations across the board. These general properties are spelled out in (150) and proved in (151).

(150) STRICT QTREE REFINEMENTS AND INCREMENTAL Q-RELEVANCE. Let T and T' be Qtrees that are more than just one root, whose verifying nodes are leaves and s.t. T' strictly refines T. Then:

(i) no conditional Qtree can be formed out of T' (as antecedent) and T (as consequent);

(ii) while a conditional Qtree can be formed out of T (as antecedent) and T' (as consequent). This is a special case of (148).

(151) *Proof of (150).* Let T and T' be Qtrees that are more than just one root, whose verifying nodes are leaves and s.t. T' strictly refines T.

a. *Proof of (150i).* Let L' be verifying in T'. Because T' strictly refines T, L' cannot be a leaf in T, and must strictly entail a leaf in T. Therefore, $T \cap L'$ is IRRELEVANT as per the SINGLE STRICT ENTAILMENT property (145). Therefore, the conditional Qtree $T' \to T$ is not defined.

b. *Proof of (150ii).* Let L be verifying in T. Because T' strictly refines T, L also belongs to T', and has at least two children in T'. Therefore, $T' \cap L$ is the subtree of T' rooted in L, that is more than just one root. The leaves of this subtree are all leaves of T', and additionally, are not *all* leaves of T', otherwise L would have been T/T''s root, contrary to assumptions. Therefore, $T' \cap L$ is RELEVANT and the conditional Qtree $T \to T'$ is defined.

In summary, if the verifying, "restrictor" nodes provided by the antecedent Qtree, are too fine-grained for the consequent Qtree, the tree-node intersection operations performed when building a conditional Qtree, will be IRRELEVANT, and the entire derivation will crash. This leads us to mention a couple more important observations about how INCREMENTAL Q-RELEVANCE operates. First, even if INCREMENTAL Q-RELEVANCE restricts tree-node intersection, where the tree is contributed by the consequent and the restrictor node (a proposition) is contributed by the antecedent, it is indirectly sensitive to the structure of the antecedent Qtree, essentially because the verifying "restrictor" nodes passed to tree-node intersection, are determined by how fine-grained the antecedent's Qtree is. If the antecedent is fined-grained like (*not*) *Noto*, the verifying nodes of its Qtrees will be fine-grained as well, meaning, the restrictor nodes passed to tree-node intersection, will be fine-grained. This in turn will make tree-node intersection less likely to achieve RELEVANCE. In that sense, INCREMENTAL Q-RELEVANCE is conceptually different from **?**'s and **?**'s approaches to "assertive" Relevance,[8] which treated a Relevant or Irrelevant proposition simply as a set of worlds, and not as a set of nodes (i.e. a set of sets of worlds).

A second, yet observation, is that Incremental Q-Relevance being a constraint on tree-node intersection, it will have to be checked for every single tree-node intersection operation performed as part of the formation of a conditional Qtree. If one of these operations fails to be Relevant, then the derivation of the entire conditional Qtree, will be expected to fail. There will be as many such operations, as there are verifying, "restrictor" nodes in the antecedent Qtree. In particular, increasing the complexity of the antecedent, may increase the number of verifying nodes. In that respect, an interesting subcase is that of a disjunctive antecedent mixing different levels of granularity. In such cases, the finer-grained component will determine what the finest-grained restrictor nodes are, and as such, will constitute the bottleneck for Incremental Q-Relevance. We will see a few examples instantiating that observation in the next Section.

3.5 Extension to "Compatible" Hurford Conditionals

In this Section, we explore the predictions of Incremental Q-Relevance on data that might look familiar from Chapter **??**, that the other prominent account of HCs, Super-Redundancy, is shown to struggle with.

3.5.1 "Compatible" Hurford Disjunctions and their disjunctwise negated counterparts

In Chapter **??**, we introduced "Compatible" Hurford Disjunctions (henceforth **CHD**s), repeated below. Such disjunctions feature merely compatible disjuncts, and still feel odd. Chapter **??** predicted the sentences in (**??**) to be odd, due to them featuring disjuncts conveying incomparable degrees of granularity, which in turn made it impossible to derive well-formed disjunctive Qtrees for these sentences. Most if not all accounts of pragmatic oddness, including Super-Redundancy, struggle with such sentences.

(**??**) a. # SuB29 will take place in the Basque country or France. $\mathbf{q} \vee \mathbf{p}$; $\mathbf{q} \wedge \mathbf{p} \not\models \bot$

b. # SuB29 will take place in France or in the Basque country. $\mathbf{p} \vee \mathbf{q}$; $\mathbf{p} \wedge \mathbf{q} \not\models \bot$

[8] Both **?** and **?** however proposed Relevance constraints between question-types. For Roberts for instance, a follow-up question is Relevant to a QuD, if all the alternatives in the denotation of that follow-up question, are **?**-Relevant (as defined in (32)). This brings us a bit closer to what was done here. **?**'s approach is further described and analyzed in Appendix 3.7.

Just like regular HDs, CHDs have "disjunctwise" negated counterparts, given in (152). We will call such expressions **DNCHDs**. The DNCHDs in (152) still meet the description of CHDs, because, if p and q are merely compatible, so do $\neg p$ and $\neg q$.

(152) a. # SuB29 won't take place in the Basque country or won't take place in France.
$(\neg \mathbf{q}) \vee (\neg \mathbf{p})$; $(\neg \mathbf{q}) \wedge (\neg \mathbf{p}) \not\models \bot$

b. ?? SuB29 won't take place in France or won't take place in the Basque country.
$(\neg \mathbf{p}) \vee (\neg \mathbf{q})$; $(\neg \mathbf{p}) \wedge (\neg \mathbf{q}) \not\models \bot$

Our account predicts the sentences in (152) to be just as bad as those in (**??**), for the same reasons: their two disjuncts convey incomparable degrees of granularity – inherited from their unnegated counterparts. Therefore, none of the variants in (152) can evoke a well-formed disjunctive Qtree. **?**'s Super-Redundancy on the other hand, predicts both sentences to be fine. This is detailed in (153).

(153) DNCHDs are not Super Redundant (SR).
We show (152)=$(\neg \mathbf{p}) \vee (\neg \mathbf{q})$, with $(\neg \mathbf{p}) \wedge (\neg \mathbf{q}) \not\models \bot$, is not SR.
Take C = $\neg \mathbf{p}$.
We then have $(131)^{-}_{C} = \neg \mathbf{q}$.
Take $D = \bot$.

$$
\begin{aligned}
(152)_{Str(C,D)} &= (\neg(\mathbf{p} \wedge D)) \vee (\neg \mathbf{q}) \\
&\equiv (\neg(\mathbf{p} \wedge \bot)) \vee (\neg \mathbf{q}) \\
&\equiv (\neg \bot) \vee (\neg \mathbf{q}) \\
&\equiv \top \vee (\neg \mathbf{q}) \\
&\equiv \top \\
&\not\equiv \neg \mathbf{q} = (152)^{-}_{C}
\end{aligned}
$$

Exact same reasoning when taking C = $\neg \mathbf{q}$, swapping the roles of $\mathbf{p}$ and $\mathbf{q}$.

What about conditional variants of the sentences in (**??**) and (152), obtained *via* the *or*-to-*if* tautology?

3.5.2 Constructing "Compatible" HCs from CHDs

Because we assumed that the formation of conditional Qtrees is distinct from that of disjunctive Qtrees, changing the disjunctions in (**??**) and (152) into conditionals, may lead to different predictions. We now apply the *or*-to-*if* tautology to the sentences in (**??**) and (152), to create four different kinds of "Compatible" Hurford Conditionals (henceforth **CHCs**). Note that we can generate four variants instead of just two (as it was the case for simple HCs derived out of HDs and DNHDs), because the two disjuncts in CHDs and DNCHDs, are not in any kind of entailment relation, and therefore play interchangeable roles. In other words, either disjunct can appear as an antecedent or consequent in the derived conditionals. Such conditionals are given in (154).

(154) a. Derived from (**??**), using $\mathbf{q}$ as antecedent.
If SuB29 will not take place in the Basque country, it will take place in France.
$\neg \mathbf{q} \rightarrow \mathbf{p}$

b. Derived from (**??**), using $\mathbf{p}$ as antecedent.
?If SuB29 will not take place in France, it will take place in the Basque country.
$\neg \mathbf{p} \rightarrow \mathbf{q}$

c. Derived from (152) and double negation elimination, using $\neg \mathbf{q}$ as antecedent.
If SuB29 will take place in the Basque country, it will not take place in France.
$\mathbf{q} \rightarrow \neg \mathbf{p}$

d. Derived from (152) and double negation elimination, using $\neg \mathbf{p}$ as antecedent.
SuB29 will take place in France, it will not take place in the Basque country.
$\mathbf{p} \rightarrow \neg \mathbf{q}$

Interestingly, the four CHCs in (154), seem to contrast in terms of felicity. Although the judgments may be subtle, it appears that the variants featuring *France* in their antecedent ((154b) and (154d)), are less degraded than the variants featuring *the Basque country* in their antecedent ((154a) and (154c)). The felicitous variants (154b) and (154d), seem to be understandable as (155a) and (155b), respectively.

(155) a. If SuB29 will not take place in France, it will take place in the Spanish Basque country. $\neg \mathbf{p} \rightarrow (\neg \mathbf{p} \wedge \mathbf{q})$

b. If SuB29 will take place in France, it will not take place in the French Basque country. $\mathbf{p} \rightarrow \neg(\mathbf{p} \wedge \mathbf{q})$

Under the material implication hypothesis, SUPER-REDUNDANCY predicts all variants in (154) to be fine, simply because the CHDs they are derived from, are already mispredicted by this account to be fine. This is shown for (154a) in (156a) and for (154c) in (156b). This trivially extends to the two other variants (154b) and (154d) by simply swapping the roles of p and q in the proofs.

(156) CHCs are not Super Redundant (SR).

a. We show (154a)=$(\neg\mathbf{q}) \to \mathbf{p}$, with $\mathbf{p} \wedge \mathbf{q} \not\models \bot$, is not SR.
Take C = $\neg\mathbf{q}$.
We then have $(154a)^-_C = \mathbf{p}$.
Take $D = \top$.

$$\begin{aligned}(154a)_{Str(C,D)} &= (\neg(\mathbf{q} \wedge D)) \to \mathbf{p}\\ &\equiv (\neg(\mathbf{q} \wedge \top)) \to \mathbf{p}\\ &\equiv (\neg\mathbf{q}) \to \mathbf{p}\\ &\not\equiv \mathbf{p} = (154a)^-_C\end{aligned}$$

Take C = $\mathbf{p}$.
We then have $(154a)^-_C = \neg\mathbf{q}$.
Take $D = \top$.

$$\begin{aligned}(154a)_{Str(C,D)} &= (\neg\mathbf{q}) \to (\mathbf{p} \wedge D)\\ &\equiv (\neg\mathbf{q}) \to (\mathbf{p} \wedge \top)\\ &\equiv (\neg\mathbf{q}) \to \mathbf{p}\\ &\not\equiv \neg\mathbf{q} = (154a)^-_C\end{aligned}$$

b. We show (154c)=$\mathbf{q} \to (\neg\mathbf{p})$, with $\mathbf{p} \wedge \mathbf{q} \not\models \bot$, is not SR.
Take C = $\mathbf{q}$.
We then have $(154c)^-_C = \neg\mathbf{p}$.
Take $D = \top$.

$$\begin{aligned}(154c)_{Str(C,D)} &= (\mathbf{q} \wedge D) \to (\neg\mathbf{p})\\ &\equiv (\mathbf{q} \wedge \top) \to (\neg\mathbf{p})\\ &\equiv \mathbf{q} \to (\neg\mathbf{p})\\ &\not\equiv \neg\mathbf{p} = (154c)^-_C\end{aligned}$$

Take C = $\neg\mathbf{p}$.
We then have $(154a)^-_C = \mathbf{q}$.
Take $D = \top$.

$$\begin{aligned}(154c)_{Str(C,D)} &= \mathbf{q} \to \neg(\mathbf{p} \wedge D)\\ &\equiv \mathbf{q} \to \neg(\mathbf{p} \wedge \top)\\ &\equiv \mathbf{q} \to \neg\mathbf{p}\\ &\not\equiv \mathbf{q} = (154c)^-_C\end{aligned}$$

From our perspective, this pattern may also look surprising, because we just established that Relevance in conditionals was associated with "granularity" violations (specifically, cases in which the antecedent was finer-grained than the consequent), and moreover, Chapter **??** established that *SuB29 will take place in France*, and *SuB29 will take place in the Basque country*, conveyed *orthogonal* degrees of granularity, that could not be reconciled. So, at first blush, we may have expected all the conditionals in (154) to pattern the same.

3.5.3 Capturing CHCs

We will now see that our model of evoked Qtrees, complemented with Incremental Q-Relevance *actually* captures the pattern in (154). This will boil down to the fact that, although *SuB29 will take place in France*, and *SuB29 will take place in the Basque country* do not evoke Qtrees in any kind of refinement relation, *SuB29 will take place in France* can be seen as "coarser-grained" than *SuB29 will take place in the Basque country*, in the following, weaker sense: some regions like the Basque country, can be included in the union of two countries, but it is harder to think of a country that would be included in the union of two (or more) regions.[9] We will see that this observation can be related to the property of Multiple Strict Entailment in (146), that we showed caused Irrelevance in tree-node intersection. This will be enough to derive that the tree-node intersection operations performed when deriving Qtrees for (154b) and (154d), are Relevant, while those performed in when deriving Qtrees for (154a) and (154c), are not.[10]

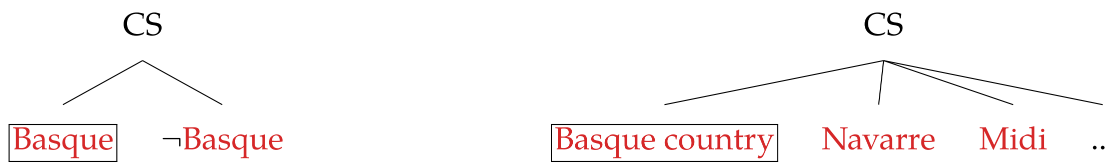
To this end, let us first repeat the Qtrees for S_p = *SuB29 will take place in France* and S_q = *SuB29 will take place in the Basque country*, already derived in Chapter **??**. Such Qtrees are given in Figure P and Q respectively.

(1) "Polar". (2) "*Wh*".

Figure P: Qtrees evoked by S_p = *SuB29 will take place in France*.

[9]Note that this difference may be even more obvious when considering similar configurations, but at different levels of granularity, e.g. by adopting **?**'s original examples involving countries like Russia, and continents like Asia. Some countries are included in the union of two continents, but no continent is included in the union of two (or more) countries. Here, we just kept France and the Basque country to stay in the theme.

[10]This prediction will follow from the concept of Relevance defined here, but does not follow from the previous version of this principle proposed in **?**.



(1) “Polar”. (2) “*Wh*”.

Figure Q: Qtrees for $S_{\mathbf{q}}$ = *SuB29 will take place in the Basque country*.

Qtrees the negations of S_p and S_q, are given in Figures R and S respectively.

(1) “Polar”. (2) “*Wh*”.

Figure R: Qtrees evoked by $\neg S_{\mathbf{p}}$ = *SuB29 won't take place in France*.

(1) “Polar”. (2) “*Wh*”.

Figure S: Qtrees for $\neg S_{\mathbf{q}}$ = *SuB29 won't take place in the Basque country*.

We can now evaluate which sentences in (154) have their conditional Qtrees violate Incremental Q-Relevance. We start with the infelicitous variant (154a). (154a)'s Qtrees should be derived by composing antecedent Qtrees for *not Basque* (in Figure S) with consequent Qtrees for *France* (in Figure P).

First, let us consider Figure S1 as antecedent and Figure P1 as consequent. In that case, the *not Basque* node acts as the only restrictor node, and gets intersected with the polar partition *France* vs. *not France*. Because the set of *not Basque* worlds is compatible with both *France* and *not France*, the intersection operation is Irrelevant, as per the Holistic Compatibility property (147).

Second, let us consider Figure S1 as antecedent and Figure P2 as consequent. In that case, the *not Basque* node again acts as the only restrictor node, and gets intersected with a by-country partition. Because the set of *not Basque* worlds is compatible with any single country (including *France* and *Spain*), the intersection operation is Irrelevant, again as per the Holistic Compatibility property (147).

Third, let us consider Figure S2 as antecedent and Figure P1 as consequent. In that case, the region-level nodes different from the Basque country act as restrictor nodes, and each gets intersected with the polar partition *France* vs. *not France*. For the intersection operation to be Relevant, any region different from the Basque country, should strictly coincide with either *France* or *not France*. This would be the only way keep one cell, and exclude one other cell, from the consequent's partition and satisfy Incremental Q-Relevance. But obviously, this cannot hold of all regions – in fact, this hold of none of them. Therefore, the intersection operation is Irrelevant.

Fourth and lastly, let us consider Figure S2 as antecedent and Figure P2. In that case, the region-level nodes different from the Basque country act as restrictor nodes, and each gets intersected with a by-country partition. For the intersection operation to be Relevant, any region different from the Basque country, should contain at least one country, and exclude at least one country. This would be the only way keep one cell, and exclude one other cell, from the consequent's partition and satisfy Incremental Q-Relevance. But obviously, this cannot hold of all regions: many regions are strictly contained in one single country. Therefore, the intersection operation is Irrelevant. We have just shown that there is no way to derive a conditional Qtree for (154a) *via* well-formed (i.e. Relevant) tree-node intersection operations. Thus, (154a) is correctly predicted to be odd.

Now turning to the other infelicitous variant (154c). (154c)'s Qtrees should be derived by composing antecedent Qtrees for *Basque* (in Figure Q) with consequent Qtrees for *not France* (in Figure R).

First, let us consider Figure Q1 as antecedent and Figure R1 as consequent. In that case, the *Basque* node acts as the only restrictor node, and gets intersected with the polar partition *France* vs. *not France*. Because the set of *Basque* worlds is compatible with both *France* and *not France*, the intersection operation is Irrelevant, as per the Holistic Compatibility property (147).

Second, let us consider Figure Q1 as antecedent and Figure R2 as consequent. In that case, the *Basque* node again acts as the only restrictor node, and gets intersected with a by-country partition. Crucially here, because the set of *Basque* worlds can be partitioned into two subsets, namely, *the French Basque country* and *the Spanish Basque country*, each of which strictly entails a leaf in the consequent's Qtree (namely, *France* and *Spain*), the intersection operation is Irrelevant, as per the Multiple Strict Entailment property (146).

Third, let us consider Figure Q2 as antecedent and Figure R1 as consequent. In that case, the *Basque* node again acts as the only restrictor node, and gets intersected with the polar partition *France* vs. *not France*. We have already established that this is Irrelevant.

Fourth and lastly, let us consider Figure Q2 as antecedent and Figure R2. In that case, the *Basque* node again acts as the only restrictor node, and gets intersected with a by-country partition. We have already established that this is Irrelevant. We have just shown that there is no way to derive a conditional Qtree for (154c) *via* well-formed (i.e. Relevant) tree-node intersection operations. Thus, (154c) is correctly predicted to be odd.

Let us now show that the felicitous variants (154b) and (154d) *can* evoke conditional Qtrees derived *via* Relevant tree-node intersection operations. In the case of (154b), let us consider the polar Qtree for *not France* in Figure R1 as antecedent Qtree, and the "*wh*" Qtree for *the Basque country* in Figure Q2, as consequent Qtree. The resulting conditional Qtree is represented in Figure T.

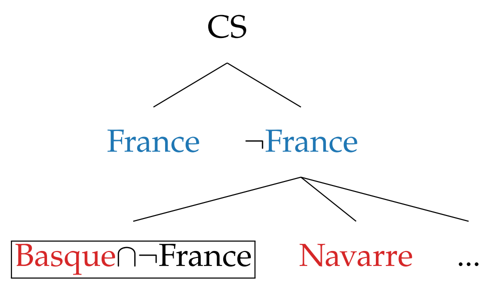


Figure T: Qtree evoked by ?(154b)=*If SuB29 will not take place in France, it will take place in the Basque country*.

In that case, the *not France* node acts as the only restrictor node, and gets intersected with a by-region partition. The result of this intersection, fully preserves all regions that are disjoint from France (e.g. *Navarre*) and fully rules out regions that are included in France (e.g. *Midi*). Regions partially not in France (e.g. *the Basque country*), are partially preserved. In any case, this intersection fully preserves one region from the original partition (e.g. *Navarre*), and fully excludes one (e.g. *Midi*). It is thus Relevant, and the resulting conditional Qtree in Figure T, is predicted to be well-formed. As a result, (154b) evokes at least one well-formed Qtree and is correctly predicted to be felicitous.

In the case of (154d), let us consider the polar Qtree for *France* in Figure P1 as antecedent Qtree, and the "*wh*" Qtree for *not Basque* in Figure S2, as consequent Qtree. The resulting conditional Qtree is represented in Figure U.

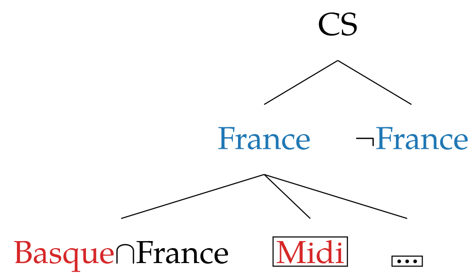


Figure U: Qtree evoked by (154d)=*If SuB29 will take place in France, it will not take place in the Basque country*.

In that case, the *France* node acts as the only restrictor node, and gets intersected with a by-region partition. The result of this intersection, fully preserves all regions included in France (e.g. *Midi*) and fully rules out regions completely out of France (e.g. *Navarre*). Regions partially in France (e.g. *the Basque country*), are partially preserved. In any case, this intersection fully preserves one region from the original partition (e.e. *Midi*), and fully excludes one (e.g. *Navarre*). It is thus RELEVANT, and the resulting conditional Qtree in Figure U, is predicted to be well-formed. As a result, (154d) evokes at least one well-formed Qtree and is correctly predicted to be felicitous.

In this Section, we have explored an interesting and relatively unexpected prediction of INCREMENTAL Q-RELEVANCE, when it comes to "Compatible" HCs. We have shown that INCREMENTAL Q-RELEVANCE, combined with our model of conveyed granularity, accounts for challenging contrasts affecting such CHCs. Interestingly, the intuitive readings of the felicitous variants (154b) and (154d), given in (155a) and (155b) respectively, are consistent with the well-formed Qtrees derived from these sentences, given in Figures T and U respectively. In these Figures, the verifying nodes are the ones contributed by the consequent, but *restricted* to the domain where the antecedent holds. For (154b), we end up with the non-French Basque country, i.e. the Spanish Basque country; for (154d), we end up with any French region that is not the French Basque country. These verifying nodes correspond to how the antecedent gets understood in (154b) and (154d).

More broadly, this result suggests that conditionals whose antecedent and consequent evoke orthogonal questions, may not always be degraded.[11] It also predicts that INCREMENTAL Q-RELEVANCE may filter out some interpretations of these conditionals, in terms of the possible questions they evoke. [12]The next Section turns to another case of "non-entailing"

[11] I thank Ivano Ciardelli for pointing out this issue to me. I was happy to realize that the concept of RELEVANCE introduced in this dissertation, unlike its previous version spelled out in **?** (which raised the initial concerns), *can* rule in such "orthogonal" conditionals, at least under certain conditions.

[12] A conditional like (i) for instance, can be felicitous granted that the proposition that *Jo gets into SuB29*

HCs, derived from familiar variants of HDs.

3.6 Conclusion and outlook

In this Chapter, we captured the challenging contrasts displayed by Hurford Conditionals, using two main ingredients. The first, was that sentences evoke questions (Qtrees) matching their degree of granularity. The second ingredient, was that the core operation behind the formation of conditional Qtrees (tree-node intersection), which is asymmetric in nature, is constrained by a new concept of Relevance. Drawing from both **?** and **?**, this new concept of Relevance was made asymmetrically sensitive to Qtree granularity. The contrast observed in HCs then boiled down to the (rough) intuitive idea that felicitous conditionals should display an antecedent that is coarser-grained than their consequent.

Beyond simple HCs, we explored more involved predictions of our Relevance constraint, and showed that, surprisingly, conditionals like CHCs, whose antecedent and consequent evoke orthogonal questions, *can* be felicitous under certain conditions. Further investigating the conditions under which these conditionals are fine, and what kind of information structure they evoke under such conditions, may give us new insights regarding the specific pragmatics of conditionals, e.g. the phenomenon of conditional perfection (**???????????**). Appendix 3.8 further extends this result to another class of HCs inspired from variants of HDs, and shows that the relevant paradigm can be explained *via* a combination of Relevance and Redundancy constraints. This captures the intuition that different sentences give rise to different flavors of oddness.

One datapoint that the current framework cannot account for, is given in (157). (157) is obtained from the infelicitous HC (125a), by simply replacing *Italy* with *France* in the consequent. The consequent *France*, unlike *Italy*, entails the antecedent not *not Noto*. This seems to lead to a sensible improvement of the judgment. Yet, our approach to Relevance does not distinguish between (125a) and (157),because it is unable to differentiate between two truth-conditionally different consequents, if they convey the same degree of

includes at least all the worlds in which Jo feels a certain emotion, and excludes all the worlds in which Jo feels another emotion. Also note that the question raised by the consequent of this sentence can only be polar (i.e. be about whether Jo is happy or not), if Jo getting into SuB29 strictly coincides with Jo being happy or Jo being not happy. This would lead to derive conditional perfection for (i). We leave a more systematic analysis of these observations for future work.

(i) If Jo gets into SuB29, they'll be very happy.

granularity. So, both sentences are predicted to be equally odd under the current view.[13]

(157) If SuB29 will not take place in Noto, it will take place in France. $\neg \mathbf{p}^{+} \rightarrow \mathbf{q}$

This Chapter, along with Chapter **??**, constituted an extensive discussion of non-scalar odd constructions, whether disjunctive or conditional in nature. The next Chapter leaves aside countries and cities (at last!) to investigate what happens in "scalar" counterparts of HDs, HCs, and some of their variants.

3.7 Appendix: ?'s Relevant Implications

We have previously reviewed **?**'s standard view on Relevance, according to which a *proposition* p is relevant to a *question* Q (partition of the CS) iff p's intersection with the CS corresponds to a (potentially empty) unions of cells in Q. But **?** also defines a concept of Relevance between two *propositions*, *via* their respective "subject matters". In more modern terms, "subject matters" correspond to questions. Under that view, two propositions are said to be Relevant to each other, iff their evoked questions are – in the sense of (158). This definition is disjunctive in nature, and therefore relatively weak. The Connection property in particular, appears quite easy to verify.

(158) **Relevance between two propositions** (rephrased in the QuD framework). Two propositions p and q, are relevant to each other, iff at least one of the following two conditions holds:

(i) Inclusion: the finest-grained question evoked by p refines the finest-grained question evoked by q.

(ii) Connection: some cell in the finest-grained question evoked by p does not overlap with some cell in the finest-grained question evoked by q.

The notable advantage of (158), is that it allows to *predict* that, in a conditional, antecedent and consequent should be Relevant to each other, in the sense of (158). Assuming a strict semantics for conditionals, i.e. that *if p then q* holds if every p-world of the CS is a q-world, **?** shows that, whenever *if p then q* holds, p and q are Relevant to each other in the sense of (158). However, when p and q are both contingent, i.e. have different

[13] The account laid out in **?** captures this datapoint, because it assigns a central role to nodes flagged as verifying by the consequent, when it comes to evaluating Relevance. The main idea, is that intersecting *Italy* with *not Noto* shrinks *Italy*, in turn causing infelicity, while intersecting *France* with *not Noto*, does not shrink *France*, thus preserving felicity. **?**'s account however, covers less ground regarding "orthogonal" conditionals, including CHCs.

truth values across different worlds of the CS, the aspect of (158) that is being used to prove this result, is CONNECTION. So, whenever *if* p *then* q holds and p and q are both contingent, p and q are connected, i.e. some cell in the finest-grained question evoked by p does not overlap with some cell in the finest-grained question evoked by q. Note that this condition is symmetric, and so is insensitive to a swap between antecedent and consequent. Therefore, whenever *if* q *then* p holds and p and q are both contingent, p and q are connected. Moreover, **?** shows that the prediction of RELEVANCE between antecedent and consequent, is insensitive to the introduction of negation. So, whenever *if* $\neg q$ *then* p holds and p and q are both contingent, p and q are connected; and whenever *if* p *then* $\neg q$ holds and p and q are both contingent, p and q are connected.

Could we use the concept of RELEVANCE between propositions introduced in (158) to tease apart HCs? HCs typically make use of two contingent propositions p and p^+, s.t. $p^+ \vDash p$. Based on (158), two HCs or the form $p \rightarrow \neg p^+$ and $\neg p^+ \rightarrow p$, will then imply the same condition of CONNECTION between p and p^+. This condition is quite weak, because it only implies that some cell of the question evoked by p be disjoint from some cell of the question evoked by p^+. But more importantly, it is the same for felicitous and infelicitous HCs. Therefore, (158) is insufficient to tease apart HCs, and a stronger approach to RELEVANCE must be posited.

3.8 Appendix: extension to "Long-Distance" Hurford Conditionals

3.8.1 "Long-Distance" Hurford Disjunctions and their disjunctwise negated counterparts

In Chapter **??**, we showed that "Long Distance" HDs (henceforth **LDHDs**, **?**, **?**), exemplified in (159), were Q-REDUNDANT. Such constructions are obtained from standard HDs of the form $p \vee p^+$ by further disjoining p^+ (that we will call "strong" disjunct) with a proposition r, which is s.t. $p^+ \vee r$ is merely compatible with p (that we will call "weak" disjunct). This can be done by choosing r to contradict p.[14] In (159) for instance, NELS55 taking place in Göttingen, which is located in Germany, is incompatible with NELS55 taking place in the US (and *a fortiori*, with NELS55 taking place in Connecticut).

[14]In fact, choosing r to be compatible with $\neg p$ would be enough to achieve the desired logical relation between the main disjuncts of an LDHD.

(159) a. # NELS55 will take place in the US, or will take place in Connecticut or in Göttingen.
$\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r})$ $\qquad \mathbf{p}^+ \vDash \mathbf{p}; (\mathbf{p}^+ \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

b. # NELS55 will take place in Connecticut or in Göttingen, or will take place in the US.
$(\mathbf{p}^+ \vee \mathbf{r}) \vee \mathbf{p}$ $\qquad \mathbf{p}^+ \vDash \mathbf{p}; (\mathbf{p}^+ \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

The infelicity of LDHDs is captured by Super-Redundancy. This is proved in (160) – adapted from **?**.

(160) LDHDs are Super-Redundant (SR).
We show (159a)=$\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r})$, with $\mathbf{p}^+ \vDash \mathbf{p}$ and $\mathbf{p} \wedge (\mathbf{p}^+ \vee \mathbf{r}) \nvDash \bot$, is SR.
Take C = $\mathbf{p}^+$.
We then have $(159a)^-_C = \mathbf{p} \vee \mathbf{r}$.

$$
\begin{aligned}
\forall D.\ (159a)_{Str(C,D)} &= \mathbf{p} \vee ((\mathbf{p}^+ \wedge D) \vee \mathbf{r}) \\
&\equiv \mathbf{p} \vee ((\mathbf{p}^+ \vee \mathbf{r}) \wedge (D \vee \mathbf{r})) \\
&\equiv (\mathbf{p} \vee \mathbf{p}^+ \vee \mathbf{r}) \wedge (\mathbf{p} \vee D \vee \mathbf{r}) \\
&\equiv (\mathbf{p} \vee \mathbf{r}) \wedge (\mathbf{p} \vee D \vee \mathbf{r}) \\
&\equiv \mathbf{p} \vee \mathbf{r} = (159a)^-_C
\end{aligned}
$$

Same proof for (159b)=$(\mathbf{p}^+ \vee \mathbf{r}) \vee \mathbf{p}$.

In a move that should now look familiar, we can derive "disjunctwise" negated LDHDs out of the sentences in (159). These negated variants are shown in (161). We will call such sentences **DNLDHD**s. Here is how DNLDHDs are constructed. In both (161a) and (161b), the weak disjunct, *NELS55 won't take place in Connecticut*, corresponds to the negation of the stronger disjunct of (159a)/(159b). Additionally, (161a) and (161b)'s stronger disjunct, *NELS55 will not take plces in the US*, corresponds to the negation of the weak disjunct of (159a)/(159b). Just like in (159), the extra proposition disjoined with the strong disjunct, *NELS55 will take place in New Haven*, is chosen to be incompatible with the weaker disjunct: NELS55 cannot take place in New Haven, and outside Connecticut.

(161) a. # NELS55 won't take place in Connecticut, or, won't take place in the US or will take place in New Haven.
$(\neg\mathbf{p}^+) \vee (\neg\mathbf{p} \vee \mathbf{s}) = \mathbf{q} \vee (\mathbf{q}^+ \vee \mathbf{s})$
With $\mathbf{q} := \neg\mathbf{p}^+; \mathbf{q}^+ := \neg\mathbf{p}$ s.t. $\mathbf{q}^+ \vDash \mathbf{q}; (\mathbf{q}^+ \vee \mathbf{s}) \wedge \mathbf{q} \nvDash \bot$

b. # NELS55 won't take place in the US or will take place in New Haven, or, won't take place in Connecticut.
$(\neg\mathbf{p} \vee \mathbf{s}) \vee (\neg\mathbf{p}^+) = (\mathbf{q}^+ \vee \mathbf{s}) \vee \mathbf{q}$
With $\mathbf{q} := \neg\mathbf{p}^+; \mathbf{q}^+ := \neg\mathbf{p}$ s.t. $\mathbf{q}^+ \vDash \mathbf{q}; (\mathbf{q}^+ \vee \mathbf{s}) \wedge \mathbf{q} \nvDash \bot$

The DNLDHDs in (161), appear extremely degraded, which intuitively seems to come the observation that they directly combine (negated) propositions conveying very different degrees of granularity; e.g. *NELS55 will take place in New Haven*, and *NELS55 won't take place in the US*. But despite this intuition, DNLDHDs are to LDHDs what DNHDs are to HDs. In particular, DNLDHDs are identical to LDHDs both in terms of their structure and in terms of the logical relations between their constitutive parts. Meaning, the LDHDs in (161) are isomorphic with the DNLDHDs in (159) – just like HDs are isomorphic with their disjunctwise negated counterparts.

For this very reason, the DNLDHDs in (161) are both predicted by Super-Redundancy to be fine, for the same reasons as DNHDs. This is proved in (162).

(162) DNLDHDs are not Super-Redundant (SR).
We show (161a) = $(\neg \mathbf{p}^+) \vee (\neg \mathbf{p} \vee \mathbf{s})$, with $\neg \mathbf{p} \vDash \neg \mathbf{p}^+$ and $(\neg \mathbf{p}^+) \wedge (\neg \mathbf{p} \vee \mathbf{s}) \nvDash \bot$, is not SR.
Take $C = \neg \mathbf{p}^+$.
We then have $(161\text{a})^-_C = (\neg \mathbf{p}) \vee \mathbf{s}$
Take $D = \top$.

$$\begin{aligned} (161\text{a})_{Str(C,D)} &= (\neg(\mathbf{p}^+ \wedge D)) \vee (\neg \mathbf{p} \vee \mathbf{s}) \\ &\equiv (\neg(\mathbf{p}^+ \wedge \top)) \vee (\neg \mathbf{p} \vee \mathbf{s}) \\ &\equiv (\neg \mathbf{p}^+) \vee (\neg \mathbf{p} \vee \mathbf{s}) \\ &\equiv (\neg \mathbf{p}^+) \vee \mathbf{s} \\ &\not\equiv (\neg \mathbf{p}) \vee \mathbf{s} = (161\text{a})^-_C \end{aligned}$$

Take $C = \neg \mathbf{p}$.
We then have $(161\text{a})^-_C = (\neg \mathbf{p}^+) \vee \mathbf{s}$.
Take $D = \bot$.

$$\begin{aligned} (161\text{a})_{Str(C,D)} &= (\neg \mathbf{p}^+) \vee (\neg(\mathbf{p} \wedge D) \vee \mathbf{s}) \\ &\equiv (\neg \mathbf{p}^+) \vee (\neg(\mathbf{p} \wedge \bot) \vee \mathbf{s}) \\ &\equiv (\neg \mathbf{p}^+) \vee (\top \vee \mathbf{s}) \\ &\equiv \top \\ &\not\equiv (\neg \mathbf{p}^+) \vee \mathbf{s} = (161\text{a})^-_C \end{aligned}$$

Take $C = \mathbf{s}$.
We then have $(161\text{a})^-_C = (\neg \mathbf{p}^+) \vee (\neg \mathbf{p}) \equiv \neg \mathbf{p}^+$.
Take $D = \top$.

$$\begin{aligned} (161\text{a})_{Str(C,D)} &= (\neg \mathbf{p}^+) \vee (\neg \mathbf{p} \vee (\mathbf{s} \wedge D)) \\ &\equiv (\neg \mathbf{p}^+) \vee (\neg \mathbf{p} \vee (\mathbf{s} \wedge \top)) \\ &\equiv (\neg \mathbf{p}^+) \vee (\neg \mathbf{p} \vee \mathbf{s}) \end{aligned}$$

$$
\begin{aligned}
&\equiv (\neg \mathbf{p}^{+}) \vee \mathbf{s} \\
&\not\equiv \neg \mathbf{p}^{+} = (161\text{a})^{-}_{C}
\end{aligned}
$$

Take $C = (\neg \mathbf{p} \vee \mathbf{s})$.

We then have $(161\text{a})^{-}_{C} = \neg \mathbf{p}^{+}$.

Take $D = \top$.

$$
\begin{aligned}
(161\text{a})_{Str(C,D)} &= (\neg \mathbf{p}^{+}) \vee ((\neg \mathbf{p} \vee \mathbf{s}) \wedge D) \\
&\equiv (\neg \mathbf{p}^{+}) \vee ((\neg \mathbf{p} \vee \mathbf{s}) \wedge \top) \\
&\equiv (\neg \mathbf{p}^{+}) \vee (\neg \mathbf{p} \vee \mathbf{s}) \\
&\equiv (\neg \mathbf{p}^{+}) \vee \mathbf{s} \\
&\not\equiv (\neg \mathbf{p}^{+}) = (161\text{a})^{-}_{C}
\end{aligned}
$$

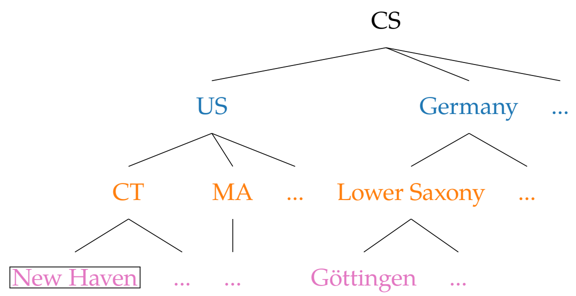
Same proof for $(161\text{b}) = (\neg \mathbf{p} \vee \mathbf{s}) \vee (\neg \mathbf{p}^{+})$.

Under our view, DNLDHDs are Q-Redundant due to their simplification $\neg p^{+} \vee s$, shown in (163).

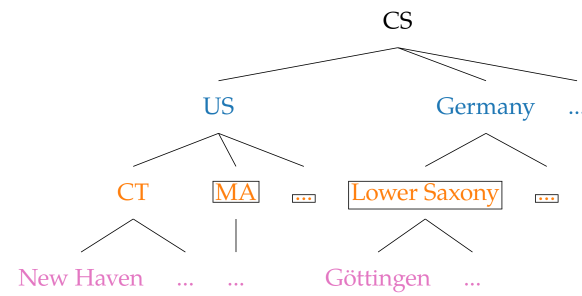
(163) NELS55 won't take place in Connecticut, or will take place in New Haven.
$(\neg \mathbf{p}^{+}) \vee \mathbf{s}$

Here is why. DNLDHDs like (161a) and (161b) display nested disjunctions, mixing three expressions associated with different levels of granularity: S_s = *NELS55 will take place in New Haven*, evokes a by-city partition; $\neg S_{p^+}$ = *NELS55 won't take place in Connecticut* evokes a by-region partition; $\neg S_p$ = *NELS55 won't take place in the US*, evokes a by-country partition.[15] The only way to properly disjoin Qtrees evoked by these expressions, is to have these Qtrees stand in a refinement relation. Specifically, the Qtree evoked by S_s = *NELS55 will take place in New Haven*, should refine the one evoked by $\neg S_{p^+}$ = *NELS55 won't take place in Connecticut*, which itself should refine the one evoked by $\neg S_p$ = *NELS55 won't take place in the US*. This is achieved by the "*wh*-articulated" Qtrees evoked by S_s, $\neg S_{p^+}$, and $\neg S_p$, respectively. Such Qtrees are represented in Figures V1, V2, and V3, and get properly disjoined in Figure V4.

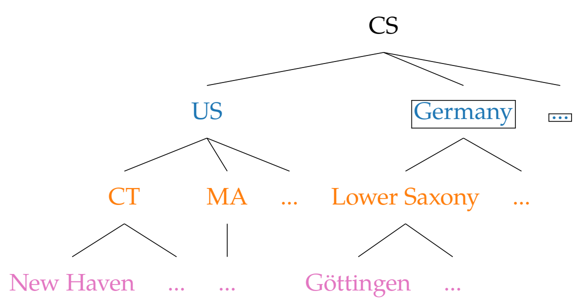
[15]Recall that negation does not have any effect on Qtree structure or conveyed granularity



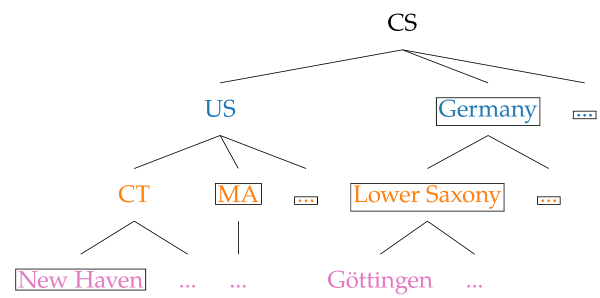
(1) “*Wh*-articulated” Qtree evoked by S_s = *NELS55 will take place in New Haven.*



(2) “*Wh*-articulated” Qtree evoked by $\neg S_{p^+}$ = *NELS55 won't take place in Connecticut.* 3



(3) “*Wh*-articulated” Qtree evoked by $\neg S_p$ = *NELS55 won't take place in the US.*



(4) Only Qtree evoked by (161a)/(161b), obtaining by disjoining Trees V1, V2, and V3.

Based on these Figures, one can also directly disjoin the Qtree evoked S_s, in Figure V1, with the one evoked by $\neg S_{p^+}$ in Figure V2. The result is represented in Figure W

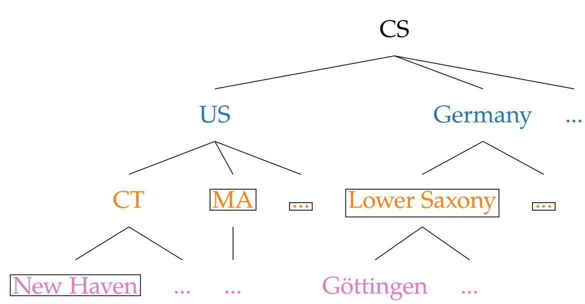


Figure W: Only Qtree evoked by (163) = $\neg S_{p^+} \vee S_s$, obtained by disjoining Trees V1 and V2.

Now, it can be shown that the Qtree corresponding to the DNLDHDs (161a)/(161b) in Figure V4, and the one corresponding to a simplification of these sentences, in Figure W, are equivalent. First, they are obviously structurally identical. And they are also characterized by the same sets of minimal verifying paths, essentially because any path from the root to a non-US region-node goes through a non-US country node. Therefore the Qtree in Figure V4 and the one in Figure W, both have the following set of minimal verifying paths: the path to *New Haven*; paths to US states different from Connecticut; paths to all non-US regions. Therefore, the only Qtree evoked by (161a)/(161b), is Q-Redundant given 161a)/(161b), and these sentences should be deemed odd.

We have just argued that our approach correctly predicts both LDHDs and DNLDHDs to be degraded, because both variants violate Q-Non-Redundancy. We proceed to explore "conditional" variants of these sentences and show that the sharp deviance of these variants is explained in our framework *via* a conspiration between Q-Non-Redundancy and Incremental Q-Relevance.

3.8.2 Constructing "Long-Distance" HCs from LDHDs

Let us now turn to the conditionals derived from LDHDs *via* the *or*-to-*if* tautology – in a way similar to how (C)HCs were derived earlier in that Chapter. To build such conditionals, we need two kinds of LDHDs: the ones in (159), and their disjunctwise negated counterparts in (161). Turning these LDHDs into conditionals *via* the *or*-to-*if* tautology, leads to the paradigm in (164). Similarly to the CHCs in Section 3.5, this paradigm is made of four different conditionals (instead of just two, as it was the case with simple HCs), because the main two disjuncts in (159) and (161), are not in any kind of entailment relation, i.e. play completely symmetric roles. As a result, it makes sense to apply the *or*-to-*if* tautology in either direction, i.e. to treat either disjunct's negation as the antecedent, and the remaining disjunct, as the consequent, of the resulting conditional.

(164) a. Derived from (159), using the simple ("weak") disjunct as antecedent.
If NELS55 won't take place in the US, it will take place in Connecticut or in Göttingen.
$\neg \mathbf{p} \rightarrow (\mathbf{p}^{+} \vee \mathbf{r})$

b. Derived from (159), using the complex disjunct as antecedent.
If NELS55 won't take place in Connecticut or in Göttingen, it will take place in the US.
$\neg(\mathbf{p}^{+} \vee \mathbf{r}) \rightarrow \mathbf{p}$

c. Derived from (161), using the simple ("weak") disjunct as antecedent.
If it's not the case that NELS55 won't take place in Connecticut, it won't take place in the US or will take place in New Haven.
$\neg(\neg \mathbf{p}^{+}) \rightarrow (\neg \mathbf{p} \vee \mathbf{s})$

d. Derived from (161), using the complex disjunct as antecedent.
\# If it's not the case that NELS55 won't take place in the US or will take place in New Haven, it won't take place in Connecticut.
$\neg(\neg \mathbf{p} \vee \mathbf{s}) \rightarrow (\neg \mathbf{p}^{+})$

All the conditionals in (164) appear infelicitous. Furthermore, it feels like these various conditionals, exhibit different "flavors" of oddness. Both (164a) and (164c) seem to convey contradictory information as part of their consequent (granted their antecedent). (164a)'s consequent entertains the idea that NELS55 will take place in Connecticut, while its antecedent said *not the US*). (164c)'s consequent entertains the idea that NELS55 won't take place in the US, while its antecedent said *Connecticut*. We will in fact show that both (164a) and (164c) are Q-Redundant, considering simplifications removing these locally contradictory pieces of information from their consequents. Turning to (164b), it just feels like its consequent does not make sense at at all, given its antecedent; we will in fact show that (164b) violates Incremental Q-Relevance. Lastly, (164d) is almost impossible to make sense of as-is. We will show (164d) violates Incremental Q-Relevance, as well.

This pattern is interesting, because, again, it is not predicted by Super-Redundancy, at least assuming conditionals are material. Indeed, under these (arguably simplifying) assumptions, the prediction made by Super-Redundancy are insensitive to transformations like the *or*-to-*if* tautology, but *are* sensitive to variable changes of the form $q := \neg p$. Super-Redundancy thus predicts the LDHCs in (164a) and (164b), derived from the LDHDs in (159), to be deviant (because the LDHDs in (159) were already predicted so). Similarly, it predicts the LDHCs in (164c) and (164d), derived from the LDHDs in (161), to be fine (because the LDHDs in (161) were already predicted so). In other words, the mispredictions of Super-Redundancy in the case of (161), spread to its conditional variants (164c) and (164d).

It is additionally worth noting that the conditionals in (164c) and (164d) may be simplified, by eliminating the double negation in the case of (164c), and by applying De Morgan's Law in the case of (164d). This is done in (165a) and (165b), respectively. In our current framework, double negation elimination, performed in (165a), is innocuous[16] It

[16]The way we defined negation makes it almost involutive, meaning, applying it twice very often leads to the same result as not applying it at all. One pathological case, is when an entire layer is flagged as verifying in the input Qtree. Applying negation to the input Qtree once, erases this flagging on the entire layer. Applying negation a second time, does not recover the flagged layer, because the flipping operation induced by negation only targets layers that have at least one verifying node. But sentences like (165a) do not belong to this pathological class.

is also inovuous from the point of Super-Redundancy, whose misprediction for (164c) carries over to (165a). However, De Morgan's Law, performed in (165b), is *a priori* not innocuous, at least for our account, because it introduces a brand new operator, conjunction, for which we have not yet devised an inquisitive contribution. And in fact, this conjunctive variant appears significantly more felicitous than the variant it is derived from. We will leave this conjunctive variant for future work.

(165) a. Derived from (164c), *via* double negation elimination.
If NELS55 will take place in Connecticut, it won't take place in the US or will take place in New Haven.
$\mathbf{p}^+ \to (\neg \mathbf{p} \vee \mathbf{r})$

b. Derived from (164d), *via* De Morgan's Law.
If NELS55 will take place in the US and not in New Haven, it won't take place in Connecticut.
$(\mathbf{p} \wedge \neg \mathbf{r}) \to (\neg \mathbf{p}^+)$

3.8.3 Capturing LDHCs

We now proceed to show that all variants in (164) – even when reasonably simplified – are predicted by our framework to be odd, for various reasons. We first focus on the two Q-Redundant cases ((164a) and (164c)/(165a)), then turn to the two Q-Irrelevant cases ((164b) and (164d)).

First, we show that (164a), repeated below, is Q-Redundant, given its simplification that omits the *Connecticut* (p^+) disjunct within its consequent; see (166)

(164a) # If NELS55 won't take place in the US, it will take place in Connecticut or in Göttingen.
$\neg \mathbf{p} \to (\mathbf{p}^+ \vee \mathbf{r})$

(166) If NELS55 won't take place in the US, it will take place in Göttingen.
$\neg \mathbf{p} \to \mathbf{r}$

S_{p^+} = *NELS55 will take place in Connecticut* is coarser-grained than S_r = *NELS55 will take place in Göttingen*, therefore both sentences evoke Qtree that can stand in a refinement relation, whose disjunction gives rise to the Qtree for $S_{p^+} \vee S_r$ in Figure X.[17]

[17]This Qtree could have involved more layers on top of the US state/region layer, but the presence/absence of such layers do not affect the final outcome.

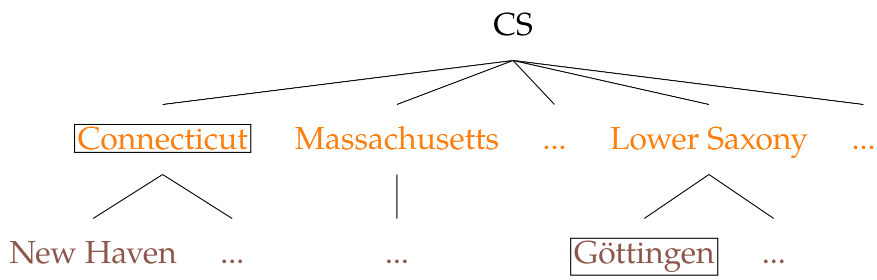
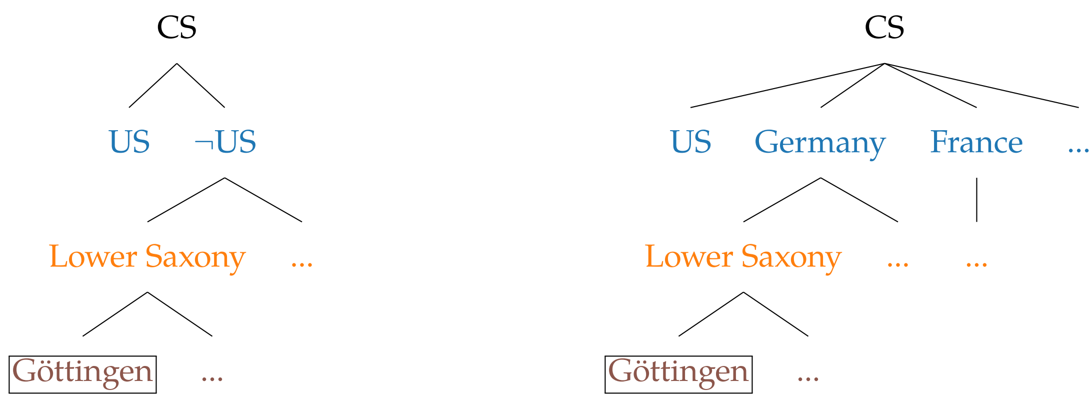


Figure X: Qtree evoked by $S_{\mathbf{p}^+} \vee S_{\mathbf{r}}$

Combining the Qtree in Figure X with an antecedent Qtree for $\neg S_p$ = *NELS55 won't take place in the US* (either "polar" or "*wh*"), yields the Qtrees in Figure Y. In such Qtrees, the *Connecticut* nodes that was verifying in the consequent, got filtered by tree-node intersection. And the Qtrees in Figure Y, are thus also evoked by the simplification $\neg S_p \to S_r$ in (166). As a result, (164a) is predicted to be odd due to violations of Q-Non-Redundancy.



(1) "Polar" antecedent Qtree $\to$ Tree X.

(2) "*Wh*" antecedent Qtree $\to$ Tree X.

Figure Y: Qtrees evoked by #(164a) = $(\neg S_{\mathbf{p}}) \to (S_{\mathbf{p}^+} \vee S_{\mathbf{r}})$.
Same as some Qtrees evoked by (166) = $\neg S_{\mathbf{p}} \to S_{\mathbf{r}}$, so Q-Redundant.

Secondly, let us show that (165a), derived from (164c) *via* double negation elimination,[18] is also Q-Redundant. This time, it is due to its simplification that omits the *not the US* ($\neg p$) disjunct within the consequent; see (167).

(165a) # If NELS55 will take place in Connecticut, it won't take place in the US or will take place in New Haven.
$\mathbf{p}^+ \to (\neg \mathbf{p} \vee \mathbf{s})$

[18] The case of (164c) is pretty obvious: it is Q-Redundant given (165a). That is why we focus on the simpler variant (165a).

(167) If NELS55 will take place in Connecticut, it will take place in New Haven.
$\mathbf{p}^{+} \rightarrow \mathbf{s}$

$\neg S_p$ = *NELS55 won't take place in the US* is coarser-grained than S_s = *NELS55 will take place in New Haven*, therefore both sentences evoke Qtree that can stand in a refinement relation, whose disjunction gives rise to the Qtree for $\neg S_p \vee S_s$ in Figure Z.[19]

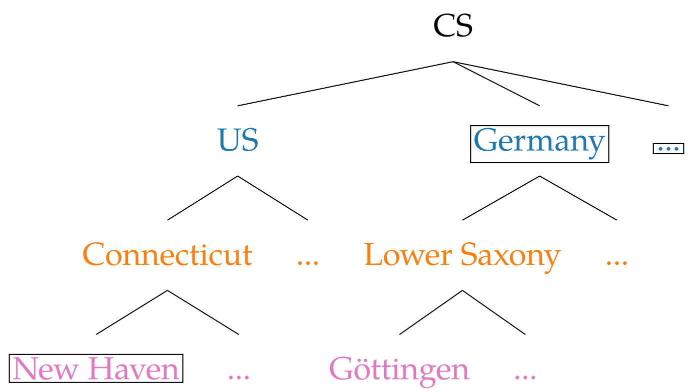
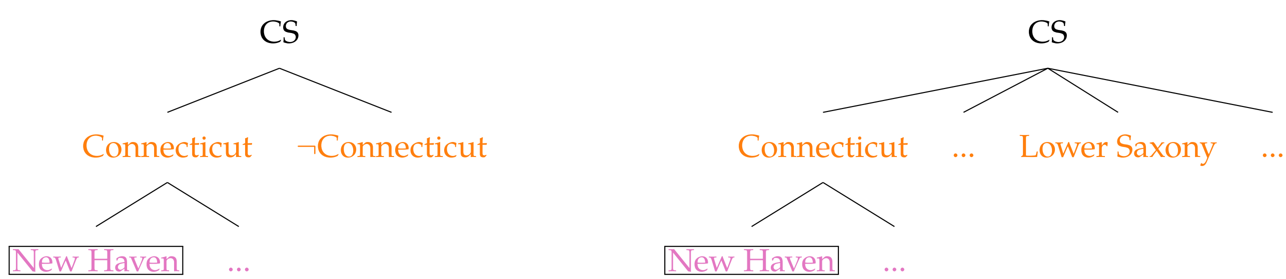


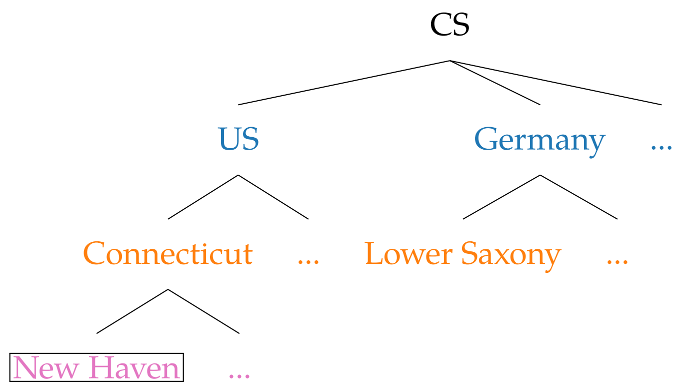
Figure Z: Qtree evoked by $\neg S_{\mathbf{p}} \vee S_{\mathbf{s}}$

Combining the Qtree in Figure Z with an antecedent Qtree for S_{p^+} = *NELS55 will take place in Connecticut* ("polar", "*wh*", or "*wh*-articulated"), yields the Qtrees in Figure AA. In such Qtrees, the non-US nodes that were verifying in the consequent, got filtered by tree-node intersection. And the Qtrees in Figure AA, are thus also evoked by the simplification $\neg S_{p^+} \rightarrow S_s$ in (167). As a result, (165a) is predicted to be odd due to violations of Q-NON-REDUNDANCY.



(1) "Polar" antecedent Qtree → Tree Z. (2) "*Wh*" antecedent Qtree → Tree Z.

[19]This Qtree may not have involved a US state/region layer, but the presence/absence of this layer (as well as the presence/absence of upper layers) does not affects the final outcome.



(3) "*Wh*-articulated" antecedent Qtree →
Tree Z.

Figure AA: Qtrees evoked by #(165a) = $S_{\mathbf{p}^+} \to (\neg S_{\mathbf{p}} \vee S_{\mathbf{s}})$.
Same as some Qtrees evoked by (167) = $S_{\mathbf{p}^+} \to S_{\mathbf{s}}$**, so Q-Redundant.**

Thirdly, we show that (164b), repeated below, turns out incrementally Irrelevant, essentially because the finest degree of granularity conveyed by its disjunctive antecedent is by-city, and as such, is finer-grained than the granularity conveyed by the consequent – incurring a violation close to the one observed in infelicitous HCs.

(164b) # If NELS55 won't take place in Connecticut or in Göttingen, it will take place in the US.
$\neg(\mathbf{p}^+ \vee \mathbf{r}) \to \mathbf{p}$

Figure AB shows the disjunctive Qtree evoked by (164b)'s antecedent, $\neg(S_{\mathbf{p}^+} \vee S_{\mathbf{r}})$ = *NELS55 won't take place in Connecticut or in Göttingen*. It is directly derived from the Qtree in Figure X, by simply flipping its verifying nodes layer-by-layer. Qtrees for (164b)'s consequent S_p = *NELS55 will take place in the US*, are given in Figure AC.

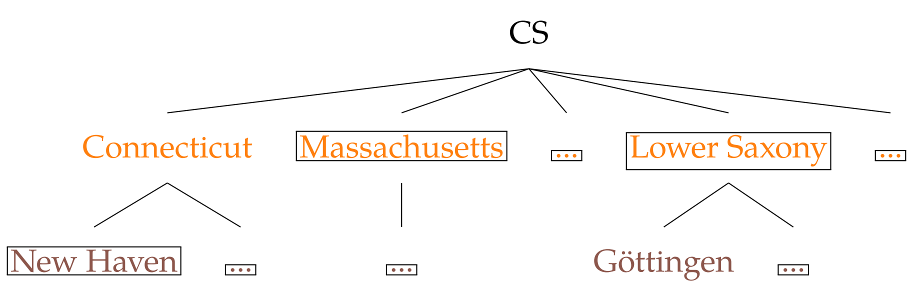


Figure AB: Qtree evoked by $\neg(S_{\mathbf{p}^+} \vee S_{\mathbf{r}})$

Figure AC: Qtrees evoked by $S_{\mathbf{p}}$.

It can be shown that combining the Qtree in Figure AB (as antecedent Qtree) with either Qtree in Figure AC (as consequent Qtree) *via* the conditional rule, violates Incremental Q-Relevance. Indeed, Incremental Q-Relevance imposes that intersecting either consequent Qtree in Figure AC with any verifying nodes from the Qtree in Figure AB, fully rules in a leaf and fully rules out another leaf. Let's consider the *New Haven* node, which is verifying in Figure AB. This node strictly entails *the US*, and so, intersecting the *New Haven* node with any Qtree from Figure AC, does not rule in any leaf, and so violates Incremental Q-Relevance. This is enough to predict that the formation of a conditional based on Figure AB as antecedent, and either Qtree in Figure AC, as consequent, will crash. As a result, (164b) is predicted to be odd due to violations of Incremental Q-Relevance.

Fourth, and lastly, we show that this last result extends to (164d), repeated below.

(164d) If it's not the case that NELS55 won't take place in the US or will take place in New Haven, it won't take place in Connecticut.
$\neg(\neg\mathbf{p} \vee \mathbf{s}) \rightarrow (\neg\mathbf{p}^{+})$

Figure AD shows the disjunctive Qtree evoked by (164d)'s antecedent, $\neg(\neg S_{\mathbf{p}} \vee S_{\mathbf{s}})$ = *It's not the case that NELS55 won't take place in the US or will take place in New Haven*. It is directly derived from the Qtree in Figure Z, by simply flipping its verifying nodes layer-by-layer. Qtrees for (164d)'s consequent $\neg S_{p^+}$ = *NELS55 won't take place in Connecticut*, are given in Figure AE.

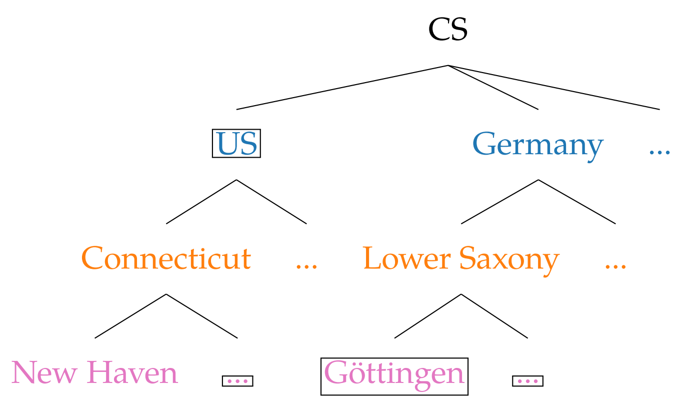


Figure AD: Qtree evoked by $\neg S_{\mathbf{p}} \vee S_{\mathbf{s}}$

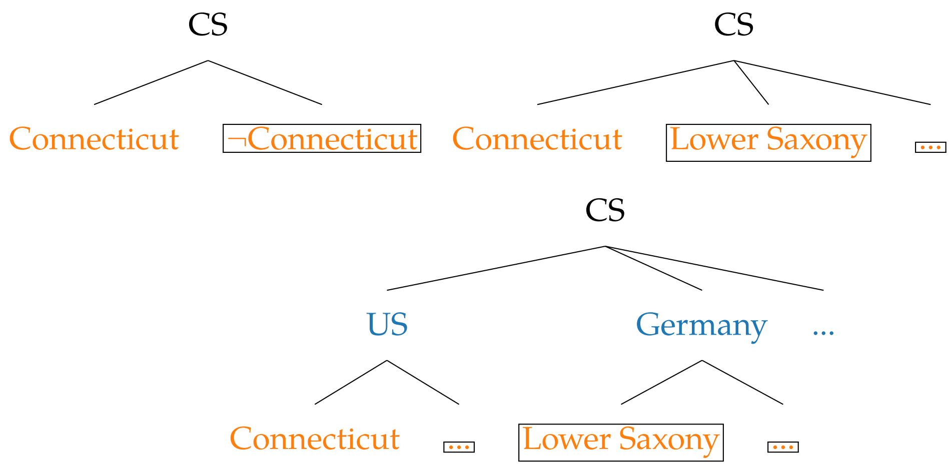


Figure AE: Qtrees evoked by $\neg S_{p^+}$.

Similarly to what was done for (164b), it can be shown that combining the Qtree in Figure AD (as antecedent Qtree) with any Qtree in Figure AE (as consequent Qtree) *via* the conditional rule, violates Incremental Q-Relevance. Let's consider the a *Connecticut*-node that is not *New Haven*. This node is verifying in Figure AD and strictly entails *Connecticut*, and so, intersecting this node with any Qtree from Figure AE, does not rule in any leaf, and so violates Incremental Q-Relevance. This is enough to predict that the formation of a conditional based on Figure AD as antecedent, and any Qtree in Figure AE, as consequent, will crash. As a result, (164d) is predicted to be odd due to violations of Incremental Q-Relevance.

In this Section, we have investigated "Long-Distance" Hurford Conditionals, derived from LDHDs. Such conditionals involve complex antecedents or consequents, and mix degree of granularity in various ways, but all appear quite degraded. They were shown to be challenging for at least one account of oddness solely based on the concept of Redundancy. Unlike this family of approach, our proposed framework talk about other ways to create long distance variants of HC by driectly disjoining the antecedent/conseunr of HCs.