

Repairing bad questions makes for good sentences: the case of (but and) at least¹

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¹Many thanks to Amir Anvari, Athulya Aravind, Gennaro Chierchia, Danny Fox, Martin Hackl, Nina Haslinger, Manfred Krifka, Viola Schmitt and Raven Zhang for their advising, feedback, or input. Thanks also to the audiences of the BerlinBrnoVienna Workshop 2024, SuB29, and AC2024 for relevant feedback on related projects. All mistakes are my own.

Some odd sentences

- Disjunctions featuring Contextually entailing disjuncts ($p^+ \models p$) are usually odd:

(1) **Hurford Disjunctions (HD; Hurford, 1974)**

- a. # Ed **studied in Paris** or **in France**. $p^+ \vee p$
- b. # Ed **studied in France** or **in Paris**. $p \vee p^+$

- Related conditionals are only odd if p^+ is in the antecedent:

(2) **Hurford Conditionals (HC; Mandelkern and Romoli, 2018)**

- a. # If Ed didn't **study in Paris**, he **studied in France**. $\neg p^+ \rightarrow p$
- b. If Ed **studied in France**, he didn't **study in Paris**. $\underbrace{p}_{\neg(\neg q^+)} \rightarrow \underbrace{\neg p^+}_q$

- We call (1-2) **Hurford Sentences**. The reason for their infelicity, whatever it is, is referred to as **Hurford's Constraint**.

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- Two existing accounts of (1-2): Kalomoiros (2024)'s SUPER-REDUNDANCY; Hénót-Mortier (to appear)'s compositional implicit QuD framework.
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In this talk, we will focus on the effect of *at least*, though we also have an analysis of *but*. We will:

1. Outline the **data** at stake, and the **challenges** it raises.
2. Sketch how the Compositional Implicit QuD framework captures the #-pattern in the **repairless Hurford Sentences** (1-2).
3. Model the **effect of at least** in that framework.
4. Show how this predicts the right **repair pattern** in (1-2).
5. Discuss and conclude with a few words on *but*.

Data and challenges

At least can repair some cases of oddness

- *At least* rescues (1a),¹ see (3a), but not (1b), see (3b).

- (3) a. Ed **studied in Paris** or at least **in France**. $p^+ \vee AL(p)$
b. # Ed at least **studied in France** or **in Paris**. $AL(p) \vee p^+$

- *At least* rescues (2a), see (4a). But it degrades (2b), see (4b).

- (4) a. If Ed didn't **study in Paris**, he at least **studied in France**.
 $\neg p^+ \rightarrow AL(p)$
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Previous insights on at least in Hurford Disjunctions

- Zhang (2022): *at least* changes the **QuD** s.t. the disjuncts in (3a) end up addressing distinct QuDs. Then, the whole disjunction no longer violates a “QuD-oriented” implementation of Hurford’s Constraint.
- Krifka (2024): *at least* weakens the embedded **Speech Act** associated with the second disjunct, s.t. the whole disjunction no longer violates a “Speech Act-oriented” variant of Hurford’s Constraint.
- Things in common: HDs exhibit **redundancy at some level** (QuD/Speech Acts), which *at least* repairs.
- Issue: no straightforward extension to the conditionals in (4).

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Challenges

- Two moving pieces:
 - (i) **constraint(s)** producing the right #-pattern in *repairless* Hurford Sentences (1-2)—already not easy;
 - (ii) a sensible **model of repairs** like *at least* whose interaction with (i), produces the right #-pattern in *repaired* Hurford Sentences (3-4).
- This is a difficult dance: (i) **differentially affects** HDs (both bad) and HCs (asymmetric)...
- ...yet **at least has a unified effect** in both constructions **p-p⁺** cases are #, **p⁺-p**-cases are good.
- We build on the Compositional Implicit QuD framework² to capture this intricate pattern.

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QuD-driven oddness in repairless cases

Take-home: oddness arises when a sentence cannot compositionally evoke an optimal QuD

Sentences evoke QuD-trees

- Assertions evoke the QuD(s) they are good answers to.³
- QuDs are nested partitions of the Context Set, represented graphically as trees.⁴
- Assertions identify how they answer each QuD by flagging nodes in the tree.

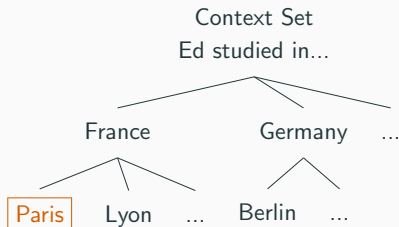


Figure 1: A QuD for Ed studied in Paris.

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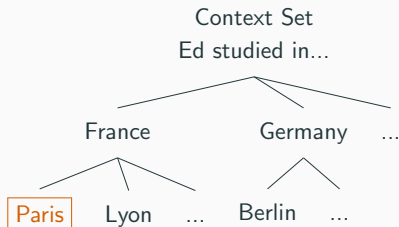


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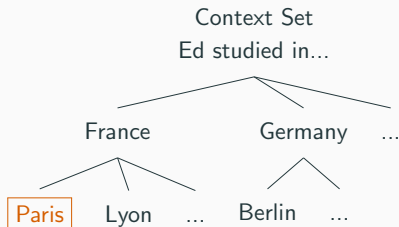


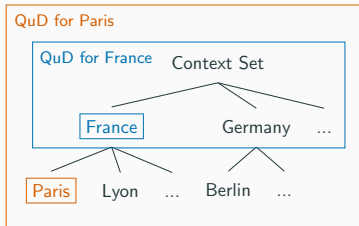
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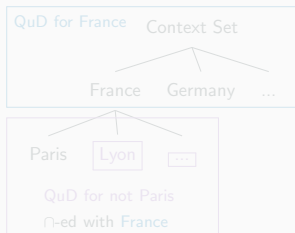
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Disjunctions and conditionals inquisitively differ

- The QuD of a disjunction **fuses** the QuDs of both disjuncts and **flags** the nodes associated with both disjuncts.⁵
- The QuD of a conditional **restricts** the QuD of the consequent, using the QuD of the antecedent; and only **flags** the nodes associated with the consequent.⁶



(a) QuD for #Paris or France.



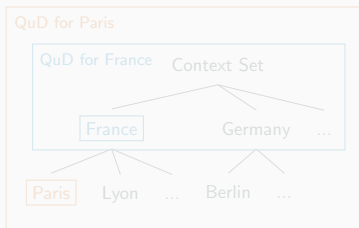
(b) QuD for if France then not Paris.

⁵Building on Simons (2001) and Zhang (2022)

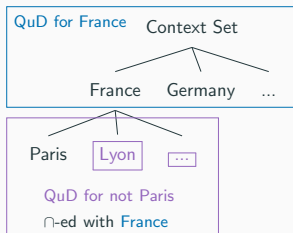
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Constraining sentences and their QuD trees

- Core idea: assertions have to be **“good answers to good questions”** (Katzir & Singh, 2015).
- Not all the QuD tree evoked by a sentence will be deemed felicitous.
- An odd sentence cannot be paired with *any* good QuD tree.
- Two constraints on QuD trees evoked by sentences:
 - Q-REDUNDANCY: roughly, do not have QuD trees flagging nodes in a “redundant” way;
 - Q-RELEVANCE: roughly, do not have QuD trees that “shrink” flagged nodes throughout their derivation.

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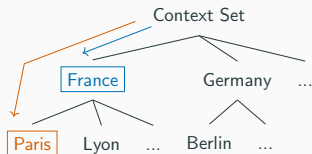
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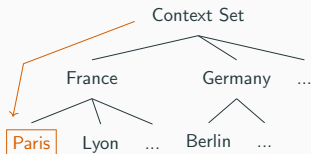
Q-Redundancy

- Q-REDUNDANCY: don't have a simpler alternative assertion evoking an “equivalent” Qtree (structure+minimal set of paths covering all boxed nodes).



(a) QuD for #Paris or France: suboptimal because equivalent to Figure (b): same structure, orange path covers both flagged nodes so is minimal.

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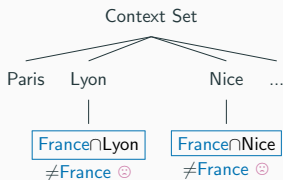


(b) QuD for Paris: same structure and minimal path as Figure (b), only Paris is simpler than Paris or France!

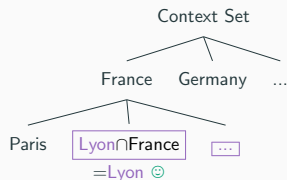
Figure 3: Q-REDUNDANCY Captures # in both HDs in (1).

Q-Relevance

- Q-RELEVANCE: don't shrink your boxed nodes when creating bigger QuD trees.



(a) QuD for (2a)=#If **not Paris** then **France**: the **France**-node flagged by the consequent cannot be “fitted” within the restriction introduced by the antecedent!⁷



(b) QuD for (2b)=If **France** then **not Paris**. The **not Paris** nodes flagged by the consequent (city-level) either are fully **France**, or fully not **France**. They “fit” the restriction introduced by the antecedent.

Figure 4: Q-RELEVANCE captures # in the HC in (2a), spares (2b).

⁷Assuming *not Paris* evokes the polar QuD [_{CS} [Paris] [¬Paris]] does not help with the “shrinkage” issue.

Taking stock

- Issue with the HDs in (1): they flag **two nodes that are on the same path** on the QuD tree (violating Q-REDUNDANCY).
- Issue with the HC in (2a): its consequent QuD is **coarser-grained than its antecedent** QuD (violating Q-RELEVANCE).
- Intuitive fix: see **France**, as a **disjunction over French cities**.
Creates a QuD tree with diverging, “non-redundant” paths, and makes **France** intuitively as fine-grained as **Paris**.

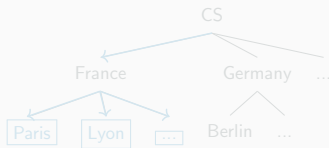


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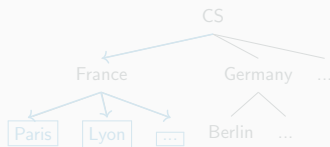


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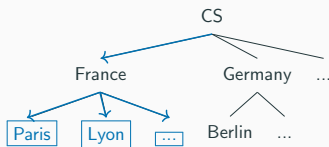


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Modeling at least

Take-home: at least p needs an antecedent QuD, and shifts the QuD of its prejacent to match the degree of granularity of the antecedent QuD.

General background on at least

- Outside of HDs, *at least* has been analyzed as giving rise to ignorance inferences about the exhausted prejacent, and some higher-ranked alternatives to the prejacent ⁸.

- (5) a. Ed is at least 30 $\leadsto \neg K(30) \wedge \neg K(30+)$
b. Ed is at least assistant professor $\leadsto \neg K(\text{assistant}) \wedge \neg K(\text{associate}+)$

- Tight link between the granularity of the QuD, and the granularity of ignorance.

- (6) a. Al: In which **city** did Ed study?
Jo: Ed studied at least in **France** (\sim some French city but I don't know which).⁹

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Bad antecedent QuDs for at least

- *At least* is bad in out-of-the-blue matrix sentences...

(7) Nobody: ...

Al: # Ed studied at least in **Paris**.

- ...When there is an overt QuD to which *at least*'s prejacent is not relevant...

(8) Jo: Is Ed in a good mood today?

Al: # Ed studied at least in **Paris**.

- ...or when the maximal answer(s) to the QuD are at most as specific as *at least*'s prejacent (alternatively: **prejacent is overinformative**).

(9) Jo: In which **country** did Ed study?

Al: # Ed studied at least in **Paris**.

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- ...or when the maximal answer(s) to the QuD are at most as specific as *at least*'s prejacent (alternatively: **prejacent is overinformative**).

(9) Jo: In which **country** did Ed study?

Al: # Ed studied at least in **Paris**.

Bad antecedent QuDs for at least

- *At least* is bad in out-of-the-blue matrix sentences...

(7) Nobody: ...

Al: # Ed studied at least in **Paris**.

- ...When there is an overt QuD to which *at least*'s prejacent is not relevant...

(8) Jo: Is Ed in a good mood today?

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- ...or when the maximal answer(s) to the QuD are at most as specific as *at least*'s prejacent (alternatively: **prejacent is overinformative**).

(9) Jo: In which **country** did Ed study?

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Good antecedent QuDs for at least

- *At least* is licensed if a QuD equally or more specific than at least's prejacent is expressed.

(10) Jo: In which **city** did Ed study?

Al: Ed studied at least in **France**.

- Claim: **such a QuD can be implicit**, if evoked by an assertion preceding *at least*, e.g. *Ed studied in Paris*—which evokes a *Which city?* kind of question. See (11)

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- Therefore, *at least p* is defined if an antecedent QuD whose tree containing a tree evoked by *p* (in terms of nodes, and edges) is **retrievable**.

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- Therefore, *at least p* is defined if an antecedent QuD whose tree containing a tree evoked by *p* (in terms of nodes, and edges) is retrievable.

Direct consequences of the QuD retrievability condition

- If *at least p* needs and antecedent QuD, then the out-of-the-blue infelicity of (3b) and (4b) follows.¹⁰

(3b) # Ed **at least studied in France** or **in Paris**.

(4b) # If Ed **at least studied in France**, he didn't **study in Paris**.

- What remains to be explained is the felicity of the “flipped” cases (3a) and (4a).

¹⁰We discuss the case of overtly provided QuDs in the Appendix.

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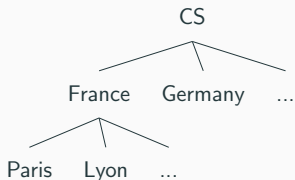
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QuD evoked by at least p

- We posit that the QuD evoked by *at least* p , is **structurally equal to the antecedent QuD tree**.
- Its flagged nodes are **leaves entailing** p .



(a) QuD tree for *In which **city** did Ed study?*

=structure of the QuD tree

evoked by *Ed studied in **Paris***

=structure of the QuD tree

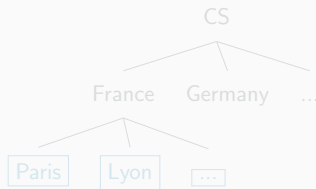
evoked by *Ed at least studied in*

France

Flagging



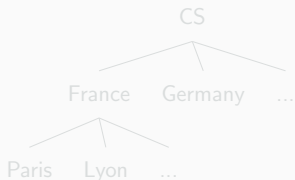
France-
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(b) QuD tree for *Ed studied at least in **France***

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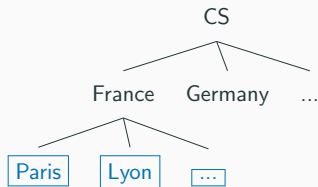
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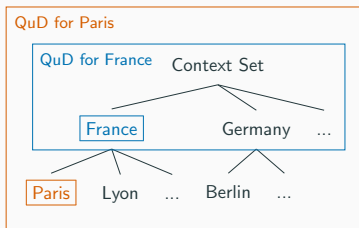
(b) QuD tree for *Ed studied at least in France*

Building up repaired QuDs

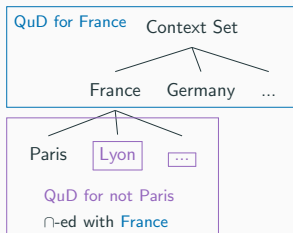
Take-home: by flagging strictly more fine-grained nodes than its prejaacent, at least obviates violations of Q-REDUNDANCY and Q-RELEVANCE.

Disjunctions and conditionals inquisitively differ, again

- The QuD of a disjunction **fuses** the QuDs of both disjuncts and **flags** the nodes associated with both disjuncts.¹¹
- The QuD of a conditional **restricts** the QuD of the consequent, using the QuD of the antecedent; and only **flags** the nodes associated with the consequent.¹²



(a) QuD for #Paris or France.

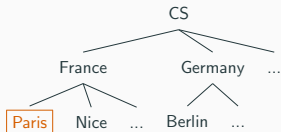


(b) QuD for if France then not Paris.

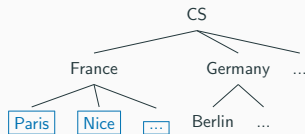
¹¹Building on Simons (2001) and Zhang (2022)

¹²Modeling “neglect-zero”; Aloni, 2022 i.a.

Building HDs repaired by at least – (3a)



(a) QuD tree for **Paris**.

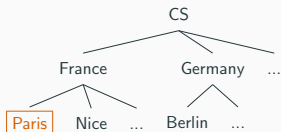


(b) QuD tree for **at least France** (if an antecedent city-level QuD is retrievable). Also QuD tree for **Paris** or **at least France**.

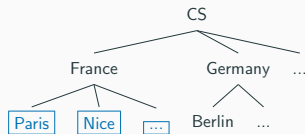
Figure 8: Deriving the QuD tree evoked by (3a)=*Ed studied in Paris or at least France*.

- **Paris** and *at least France* give rise to structurally similar QuD trees, and the flagged nodes of the former are contained in those of the latter.
- Thus, their disjunction evokes the same as QuD tree as *at least France*.

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Rescuing (3a)

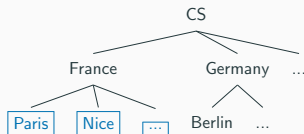


Figure 9: QuD tree for (**Paris or**) at least **France**.

- The fact (**Paris or**) at least **France**, give rise to the same QuD tree, may seem problematic, because *at least France* is strictly simpler, so **Paris or at least France** should be Q-REDUNDANT.
- But recall *at least p* is licensed **only if a suitable antecedent QuD is retrievable**.
- If (3a) gets simplified into *at least France*, *at least* “loses” its antecedent QuD, and therefore, is not a legit competitor to **Paris or at least France**.¹³ And Q-REDUNDANCY does not kick in.

¹³Conundrum of overt QuDs discussed in the Appendix.

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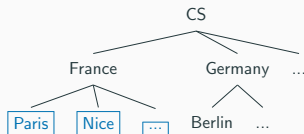


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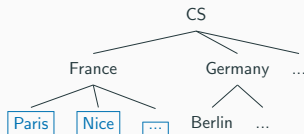


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Building HCs repaired by at least – (4a)

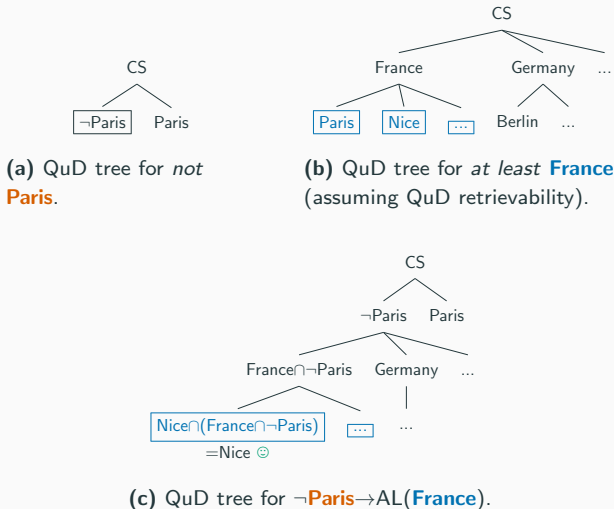


Figure 10: Deriving the QuD tree evoked by (4a) = *If Ed did not study in Paris, he at least studied in France*.

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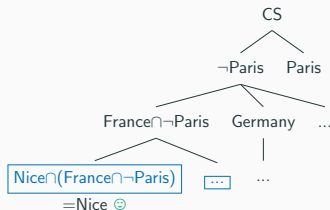


Figure 11: QuD tree evoked by (4a) = *If Ed did not study in Paris, he at least studied in France.*

- The **France**-node still gets shrunk in the process, but since it is no longer flagged, Q-RELEVANCE doesn't mind.
- None of the finer-grained, city-level flagged nodes (e.g. Nice) gets shrunk.
- It is crucial that *at least* completely shifts the QuD to a more fine-grained one, and erases the at-issueness of its prejacent.

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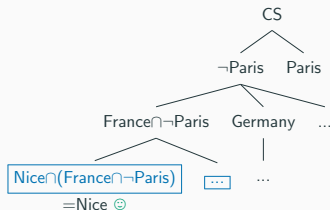


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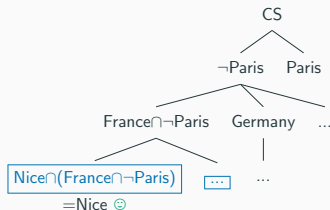


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Concluding remarks

Connection to previous accounts of at least

- Connections between our model of *at least* and the idea of Speech Act weakening (Krifka, 2024) can be drawn.
- In our model, **at least shifts the question to something more specific** than what was originally raised by its prejacent.
- Nevertheless, unioning the flagged nodes of *at least*'s output QuD tree (e.g. French cities), yields a proposition that is equal to *at least*'s prejacent (e.g. **France**).
- This interaction between the message and its packaging is reminiscent of Speech Act weakening: because *at least* increases the specificity of the implicit QuD, the intension of the prejacent becomes less determinate; it gets split across different nodes of the QuD tree, as if *at least p* was disjunctive.

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A few words on but

- *But* rescues HDs,¹⁴ but not HCs!

- (12) a. Ed **studied in Paris** or **in France** but not Paris. $p^+ \vee (p \text{ B } \neg p^+)$
b. ? Ed **studied in France** but not Paris or **in Paris**. $(p \text{ B } \neg p^+) \vee p^+$
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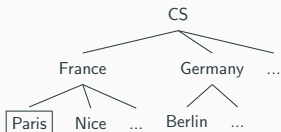


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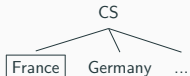
Appendix

Building repairless HDs (1)

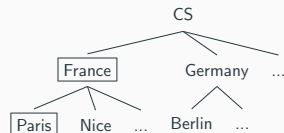
- To build **disjunctive QuD trees** for LFs of the form $X \vee Y$, one just builds all the possible **unions** of QuD trees for X and for Y , and retains the ones that are well-formed. Flagged nodes are **inherited** from the two unioned QuD trees.
- The only possible QuD tree derived from the repairless HDs in (1), is given below, along with the QuD trees used to build it.



(a) QuD tree for **Paris**.



(b) QuD tree for **France**.

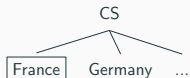


(c) QuD tree for **Paris** \vee **France** / **France** \vee **Paris**.

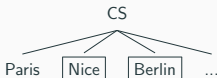
Figure 12: Deriving the QuD tree evoked by (1a)=#Ed studied in **Paris** or **France**, or (1b)=#Ed studied in **France** or **Paris**.

Building repairless HCs: felicitous case (2b)

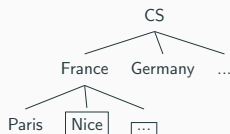
- To build **conditional QuD trees** for LFs of the form $X \rightarrow Y$, one takes a QuD tree for X and replaces each of its flagged nodes, by its intersection with a QuD tree for Y . Flagged nodes are **inherited** from the QuD tree evoked by Y that was used to perform the intersection operation.
- A possible QuD tree for the felicitous repairless HC (2b), is given below, along with the QuD trees used to build it. Other trees are possible, but don't jeopardize the general result.



(a) QuD tree for **France**.



(b) QuD tree for \neg **Paris**.

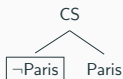


(c) QuD tree for $\text{France} \rightarrow \neg \text{Paris}$.

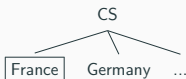
Figure 13: Deriving the QuD tree evoked by (2b)=*If Ed studied in **France**, he did not study in **Paris***.

Building repairless HCs: infelicitous case (2a)

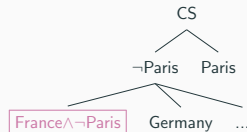
- A possible QuD tree for the infelicitous repairless HC (2a), is given below, along with the QuD trees used to build it.
- Other trees are possible, but don't jeopardize the general result (i.e. end up all equally problematic, due to a shrinkage of the flagged **France**-node).



(a) QuD tree for $\neg\text{Paris}$.



(b) QuD tree for France .



(c) QuD tree for $\neg\text{Paris} \rightarrow \text{France}$.

Figure 14: Deriving the QuD tree evoked by (2a)=#If Ed did not study in **Paris**, he studied in **France**.

Repairing oddness: but

- Contrasting the weaker disjunct with the negation of the stronger one (using *but* as a connective), rescues both HDs in (1), cf. (12). Such structures were dubbed Quasi HDs (Marty & Romoli, 2022).

- (12) a. Ed **studied in Paris** or **in France but not Paris**. $p^+ \vee (p \text{ B } \neg p^+)$
b. ? Ed **studied in France but not Paris** or **in Paris**. $(p \text{ B } \neg p^+) \vee p^+$

- Zhang proposed an additional constraint on given material to capture Quasi HDs. This is nice, but did not come for free.
- Additionally, *but* does not rescue the HC (2a) (cf. (13a)), and happens to degrade the HC (2b) (cf. (13b))!

- (13) a. # If Ed didn't **study in Paris**, he **studied in France but not Paris**. $\neg p^+ \rightarrow (p \text{ B } \neg p^+)$
b. # If Ed **studied in France but not Paris**, he didn't **study in Paris**. $(p \text{ B } \neg p^+) \rightarrow \neg p^+$

Summary of the data, and preview of my predictions

Structure	Ordering	Felicity	My prediction
HD (1)	sw/ws	X	Redundant
HC (2)	sw	X	Irrelevant
	ws	✓	
HD+AL (3)	sw	✓	
	ws	X	Plain odd/Redundant ¹⁵
HC+AL (4)	sw	✓	
	ws	X	Plain odd/Redundant
HD+But (12)	sw/ws	✓	
HC+But (13)	sw	X	Irrelevant
	ws	X	Redundant/Irrelevant

¹⁵Plain odd if no overt QuD provided (impossible for the sentence to evoke a QuD in the first place); redundant otherwise

The licensing and effects of but

- *But* is a contrastive operator that can give rise to (asymmetric) Hurford-like effects (Tomioka, 2021).

- (16) a. # Ed studied in **PARIS** but Al studied in **FRANCE**.
b. Ed studied in **FRANCE** but Al studied in **PARIS**.

- Although I don't have a full account of (16), these examples suggest that *but* is **sensitive to the relative degrees of specificity** conveyed by its two arguments (just like HCs¹⁶), but also, make its **two arguments at-issue** (just like HDs).

¹⁶It's worth noting that *but* could be replaced by *if* in (16), while retaining more or less the same meaning.

The licensing and effects of *but*

- What does *but* do to the QuD(s) evoked by its two arguments?
- *But* is ok out-of-the-blue, and can answer questions about its first argument, when the second argument is also somehow important.¹⁷

(17) a. Jo: In which **country** did Ed study? # I don't care which **city**.

Al: Ed studied in **France** but not **Paris**

b. Jo: In which **city** # (or **country**) did Ed study?

Al: Ed studied in **France** but not **Paris**

¹⁷The 1st argument still has some extra prominence as opposed to the 2nd, because any overt question answered by *but* has to mention the level of specificity of *but*'s 1st argument (cf. (17b)), even if the 2nd argument is more specific (and therefore settling it settles everything). We abstract over any difference in prominence here, and simply treat both arguments of *but* as at-issue.

Modeling but: general case

- We model the QuD evoked by X *but* Y in the following way:
 - Build a QuD tree for X and replace all its flagged nodes by their intersection with a QuD tree for Y .
 - The intersection between a tree T and a node N , is T whose nodes are each intersected with N (and empty nodes and trivial edges are removed).
 - Retain the flagged nodes of *both* X 's and Y 's QuD trees.
- We will see that this building process:
 - retains the **structural** aspects of the building of **conditional** QuD trees (in that a QuD tree for the 2nd argument is “plugged” into a QuD tree for the 1st)...
 - and the **“flagging”** aspects of the building of **disjunctive** QuD trees (in that both arguments are made at-issue).
- In other words, *but* is “inquisitively” like a **conditional that assigns equal at-issueness to both its arguments**.

Modeling Ed studied in **France** but not **Paris**

- A QuD tree for **France** but not **Paris**, is a QuD tree for **France**, where the **France**-node is replaced by its intersection with a QuD tree for \neg **Paris**.
- **France** and city nodes different from **Paris** are flagged. This is all done in Figure 15.

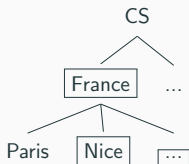
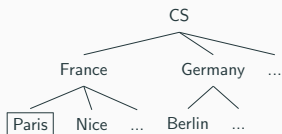


Figure 15: QuD tree evoked by **France** but not **Paris**.

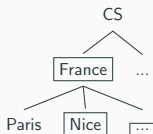
- Other trees are also possible, because **France** and **Paris** are themselves compatible with multiple trees. We omit these extra trees here because they don't jeopardize the final result.
- Contrary to the *at least* case, **the at-issueness of France is retained**.
- Now that we have locally derived QuD trees for the repaired fragments of the sentences at stake, we need to compute the effects of these repairs at the global level.

Building HDs repaired by but (12)

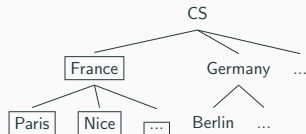
- The effect of *but* in (12a-b) is computed below.¹⁸
- The only difference with *at least*-repairs in HDs, is that **France** remains flagged in the case of *but*-repairs. We'll see that this extra flagged node does not have any consequence in HDs, in terms of redundancy.



(a) QuD tree for **Paris**.



(b) QuD tree for **France** B **not Paris**.



(c) QuD tree for **Paris** \vee **France** B **not Paris** / (**France** B **not Paris**) \vee **Paris**.

Figure 16: Deriving the QuD tree evoked by (12a)=*Ed studied in Paris or France but not Paris*, or (12b)=*Ed studied in France but not Paris*, or in **Paris**.

¹⁸Even if we omitted some possible QuD trees for **France** but not **Paris**, such trees are filtered out when building a QuD tree for the whole disjunction, as done in the above Figure.

“But” rescues HDs

- Figure 17 repeats the QuD tree derived for the repaired HDs in (12).
- It's easy to see this tree does not violate Q-RELEVANCE, because no node got shrunk throughout its derivation.
- And it's not Q-REDUNDANT either. For it to be Q-REDUNDANT given (12a)/(12b), we'd need to find a simplification of (12a)/(12b) leading to the same tree structure and, more importantly, same minimal paths to flagged nodes.

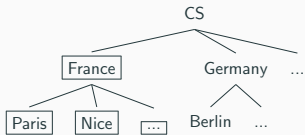


Figure 17: QuD tree for (12a)=*Ed studied in Paris or France but not Paris*, or (12b)=*Ed studied in France but not Paris or Paris*

- This means that a simplification of (12a)/(12b) should flag all the French city nodes. Neither **Paris**, **France**, or **France** but not **Paris**, achieve this.
- Both repaired HDs in (12) are therefore rescued from oddness.

Building HCs tentatively repaired by but: unrescuable case (13a)

- The effect of *but* in (13a) is computed below.
- Other trees are possible, but don't jeopardize the general result (i.e. end up all equally problematic, due to a shrinkage of the flagged **France**-node).

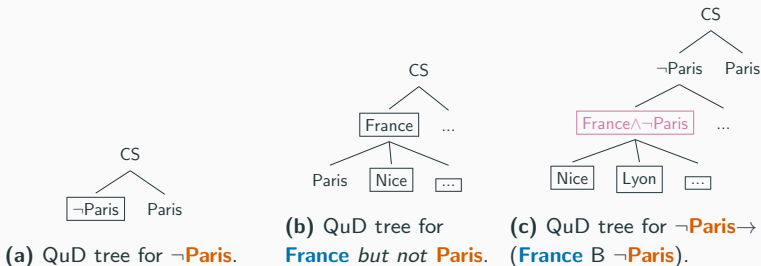
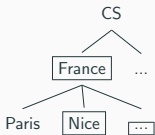


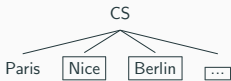
Figure 18: Deriving the QuD tree evoked by (13a)=#If Ed did not study in Paris, he studied in France but not Paris.

Building HCs tentatively repaired by but: degraded case (13b)

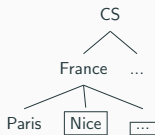
- The effect of *but* in (13b) is computed below. The tree obtained by this process ends up being equal to a tree evoked by the repairless simplification of (13b), (2b).
- The effect of *but* is obfuscated, due to the fact **France** is flagged in the antecedent QuD tree, and so gets *replaced* by its intersection with a consequent QuD tree when the conditions tree gets built. This removes the extra city-partitioning and flagging locally introduced by *but*. Other possible trees don't change this result.



(a) QuD tree for **France** but not **Paris**.



(b) QuD tree for \neg **Paris**.



(c) QuD tree for (**France** B \neg **Paris**) \rightarrow \neg **Paris**.

Figure 19: Deriving the QuD tree evoked by (13b)=*If Ed studied in France but not Paris, he did not study in Paris*.

“But” does not rescue infelicitous HCs

- Figure 20 repeats the QuD tree derived for the HC in (13a).
- Other trees are possible for this sentence, but they will always be evoked by the repairless counterpart of (13a), (2a).
- This is because *but* in the antecedent, preserves the at-issueness of **France**, and so causes a replacement of this node by its intersection with a QuD tree for the consequent (\neg **Paris**), when the conditional QuD tree gets built.

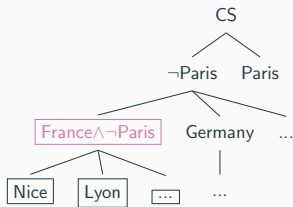


Figure 20: QuD tree for (13a)=#*If Ed did not study in Paris, he studied in France but not Paris.*

- In other words, whatever *but* added beyond **France** at the level of local QuD trees, gets overwritten. In particular, the inquisitive contribution of its second argument is totally ignored.
- (13b) is therefore correctly predicted to be odd.

“But” degrades otherwise felicitous HCs

- Figure 17 repeats the QuD tree derived for the HD in (13b).
- As evoked by (13b), this tree is Q-REDUNDANT, because it is equal to a tree evoked by the repairless simplification of (13b), (2b).
- (13b) is thus correctly predicted to be odd.

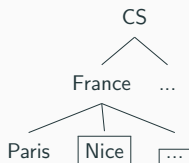


Figure 21: QuD tree evoked by (12b)=*#If Ed studied in France but not Paris, he did not study in Paris.*

A note on corrective at least

- The use of *at least* in HDs and HCs may feel “corrective”. Zhang (2022) puts forth arguments against this view in English.
 - I’d like to concur and discuss a potential argument from French, where corrective and non-corrective uses of *at least* may be teased apart by the lexicon.
 - In addition to *au moins* (lit. *at least*), French displays other strategies to achieve similar effects: *(tout) du moins* and *(tout) au moins*.
 - (18) shows that the former seems purely corrective (it cannot be used in simplex answers), while the latter does not, and still has a repairing effect in HDs and HCs (however, it might still be ambiguous, despite the existence of *tout du moins/au moins...*).
- (18) a. Dans quelle **ville** Ed a étudié?
–Il a (tout) au/#du moins étudié en **France**.
- b. Ed a étudié à **Paris**, ou (tout) au/du moins en **France**.
- c. Si Ed n’a pas étudié à **Paris**, il a (tout) au/du moins étudié en **France**.

Deriving the left-right asymmetry of at least repairs (3b)/(4b)

(3b) # Ed **at least** studied in France or in Paris. $AL(p) \vee p^+$

(4b) # If Ed **at least** studied in France, he didn't study in Paris.

$AL(p) \rightarrow \neg p^+$

- Note that the HD (3b), where *at least* occurs in the 1st disjunct, cannot give rise to any QuD out-the-blue, because *at least* lacks an antecedent QuD to do its job in that case. (3b) is thus correctly predicted to be odd out-of-the-blue.
- This extends to cases where an overt QuD is retrievable. In that case, (3b) is Q-REDUNDANT, given its simplification *at least* France, which gives rise to the same QuD tree.
- And this extends to the HC (4b), where *at least* occurs in the antecedent. Either *at least* cannot give rise to a QuD tree and we get oddness “for free” (out-of-the-blue case), or it can, but ultimately produces a QuD tree that makes (4b) Q-REDUNDANT given its repairless simplification (2b).

Issues with HDs repaired by at least, given an overt QuD

- We have argued that (3a) could be rescued by *at least*, because, even though it gives rise to the same QuD tree as its simplification *at least* **France**, competition between the two forms and their QuD trees does not take place.
- This is because such a simplification can be said to lack an antecedent QuD, and so cannot itself evoke a QuD. It is therefore not a valid competitor. This **relies on the assumption that QuD dependencies are resolved post-simplification**.
- But what if an overt QuD e.g. *in which* **city** *did Ed study?* is independently provided? (3a) still seems fine (cf. 19).

(19) Jo: In which **city** did Ed study?

Al: Ed studied in **Paris**, or at least **France**.

- In that context, we'd predict the *at least*-simplification of (3a) to be a valid competitor when evaluating Q-REDUNDANCY, and so (3a) should be odd... the only way around it is to assume **simplifications are also blind to contextually-provided QuDs!**

Unpacking Q-Redundancy

- (20) **Q-Redundancy:** LF X is Q-REDUNDANT iff there is a formal simplification X' of X obtained *via* constituent-to-subconstituent substitution, s.t. $\text{Qtrees}(X) \leq \text{Qtrees}(X')$.
- (21) **Equivalent Sets of Qtrees:** $S \leq S'$ iff $\forall T \in S. \exists T' \in S'. T \equiv T'$ (note: it is an asymmetric relation!)
- (22) **Equivalent Qtrees:** $T \equiv T'$ iff T and T' have same structure and same set of maximal verifying paths.
- (23) **Verifying paths:** set of paths (=ordered list of nodes) from the root to each flagged node.
- (24) **Path containment:** $p \subseteq p'$ iff p is a prefix of p' .
- (25) **Maximal Verifying Paths (P^*):** if P is a set of verifying paths, P^* is the set of maximal elements of P w.r.t. path containment.

Rephrasing Relevance

- Under the partition-based view of questions, a proposition p is relevant given a question, if it **does not cut across cells**. We want some generalization of this to apply as a filter during Q-tree derivation.

(26) Q-RELEVANCE: If T'' is derived from T and T' via Q-tree composition, then $\mathbb{N}_{T''}^+ \subseteq \mathbb{N}_T^+ \cup \mathbb{N}_{T'}^+$.

- This means that verifying nodes coming from the Q-trees passed as input to a binary Q-tree composition rule should be either fully ruled-out, or fully preserved in the output Q-tree, i.e., they should not be cut-across.
- A correlate in our $\{\neg, \vee, \rightarrow\}$ -fragment (trust me on the meaning of \vee at that point):

(27) Q-RELEVANCE (correlate): If tree T gets intersected with node N , $\mathbb{N}_{T \cap N}^+ \subseteq \mathbb{N}_T^+$