"One tool to rule them all"? An integrated model of the QuD for Hurford sentences[1]

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Abstract. A recent line of research (Katzir and Singh (2015) a.o.) develops the idea that felicitous sentences should be possible answers to a "good" Question under Discussion (QuD, Roberts (1996); Van Kuppevelt (1995)). It remains a bit unclear whether a QuD model is *needed* as an additional explanatory tool for pragmatics, partly because the formalization of QuD composition at the subsentential level remains understudied. In this paper, we develop a compositional machinery linking assertions to the implicit questions they evoke, and show that relocating a number of pragmatic principles previously associated to assertions, in the domain of their implicit questions, allows to solve puzzles pertaining to Hurford Disjunctions and variants thereof, in an intuitive way.

Keywords: redundancy, relevance, question under discussion

1. Introduction

Hurford Disjunctions (henceforth, HD, Hurford (1974)), such as (1a-1b), are disjunctions which typically feature entailing disjuncts. Such constructions, at least when they do not involve scalemates such as *some* and *all*, appear redundant regardless on the order of the weak (p) vs. strong (p^+) disjunct.

(1) a. # SuB29 will take place in Noto[2] or Italy. $p^+ \vee p$

b. # SuB29 will take place in Italy or Noto. $p \vee p^+$

Such constructions have been a long-standing puzzle for pragmatic theory, because it appears difficult to devise a single principle accounting for them, as well as all their variants (Marty and Romoli, 2022). Hurford Conditionals (henceforth HC, (Mandelkern and Romoli, 2018)) like (2a-2b) for instance, exhibit an asymmetry that is challenging for existing accounts of Hurford Sentences, due to the fact that (2a-2b) are directly derived from (1a) *via* the *or-to-if* tautology and basic principles of classical logic (cf. 3).

(2) a. # If SuB29 will not take place in Noto, it will take place in Italy. $\neg p^+ \rightarrow p$

b. If SuB29 will take place in Italy, it will not take place in Noto. $p \rightarrow \neg p^+$

(3) *Equivalence between HDs and HCs*

a. (2a) $\equiv \neg p^+ \rightarrow p \overset{\clubsuit}{\equiv} \neg(\neg p^+) \vee p \overset{\spadesuit}{\equiv} p^+ \vee p \equiv$ (1a)

b. (2b) $\equiv p \rightarrow \neg p^+ \overset{\clubsuit}{\equiv} (\neg p) \vee (\neg p^+) \overset{\heartsuit}{\equiv} q^+ \vee q \equiv$ (1a)

$\clubsuit$: *or-to-if* tautology; $\spadesuit$: double-negation elimination; $\heartsuit$: variable change of the form $\neg p := q^+; \neg p^+ := q$, with $q^+ \vdash q$.

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[2]Noto is located in Italy and is where the main session of SuB29 was actually organized.

Previous accounts of the above contrast build on the idea that overt negation has a special status when it come to evaluating redundancy. In this paper, I want to argue for an alternative view that is perhaps more in line with basic intuitions on might have about HCs and HDs – namely the fact that disjunctions and conditionals package information differently, in terms of the potential questions they evoke, and as such are not equally sensitive to the "granularity" of their arguments.

2. Previous approaches

In this section I briefly present three existing accounts of Hurford Sentences: Local Redundancy Checking, Local Contexts, Super-Redundancy. I show how the first two fall short in explaining HCs, even if the conditional is understood as non-material. I then show how the last account captures the contrast between HDs and HCs.

2.1. Local Redundancy Checking

2.2. Local Contexts

2.3. Super-Redundancy

HDs feel redundant; while HCs sound locally irrelevant. Talk about repairs: the fact the repairs are differnt suggets the violation stems from a different source.

3. Linking assertions to questions

To explain the contrast between HDs and HCs, I propose a compositional machinery linking Logical Forms of assertive sentences to the implicit questions they may raise. One sentence might be associated to multiple potential questions. This kind of machinery is independently motivated by the fact that sentences are never uttered in and of themselves; their purpose is to answer a question, overt or not, and to induce further questions. A pragmatic model of assertion therefore needs to integrate what sentences mean, but also what kind of information *structure* they evoke. I will start by defining questions evoked by simplex LFs, containing no operator, quantifier or connective. Once this is done, I will extend the model inductively, by assigning a semantics to negation, disjunction, and implication, in terms of how they manipulate questions and create more complex ones.

3.1. Background assumptions on question semantics

Let us start by reviewing the standard view on questions. Questions are usually seen as the set of their potential answers Hamblin (1973), i.e. as partitions of the Context Set (henceforth CS, Stalnaker (1974)). This is formalized in (4).

(4) *Standard semantics for questions*
Given a Context Set S, i.e. a set of worlds compatible with the premises of the conversation, a question on S is a partition of S, i.e. a set of subsets of S ("cells") $\{c_1,...,c_k\}$ s.t.:

- "No empty cell": $\forall i \in [1;k].\ c_i \neq \emptyset$
- "Full cover": $\bigcup_{i\in[1;k]} c_i = S$
- "Pairwise disjointness": $\forall (i,j) \in [1;k]^2.\ i \neq j \Rightarrow c_i \cap c_j = \emptyset$

Given a Context Set S, and a set of propositions $P = \{p_1,...,p_l\}$ a partition of S can be induced by grouping together the worlds of S which agree on all the propositions of P. This is formalized in (5).

(5) *Partition induced by a set of propositions.*
Given a Context Set S and a set of propositions $\{p_1, ..., p_l\}$, one can define:
- an equivalence relation $\equiv_P$ s.t. $\forall (w, w') \in S.\ w \equiv_P w' \Leftrightarrow \forall p \in P.\ p(w) = p(w')$
- a partition of S induced by P as the set of equivalence classes of $\equiv_P$ on S, i.e. the set $\{\{w' | w' \in S \wedge w \equiv_P w'\} | w \in S\}$.

We call PARTITION(S, P) the partition on S induced by P.

We can then define the questions evoked by a proposition p as the partitions evoked either by p alone, or by p and relevant alternatives to p. If p is not settled in the CS, the former partition takes the form $\{p, \neg p\}$ and amounts to the question of *whether p*. If the set $\mathscr{A}_p$ of relevant alternatives to p contains mutually exclusive propositions covering the CS, then the latter partition is simply $\mathscr{A}_p$ and amounts to a *wh*-question for which p is a felicitous answer.

3.2. Questions evoked by simplex LFs
Let us now go one step further and adapt this definition to a more elaborate model of questions, which will eventually reflect the intuition that logically equivalent sentences can "package" information differently. Building on (Büring, 2003; Riester, 2019; Onea, 2016; Zhang, 2024), we take questions to denote *parse trees* of the CS, i.e. ways to hierarchically organize the worlds that are compatible with the premises of the conversation. Such trees ("Qtrees") have the structure defined in (6).

(6) *Structure of Question-trees (Qtrees)*
Qtrees are trees whose nodes are all subsets of the CS and s.t.:
- Their root generally[3] denotes the CS;
- Any intermediate node is partitioned by the set of its children.

The nodes of such trees can be assigned the following interpretation. The root denotes a tautology over the CS, and any other node, a possible answer to the global question denoted by the tree. Intermediate nodes can generally be seen as non-maximal answers, while leaves can generally be seen as maximal answers.[4] By construction, the leaves of such trees form a partition of the CS, and allow to retrieve the previous notion of question-as-partition. In those trees, any subtree rooted in a node N can be understood as conditional question taking for granted the proposition denoted by N. Finally, a path from the root to any node N can be seen as a strategy of inquiry (or a sequence of conditional questions) leading to the answer denoted by N.

We now use this definition to define the possible Qtrees a simplex LF is compatible with. Before doing this, let us add one last ingredient to the current model, which is that, sentences also distinguish specific nodes (typically leaves) within the Qtrees they evoke, namely the nodes that verify the proposition denoted by the sentence (*prejacent*). In other words, a Qtrees associated with an assertion not only specifies which question the assertion addresses, but also how the

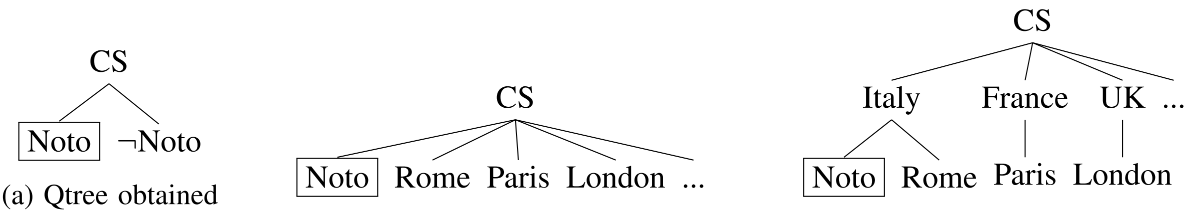
[3] We assume this is the case in the absence of extra presuppositions. In this paper, we will focus on presupposition-less sentences, so all Qtrees will have the same CS as root. But it is reasonable to think that a sentence carrying a presupposition p introduces a questions whose root denotes the CS intersected with p.
[4] We say "generally" here because we think some operators like *at least* can actually influence the relevant level of granularity addressed by a Qtree, such that intermediate nodes can sometimes end up being seen as maximal answers.

assertion actually answers the question. We assume that if a Qtree evoked by a sentence ends up being associated with an empty set of verifying nodes at some point of the Qtree-derivation process, this Qtrees should be deemed ill-formed.

(7) *Qtrees for simplex LFs*
Let X be a simplex LF (no negation, no connective, no quantification) denoting p, not settled in the CS. Let $\mathscr{A}^{g}_{p,X}$ be a set of relevant alternatives to p, obtained from formal alternatives to X derived *via* the substitution of focused material by a same-granularity alternatives. We assume $\mathscr{A}^{g}_{p,X}$ partitions the CS. A Qtree for X is either:
   (i) A depth-1 Qtree whose leaves denote PARTITION(CS, $\{p\}$) = $\{p, \neg p\}$
   (ii) A depth-1 Qtree whose leaves denote PARTITION(CS, $\mathscr{A}^{g}_{p,X}$) = $\mathscr{A}^{g}_{p,X}$.
   (iii) A depth-k Qtree ($k > 1$), whose leaves denote $\mathscr{A}^{g}_{p,X}$, and such that removing those leaves yields a Qtree for an LF Y which is a formal alternative to X associated with a strictly coarser granularity.

In any case, the set of $\boxed{\text{verifying nodes}}$ is defined as the set of p-leaves.

Let us see how this applies to LFs such as X^{+}=*SuB29 will take place in Noto* and X=*SuB29 will take place in Italy*. The same-granularity alternatives to X^{+} different from X^{+} are of the form $\{$*SuB29 will take place in Rome*, *SuB29 will take place in Paris ...* $\}$ where *Noto* is replaced by city-level alternatives. The same-granularity alternatives to X different from X are of the form $\{$*SuB29 will take place in France*, *SuB29 will take place in the UK ...* $\}$ where *Italy* is replaced by country-level alternatives. Moreover, X can be seen as a coarser-grained alternative to X^{+}. This implies that X^{+} and X are respectively compatible with the Qtrees in Figures 1 and 2. In such trees, we assume each node denotes the proposition it is labeled after, properly intersected with the CS. Boxed node represent verifying nodes, as induced by the prejacent proposition. Because X is coarser grained than X^{+}, the Qtrees obtained *via* principle (7iii) for X^{+} will always contain Qtrees obtained for X *via* the same principle (where containment is understood as Qtree equality *modulo* leaf trimming).



(a) Qtree obtained from (7i)

(b) Qtree obtained from (7ii)

(c) Qtree obtained from (7iii)[5]

Figure 1: Qtrees for X^{+}=*SuB29 will take place in Noto*

[5]Note that in principle more tiers can be added to that kind of Qtree, according to principle (7iii). For simplicity we only consider a city vs. country distinction here. The crucial point is that both X and X^{+} are parametrized by the same tiers of same-granularity alternatives, whatever they are.

(a) Qtree obtained from (7i) (b) Qtree obtained from (7ii)

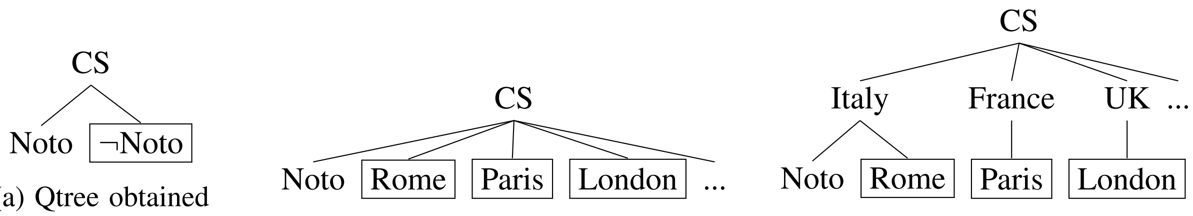
Figure 2: Qtrees for *X=SuB29 will take place in Italy*

3.3. Questions evoked by negated LFs

We assume negated LFs evoke questions that are structurally similar to those evoked by their non-negated counterpart. The only difference resides in the set of verifying nodes, which is flipped by negation. This is formalized in (8).

(8) *Qtrees for negated LFs*
A Qtree T' for $\neg X$ is obtained from a Qtree T for X by:
- retaining T's structure;
- defining the set of T''s verifying nodes, $\mathbb{N}^+(T')$ as $\{N' | N' \notin \mathbb{N}^+(T) \wedge \exists N \in \mathbb{N}^+(T).\ d(N',T') = d(N,T)\}$, where $d(N,T)$ denotes the depth of a node N in a tree T.[6]

Qtrees corresponding to $\neg X^+$=*SuB29 will not take place in Noto* are given in Figure 3. They are derived by simply swapping verifying and non-verifying leaves in the Qtrees from Figure 1, corresponding to X^+=*SuB29 will take place in Noto*.



(a) Qtree obtained from (7i) (b) Qtree obtained from (7ii) (c) Qtree obtained from (7iii)[7]

Figure 3: Qtrees for $\neg X^+$=*SuB29 will not take place in Noto*

3.4. Questions evoked by disjunctive LFs

Building on Simons (2001); Zhang (2024), we assume disjunctive LFs raise questions pertaining to both disjuncts *in parallel*. In other words, disjuncts should mutually address each-other's questions. This is modeled by assuming that disjunctions return all possible unions of the Qtrees evoked by both disjuncts, filtering out the outputs that do not qualify at Qtrees.

(9) *Qtrees for disjunctive LFs*
A Qtree T for $X \vee Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:
- unioning the nodes, edges, and verifying nodes of T_X and T_Y;

[6]Note that, if all verifying nodes are leaves, this definition is simplified: $\{N' | N' \notin \mathbb{N}^+(T) \wedge leaf(N')\}$. Moreover, because T and T' have same structure, the tree-argument is irrelevant to determine node depth in that particular case: $\forall N.\ d(N,T') = d(N,T)$. We keep it because, in the general case, node-depth depends on tree structure.

- returning the output only if it is a Qtree.

In other words, $Qtrees(X \vee Y) = \{T_X \cup T_Y | T_X \cup T_Y \text{verifies } (6) \wedge (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y)\}$

GIVE PROBLEMATIC CONFIGURATION

3.5. Questions evoked by conditional LFs

Building on insights from the psychology literature which revealed that subjects tend to massively overlook the eventualities where the antecedent is false when verifying the truth conditions of conditionals Wason (1968), we assume conditional LFs preferentially raise questions pertaining to their consequent, in the domain(s) of the CS where the antecedent holds, as defined by a Qtree for the antecedent. This is modeled by assuming that implications return Qtrees evoked by their antecedent whose verifying nodes get replaced by their intersection with a Qtree evoked by their consequent. Similarly to disjunctions, this process is assumed to filter out the outputs that do not qualify at Qtrees.

(10) *Qtrees for conditional LFs*

A Qtree T for $X \rightarrow Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

- replacing each node N of $T_X$that is in $\mathbb{N}^+(T_X)$ by $N \cap T_Y$, where $N \cap T_Y$ is defined as T_Y where each node gets intersected with N and empty nodes as well as trivial ("only child") links get removed;
- returning the output only if it is a Qtree.

In other words, $Qtrees(X \rightarrow Y) = \{T_X \cup (N \cap T_Y) | T_X \cup T_Y \text{verifies } (6) \wedge (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y)\}$

GIVE PROBLEMTIC CONFIGURATION casesin whic replacement kicksin non-terminal node: structure gets overwritten if Germany or Paris then

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