Contents

Chapter 1

Assertions and Questions

Assertions and questions can be seen as the two sides of the same coin, as they form the two core building blocks of any given conversation. Question typically request information, while assertions typically provide information. (1a) for instance, is a question that requests information about the country where Jo grew up (presupposing there is one such country). (1b) can be seen as a good (assertive) answer to this question, providing the piece of information that Jo grew up in France. Semanticists have observed that the pairs formed by questions and answers are restricted: some are obviously good, while some others are (sometimes surprisingly) odd. So, questions and answers have to be somewhat *congruent*. For instance, (1c) cannot be seen as a suitable answer to (1a), even if it seems to indicate something about Jo's nationality.

(1) a. In which country did Jo grow up?
b. –Jo grew up in France.
c. # –Jo speaks French natively.

This Chapter motivates and lays the foundation of the main contribution of this dissertation: a constrained machinery "retro-engineering" questions out of assertions, allowing to capture intricate patterns in the domain of pragmatic oddness, that were not previously seen as an issue of question-answer congruence. This Chapter is organized as follows. Section 1.1 provides a broad overview of the semantics of assertions, and discusses to what extent they can meaningfully contribute to a conversation. Section 1.2 turns to the semantics and pragmatics of questions and highlights how questions relate to alternative assertions, and their possible answers. Section 1.3 bridges Sections 1.1 and 1.2, by discussing how questions further constrain which assertions should matter in a given conversation. It also points out a few cases in which question-answer is (seemingly) unhelpful. Section 1.5 constitutes a more technical appendix sketching how the semantics of questions is standardly derived. This whole Chapter heavily builds on the section of von Fintel and Heim (2023) dedicated to Questions.

1.1 Assertions provide information in the form of propositions

1.1.1 Extension and intension of assertions

When studying the semantics of natural language expressions, one usually starts with assertions, because they appear intuitively simpler. We will use the simple assertion in (1b), as a running example. At the most basic level, assertions are truth-conditional, i.e. their meaning corresponds to the set of conditions under which they hold. For instance, *Jo grew up in France* will be true if and only if whoever *Jo* is, grew up in whatever geographical entity *France* is. The *extension* of an assertion is therefore of type $\texttt{t}$, the type of truth-values.

Additionally, the truth-conditions of a sentence are parametrized by (at least) a world variable.[1]. So, *Jo grew up in France* will be true as evaluated against a world w_0 if and only if whoever *Jo* is in w_0, grew up in w_0 in whatever geographical entity *France* is in w_0. One can then abstract over this world-parameter, and define the *intension* of an assertion as a function from worlds to truth-values. Such functions are called *propositions*, and have type $\langle \texttt{s}, \texttt{t} \rangle$, where $\texttt{s}$ is the type of world-variables. So, the intension, or propositional content of *Jo grew up in France*, will be a function mapping any world variable w, to true if and only if, whoever *Jo* is in w, grew up in w in whatever geographical entity *France* is in w. This is formalized (with some simplifications) in (2).

(2) $$\llbracket \text{ Jo grew up in France } \rrbracket = \lambda w.\ \text{Jo grew up in France in } w \\ : \langle \texttt{s}, \texttt{t} \rangle$$

Propositions can receive an alternative, equivalent interpretation in terms of sets, based on the idea that any function with domain D and range R is just a (potentially infinite) set of pairs of elements in $D \times R$. A proposition is then simply the set of worlds in which it holds. This interpretation of propositions will be heavily used throughout the dissertation, and is outlined in (3).

(3) $$\llbracket \text{ Jo grew up in France } \rrbracket = \lambda w.\ \text{Jo grew up in France in } w \\ \simeq \{ w \mid \text{ Jo grew up in France in } w \ \}$$

[1]Other parameters can also be relevant, like times, and assignments. But we choose to keep things simple here.

1.1.2 Assertions in conversation

Propositions either denote functions of type $\langle \mathtt{s}, \mathtt{t} \rangle$, or subsets of the set of elements of type $\mathtt{s}$. Should all elements of type $\mathtt{s}$ be considered when evaluating such functions, or computing such subsets? It is commonly assumed that the worlds under consideration at any point of a conversation, are the ones that are compatible with the premises of the said conversation (Stalnaker, 1974, 1978). For instance, if two people have a discussion about *France*, it is often reasonable to assume that they agree on what geographical area *France* encompasses, and more generally about the topology of Earth. Moreover, they agree that they agree on this; and agree that they agree that they agree on this; etc. Propositions subject to this recursive, mutual, tacit agreement pattern, form what is called a Common Ground (henceforth **CG**, (Stalnaker, 1978)). Each conversation has its own CG, as defined in (4). The set of worlds in which all the propositions of the CG hold, is called the Context Set (henceforth **CS**). The CS associated with a conversation is therefore a subset of the set of all possible worlds; and can also be seen (under the set interpretation of propositions) as the grand intersection of the propositions in the CG. This is defined in (5).

(4) **Common Ground (CG).** Let $\mathcal{C}$ be a conversation between participants $\{P_1, ..., P_k\}$. Let $K(x,p)$ is a proposition meaning that individual x knows p, and p is a proposition. The Common Ground of $\mathcal{C}$ is the set of propositions that are recursively taken for granted by all the participants in $\mathcal{C}$:
$p \in CG(\mathcal{C}) \iff \forall n \in \mathbb{N}^*.\ \forall \{k_1, ...k_n\} \in [1;k]^n.\ K(P_{k_1}, K(P_{k_2}, ...K(P_{k_n}, p)...)$

(5) **Context Set (CS).** Let $\mathcal{C}$ be a conversation between participants $\{P_1, ..., P_k\}$. Let CG $CG(\mathcal{C})$ be the Common Ground of this conversation. Under a set interpretation of propositions, the resulting Context Set $CS(\mathcal{C})$ is the set of worlds verifying all propositions of the CG, i.e.:
$CS(\mathcal{C}) = \bigcap \{p \mid p \in CG(\mathcal{C})\}$.

The concepts of CG and CS help delineate which worlds to focus on when evaluating an assertion in context, and determining to what extent this assertion is informative. If uttering an assertion is akin to *adding* it to the CG, then, it also amounts to *intersecting* this assertion with the CS.

(6) *Updating the Common Ground.* Let $\mathcal{C}$ be a conversation, and $CG(\mathcal{C})$ its Common Ground. If a sentence S denoting p is uttered, then p is added to $CG(\mathcal{C})$ to form a new Common Ground $CG'(\mathcal{C})$:
$CG'(\mathcal{C}) = CG(\mathcal{C}) \cup \{p\}$

(7) *Updating the Context Set.* Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S denoting p is uttered, then a new Context Set $CS'(\mathcal{C})$ is derived by intersecting $CS(\mathcal{C})$ with p:
$CS'(\mathcal{C}) = CS(\mathcal{C}) \cap p$

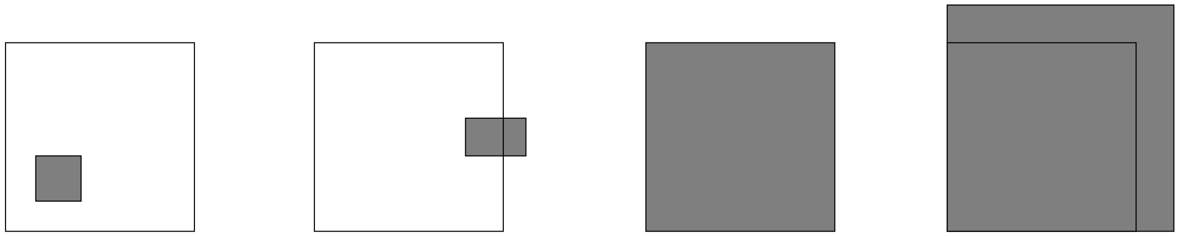
(8) *Link between the two updates.* (7) can be derived from (6) and the definition of the CG in (5):
$$\begin{aligned} CS'(\mathcal{C}) &= \bigcap\{q \mid q \in CG'(\mathcal{C})\} \\ &= \bigcap\{q \mid q \in CG(\mathcal{C}) \cup \{p\}\} \\ &= \bigcap\{q \mid q \in CG(\mathcal{C})\} \cap p \\ &= CS(\mathcal{C}) \cap p \end{aligned}$$

Note that updating the CG will always create a bigger set, because the CG is simply a collection of propositions. For instance, if *Jo grew up in Paris* is already in the CG, then, adding the proposition denoted by *Jo grew up in France* to the CG will mechanically expand it. Updating the CS however, does not always lead to a different, smaller CS. For instance, taking for granted that Paris is in France (i.e., all the *Paris*-worlds of the CS are *France*-worlds), and assuming that *Jo grew up in Paris* is already common ground, intersecting the CS with the proposition that *Jo lives in France* will not have any effect. This seems to capture the idea that a proposition like *Jo lives in France* is *uninformative* once it is already known by all participants that *Jo lives in Paris*.

More generally, if it is Common Ground that p, and a sentence S denoting p^- s.t. $p \vDash p^-$ is uttered, then S will feel uninformative. An informative assertion should lead to a non-vacuous update of the CS, i.e. it should properly *shrink* the CS. This is spelled out in (9).

(9) **INFORMATIVITY** (propositional view). A sentence S denoting a proposition p is informative in a conversation $\mathcal{C}$, iff $CS(\mathcal{C}) \cap p \subset CS(\mathcal{C})$.

In that framework, an assertion provides information in the sense that it reduces the set of live possibilities, and allows to better guess which world is the "real" one. Figure A illustrates how an asserted proposition can be informative or uninformative, depending on its set-theoretic relationship to the CS.



(1) Informative update (p strictly contained in the CS).

(2) Informative update (p strictly overlaps with the CS).

(3) Uninformative update (p equal to the CS).

(4) Uninformative update (p containing the CS).

Figure A: A few examples of informative and uninformative updates of the CS. The big squares represent the CS. The grey shapes refer to p, the proposition added to the CG (and intersected with the CS to update it).

1.1.3 Dynamic Semantics

So far, we have mainly considered "simplex" assertions that did not make use of operators, connectives or quantifiers. But what about sentences like those in (10)? How should they interact with the Context Set?

(10) a. Jo did not grow up in France.

b. Jo grew up in France or Belgium.

c. Jo grew up in France and Ed in Belgium.

The simplest way to deal with these sentences, would be to compute their intension (the proposition they denote) based on the semantics of negation, disjunction, and conjunction, and then, intersect the resulting proposition with the Context Set. We will call this approach the naive "bulk" CS update. There is evidence, coming from the behavior of presuppositions, that this might not be the way to go, and that complex assertions should be added to the Context Set "bit by bit" (Heim, 1982, 1983; Haim, 1983).

To see this, let us consider the pair in (11). The sentences in (11) are conjunctive and only vary in the order of their conjuncts. Additionally, one of their conjuncts contains the presupposition trigger *too*, associated with the predicate *grew up in France*. In the felicitous variant (11a), *too* occurs in the second conjunct; in the the infelicitous variant (11a), *too* occurs in the first conjunct. Intuitively, *X too VP* imposes that whatever predicate *VP* denotes be true of at least one individual different from the one *X* denotes. This presupposition can be seen as a precondition on the Context Set (as defined prior to the update step). In the case of (11a) and (11a), *Ed too grew up in France* then imposes that the Context Set at the time of the update entail that somebody other than Ed (e.g., Jo) grew up in France.

(11) a. Jo grew up in France, and Ed too grew up in France.

b. # Ed too grew up in France, and Jo grew up in France.

Let us attempt a naive "bulk" CS update with sentences (11a)/(11b). The first step is to compute (11a)/(11b)'s presuppositions and (propositional) assertions. The CS, as defined prior to the utterance of (11a)/(11b), then gets updated, provided that it verifies (11a)/(11b)'s presupposition. Let us start with (11a) and (11b)'s presuppositional component. We can assume that the presupposition that somebody other than Ed grew up in France projects from inside the conjunctive operator. Under this assumption, both (11a) and (11b) end up imposing that the CS prior to their utterance entail that somebody other than Ed grew up in France. This will in principle *not* be verified. So, the naive "bulk" Context Set update correctly predicts the infelicity of (11b), but, also, incorrectly predicts (11a) to be odd. Assuming the presupposition does not project does not address the issue. Under this assumption, both (11a) and (11b) end up being presuppositionless, and the naive "bulk" CS update correctly predicts (11a)'s felicity, but also incorrectly predicts (11b) to be just as felicitous. So, regardless of how presupposition should exactly behave in complex sentences, the asymmetry between (11a) and (11b) does not seem to be captured by the naive "bulk" CS update.

The linear asymmetry in (11) in fact suggests an alternative, "bit by bit" update strategy for complex sentences like conjunctions. If each conjunct were to update the CS one at a time, following the linear order of the sentence, then, the first conjunct of (11a) would create an updated CS that would incorporate the information that *Jo grew up in France*, and as such verify the presupposition of (11a)'s second conjunct (that somebody other than Ed grew up in France). This would allow (11a)'s second conjunct to be subsequently intersected to the CS, and would predict the whole conjunction in (11a) to be felicitous. By contrast, (11b)'s first conjunct would still be problematic in this framework, because its presupposition would not be satisfied by the original CS.

In this toy example, a presupposition was used as a diagnostic to better determine the nature of the CS update triggered by a conjunctive sentence. The conclusion is that the update should be dynamic: the two conjuncts should be intersected with the CS one by one, in the order in which they appear. This should apply to presuppositionless sentences as well; and is summarized in (12).

(12) *Conjunctive update of the CS.* Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S of the form $X \wedge Y$, with $[\![X]\!] = p$ and $[\![Y]\!] = q$ is uttered, then a new Context Set $CS''(\mathcal{C})$ is derived by, first intersecting $CS(\mathcal{C})$ with p to create $CS'(\mathcal{C})$, and second, intersecting $CS'(\mathcal{C})$ with q to create $CS''(\mathcal{C})$:
$CS''(\mathcal{C}) = (CS(\mathcal{C}) \cap p) \cap q = CS'(\mathcal{C}) \cap q$

The potential presuppositions of X and Y are tested on the CS at the time of their respective update, i.e. on $CS(\mathcal{C})$ and $CS'(\mathcal{C})$ respectively.

Dynamic Semantics is a framework that proposes to extend this view to other kinds of complex sentences, e.g. disjunctive and conditional sentences. In Dynamic Semantics, sentences give rise to different kinds of CS updates, depending on how they are constructed. More fundamentally, Dynamic Semantics proposes a shift of perspective when it comes to the meaning of assertions: assertions no longer denote propositions, instead they denote proposals to update the CS in specific ways. In that sense, assertions can be seen as functions from an input CS, to an output CS–sometimes called Context-Change Potentials (**CCP**). CCPs for disjunctive and conditional sentences are spelled out in (13) and (14) respectively.

(13) *Disjunctive update of the* CS. Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S of the form $X \vee Y$, with $[\![X]\!] = p$ and $[\![Y]\!] = q$ is uttered, then a new Context Set $CS'(\mathcal{C})$ is derived by intersecting $CS(\mathcal{C})$ with $p \cup q$:
$CS'(\mathcal{C}) = CS(\mathcal{C}) \cap (p \cup q)$
The potential presuppositions of X and Y are tested on, respectively, $CS(\mathcal{C})$ and $CS(\mathcal{C}) \cap \neg p$.[2]

(14) *Conditional update of the* CS. Let $\mathcal{C}$ be a conversation and $CS(\mathcal{C})$ its Context Set. If a sentence S of the form *if X then Y*, with $[\![X]\!] = p$ and $[\![Y]\!] = q$ is uttered, then a new Context Set $CS''(\mathcal{C})$ is derived by, first intersecting $CS(\mathcal{C})$ with p to create $CS'(\mathcal{C})$, and second, intersecting $CS'(\mathcal{C})$ with q to create $CS''(\mathcal{C})$:
$CS''(\mathcal{C}) = (CS(\mathcal{C}) \cap p) \cap q = CS'(\mathcal{C}) \cap q$
The potential presuppositions of X and Y are tested on the CS at the time of their respective update, i.e. on $CS(\mathcal{C})$ and $CS'(\mathcal{C})$ respectively.

This incremental view of assertions leads to a revised, incremental definition of informativity, given in (15).

(15) **Informativity** (CCP view). A sentence S is informative in a conversation $\mathcal{C}$, iff all the updates of $CS(\mathcal{C})$ it gives rise to are non-vacuous.

In sum, assertions can be seen as proposals to update (shrink) the CS. The specific update they give rise to is compositionally derived, and incrementally performed, following the structure of the sentence. We will use a similar approach in Chapter **??** when defining

[2]There is a debate on whether or not disjunctions should behave symmetrically w.r.t. the presupposition(s) carried by their disjuncts. An alternative, symmetric way to evaluate X and Y's potential presuppositions, would be to test them against $CS(\mathcal{C}) \cap \neg q$ and $CS(\mathcal{C}) \cap \neg p$ respectively.

questions *evoked* by assertions. But this first requires to define what questions mean. This is what we do in the next section, in which we show that questions influence, not the size, but rather, the topology of the CS.

1.2 Questions indicate which kind of information is worth providing

1.2.1 Questions as answerhood conditions

Participants in a conversation utter assertions to shrink the CS, and hopefully, jointly figure out which world they are in. But this allows for very unnatural interactions like (16), taking the forms of sequences of intuitively unrelated sentences–as long as each of them denotes propositions shrinking the CS!

(16) –Jo grew up in France.
–I like cheese.
–Al is arriving tomorrow.

This is where questions enter the game. Intuitively, a question indicates an interest in *which* proposition(s) hold, among a restricted set. The proposition at stake are typically possible answers to the question Hamblin (1973); Dayal (1996). Questions therefore denote sets of sets of worlds (equivalent to a type $\langle\langle \mathtt{s},\mathtt{t}\rangle,\mathtt{t}\rangle$), and constrain which kind of (informative) propositions can be uttered as a follow-up. For instance, a polar question such as *Is it raining?* will typically request information of the form *It is raining*, or *It is not raining*, see (17).

(17) –Is it raining?
–Yes, it is raining. / No, it is not raining.

The question *Is it raining?* can thus be represented as a set made of two propositions, namely, the proposition that *it is raining*, and the proposition that *it is not raining*.

(18) $$\begin{aligned} [\![\text{ Is it raining? }]\!] &= \{\ [\![\text{ It is raining }]\!],\ [\![\text{ It is not raining }]\!]\ \} \\ &= \{\lambda w.\ \text{ it is raining in } w,\ \lambda w.\ \text{ it is not raining in } w\} \\ &: \langle\langle \mathtt{s},\mathtt{t}\rangle,\mathtt{t}\rangle \end{aligned}$$

In the case of the question *is it raining?*, the set of possible answers is fairly simple: it only contains two elements. These two elements cover the space of all possibilities,[3] and

[3]This is the case assuming there is no vagueness-induced "grey area", i.e. any salient situation is either a *raining*-situation, or a *not raining*-situation

are *exclusive*: if it's the case that it's raining (at a salient place, at a salient time) in w, then, it's not the case that it is not raining (at the same place, at the same time), in w. We will see in the next section that this configuration amounts to a partition of the CS. A definition of exclusivity under the set interpretation of propositions is given in (19).

(19) *Exclusive propositions.* $p : \langle \mathtt{s}, \mathtt{t} \rangle$ and $q : \langle \mathtt{s}, \mathtt{t} \rangle$ are exclusive if $p \cap q = \emptyset$.

But questions may not always intuitively request information about exclusive propositions. For instance, a *wh*-question like *Which students passed the class?* expects answers that convey a subset of students who passed the class, see (20). But there are many possible, overlapping subsets of students, so, the corresponding propositions will be overlapping as well. For instance, the proposition that *Jo passed the class*, denotes the set of worlds in which Jo passed the class, and this set happens to contain the set of worlds where both Jo and Al passed the class. It also overlaps with the set of worlds in which Al passed the class.

(20) Which students passed the class?
–Jo did.
–Al did.
–Jo and Al did.

We will call propositions like *Jo passed the class*, and *Jo and Al passed the class*, alternatives associated to the question *Which students passed the class?* Alternatives may be overlapping; and, as we will see, can be obtained from the original question by substituting its *wh*-component (e.g., *which students*), with relevant, same-type material (e.g., students or groups of students).[4]

(21) Question : $[\![$ Which students passed the class? $]\!]$
Alternatives: $\{[\![$ Jo passed $]\!]$, $[\![$ Al passed $]\!]$, $[\![$ Jo and Al passed $]\!]$ ... $\}$

Why would this overlap between alternative answers be an issue in modeling the meaning of questions? The fact that entailing or merely overlapping propositions should be considered equally good answers does not capture the idea that more specific propositions constitute more exhaustive answers than less specific ones. For instance, answering that *Jo passed*, in theory leaves the fate of the other students undecided–for instance, it does

[4]It is worth mentioning that the set $\{\lambda w.$ it is raining in w, $\lambda w.$ it is not raining in $w\}$ does not strictly speaking correspond to the set of alternatives raised by *Is it raining?* Section 1.5 further specifies how alternatives get compositionally derived, and predicts that *Is it raining?* should only give rise to one alternative: $\lambda w.$ it is raining in w. The set $\{\lambda w.$ it is raining in w, $\lambda w.$ it is not raining in $w\}$ is derived from this singleton alternative *via* the "pragmatic" process presented in (24), in the next Section.

not settle if *Al passed*, or not. Answering that *Jo and Al passed* by contrast, settles Al's fate, in addition to Jo's. Ideally, an answer to *Which students passed?* should explicitly address whether *each* student of the class passed, or not. That would be an exhaustive answer.

1.2.2 Questions as partitions of the Context Set

We have just discussed that, at the semantic level, questions characterize the conditions under which they are answered, i.e. denote a set of potentially overlapping propositions. But, just like we did with assertions, the effect of this semantics on the Context Set has to be defined. There is in fact a deterministic way to change a set of overlapping propositions P (i.e. a set of subsets of the CS), into a set of exclusive subsets of the CS (called *cells*, for reasons made clear in (24)). To do so, one can group in the same cell the worlds of the Context Set that all "agree" on all propositions in P. This "agreement" property amounts to the same-cell relation in (22). This relation is reflexive, symmetric and transitive, i.e. is an equivalence relation (see proof in (23)). From this, we can conclude that the set of subsets of the CS induced by P, obtained by grouping worlds of the CS according to the same-cell relation, forms a partition of the Context Set (see proof in (24)).[5] So, on top of being exclusive, cells are non-empty and together cover the CS. We assume that the process changing the set of alternative propositions raised by a question, to a partition of the CS, belongs to pragmatics. So, questions *denote* sets of alternative propositions, and this set *pragmatically induces* a partition structure on the CS.

(22) *Same-cell relation $\equiv_P$.* Let P be a set of propositions, i.e. a set of subsets of the Context Set ($P \in \mathcal{P}(\mathcal{P}(CS))$, with $\mathcal{P}$ the powerset operation). Let w and w' be two worlds of the Context Set. $w \equiv_P w'$ iff, $\forall p \in P.\ p(w) = p(w')$.

(23) $\equiv_P$ is an equivalence relation, no matter what P is. Let $\forall P \in \mathcal{P}(\mathcal{P}(CS))$.

- $\equiv_P$ is reflexive: $\forall w \in CS.\ \forall p \in P.\ p(w) = p(w)$.
- $\equiv_P$ is symmetric. Let $\forall (w, w') \in CS^2$.
  $\forall p \in P.\ p(w) = p(w')$ iff $\forall p \in P.\ p(w') = p(w)$.
- $\equiv_P$ is transitive. Let $\forall (w, w', w'') \in CS^3$.
  We assume $\forall p \in P.\ p(w) = p(w')$ and $\forall p \in P.\ p(w') = p(w'')$.
  Let $\forall p \in P$. We have $p(w) = p(w')$ and $p(w') = p(w'')$, so $p(w') = p(w'')$.
  So, $\forall p \in P.\ p(w) = p(w'')$

[5] Cells as we defined them are also called equivalence classes. It's a general property that equivalence classes induced by an equivalence relation on a certain set on which this relation is defined, will create a partition of the set.

(24) *Partition of the CS induced by P.*[6] Let P be a set of propositions. The partition induced by P in the Context Set is the set of subsets of the CS (cells): $\mathfrak{P}_{P,CS} = \{\{w' \mid w' \in CS \wedge w' \equiv_P w\} \mid w \in CS\}$. This set partitions the CS.

- No cell c of $\mathfrak{P}_{P,CS}$ is empty. Let $c \in \mathfrak{P}_{P,CS}$. There is a $w \in CS$ s.t. $c = \{w' \mid w' \in CS \wedge w' \equiv_P w\}$. Then at least $w \in c$, because $w \equiv_P w$.
- Cells cover the CS. Let $w \in CS$. $\mathfrak{P}_{P,CS}$ contains a cell $c = \{w' \mid w' \in CS \wedge w' \equiv_P w\}$. Then $w \in c$ because $w \equiv_P w$.
- Cells are disjoint. Let $(c, c') \in \mathfrak{P}_{P,CS}$, s.t. $c \cap c' \neq \emptyset$. We show $c = c'$. c and c' have resp. the form $c = \{w'' \mid w'' \in CS \wedge w'' \equiv_P w\}$ and $c = \{w'' \mid w'' \in CS \wedge w'' \equiv_P w'\}$, for $(w, w') \in CS^2$. Let $w''' \in c \cap c'$. Then $w''' \equiv_P w$ and $w''' \equiv_P w'$, and so by symmetry and transitivity, $w \equiv_P w'$, and $c = c'$.

It is easy to show that, in the polar example (17), the subsets of the CS defined by *It is raining* and *It is not raining*, which we said were intuitive answers to the question, form a partition of the CS. Section 1.5 will in fact show that polar questions of the form p? *denote* the singleton set formed by p, and *induce* a 2-cell partition of the form $\{p, \neg p\}$.

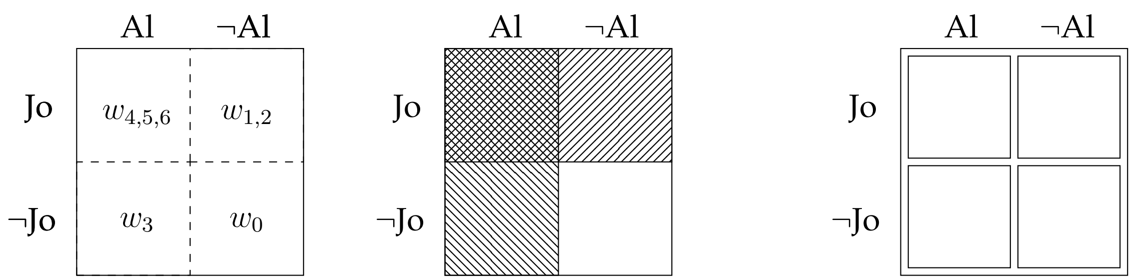
Let us now see how the above definitions apply to a *wh*-question like *Which students passed?* in (20). Let's assume there are only two salient students, Jo and Al. We assume that the alternatives the question raises (labeled P), are the proposition that *Jo passed*, and the proposition that *Al passed*. We assume that the CS contains six possible worlds, which vary according to whether Jo, Al, both, or none passed the class. The worlds may vary in other respects, that are not relevant to us here. The alternatives and cells associated with this question are given in (25). The alternative set P then corresponds to two subsets of the CS, which do not cover it. In particular, the world in which nobody passed (w_0) is included in none of the two alternatives. Moreover, the two subsets are overlapping: both *Jo passed* and *Al passed* contain w_4, w_5, and w_6. Now turning to the cells induced by P on the CS, we notice that there are four of them, which correspond to worlds where nobody, only Jo, only Al, or both Jo and Al passed the class. Such cells cover the CS, are disjoint, and non-empty, so correctly form a partition of the CS. They also fully specify, for *both* Jo and Al, if they passed the class; and as such constitute exhaustive answers to the original question.

(25) Question : *Which students passed the class?*
Context Set: $\{w_0, w_1, w_2, w_3, w_4, w_5, w_6\}$, s.t.:

[6] *Fox (2018) proposes an alternative way to derive a partition of the CS from a set of alternative propositions, leveraging the covert operator exh*

- Nobody passed in w_0;
- Only Jo passed in w_1 and w_2;
- Only Al passed in w_3;
- Both Jo and Al passed in w_4, w_5, and w_6.

Alternatives (P): $\{[\![\text{Jo passed}]\!], [\![\text{Al passed}]\!]\} = \{\{w_1, w_2, w_4, w_5, w_6\}, \{w_3, w_4, w_5, w_6\}\}$
Cells induced by $\equiv_P$: $\{\{w_0\}, \{w_1, w_2\}, \{w_3\}, \{w_4, w_5, w_6\}\}$



(1) Distribution of $w_0 ... w_6$ in the CS.

(2) Alternatives: defines *Al passed* and defines *Jo passed*.

(3) Partition induced by and . Cells correspond to the subsets of Figure B2 featuring the same overall pattern.

Figure B: Partitioning of the CS defined in (25) according to the alternatives *Jo passed* and *Al passed*. The CS is organized as follows: counter-clockwise, quadrant I is made of *Jo but not Al*-worlds; quadrant II, *Jo and Al*, quadrant III, *Al but not Jo*, and quadrant IV, *neither Jo nor Al*.

To summarize, at the pragmatic level questions are partitions of the Context Set, as formalized in (26).[7] The cells of such partitions constitute maximal answers to the questions. Unions of two or more cells constitute non-maximal answers, as defined in (27).

(26) *Standard semantics for questions (Jäger, 1996; Hulstijn, 1997; Groenendijk and Stokhof, 1984; Groenendijk, 1999).* Given a conversation $\mathcal{C}$ and a Context Set $CS(\mathcal{C})$, a question on $CS(\mathcal{C})$ is a partition of $CS(\mathcal{C})$, i.e. a set of subsets of $CS(\mathcal{C})$ ("cells") $\{c_1, ..., c_k\}$ s.t.:

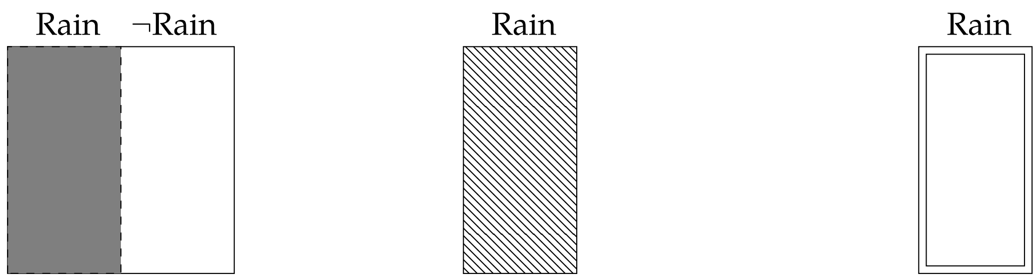
- "No empty cell": $\forall i \in [1;k].\ c_i \neq \emptyset$
- "Full cover": $\bigcup_{i \in [1;k]} c_i = CS(\mathcal{C})$
- "Pairwise disjointness": $\forall (i,j) \in [1;k]^2.\ i \neq j \Rightarrow c_i \cap c_j = \emptyset$

[7] It is important to note that questions may be taken to have a partition *semantics*. But we do not cover this here.

(27) *Maximal and non-maximal answers to a question.* Given a conversation $\mathcal{C}$, a Context Set $CS(\mathcal{C})$, and a question Q forming a partition $\{c_1, ..., c_k\}$ of $CS(\mathcal{C})$:

- Any $c \in \{c_1, ..., c_k\}$ constitutes a maximal answer to Q;
- Any c' s.t. $\exists C \subseteq \{c_1, ..., c_k\}.\ |C| > 1 \wedge c' = \bigcup C$ is a non-maximal answer to Q.

Just like we did with assertions, let us clarify further what it means to be a good question. We have established that the idea of a partition is a good candidate to model the effect of questions on a given CS. But what if the CS is already such that the partition induced by the question's alternatives is just made of one big cell? Such a configuration suggests that the question is already *settled*, meaning, the CS already makes one maximal answer trivial. For instance, if it is already common ground between the conversation's participants that *it is raining* (at the salient place and time) in (17), then, the question *Is it raining?* appears completely trivial. This is illustrated in Figure C and generalized in (28).



(1) Interecting the proposition that *It's raining* () with a CS that is agnostic about the weather.

(2) Alternatives on the restricted CS: defines *It's raining*.

(3) Partition induced by on the CS. Cells correspond to the subsets of Figure C2 featuring the same overall pattern (1 such subset).

Figure C: Updating the CS with the proposition that *It's raining*, and then computing the partition induced by *Is it raining* on the resulting "shrunk" CS. The outcome is a single-cell partition, i.e., the question has a trivial pragmatics.

(28) **Trivial Question**. Let $\mathcal{C}$ be a conversation, $CS(\mathcal{C})$ its associated Context Set, and Q a question. Q is trivial given $CS(\mathcal{C})$ iff the partition induced by Q on $CS(\mathcal{C})$ is made of a singleton cell, i.e. has cardinal 1.

We now have a basic notion of what it mean to be a good assertion, given a CS, and a good question, given a CS. A good assertion has to be informative, i.e. properly shrink the CS (as per (9)/(15)). A good question has to induce a non-trivial, multiple-cell partition on the CS (as per (28)). But being a good question or a good assertion, does not *only* depend on the state of the CS! In particular, good assertions also have to be good answers to good questions. This principle, dubbed *Question-Answer Congruence*, is given in (29).

(29) **Question-Answer Congruence** (Katzir and Singh, 2015). A felicitous assertion has to be a good answer to a good question.

The next Section presents what can be seen as a partial implementation of this principle, in the form of a general principle dubbed Relevance. It also points out the limitations of this principle.

1.3 Assertions as good answers to questions

1.3.1 Relevance mediates questions and assertions

Now that we precisified what assertions and questions are, it becomes possible to (at least partially) define what a good assertion should be, given a question. The principles we introduce in this Section are based on the general concept of Relevance. They will eventually rule out informative but "unnatural" sequences of assertions like (16), but also, more generally, a wide range of odd question-answer pairs.

Following much previous literature (van Kuppevelt, 1995a,b; Roberts, 1996, 2012; Ginzburg, 1996; Büring, 2003), we call the question against which assertions are evaluated, *Question under Discussion* (henceforth **QuD**). QuDs are typically seen as partitions of the CS. In (27), we defined cells an unions of cells as respectively maximal and non-maximal answers to a question. Very broadly, Relevance constrains what a proposition should do to the cells of the QuD. Let us now unpack this with an example.

If for instance the QuD is about which country Jo grew up in (as in (30)), the CS will be partitioned according to propositions of the form *Jo grew up in c*, with *c* a country. Utterances such as (30a) or (30b), both seem relevant to that kind of QuD, and both constitute answers to the QuD–maximal, or not. By contrast, utterances such as (30c), (30d) or (30e), do not appear relevant, and do *not* constitute answers to the QuD: there are native and non-native French speakers in virtually all countries; same holds for wine-lovers and wine-haters; as for (30e) it seems completely independent from the subject matter.[8] These various configurations are sketched in Figure D.

(30) QuD: In which country did Jo grow up?

a. Jo grew up in France.

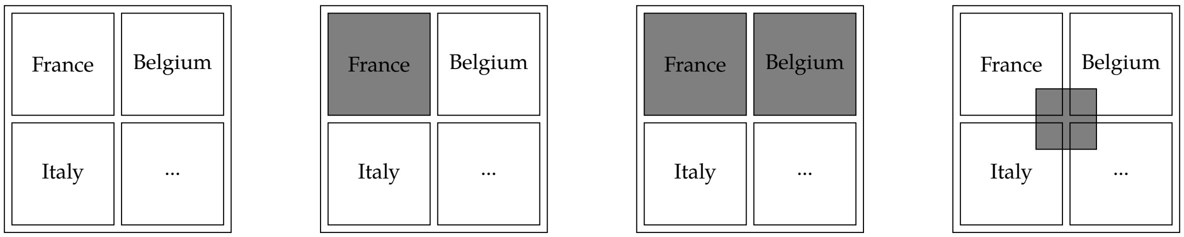
[8] It is interesting to note that (30c) and (30d) can be more easily coerced into relevance than (30e). For instance with (30c), one might consider that France is the country which, in proportion, comprises the most native French speakers, and so (30c) may be understood as *It is likely that Jo grew up in France*–which constitutes a modalized answer to the QuD. This kind of reasoning is harder (if not impossible) to perform when facing an utterance like (30e).

b. Jo grew up in France or Belgium.

c. ?? Jo speaks French natively.

d. ?? Jo enjoys wine.

e. # The cat went outside.



(1) QuD for *In which city did Jo grow up?* (2) Utterance: *Jo grew up in France.* (3) Utterance: *Jo grew up in France or Belgium.* (4) Utterance: (30c), (30d) or (30e).

Figure D: QuD-utterance configurations for a QuD like *In which country did Jo grow up*, and possible follow-up utterance.

From this, we can conclude that a proposition is "relevant" to a question, if it constitutes a maximal or a non-maximal answer to the question. This is similar in spirit to the notion of *Aboutness* developed Lewis (1988), according to which a proposition p is about a subject matter (in modern terms, a QuD), if and only if the truth value of that proposition supervenes on that subject matter (i.e. p should not introduce truth-conditional distinctions between cellmates, i.e. p does not "cut across" cells). This is rephrased in (31).

(31) **Lewis's Relevance** (rephrased in the QuD framework). Let $\mathcal{C}$ be a conversation, Q a QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is Lewis-Relevant to Q, iff $\exists C \subseteq Q.\ p \cap CS(\mathcal{C}) = C$

A typical Lewis-Relevant configuration is exemplified in Figure F1. Note however two edge cases. The first, is that of a proposition whose intersection with the CS is empty (a contextual contradiction). This kind of proposition verifies (31), because the empty set is a subset of any set, including the set of propositions defined by the QuD–whatever it is. Figure F2 exemplifies this kind of configuration. The second edge case, is that of a proposition whose intersection with the CS is the entire CS (a contextual tautology, uninformative as per (9)). This kind of proposition also verifies (31), because the entire CS corresponds to the unions of all cells of any given QuD defined on that CS. Figure F3 exemplifies this kind of configuration.

But, coming back to the QuD *In which country did Jo grow up?*, what about an utterance of the form *Jo grew up in Paris?* Although overinformative (the QuD was only asking about countries, not cities!), this utterance appears relevant, because it allows to infer that Jo grew up in France, and not, say, Belgium. This kind of configuration is sketched in Figure E.

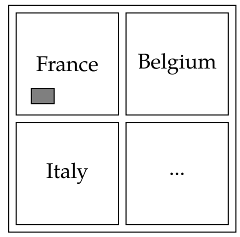
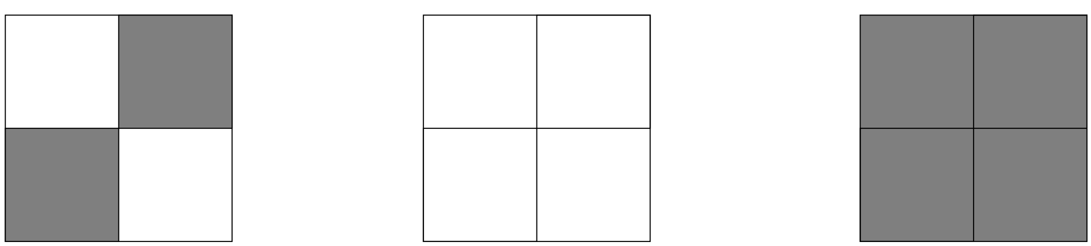


Figure E: QuD-utterance configuration for a QuD like *In which country did Jo grow up?* and an utterance like *Jo grew up in Paris*.

The view of relevance, developed by Roberts (2012), captures this intuition, by stating that a relevant proposition has to rule out at least one maximal answer conveyed by the QuD. In other words, a relevant proposition has to be incompatible with at least one cell of the QuD. This is summarized in (32). This definition makes uninformative propositions irrelevant (see Figure F3), but allows certain propositions that do not coincide with the grand union of a subset of the QuD's cells, to be relevant (see Figures F4 and F5). In other words, relevant propositions in the sense of Roberts may introduce truth-conditional distinctions between cellmates–as long as they rule out a cell. A particular case is that of propositions like *Jo grew up in Paris*, when the QuD is about countries, which strictly entail a specific cell of the QuD, i.e. are strictly contained in one single cell (see Figure F4).

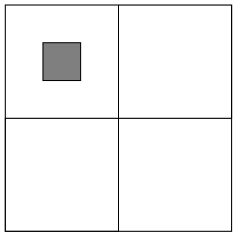
(32) **ROBERTS'S RELEVANCE** (Roberts, 2012). Let $\mathcal{C}$ be a conversation, Q a (non-trivial) QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is ROBERTS-RELEVANT to Q, if $\exists c \in Q.\ p \cap c = \emptyset$.



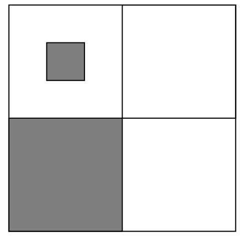
(1) Informative
LEWIS-RELEVANT
ROBERTS-RELEVANT.

(2) Informative
LEWIS-RELEVANT
ROBERTS-RELEVANT.

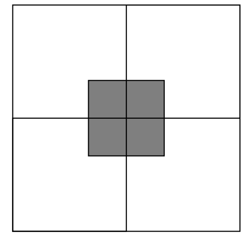
(3) Uninformative
LEWIS-RELEVANT
not ROBERTS-RELEVANT.



(4) Informative
not Lewis-Relevant
Roberts-Relevant.



(5) Informative
not Lewis-Relevant
Roberts-Relevant.



(6) Informative
not Lewis-Relevant
not Roberts-Relevant.

In sum, the concept of Relevance (whether it follows Lewis's or Roberts's implementation) allows to rule-out a wide range of QuD-utterance pairs, by stating that propositions should properly relate to an existing question. We will not discuss which approach between Lewis's and Roberts's is best here, and will propose an incremental variant of this core concept in Chapter **??**, to deal with certain complex, out-of-the blue sentences. The next two section outline a few limitations of relevance.

1.3.2 A few conceptual shortcomings of Relevance

Regardless on which view of Relevance is adopted, relevant propositions can be added to the Common Ground, and as such, trigger an update of the CS. This, in turn, updates the QuD, which must remain a partition of the CS. It is easy to show, given how partition are "induced" on a set (see definition (24)), that the updated QuD on the smaller CS corresponds to the previous QuD, whose cells are pointwise intersected with the newly added proposition, and such that empty cells are filtered. This is formalized in (33).

(33) *Updating the partitioned Context Set.* Let $\mathcal{C}$ be a conversation, $CS(\mathcal{C})$ its Context Set, and let Q be a partition of $CS(\mathcal{C})$. If a sentence S denoting p is uttered and relevant given Q (as per (31) or (32)), then a new Context Set $CS'(\mathcal{C})$ is derived by intersecting $CS(\mathcal{C})$ with p, and this new context set is partitioned by Q', s.t.:
$Q' = \{c' \mid \exists c \in Q.\ c' = c \cap p \wedge c' \neq \emptyset\}$

There are two shortcomings to the current framework. First, adding a proposition to the CG "mechanically" leads to an update of the CS and of the QuD, but does not directly affect the *structure* of this QuD: even if some cells should shrink, the *limits* of each cell remain the same. This goes against the intuition that sometimes, sentences give rise rise to brand new QuDs, as exemplified by the exchange in (34).

(34) –Is it raining?
–Yes, I think so. I just so Ed come in with this very pretty umbrella.

(Likely follow-up: Where did Ed find this umbrella?)

Second, and relatedly, one can wonder what is supposed to happen in the case of out-of-the-blue sentences, i.e. sentences for which there is no explicit QuD. In such cases, it is generally assumed that a reasonable QuD is somehow inferred. But, given the fact that a QuD is merely a partition of the current CS, there exists many options. This dissertation will focus on how exactly QuDs are inferred, what additional constraints hold between an assertion and a QuD, and what the consequences are for pragmatic theory.

1.3.3 Relevance and the packaging of information

We start by showing that the felicity of disjunctions and conditionals is sensitive to *overt* QuDs – but in different ways. We take this as evidence that out-of-the-blue disjunctions and conditionals accommodate different kinds of implicit QuDs.

If a context contrasting *Paris* and *France but not Paris* is set as in (35), (**??**) and (**??**) improve (see Haslinger (2023) for similar effects on disjunctions and conjunctions). This is strange: even if the context and question made *Paris* (but no other French city) a relevant alternative to *France*, *exh* would remain IW in the consequent of (**??**): *if Jo did not grow up in Paris, she grew up in France but not Paris*, is equivalent to *if Jo did not grow up in Paris, she grew up in France*. In other words, *exh* (as constrained by IW) cannot leverage the contextually provided alternatives to make (**??**) escape SR in (35). The same applies to (**??**).

(35) *Context: French accents vary across countries and between Paris the rest of France.*
Al: I'm wondering where Jo learned French.
Lu: I'm not completely sure but... (**??**) ✓(**??**) ✓

This suggests that a purely LF-based view of redundancy such as SR, may be insufficient to capture the interaction between HCs and how their context of utterance packages information. Rather, it seems that the context of (35) makes a specific partition of the CS salient, and that this partition can be used to make otherwise infelicitous assertions accommodate a different question than the one they would evoke out-of-the-blue.

Additionally, conditionals and disjunctions seem to accommodate distinct QuDs. To show this, we use the construction *depending on Q, p* (Karttunen (1977); Kaufmann (2016)), where Q is a question and p a proposition. This construction has been argued to force the partition conveyed by Q to match specific live issues raised by p. We understand such "live issues" as the maximal true answers of the QuD evoked by p. The contrast between (36a) and (36b) then suggests that the *France* and *Belgium* answers can be matched against Q in the disjunctive, but not in the conditional case. This in turn means that a disjunction introduces a QuD making both disjuncts maximal true answers, while a conditional does not do the same with its consequent and the negation of its antecedent.

(36) Depending on [how her accent sounds like]$_Q$...

a. Jo grew up in France **or** in Belgium. $\mathbf{p} \vee \mathbf{q}$

b. ?? **if** Jo didn't grow up in France she grew up in Belgium. $\neg\mathbf{p} \rightarrow \mathbf{q}$

c. ? **if** Jo didn't grow up in France, she grew up in Belgium **or** in Québec. $\neg\mathbf{p} \rightarrow (\mathbf{q} \vee \mathbf{r})$

d. ?? **if** Jo didn't grow up in France **or** Belgium, she grew up in Québec. $\neg(\mathbf{p} \vee \mathbf{q}) \rightarrow \mathbf{r}$

The existence of an improvement between (36b) and (36c), and the absence of a similar improvement in between (36b) and (36d), also implies that the answers targeted by *depending on Q*, when p is conditional, are the ones made available by the consequent of p (which is appropriately disjunctive in (36c), but not (36d)).

More generally, this predicts "connectivity effects" in disjunctions-of-conditionals, in that the antecedents and consequents respectively have to address similar QuDs; and no such effect in conditionals-of-disjunctions, in that disjuncts coming from the antecedent and consequent may be inquisitively unrelated.

1.4 Roadmap of the dissertation

Specifically, we will claim that instead of being a "good" answer to *some* QuD, an out-of-the-blue sentence must be a good answer to a *good* QuD, following insight by Katzir and Singh (2015). We will show that operationalizing this principle allows to account for a wider range of oddness phenomena, that previous approaches struggled to capture under the same umbrella.

"Good" QuDs are determined from the shape of the assertive sentence itself. This is pushing the idea that assertions evoke alternatives one step further, in the sense that sentences will be taken to evoke questions (themselves derived from alternatives). These evoked questions will have a structure that consists in a generalization of the partition structure, namely, they will take the form of parse tree of the CS. We will additionally claim the process deriving good questions from good answers, is subject to constraints that go beyond relevance, and cover concepts such as redundancy. These constraints will make way for a "lifted" view of pragmatic oddness, under which an assertion is not odd *per se*, but rather, is odd due to its interaction with the QuDs it evokes. Before presenting the core components of our model, we will briefly present two recent accounts of oddness based on similar ideas.

1.5 Appendix: computing questions from propositions

So far, we have described what could be a reasonable model for questions, in the form of partitions of the CS. But this was done without explaining how exactly such partitions are derived from the Logical Form of questions. This sketches how this is done, while further clarifying the distinction between propositions, alternatives, and questions. We will show that questions are standardly derived from closely related propositions, by abstracting over specific variables.

We will use the question *In which country did Jo grow up?* as an example. The LF associated with this question is given in Figure G.

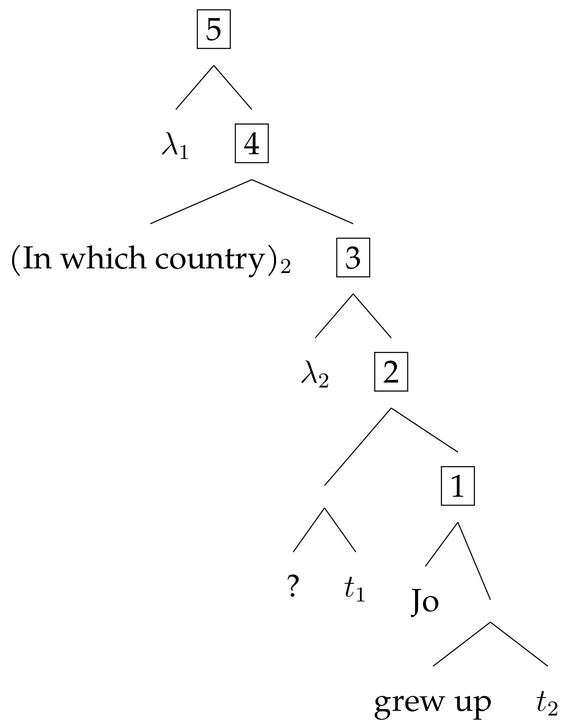


Figure G: LF of the question *In which country did Jo grow up?*

This question involves a *wh*-phrase (*in which country*), which syntactically originates in an adjunct of *grow up*. It is assumed that the *wh*-phrase leaves a trace t_2 in this position. The semantics assigned to the *wh*-phrase is existential, and akin to *some country*. Specifically, *in which country* takes a predicate of type $\langle \mathtt{e}, \mathtt{t} \rangle$ as argument, and returns the quantified statement that *some country* verifies the predicate.

(37) $[\![\text{In which country}]\!]^w = \lambda P.\ \exists l.\ l \text{ is a country in } w \wedge P(l) = 1$

The *wh*-phrase outscopes another "proto-question" operator (Karttunen, 1977). This operator takes two propositions (here, the trace t_1 and the proposition that *Jo grew up in* t_2), and simply equates them.

(38) $[\![?]\!]^w = \lambda p.\ \lambda q.\ p = q$

Applying this operator successively to t_1 and the intension of $\boxed{1}$, yields the following.

(39) $\boxed{1} = [\![\text{Jo grew up } t_2]\!]^w = 1$ iff Jo grew up in t_2 in w

(40) $\boxed{2} = [\![?\ t_1 \text{ Jo grew up } t_2]\!]^w = 1$ iff $t_1 = \lambda w'$. Jo grew up in t_2 in w'

Abstraction then applies to $\boxed{2}$, binds t_2 and yields a predicate that can then serve as an argument of the *wh*-phrase. The *wh*-phrase then turns this predicate into an existentially quantified expression targeting the element being questioned (here, a country).

(41) $\boxed{3} = [\![\ \lambda_2\ ?\ t_1 \text{ Jo grew up } t_2]\!]^w = \lambda l.\ t_1 = \lambda w'$. Jo grew up in l in w'

(42) $\boxed{4} = [\![\text{ In which country ... Jo grew up } t_2]\!]^w$
$= \exists l.\ l$ is a country in $w \wedge t_1 = \lambda w'$. Jo grew up in l in w'

Lastly, a t_1 gets bound to produce a set of propositions, namely, the set of propositions that coincide with the proposition that *Jo grew up in l*, for some country l.

(43) $\boxed{5} = [\![\ \lambda_1 \text{ In which country ... Jo grew up } t_2]\!]^w$
$= \lambda p.\ \exists l.\ l$ is a country in $w \wedge p = \lambda w'$. Jo grew up in l in w'
$\simeq \{p \mid \exists l.\ l$ is a country in $w \wedge p = \lambda w'$. Jo grew up in l in $w'\}$

This example showed that the semantics of a question is derived from that of its "assertive counterpart", where the *wh*-phrase is replaced by a quantified variable. Combined with the proto-question operator and λ-abstraction, this allows to generate a set of propositions, which only vary in terms of the variable being questioned. This set of propositions (alternatives) can then be used to induce a partition of the CS, as per (**??**).

Chapter 2

Accommodating QuDs: Qtrees

This Chapter introduces a model of questions that is more sophisticated than standardly assumed (cf. Chapter 1). Questions are defined as recursive partitions, or parse trees of the Context Set. This model is shown to capture fine-grained information about how questions relate to each other in terms of specificity, and what it means to answer a question. The Chapter then describes how such questions can be "retro-engineered" from assertions, in a compositional way. Lastly, we suggest how this model of questions can eventually make novel predictions in the domain of pragmatic oddness.

2.1 Making sense

2.1.1 Oddness despite relevance and informativeness

In Chapter 1, we have seen that assertive sentences should be informative, i.e. lead to an incremental shrinkage of the Context Set (**CS**) (Stalnaker, 1978; Heim, 1982). We have also seen that they should be relevant, i.e. shrink the CS in a way consistent with the Question under Discussion (**QuD**) (Lewis, 1988; Roberts, 2012). But sometimes, it is unclear what the QuD should be. For instance, the exchange in (44) already settles the overt QuD (*Have you seen Jo today?*), and intersects the CS with the set of worlds in which Ed has not seen Jo on the day that *today* refers to. But one could imagine many possible continuations to Ed's utterance. Any such continuation should be informative and relevant to *some* QuD, but it is unclear how this QuD should be determined. In principle, it could be any non-vacuous partition of the newly updated CS. But there are many such partitions. How to know which one to pick?

(44) Al: Have you seen Jo today?
Ed: No I haven't...

Let us consider the following felicitous follow-up to (44). This continuation is felicitous, so, should be both informative and relevant. To be relevant, the sentence has to relate to a QuD. But, as mentioned earlier, the overt QuD *Have you seen Jo?* is at that point already settled. This suggests that, when no overt QuD is on the table, a "reasonable" QuD is chosen among all the possible non-vacuous partitions of the CS, and is such that the sentence under consideration properly answers it. This is motivated by the idea that sentences are never uttered in and of themselves; their purpose is to answer a question, overt or not, and to induce further questions Roberts (1996). A pragmatic model of assertion therefore needs to integrate what sentences mean, but also what kind of information structure they evoke. Assuming such a "reasonable" QuD is along the lines of *Where is Jo?*, then, the continuation in (45) is predicted to be both informative (it says that Jo is sick or at a conference), and relevant.

(45) –Have you seen Jo today?
–No I haven't... Either she is sick, or if she's not sick, she is at a conference.

But even if some implicit "reasonable" QuD can be inferred in the absence of an overt one, some cases of oddness remain mysterious. The follow-up sentence in (46) for instance, is equivalent to the one in (45) assuming implication is material, and so should in principle evoke the same QuD. (45) is thus predicted to be both informative and relevant, just like (45). Yet, this follow-up is sharply odd.

(46) –Have you seen Jo today?
–No I haven't... # Either she is sick, or if she's not at a conference, she is sick.

These two datapoints outline the following desideratum: if the contrast between (45) and (46) is due to the nature of the "reasonable" QuDs inferred from these sentences, then one must devise a way to systematically derive QuDs from out-of-the-blue assertions, in such a way that semantically similar, yet structurally distinct assertions, sometimes give rise to distinct QuDs.

This Chapter will address this desideratum and introduce a pragmatic model of these sentences in which (i) they package information differently in terms of their evoked QuDs, and (ii) unlike (46), (45), packages information in a way that is pragmatically optimal.

2.1.2 Overview and motivation of the Chapter

The machinery we introduce in this Chapter aims to account for the above datapoints (among others), by relating their felicity or oddness to the QuD(s) inferred from them.[1] The fundamental principle we want to operationalize is *Question-Answer Congruence* (henceforth **QAC**), as formalized by Katzir and Singh (2015),[2] and given in (47).

(47) *Question-Answer Congruence* (***QAC***). A felicitous assertion has to be a good answer to a good question.

This take on QAC is interesting because it roots this principle in pragmatics, and is broad enough to encompass a variety of constraints that were previously not grouped under the same umbrella. Chapter 1 for instance, showed that RELEVANCE could rule out a wide range of question-answer pairs, and as such could constitute a partial implementation of QAC. But QAC may in principle involve other constraints applying to question-answer pairs. This dissertation will show that, under a certain interpretation of "good answer" and "good question", many more cases of pragmatic oddness can be understood as an accross-the-board failure of QAC.

In this Chapter, we will lay out the groundwork for this more general pragmatic theory of question-answer well-formedness. We begin by introducing a more new model of questions, based on nested partitions, instead of mere partitions of the CS (as discussed in Chapter 1). This model is building on Büring (2003); Ippolito (2019); Zhang (2022), among many others. Next, equipped with this model of questions, we will show that questions can be evoked by assertions in a compositional way. As a result, sentences involving different operators (specifically, disjunctions and conditionals), give rise to different kinds of questions. Crucially in this model, each sentence may be associated with multiple potential questions. Finally, we will sketch what a pragmatics for question-answer pairs should look like in that framework. In line with QAC, a sentence which cannot be felicitously paired with *any* question will be deemed odd. This can happen if *all* the pairs formed by a sentence and a question it evokes, are themselves ill-formed.

We now proceed to define questions, not just as partition, but rather, as parse trees of the Context Set, that we will call Qtrees.

[1] We will not talk extensively about cases in which an assertive sentence constitutes a direct answer to an *overt* QuD. There is in fact an interesting line of work showing that overt QuDs can influence pragmatic oddness, especially when it comes to matters of redundancy (Haslinger, 2023).

[2] This principle has been discussed in several forms for many years, within and outside the field of generative linguistics. See for instance Rooth (1992) for a discussion on how focused assertions and questions can be systematically related in terms of their *semantics*.

2.2 Structure of Question Trees

2.2.1 From partitions to recursive partitions, to parse trees

Building on the standard model presented in Chapter 1, we introduce a more elaborate view of the pragmatics of questions. This model will incorporate the idea that questions have internal structure, and specifically, are hierarchically organized. This hierarchical organization is meant to capture the intuition that a question such as (48a) for instance, appears more *fine-grained*, than a question like (48b). Alternatively, whatever proposition identifies a cell in (48a), also identifies a cell in (48b). Crucially, this intuition will be incorporated in the pragmatics of questions, and so will be made directly accessible to the grammar.

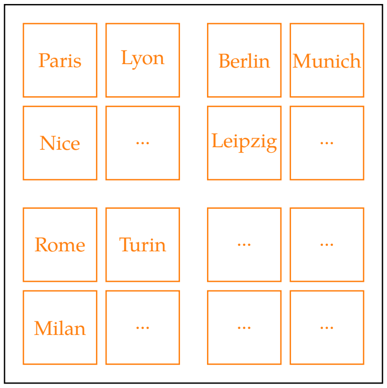
(48) a. In which city did Jo grow up?
 b. In which country did Jo grow up?

First, let us observe that these intuitions about question-specificity are *not* readily cashed out by standard partitions or alternative sets associated with questions. (48a)'s and (48b)'s sets of alternatives, given in (49a) and (49b) respectively, are made of disjoint, non empty propositions which, at a certain level of approximation, cover the space of all possibilities.[3] In other words, these alternatives already partition the set of *all* worlds. The partition that (49a) (resp. (49b)) induces on the CS is therefore obtained from (49a) (resp. (49b)) by simply intersecting each of its elements (a proposition/cell) with the CS – discarding empty sets.

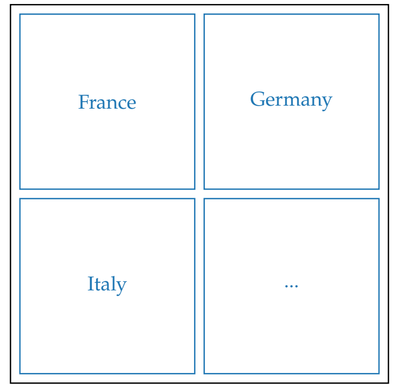
(49) a. $[\![$ In which city did Jo grow up?$]\!]^w =$
 $\{p \mid \exists l.\ l$ is a city $\wedge\ p = \lambda w'.$ Jo grew up in l in $w'\}$
 b. $[\![$ In which country did Jo grow up?$]\!]^w =$
 $\{p \mid \exists l.\ l$ is a country $\wedge\ p = \lambda w'.$ Jo grew up in l in $w'\}$

(48a) therefore induces a by-city partition of the CS (see Figure A1), while (48b) induces a by-country partition (see Figure A2). But nothing in (48a)'s partition signals that each of its cells is properly contained in a cell of (48b)'s partition. This property can de derived from the two structures, but is not readily *encoded* by them.

[3] We will assume here, that any point on Earth is associated with one single country, and one single city, in a Voronoi fashion. At this level of approximation, there is no countryless or cityless area. Alternatively, one could assume that there are cityless areas, but that the possibility of Jo growing up in such areas is ruled-out by the presupposition carried by *which*-questions like (48a). Under this assumption, *where*-questions may require more work.



(1) By-city partition associated with (49a). Cells are ordered on a grid for clarity only.



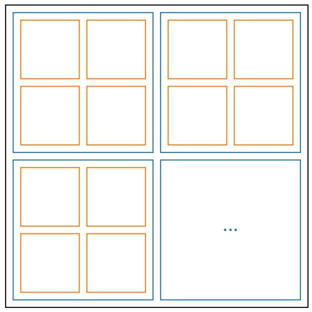
(2) By-country partition associated with (49b). Cells are ordered on a grid for clarity only.

Figure A: Standard partitions induced by a fine-grained (49a) and a coarser-grained question (49b).

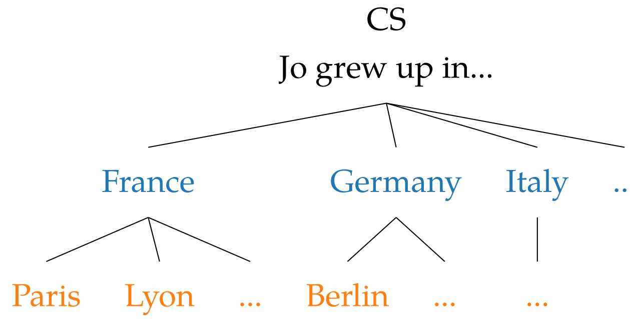
Intuitively, grouping together the propositions listed in (49a) talking about cities belonging to the same country, would help capture the desired property. This is done in (50). (50) then defines a set of sets of propositions.

(50) $[\![$ In which city did Jo grow up?$]\!]^w =$
$\{\{p \mid \exists l.\ l$ is a city in $l' \wedge\ p = \lambda w'.$ Jo grew up in l in $w'\} \mid l'$ is a country$\}$

Grouping together cells within bigger sets (which are cells themselves), amounts to building a *nested* partition of the CS. In our example, the "outer" partition is by-country, and the "inner" partition, is by-city. Graphically, this is equivalent to adding the "blue rectangles" from Figure A2, to Figure A1. This operation is performed in Figure B1. The tree in Figure B2 is yet another, more readable way to represent the same thing. In this tree, each node refers to a proposition of the form *Jo grew up in l*, l denoting a city or a country. Each node is understood as intersected with the CS, which corresponds to the root of the tree. Therefore, each node forms a proper subset of the CS. Nodes appearing at the same level (forming a "layer"), partition the CS. Deeper layers, correspond to finer-grained partitions. Tree like Figure B2 will be used throughout the dissertation to represent nested partitions like Figure B1. One must always keep in mind that the two representations are equivalent. (51) formally defines the bijective mapping between nested sets of propositions (dubbed *inductive propositions*) like (50), and tree structures like Figure B2.



(1) Recursive partition view



(2) Tree view

Figure B: Alternative representations of the CS corresponding to the nested sets of (50).

(51) *Set-to-tree bijection.* To define this bijection, we first define inductive propositions, and their propositional content. S is an inductive proposition if either:

- S is a set of worlds (i.e. a proposition);
- S is a set of inductive propositions.

The propositional content of an inductive proposition is then defined as:

- If S is a proposition: S;
- If S is a set of inductive propositions: the grand union of the propositional contents of S's elements.

Any inductive proposition S is in a bijection with a tree structure whose nodes are propositions, and defined as:

- If S is a proposition: the tree node denoting S;
- If S is a set of inductive propositions: the tree whose root denotes S's propositional content, and whose children are the tree structures induced by each of S's elements.

So far, we have shown that the standard view linking questions to partitions, fails to account for the intuition that questions differing in terms of specificity, stand in some kind of inclusion relation encoded in their structure. We proposed a way to cash out this intuition, by appealing to recursive partitions, that we represent as trees for clarity.

We now proceed to generalize these observations about the structure of questions. Building on Büring (2003); Riester (2019); Onea (2016); Ippolito (2019); Zhang (2022) (among others), we take questions to denote *parse trees* of the CS, i.e. structures that hierarchically organize the worlds of the CS. Such trees (abbreviated **Qtrees**) are defined in (52).

(52) *Structure of Question-trees* (***Qtrees***). Qtrees are rooted trees whose nodes are all subsets of the CS and s.t.:

- Their root generally[4] refers to the CS;
- Any intermediate node is a proposition, which is partitioned by the set of its children.

A Qtree can be bijectively mapped to a nested partition of the CS as defined in (53). Due to this equivalence, we will mostly use Qtrees in the rest of this dissertation.

(53) *Nested partition.* A nested partition P of a set S is a kind of inductive proposition, s.t.:

- If P is a set of inductive propositions, then the propositional contents of P's elements partition P's propositional content. Additionally, P's elements are nested partitions of their own propositional content.

Before investigating the interpretation and the structural properties of model of questions, the next Section covers a few core concepts from graph theory that will be useful in the rest of the Chapter and beyond.

2.2.2 A brief refresher on graph theory (and a few useful concepts for Qtrees)

(52) defines Qtres as rooted trees. Linguists typically understand trees as relations between parent nodes and their children, along the lines of (65).

(54) *Rooted tree* (*inductive version*). A tree rooted in N is either:

- N (single, childless node);
- N, along with N's children, which are all rooted trees.

But we will see throughout this dissertation that it is also useful to see a tree as a specific kind of graph. We will first define graphs, then define trees as a subkind of graph, and lastly, show the importance of defining a root in such trees. The definition of a graph is given in (55). A graph is a way to represent a binary relation, which by default will be

[4] In the case of sentences carrying presuppositions, the root will be assumed to correspond to the intersection between the CS and the sentence's presupposition. In fact, the whole Qtree will be intersected with the presupposition. This will be put to use in Chapters 7 and **??**. But the examples we will see before this, will all involve Qtree rooted in the CS.

symmetric[5]. Elements in the domain of the relation are modeled as nodes, and unordered pairs of nodes are connected with an edge, iff they verify the relation. A graph therefore amounts to a set of nodes, and a set of edges between these nodes. This is illustrated in Figure C.

(55) *Graph.* A graph is defined by a set of nodes $\mathcal{N}$ and by a set of edges $\mathcal{E}$ between elements of $\mathcal{N}$. Edges are defined as unordered pairs of nodes: $\mathcal{E} \subseteq \{\{N_1, N_2\} \mid (N_1, N_2) \in \mathcal{N}^2\}$

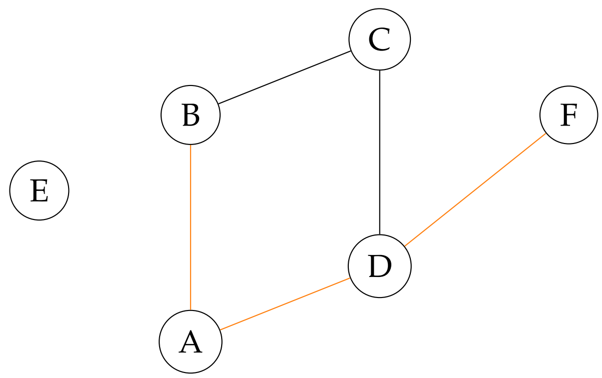


Figure C: A graph $G = (\mathcal{N}, \mathcal{E})$, with $\mathcal{N} = \{A, B, C, D, E, F\}$ and $\mathcal{E} = \{\{A, B\}, \{A, D\}, \{B, C\}, \{C, D\}, \{D, F\}\}$.

This definition allows to define rooted trees as a kind of graph with a few extra properties: connectivity, acyclicity, and rootedness; see (56). We now unpack what these three extra properties mean for graphs. This will lead us to define a few useful concepts applying to trees, namely paths, ancestry, and depth.

(56) *Rooted tree* (*graph version*). A rooted tree is a graph that is connected and acyclic, and features a distinguished node called root.

In graphs, sequences of adjacent edges form paths. For instance, in Figure C, the ordered sequence $[\{A, B\}, \{A, D\}, \{D, F\}]$ forms a path, between node A and node F. This is generalized in (57).

(57) *Path.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. Let $(N_1, N_2) \in \mathcal{N}^2$ be two nodes of G. There is a path in G between N_1 and N_2 (abbreviated $N_1 \overset{G}{\leadsto} N_2$) iff N_1 and N_2 can be connected by a series of edges in G, i.e. $\exists (e_1, ...e_k) \in \mathcal{E}^k.\ N_1 \in e_1 \wedge N_2 \in e_k \wedge \forall i \in [1; k-1].\ |e_i \cap e_{i+1}| = 1$, where $|.|$ is the cardinality operator.

[5] *Undirected* graphs, that we will simply call graphs, implement symmetric relations, while *directed* graphs implement asymmetric relations.

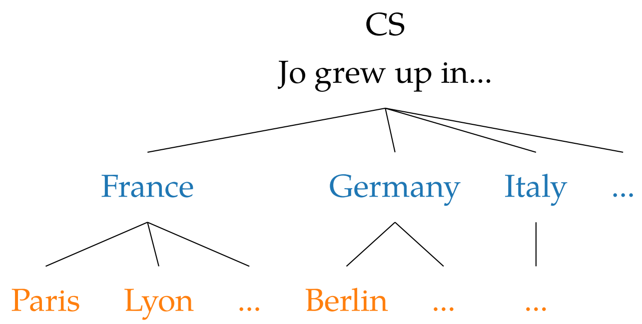
In Figure C, it is easy to see that nodes A, B, C, D and F are all connected to each other by at least one path (in fact, infinitely many of them that cycle through these nodes). Node E on the other hand, is isolated. So, Figure C represents a graph that is *not* connected. If E were removed from the set of nodes, and the edges remained the same, the resulting graph would be connected. This concept of connectivity is generalized in (58). If a graph is a tree, then, it is connected.

(58) *Connectivity.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. G is connected, iff there is a path in G between any pair of nodes in $\mathcal{N}$, i.e. $\forall (N_1, N_2) \in \mathcal{N}^2.\ N_1 \overset{G}{\rightsquigarrow} N_2$.

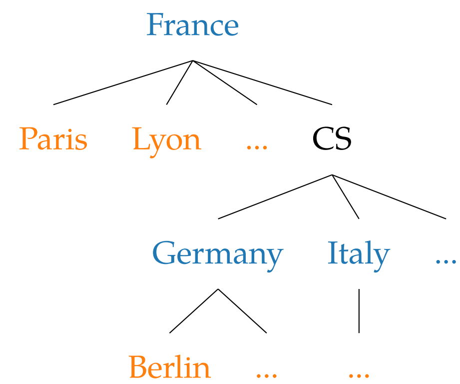
Another thing to note about Figure C, is that nodes A, B, C, and D form a "cycle", there is a path that starts at one of these nodes (e.g., C), and ends at this very same node, *via* B, A, and D. Because of this cycle, there are infinitely many paths between A, B, C, and D, and also between each of these nodes, and F. Removing the edge between, say, A and B, would break the cycle (yet, interestingly, maintain connectivity between A, B, C, and D). The resulting graph would be acyclic. The general definition of an acyclic graph, is given in (59). If a graph is a tree, then, it is acyclic. Moreover, connectivity and acyclicity, are necessary and sufficient for a graph to be a tree.

(59) *Acyclicity.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. G is acyclic, iff no node N of $\mathcal{N}$ is s.t. there is a path starting and ending at N in G, i.e. $\neg \exists N \in \mathcal{N}.\ N \overset{G}{\rightsquigarrow} N$.

We now have a definition of what kind of data structure a tree is. But why do we need Qtrees to be "rooted"? To understand why, let us go back to the tree in Figure B2, repeated in Figure D1 below. The way this tree is represented on paper, is somehow misleading. Recall that, from the point of view of graph theory, a tree is just an undirected graph, with a few extra properties constraining its edges. If the tree represented in Figure D1 were not "rooted", nothing would prevent us from representing it in the form of Figure D2: the nodes and edges are strictly the same, but in Figure D2, *France* "appears" to be the root of the tree, because visually, it is represented at the top. To avoid this confusion, the fact that the CS node should be "at the top" is made part of the representation of the tree – which then becomes a *rooted* tree. So, a rooted tree is just a tree, plus one distinguished node that serves as root.



(1) The "intuitive" view, in which the CS appears at the top.



(2) An alternative "counter-intuitive" view, in which the CS is *not* at the top, yet all edges and nodes are the same.

Figure D: Two equivalent ways to represent the tree corresponding to the question in (48a); assuming trees were connected, acyclic graphs, but not rooted.

The notion of a distinguished root in fact allows to define a few interesting properties on trees that linguist may be more familiar with, and that will be used throughout this dissertation. First, once a tree is rooted, it is possible to define a measure of distance between each node of the tree, and the root. This corresponds to the concept of depth, defined in (60a). In Figure D1 for instance, the CS has depth 0, *Germany* depth 1, and *Lyon* depth 2. This also allows to define the global "size" of the tree, in the form of its maximal depth; see (60b). Figure D1 for instance, is a tree of depth 2.

(60) a. *Depth of a node in a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree, with root R. Let $N \in \mathcal{N}$. The depth of N in T ($d(N,T)$) corresponds to the length of the minimal path between R and N if $N \neq R$,[6] and is set to 0 if $N = R$.

b. *Depth of a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree, with root R. The depth of T ($d(T)$) is the maximal depth of a node in T: $d(T) = max_{N \in \mathcal{N}}(d(N,T))$.

Having a distinguished root, and the derived concepts of depth, gives us the parent-child relation between nodes for free.[7] This relation is defined based on depth and edges in (61), and its transitive closure (the ancestor relation) is defined in (62), in two possible ways.

(61) *Parent-child relation in a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is the parent of N_2 (and N_2 is the child of N_1), iff $\{N_1, N_2\} \in \mathcal{E}$ and $d(N_1,T) < d(N_2,T)$.

[6] This path can be determined using a simple Depth-First Search algorithm starting from the root.
[7] in the next Section, we will introduce another definition of tree, that takes this relation as a primitive

(62) a. *Ancestor relation (recursive version).* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is an ancestor of N_2 iff either:

- N_1 is the parent of N_2;
- or N_1 is the parent of an ancestor of N_2.

b. *Ancestor relation (path version).* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is an ancestor of N_2 iff $N_1 \overset{T}{\rightsquigarrow} N_2$ and $d(N_1, T) < d(N_2, T)$.

Lastly, in the rest of this dissertation, we will extensively use the concept of *layer*, that we define as a the maximal set of same-depth nodes in a rooted tree; see (63). Figure D1 features a country-layer at depth 1, and a city-layer at depth 2. Layers therefore reflect an intuitive notion of granularity.

(63) *Depth-k layer of a rooted tree.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree, with root R. Let k be an integer s.t. $0 \geq k < d(T)$. The depth-k layer of T is the set of nodes in $\mathcal{N}$ whose depth is k, i.e. $\{N \in \mathcal{N} \mid d(N, T) = k\}$.

Now that we have defined the core structure of Qtrees along with a few related properties and metrics, we proceed to assign an interpretation to this kind of structure.

2.2.3 Interpreting Qtrees

At the end of Section 2.2.1, we showed that question should better be represented as nested partition, in order to encode their degree of specificity, and the grammar sensitive to how more or less fine-grained questions relate to each other. We also discussed how nested partitions could be unequivocally represented as Qtrees. It is easy to see that the tree in Figure B2/D1, repeated again in Figure E, is a Qtree according to (52). We saw that this Qtree intuitively capture the idea that a *Which country?* kind of question, is contained in a *Which city?* kind of question. We now investigate how to exploit this hierarchy in a meaningful way. We will use Figure E as an example, and now assign an interpretation to nodes and paths in such structures. We will focus on three meaningful aspects of Qtrees: answer-granularity (understood as node depth), strategies of inquiry (understood as paths), and question refinement (understood as tree inclusion)

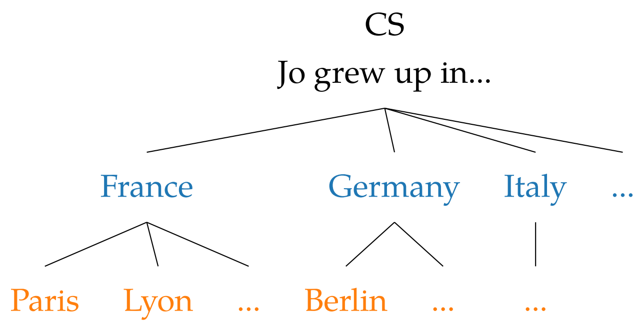


Figure E: "Intuitive" Qtree for *Which city did Jo grow up in?*

We start with the interpretation of nodes as possible answers, with different granularities. The root of Figure E for instance, which corresponds to the entire CS, defines a tautology: it is a proposition which is true of all worlds of the CS, because it simply coincides with it.[8] It can be understood as identifying the unique cell of coarsest-grained partition of the CS, that is, the CS itself. By contrast, leaves like *Paris*, *Lyon*, *Berlin* in Figure E, correspond to the "smallest" cells of the recursive partition that the Qtree defines. They can be seen as maximal answer to the underlying question, e.g., *In which city did Jo grow up?*. Intermediate nodes like *France* or *Germany* in Figure E, form cells of "intermediate" size, and can always be seen as unions of leaves. They appear to correspond to non-maximal answers. Because Qtrees can be made of many layers, they induce a hierarchy between non-maximal answers: an non-maximal answer p is "more maximal" than another non-maximal answer q, iff the node corresponding to p is located deeper in the Qtree than the node corresponding to q. This is formalized in (64).

(64) *Answer granularity.* Let T be a Qtree and (N_1, N_2) be two nodes in T. N_1 constitutes a finer-grained answer than N_2 iff $d(N_1, T) > d(N_2, T)$. This implies that leaves of T correspond to the finest-grained answers (maximal answers) to the question T represents.

Next, we discuss how Qtree encapsulate Roberts's notion of *Strategy of Inquiry*. To this end, we observe that nodes in a tree can receive a "recursive" interpretation, that incorporates everything the node dominates. Under this interpretation, a node N in a Qtree is not only what N denotes; it is the whole subtree ($\sim$subquestion) rooted in N, as defined in (65)

(65) *Recursive interpretation of tree nodes*. Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a rooted tree. Let $N \in \mathcal{N}$ be a node of T. N's recursive interpretation corresponds to:

[8] Chapter 1 moreover identifies it as an uninformative proposition that is Lewis-relevant but not Roberts-relevant.

- N, if N is a leaf;
- the subtree of T rooted in N, otherwise.

This point of view originates from the inductive definition of a rooted tree given in (54) and repeated below.

(54) *Rooted tree* (*inductive version*). A tree rooted in N is either:

- N (single, childless node);
- N, along with N's children, which are all rooted trees.

If T is a Qtree, then N's recursive interpretation will be the Qtree rooted in N. This Qtree's root can be seen as a "local" CS, which is equal to the global CS, updated with N. For instance, the recursive interpretation of the *France*-node in Figure E, corresponds to the subtree of Figure E rooted in *France*. This subtree, given in Figure F, amounts to the question *In which city did Jo grow up?*, granted that *Jo lives in France*, since its root corresponds to the CS intersected with the proposition that *Jo lives in France*.

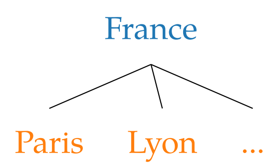
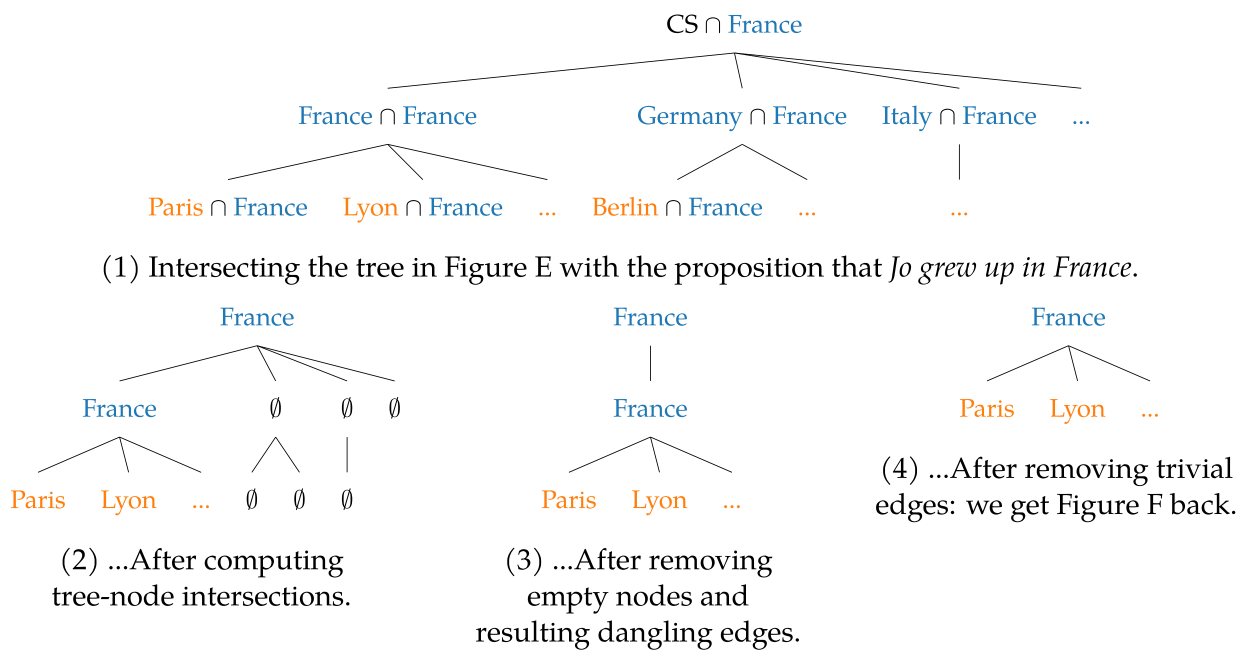


Figure F: "Recursive" interpretation of the *France*-node in Figure E.

In fact, this subtree as a whole, can be understood as the tree-node intersection of Figure E and the proposition that *Jo grew up in France*. Tree-node intersection is defined in (66). This operation takes a Qtree and a proposition p, and creates a Qtree whose nodes are each intersected with p, and resulting empty nodes are removed. Edges from the original Qtree are retained, as long as the nodes they connect are still part of the newly formed Qtree. Note that, because the nodes and edges of a tree form *sets* (and not *multisets*), tree-node intersection automatically collapses nodes from the original tree whose intersections with p yield the same result; and it also collapses the edges between such nodes. Figure G provides a decomposition of this procedure, computing the tree-node intersection between Figure E and the proposition that *Jo grew up in France*, and illustrating how nodes and edges may "collapse". (67) generalizes this point, by stating that the subtree of a Qtree rooted in a node N, can be reconstructed by tree-node intersecting the entire Qtree with the proposition N corresponds to. In other words, the subquestion corresponding to a node N, can be seen as a *restriction* of the entire Qtree, taking N for granted. This is proved in (68).

(66) *Tree-node intersection.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a Qtree. Let p be a proposition. The tree-node intersection between T and p, noted $T \cap p$, is defined iff $R \cap p \neq \emptyset$ and, if so, is the Qtree $T' = (\mathcal{N}', \mathcal{E}', R')$ s.t.:

- $\mathcal{N}' = \{N \cap p \mid N \in \mathcal{N} \wedge N \cap p \neq \emptyset\}$
- $\mathcal{E}' = \{\{N_1 \cap p, N_2 \cap p\} \mid \{N_1, N_2\} \in \mathcal{E} \wedge (N_1 \cap p) \neq (N_2 \cap p) \wedge N_1 \cap p \neq \emptyset \wedge N_2 \cap p \neq \emptyset\}$
- $R' = R \cap p$



(1) Intersecting the tree in Figure E with the proposition that *Jo grew up in France*.

(2) ...After computing tree-node intersections.

(3) ...After removing empty nodes and resulting dangling edges.

(4) ...After removing trivial edges: we get Figure F back.

Figure G: The recursive interpretation of a node, can be obtained by intersecting the whole Qtree with that node, removing empty nodes and trivial edges (formed by a parent node and its only child).

(67) *Recursive interpretation and CS update.* Let T be a Qtree. Let N be a node of T. N's recursive interpretation corresponds to the tree-node intersection of T with N.

(68) *Proof of (67).* Let T be a Qtree. Let N be a node of T. Because T is a Qtree, any node N dominates is a subset of N; any node dominating N, is a superset of N, and any node that is neither dominated nor dominating N, is disjoint from N. By definition, N's recursive interpretation is the subtree of T rooted in N, noted T'. We show that T' corresponds to the tree-node intersection between T and N, $T \cap N$. Let N' be N or a node dominated by N. $N' \subseteq N$, so $N' \cap N = N'$. This holds for any N' dominated by N or equal to N. So $T \cap N$ preserves T'. Let N' be an ancestor of N. $N \subseteq N'$ so $N' \cap N = N$. So any ancestor of N in T, is reduced

to N in $T \cap N$. Let N' be a node in T that is neither dominated nor dominating N. $N \cap N' = \emptyset$, and so any sibling/uncle/cousin of N in T is absent in $T \cap N$, along with any incident edges. Therefore, $T \cap N$ ends up being just T'.

We have just seen that under the recursive interpretation of nodes, each node N can be seen as a subquestion of the whole Qtree, which takes N's propositional content for granted. Under this interpretation, a path from the root (CS) to any node N, can then be seen as a series of subquestions, taking for granted increasingly strong propositions. In Figure E for instance, a path of the form [*CS*, *France*, *Paris*], can be interpreted as a series of inquiries of the form: [*In which city did Jo grow up* (*I have no idea*)?, *In which city did Jo grow up* (*given Jo grew up in France*)?, *Jo grew up in Paris*]. If the path terminates on a leaf, then the series of inquiries converges to a maximal answer. We will call such paths complete strategies of inquiry.

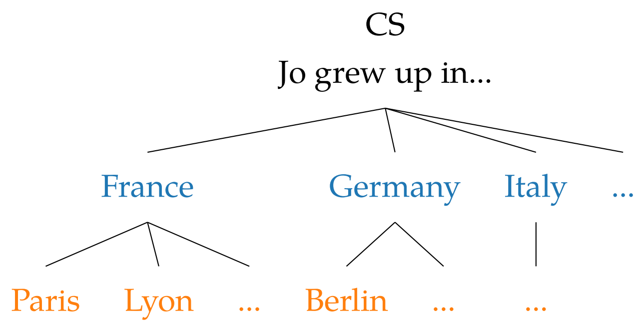
(69) *Complete Strategy of Inquiry.* Let T be a Qtree. A complete strategy of inquiry on T is a path from T's root to one of T's leaves.

This model is very close to what the previous literature had posited at the conversational level, whereby sentences answer questions and sometimes evoke new, finer-grained questions. The key difference here, is that individual questions are assumed to encapsulate the same kind of dynamic, hierarchical information. How does this relate to question granularity? Note that intuitively finer-grained questions yield deeper Qtrees than intuitively coarser-grained ones. Additionally, (60b) defined Qtree depth as the maximal length of a path from the root to a leaf in the tree. This leads to the equivalence in (70).

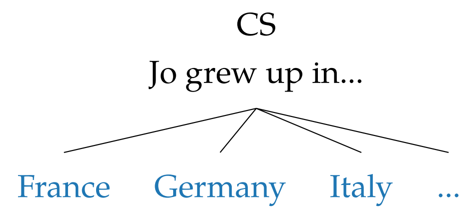
(70) *Depth and Complete Strategies of Inquiry.* Let T be a Qtree. T's depth can be recovered by finding the length of its longest complete strategy of inquiry, dubbed maximal complete strategy of inquiry.

In other words, finer-grained questions are linked to deeper Qtrees, which are characterized by a longer, maximal complete strategy of inquiry. In sum, a fine-grained question is a question for which converging to a maximal answer may require a lot of intermediate steps, or subquestions. This is useful as an absolute measure of question-complexity, but probably not enough to determine if a question is finer-grained than another question. For instance, this incorrectly predicts two completely independent questions to be comparable in terms of granularity, just because they give rise to Qtree of different depths.

There is in fact another way in which the recursive interpretation of nodes can help clarify in what sense a *Which city* kind of question, is more fine-grained than a *Which country* kind of question, in the current framework. Figure H shows what a Qtree for (49a) and a Qtree for (49b) should intuitively look like.



(1) "Intuitive" Qtree for (49a) = *Which city did Jo grow up in?*



(2) "Intuitive" Qtree for (49b) = *Which country did Jo grow up in?*

Figure H: Comparing *Which city* and *Which country* Qtrees.

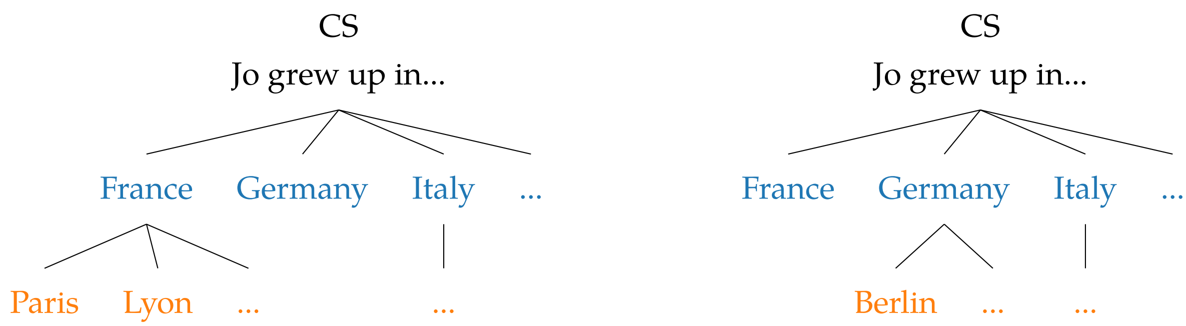
The Qtree for (49a) stops at the city-level, because cities should constitute maximal answer to that kind of question; the Qtree for (49b) on the other hand, stops at the country level, for similar reasons. And it is easy to notice that the Qtree for (49b) somehow forms a "subset" of the Qtree for (49a): it forms a subset of the nodes, and a subsets of the edges, of the Qtree for (49a). Additionally, it is not a random subgraph of Figure H1 (as defined in (71)). It remains a Qtree, that constitues a refinement of Figure H1, as defined in (72). It can also be shown, that all the possible refinements of a Qtree T, correspond to all the possible subgraphs of T that have the Qtree property.[9]

(71) *Subgraph.* Let $G = (\mathcal{N}, \mathcal{E})$ and $G' = (\mathcal{N}', \mathcal{E}')$ be two graphs. $G' \subseteq G$, iff $\mathcal{N}' \subseteq \mathcal{N}$ and $\mathcal{E}' \subseteq \mathcal{E}$.

[9]We identify the refinement operation between T and T', as a set $\mathcal{T}$ of subtrees of T, that is closed under root-sisterhood. We assume T is a refinement of a Qtree T' and show T' is a subgraph of T with the Qtree property. T' is obtained from T by removing the subtrees in $\mathcal{T}$ from T. So it is obviously a subgraph of T. We now show T' is a Qtree. $\mathcal{T}$ cannot contain the tree rooted in T, otherwise T' would be empty. So T' has same root as T, and this root is the CS. Let N be an intermediate node in T'. N has at least one child N', which means that $\mathcal{T}$ cannot contain the subtree of T rooted in N'. To be partitioned by its children, N in T' must have the same children as N in T, i.e. $\mathcal{T}$ should not contain any tree rooted in a child of N. If $\mathcal{T}$ did, then $\mathcal{T}$ would also contain the subtree of T rooted in N', du to its closure property. Contradiction. So N retained all its children from T, and is partitioned by them, given that T is a Qtree.
We assume T' is a subgraph of T with the Qtree property and show T is a refinement of a Qtree T'. T' is a subgraph of T, so we can define S the set of nodes of T not in T'. We then define $\mathcal{T}$ as the set of subtrees of T rooted in a maximal element of S w.r.t. the ancestor relation, as induced by T''s edges. We show this set is closed under root-sisterhood. Let $T'' \in \mathcal{T}$. It is subtree of T rooted in some node N, and N is maximal in S w.r.t. the ancestor relation. If N has no sister in T, then the closure property is trivially verified. If N has a sister N' in T, we show the closure property by contradiction. If the subtree of T rooted in N' did not belong to $\mathcal{T}$, then, either N' would be part of a subtree of $\mathcal{T}$ (but not as root), or, N' would be a node in T'. The former option would imply that some common ancestor of N and N' would be the root of a subtree in $\mathcal{T}$. But then both N and some ancestor of N, would be maximal in S w.r.t. the ancestor relation. Contradiction. The former option would mean that N''s parent in T', would have N', but not N has child, and so would not be partitioned by its children. Therefore, T' would not be a Qtree. Contradiction.

(72) *Qtree refinement.* Let T and T' be Qtrees. T is a refinement of T' (or: T is finer-grained than T'), iff T' can be obtained from T by removing a subset $\mathcal{T}$ of T's subtrees, s.t., if $\mathcal{T}$ contains a subtree rooted in N, then, for each node N' that is a sibling of N in T, the subtree of T rooted in N', is also in $\mathcal{T}$.

Two other possible refinements of Figure H1 are given below. It is worth noting that the process deriving a refinement from a Qtree need not remove entire layers.



(1) A refinement where the children of the *Germany*-node are deleted.

(2) A refinement where the children of the *France*-node are deleted.

Figure I: Possible refinements of the Qtree for *Which city did Jo grow up in?* in Figure H1.

2.2.4 Flagging Qtrees

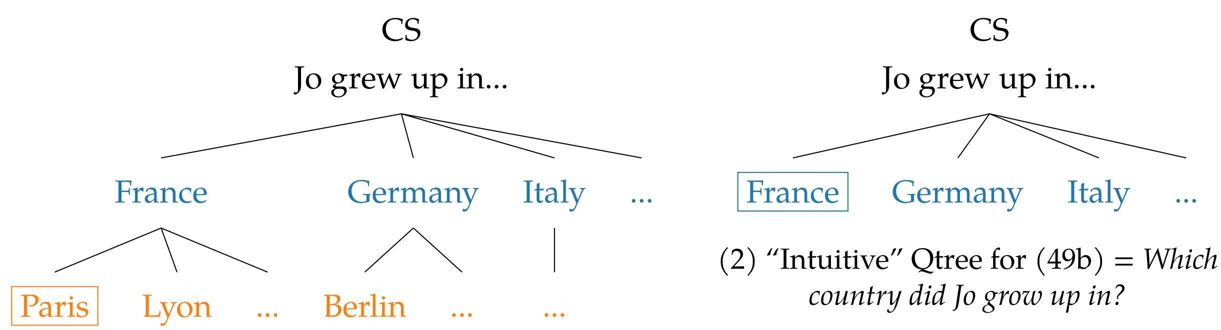
So far, we have considered Qtrees directly associated with questions, like (48a) and (48b). But, as suggested in the introduction to this Chapter, we want to go one step further, and posit that assertive sentences, like (73a) and (73b), also evoke questions in the form of Qtrees. Such questions will correspond to the ones a given assertion could be a good answer to.

(73) a. Jo grew up in Paris.
b. Jo grew up in France.

So (73a) and (73b) for instance, should evoke Qtrees associated with questions like (48a) and (48b), respectively. We sketched an intuitive representation of these Qtrees in Figure H. Are these Qtrees representing everything that the assertions in (73a) and (73b) convey though? One major difference between questions and assertions, is that questions are ignorant of the answer, while assertions provide such an answer. So, if (73a) were directly mapped to the Qtree in Figure H1, the information that (73a) actually answers the question by identifying the *Paris*-node, would be lost. Another way to see the issue, is to observe that (73a) and (74) would then be associated with the exact same Qtree.

(74) Jo grew up in Berlin.

To avoid such collisions in the case of assertive sentences, we define an extra piece of machinery on top of the Qtree architecture, that consists in a set of "verifying" nodes keeping track of *how* the assertion answers the question it evokes. I Figures, these nodes will be represented in boxes; given a Qtree T, T's set of verifying nodes will be referred to as $\mathcal{N}^+(T)$. (73a) and (73b) for instance, will intuitively evoke Qtree that *structurally* match those in Figure H, but whose *Paris* and *France* noes respectively, are "boxed", i.e. flagged as verifying.



(1) "Intuitive" Qtree for (49a) = *Which city did Jo grow up in?*

(2) "Intuitive" Qtree for (49b) = *Which country did Jo grow up in?*

Figure J: Comparing *Which city* and *Which country* Qtrees.

In that particular case, the nodes that are flagged as verifying in both Qtrees, strictly coincide with the proposition conveyed by the assertions, namely, that *Jo grew up in Paris*, and that *Jo grew up in France*. For an assertion like (74), the only flagged node would be *Berlin*. But we will not take this strict equivalence between prejacent proposition and verifying nodes to be a generality. In the model laid out in the next Section, we will assume that verifying nodes, just like Qtree structure, are compositionally "retro-engineered" from the structure and meaning of the sentence. As a result, there may be more than on verifying node in a given Qtree, and, the grand union of a Qtree's verifying nodes, may not always coincide with the proposition denoted by the assertion.[10] The next Section therefore introduces a more systematic way to derive Qtrees and their verifying nodes from simplex sentences.

Moreover, an accommodated Qtree should allow the sentence evoking it to properly answer it; that is why we assume that any well-formed Qtree derived from a sentence should come with a non-empty set of verifying nodes.(see (75)). More generally, we as-

[10]This will in particular be true of conditional assertions.

sume that oddness results from the fact that a given sentence, through its LF, cannot give rise to any well-formed Qtree. This is summarized in (76).

(75) *Empty labeling of verifying nodes.* If a sentence S evokes a Qtree T but does not flag any node as verifying on T, then T is deemed odd given S.

(76) *Oddness of a sentence.* A sentence S is odd if any Qtree T it evokes is odd given S.

2.3 Compositional Qtrees: base case

So far, we have established that assertive sentences can evoke the questions they are good answers to, in the form of Qtrees. And we used "intuitive" Qtrees like the ones in Figures J1 and J2 to show how such structures could capture a wide range of properties connecting questions and answers, and questions to each other. We now introduce a principled algorithm to derive Qtrees out of assertive sentences. This will heavily build on the notion of alternatives and how they can be ordered in terms of specificity. We will show that this ordering of alternatives induces a specific "layering" on Qtrees.

2.3.1 Alternatives

It is quite uncontroversial that assertive sentences evoke alternatives (Rooth, 1992; Katzir, 2007; Fox and Katzir, 2011). An alternative is a sentence that is sufficiently "similar" to the sentence it is evoked by, but may have a different meaning. For instance, (77a) is felt to have (77b) as alternative, and vice versa. Pre-theoretically, this is because, in many contexts, (77a) is utterable iff (77b) is, too. The same holds for (78a) and (78b).

(77) a. Jo ate all of the cookies.
     b. Jo ate some of the cookies.

(78) a. Jo grew up in Paris.
     b. Jo grew up in Lyon.

Moreover, the computation of alternatives is driven by focus. In (77) and (78), it was implicitely assumed that quantifiers and city names respectively, were focused, and so gave rise to alternative varying in terms of the focused element. But note that, if the object of (77a) and the verb of (78a) had been focused instead, the alternatives to these sentences would have been different, along the lines of (79b) and (80b), respectively.

(79) a. Jo ate all of the COOKIES.

b. Jo ate all of the muffins.

(80) a. Jo GREW UP in Paris.

b. Jo resides in Paris.

We define what counts as an alternative to a given sentence (or more generally an LF) in (123), based on Rooth (1992) and Katzir (2007).

(81) *Structural alternatives.* Let X be an LF containing a focused constituent. The set of X's alternatives is the set $\mathcal{A}_X$ of LFs Y, obtained by substituting X's focused constituent with an element that is at most as complex.

(82) *Structural complexity.* Let X and Y be two LFs. X is at most as complex as Y iff X can be obtained from Y *via* substitutions of lexical items with other lexical items, or constituent-to-subconstituent substitutions.

Structural alternatives to a given LF (which are also LFs), induce a set of propositional alternatives, as defined in (83).

(83) *Propositional alternatives.* Let X be an LF denoting a proposition p. The set of X's propositional alternatives is the set $\mathcal{A}_{p,X}$ of propositions denoted by X's structural alternatives: $\mathcal{A}_{p,X} = \{q \mid \exists Y \in \mathcal{A}_X.\ [\![Y]\!] = q\}$

Propositional alternatives can be related to each other by entailment – forming a partial order. This partial order between propositional alternatives can be graphically represented in the form of a Hasse diagram. Hasse diagrams for two possible sets of propositional alternatives, roughly corresponding to (79) and (78), are given in Figure K.

(1) Hasse diagram for $\{\mathit{all}, \mathit{some}, \mathit{none}\}$

(2) Hasse diagram for $\{\mathit{Europe}, \mathit{France}, \mathit{Germany}, \mathit{Paris}, \mathit{Lyon}, \mathit{Berlin}\}$

Figure K: Hasse diagrams generated by $\vDash$ on two possible sets of propositional alternatives.

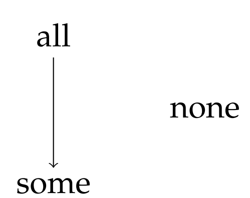
How are these diagrams obtained from propositional alternatives, and the entailment relations between them? Formally, a Hasse diagram is a directed graph, as defined in (84). The only difference between a graph and a directed graph, is that the edges of a directed graph have a direction, i.e. they correspond to ordered pairs instead of sets of cardinality 2. If $[N_1, N_2]$ is a directed edge, $[N_1, N_2]$ is visually represented as $N_1 \to N_2$. Paths are also directed, and so is the ancestor relation (see (85) and (86)). Directed graphs are designed to model *asymmetric* relations, like $\vDash$.

(84) *Directed graph.* A directed graph is defined by a set of nodes $\mathcal{N}$ and by a set of directed edges $\mathcal{E}$ between elements of $\mathcal{N}$. Directed edges are defined as ordered pairs of nodes: $\mathcal{E} \subseteq \{[N_1, N_2] \mid (N_1, N_2) \in \mathcal{N}^2\}$

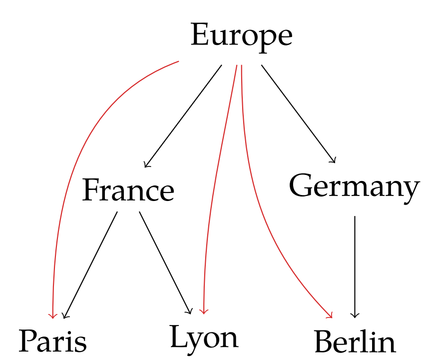
(85) *Directed Path.* Let $G = (\mathcal{N}, \mathcal{E})$ be a directed graph. Let $(N_1, N_2) \in \mathcal{N}^2$ be two nodes of G. There is a path in G between N_1 and N_2 (abbreviated $N_1 \overset{G}{\leadsto} N_2$) iff N_1 can be connected to N_2 by a series of directed edges in G, i.e. $\exists (e_1, ...e_k) \in \mathcal{E}^k.\ e_1^{(0)} = N_1 \wedge e_k^{(1)} = N_2 \wedge \forall i \in [1; k-1].\ e_i^{(1)} = e_{i+1}^{(0)}$, where, for any edge e, $e = [e^{(0)}, e^{(1)}]$.

(86) *Ancestor relation (directed path version).* Let $G = (\mathcal{N}, \mathcal{E})$ be a directed graph. Let $(N_1, N_2) \in \mathcal{N}^2$. N_1 is an ancestor of N_2 iff $N_1 \overset{G}{\leadsto} N_2$.

The directed graphs (not yet the Hasse diagrams) induced by $\vDash$ on the sets of alternatives from Figure K, are given in Figure L. In these graphs, there is a directed edge $[p, q]$ between two nodes corresponding to propositions p and q, iff $q \vDash p$. Figure L1 already looks like the corresponding Hasse diagram in Figure K1, but Figure L2 does not: it features a few more directed edges (in red) than its Hasse counterpart in Figure K2.



(1) Directed graph induced by $\vDash$ on $\{\textit{all}, \textit{some}, \textit{none}\}$



(2) Directed graph induced by $\vDash$ on $\{\textit{Europe}, \textit{France}, \textit{Germany}, \textit{Paris}, \textit{Lyon}, \textit{Berlin}\}$

Figure L: Directed graphs generated by $\vDash$ on two possible sets of propositional alternatives.

How do Hasse diagrams eliminate these few superfluous edges? The Hasse diagrams we are interested in correspond to the transitive reduction of the graphs in Figure L, which were induced by $\vDash$ on sets of propositional alternatives. The transitive reduction operation precisely gets rid of the red edges in Figure K2, based on the idea that such edges correspond to paths formed by the black ones. The formal (though, non constructive) definition of a transitive reduction, is given in (87). This definition maps the graphs in Figure L, to the Hasse diagrams in Figure K.

(87) *Transitive reduction of a graph.* Let $G = (\mathcal{N}, \mathcal{E})$ be a graph. The transitive reduction G' of G is the graph:

- Whose set of nodes is $\mathcal{N}$;
- Whose edges are the smallest set $\mathcal{E}'$ s.t. $\forall(N_1, N_2) \in \mathcal{N}.\ N_1 \overset{G}{\rightsquigarrow} N_2 \iff N_1 \overset{G'}{\rightsquigarrow} N_2$

The Hasse diagram in Figure K2 is basically a rooted tree, and may look like a Qtree, but this is not a generality. For instance, the Hasse diagram in Figure K1 is not connected, so is not even a tree. Additionally, not all sets of propositional alternatives, even if their Hasse diagram is tree-like, are guaranteed to verify the partition property of Qtrees. The next Section focuses on how Hasse diagrams can be used to determine how alternatives relate to each other in terms of granularity. This will eventually allow us to encode granularity in the structure of Qtree.

2.3.2 Alternatives and granularity

In this Section, we use a notion of granularity to constrain what kind of Qtree can be evoked by a simplex assertive LFs. We consider simplex LFs to be LFs which do not contain a node of type $\texttt{t}$ besides their root.[11] The goal is to organize the layers of a Qtree in terms of how specific the nodes in this layer are. Why is an external notion of granularity needed to structure Qtree? In the Qtree sketched in e.g. Figure J1, each layer corresponds to an intuitive degree of specificity: a by-country layer dominates a by-city layer. Though intuitive, this kind of configuration is not the only one to verify the Qtree property. The tree in Figure M, where the *Germany*-node is replaced by its children, is also a Qtree: at our level

[11]Nina Haslinger suggested that this condition *may* be relaxed under certain contexts. For instance, a disjunctive LF denoting p, may sometimes be understood as simplex, and generate Qtrees based on the recipe presented later in this Chapter. The exact circumstances under which this is possible, should be fleshed out, but will not be the focus of this dissertation. One intuition, suggested by Nina, is that an overt QuD may indicate what is relevant, and in turn determine what should be considered "simplex" when computing implicit Qtrees (and comparing such Qtrees to the overt QuD). Another intuition, is that the "simplex" character of an LF, may be partly driven by focus.

of approximation, all countries but Germany, plus all the German cities, partition the set of all possible locations, and, each country represented in this tree is properly partitioned by the set of its cities.

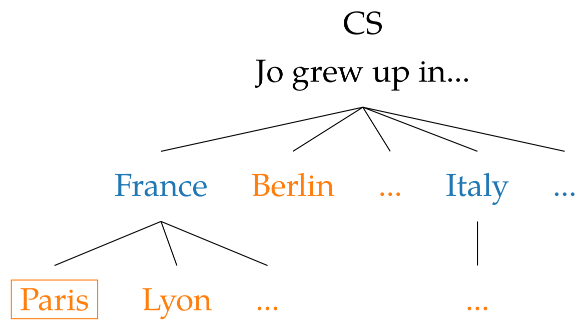


Figure M: "Unintuitive" Qtree for (49a) = *Which city did Jo grow up in?*, where layers exhibit "mixed granularity".

To derive Qtrees like Figure J1, and rule-out Qtrees like Figure M, we need a notion of granularity that can transfer into the Qtree layers. We now show that a relation of same-granularity can be derived from Hasse diagrams induced by $\vDash$ on "complete" sets of propositional alternatives. Considering such diagrams, where *all* relevant alternatives are considered, we take that two nodes (two propositional alternatives) have same granularity if they are equidistant (in terms of path length) to a common ancestor. This relation, defined in (88) is close in spirit to Ippolito (2019)'s *Specificity Condition*.[12] Note that this definition is conditional, and not biconditional: nodes that do not have a common ancestor, may or may not be seen as same-granularity.[13]

(88) *Same granularity relation* $\sim_g$. Let p and q be two propositions belonging to the same set of propositional alternatives. Let H be the Hasse diagram induced by $\vDash$ on this set of alternatives. If p and q have a common ancestor r in H, and the paths from r to p and r to q have same length, then $p \sim_g q$.

We now leverage this relation between propositions to define the layers of a Qtree as partitions induced by sets of same-granularity alternatives. We first observe that the relation $\sim_g$ defined in (88) can be used to divide the set of propositional alternatives to a

[12]However, the structure on which this condition operates in Ippolito's model, appears slightly different (*Structured Sets of Alternatives*). Also, the *Specificity Condition* is not taken to be a relation, but a rather, a constraint defining which kind of alternative can be raised in e.g. disjunctive environments.

[13]This will be discussed in more detail when dealing with scalar alternatives such as $\langle$*some*, *all*$\rangle$, in Chapter 7. It will be crucial that such alternatives, which do *not* have a common ancestor in their Hasse diagram (see Figure K1), *can* be seen as same-granularity.

given LF, into subsets sharing the same level of granularity. This gives rise to a "tiered" set of alternatives, as defined in (89).

(89) *Tiered set of propositional alternatives.* Let X be a sentence denoting a proposition p. A tiered set of propositional alternatives to X, is the set of sets of propositions, whose elements are the maximal sets of propositions related by the same-granularity relation. In other words, $\mathcal{A}_{p,X}^{\sim_g} = \{\{r \in \mathcal{A}_{p,X} \mid r \sim_g q\} \mid q \in \mathcal{A}_{p,X}\}$. If $\sim_g$ is an equivalence relation, $\mathcal{A}_{p,X}^{\sim_g}$ is a partition.

Tiered sets of alternatives are quite close to the *Structured Sets of Alternatives* defined in Ippolito (2019). One difference however, is that tiered sets of propositional alternatives are *not* assumed to include propositions corresponding to alternatives that are more complex than the original LF. The elements of a tiered set of propositional alternatives are sets of propositions and form same-granularity "tiers", as defined in (90). These tiers will be used to form Qtree layers. If $\sim_g$ is an equivalence relation when restricted to a specific set of propositional alternatives, then the resulting tiered set of alternatives will partition it, i.e. same-granularity tiers will be cells.

(90) *Same-granularity tier.* Let X be a sentence denoting a proposition p, and $\mathcal{A}_{p,X}$ its set of propositional alternatives. Let $q \in \mathcal{A}_{p,X}$. The set of same-granularity alternatives to q (in $\mathcal{A}_{p,X}$), is the set of propositions in $\mathcal{A}_{p,X}$ sharing same-granularity with q. We call this set $\mathcal{A}_{p,X}^{q}$. $\mathcal{A}_{p,X}^{q} = \{r \in \mathcal{A}_{p,X} \mid r \sim_g q\}$. $\mathcal{A}_{p,X}^{q}$ is a subset of the tiered set of propositional alternatives to X, $\mathcal{A}_{p,X}^{\sim_g}$. Moreover, if $\sim_g$ is an equivalence relation, then $\mathcal{A}_{p,X}^{q}$ constitutes a cells of $\mathcal{A}_{p,X}^{\sim_g}$.

2.3.3 Leveraging alternatives to generate Qtrees

We are now equipped to devise a recipe generating Qtrees out of simplex sentences, based on tiered sets of propositional alternatives. We start by considering the standard constraint on question-answer pairs, given in (91). This constraint establishes a connection between the standard set of alternatives derived from a sentence involving focus, and the kind of question this sentence answers.

(91) *Constraint on question-answer pairs* (*Rooth, 1992, to be revised*). A good question-answer pair (Q, A) is s.t. $[\![Q]\!] \subseteq [\![A]\!]^f$, where:

- $[\![Q]\!]$ corresponds to the alternative semantics of the question;
- $[\![A]\!]^f$ corresponds to the focus semantic value of the answer, i.e. the set of propositions denoted by LFs obtained from A *via* the substitution of A's focused material by a same-type element.

Let us show that this constraint is not sufficient (though, a good starter) for a model of questions evoked by assertions. We assume that A corresponds to the sentence *Jo grew up in PARIS*, where *PARIS* is focused. The focus semantic value of A then involves propositions denoted by LFs of the form *Jo grew up in l*, with *l* a location, e.g. *Paris*, *France*, or *Germany*. If the only constraint on the question Q accommodated from A was that $[\![Q]\!]$ should be a subset of $[\![A]\!]^f$, then, in principle, $[\![Q]\!]$ could be made of the three propositions that *Jo grew up in Paris*, *Jo grew up in France*, and *Jo grew up in Germany*. Granted that Paris is in France, and that France and Germany are disjoint, this set of alternatives would induce a partition of the CS of the form $\{\neg\mathit{France} \wedge \neg\mathit{Germany}, \mathit{Germany}, \mathit{France} \wedge \neg\mathit{Paris}, \mathit{Paris}\}$. This appears similar to the mixed-granularity layer that we said was problematic in Figure M. So not all questions allowed by (91), given a fixed assertion, appear to make sense. There are two ways to alter (91) to avoid that kind of configuration: modify the relation between $[\![Q]\!]$ and $[\![A]\!]^f$, and/or, change $[\![A]\!]^f$ into something else.

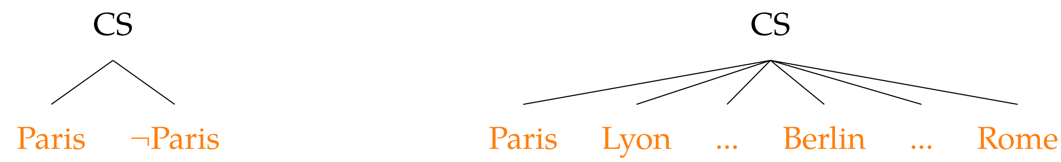
We in fact opt for both options, and reuse the ideas presented in the previous Section. Specifically, we consider two subcases: the case in which $[\![Q]\!]$ simply corresponds to $\{[\![A]\!]\}$, and induces a partition of the CS of the form $\{[\![A]\!], \neg[\![A]\!]\}$; and the case foreshadowed in the previous Section, in which $[\![Q]\!]$ corresponds to same-granularity alternatives to $[\![A]\!]$. These two cases are repeated in (92).

(92) *Constraint on question-answer pairs* (*first revision*). Let X be a LF denoting p. X evokes a question that is either:

(i) $[\![Q]\!] = \{p\}$;

(ii) $[\![Q]\!] = \mathcal{A}^p_{p,X}$, the set of same-granularity alternatives to p.

(92) allows assertions to evoke multiple potential Qtrees. According to (92), an assertion such as *Jo grew up in Paris*, will either evoke the question $[\![Q]\!] = \{\mathit{Paris}\}$, inducing a partition of the CS of the form $\{\mathit{Paris}, \neg\mathit{Paris}\}$, and corresponding to the polar question of whether or not Jo grew up in Paris; or, $[\![Q]\!] = \{\mathit{Paris}, \mathit{Lyon}, \mathit{Nice}, ..., \mathit{Berlin}, ..., \mathit{Rome}, ...\}$, inducing a similar partition of the CS, and corresponding to the *wh*-question *In which city did Jo grow up?*. These partitions are represented in Figure N.



(1) A Qtree for (73a) assuming (92i)

(2) A Qtree for (73a) assuming (92ii)

Figure N: One-layer Qtrees generable from (92) and the sentence (73a)=*Jo grew up in Paris*.

The above partitions seem more in line with intuitions than the pathological ones generable from (91). However, they still do not form layered Qtrees. Therefore, (92) is still not powerful enough to capture the specificity differences sketched in Figure H among others. Going one step further, we can assume that the Qtrees compatible with a sentence, are either generated by the proposition p denoted by the sentence (thus creating a Qtree like Figure N1), or, by the sentence's tiered set of propositional alternatives, as defined in (89). Specifically in the latter case, it will be assumed that each layer of the Qtree corresponds to the partition induced on the CS by a same-granularity tier of propositional alternatives, and that layers are ordered in terms of granularity. Figure N2 constitutes the simplest subcase of this principle, in which only one layer gets generated out of same-granularity alternatives to p. In any case, verifying nodes are defined as the leaves of the tree entailing p (i.e. contained in p). This is formalized in (93). In this definition, (93ii) may be seen as a subcase of (93iii), in which the p-chain set to p only.

(93) *Qtrees for simplex LFs (to be further generalized in Chapter 7).* Let X be a simplex LF denoting p, not settled in the CS. Let $\mathcal{A}_{p,X}$ be the set X's propositional alternatives. For any $q \in \mathcal{A}_{p,X}$, let $\mathcal{A}^q_{p,X} \subseteq \mathcal{A}_{p,X}$ be the set of alternatives from $\mathcal{A}_{p,X}$ sharing same granularity with q. We assume for simplicity that for any q, $\mathcal{A}^q_{p,X}$ partitions the CS. A Qtree for X is either:

   (i) A depth-1 Qtree whose leaves denote $\mathfrak{P}_{\{p\},CS} = \{p, \neg p\}$

   (ii) A depth-1 Qtree whose leaves denote $\mathfrak{P}_{\mathcal{A}^p_{p,X},CS} = \mathcal{A}^p_{p,X}$.

   (iii) A depth-k Qtree ($k > 1$) constructed in the following way:

   - Formation of a "p-chain" $p_0 = p \subset p_1 \subset ... \subset p_n$ where $p_0, ..., p_n$ are all in $\mathcal{A}_{p,X}$ but belong to different granularity tiers in $\mathcal{A}^{\sim_g}_{p,X}$: $\mathcal{A}^{p_0}_{p,X} \neq \mathcal{A}^{p_1}_{p,X} \neq ... \neq \mathcal{A}^{p_n}_{p,X}$.
   - Generation of the "layers" of the Qtree, based on the partitions induced by the granularity tiers corresponding to each element of the p-chain: $\left\{\mathfrak{P}_{\mathcal{A}^{p_i}_{p,X},CS} \mid i \in [0; n]\right\}$.

- Determination of the edges between nodes (cells) of adjacent layers (and between the highest layer and the root), based on the subset relation.[14]

In any case, $\boxed{\text{verifying nodes}}$ are defined as the set of leaves entailing p.

2.3.4 Applying the recipe to two simple sentences

We can now apply (93) to sentences like (73a) and (73b), repeated below.

(73) a. Jo grew up in Paris.
b. Jo grew up in France.

We start with (73a), and assume that its alternatives are of the form *Jo grew up in l*, with *l* a city or a country. Taking for granted that "city" propositions and "country" propositions form two distinct granularity tiers, the tiered set of propositional alternatives to (73a), will be as in (94).

(94) $$\begin{aligned}\mathcal{A}^{\sim g}_{\textit{Paris},(73a)} &= \{\{\textit{Paris}, \textit{Lyon}, ..., \textit{Berlin}, ...\}, \{\textit{France}, \textit{Germany}, ...\}\} \\ &= \{\{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\}, \\ &\qquad \{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\}\}\end{aligned}$$

First, we can generate a Qtree for (73a) using principle (93i). This Qtree will have the CS as root, and two leaves corresponding to the propositions that *Jo grew up in Paris*, and *Jo did not grow up in Paris* (assuming this matter is not settled in the CS). This Qtree is depicted in Figure O1. Intuitively, it corresponds to the question of whether or not Jo grew up in Paris.

Second, we can use principle (93ii). To do so, we must determine the set of same-granularity alternatives to the prejacent proposition that *Jo grew up in Paris*. This set, labeled $\mathcal{A}^{\textit{Paris}}_{\textit{Paris},(73a)}$, corresponds to the first element of the tiered set of propositional alternatives $\mathcal{A}^{\sim g}_{\textit{Paris},(73a)}$ in (94). It is repeated in (95). The alternatives contained in $\mathcal{A}^{\textit{Paris}}_{\textit{Paris},(73a)}$ are all exclusive (cities are spatially disjoint), and moreover cover the space of possibilities. So, once intersected with the CS, they already form a partition of the CS. According to principle (93ii), this partition correspond to the leaves of the resulting Qtree. This Qtree is depicted in Figure O2. Intuitively, it corresponds to the question of which city Jo grew up in.

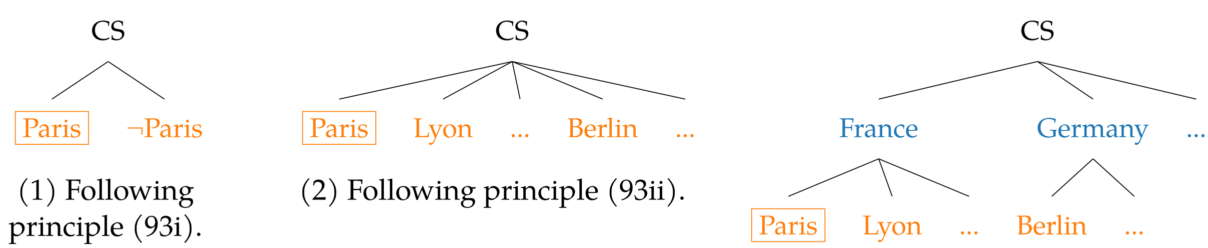
(95) $$\begin{aligned}\mathcal{A}^{\textit{Paris}}_{\textit{Paris},(73a)} &= \{\textit{Paris}, \textit{Lyon}, ..., \textit{Berlin}, ...\} \\ &= \{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\} \\ &= \mathfrak{P}_{\{\textit{Paris},\textit{Lyon},...,\textit{Berlin},...\},CS}\end{aligned}$$

[14]This may not always create well-formed Qtrees. Chapter 7 will explore such cases update (93) in consequence.

Third and lastly, we can use principle (93iii), which constitutes are multi-layer generalization of principle (93ii). To do so, we need to define a p-chain of propositions entailed $p = \lambda w.$ *Jo grew up in Paris in* w. The tiered set of alternatives posited in (94) contains one such proposition, namely $p' = \lambda w.$ *Jo grew up in France in* w. The resulting Qtree will therefore be made of three layers: the CS (root), the partition generated by the same granularity alternatives to p', and the partition generated by the same granularity alternatives to p, already defined in (95). The set of same-granularity alternatives to p', labeled $\mathcal{A}^{France}_{Paris,(73a)}$, corresponds to the second element of the tiered set of propositional alternatives $\mathcal{A}^{\sim g}_{Paris,(73a)}$ in (94). It is repeated in (96). The alternatives contained in $\mathcal{A}^{France}_{Paris,(73a)}$ are all exclusive (country are spatially disjoint), and moreover cover the space of possibilities. So, once intersected with the CS, they already form a partition of the CS.

(96) $$\begin{aligned}\mathcal{A}^{France}_{Paris,(73a)} &= \{France, Germany, ...\} \\ &= \{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w. \text{ Jo grew up in } l \text{ in } w\} \\ &= \mathfrak{P}_{\{France, Germany, ...\}, CS}\end{aligned}$$

As per principle (93iii), a Qtree evoked by (73a) will then have the CS as top layer, the nodes corresponding to the partition in (96) as middle layer, and the nodes corresponding to the partition in (95) as bottom (leaf) layer. Connectivity between layers is straightforward: it corresponds to the inclusion relation between cities and countries, and between countries and "the whole world" (~CS). The resulting Qtree is given in Figure O3. Intuitively, it corresponds to the question of which city Jo grew up in, but such that this question is decomposed into two subquestions: first, which country Jo grew up in; then, knowing the country, which city Jo grew up in, in that country.



(1) Following principle (93i).

(2) Following principle (93ii).

(3) Following principle (93iii).

Figure O: Possible Qtrees evoked by the assertion (73a)=*Jo grew up in Paris*.

Of course, if more alternatives to (73a) had been posited in the first place, principle (93iii) would have produced more Qtrees. For instance, if continent alternative had been considered, the tiered set of propositional alternatives to (73a), $\mathcal{A}^{\sim g}_{Paris,(73a)}$, would have been as in (97), and the Qtrees generated by principle (93iii), would have been the one in Figure O3, plus the one in Figure P.

(97) $\mathcal{A}^{\sim g}_{\textit{Paris},(73a)} = \{\{\textit{Paris}, \textit{Lyon}, ..., \textit{Berlin}, ...\}, \{\textit{France}, \textit{Germany}, ...\}, \{\textit{Europe}, \textit{Asia}, ...\}\}$
$= \{\{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\},$
$\{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\}$
$\{p \mid \exists l.\ l \text{ is a continent} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\}\}$

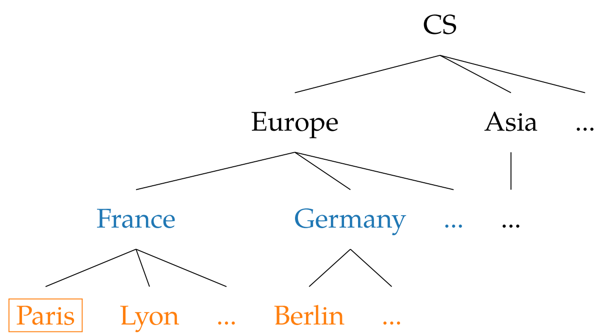


Figure P: An extra Qtree for (73a), generated by principle (93iii), assuming that (73a)'s tiered set of propositional alternatives is as in (97). .

For simplicity and ease of comparison, we will stick to a tiered set of alternatives involving city- and country-tiers, as defined in (94). Similarly, we can derive Qtrees for (73b)=*Jo grew up in France*. This assertion will in fact require less work, because it appears coarser-grained than (73a). To ensure that (73a) and (73b) are analyzed at the same level of approximation, we assume that (73b)'s alternatives are also of the form *Jo grew up in l*, with *l* a city or a country. The tiered set of propositional alternatives to (73b) is therefore identical to that of (73a), and given in (98).

(98) $\mathcal{A}^{\sim g}_{\textit{France},(73b)} = \{\{\textit{Paris}, \textit{Lyon}, ..., \textit{Berlin}, ...\}, \{\textit{France}, \textit{Germany}, ...\}\}$
$= \{\{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\},$
$\{p \mid \exists l.\ l \text{ is a country} \wedge\ p = \lambda w.\text{ Jo grew up in } l \text{ in } w\}\}$
$= \mathcal{A}^{\sim g}_{\textit{Paris},(73a)}$

First, we can generate a Qtree for (73b) using principle (93i). This Qtree will have the CS as root, and two leaves corresponding to the propositions that *Jo grew up in France*, and *Jo did not grow up in France* (assuming this matter is not settled in the CS). This Qtree is depicted in Figure Q1. Intuitively, it corresponds to the question of whether or not Jo grew up in France.

Second, we can use principle (93ii). To do so, we must determine the set of same-granularity alternatives to the prejacent proposition that *Jo grew up in France*. This set, labeled $\mathcal{A}^{France}_{France,(73b)}$, corresponds to the second element of the tiered set of propositional alternatives $\mathcal{A}^{\sim g}_{France,(73b)}$. It is repeated in (99). This set is also equal to the set of same-granularity alternative to *France*, when the prejacent was *Paris* (see (96)). Thus, the alternatives in this set, once intersected with the CS, already form a partition of the CS. According to principle (93ii), this partition correspond to the leaves of the resulting Qtree. This Qtree is depicted in Figure Q2. Intuitively, it corresponds to the question of which country Jo grew up in.

(99) $$\begin{aligned} \mathcal{A}^{France}_{France,(73b)} &= \{\mathit{France}, \mathit{Germany}, ...\} \\ &= \{p \mid \exists l.\ l \text{ is a city} \wedge\ p = \lambda w.\ \text{Jo grew up in } l \text{ in } w\} \\ &= \mathfrak{P}_{\{\mathit{France}, \mathit{Germany}, ...\}, CS} \\ &= \mathcal{A}^{France}_{Paris,(73a)} \end{aligned}$$

Third and lastly, we could use principle (93iii), but this principle would in fact give us nothing more than principle (93ii), given our assumptions about (73b)'s tiered set of propositional alternatives. This is because no proposition in $\mathcal{A}^{\sim g}_{France,(73b)}$ is weaker than $p{=}\lambda w.$ *Jo grew up in France in* w, and therefore, the only p-chain available in the case of (73b), is made of simply p. This p-chain would generate one single country-layer beyond the CS root, and the resulting Qtree, would simply be the one in Figure Q2.[15]

(1) Following principle (93i).

(2) Following principle (93ii).

Figure Q: Possible Qtrees evoked by the assertion (73b)=*Jo grew up in France*.

Before moving on to "compositional" Qtrees, let us take stock.

First, the recipe in (93) defines way to determine which parse of the CS assertive sentences evoke. We have discussed in Chapter 1 that questions typically correspond to partitions of the CS *in the pragmatic domain*. Semantically, questions are taken to be sets of alternatives. In that sense, Qtrees evoked by sentences should be understood as a form of "inquisitive pragmatics" rather than "inquisitive semantics". Tiered sets of propositional alternatives may be closer to the latter concept.

[15]Of course, if we had considered continent-level alternatives as well, principle (93iii) would have generated an extra Qtree for (73b), characterized by a continent-layer on top of a country-layer. But this would have led us to do the same move for (73a), and thus to generate the Qtree in Figure P for that sentence.

Second, the recipe in (93) typically generates *multiple* Qtrees out of one assertion. Under our current assumptions, (73a) gives rise to three possible Qtrees, and (73b), to two. So there is some degree of uncertainty about which Qtree any given sentence actually answers. Very roughly, evoked Qtrees can be "polar" (principle (93i)), "*wh*" (principle (93ii)), or "*wh*-articulated" (principle (93iii)). This optionality contrasts with frameworks like inquisitive semantics, in which any given sentence is mapped to a single nonempty downward-closed set of propositions. Given this, our recipe generates more Qtrees than intuitively assumed in the previous Sections. Additionally, this leads us to define the oddness of a sentence as equivalent to the oddness of *all* sentence-Qtree pairs to sentences can generate. This was already defined in (76), repeated below.

(76) *Oddness of a sentence.* A sentence S is odd if any Qtree T it evokes is odd given S.

Third, we mentioned that (73a) and (73b), beyond the fact that they are obviously in a relation of logical entailment, are such that (73a) feels more "fine-grained" than (73b). This is somehow cashed out by the kind of Qtrees these sentences evoke. Specifically, we observe that some Qtrees (73a) evokes (Figure O3) constitute refinements of some Qtree (73b) evokes (Figure Q2), where refinement is defined as in (72), repeated below. This implication does not hold in the opposite direction: no Qtree (73b) evokes, constitutes a refinement of a Qtree (73a) evokes. So (in a very weak sense) finer-grained assertions evoke finer-grained Qtrees. This observation will be crucial in Chapter **??**.

(72) *Qtree refinement.* Let T and T' be Qtrees. T is a refinement of T' (or: T is finer-grained than T'), iff T' can be obtained from T by removing a subset $\mathcal{T}$ of T''s subtrees, s.t.:

- if $\mathcal{T}$ contains a subtree rooted in N, then, for each node N' that is a sibling of N in T, the subtree of T rooted in N', is also in $\mathcal{T}$.

We now proceed to define Qtree for complex sentences belonging to the $\{\neg, \vee, \rightarrow\}$-fragment of the language. We will do so inductively, using our recipe for simplex sentences (93) as base case, along with specific combination rules corresponding to the inquisitive effect of each operator.

2.4 Compositional Qtrees: inductive step

In the previous Section, we have seen how to derive Qtrees from simplex sentences, containing no operator or connective. In this Section, we clarify how complex sentences, that may be equally informative, and may even have same propositional meaning, may end up packaging information differently from one another, in terms of their evoked Qtrees. This difference in information packaging, will allow us to derive different felicity profiles for these sentences. We start with Qtrees evoked by negated LFs, before moving on to Qtrees evoked by disjunctions and conditionals.

2.4.1 Questions evoked by negated LFs

We assume negated LFs evoke questions that are structurally similar to those evoked by their non-negated counterpart. The only difference resides in the set of verifying nodes, which is "flipped" by negation. This is formalized in (100).[16]

(100) *Qtrees for negated LFs.* Let T be a Qtree evoked by a LF X. A Qtree $T_\neg$ for $\neg X$ is obtained from T by:

- retaining T's structure; i.e. if $T = (\mathcal{N}, \mathcal{E}, R)$, then $T_\neg = (\mathcal{N}, \mathcal{E}, R)$, too;
- defining $T_\neg$'s set of verifying nodes $\mathcal{N}^+(T_\neg)$ as the set of $T_\neg$'s nodes N that are not verifying in T ($N \notin \mathcal{N}^+(T)$) but belong to a layer containing at least one verifying node N' in T ($N' \in \mathcal{N}^+(T)$). In other words:
  $\mathcal{N}^+(T_\neg) = \{N \mid N \notin \mathcal{N}^+(T) \wedge \exists N' \in \mathcal{N}^+(T).\ d(N, T_\neg) = d(N', T_\neg)\}$
  With $d(N, T)$ the depth of a node N in a tree T (see (60b)).[17]

The recipe in (100) is exemplified in the abstract Qtrees in Figure R.

[16] This approach is perhaps a bit naive; uttering p vs. $\neg p$, does not seem to preferentially answer the same kind of question. More specifically, it seems that uttering negative statements in general conveys the idea that the original question was more likely to be a polar question of the form *whether p?* – as opposed to a *wh* kind of question. We discuss this more in depth in Chapter 7.

[17] Because T and $T_\neg$ have same structure, it does not matter which Qtree among T and $T_\neg$ is passed as argument to the depth function; in that particular case, $\forall N \in \mathcal{N}.\ d(N, T) = d(N, T_\neg)$.

(1) An abstract Qtree for X

(2) An abstract Qtree for $\neg X$, derived from Figure R1.

Figure R: An abstract Qtree for X and the abstract Qtree for $\neg X$ derived from it, *via* (100).

It may not seem obvious at this point why and how verifying nodes would occur at intermediate levels in a Qtree; after all, all the Qtrees we have seen so far (derived from simplex sentences) had their verifying nodes at the leaf level. But we will see that Qtrees derived from complex sentences (typically, involving disjunctions and conditionals) can in principle feature intermediate verifying nodes, because such nodes are also derived compositionally. Now, granted that verifying nodes may indeed occur at different levels, the intuition behind the "flipping" algorithm in (100) is the following. If a node N is verifying in a Qtree T corresponding to an LF X, and N is located at depth k in T, then somehow the k-layer of T is "addressed" by X. We aim for a pair $(\neg X, T_\neg)$ to address the same layers as (X, T), so $T_\neg$'s verifying nodes should have similar a similar depth distribution as T's verifying nodes. But of course, the two sets of nodes need to be distinct, because negation standardly flips truth values – hence the by-layer flipping.

It is additionally worth mentioning that, if all verifying nodes in the original Qtree T are leaves, (100) is simplified: $T_\neg$'s set of verifying nodes is simply the set of leaves in $T/T_\neg$ that are not verifying in T. This is summarized in (101).

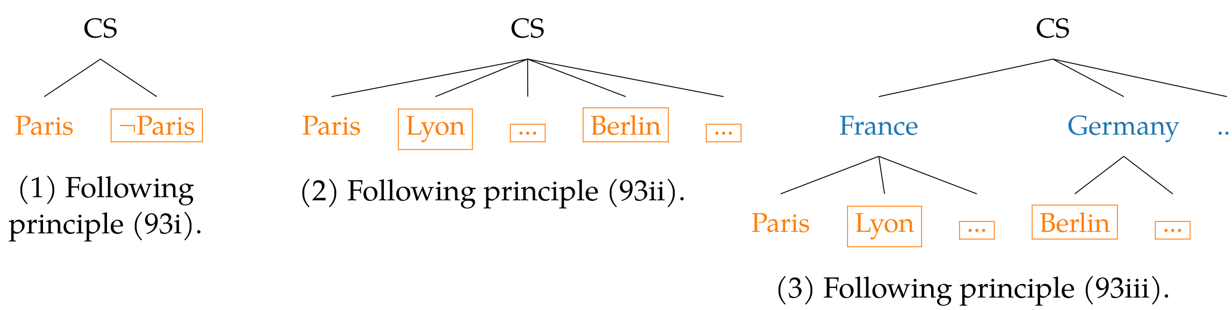
(101) *Qtrees for negated LFs* (*leaf-only version, subcase of* (*100*)). Let T be a Qtree evoked by a LF X s.t. $\mathcal{N}^+(T) \subseteq \mathcal{L}(T)$, where $\mathcal{L}(T)$ refers to T's leaves. A Qtree $T_\neg$ for $\neg X$ is obtained from T by:

- retaining T's structure;
- defining $T_\neg$'s set of verifying nodes as the complement set of $\mathcal{N}^+(T)$ within $\mathcal{L}(T)$: $\mathcal{N}^+(T_\neg) = \{N \in \mathcal{L}(T) \mid N \notin \mathcal{N}^+(T)\}$

Following this simplified recipe, Qtrees for (102), which correspond to the negation of (73a), are given below. They are obtained from Figure O, by simply flipping boxed nodes at the leaf level. These new Qtree capture the intuition that (102) can answer three kinds of question: a question about whether or not Jo grew up in Paris; a question about which city Jo grew up in; and a question about which city Jo grew up in question of which city Jo grew up in, but such that this question is decomposed into two subquestions: first, which country Jo grew up in; then, knowing the country, which city Jo grew up in, in that country. The nodes that get flagged as verifying, correspond to sets of worlds disjoint from $\lambda w.$ *Jo grew up in Paris in* w. Interestingly, negation preserves Qtree granularity, simply because it preserves Qtree structure.

(73a) Jo grew up in Paris.

(102) Jo did not grow up in Paris.



(1) Following principle (93i).

(2) Following principle (93ii).

(3) Following principle (93iii).

Figure S: Possible Qtrees evoked by the assertion (102)=*Jo did not grow up in Paris*.

2.4.2 Questions evoked by disjunctive LFs

Let us consider the disjunction in (103). Intuitively, this sentence is a good, non-maximal answer to a question like (48a), repeated below. It identifies two cities in which Jo could have grown up in, and conveys ignorance about which city Jo actually grew up in. Note that either disjunct taken in isolation, *Jo grew up in Paris*, or *Jo grew up in Lyon*, constitutes a *maximal* answer to (48a).

(48a) In which city did Jo grow up?

(103) Jo grew up in Paris or Lyon.

This observation is consistent with the idea that, in a felicitous disjunction, both disjuncts must answer the same kind of question (Simons, 2001; Zhang, 2022). Ou rephrasing of this observation is spelled out in (104).

(104) *Disjunctive answer.* Let $X = Y \vee Z$ be a disjunctive LF. If X is a felicitous assertion, then the set of questions Y answers is equal to the set of questions Z answers. Additionally, if Y/Z answer a question, then X answers it too.

A way to further specify this intuition in our model, is to assume that a Qtree for $X = Y \vee Z$, must contain a Qtree for Y and a Qtree for Z. Containment is understood as the subgraph relation (defined in (71)). This ensures that any node in Y's Qtree is also in X's Qtree, and any node in Z's Qtree, is also in X's Qtree. So, whatever answers Y or Z, also answers X. This is modeled by assuming that the Qtrees evoked by a disjunction are all the possible well-formed unions of Qtrees evoked by each disjunct. This is spelled out in (105). In this definition, Qtree union builds on the notion of graph-union, as formalized in (106).[18] On top of this, Qtree union involves the union of verifying nodes, and the determination of a root node for the output Qtree, defined as the maximum between the two roots of the input Qtrees.

(105) *Qtrees for disjunctive LFs.* A Qtree $T_\vee$ for $X \vee Y$, if defined, is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

- graph-unioning T_X and T_Y;
- defining $T_\vee$'s root as the maximal element (i.e. the weaker proposition) between the root of T_X and the root of T_Y. This will typically be the entire CS. If there is no such maximum, then the output cannot be a Qtree. [19]
- defining $T_\vee$'s verifying nodes as the union of T_X's and T_Y's verifying nodes: $\mathcal{N}^+(T_\vee) = \mathcal{N}^+(T_X) \cup \mathcal{N}^+(T_Y)$.
- returning the output only if it is a Qtree.

In other words, $Qtrees(X \vee Y) = \{T_X \cup T_Y \mid T_X \cup T_Y \text{ verifies (52)} \wedge (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y)\}$

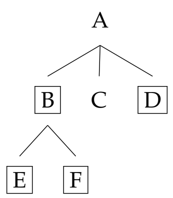
(106) *Graph union.* Let $G = (\mathcal{N}, \mathcal{E})$ and $G' = (\mathcal{N}', \mathcal{E}')$ be two graphs. The union of G and G', noted $G \cup G'$, is the graph $G'' = (\mathcal{N}'', \mathcal{E}'')$ s.t.:

[18] I thank Amir who helped me see this.

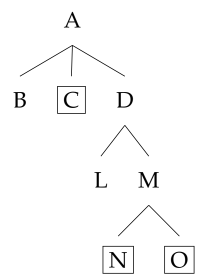
[19] Indeed, suppose R_X and R_Y are the roots of respectively T_X and T_Y, and that R_X and R_Y are not in any kind of inclusion relation. We show by contradiction that $T_X \cup T_Y$ cannot be a Qtree. If $T_X \cup T_Y$ were a Qtree, then, R_X and R_Y would not be in an ancestry relation, meaning, R_X would not be an ancestor of R_Y, and R_Y would not be an ancestor of R_X. So, neither R_X nor R_Y could be the root of $T_X \cup T_Y$, because the root is an ancestor of all the other nodes. Let's call R this root. R is a common ancestor of both R_X and R_Y in $T_X \cup T_Y$. So R must be a strict superset of R_X and R_Y. Also, because $T_X \cup T_Y$ is obtained *via* node- and edge-union, we must have, in the input Qtrees: $R_X \overset{T_X}{\rightsquigarrow} R$ and $R_Y \overset{T_Y}{\rightsquigarrow} R$. In other words, R_X is an ancestor of R in T_X, and R_Y is an ancestor of R in T_Y. Because T_X and T_Y are Qtrees, this implies that R is a strict subset of R_X, and also strict subset of R_Y. Contradiction.

- $\mathcal{N}'' = \mathcal{N} \cup \mathcal{N}'$
- $\mathcal{E}'' = \mathcal{E} \cup \mathcal{E}'$

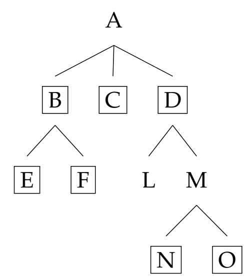
Figure T below exemplifies Qtree union applied to two abstract Qtrees, represented in Figures T1 and T2. In these Qtrees, nodes with different labels are assumed to correspond to a different propositions. By definition, $\{B, C, D\}$ partitions A; $\{E, F\}$ partitions B, $\{L, M\}$ partition D, and $\{N, O\}$ partition M. The disjunction of Figures T1 and T2 is shown in Figure T3. Nodes, edges, and verifying nodes, are unioned, and the output is a Qtree, that contains the two input Qtrees. So, whatever answered either Qtree in Figures T1 and T2, also answers their disjunction in Figure T3.



(1) An abstract Qtree for X



(2) An abstract Qtree for Y.



(3) An abstract Qtree for $X = Y \vee Z$, derived from Figures T1 and T2.

Figure T: Successful attempt at deriving a Qtree from the union of two Qtrees.

It can be shown that if a disjunctive Qtree $T_\vee$ is well-formed and results from the union of two Qtrees T and T' sharing the same root, $T_\vee$ will always constitute a refinement of both T and T'.

What about cases in which the union of two Qtrees, is not a well-formed Qtree? A prediction of (105) is that two Qtrees sharing the same root can be properly disjoined iff they do not involve a common node that gets partitioned in two different ways in the two different input Qtrees.[20] We call this problematic configuration a partition "clash" (or simply a clash). It is formally defined in (107), and related to disjoinability in (108).

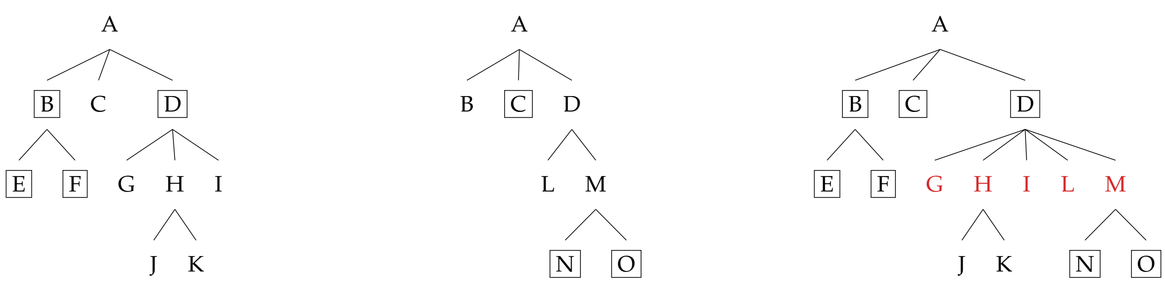
[20]We show that if T and T' exhibit such a clash, their disjunction is not a Q-tree. Let's call C and C' the sets of nodes of resp. T and T' that induce a bracketing clash; by assumption, C and C' are s.t. $C \neq C'$, and have mothers N and N' s.t. $N = N'$. Because $\vee$ achieves graph-union, $T \vee T'$ will have a node N with $C \cup C'$ as children, and because $C \neq C'$, $C \cup C' \supset C, C'$. Given that both C and C' are partitions of N, $C \cup C'$ cannot be a partition of N. Conversely, if two Q-trees T and T' sharing the same CS as root are s.t. their union $T \cup T'$ is not a Qtree, it must be because T and T' had a bracketing clash. Indeed, under those assumptions, $T \cup T'$ not being a Qtree means one node N in $T \cup T'$ is not partitioned by its children. Given N is in $T \cup T'$, N is also in T, T', or both. If N was only in, say, T, then it means N's children are also only in T, but then, T itself would have had a node not partitionned by its children, contrary to the assumption T is a Qtree. The same holds *mutatis mutandis* for T', so, N must come from *both* T and T'. Let us call C and

(107) *Partition clash.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}', R')$ be two Qtrees. T and T' feature a partition clash iff there is $N \in \mathcal{N}$ and $N' \in \mathcal{N}'$ s.t. $N = N'$ but the sets of children of N and N' differ.

(108) *Partition clashes and Qtree disjoinability.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}', R)$ be two Qtrees. T and T' are disjoinable (i.e., their union is a well-formed Qtre) iff T and T' do not exhibit any partition clash.

So, under a recursive interpretation of nodes, two Qtrees with the same root can be disjoined iff, for each node N present in both Qtrees, N's recursive interpretation is the same across Qtrees, or one interpretation constitutes a refinement of the other. This means that, to be disjoinable Qtrees should not introduce different subquestions at the local level.

Figure U illustrates a degenerate case of Qtree union, arising from a partition clash between two abstract input Qtrees. The two input Qtrees, represented in Figures U1 and U2, minimally differ from those in Figures U1 and U2: Figures T2 and U2 are the same, but, in Figure U1, $\{G, H, I\}$ are extra nodes that partition D, and $\{J, K\}$ partitions H. The "clash" between the Qtrees in Figures U1 and U2 comes from the $\{G, H, I\}$ nodes in Figure T1 and the $\{L, M\}$ nodes in Figure T2: these two sets partitions node D in different ways. As a result, the union of these two sets of nodes *cannot* partition D. Figure U3, which represents the disjunction of Figures U1 and U2, thus features nodes $\{G, H, I, L, M\}$ as children of node D, and this configuration violates the partition property of Qtrees. This prevents the tree in Figure U3 from being a well-formed Qtree.



(1) An abstract Qtree for X

(2) An abstract Qtree for Y.

(3) An abstract Qtree for $X = Y \vee Z$, derived from Figures U1 and U2.

Figure U: Unsuccessful attempt at deriving a Qtree from the union of two Qtrees exhibiting a bracketing clash.

The badness of this kind of configuration, captures the intuition that two disjoined

C' the partitioning introduced by N in resp. T and T'. The fact C, C', but not $C \cup C'$ partition N entails $C \neq C'$, i.e. T and T' feature a bracketing clash.

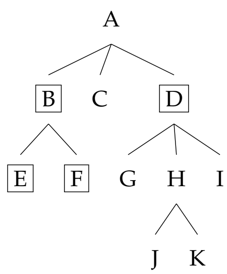
Qtrees should not raise orthogonal issues locally. We call two issues (partitions) orthogonal if they involve two nodes/cells that strictly overlap; see (109). This definition can be shown to be equivalent to that of a partition clash,[21] It is interesting, because it can be more directly related to some concept of Relevance discussed in Chapter 1; see (110).

(109) *Orthogonal partitions.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}, R)$ be two depth-1 Qtrees sharing the same root R (equivalently, two partitions of the same CS). T and T' are orthogonal iff they involves two nodes that are strictly overlapping, i.e. $\exists (N, N') \in \mathcal{N} \times \mathcal{N}'.\ N \cap N' \neq \emptyset \wedge N \neq N'$. T and T' are orthogonal iff T and T' exhibit a partition clash.

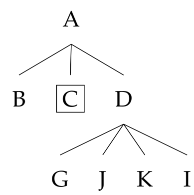
(110) *Orthogonal partitions and relevance.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}, R)$ be two depth-1 Qtrees sharing the same root R. T and T' are orthogonal iff some maximal answer (leaf) of T is not Lewis-Relevant to T'.

Figure V illustrates yet another degenerate case, that may seem more subtle when looking at the two input Qtrees, but with more drastic consequences when looking at the output structure, that is not even a tree. In this example, the two input Qtrees, represented in Figures V1 and V2 clash again at the level of the D node: both $\{G, H, I\}$ $\{G, J, K, I\}$ partition D, but in different ways, since the latter partition is finer grained ($\{J, K\}$ partitions H). This kind of clash, though subtle, generates a disjunctive Qtree that is not even a tree: in Figure V3, J/K is connected to D *via* two distinct paths: directly, and *via* H. So Figure V3 is not acyclic. Zooming out, this degenerate configuration stems from the fact that Qtree union "collapsed" the J and K nodes from the two input Qtrees, and that these nodes, being located at different levels in the two Qtree, were connected differently to the other nodes. This example outlines the idea that, in order to be disjoinable, two Qtrees must match in terms of their layering, i.e. in terms of their degrees of granularity.

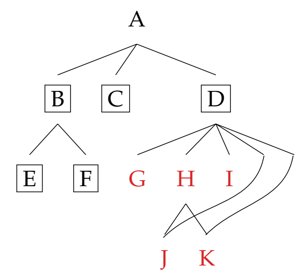
[21] Let us show that if two partitions are different (i.e. involve different cells), then, there is one cell from the former partition and one cell from the latter partition that strictly overlap. Let us assume two partitions P_1 and P_2 are distinct. We show that there is a cell in P_1 and a cell in P_2 that strictly overlap. We consider P_1' and P_2' the partitions obtained from P_1 and P_2 by removing the cells P_1 and P_2 have in common. P_1' and P_2' are not empty, because otherwise P_1 and P_2 would be identical. Moreover, there must be 2 cells c_1 and c_2 in P_1' and P_2' that overlap, because P_1' and P_2' are partitions and as such must be fully covered by their cells. Moreover, c_1 and c_2 cannot be the same, otherwise, they would not be in P_1' and P_2' by construction. So c_1 and c_2 strictly overlap. The other direction of the proof is trivial: if two partitions of the same space P_1 and P_2 involve two strictly overlapping cells, then these two cells must be distinct, and so P_1 and P_2 must be different sets.



(1) An abstract Qtree for X



(2) An abstract Qtree for Y.



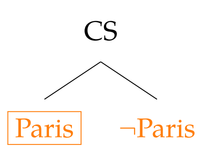
(3) An abstract Qtree for $X = Y \vee Z$, derived from Figures V1 and V2.

Figure V: Yet another unsuccessful attempt at deriving a Qtree from the union of two Qtrees exhibiting a bracketing clash.

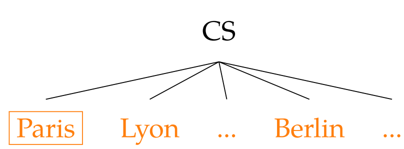
Now that we have defined how disjunctive Qtrees are formed and what the well-formedness conditions for such trees are, we come back to our more concrete disjunctive example (103), repeated below.

(103) Jo grew up in Paris or Lyon.

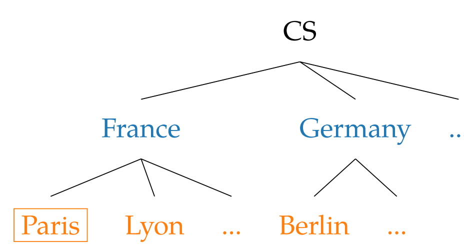
To derive the Qtrees evoked by this disjunctive LF, one must first derive the Qtrees evoked by its two disjuncts, abbreviated *Paris* and *Lyon*. This has been done already in Figure O (repeated in Figure W) for *Paris*. Additionally, *Paris* and *Lyon* have same granularity, and therefore, give rise to the same tiered set of propositional alternatives. This in turn ensures that both *Paris* and *Lyon* give rise to similar Qtrees, that mostly differ in terms of their verifying nodes: *Paris* will flag *Paris*-nodes, and *Lyon*, *Lyon*-nodes. The Qtrees evoked by *Lyon* can be found in Figure X.



(1) Following principle (93i).

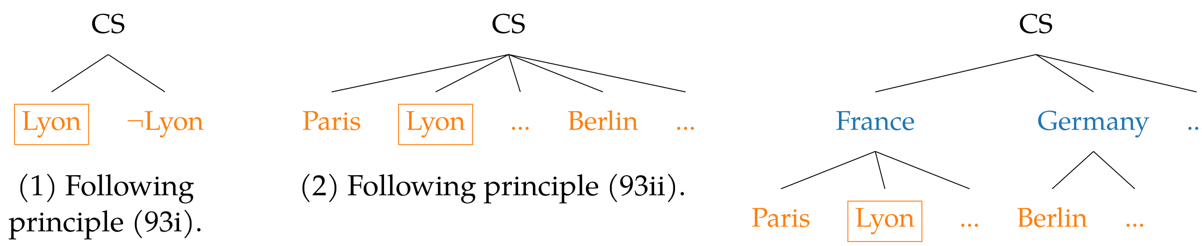


(2) Following principle (93ii).



(3) Following principle (93iii).

Figure W: Possible Qtrees evoked by the assertion (73a)=*Jo grew up in Paris*.



(1) Following principle (93i).

(2) Following principle (93ii).

(3) Following principle (93iii).

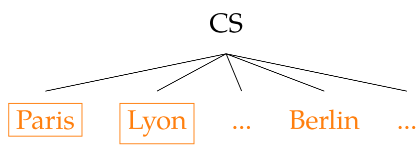
Figure X: Possible Qtrees evoked by the assertion *Jo grew up in Lyon*.

We could now compute all possible unions of the Qtrees in Figures W and X, and retain those that are Qtrees. This would effectively yield the Qtree evoked by (103). But instead of computing all these unions, let us use the notion of a partition clash to retain the input Qtrees that will in fact give rise to well-formed disjunctive Qtrees. We now evaluate all pairs of Qtrees from Figures W and X for partition clashes, and compute unions only if no clash is detected.

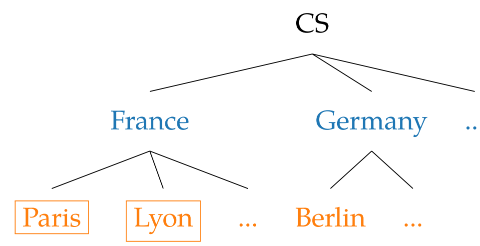
Starting with the two "polar" Qtrees W1 and X1, we notice an obvious clash between the 2-cells partitions $\{Paris, \neg Paris\}$ and $\{Lyon, \neg Lyon\}$. So we can ignore the union of these two Qtrees. Qtrees W1 and X2 also clash, because $\{Paris, \neg Paris\}$ and $\{Paris, Lyon...\}$ are different partitions. Again, we ignore this combination. Same holds for Qtrees W1 and X3, because $\{Paris, \neg Paris\}$ and $\{France, Germany...\}$ are different. We thus once again ignore this combination. From this, we conclude that the "polar" Qtree for *Paris* W1 is not disjoinable with any Qtree *Lyon* evokes. Reciprocally, the "polar" Qtree for *Lyon* X1 is not disjoinable with any Qtree *Paris* evokes.

Moving on to the "*wh*" Qtree for *Paris* W2, it is structurally identical to the "*wh*" Qtree for *Lyon* X2. Therefore, these two Qtrees do not clash, and can de disjoined. Their union is given in Figure Y1. Because the two input Qtrees are structurally identical, the structure of the disjunctive output Qtree is also similar. The only difference between inputs and output, is that the nodes flagged as verifying by the output Qtree, are both the *Paris* and the *Lyon* node. Considering now the "*wh*" Qtree for *Paris* W2, and the "*wh*-articulated" Qtree for *Lyon* X3, we notice yet another partition clash: $\{Paris, Lyon, ...\}$ is different from $\{France, Germany, ...\}$. So these two Qtrees cannot be disjoined. Reciprocally, the "*wh*" Qtree for *Lyon* X2, and the "*wh*-articulated" Qtree for *Paris* W3, will not be disjoinable.

This leaves us with one last pair to evaluate, namely the pair made by the two "*wh*-articulated" Qtrees in Figures W3 and X3. These two Qtrees are structurally identical, and so can be disjoined. The result of their union is given in Figure Y2. Because the two input Qtrees are structurally identical, the output is also similar. The only difference between inputs and output, is that the nodes flagged as verifying by the output disjunctive Qtree, are both the *Paris* and the *Lyon* node.



(1) Qtree W2 $\vee$ Qtree X2



(2) Qtree W3 $\vee$ Qtree X3

Figure Y: Possible Qtrees evoked by the assertion (103)=*Jo grew up in Paris or Lyon*.

Figure Y capture the idea that a disjunction like (103), evokes the same Qtrees as the *wh*-question *In which city did Jo grow up?*: either a simple "*wh*" Qtree partitioning the CS according to cities, or a more complex "*wh*-articulated" Qtree corresponding to the question of which city Jo grew up in, but such that this question is decomposed into two subquestions: first, which country Jo grew up in; then, knowing the country, which city Jo grew up in, in that country.[22] We will see more examples of Qtree disjunctions in the next Chapters, including pathological cases in which the disjuncts may not share the same degree of specificity. We now proceed to define Qtree evoked by conditionals. Crucially, the way such Qtrees will be defined, will not be a function of the "recipes" we just devised for negated and disjunctive LFs. In other words, conditional Qtrees will not be "material".

2.4.3 Questions evoked by conditional LFs

Material implication, defined in (111) is perhaps the simplest way to analyze natural language conditionals.

(111) *Material Implication.* Let X and Y be two LFs denoting p and q respectively. Under the material analysis, $[\![$ *If* X *then* $Y]\!]$ is true iff $\neg p \vee q$ is true. $\rightarrow$ is used as a shorthand for $\lambda p.\ \lambda q.\ \lambda w.\ \neg p \vee q$, s.t. $\neg p \vee q \equiv p \rightarrow q$.

It may be tempting to adapt this definition to the domain of Qtrees evoked by assertions. This tentative translation is given in (112).

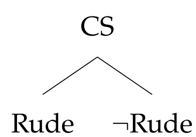
[22]One might wonder at this point why a condition on Qtree disjoinability should not involve structural equality between inputs. After all, the two Qtrees we just derived, depicted in Figure Y, were associated with structurally identical inputs. Chapter **??** will discuss why this identity condition might be too strong, on top of being stipulative.

(112) *"Material" Conditional Qtrees.* Let X be an LF of the form *If Y then Z*. A Qtree for X is a Qtree for $\neg Y \vee Z$.

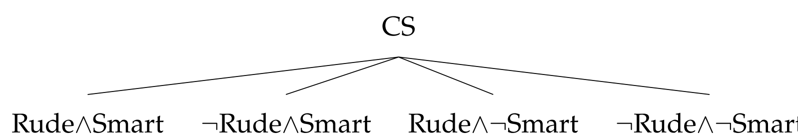
Because we defined Qtrees for negated and disjunctive LFs in the previous Sections, we already have the tools to understand what (112) would predict for Qtrees evoked by natural language conditionals. In particular, we noted that negation preserves Qtree structure, and that disjunction forces the two disjuncts to evoke structurally similar Qtrees. These properties combined, predict that, under (112) the antecedent and consequent of a conditional, should evoke similar Qtrees, devoid of any partition clash. In other words, two Qtrees evoked by X and Y should be "conditionalizable" (in the material sense) iff they are disjoinable. This does not seem to match intuitions about conditionals. (113a) for instance, sounds fine, even if the antecedent *Jo is rude* and the consequent *Jo grew up in Paris*, appear to evoke Qtrees with very different structures. The previous Section already detailed what the latter Qtrees for *Jo grew up in Paris* should looks like, and Figure Z sketches how Qtrees for *Jo is rude* should look like. Clearly, partitions of the CS induced by personality traits, are unlikely to match partitions induced by countries, so under the material analysis, a sentence like (113a) should behave exactly like (113b) at the inquisitive level. Therefore, it should not give rise to any Qtree and should be deemed odd.

(113) a. If Jo is rude, she grew up in Paris.

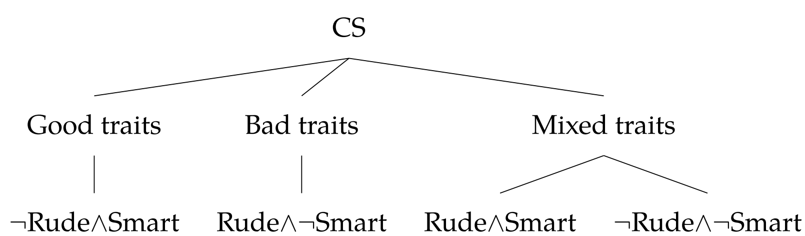
b. # Jo is not rude, or she grew up in Paris.



(1) Following principle (93i).



(2) Following principle (93ii).



(3) Following principle (93iii).

Figure Z: Possible Qtrees evoked by the assertion *Jo is rude*.

This empirical difference between conditionals and disjunctions regarding the questions they evoke, motivates a non-material model of conditionals at the inquisitive level.

Intuitively, what a conditional statement like (113a) seems to convey, is that figuring out Jo's rudeness may help narrow down where Jo grew up. So, (113a) seems to primarily answer a question about where Jo grew up, taking for granted that she is a rude person. This introduces an asymmetry between antecedent and consequent; it seems that the question evoked by the consequent gets *restricted* to the CS updated with the antecedent. A Qtree for (113a) would then look like the one in Figure AA. In this tree, the Qtree corresponding to the consequent, is "plugged" into the node corresponding to the *Jo is rude* worlds.

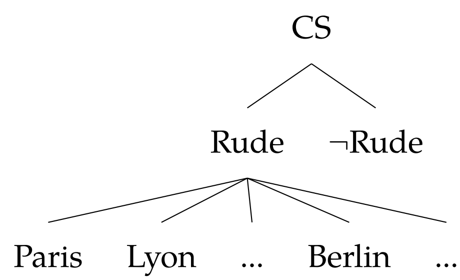


Figure AA: An intuitive Qtree for (113a)= *If Jo is rude, she grew up in Paris*.

We already have the tools to cash out this intuition in a compositional way. Recall that in Section 2.2.3, we discussed how a subtree rooted in N in a given Qtree, could be interpreted as the intersection between the entire Qtree and N, following (66). The definition of this operation is repeated below.

(66) *Tree-node intersection.* Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a Qtree. Let p be a proposition. The tree-node intersection between T and p, noted $T \cap p$, is defined iff $R \cap p \neq \emptyset$ and, if so, is the Qtree $T' = (\mathcal{N}', \mathcal{E}', R')$ s.t.:

- $\mathcal{N}' = \{N \cap p \mid N \in \mathcal{N} \wedge N \cap p \neq \emptyset\}$
- $\mathcal{E}' = \{\{N_1 \cap p, N_2 \cap p\} \mid \{N_1, N_2\} \in \mathcal{E} \wedge (N_1 \cap p) \neq (N_2 \cap p) \wedge N_1 \cap p \neq \emptyset \wedge N_2 \cap p \neq \emptyset\}$
- $R' = R \cap p$

Tree-node intersection, seen as a form of contextual restriction, in fact allows to "plug" specific Qtrees into the node(s) of another Qtree – producing an output that is still a well-formed Qtree. This gives rise to the definition of conditional Qtrees in (114). This definition defines a Qtree evoked by a conditional *If* X *then* Y, as a Qtree for X whose verifying nodes get replaced by their intersection with a Qtree evoked by Y.

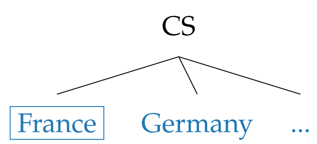
(114) *Qtrees for conditional LFs.* A Qtree T for $X \rightarrow Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

- replacing each node N of T_X that is in $\mathcal{N}^+(T_X)$ with $N \cap T_Y$ (see (**??**));
- returning the result only if it is a Qtree.

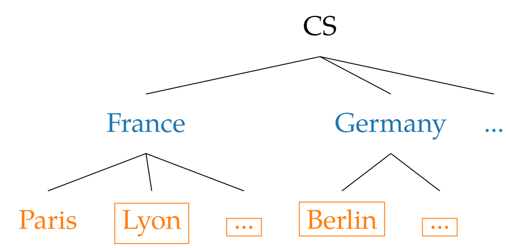
In other words, $Qtrees(X \rightarrow Y) = \{T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)} (N \cap T_Y) | (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y) \wedge T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)} (N \cap T_Y)$verifies (52)$\}$, and $\mathcal{N}^+(T_X \rightarrow T_Y) = \{N \cap N' | (N, N') \in \mathcal{N}^+(T_X) \times \mathcal{N}^+(T_Y) \wedge N \cap N' \neq \emptyset\}$.

Let us see what (114) predicts for a conditional Qtree corresponding to (115), assuming the input Qtrees for the antecedent *France*, and the consequent *not Paris*, are those in Figure AB.[23]

(115) If Jo grew up in France, she did not grow up in Paris



(1) A Qtree for(73b)=*Jo grew up in France*, repeated from Figure Q2.

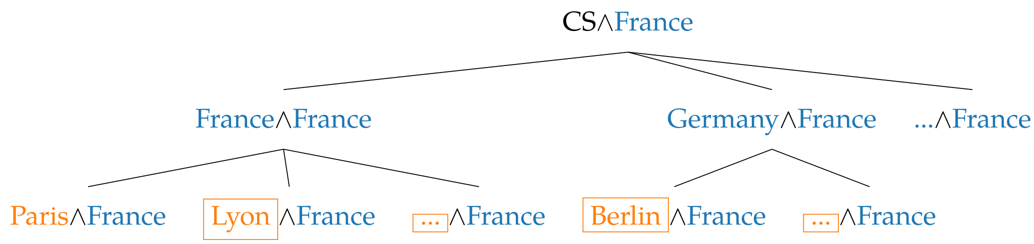


(2) A Qtree for (102)=*Jo did not grow up in Paris*, repeated from Figure S3.

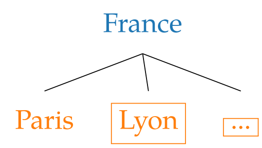
Figure AB: Qtrees corresponding to the antecedent and consequent of (115).

According to definition (114), the conditional Qtree generated from Figure AB, should be Figure AB1, whereby the *France* verifying node gets replaced by its intersection with Figure AB2. This intersection is computed in Figure, following the tree-node intersection intersection principle (66). The result of this operation, is the subtree of Figure AB2 rooted in *France*, in other words, the recursive interpretation of the *France*-node in Figure AB2 – in line with property (67).

[23]Of course, more Qtrees are available for the antecedent and the consequent, and each pairings should be considered when generating Qtrees corresponding to the conditional. We focus on one possible pairing here.



(1) Intersecting all nodes of Figure AB2 with *France*.



(2) Filtering out empty nodes and removing trivial edges.

Figure AC: A Qtree for (102)=*Jo did not grow up in Paris*, intersected with the *France*-node.

The Qtree in Figure AC, can then replace the original *France*-node of Figure AB1 (the antecedent Qtree), to create a Qtree for the conditional assertion (115). This is done in Figure AD.

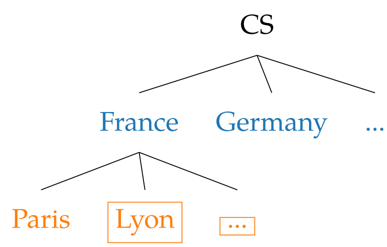


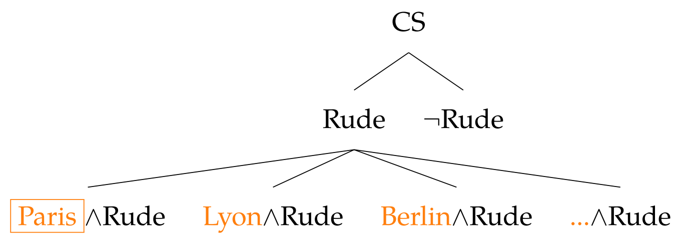
Figure AD: A Qtree for (73b)=*Jo grew up in France*, whose verifying node gets replaced by its intersection with a Qtree for (102)=*Jo did not grow up in Paris*. This creates a Qtree for (115)=*If Jo grew up in France, she did not grow up in Paris*.

It is worth noting that replacing the *France*-node by its intersection with a consequent Qtree in the above example, "erased" the verifying character of the *France*-node. In other words, the verifying nodes of a conditional Qtree, are inherited from its consequent only. The nodes that were verifying in the antecedent Qtree, but were not so in the consequent Qtree, are no longer verifying in the output conditional Qtree.

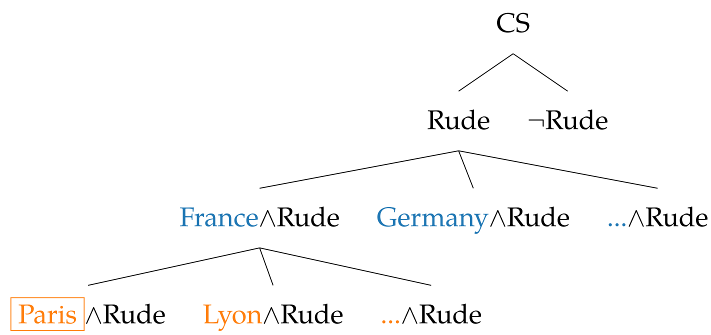
The core idea behind this operation is that conditionals do not make antecedent and consequent QuDs at issue at the same time; rather, they introduce a hierarchy between these two objects, by raising the consequent QuD only in the cells of the CS (as defined by the antecedent QuD), where the antecedent holds. Yet another way to phrase this is by saying that, through the process of Qtree-conditionalization, the consequent Qtree gets *re-*

stricted by the antecedent Qtree. This view is consistent with influential finding in psychology, showing that when asked to verify the truth of a conditional statement, participants tend to massively overlook the eventualities falsifying the antecedent (Wason, 1968) It is also consistent with insights from the recent linguistic literature, which argues that zero-models should be disregarding by the semantic module (Aloni, 2022) – a model where the antecedent of a conditional does not hold is an example of such a zero-model.

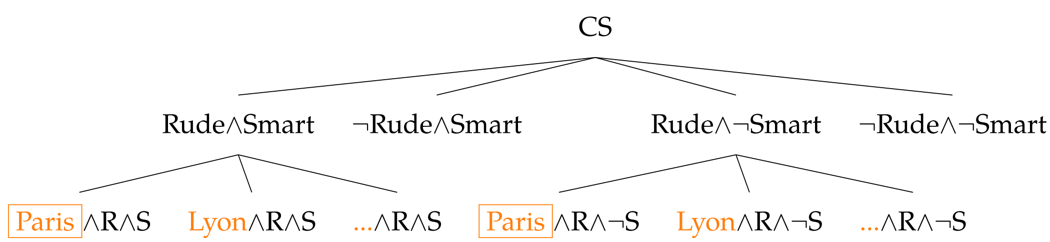
Coming back to the example that motivated this non-material analysis of conditionals at the inquisitive level, a Qtree for (113a), is given in Figure



(1) Conditional Qtree generated from Figure Z1 (antecedent) and O2 (consequent).



(2) Conditional Qtree generated from Figure Z1 (antecedent) and O3 (consequent)



(3) Conditional Qtree generated from Figure Z2 (antecedent) and O2 (consequent)

Figure AE: Possible conditional Qtrees corresponding to (113a)=*If Jo is rude, she grew up in Paris*. Other Qtrees are possible.

Lastly, we observe that node-Qtree intersection between a (non-empty) node N and a tree T, is "vacuous" (i.e., equal to N), iff N entails a specific leaf in T.[24]

[24]Vacuousness is defined structurally only. The verifying status of N will still depends on T's verifying

(116) *Vacuous tree-node intersection.* Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). $T \cap N = N$ iff $\exists N' \mathcal{L}(T).\ N \vDash N'$.

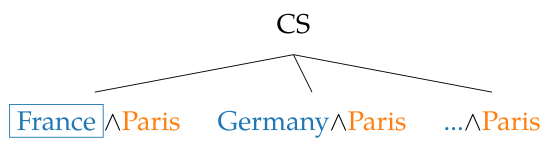
(117) *Proof of (116).* Let T be a Qtree and N a node entailing some leaf in T. T being a Qtree, each node of T is s.t. its ancestors are exactly the nodes entailed by it, its descendants are exactly the nodes entailing it, and other nodes are incompatible with it. If N entailed two or more leaves in T, then N would entail a contradiction, i.e. be empty. So N entails a single leaf L, and all the nodes in T entailed by N must correspond to a path from CS root, to L. Intersecting all such nodes with N, yields N. Intersecting N with any other node, yields the empty set. Therefore, intersecting N with T leads to a single N-root.
Let T be a Qtree and N a node s.t. $T \cap N = N$. We show that N entails some leaf in T by contradiction. If no leaf in T were entailed by N, then at least some leaf in T, when intersected with N, would yield a node different from N. Therefore, $T \cap N$, cannot be equal to N. Contradiction.

The whole conditional Qtree formation process will then be vacuous if each verifying leaf in the antecedent Qtree entails a specific leaf of the consequent Qtree. This configuration is exemplified when considering a sentence like (118).

(118) ?? If Jo grew up in Paris, she grew up in France.

In that sentence, Qtrees corresponding to the antecedent stop at the city-level and mark the *Paris*-node as verifying. Qtrees corresponding to the consequent stop at the country-level, and contain a *France*-leaf. Because *Paris* entails *France*, intersecting a Qtree for the consequent with the "restrictor" *Paris*-node from the antecedent, will simply yield the Paris node. This is shown in Figure AF. In that case, the node resulting from the intersection operation inherits its verifying status from the *France*-node, flagged by the input (consequent) Qtree. Replacing the verifying *Paris*-node in the antecedent Qtree by this node, does not have any effect. Therefore, the output conditional Qtree will be identical to the antecedent Qtree.

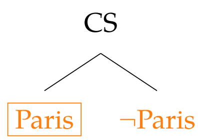
nodes.



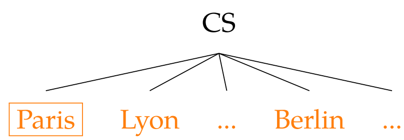
(1) Intersecting all nodes of Figure Q2 with *Paris*.

(2) Filtering out empty nodes and removing trivial edges.

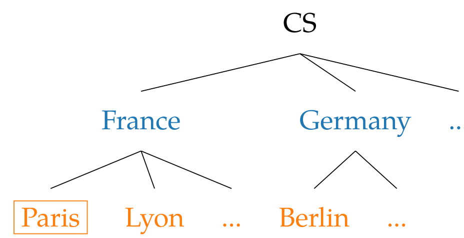
Figure AF: A Qtree for (73b)=*Jo studied in France*, intersected with the *Paris*-node, is just the *Paris*-node itself.



(1) .



(2) .



(3) .

Figure AG: Possible Qtrees evoked by the conditional assertion (118)=*If Jo grew up in Paris, she grew up in France*. Same as those evoked by (73a)=*Jo grew up in Paris*, due to tree-node intersection being vacuous across-the-board.

Moreover, if each verifying leaf in the antecedent Qtree entails a specific *non-verifying* leaf of the consequent Qtree, the output Qtree will be structurally identical to the antecedent Qtree but, will be left with *no* verifying node. Such a tree will be deemed ill-formed as per principle (75) (pertaining to the empty labeling of verifying nodes).

2.5 Conclusion

In this Chapter, we have build a model of overt and evoked questions that incorporates the concept of specificity *within* the formalism. We have defined how this model derives implicit questions evoked by simple, negated, disjunctive and conditional sentences, in a compositional way. Doing so we attempted to maintain a few intuitions about the truth-conditional effect of negation, disjunction, and conditionals. For instance, negation "flips" verifying nodes as it would flip truth-values; disjunction forces some parallelisms between disjuncts; and conditional act as questions "restrictors".

This mode differs from inquisitive semantics (Mascarenhas, 2008; Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli et al., 2018) in at least two ways. First, Inquisitive Semantics proposes a *unified* view of questions and assertions at the semantic level, while what is propoed here, is a form of inquisitive *pragmatics*. Sentences are still assigned "standard" extensional/intensional meaning, but also have an inquisitive contribution at the pragmatic level, which, as we started to see, drives feelings of oddness (as opposed to falsity/contradiction). The second dimension along which our view differs from inquisitive semantics, is that it makes directional predictions about how sentences may partition the context set: first, the partitioning must be nested; and second, its specificities depends on how the sentence is constructed, with a clear contrast between disjunctions and conditionals, and the property that negation preserves Qtree structure. Inquisitive Semantics on the other hand, is less restricted in the sense an inquisitive proposition in that framework, is a nonempty downward-closed set of information states (sets of worlds), which may not constitute a nested partition/Qtree. Additionally, negation is assumed to flatten the original topology of the prejacent inquisitive proposition, by collecting the information states incompatible with it.

In fact, from a conceptual perspective, the machinery presented here may be closer in spirit to Dynamic Semantics (Heim, 1983; Haim, 1983), where different operators give rise to different incremental updates of the Context Set. Under our view, different operators will give rise to different *parses* of the Context Set, at the inquisitive level. This will eventually allow to capture a contrast between (45), (46) (see Chapter 4), and many other cases, including (115), (118) (see Chapter **??**).

Chapter 3

Comparison of the Qtree model to earlier similar approaches

Abstract. This Chapter consists in a literature review and compares the model of questions introduced in Chapter 2 to earlier approaches accounting for oddness phenomena *via* theories of questions or alternatives. It is shown that these earlier models differ from the current framework in three possible ways: (i) the core model is technically very similar, but at the conceptual level assertions are not taken to evoke full-fledged questions (Ippolito, 2019), or (ii) the machinery proposed *is* based on evoked QuDs but not fully compositional (Zhang, 2022), or (iii) question semantics is taken to fully *replace* standard propositional content (the Inquisitive Semantics framework).

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3.1 Inquisitive Semantics

inquisiyive semantics says that sentences are more or less questions, they raise issues. but paradox:sentences and qs are the same kind of thing, but then, sentences get impoverished to be made diff from qs eventually

3.2 Ippolito's contribution

Ippolito (2019) proposes a model of alternatives that is very close in its implementation to the Qtree model proposed in the first half of this Chapter. Under Ippolito's view, the way alternatives are structured is seen as a source of oddness. But, as a whole, the account will be shown to differ from ours in at least three respects: first, sentences are not taken to evoke full-fledged questions (a mainly conceptual difference); second, it leaves unexplained when, and how, sets of alternatives can be combined, cross-sententially and in biclausal sentences; third, under this view oddness arises from a purely structural constraint (the *Specificity Constraint*), that appears independent from familiar competition-based pragmatic principles. The current section will present the account and outline the first two differences. Chapter 4 will further clarify the third difference, by introducing a new, competition-based REDUNDANCY constraint on LF-Qtree pairs.

3.2.1 The data

Ippolito (2019)'s goal was to provide a unified analysis of a number of seemingly independent instances of pragmatic oddness, taking the form of Sobel sequences (119), sequences of superlatives (120), and Hurford Disjunctions (121).

(119) a. If the USA had thrown their nuclear weapons into the sea, there would have been war. But if all the nuclear powers had thrown their weapons into the sea, there would have been peace.

b. # If all the nuclear powers had thrown their nuclear weapons into the sea, there would have been peace. But if the USA had thrown their weapons into the sea, there would have been war.

(120) a. The closest gas stations are crummy; but the closest Shell stations are great.

b. # The closest Shell stations are great; but the closest gas stations are crummy.

(121) a. John ate some of the cookies or all of them.

b. # John ate all of the cookies or some of them.

These three families of sentences share commonalities. In all three configurations, two sentences or fragments are being contrasted using connectives like *but* and *or*. For instance, in the Sobel case (119a), *If the USA had thrown their nuclear weapons into the sea, there would have been war* gets contrasted with *If all the nuclear powers had thrown their nuclear weapons into the sea, there would have been peace*. Additionally, in all three cases, the two sentences being contrasted exhibit some degree of parallelism, in the sense that they each contain a subconstituent C/C^+, such that $[\![C^+]\!] \vdash [\![C]\!]$. For instance, *all the nuclear powers had thrown their nuclear weapons into the sea*, entails that *the USA had thrown their nuclear weapons into the sea*. Lastly, all configurations are such that the a. examples, which start with the sentences containing the "weaker" C, appear more felicitous than the b. examples, which start with the sentences containing the "stronger" C^+.

3.2.2 Structured Sets of Alternatives

To account for these asymmetries, Ippolito (2019) submits that the alternatives evoked by assertive sentences form "structured sets" (henceforth **SSA**). Such sets are defined in (122). The kind of structures generated by this definition are in essence recursive partitions of the CS, or Qtrees, as defined in (52).[1]

(122) *Structured Set of Alternatives* (*SSA*) (*Ippolito, 2019*). $T_{\mathcal{A}}$ is a well-formed structured set of alternatives iff the following conditions are met:

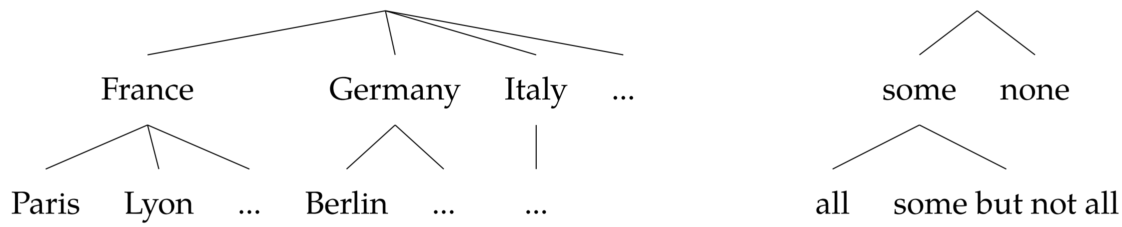
- Strength: for any two alternatives α, $\beta \in \mathcal{A}$, β is the daugther of α in $T_{\mathcal{A}}$ just in case $[\![\beta]\!] \subset [\![\alpha]\!]$.
- Disjointness: for any two alternatives β_1, $\beta_2 \in \mathcal{A}$, if β_1 and β_2 are sisters in $T_{\mathcal{A}}$, then $[\![\beta_1]\!] \cap [\![\beta_2]\!] = \emptyset$
- Exhaustivity: for any alternative α with daughters $\beta_1, ...\beta_n$, in $T_{\mathcal{A}}$, $[\![\beta_1]\!] \cup [\![\beta_2]\!] \cup ... \cup [\![\beta_n]\!] = [\![\alpha]\!]$

Alternatives evoked by an assertion are modeled following Rooth (1992), i.e. assumed to be obtained by substituting the original sentence's focused material by any expression of the same type. This is spelled out in (123).

(123) *Focus alternatives* (*Rooth, 1992*). Let S be a sentence containing a focused element α. The set of focus alternatives to $[\![S]\!]$ is the set of propositions $[\![S']\!]$, where S' is obtained from S by substituting α with any element of the same type as α.

Figure A illustrates SSAs for simple sentences containing scalar and non-scalar alternatives. It is worth noting that sentences associated with different degrees of granularity (e.g. *Jo grew up in Pairs* vs. *Jo grew up in France*) are not expected to give rise to different SSAs, as shown in Figure A1. Same holds for scalar sentences in an entailment relation (e.g. *Jo ate some of the cookies* vs. *Jo ate all of the cookies*).

[1] This is what at least is argued in Ippolito (2019). It is worth mentioning however, that the definition in (122) does not in itself guarantee that any Structured Set of Alternatives should form a tree. Instead, it guarantees that any branching of the form $[_{\alpha}\beta_1...\beta_n]$ is s.t. $(\beta_i)_{i\in[1;n]}$ partitions α. But nothing in principle guarantees the connectedness of the structure: if specific alternatives happen to be "missing" (for relevance/QuD-related reasons, or perhaps due to a missing lexicalization), then, the resulting Structured Set of Alternatives may end up being a forest, instead of a single tree.



(1) SSA associated with *Jo grew up in Paris$_F$/France$_F$/Italy$_F$* etc.

(2) SSA associated with *Jo ate some$_F$/all$_F$/none$_F$ of the cookies*.

Figure A: SSAs for simple focused sentences.

Additionally, alternatives are assumed to be constrained by "the" QuD. This constitutes the first, conceptual difference with our account introduced earlier in this Chapter: under Ippolito's view, assertions are not assumed to help determine "the" QuD; instead, they are assumed to evoke alternatives, which are themselves constrained by "the" QuD. In other words, SSAs are not expected to help determine what "the" QuD is–they are partially derived from it. This is far from an esoteric perspective, and appears in line with much past literature. What we want to propose instead, is the reverse perspective: assertions and their alternatives are the primitive, and help *derive* potential QuDs (along with contrasts in pragmatic oddness).

3.2.3 The Specificity Constraint

Ippolito (2019) then proposes that oddness arises from certain SSA configurations. In particular, sequences of sentences belonging to the same SSA are subject to a Specificity Constraint (henceforth **SC**), spelled out in (124). The SC states that the two alternatives in the sequence, should be dominated by the same number of nodes in their common SSA. This is equivalent to saying that two alternatives being contrasted should match in terms of their degree of specificity, or granularity.

(124) *Specificity Condition* (*Ippolito, 2019*). A sequence $\Sigma =< [_{S_i} ...\alpha_F...], [_{S_j} ...\beta_F...] >$, s.t. both S_i and S_j are answers to the same QuD and β is in the structured set of alternatives evoked by α ($T_{\mathcal{A}_\alpha}$), is felicitous if either:

- α or β is the only node on its branch in $T_{\mathcal{A}_\alpha}$, or
- α and β are dominated by the same number of nodes in $T_{\mathcal{A}_\alpha}$.

A sentence like (121b) then violates the SC, because its "*all*" and its "*some*" disjunct are respectively dominated by 2, and 1 node in the corresponding SSA from Figure A2. The SC therefore correctly predicts (121b) to be odd. But, because (121a) only differs from (121b) in how the disjuncts are ordered, the SC also incorrectly predicts (121a) to be odd–at least in the absence of any additional assumptions.

The felicity of (121a) is captured in Ippolito (2019)'s framework based on the familiar idea that violations of the SC can be avoided by strengthening the weaker alternative (Gazdar, 1979; Singh, 2008b,a; Chierchia et al., 2009; Fox, 2018). To retain the *contrast* between (121b) and (121a), it is assumed that covert strengthening is governed by an economy condition, which disallows it in (121b). This is shown to generalize to the a. and b. sequences in (119-120).

Even though the SC appears like a reasonable constraint, the deep reason why contrast alternatives with different degrees of specificity should be disallowed, remains relatively mysterious. In particular, the account does not directly relate the SC to general pragmatic principles based on competition *between* sentences: the SC is a constraint that is only sensitive to the SSA associated with the target sentence, independently of the sentence's competitors and their own SSAs. In that respect, it remains close to Hurford's original constraint. Moreover, the constraint amounts to counting the number of parent nodes for each contrasted alternative, and as such is not sensitive to the relative positions of the two alternatives within their common SSA. This perspective might be slightly reductive, and would not capture the observation that oddness gets stronger if the two alternatives are in a dominance relation, as shown by gradience of the judgments in (125).

(125) a. # Jo grew up in Paris or France. — Different specificity, dominance
b. ? Jo grew up in Paris or Germany. — Different specificity,no dominance
c. Jo grew up in France or Germany. — Same specificity,no dominance

In Chapter 4, we will propose a constraint akin in effect to the SC, but that will constitute a more direct extension of earlier Redundancy-based constraints used to capture Hurford Disjunctions. We will show how it applies to basic (non-scalar) Hurford Disjunctions and extends to another challenging family of intuitively redundant sentences. Chapters 7 and **??** will discuss the particular case of scalar Hurford Disjunctions like (121), and extend the account to scalar Sobel sequences.

3.3 Zhang's

Chapter 4

Redundancy under Discussion[1]

This chapter presents novel data derived from the logical form $p \vee p \vee q$, *via* the *or*-to-*if* tautology and core properties of disjunction (commutativity, associativity). The sentences at stake exhibit differing degrees of pragmatic oddness, which is shown to represent a challenge for existing theories of oddness. Building on the machinery defined in Chapter 2, we propose a solution the this paradigm in term of QuD-driven Redundancy. More broadly, this Chapter motivates the use of implicit QuDs, which are semantically richer than the Logical Forms they are derived from, to evaluate pragmatic oddness.

4.1 A problematic dataset

The disjunctive sentences in (126), which are logically related to each other *via* applications of $\vee$-commutativity and $\vee$-associativity, appear sharply infelicitous.[2] Such sentences can be seen as pragmatically odd due to them being contextually equivalent to their complex disjunct, whether it is $p \vee q$, or $q \vee p$ (Katzir and Singh, 2014).

(126) *Context: Jo is supposed to attend Sinn und Bedeutung in Italy, but is also busy writing his MIT dissertation. Jo's friend are at the conference and wonder if he is around.*

a. # Either Jo is at SuB, or else he is at SuB or in Cambridge. $\mathbf{p} \vee (\mathbf{p} \vee \mathbf{q})$

b. # Either Jo is at SuB, or else he is in Cambridge or at SuB. $\mathbf{p} \vee (\mathbf{q} \vee \mathbf{p})$

c. # Either Jo is at SuB or in Cambridge, or else he is at SuB. $(\mathbf{p} \vee \mathbf{q}) \vee \mathbf{p}$

[1] This Chapter constitutes a longer and hopefully more readable adaptation of Hénot-Mortier (to appear). I would like to thank the audience and reviewers of SuB29 and of the 2024 BerlinBrnoVienna Workshop, in particular Itai Bassi, for relevant questions, datapoints and suggestions regarding earlier iterations of this project.

[2] More variants could be derived, for instance $q \vee (p \vee p)$. Here, we focus on the less obvious variants where two instances of p do not directly combine together.

d. # Either Jo is in Cambridge or at SuB, or else he is at SuB. $(\mathbf{q} \vee \mathbf{p}) \vee \mathbf{p}$

(127-130) show variants of (126a-126d) obtained *via* the *or*-to-*if* tautology. In each pair of sentences, the a. instances are derived by modifying the outer disjunction, while the the b. instances are derived by modifying the inner disjunction.[3]. Note that we do not intend to commit to a material analysis of the conditionals featured in these examples. Throughout this Chapter, $\rightarrow$ will be used as a mere shorthand for *if... then...*, i.e. will not imply that the conditionals under consideration are necessarily material.

(127) Derived from (126a):

a. # If Jo is not at SuB then he is at SuB or in Cambridge.
$\neg\mathbf{p} \rightarrow (\mathbf{p} \vee \mathbf{q})$

b. Either Jo is at SuB or if he is not at SuB then he is in Cambridge.
$\mathbf{p} \vee (\neg\mathbf{p} \rightarrow \mathbf{q})$

(128) Derived from (126b):

a. # If Jo is not at SuB then he is in Cambridge or at SuB.
$\neg\mathbf{p} \rightarrow (\mathbf{q} \vee \mathbf{p})$

b. # Either Jo is at SuB or if he is not in Cambridge then he is at SuB.
$\mathbf{p} \vee (\neg\mathbf{q} \rightarrow \mathbf{p})$

(129) Derived from (126c):

a. # If it's not true that Jo is at SuB or in Cambridge, then he is at SuB.
$\neg(\mathbf{p} \vee \mathbf{q}) \rightarrow \mathbf{p}$

b. ? Either Jo is in Cambridge if not at SuB, or he is at SuB.
$(\neg\mathbf{p} \rightarrow \mathbf{q}) \vee \mathbf{p}$

(130) Derived from (126d):

a. # If it's not true that Jo is in Cambridge or at SuB, then he is at SuB.
$\neg(\mathbf{q} \vee \mathbf{p}) \rightarrow \mathbf{p}$

b. # Either Jo is at SuB if not in Cambridge, or he is at SuB.
$(\neg\mathbf{q} \rightarrow \mathbf{p}) \vee \mathbf{p}$

Surprisingly, these variants exhibit different degrees of oddness: (127b) and (129b) escape infelicity, while the other variants do not.[4] This is unexpected given that all the

[3] One could also apply the *or*-to-*if* tautology to *both* the inner and the outer disjunction in the sentences in (126). Nested conditionals however, are hard to judge. We will briefly cover them in Section 4.6.2.

[4] (129b) sounds more degraded than (127b) however. We come back to this contrast in Section **??**.

sentences in (127-130) have same logical structure as the infelicitous sentences in (126a-126d), assuming implications are material (we will see that, in fact, issues remain when conditionals are *not* treated as material). Particularly puzzling is the existence of a contrast *between* the different b. examples in (127-130), which are derived from (126a-126d) using the *same* transformation.

The descriptive generalization seems to be the following: the sentences in (127-130) that retain an outer disjunction, and whose complex (conditional) disjunct has the negation of their simple disjunct as antecedent, are rescued.

In this Chapter, we propose that this descriptive generalization follows from the idea that oddness arises when sentences cannot evoke any well-formed implicit QuD, as defined in Chapter 2. The crucial point proposed in Chapter 2 that we will exploit here, is that disjunctions and conditionals give rise to different implicit QuDs. This model of accommodated QuDs will lead us to introduce a new notion of redundancy, Q-Non-Redundancy, that applies to pairs formed by LFs and accommodated QuDs – instead of just LFs. Under this view, sentences like (127b) and (129b) whose re-occurring material (p), each time plays different roles w.r.t. the QuD (typically, as a disjunct, or as a conditional "restrictor"), can escape Q-Non-Redundancy.

Assuming that the sole application $\vee$-commutativity does not affect oddness (generally in line with the data presented here), we now focus on sentences (126a), (127a), (127b), (128b), and (129a), repeated in that order in (131).

(131) a. # Either Jo is at SuB, or else he is at SuB or in Cambridge.
$\mathbf{p} \vee (\mathbf{p} \vee \mathbf{q})$

b. # If Jo is not at SuB then he is at SuB or in Cambridge.
$\neg\mathbf{p} \rightarrow (\mathbf{p} \vee \mathbf{q})$

c. Either Jo is at SuB or if he is not at SuB then he is in Cambridge.
$\mathbf{p} \vee (\neg\mathbf{p} \rightarrow \mathbf{q})$

d. # Either Jo is at SuB or if he is not in Cambridge then he is at SuB.
$\mathbf{p} \vee (\neg\mathbf{q} \rightarrow \mathbf{p})$

e. # If it's not true that Jo is at SuB or in Cambridge, then he is at SuB.
$\neg(\mathbf{p} \vee \mathbf{q}) \rightarrow \mathbf{p}$

The rest of this Chapter is structured as follows. The next Section reviews why some of the sentences in (131) are problematic for existing accounts of oddness. Section 4.3 briefly summarizes the model of implicit QuDs laid out in Chapter 2 and shows how it derives different QuDs for the disjunctions and conditionals at stake. Section 4.4 defines a new NON-REDUNDANCY constraint targeting pairs formed by LFs and their accommodated QuD, and shows how this constraint captures the contrasts in (131). Section 4.5 compares the constraint to those posited by similar earlier accounts and further connects it to Grice's MAXIM OF MANNER. Section 4.6 discusses a few additional datapoints related to (131), and Section 4.7 concludes the Chapter.

4.2 Previous accounts of oddness, and their shortcomings

In this section we present four existing accounts of oddness: GLOBAL NON-REDUNDANCY, LOCAL NON-REDUNDANCY, SUPER-REDUNDANCY, and NON-TRIVIALITY (see Marty and Romoli, 2022 for a more complete overview of these principles). The first three accounts are based on the notion of redundancy, which can be traced back to Grice's Maxim of Manner (submaxim of BREVITY, Grice, 1975). The last account exploits the notion of triviality (Stalnaker, 1974). We show that all accounts straightforwardly capture the double disjunction case (131a). However, the first three accounts fall short in explaining the contrast between the felicitous (131c) and the infelicitous (131b), (131d), and (131e). The last account on the other hand, *can* capture the pattern in (131), but at the cost of mispredicting the classic pattern of so-called Hurford Disjunctions (Hurford, 1974).

4.2.1 Global Non-Redundancy

REDUNDANCY-based accounts of oddness based on Grice's submaxim of BREVITY, which is part of the maxim of MANNER, defined in (132).

(132) **MAXIM OF MANNER** (Grice, 1975). Be clear, meaning:

1. Avoid obscurity of expression – i.e., avoid language that is difficult to understand;
2. Avoid ambiguity – i.e., avoid language that can be interpreted in multiple ways;
3. Be brief – i.e., avoid unnecessary verbosity;
4. Be orderly – i.e., provide information in an order that makes sense, and makes it easy for the recipient to process it.

According to this view, sentences that feature unnecessary verbosity should be deemed odd. This is cashed out in (133), which states that, if two sentences are contextually equivalent, then the simpler one should be preferred, and the more complex one should be deviant. Simplicity is understood as structural, following Katzir (2007) – see (134). We dub the principle in (133) Global Non-Redundancy, because contextual equivalence and simplicity are evaluated at the level of the entire sentence, and not locally.[5]

(133) **Global Non-Redundancy** (Meyer, 2013; Mayr and Romoli, 2016). A sentence S cannot be used in context c if there is a sentence S' s.t. S' is a simplification of S and $S' \equiv_c S$.

(134) **Structural simplicity** (Katzir, 2007). S' is a simplification of S if S' can be derived from S by replacing nodes in S with their subconstituents.

(133) predicts the double disjunction (131a) to be deviant, because it is contextually equivalent to its complex disjunct $(p \vee q)$. The same can be said of all other variants in (131), see (135). In brief, (133) does not derive the expected contrast between the felicitous (131c) on the one hand, and the infelicitous (131a), (131b), (131d), and (131e), on the other.

(135) Applying Global Non-Redundancy (abbreviated GNR) to the sentences in (131), under the assumption conditionals are material. Underlined constituents are the ones the entire expressions end up being contextually equivalent to.

a. (131a): $\mathbf{p} \vee (\underline{\mathbf{p} \vee \mathbf{q}}) \equiv \mathbf{p} \vee \mathbf{q}$ GNR ✗

b. (131b): $\neg\mathbf{p} \rightarrow (\underline{\mathbf{p} \vee \mathbf{q}}) \equiv \mathbf{p} \vee (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q}$ GNR ✗

c. (131c): $\mathbf{p} \vee (\underline{\neg\mathbf{p} \rightarrow \mathbf{q}}) \equiv \mathbf{p} \vee (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q} \equiv \neg\mathbf{p} \rightarrow \mathbf{q}$ GNR ✗

d. (131d): $\mathbf{p} \vee (\underline{\neg\mathbf{q} \rightarrow \mathbf{p}}) \equiv \mathbf{p} \vee (\mathbf{q} \vee \mathbf{p}) \equiv \mathbf{p} \vee \mathbf{q} \equiv \neg\mathbf{q} \rightarrow \mathbf{p}$ GNR ✗

e. (131e): $\neg(\underline{\mathbf{p} \vee \mathbf{q}}) \rightarrow \mathbf{p} \equiv (\mathbf{p} \vee \mathbf{q}) \vee \mathbf{p} \equiv \mathbf{p} \vee \mathbf{q}$ GNR ✗

Assuming conditionals a non-material does not help. Under this assumption, the prediction for the double disjunction (131a) does not change, but those for the "conditional" variants (131b-131e), may change. Since a non-material conditional is never contextually equivalent to its antecedent or consequent, regardless of what they denote, one can focus on disjunct simplifications of (131b-131e) when evaluating (133). Moreover, one can focus on simplifications retaining an occurrence of q, since these are the only ones which may turn out equivalent to the entire expressions (which involve q). Such simplifications are collected in (136).

(136) Computing potentially equivalent simplifications of (131b-131e)

a. (131b): $\neg\mathbf{p} \rightarrow (\cancel{\mathbf{p} \vee}\mathbf{q}) = \neg\mathbf{p} \rightarrow \mathbf{q}$

[5]A local variant of this principle will be investigated in the next Section.

b. (131c): $\cancel{p \vee} (\neg p \to q) = \neg p \to q$

c. (131d): $\cancel{p \vee} (\neg q \to p) = \neg q \to p$

d. (131e): $\neg(\cancel{p \vee} q) \to p = \neg q \to p$

One can then evaluate the contextual equivalence between the candidate simplifications in (136), and the entire expressions in (131b-131e). Assuming conditionals are strict, we find that (131b) and (131e) are equivalent to their simplifications in (136a) and (136d), respectively, so should be deemed deviant, in line with intuitions. Both (131c) and (131d) however, are logically independent from their only reasonable simplifications (136b) and (136c), and therefore, should both be felicitous. This is a good prediction for (131c), but a bad prediction for (136c). In brief, assuming conditionals are strict, improves the "fit" of principle (133), but still, fails at capturing the entire picture.

(137) Applying GLOBAL NON-REDUNDANCY (abbreviated GNR) to (131b-131e), under the assumption conditionals are strict, and considering the candidate simplification computed in (136).

a. (131b) $\equiv$ every $\neg p$-world is a p- or a q-world
$\equiv$ every $\neg p$-world is a q-world
$\equiv$ (136a) GNR ✗

b. (131c) $\equiv$ p or every $\neg p$-world is a q-world
$\not\equiv$ every $\neg p$-world is a q-world $\equiv$ (136b) GNR ✓

c. (131d) $\equiv$ p or every $\neg q$-world is a p-world
$\not\equiv$ every $\neg q$-world is a p-world $\equiv$ (136c) GNR ✓

d. (131e) $\equiv$ every $\neg(p \vee q)$-world is a p-world
$\equiv$ every $(\neg p \wedge \neg q)$-world is a p-world
$\equiv$ every $\neg q$-world is a p-world
$\equiv$ (136d) GNR ✗

Under a variably strict analysis, all cases but (131b) are predicted to be felicitous – see (138). Again, this is not the expected contrast. In fact, a variably strict analysis seems to worsen the "fit" of (133), as compared to a strict analysis.

(138) Applying GLOBAL NON-REDUNDANCY (abbreviated GNR) to (131b-131e), under the assumption conditionals are variably strict, and considering the candidate simplification computed in (136).

a. (131b) $\equiv$ every closest $\neg p$-world is a p- or a q-world
$\equiv$ every closest $\neg p$-world is a q-world
$\equiv$ (136a) GNR ✗

b. (131c) $\equiv$ **p** or every closest $\neg$**p**-world is a **q**-world
$\not\equiv$ every closest $\neg$**p**-world is a **q**-world $\equiv$ (136b) GNR ✓

c. (131d) $\equiv$ **p** or every closest $\neg$**q**-world is a **p**-world
$\not\equiv$ every closest $\neg$**q**-world is a **p**-world $\equiv$ (136c) GNR ✓

d. (131e) $\equiv$ every closest $\neg$(**p** $\vee$ **q**)-world is a **p**-world
$\equiv$ every closest ($\neg$**p** $\wedge$ $\neg$**q**)-world is a **p**-world
$\not\equiv$ every closest $\neg$**q**-world is a **p**-world $\equiv$ (136d) GNR ✓

Deep down, the issue with principle (133) stems from the observation that it is not sensitive to the fact that the subformula felt to cause redundancy (p), should be "disregarded" when present in the antecedent of a conditional. This is because, in the material case, there is no clear notion of what is antecedent is (due to the *or*-to*if* tautology), and, in the non-material cases, conditionals are by construction non-redundant, *whether or not* the problematic subformula occurs in the antecedent. This predicted (131c) and (131d) to *both* satisfy (133).

4.2.2 Local Non-Redundancy

Katzir and Singh (2014) propose a local implementation of Global Non-Redundancy, stating that the semantic computation evaluates, at certain nodes, whether the composition principle that applies there is non-vacuous. This gives rise to the principle in (139).

(139) **Local Non-Redundancy** (Katzir and Singh, 2014). S is deviant if S contains γ s.t. $[\![\gamma]\!] = [\![O(\alpha, \beta)]\!] \equiv_c [\![\zeta]\!]$, $\zeta \in \{\alpha, \beta\}$.

This predicts the double disjunction (131a) to be deviant, because, at the level of the highest disjunction, it is contextually equivalent to its complex disjunct ($p \vee q$). It also predicts all variants but (131e) to be deviant, assuming conditionals denote material implications. The felicity of (131c) is therefore not derived, and that of (131e), mispredicted. (140) details the computations leading to these predictions.

(140) Applying Local Non-Redundancy (abbreviated LNR) to the sentences in (131), under the assumption conditionals are material. Underlined binary operators are the ones being evaluated.

a. (131a): $\mathbf{p} \mathbin{\underline{\vee}} (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q}$ LNR ✗

b. (131b): $\neg\mathbf{p} \mathbin{\underline{\rightarrow}} (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q}$ LNR ✗

c. (131c): $\mathbf{p} \mathbin{\underline{\vee}} (\neg\mathbf{p} \rightarrow \mathbf{q}) \equiv \mathbf{p} \vee (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q} \equiv \neg\mathbf{p} \rightarrow \mathbf{q}$ LNR ✗

d. (131d): $\mathbf{p} \mathbin{\underline{\vee}} (\neg\mathbf{q} \rightarrow \mathbf{p}) \equiv \mathbf{p} \vee (\mathbf{q} \vee \mathbf{p}) \equiv \mathbf{p} \vee \mathbf{q} \equiv \neg\mathbf{q} \rightarrow \mathbf{p}$ LNR ✗

e. (131e): $\neg(\mathbf{p} \vee \mathbf{q}) \underline{\rightarrow} \mathbf{p} \equiv (\mathbf{p} \vee \mathbf{q}) \vee \mathbf{p} \equiv \mathbf{p} \vee \mathbf{q} \not\equiv \neg(\mathbf{p} \vee \mathbf{q}), \mathbf{p}$ LNR ✓
$\mathbf{p} \underline{\vee} \mathbf{q} \not\equiv \mathbf{p}, \mathbf{q}$ LNR ✓

The issue in fact persists if we adopt a non-material analysis of conditionals. Under this assumption, a conditional is never contextually equivalent to its antecedent or consequent, regardless of what they denote. So, one can focus on disjunctive nodes when evaluating (139). Under a (variably) strict analysis of conditionals, none of the disjunctive nodes in (131) are predicted to be equivalent to one of their daughters, as shown in (141). Therefore, none of the sentences in (131) is expected to be deviant, as per (139). Although the felicity of (131c) is predicted under these assumptions, all the other conditional variants are predicted to be felicitous, as well. Again, this is not the expected pattern.

(141) Applying Local Non-Redundancy (abbreviated LNR) to the sentences in (131), under the assumption conditionals are non-material (strict, or variably strict), and hence, focusing on disjunctive nodes. Underlined binary operators are the ones being evaluated.

a. (131a): $\mathbf{p} \underline{\vee} (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q}$ LNR ✗
b. (131b): $\mathbf{p} \underline{\vee} \mathbf{q} \not\equiv \mathbf{p}, \mathbf{q}$ LNR ✓
c. (131c): $\mathbf{p} \underline{\vee} (\neg\mathbf{p} \;\Box\!\!\rightarrow \mathbf{q}) \not\equiv \mathbf{p}, (\neg\mathbf{p} \;\Box\!\!\rightarrow \mathbf{q})$ LNR ✓
d. (131d): $\mathbf{p} \underline{\vee} (\neg\mathbf{q} \;\Box\!\!\rightarrow \mathbf{p}) \not\equiv \mathbf{p}, (\neg\mathbf{q} \;\Box\!\!\rightarrow \mathbf{p})$ LNR ✓
e. (131e): $\mathbf{p} \underline{\vee} \mathbf{q} \not\equiv \mathbf{p}, \mathbf{q}$ LNR ✓

The issue with this local principle appears similar to that of its "global" predecessor (133): the semantics assigned to conditionals, whether or not it is material, cannot make Local Non-Redundancy sensitive to whether or not the problematic subformula (p) is present in the antecedent of a conditional.

4.2.3 Super-Redundancy

Kalomoiros (2024), elaborating on Katzir and Singh (2014)'s view, introduces Super-Redundancy. Roughly, a sentence S is super-redundant if it features a binary operation taking a constituent C as argument, and moreover there is no way of strengthening C to C^+ that would make the resulting sentence S^+ non-redundant (i.e., non-equivalent to its counterpart where C^+ got deleted).

(142) **Super-Redundancy** (Kalomoiros, 2024). A sentence S is infelicitous if it contains $C * C'$ or $C' * C$, with $*$ a binary operation, s.t. $(S)^-_C$ is defined and for all D, $(S)^-_C \equiv S_{Str(C,D)}$. In this definition:

- $(S)^-_C$ refers to S where C got deleted;
- $Str(C, D)$ refers to a strengthening of C with D, defined inductively and whose key property is that it commutes with negation ($Str(\neg\alpha, D) = \neg(Str(\alpha, D))$), as well as with binary operators ($Str(O(\alpha, \beta), D) = O(Str(\alpha, D), Str(\beta, D))$);
- $S_{Str(C,D)}$ refers to S where C is replaced by $Str(C, D)$.

Because strengthenings can be understood to "project" from inside negation, this account predicts that overt negation influences the evaluation of redundancy. More specifically, it predicts that two LFs that have same logical structure *modulo* double-negation introduction and a variable change of the form $p' := \neg p$, may be such that one is Super-Redundant (henceforth **SR**) while the other is not. However, this account fails to predict any contrast for (131) assuming conditional are material, precisely because those sentences are logically isomorphic *without* any appeal to double negation introduction. The only operations used to derive these variants were commutativity, and the *or*-to*if* tautology.

(143) shows that under the material implication hypothesis, all the sentences in (131) can have one of their p-denoting constituents locally strengthened to yield an expression equivalent to the sentence without this p-constituent. Meaning, the sentences in (131) are all predicted to be SR.

(143) All the sentences in (131) are SR for the same reason (material case).

a. We show (131a)=$\mathbf{p} \vee (\mathbf{p} \vee \mathbf{q})$ is SR.
   Take C = (131a)'s 1st disjunct = $\mathbf{p}$.
   We then have $(131a)^-_C = \mathbf{p} \vee \mathbf{q}$
   $\forall D.\ (131a)_{Str(C,D)} = (\mathbf{p} \wedge D) \vee (\mathbf{p} \vee \mathbf{q}) \equiv (\mathbf{p} \vee \mathbf{q}) \wedge (D \vee \mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q} = (131a)^-_C$

b. We show (131b)=$\neg\mathbf{p} \to (\mathbf{p} \vee \mathbf{q})$ is SR.
   Take C = $\mathbf{p}$ in (131b)'s antecedent.
   We then have $(131b)^-_C = \mathbf{p} \vee \mathbf{q}$
   $\forall D.\ (131b)_{Str(C,D)} = \neg(\mathbf{p} \wedge D) \to (\mathbf{p} \vee \mathbf{q}) \equiv (\mathbf{p} \wedge D) \vee (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q} = (131b)^-_C$

c. We show (131c)=$\mathbf{p} \vee (\neg\mathbf{p} \to \mathbf{q})$ is SR.
   Take C = (131c)'s first disjunct = $\mathbf{p}$.
   We then have $(131c)^-_C = \neg\mathbf{p} \to \mathbf{q}$
   $\forall D.\ (131c)_{Str(C,D)} = (\mathbf{p} \wedge D) \vee (\neg\mathbf{p} \to \mathbf{q}) \equiv (\mathbf{p} \wedge D) \vee (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q} = (131c)^-_C$

d. We show (131d)=$\mathbf{p} \vee (\neg\mathbf{q} \rightarrow \mathbf{p})$ is SR.
Take C = (131d)'s first disjunct = $\mathbf{p}$.
We then have $(131d)^-_C = \neg\mathbf{q} \rightarrow \mathbf{p}$
$\forall D.\ (131d)_{Str(C,D)} = (\mathbf{p} \wedge D) \vee (\neg\mathbf{q} \rightarrow \mathbf{p}) \equiv (\mathbf{p} \wedge D) \vee (\mathbf{p} \vee \mathbf{q}) \equiv \mathbf{p} \vee \mathbf{q} = (131d)^-_C$

e. We show (131e)=$\neg(\mathbf{p} \vee \mathbf{q}) \rightarrow \mathbf{p}$ is SR.
Take C = (131e)'s consequent = $\mathbf{p}$.
We then have $(131e)^-_C = \neg(\mathbf{p} \vee \mathbf{q})$
$\forall D.\ (131e)_{Str(C,D)} = \neg(\mathbf{p} \vee \mathbf{q}) \rightarrow (\mathbf{p} \wedge D) \equiv (\mathbf{p} \vee \mathbf{q}) \vee (\mathbf{p} \wedge D) \equiv \mathbf{p} \vee \mathbf{q} = (131e)^-_C$

The problem persists under a strict analysis of conditionals. In that case, contrasts are predicted, but not between the right sentences: (131a), (131b) and (131e) are correctly predicted to be SR, (131c) is correctly predicted to be non-SR, but (131d) is *incorrectly* predicted to be non-SR. This is shown in (144). It appears that SR does not distinguish between the felicitous case where re-occurring material (p in our case) is in the antecedent of a conditional, and the odd case where it appears in the consequent. Both Global and Local Non-Redundancy were already characterized by the same shortcoming.

(144) Super-Redundancy and strict conditionals incorrectly predict (131d) to be non-SR.[6]

a. We show (131b)=$\neg\mathbf{p} \Box\!\!\rightarrow (\mathbf{p} \vee \mathbf{q})$ is SR.
Take C = $\mathbf{p}$ in (131b)'s disjunction.
We then have $(131b)^-_C = \neg\mathbf{p} \Box\!\!\rightarrow \mathbf{q}$
$\forall D.\ (131b)_{Str(C,D)} = \neg\mathbf{p} \Box\!\!\rightarrow ((\mathbf{p} \wedge D) \vee \mathbf{q}) \equiv \neg\mathbf{p} \Box\!\!\rightarrow ((\mathbf{p} \vee \mathbf{q}) \wedge (D \vee \mathbf{q})) \equiv \neg\mathbf{p} \Box\!\!\rightarrow (\mathbf{q} \wedge (D \vee \mathbf{q})) \equiv \neg\mathbf{p} \Box\!\!\rightarrow \mathbf{q} = (131b)^-_C$

b. We show (131c)=$\mathbf{p} \vee (\neg\mathbf{p} \Box\!\!\rightarrow \mathbf{q})$ is not SR.
Take C = (131c)'s first disjunct = $\mathbf{p}$.
We then have $(131c)^-_C = \neg\mathbf{p} \Box\!\!\rightarrow \mathbf{q}$.
Take $D = \top$.
$(131c)_{Str(C,D)} = (\mathbf{p} \wedge D) \vee (\neg\mathbf{p} \Box\!\!\rightarrow \mathbf{q}) \equiv \mathbf{p} \vee (\neg\mathbf{p} \Box\!\!\rightarrow \mathbf{q}) \not\equiv \neg\mathbf{p} \Box\!\!\rightarrow \mathbf{q} = (131c)^-_C$
Taking C to be any other candidate constituent, would create an instance of $(131c)^-_C$ that would not contain the operator $\Box\!\!\rightarrow$. Given this, setting $D = \top$ will always give rise to a lack of equivalence between $(131c)_{Str(C,D)}$ ($\equiv$ (131c) in that case), and $(131c)^-_C$.

[6]Note that, to prove (131c) and (131d) are non-SR, we focus on setting C as the constituent that combines with a binary operator that is not conditional – typically here, the disjunctive operator. We do this, despite the fact that Super-Redundancy in principle has to be checked for every binary operand. But locally strengthening the other possible constituents would lead to compare a (locally strengthened) strict conditional to a disjunction. Such a comparison trivially leads to non-equivalence.

c. We show (131d)=$\mathbf{p} \vee (\neg\mathbf{q} \mathrel{\Box\!\!\rightarrow} \mathbf{p})$ is not SR.
Take C = (131d)'s first disjunct = $\mathbf{p}$.
We then have $(131\text{d})^{-}_{C} = \neg\mathbf{q} \mathrel{\Box\!\!\rightarrow} \mathbf{p}$.
Take $D = \top$.
$(131\text{d})_{Str(C,D)} = (\mathbf{p} \wedge D) \vee (\neg\mathbf{q} \mathrel{\Box\!\!\rightarrow} \mathbf{p}) \equiv \mathbf{p} \vee (\neg\mathbf{q} \mathrel{\Box\!\!\rightarrow} \mathbf{p}) \not\equiv \neg\mathbf{q} \mathrel{\Box\!\!\rightarrow} \mathbf{p} = (131\text{d})^{-}_{C}$
Taking C to be any other candidate constituent, would create an instance of $(131\text{d})^{-}_{C}$ that would not contain the operator $\mathrel{\Box\!\!\rightarrow}$. Given this, setting $D = \top$ will always give rise to a lack of equivalence between $(131\text{d})_{Str(C,D)}$ ($\equiv$ (131d) in that case), and $(131\text{d})^{-}_{C}$.

d. We show (131e)=$\neg(\mathbf{p} \vee \mathbf{q}) \mathrel{\Box\!\!\rightarrow} \mathbf{p}$ is SR.
Take C = $\mathbf{q}$.
We then have $(131\text{e})^{-}_{C} = \neg\mathbf{p} \mathrel{\Box\!\!\rightarrow} \mathbf{p} = \bot$ if $\mathbf{p} \neq \emptyset$ else $\top$
$\forall D.\ (131\text{e})_{Str(C,D)} = \neg(\mathbf{p} \vee (\mathbf{q} \wedge D)) \mathrel{\Box\!\!\rightarrow} \mathbf{p} \equiv (\neg\mathbf{p} \wedge \neg(\mathbf{q} \wedge D)) \mathrel{\Box\!\!\rightarrow} \mathbf{p} = \bot$ if $\mathbf{p} \neq \emptyset$ else $\top = (131\text{e})^{-}_{C}$

Lastly, testing the predictions of SR on our sentences, under the assumption that conditionals are variably strict, would not fundamentally help, given that Kalomoiros (2024) observed that SR coupled with variably strict conditionals fails to capture the Hurford Conditionals SR was originally designed to account for.

4.2.4 Non-Triviality

A different line of work (Mayr and Romoli, 2016 i.a.), building on the notion of Local Contexts (Schlenker, 2009), associates oddness with triviality in the sense of (Stalnaker, 1999). This view is summarized in (145).

(145) **Non-Triviality** (Mayr and Romoli, 2016). A sentence S cannot be used in a context c if some part π of S is entailed or contradicted by the Local Context of π in c (abbreviated LC(π, c)).

(146) *Local Context.* The Local Context of an expression π in a sentence S is the smallest domain that one may restrict attention to when assessing E without jeopardizing the truth conditions of S. Let c be the global context of S. The above definition derives the following facts for disjunctions and conditionals:

a. If S is a conditional of the form $\Phi \to \Psi$, LC(Φ, c) = c and LC(Ψ, c) = $c \cap \Phi$.

b. If S is a disjunction of the form $\Phi \vee \Psi$, and LCs are assumed to be computed incrementally (left-to-right), LC(Φ, c) = c and LC(Ψ, c) = $c \cap \neg\Phi$

c. If S is a disjunction of the form $\Phi \vee \Psi$, and LCs are assumed to be computed symmetrically (left-to-right and right-to-left), LC$(\Phi, c) = c \cap \neg\Psi$ and LC(Ψ, c) $= c \cap \neg\Phi$.

Assuming LCs are computed incrementally for disjunctions (see (146b)), (145) predicts the right pattern for the sentences in (131). This is detailed in (147).

(147) All the sentences in (131) are locally trivial, for the same reason (assuming asymmetric Local Contexts for $\vee$).

a. We show (131a)=$\mathbf{p} \vee (\mathbf{p} \vee \mathbf{q})$ is locally trivial.
Take $\pi = \mathbf{p}$ in (131a)'s inner disjunction.
LC(π, c) = LC$(\mathbf{p}, c)$= $c \cap \neg\mathbf{p}$ (negation of 1st disjunct), contradiction.

b. We show (131b)=$\neg\mathbf{p} \to (\mathbf{p} \vee \mathbf{q})$ is locally trivial.
Take $\pi = \mathbf{p}$ in (131a)'s disjunction.
LC(π, c) = LC$(\mathbf{p}, c)$= $c \cap \neg\mathbf{p}$ (antecedent), contradiction.

c. We show (131c)=$\mathbf{p} \vee (\neg\mathbf{p} \to \mathbf{q})$ is not locally trivial.
Take π = (131a)'s 1st disjunct = $\mathbf{p}$. LC$(\pi, c) = c$, consistent.
Take π = (131a)'s antecedent = $\neg\mathbf{p}$. LC(π, c) = LC$(\neg\mathbf{p}, c)$ = $\neg\mathbf{p}$ (negation of 1st disjunct), consistent.
Take π = (131a)'s consequent = $\mathbf{q}$. LC(π, c) = LC$(\mathbf{q}, c)$ = $\neg\mathbf{p}$ (negation of 1st disjunct / antecedent), consistent.

d. We show (131d)=$\mathbf{p} \vee (\neg\mathbf{q} \to \mathbf{p})$ is locally trivial.
Take π = (131a)'s consequent = $\mathbf{p}$.
LC(π, c) = LC$(\mathbf{p}, c)$= $c \cap \neg\mathbf{p} \cap \neg\mathbf{q}$ (negation of 1st disjunct and antecedent), contradiction.

e. We show (131e)=$\neg(\mathbf{p} \vee \mathbf{q}) \to \mathbf{p}$ is locally trivial.
Take π = (131a)'s consequent = $\mathbf{p}$.
LC(π, c) = LC$(\mathbf{p}, c)$= $c \cap \neg(\mathbf{p} \vee \mathbf{q}) = c \cap \neg\mathbf{p} \wedge \neg\mathbf{q}$ (antecedent), contradiction.

This might suggest that Non-Triviality is a good enough theory of oddness, given what we have seen so far. There are however two caveats coming with this result. First, the result is not maintained if we assume LCs are computed symmetrically (see (148)).

(148) Assuming symmetric Local Contexts, we show (131c)=$\mathbf{p} \vee (\neg\mathbf{p} \to \mathbf{q})$ is locally trivial.
Take π = (131a)'s 1st disjunct = $\mathbf{p}$.
LC$(\pi, c) = c \cap \neg(\neg\mathbf{p} \to \mathbf{q}) = c \cap (\neg\mathbf{p} \wedge \neg\mathbf{q})$, contradiction with $\mathbf{p}$.

But making such an assumption is independently needed to account for Hurford Disjunctions (Hurford, 1974), featuring contextually entailing disjunct, and which appear infelicitous regardless of the linear order of such disjuncts. Strong-to-weak Hurford Disjunctions such as (149a) in particular, *require* symmetric Local Contexts when evaluating NON-TRIVIALITY, so that the stronger disjunct (left disjunct) gets interpreted in a context entailing the negation of the weaker one – leading to a contradiction. This is shown in (150-151). Note that (151) straightforwardly extends to weak-to-strong Hurford Disjunctions like (149b).

(149) a. # Jo is in Cambridge or in Massachusetts. $\mathbf{p}^+ \vee \mathbf{p}$

b. # Jo is in Massachusetts or in Cambridge. $\mathbf{p} \vee \mathbf{p}^+$

(150) Assuming asymmetric Local Contexts, we show Hurford Disjunctions of the form $\mathbf{p}^+ \vee \mathbf{p}$ are incorrectly predicted to be felicitous.
Take $\pi =$ (149a)'s 1st disjunct = $\mathbf{p}^+$.
LC$(\pi, c) = c$, consistent.
Take $\pi =$ (149a)'s 2nd disjunct = $\mathbf{p}$.
LC$(\pi, c) = c \cap (\neg \mathbf{p}^+)$, consistent.

(151) Assuming symmetric Local Contexts, we show Hurford Disjunctions of the form $\mathbf{p}^+ \vee \mathbf{p}$ are correctly predicted to be infelicitous.
Take $\pi =$ (149a)'s 1st disjunct = $\mathbf{p}^+$.
LC$(\pi, c) = c \cap (\neg \mathbf{p})$, contradiction.

In brief, NON-TRIVIALITY makes correct predictions for the sentences we focus on in this Chapter, but *modulo* assumptions on the computation of Local Contexts that cannot be maintained once additional data is considered. The second caveat, will be presented in more detail in Chapter 6, and has to do with the Hurford *Conditionals* (Mandelkern and Romoli, 2018) exemplified in (152). Chapter 6 will show that NON-TRIVIALITY cannot capture the contrast in (152), while an extension of the account we put forth here, can.

(152) a. # If Jo is in not Cambridge, he in Massachusetts. $\neg \mathbf{p}^+ \vee \mathbf{p}$

b. Jo is in Massachusetts, he is not in Cambridge. $\mathbf{p} \rightarrow \mathbf{p}^+$

We have just reviewed four prominent accounts of pragmatic oddness, and showed that they either cannot capture the pattern in (131), or can, but *modulo* assumptions jeopardizing previously established results pertaining to other famously odd expressions. In the next Section, we summarize the core ingredients of the framework introduced in Chapter 2, that we will reuse here in combination with a novel NON-REDUNDANCY constraint, to account for the target pattern.

4.3 A QuD-based approach to the data at stake

4.3.1 Overview

In Chapter 2, we argued that Logical Forms evoke the implicit QuDs they could felicitously answer. Such QuDs were modeled as parse trees of the Context Set, following insights from Büring (2003); Ippolito (2019); Zhang (2022) (i.a.). We called these stuctures Qtrees. Chapter 2 additionally proposed that the Qtrees evoked by a complex LF be derived from the ones evoked by this LF's constitutive parts. As a result, disjunctions and conditionals were assumed to evoke distinct Qtrees. Roughly, disjunctions were taken to evoke trees that make both disjuncts at issue at the same time (based on Simons, 2001; Westera, 2018; Zhang, 2022 i.a.), while conditionals were taken to evoke trees that makes the consequent at issue in the domain(s) of the CS where the antecedent holds (based on insights from Enguehard, 2021; Aloni, 2022 i.a.).

In this Chapter, we supplement this compositional machinery with a NON-REDUNDANCY constraint, which applies to pairs formed by (i) an LF and (ii) one of its possible Qtrees. Roughly, this constraint will rule out a Qtree derived from an LF, if this Qtree is also evoked by a simplification of the LF. This NON-REDUNDANCY constraint will predict that certain LFs, though informative, do *not* evoke any well-formed Qtree, and as such, should be deemed odd.

The fact that conditionals, unlike disjunctions, evoke an "asymmetric" Qtree, which disregards the at-issueness of the antecedent, will prevent (131c) from violating our new NON-REDUNDANCY constraint, but will correctly predict all other variants to be deviant. Specifically, we will show that this model predicts all the Qtrees evoked by (131a) to be ill-formed, due to them being evoked by (131a)'s simplification $p \vee q$. Likewise, all the Qtrees evoked by (131b) will be ruled out because they are all evoked by the q / $\neg p \rightarrow q$ simplifications of (131b). The Qtrees evoked by (131d) will be as well, due to the p simplification of (131d). Lastly, the Qtree evoked by (131e) will be deemed ill-formed due to being "answerless" (as per (75)). Crucially, (131c) will be correctly ruled-in, essentially because its redundant material (p) is nested with the antecedent of a conditional. Our model of conditional Qtrees, and of Non-Redundancy, will ensure that Non-Redundancy is "blind" to the presence of this intuitively redundant element, in that particular position.

4.3.2 General structure and interpretation of Question Trees

Based on Chapter 2 and on previous literature (Büring, 2003; Riester, 2019; Ippolito, 2019; Onea, 2016; Zhang, 2022), we take questions to denote recursive partitions, or *parse trees*, of the Context Set (Stalnaker, 1974; henceforth **CS**). This is specified in (52), repeated below.

(52) *Structure of Question-trees* (***Qtrees***). Qtrees are rooted trees whose nodes are all subsets of the CS and s.t.:

- Their root generally refers to the CS;
- Any intermediate node is a proposition, which is partitioned by the set of its children.

In such trees, the root can be seen as a tautology over the CS, and any other node, as a possible answer to the global question denoted by the tree. Intermediate nodes can generally be seen as non-maximal answers, while leaves can generally be seen as maximal ones. Any subtree rooted in a node N can be understood as conditional question taking for granted the proposition denoted by N. Finally, a path from the root to any node N can be seen as a strategy of inquiry (or a sequence of conditional questions) leading to the answer denoted by N.

When evoking a given Qtree, an LF "flags" specific nodes on the tree as maximal true answers. These nodes, dubbed *verifying nodes* in Chapter 2, are typically the leaves of the Qtree which entail the proposition denoted by the LF. They will appear in $\boxed{\text{boxes}}$ throughout this dissertation. Just like Qtree structure, verifying nodes are compositionally derived. Moreover, Chapter 2 argued that an accommodated Qtree should allow the sentence evoking it to properly answer it; that is why we assume that any well-formed Qtree derived from a sentence should come with a non-empty set of verifying nodes. This is repeated in (75) below. More generally, Chapter 2 proposed that pragmatic oddness results from the fact that a given sentence, through its LF, cannot give rise to any well-formed Qtree. This is repeated in (76).

(75) **Empty labeling**. If a sentence S evokes a Qtree T but does not flag any node as verifying in T, then T is deemed odd given S.

(76) *Oddness of a sentence.* A sentence S is odd if any Qtree T it evokes is odd given S.

We now proceed to deriving Qtrees for the two simplex LFs at stake in this Chapter: S_p = *Jo is at SuB* and S_q = *Jo is in Cambridge*. We do not go over the entire recipe for generating such Qtrees here; for a more complete overview, see Chapter 2, Section 2.3.

4.3.3 Simplex Qtrees

We assume here that S_p and S_q constitute same-granularity alternatives. S_p gives rise to two kinds of Qtree: a "polar-question" depth-1 Qtree whose leaves are the p and $\neg p$ worlds; and a "*wh*-question" depth-1 Qtree whose leaves are p and relevant, mutually exclusive alternatives to p.[7] Moreover, verifying nodes are defined on such trees as the leaves entailing p. The same can be said of S_q, *mutatis mutandis*.

The Qtrees compatible with S_p and S_q are given in Figures A and B. The two Figures are equivalent *modulo* a permutation of p and q. Figures A1 and B1, respectively model polar questions of the form: *Is Jo at SuB? Is Jo in Cambridge?* Figures A2 and B2 on the other hand, model a *wh*-question of the form: *Where is Jo?* At that point, it is worth observing that the "*wh*" Qtrees raised by S_p and S_q have similar structures (ignoring verifying nodes); while the corresponding "polar" Qtrees do not.

(1) "Polar". (2) "*Wh*".

Figure A: Qtrees for $S_{\mathbf{p}}$ = *Jo is at SuB*. $\boxed{\text{Boxed}}$ nodes are verifying.

(1) "Polar". (2) "*Wh*".

Figure B: Qtrees for $S_{\mathbf{q}}$ = *Jo is in Cambridge*. $\boxed{\text{Boxed}}$ nodes are verifying.

We now derive Qtrees raised by more complex LFs, in particular, negated, disjunctive, and conditional LFs derived from S_p and S_q.

[7] This is a simplification; Chapter 2 assumed that even simplex LFs can could give rise to layered Qtrees, whose layers are ordered by some notion of granularity. But this assumption is not relevant here.

4.3.4 Negated Qtrees

As discussed in Chapter 2, a negated LF is assumed to evoke the same kind of question as its positive counterpart, but flags a disjoint set of verifying nodes. Given an LF X, evoking a Qtree T, a Qtree T' for $\neg X$ is obtained by retaining T's structure (nodes and edges), and "swapping" T's verifying nodes, by replacing any set of same-level verifying nodes in T by the set of non-verifying nodes at the same level in T. If the verifying nodes are all leaves, this operation simply corresponds to set complementation in the domain of leaves. This is done for $\neg S_p$ and $\neg S_q$ in Figures C and D.

(1) "Polar". (2) "*Wh*".

Figure C: Qtrees for $\neg S_{\mathbf{p}}$ = *Jo is not at SuB*. $\boxed{\text{Verifying nodes}}$ are the sisters of the **p**-node.

(1) "Polar". (2) "*Wh*".

Figure D: Qtrees for $\neg S_{\mathbf{q}}$ = *Jo is not in Cambridge*. $\boxed{\text{Verifying nodes}}$ are the sisters of the **q**-node.

These two Figures maintain the parallelism observed in the non-negated cases: the "polar" Qtrees for $\neg S_p$ and $\neg S_q$ have different structures, while their "*wh*" Qtrees are structurally similar.

4.3.5 Disjunctive Qtrees

A disjunctive Qtree should address the questions evoked by each disjunct *in parallel*, making them both at issue (Simons, 2001; Zhang, 2022). Chapter 2 therfore argued that disjunctive Qtrees are well-formed unions of Qtrees evoked by the individual disjuncts. "Union" refers to that of nodes and edges; it is thus symmetric. The union of two Qtrees T and T' will be well-formed if there is no node N present in both T and T' that introduces different partitionings in T and T'.[8] The sets of verifying nodes attached to the two disjoined Qtrees, are also unioned.

The only possible Qtree for $S_p \vee S_q$ / $S_q \vee S_p$ is given in Figure E. It is obtained from Qtrees A2 and B2, which have similar structures and as such can be properly unioned. Other possible unions of Qtrees are shown in Figure F but are ill-formed, because their leaves do not partition the CS.

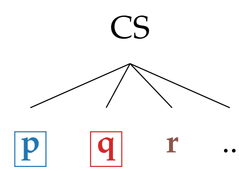
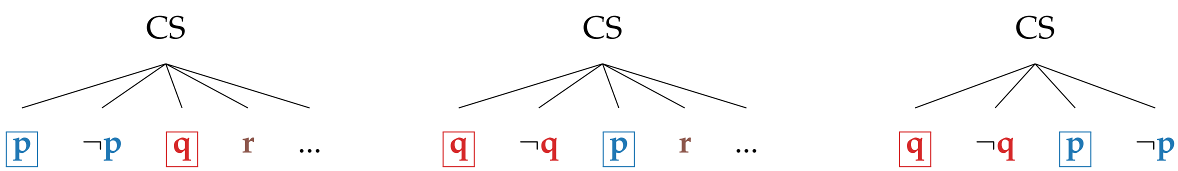


Figure E: Only well-formed Qtree evoked by $S_{\mathbf{p}} \vee S_{\mathbf{q}}$ = *Jo is at SuB or in Cambridge*, obtained from Qtrees A2 and B2. This Qtree is also the only Qtree compatible with #(131a) = *Jo is at SuB or at SuB or in Cambridge*.



(1) Tree A1 ∨ Tree B2. (2) Tree A2 ∨ Tree B1. (3) Tree A1 ∨ Tree B1.

Figure F: Ill-formed Qtrees resulting from the union of the Qtrees in Figures A and B. The leaves of these Qtrees do not properly partition the CS.

Likewise, the Qtrees in Figures A2 (for S_p) and E (for $S_p \vee S_q$), can be unioned to derive the only possible Qtree for (131a) = $S_p \vee (S_p \vee S_q)$. Because Qtree E has same structure as Qtree A2, and its set of verifying nodes contains those of Qtree A2, the result of their union is simply Qtree E. In sum, Qtree E turns out to be compatible with *both* the simple disjunction $S_p \vee S_q$ and the more complex disjunction (131a). We will show in Section 4.4 that this makes Qtree E "redundant" given (131a) – an in turn, predicts (131a) to be odd. Before this, let us turn to conditional LFs, in order to derive Qtrees for the other sentences in (131).

[8] Chapter 2, Section 2.4.2, provides a few illustrations of (un)problematic cases of Qtree disjunction.

4.3.6 Conditional Qtrees

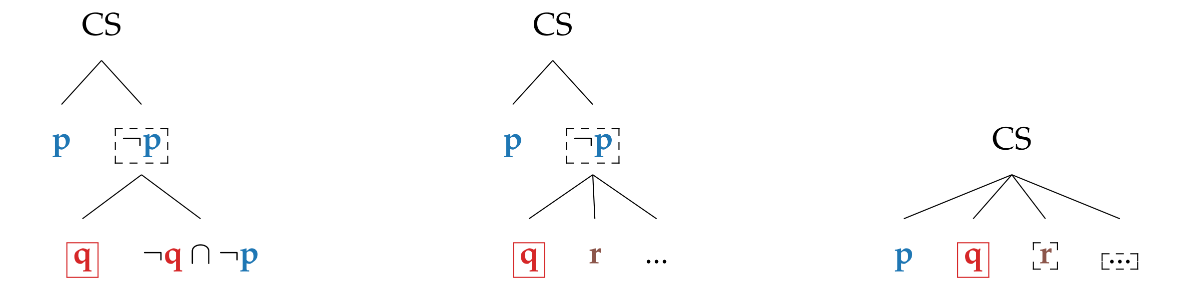
Chapter 2 proposed that conditionals evoke questions pertaining to their consequent, set in the domain(s) of the CS where the antecedent holds. This was modeled by assuming that conditional Qtrees are derived by "plugging" a consequent Qtree T_C into the verifying nodes of antecedent Qtrees T_A. More concretely, for each verifying node N of T_A, N gets replaced by $N \cap T_C$, where $\cap$ refers to tree-node intersection. This operation is formally defined in (66), but let us just note here that, from an algorithmic perspective, node-wise intersection betwen T and N can be achieved by (i) intersecting all nodes of T with N; (ii) removing resulting empty nodes; (iii) removing resulting dangling and unary edges.[9]

Additionally, Chapter 2 assumed some form of "neglect-zero" effect (Aloni, 2022; Flachs, 2023) in conditional Qtrees, in the sense that only the consequent of a conditional contributes verifying nodes in the resulting conditional Qtree. In particular, nodes falsifying the antecedent are not considered verifying in the resulting conditional Qtree.[10] This will be crucial to derive the absence of oddness in the case of (131c): disjoining $\neg p \rightarrow q$ with p creates a Qtree where p is at-issue (verifying), whereas in the Qtrees evoked by the simpler conditional statement $\neg p \rightarrow q$, p is "neglected" (non-verifying). In other words, disjunction has a non-redundant effect in (131c). This will *not* hold of the other (infelicitous) variants in (131).

The core idea behind this operation is that conditionals introduce a hierarchy between antecedent (backgrounded) and consequent (at-issue): the consequent Qtree gets *restricted* by the antecedent Qtree. Applying this to $\neg S_p \rightarrow S_q$, using the Qtrees for $\neg S_p$ from Figure C as antecedent Qtrees, and the Qtrees for S_q from Figure B as consequent Qtrees, leads to the conditional Qtrees in Figure G. In this Figure, nodes in dashed boxes refer to those that were verifying in the antecedent Qtree, but are no longer verifying in the conditional Qtree. They can be seen as "restrictor" nodes, which define the domain(s) of the CS in which the consequent Qtree is introduced. Nodes in $\boxed{\text{solid boxes}}$ refer to the nodes that are verifying in the consequent Qtree, and are thus still verifying in the conditional Qtree. Figures H and I further detail how the conditional Qtrees in Figures G1 and G3 were derived.

[9]This operation is structurally idle if N entails a leaf of T_C, because in this case, $N \cap T_C$ reduces to a root N, and replacing N by N in T_A is idle. However, it might still affect verifying nodes.

[10]This predicts that a sentence whose antecedent is falsified in the CS, evokes a Qtree without any verifying node, and thus checks the Empty Labeling condition (75) – in turn causing oddness.



(1) Tree C1 → Tree B1. (2) Tree C1 → Tree B2. (3) Tree C2 → Tree B1 / B2.[11]

Figure G: Qtrees for $\neg S_{\mathrm{p}} \to S_{\mathrm{q}}$ = *If Jo is not at SuB then he is in Cambridge.*

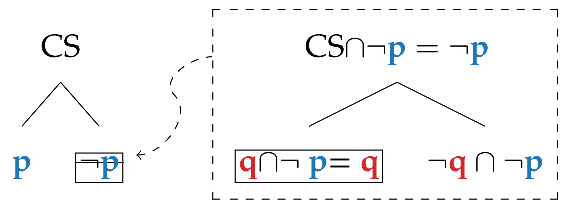


Figure H: Breakdown of the derivation of Figure G1 (can be easily adapted to Figure G2).

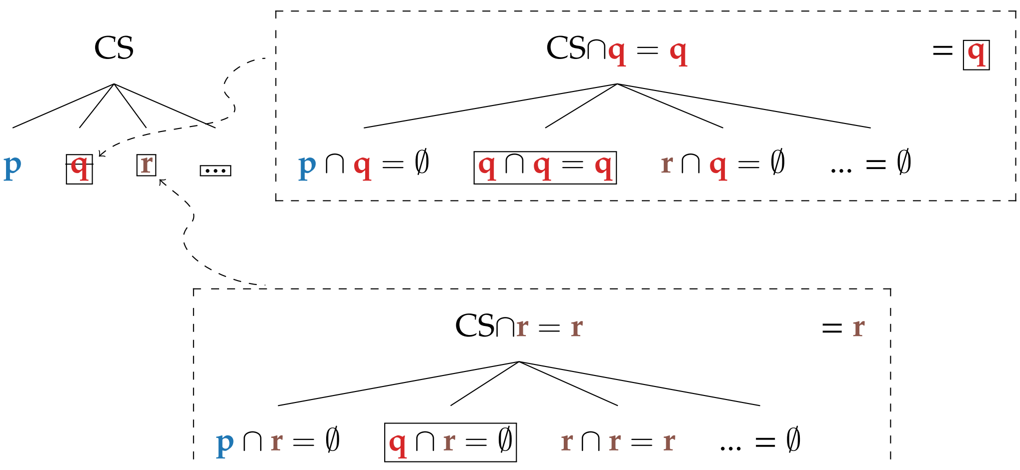
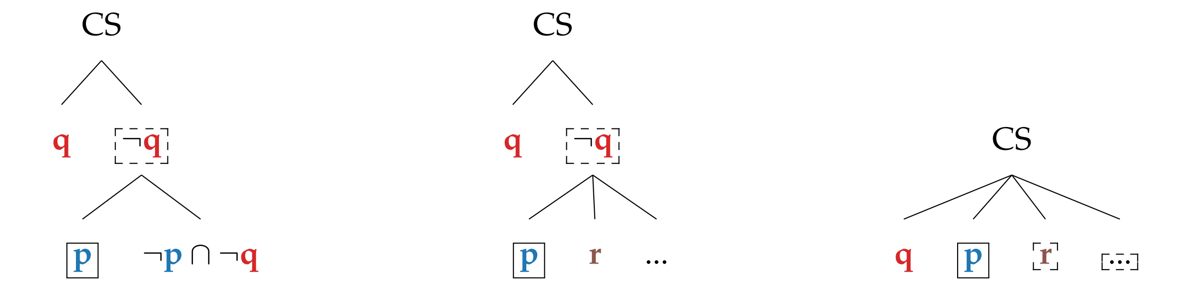


Figure I: Breakdown of the derivation of Figure G3.

At this point, we observe that the Qtrees for $\neg S_p \to S_q$ do not flag their p-node as verifying. This is because we assumed conditional Qtrees disregard the falsity of their antecedent when it comes to flagging/at-issueness. This will be crucial to predict that (131c) = $S_p \vee (\neg S_p \to S_q)$ is not "redundant". Lastly, Qtrees for $\neg S_q \to S_p$, can be obtained from Figure G, by simply swapping the p and q nodes. This is done in Figure J.



(1) Tree D1 → Tree A1. (2) Tree D1 → Tree A2. (3) Tree D2 → Tree A1 / A2.

Figure J: Qtrees for $\neg S_{\mathrm{q}} \to S_{\mathrm{p}}$ = *If Jo is not in Cambridge then he is at SuB*; obtained *mutatis mutandis* from Figure G.

Now that the compositional Qtree machinery is set up, it can be used to derive Qtrees for all the sentences in (131), by compositing the rules for negated, dijunctive, and conditional Qtree formation. Before doing this, we introduce in the next Section a new version of Non-Redundancy which applies to LF-Qtree pairs. We will then derive the Qtrees for the sentences at stake, and discuss when, and how, they comply with our newly defined constraint. This will eventually enable us to derive the right pattern of felicity in (131).

4.4 Capturing the target cases

4.4.1 Non-Redundancy as a constraint on LF-Qtree pairs

In the previous Section, we noted cases where the Qtrees derived from our target sentences turned out to be identical to Qtrees evoked by other, "simpler" sentences. For instance, we observed that the Qtree evoked by (131a) $= S_p \vee S_p \vee S_q$ (in Figure E), was the same as the Qtree evoked by the simpler sentence $S_p \vee S_q$. We now argue that such configurations constitute violations of a specific implementation of Non-Redundancy, inspired by previous Redundancy-based approaches to oddness (Meyer, 2013; Katzir and Singh, 2014; Mayr and Romoli, 2016). The crucial difference between our constraint, and the earlier ones, is that our constraint will be designed to operate on LF-Qtree pairs, instead of just LFs and their propositional meanings. Crucially, this constraint will be sensitive, not only to what sentences mean, but to how they "package" information *via* their implicit Qtree. This will eventually allow us to introduce the right felicity contrasts between the sentences in (131), which, as we will see, evoke distinct Qtrees.

Our implementation of Non-Redundancy, dubbed Q-Non-Redundancy, goes as follows. If a Qtree Q is evoked by a sentence S and also by one of the sentence's formal simplifications S', then Q is deemed Q-REDUNDANT given S. This is formalized in (153a-153b) and is based on the concept of structural simplicity, repeated in (134) below.

(153) a. **Q-NON-REDUNDANCY** (to be revised in Chapter 5). Let X be a LF and let $Qtrees(X)$ be the set of Qtrees evoked by X. For any $T \in Qtrees(X)$, T is deemed Q-REDUNDANT given X (and thus, odd given X) iff there exists a formal simplification of X, X', and $T' \in Qtrees(X')$, such that $T = T'$.

b. *Qtree equality*. $T = T'$ iff T and T' have same structure and same verifying nodes.[12]

(134) *Structural simplicity (Katzir, 2007)*. S' is a simplification of S if S' can be derived from S by replacing nodes in S with their subconstituents.

A *sentence* S will be deemed Q-REDUNDANT if *all* the Qtrees it evokes, are Q-REDUNDANT given S. This constitutes a special case of sentence oddness (as defined in (76)).

The double disjunction (131a) = $S_p \vee S_p \vee S_q$ is thus odd, because the only possible Qtree it evokes (in Figure E, repeated in Figure K below), is also evoked by $S_p \vee S_q$, and $S_p \vee S_q$ can be obtained from (131a) by substitution (regardless of bracketing). So our constraint captures the basic case case that previous approaches already accounted for.

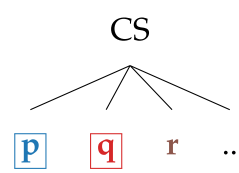


Figure K: Only well-formed Qtree evoked by $S_{\mathbf{p}} \vee S_{\mathbf{q}}$ = *Jo is at SuB or in Cambridge*, obtained from Qtrees A2 and B2. This Qtree is also the only Qtree compatible with #(131a) = *Jo is at SuB or at SuB or in Cambridge*.

We now proceed to deriving the Qtrees evoked by the infelicitous sentences (131b), (131d), and 131e), which appeared more problematic for previous approaches to oddness. We will show that the first two sentences ((131b) and (131d)) are both Q-REDUNDANT in our framework. The last sentence (131e), will be problematic for not flagging any node as verifying – triggering the EMPTY LABELING condition.

[12] This is sufficient for our purposes here, but needs to be generalized to cover other cases of REDUNDANCY in this QuD-driven framework. The generalized concept of Qtree equality ("equivalence"), is based on structural equality, and equality between sets of minimal verifying paths; see Chapter 5.

4.4.2 Ruling out the infelicitous (131b), (131d), and (131e)

For conciseness, we now use p and q as shorthands for the sentences denoting p and q, previously noted S_p = *Jo is at SuB* and S_q = *Jo is in Cambridge*.

We start with (131b) $= \neg p \to (p \vee q)$, whose Qtrees are given in Figure L, and are derived using Figures C (for $\neg p$) and E (for $p \vee q$), along with the combination rule for conditional Qtrees.

(1) Tree C1 $\to$ Tree E.
Same as Qtree G2 evoked by $\neg$**p** $\to$ **q**.

(2) Tree C1 $\to$ Tree E.[13]
Same as Qtree B2 evoked by q.

Figure L: Qtrees for (131b) $= \neg \mathbf{p} \to (\mathbf{p} \vee \mathbf{q})$.

In both cases, the output Qtree is also evoked by a simpler expression, $\neg p \to q$ and q respectively. This stems from two features of conditional Qtree formation: (i) p in the disjunctive consequent of (131b) gets "ignored" at the Qtree level, due to the antecedent restricting the consequent Qtree to the $\neg p$-domain; and (ii) nodes falsifying the antecedent (here, p) are *not* treated as verifying. As a result, both Qtrees in Figure L are Q-Redundant given (131b), and (131b) turns out to be a Q-Redundant sentence.

Turning to (131d) $= p \vee (\neg q \to p)$, its only possible Qtree, shown in Figure M, is obtained using Figures A (for p) and J (for $\neg q \to p$) along with the union rule for disjunctive Qtrees. Because both input Qtrees in Figures A and J are the same, the (well-formed) output Qtree in M is also similar. It is therefore Q-Redundant given (131d).

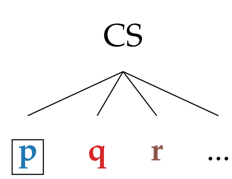
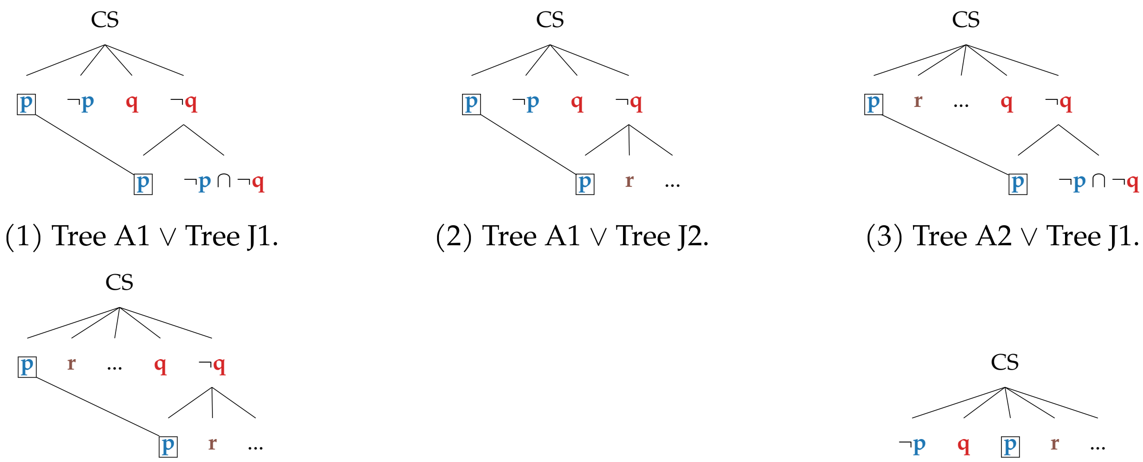


Figure M: Only possible Qtree for (131d) $= \mathbf{p} \vee (\neg \mathbf{q} \to \mathbf{p})$, obtained from Tree A2 $\vee$ Tree J3.**Same as Qtree A2 evoked by** p.

Other Qtree combinations for p and $\neg q \rightarrow p$ cannot be properly disjoined (unioned), because the partitionings introduced by p and that introduced by $(\neg q \rightarrow p)$ at depth 1 differ from each other, leading to the kind of ill-formedness issue described in Figure F. This is detailed in Figure N. In summary, (131d) is not compatible with any well-formed Qtree, and therefore should be deemed odd.



(1) Tree A1 $\vee$ Tree J1. (2) Tree A1 $\vee$ Tree J2. (3) Tree A2 $\vee$ Tree J1. (4) Tree A2 $\vee$ Tree J2. (5) Tree A1 $\vee$ Tree J3.

Figure N: Other, ill-formed Qtrees for (131d) = $\mathbf{p} \vee (\neg \mathbf{q} \rightarrow \mathbf{p})$, obtained by disjoining Qtrees from Figure A with Qtrees from Figure J. In all cases, the children of the CS root do not properly partition the CS. Additionally, four of these "trees" actually feature a cycle!

Finally, the only possible Qtree associated with (131e) is given in Figure O. Figure P further details how this Qtree was derived *via* the conditional Qtree formation rule. The Qtree in Figure O is such that no verifying node remains after the conditional rule applies. It is thus considered ill-formed due to Empty Labeling (see (75)) and (131e) is in turn predicted to be odd as per (76).

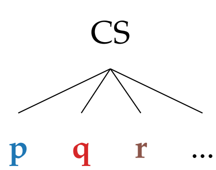


Figure O: Only possible Qtree for (131e) = $\neg(\mathbf{p} \vee \mathbf{q}) \rightarrow \mathbf{p}$, obtained from $\neg$(Tree E) $\rightarrow$ Tree A1 / A2. **Odd due to Empty Labeling.**

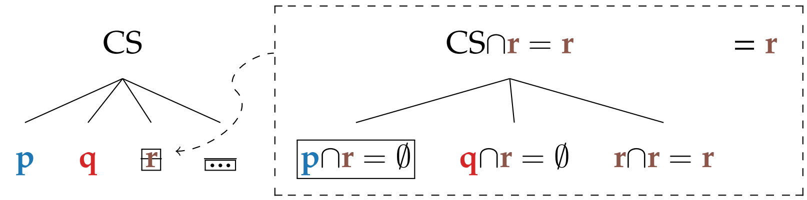
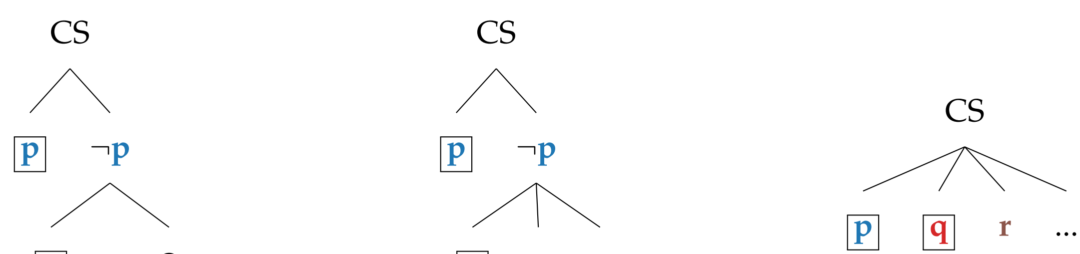


Figure P: Breakdown of the derivation of Figure O.

Now that we have accounted for the three challenging odd sentences from our dataset, we turn to the only variant that gets rescued from infelicity, namely (131c).

4.4.3 Ruling in (131c)

We now show that Q-NON-REDUNDANCY spares the felicitous (131c) $= p \vee (\neg p \rightarrow q)$. The relevant Qtrees, shown in Figure Q, are obtained using Figures A (for p) and G (for $\neg p \rightarrow q$), combined with the union rule for disjunctive Qtrees. Because the Qtrees evoked by p that are properly disjoinable with those evoked by $\neg p \rightarrow q$, are structurally contained in them, the Qtrees in Figure Q are structurally similar to those from Figure G (corresponding to $\neg p \rightarrow q$), but, crucially, display an extra verifying p-leaf, contributed by the first disjunct of (131c).



(1) Tree A1 $\vee$ Tree G1. (2) Tree A1 $\vee$ Tree G2. (3) Tree A2 $\vee$ Tree G3. **Same as Qtree E** evoked by $p \vee q$.

Figure Q: Qtrees for (131c) $= \mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$.

This extra leaf guarantees that Qtrees Q1 and Q2 *cannot* be evoked by a simplification of sentence (131c). To show this, let us review the nine possible simplifications of (131c), which for the sake of clarity we divide into three groups – see Table 4.1.

Group	Simplifications
1	$\mathbf{p}$, $\mathbf{q}$, $\neg\mathbf{p}$, $\mathbf{p} \vee \mathbf{q}$, $\mathbf{p} \vee \mathbf{p}$, $\mathbf{p} \vee \neg\mathbf{p}$
2	$\neg\mathbf{p} \to \mathbf{q}$
3	$\mathbf{p} \to \mathbf{q}$, $\mathbf{p} \vee (\mathbf{p} \to \mathbf{q})$

Table 4.1: Gathering the formal simplification of $p \vee (\neg p \to q)$.

Starting with the simplifications in group 1, we notice that their corresponding Qtrees are always of depth 1, because they correspond to Figures A, B, or E, or to slight variants thereof, where only the verifying nodes change (due to the effect negation, or union). Such Qtrees are thus obviously distinct from Qtrees Q1 and Q2, which have depth 2. So the simplifications in group 1 cannot make Qtrees Q1 and Q2 Q-Redundant given $(131c)$.

Regarding the $\neg p \to q$ simplification (group 2), it was shown to gives rise to the Qtrees in Figure G. Crucially, these Qtrees do not count the p-leaf as verifying–unlike those in Figures Q1 and Q2. So they cannot be used to trigger Q-Non-Redundancy.

As for the simplifications in group 3, we show in that the Qtree for $p \to q$ has only one layer, and moreover displays Empty Labeling.

(1) Tree A1 $\to$ Tree B1 / B2.

(2) Tree A2 $\to$ Tree B1 / B2.

Figure R: Qtrees for $\mathbf{p} \to \mathbf{q}$. **Odd due to Empty Labeling.**

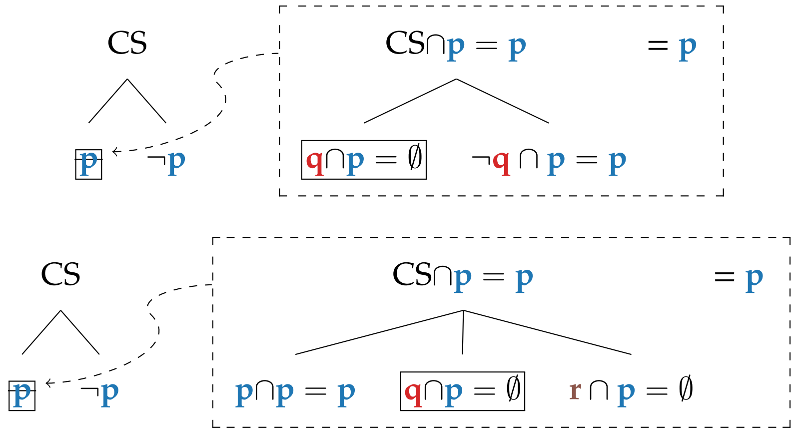


Figure S: Breakdown of the derivation of Figure R1 (can be easily adapted to Figure R2).

Again, this kind of Qtree is clearly distinct from the Qtrees for $p \vee (\neg p \rightarrow q)$ in Figures Q1 and Q2. Disjunction with a Qtree for p, to form a Qtree for $p \vee (p \rightarrow q)$ (the last simplification in group 3), does not help either: the resulting Qtree gains one verifying p-leaf, but retains one single layer, making it distinct from Qtrees Q1 and Q2.

Therefore, no simplification of (131c) gives rise to Qtrees like Q1 and Q2, and, as a result, such Qtrees are *not* Q-Redundant given (131c). This means that (131b) should not be deemed odd on the basis of Q-Non-Redundancy, in line with intuitions.

To sum up, we accounted for the pattern in (131) by appealing to a model of compositional QuDs assigning disjunctions and conditionals different "inquisitive" contributions, and by redefining Non-Redundancy on pairs formed by sentences and their possible implicit QuD trees. In the next Section, we discuss how this new model relates to earlier similar approaches and to the Maxim of Manner.

4.5 Taking stock

4.5.1 Comparison with similar approaches

Previous approaches to redundancy were focusing on Hurford phenomena (Hurford, 1974; Marty and Romoli, 2022; Mandelkern and Romoli, 2018 i.a.), in particular Hurford Disjunctions, whereby one disjunct contextually entails the other. We briefly mentioned such constructions when discussing the shortcomings of Non-Triviality in Section 4.2.4;

the critical datapoint is repeated in (149) below.[14]

(149) a. # Jo is in Cambridge or in Massachusetts. $\mathbf{p}^{+} \vee \mathbf{p}$

b. # Jo is in Massachusetts or in Cambridge. $\mathbf{p} \vee \mathbf{p}^{+}$

Many approaches to such phenomena were exploiting ideas similar to those presented here. Ippolito (2019) and Zhang (2022) in particular, both use structures very close to Qtrees, and propose that oddness can arise from specific configurations in these structures. In Zhang's framework, this takes the form of a distinctness constraint between answers to the same question; in Ippolito's, this is cashed out in terms of matching specificity between disjoined alternatives. In both cases, the constraints posited are mostly structural and only target *the* implicit alternative set/QuD evoked by any given sentence. Even if the constraints posited are very sensible, they are not directly motivated by familiar, competition-based, pragmatic principles. As we will further show in the next Section (and also, in the next Chapter), our implementation of Q-Non-Redundancy fills this gap and provides a perhaps more explanatory account of QuD-driven cases of oddness.[15]

Additionally, previous accounts were mostly focused on disjunctions, or more generally, configurations where two subconstituents could be taken to answer the same QuD.[16] But it remained unclear how the overarching question was derived in each case, and whether it should be in the first place. Our system also fills this gap, in providing a set of recipes to compositionally derive implicit QuDs, instead of taking them for granted. Together with Q-Non-Redundancy, this machinery captured the target contrasts. The next Chapters will further show how it generalizes beyond the datapoints discussed here.

Our approach also differs from Inquisitive Semantics (Mascarenhas, 2008; Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2017; Ciardelli et al., 2018; Zhang, to appear), whereby assertive sentences and questions are fundamentally the same kinds of objects. Under the current view, sentences retain a "semantic", truth-conditional component, and evoke Qtrees at a distinct "inquisitive" level. While the semantic module is sensitive to truth conditions, the pragmatic module is assumed to be sensitive to the interaction between form, meaning, and inquisitive content. So our approach may be seen

[14]This example will be the focus of Chapter 5, and will lead us to introduce a slightly more involved version of Q-Non-Redundancy.

[15]Zhang (to appear) proposes a different account that also goes in this direction.

[16]Ippolito (2019) in fact argues that this comprises Sobel Sequences and sequences of superlatives. We will discuss SObel cases in Chapter **??**.

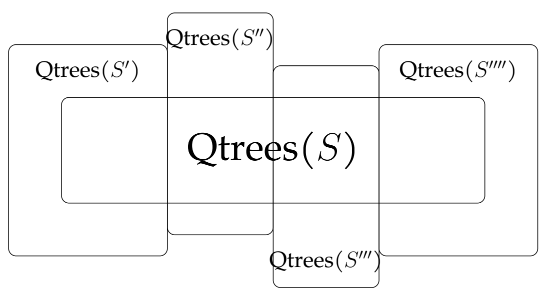
as an “inquisitive pragmatics”. The difference between the two frameworks is particularly visible when it comes to negation: in Inquisitive Semantics, negation removes structure by collecting and collapsing all information states incompatible with those of the prejacent. In our framework, negation retains structure, and simply flips verifying nodes.

4.5.2 An “inquisitive” Maxim of Manner?

Earlier definitions of Redundancy were linking this notion to Grice’s Maxim of Manner (Manner for short, see (132)). Roughly, this Maxim states that if two sentences have the same logical contribution, then the more concise one should be preferred. Is Q-Non-Redundancy a proper extension of Manner at the inquisitive level? At first blush, not exactly. In particular, Q-Non-Redundancy does not state that, for a sentence S to be Q-Redundant, *all* Qtrees compatible with S should be identified (*via* a bijective operation) to *all* Qtrees compatible with some simplification of S. This perhaps would have been the most intuitive extension of Manner as the QuD level, and is depicted in Figure T1. Instead, Q-Non-Redundancy states that for a sentence S to be Q-Redundant, *each* Qtree compatible with S should be identified with *some* Qtree generated by *some* simplification of S. This configuration, depicted in Figure T2, is significantly less strong, i.e. predicts *more* sentences to be redundant. For instance, we concluded that (131b) was Q-Redundant because each of its Qtrees could be identified with Qtrees evoked by *distinct* simplifications of (131b) – namely q and $\neg p \to q$. Moreover, q and $\neg p \to q$ themselves led to Qtrees that were *not* compatible with (131b).

$$\boxed{\text{Qtrees}(S) = \text{Qtrees}(S')}$$

(1) What a more “intuitive” version of Q-Non-Redundancy could have been (S' refers to some simplification of S).



(2) What it takes for S to be Q-Redundant (S', S'', S''', and S'''' refer to simplifications of S).

Figure T: Comparing Q-Non-Redundancy to a more “intuitive” extension of Manner to the QuD domain.

Our definition of Q-Non-Redundancy also leaves space for other Qtree well-formedness constraints to contribute to a sentence’s oddness. For instance a sentence S may be deemed odd because *some* Qtrees compatible with S can be identified with *some*

Qtree generated by *some* simplification of S (violating Q-NON-REDUNDANCY), and the other Qtrees compatible with S are ruled-out EMPTY LABELING, or any other relevant constraint.[17] This mixed-oddness profile is schematized in Figure U.

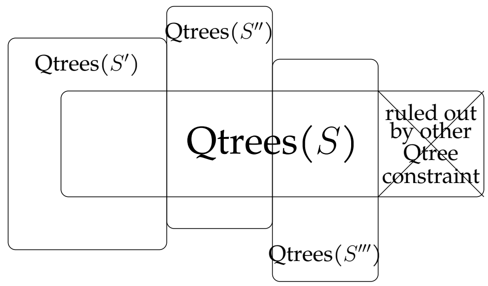


Figure U: What it means for S to be odd partly due to Q-NON-REDUNDANCY, partly due to other Qtree well-formedness constraints, e.g. EMPTY FLAGGING.

If Q-NON-REDUNDANCY at the sentential level is not an intuitive extension of MANNER, Q-NON-REDUNDANCY defined on LF-Qtree pairs (see 153a), is. To see this, one must define the simplification of a LF-Qtree *pair* (S, T), where T is a Qtree evoked by S, as a pair (S', T') where S' is a formal simplification of S in the sense of (**??**), and T' is a Qtree evoked by S'. Additionally, one must define equivalence between LF-Qtree pairs as equivalence between their Qtree-component. This yields a definition of Q-RELEVANCE-as-MANNER, given in (154) that is set as a two-dimensional optimization problem on both LFs (which calibrate conciseness, and, indirectly, informativeness) and Qtrees (which calibrate informativeness).

(154) a. *LF-Qtree pair.* (S, T) is a well-formed LF-Qtree pair iff S evokes T.

b. **Q-REDUNDANCY *as* MANNER. If (S, T) and (S', T') are two LF-Qtree pairs that are equivalent to each other, then the most concise of the two should be preferred.**

c. *Conciseness of a LF-Qtree pair.* If (S, T) and (S', T') are two LF-Qtree pairs, (S', T') is more concise than (S, T) iff S' is a formal simplification of S as per (134).

d. *Equivalence between LF-Qtree pairs.* If (S, T) and (S', T') are two LF-Qtree pairs, (S', T') is equivalent to (S, T) iff $T = T'$.[18]

Now that we clarified how Q-NON-REDUNDANCY can be seen as a proper extension of earlier pragmatic principles to the domain of LF-Qtree pairs, we discuss the effect of dis-

[17]Chapter **??** will introduce another such constraint, Q-RELEVANCE, which will enter the mix when evaluating sentence oddness.

[18]This is simplified for the purposes of this paper: equality could be replaced by any more elaborate relation between Qtrees; see footnote 12, and Chapter 5.

junct ordering on the felicitous (131c), as well as a few more interesting cases derived from the sentences in (131).

4.6 Exploring elaborations of the target sentences

4.6.1 Effect of disjunct ordering in the felicitous case

First, let us briefly come back to the pair (129b)-(127b), repeated in (155). The two sentences in (155) only differ in the ordering of their disjuncts. At this point, we predict both to escape Q-Non-Redundancy, and more generally oddness. This, again, is because the introduction of p as a disjunct makes it at-issue, while it would be "neglected" if the sentence were simplified into its conditional disjunct. (155b) however, sounds worse than (155a), and even more so if the conditional disjunct did not feature inversion.

(155) a. Either Jo is at SuB or if he is not at SuB then he is in Cambridge.
$\mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$

b. ? Either Jo is in Cambridge if not at SuB, or he is at SuB.
$(\neg \mathbf{p} \rightarrow \mathbf{q}) \vee \mathbf{p}$

We suggest this contrast is caused by an independent, incremental constraint targeting Qtree derivations. As observed in Section 4.4.3, (155a) is compatible with two well-formed Qtrees, repeated in Figure V. Figures W and X summarize the "ingredients" used to derive these two disjunctive Qtrees: a Qtree for p, and two possible Qtrees for $\neg p \rightarrow q$.

(1) Tree A1 $\vee$ Tree G1.

(2) TreeA1 $\vee$ Tree G2.

Figure V: Non Q-Redundant Qtrees for (131c)/(155a) = $\mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$.

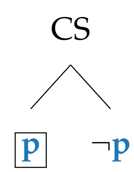


Figure W: Qtree for $\mathbf{p}$ = *Jo is at SuB* (first disjunct of (131c)/(155a)), used to derive the Qtrees in Figure V.

(1) Tree C1 $\rightarrow$ Tree B1. (2) Tree C1 $\rightarrow$ Tree B2.

Figure X: Qtrees for $\neg\mathbf{p} \rightarrow \mathbf{q}$ = *If Jo is not at SuB then he is in Cambridge* (second disjunct of (131c)/(155a)), used to derive the Qtrees in Figure V.

We observe that the Qtree in Figure W, which is evoked by p and corresponds to the first disjunct of (155a), is structurally contained in the Qtrees in Figure X, which are evoked by $\neg p \rightarrow q$ and correspond to the second disjunct of (155a). In other words, (155a)'s first disjunct evokes a Qtree that is coarser-grained (i.e. less specific) than the Qtrees evoked by (155a)'s second disjunct. The opposite holds for (155b). In other words, the disjunction in (155a) takes two Qtrees of increasing specificity as input (from left to right), while the disjunction in (155b) takes two Qtrees of decreasing specificity. And it is reasonable to think that the latter order should be preferred. This is supported by the sequences of questions in (156): it appears more natural to ask a less specific question (e.g., about countries), before a more specific one (e.g., about cities), than the other way around.[19] The latter ordering in fact seems to suggest that the more specific question would not allow to infer the exact answer to the less specific one.

(156) a. In which country does Jo live? And in which city?

b. ? In which city does Jo live? And in which country?

[19]Similar considerations will ground our definition of Q-Relevance in Chapter 6, in the context of Hurford Conditionals. Q-Relevance will yield slightly more subtle predictions than (157).

Assuming that the country-level questions in (156) are structurally contained in the city-level questions (as argued in Chapter 2), we derive the following generalization covering both (155) and (156).[20]

(157) **Incremental Qtree Containment.** Let X and Y be LFs, and $\circ$ be a binary operator. If $X \circ Y$ and $Y \circ X$ have same meaning and same evoked Qtrees, and if $\forall T \in Qtrees(X \circ Y)$, T is obtained from $T' \in Qtrees(X)$ and $T'' \in Qtrees(Y)$ with $T' \subset T''$, then $X \circ Y$ should be preferred over $Y \circ X$.

4.6.2 Double or-to-if

Before wrapping this Chapter, let us briefly discuss more complex variants of (126), derived *via* two applications of the *or-to-if* tautology. (158a) sounds clearly redundant, while (158b) and (158b) somehow feel contradictory. (158d) and (158d) appear very tough to make sense of.

(158) a. # If Jo isn't at SuB then, if he isn't at SuB then he is in Cambridge.
$\neg \mathbf{p} \rightarrow (\neg \mathbf{p} \rightarrow \mathbf{q})$

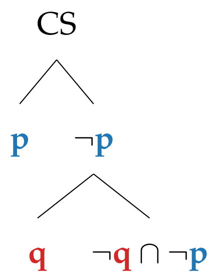
b. # If Jo isn't at SuB then, if he isn't in Cambridge then he is at SuB.
$\neg \mathbf{p} \rightarrow (\neg \mathbf{q} \rightarrow \mathbf{p})$

c. ?? If it's not that Jo is in Cambridge if not at SuB, then he isn't at SuB.
$\neg(\neg \mathbf{p} \rightarrow \mathbf{q}) \rightarrow \mathbf{p}$

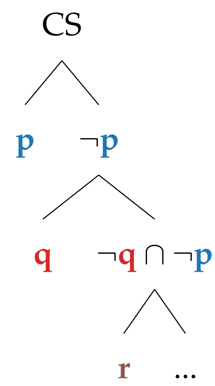
d. # If it's not that Jo is at SuB if not in Cambridge, then Jo is at SuB.
$\neg(\neg \mathbf{q} \rightarrow \mathbf{p}) \rightarrow \mathbf{p}$

The model laid out in this paper predicts all the sentences in (158) to be odd, for different reasons. (158a) turns out Q-Redundant, because all the Qtrees it gives rise to are the same as the ones generated by its consequent $\neg p \rightarrow q$ (see Figure G). Both (158b) and (158d) generate Qtrees that invariably display Empty Flagging, see Figures Y and Z. In both cases, this can be traced back to the fact that the "restrictor" nodes in which the consequent Qtree is "plugged", are sets of $\neg p$-worlds, and p is precisely what the consequent Qtree would have contributed as verifying node.

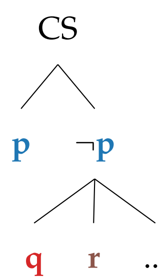
[20] (156) may seem reminiscent of Hurford Disjunctions (Hurford, 1974). Chapter 5 will predict Hurford Disjunctions to be bad in *both* orders due to an updated version of Q-Non-Redundancy, i.e., *independently* of the constraint in (157).



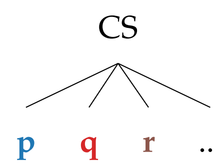
(1) Tree C1 → Tree J1.



(2) Tree C1 → Tree J2.

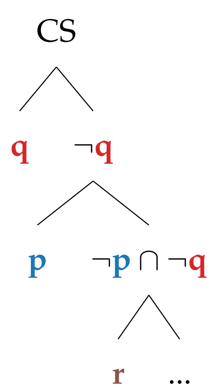


(3) Tree C1 → Tree J3.

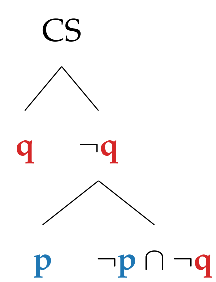


(4) Tree C2 → Tree J1 / J2 / J3.

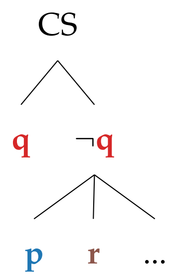
Figure Y: Qtrees for #(158b) = $\neg p \rightarrow (\neg q \rightarrow p)$. **Odd due to Empty Labeling.**



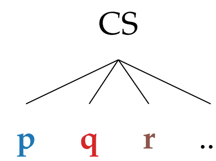
(1) ¬(Tree J1) → Tree A2.
Empty Labeling.



(2) ¬(Tree J1) → Tree A1.
Empty Labeling.



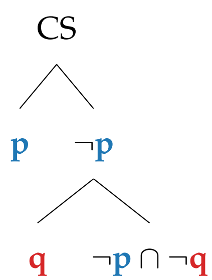
(3) ¬(Tree J2) → Tree A1 / A2.
Empty Labeling.



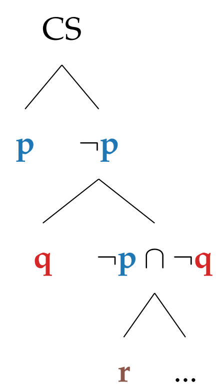
(4) ¬(Tree J3) → Tree A1 / A2.
Empty Labeling.

Figure Z: Qtrees for #(158d) = $\neg(\neg q \rightarrow p) \rightarrow p$. **Odd due to Empty Labeling.**

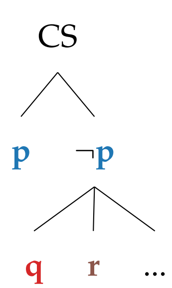
Lastly, (158c) represents a mixed case of oddness: most of the Qtrees it evokes display Empty Flagging, and one tree turns out Q-Redundant due to the p-simplification of the sentence. This is further detailed in Figure AA.



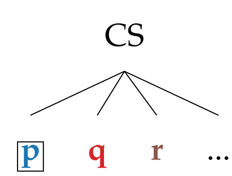
(1) ¬(Tree G1) → Tree A1.
Empty Labeling.



(2) ¬(Tree G1) → Tree A2.
Empty Labeling.



(3) ¬(Tree G2) → Tree A1 / A2.
Empty Labeling.



(4) ¬(Tree G3) → Tree A1 / A2.
Same as Figure A2.

Figure AA: Qtrees for ??(158c) = $\neg(\neg p \rightarrow q) \rightarrow p$. **Odd for mixed reasons (Empty Labeling/Q-Non-Redundancy).**

It is worth mentioning that some of the Qtrees in Figure AA, were computed from Qtrees that *themselves* were not complying with Qtree well-formedness constraints. For instance, the Qtree in Figure AA4 (which turns out Q-Redundant given (158c)), is derived using the Qtree in Figure G3, which is itself Q-Redundant given $\neg q \to p$. Maybe this derivation should have been blocked altogether, in virtue of one of its inputs being Q-Redundant. More generally, this raises the question whether Non-Q-Redundancy should apply globally (*à la* Global Redundancy), i.e. once all the possible Qtrees for a given sentence are derived, or, locally (*à la* Local Redundancy), i.e. each time a Qtree is compositionally derived from input Qtree(s). We leave this question for future work.

4.7 Conclusion and outlook

In this Chapter, we presented novel data based on logical variants of $p \lor p \lor q$, that appeared challenging to account for while retaining classic results on other families of odd sentences, in particular Hurford Disjunctions and Conditionals (Hurford, 1974; Mandelkern and Romoli, 2018). In particular, these data appeared problematic for Kalomoiros's recent account, otherwise characterized by a wide empirical coverage, including the very challenging Hurford Conditionals.

This Chapter then proposed an account of the problematic paradigm in the QuD-framework, based on two core ideas introduced in Chapter 2, namely, that sentences have to be good answer to good questions (Katzir and Singh, 2015), and that disjunctions and conditionals evoke distinct kinds of questions (Qtrees). Beyond these two insights, we devised a novel implementation of Non-Redundancy, which was made sensitive to how sentences "package" information *via* their Qtree. The next two Chapters extend this framework to capture Hurford Disjunctions and Conditionals (Hurford, 1974; Mandelkern and Romoli, 2018) – leading us to update Q-Non-Redundancy, and to introduce yet another Qtree well-formedness constraint, in the form of Q-Relevance. More broadly, this approach suggests that oddness may come in different "flavors" and that sentences may be odd due to a conspiration of these various factors.

A WORD ON THE CONJUNCTIVE ALTERNATIVE FROM MAYR AND ROMOLI??

Chapter 5

All the paths lead to Noto: oddness in non-scalar Hurford Disjunctions[1]

This Chapter constitutes a direct follow-up of Chapter 4, and proposes to generalize the NON-REDUNDANCY constraint introduced in that Chapter, in order to account for a range of Hurford Disjunctions (Hurford, 1974; Marty and Romoli, 2022). Specifically, we will argue that the proper notion of equivalence between Qtrees that feeds Q-NON-REDUNDANCY is not exactly equality (of nodes and edges) as proposed in Chapter 4, but instead, structural equality combined with equality between minimal strategies of inquiry – defined in terms of optimal paths from the CS-root to verifying nodes. This will preserve the results obtained for the dataset presented in Chapter 4, and additionally cover the relevant Hurford cases.

5.1 Introducing Huford Disjunctions

In Chapter 4, we introduced a new NON-REDUNDANCY constraint accounting for a problematic dataset. We showed that previous approaches to oddness either could not fully capture these data (Katzir and Singh, 2014, Kalomoiros, 2024), or could (Mayr and Romoli, 2016), but at the cost of jeopardizing results previously obtained on other famously odd sentences. Among these famously odd sentences, are Hurford Disjunctions (Hurford, 1974), henceforth **HD**s, exemplified in (159). HDs typically feature contextually entailing

[1]This Chapter is partly based on Hénot-Mortier (to appear), and hopefully spells out the model and arguments more extensively and explicitly. I would like to thank the audience and reviewers of SuB29 and of the 2024 BerlinBrnoVienna Workshop, in particular Alex Kalomoiros, Flavia Naehrlich, Jacopo Romoli, Benjamin Spector, and Raven Zhang, for relevant questions, datapoints and suggestions regarding earlier iterations of this project.

disjuncts (abbreviated $p^+ \vDash p$), and are generally odd regardless of the order of the weak (p) vs. strong (p^+) disjunct.[2]

(159) a. # SuB29 will take place in Noto[3] or Italy. $\mathbf{p^+} \vee \mathbf{p}$

b. # SuB29 will take place in Italy or Noto. $\mathbf{p} \vee \mathbf{p^+}$

Accounting for (159) in an explanatory way, i.e. without simply stating that disjunctions should not feature entailing disjuncts, is challenging. First, identifying a specific, well-motivated pragmatic principle susceptible to account for (159) is not necessarily easy (though, Chapter 4, Section 4.2, already foreshadowed several potential solutions). Second, pre-theoretically similar instances of oddness seem to affect expressions in which disjuncts are *not* in a relation of contextual entailment. For instance, given that the Basque country encompasses Northern Central Spain and Southwestern France, saying that *SuB29 will take place in the Basque country* is *compatible* with saying that *SuB29 will take place in France*. Yet, the sentences in (160) still sound quite odd (Singh, 2008b). We will call such expressions Compatible Hurford Disjunctions, henceforth **CHD**s.

(160) a. # SuB29 will take place in the Basque country or France. $\mathbf{q} \vee \mathbf{p}$; $\mathbf{q} \wedge \mathbf{p} \nvDash \bot$

b. # SuB29 will take place in France or in the Basque country. $\mathbf{p} \vee \mathbf{q}$; $\mathbf{p} \wedge \mathbf{q} \nvDash \bot$

Relatedly, disjunctions derived from HDs by further disjoining the stronger disjunct with a proposition incompatible with the weaker disjunct, also appear quite odd (Marty and Romoli, 2022). This is shown in (161). This is surprising essentially because, just like CHDs, such expressions feature merely compatible disjuncts. The issue seems to come from the observation that one disjunct involves a sub-expression (p^+) entailing the other disjunct (p). For this reason, these disjunctions were dubbed Long-Distance Hurford Disjunctions, henceforth **LDHD**s.

(161) a. # Either SuB29 will take place in Noto or Paris, or it will take place in Italy.
$(\mathbf{p^+} \vee \mathbf{r}) \vee \mathbf{p}$ $\mathbf{p^+} \vDash \mathbf{p}$; $(\mathbf{p^+} \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

b. # Either SuB29 will take place in Italy, or it will take place in Noto or Paris.
$\mathbf{p} \vee (\mathbf{p^+} \vee \mathbf{r})$ $\mathbf{p^+} \vDash \mathbf{p}$; $(\mathbf{p^+} \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

The rest of this Chapter proposes an account of these data, and is organized as follows. Section 5.2 reviews four existing accounts of pragmatic oddness, and shows that they can

[2]A notable exception is when the two disjuncts are the same *modulo* scalemate expressions, such as *some* vs. *all*; in that case, HDs may be rescued from infelicity (Gazdar, 1979; Singh, 2008a; Fox, 2018; Ippolito, 2019; Hénot-Mortier, 2023 i.a.). We do not cover these cases here, but Chapters 7 and **??** provide an overview of the challenges raised by these "scalar" variants, and sketch solutions based on the current framework.

[3]Noto is located in Italy and is where the main session of SuB29 was organized.

only partially account for the data presented in this introduction. Section 5.3 presents Q-Non-Redundancy, as defined in Chapter 4, shows that it is also insufficient in its current form, and proposes to update it in a "conservative" way, based on a new notion of equivalence between Qtrees. Section 5.4 shows that this newly defined constraint captures both HDs and LDHDs, and additionally discusses how the Qtree model in fact captures CHDs, independently of Q-Non-Redundancy. Section 5.5 discusses a few conceptual and empirical implications of this analysis, and in particular compares it to previous Redundancy-based and QuD-tree-based accounts of HDs. Section 5.6 concludes this Chapter.

We now proceed to reviewing why some of the data presented in this introduction are problematic for previous accounts of oddness (excluding the new QuD-driven approach introduced in Chapter 4). We will follow the same structure as in Chapter 4, Section 4.2.

5.2 Previous accounts of oddness, and their shortcomings

The Section presents four existing accounts of oddness already covered in Chapter 4: Global Non-Redundancy, Local Non-Redundancy, Super-Redundancy, and Non-Triviality.[4] We show that all accounts straightforwardly capture HDs, but struggle accounting for CHDs. Some, but not all approaches, manage to capture the case of LDHDs.

5.2.1 Global Non-Redundancy

This approach builds on the idea that sentences featuring unnecessary verbosity should be deemed odd. This is cashed out in (133) (repeated from Chapter 4). Roughly, (133) states that, if two sentences are contextually equivalent, then the simpler one should be preferred, and the more complex one should be deviant. Simplicity is understood as structural, following Katzir (2007) – see (134), repeated below. We dub the principle in (133) Global Non-Redundancy, because contextual equivalence and simplicity are evaluated at the level of the entire sentence, and not locally.[5]

(133) **Global Non-Redundancy** (Meyer, 2013; Mayr and Romoli, 2016). A sentence S cannot be used in context c if there is a sentence S' s.t. S' is a simplification of S and $S' \equiv_c S$.

[4]This Section relies extensively on Marty and Romoli's overview paper (Marty and Romoli, 2022).
[5]A local variant of this principle will be investigated in the next Section.

(134) *Structural simplicity* (*Katzir, 2007*). S' is a simplification of S if S' can be derived from S by replacing nodes in S with their subconstituents.

(133) correctly predicts HDs to be deviant in both orders, because they are contextually equivalent to their weaker disjunct ($p^+ \vee p \equiv p \vee p^+ \equiv p$). However, it predicts CHDs and LDHDs to be fine. This is proved in (162) and (163) respectively. The issue seems to be that (133) is too "global" and thus not sensitive to the internal organization of the disjuncts. This is especially obvious in the case of LDHDs, whereby infelicity seems to arise from the logical relation between a "high", weak disjunct, and a "low", strong disjunct embedded within an inner disjunction.

(162) (133) predicts CHDs to be fine.
Let $\mathbf{p}$, $\mathbf{q}$ be s.t. $\mathbf{p} \wedge \mathbf{q} \nvDash \bot$ (compatible), $\mathbf{p} \nvDash \mathbf{q}$ and $\mathbf{q} \nvDash \mathbf{p}$ (non-entailing in any direction).
If $\mathbf{p} \vee \mathbf{q} \equiv \mathbf{p}$, then we would have $\mathbf{q} \vDash \mathbf{p} \vee \mathbf{q} \equiv \mathbf{p}$; contradiction.
If $\mathbf{p} \vee \mathbf{q} \equiv \mathbf{q}$, then we would have $\mathbf{p} \vDash \mathbf{p} \vee \mathbf{q} \equiv \mathbf{q}$; contradiction.
So $\mathbf{p} \vee \mathbf{q} \not\equiv \mathbf{p}$ and $\mathbf{p} \vee \mathbf{q} \not\equiv \mathbf{q}$.

(163) (133) predicts LDHDs to be fine.
Let $\mathbf{p}$, $\mathbf{p}^+$and $\mathbf{r}$ be s.t. $\mathbf{p}^+ \vDash \mathbf{p}$ ("long-distance" entailment), and $\mathbf{p} \wedge \mathbf{r} \vDash \bot$.
We define $\mathbf{q}$ as $\mathbf{p}^+ \vee \mathbf{r}$.
Then, $\mathbf{p}$, $\mathbf{q}$ are s.t. $\mathbf{p} \wedge \mathbf{q} \nvDash \bot$ (compatible), $\mathbf{p} \nvDash \mathbf{q}$ and $\mathbf{q} \nvDash \mathbf{p}$ (non-entailing in any direction).
Therefore $\mathbf{p} \vee \mathbf{q} \nvDash \mathbf{p}$ and $\mathbf{p} \vee \mathbf{q} \nvDash \mathbf{q}$, as per proof (162).
Therefore $\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r}) \nvDash \mathbf{p}$ and $\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r}) \nvDash \mathbf{p}^+ \vee \mathbf{r}$.

5.2.2 Local Non-Redundancy

Katzir and Singh (2014) propose a local implementation of Global Non-Redundancy, stating that the semantic computation evaluates, at certain nodes, whether the composition principle that applies there is non-vacuous. This gives rise to the principle in (139).

(139) **Local Non-Redundancy** (Katzir and Singh, 2014). S is deviant if S contains γ s.t. $[\![\gamma]\!] = [\![O(\alpha, \beta)]\!] \equiv_c [\![\zeta]\!]$, $\zeta \in \{\alpha, \beta\}$.

Just like Global Non-Redundancy, (139) correctly predicts HDs to be deviant in both orders, because they are contextually equivalent to their weaker disjunct (p). But it also incorrectly predicts CHDs to be fine, for the same reason as Global Non-Redundancy; see proof (162). The same incorrect prediction hold for LDHDs, as shown in (164).

(164) (139) predicts LDHDs to be fine.

Let $\mathbf{p}$, $\mathbf{p}^+$and $\mathbf{r}$ be s.t. $\mathbf{p}^+ \vDash \mathbf{p}$ ("long-distance" entailment), and $\mathbf{p} \vee \mathbf{r} \vDash \bot$.

Evaluating the outer disjunction: $\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r}) \nvDash \mathbf{p}$ and $\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r}) \nvDash \mathbf{p}^+ \vee \mathbf{r}$ (see proof (163)). Not locally redundant.

Evaluating the inner disjunction: given that $\mathbf{p}^+ \vDash \mathbf{p}$ and $\mathbf{p} \wedge \mathbf{r} \vDash \bot$, $\mathbf{p}^+ \wedge \mathbf{r} \vDash \bot$ (i.e. the two disjuncts of the inner disjunction are incompatible). Therefore, $\mathbf{p}^+ \vee \mathbf{r} \not\equiv \mathbf{p}^+$ and $\mathbf{p}^+ \vee \mathbf{r} \not\equiv \mathbf{r}$. Not locally redundant.

Even if Local Non-Redundancy was designed to be more "local", it still cannot capture the "long-distance" logical dependency between p and p^+ in LDHDs. The case of CHDs also remains problematic, due to their apparent "opacity": structurally, CHDs amount to one single disjunction, and as such, cannot be better captured by Local Non-Redundancy (as compared to Global Redundancy).

5.2.3 Super-Redundancy

Kalomoiros (2024), elaborating on Katzir and Singh (2014)'s view, introduces Super-Redundancy. Roughly, a sentence S is super-redundant if it features a binary operation taking a constituent C as argument, and moreover there is no way of strengthening C to C^+ that would make the resulting sentence S^+ non-redundant (i.e., non-equivalent to its counterpart where C^+ got deleted).

(142) **Super-Redundancy** (Kalomoiros, 2024). A sentence S is infelicitous if it contains $C * C'$ or $C' * C$, with $*$ a binary operation, s.t. $(S)^-_C$ is defined and for all D, $(S)^-_C \equiv S_{Str(C,D)}$. In this definition:

- $(S)^-_C$ refers to S where C got deleted;
- $Str(C, D)$ refers to a strengthening of C with D, defined inductively and whose key property is that it commutes with negation ($Str(\neg\alpha, D) = \neg(Str(\alpha, D))$), as well as with binary operators ($Str(O(\alpha, \beta), D) = O(Str(\alpha, D), Str(\beta, D))$);
- $S_{Str(C,D)}$ refers to S where C is replaced by $Str(C, D)$.

Interestingly, this principle predicts both HDs and LDHDs to be degraded (see (165)[6] and (166)). Thus, it constitutes an improvement over Global/Local Non-Redundancy.

(165) HDs are Super Redundant (SR).

We show (159a)=$\mathbf{p}^+ \vee \mathbf{p}$ and (159b)=$\mathbf{p} \vee \mathbf{p}^+$ are SR.

[6]Proof adapted from the original paper (Kalomoiros, 2024).

In either case, take C = $\mathbf{p}^+$.

We then have $(159a)^-_C = (159b)^-_C = \mathbf{p}$

$$
\begin{aligned}
\forall D.\ (159a)_{Str(C,D)} = (159b)_{Str(C,D)} &= (\mathbf{p}^+ \wedge D) \vee \mathbf{p} \\
&\equiv (\mathbf{p}^+ \vee \mathbf{p}) \wedge (D \vee \mathbf{p}) \\
&\equiv \mathbf{p} \wedge (D \vee \mathbf{p}) \\
&\equiv (\mathbf{p} \wedge D) \vee \mathbf{p} \\
&\equiv \mathbf{p} = (159a)^-_C = (159b)^-_C
\end{aligned}
$$

(166) LDHDs are Super Redundant (SR).

We show (161a)=$(\mathbf{p}^+ \vee \mathbf{r}) \vee \mathbf{p}$ and (161b)=$\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r})$ are SR.

In either case, take C = $\mathbf{p}^+$.

We then have $(161a)^-_C = (161b)^-_C = \mathbf{p} \vee \mathbf{r}$

$$
\begin{aligned}
\forall D.\ (161a)_{Str(C,D)} = (161b)_{Str(C,D)} &= ((\mathbf{p}^+ \wedge D) \vee \mathbf{r}) \vee \mathbf{p} \\
&\equiv (\mathbf{p} \vee \mathbf{r}) \vee (\mathbf{p}^+ \wedge D) \\
&\equiv (\mathbf{p} \vee \mathbf{r} \vee \mathbf{p}^+) \wedge (\mathbf{p} \vee \mathbf{r} \vee D) \\
&\equiv (\mathbf{p} \vee \mathbf{r}) \wedge (\mathbf{p} \vee \mathbf{r} \vee D) \\
&\equiv (\mathbf{p} \vee \mathbf{r}) \vee (\bot \wedge D) \\
&\equiv (\mathbf{p} \vee \mathbf{r}) \vee \bot \\
&\equiv \mathbf{p} \vee \mathbf{r} = (161a)^-_C = (161b)^-_C
\end{aligned}
$$

However, this account still cannot capture the case of CHDs, again due to their structural "opacity". This is shown in (167).

(167) CHDs are not Super Redundant (SR).

We show (160a)=$\mathbf{q} \vee \mathbf{p}$ and (160b)=$\mathbf{p} \vee \mathbf{q}$ are not SR.

Take C = $\mathbf{p}$.

We then have $(160a)^-_C = (160b)^-_C = \mathbf{q}$.

Take $D = \top$.

$$
\begin{aligned}
(160a)_{Str(C,D)} = (160b)_{Str(C,D)} &= (\mathbf{p} \wedge D) \vee \mathbf{q} \\
&\equiv (\mathbf{p} \wedge \top) \vee \mathbf{q} \\
&\equiv \mathbf{p} \vee \mathbf{q} \\
&\not\equiv \mathbf{q} = (160a)^-_C = (160b)^-_C.
\end{aligned}
$$

Same proof if C = $\mathbf{p}$, swapping the roles of $\mathbf{p}$ and $\mathbf{q}$.

Solving the puzzle of CHDs most likely requires us to say something *explanatory* about compatible disjuncts, going beyond their intensional semantics. The Chapter will suggest that our QuD framework can essentially get CHDs "for free", based on how the concept of granularity gets incorporated in Qtrees, and on how disjunctive Qtrees get computed.

5.2.4 Non-Triviality

A different line of work (Mayr and Romoli, 2016 i.a.), building on the notion of Local Contexts (Schlenker, 2009), associates oddness with triviality in the sense of (Stalnaker, 1999). This view is summarized in (145), repeated from Chapter 4.

(145) **Non-Triviality** (Mayr and Romoli, 2016). A sentence S cannot be used in a context c if some part π of S is entailed or contradicted by the Local Context of π in c (abbreviated LC(π, c)).

(168) *Local Context.* The Local Context of an expression π in a sentence S is the smallest domain that one may restrict attention to when assessing E without jeopardizing the truth conditions of S. Let c be the global context of S. The above definition derives the following facts for disjunctions and conditionals:

a. If S is a conditional of the form $\Phi \rightarrow \Psi$, LC$(\Phi, c) = c$ and LC$(\Psi, c) = c \cap \Phi$.

b. If S is a disjunction of the form $\Phi \vee \Psi$, and LCs are assumed to be computed incrementally (left-to-right), LC$(\Phi, c) = c$ and LC$(\Psi, c) = c \cap \neg\Phi$

c. If S is a disjunction of the form $\Phi \vee \Psi$, and LCs are assumed to be computed symmetrically (left-to-right and right-to-left), LC$(\Phi, c) = c \cap \neg\Psi$ and LC(Ψ, c) $= c \cap \neg\Phi$.

Chapter 4 briefly mentioned the tension between HDs and the dataset presented in that Chapter. Let us briefly review the arguments here.

Assuming LCs are computed *symmetrically* for disjunctions (see (146c)), (145) predicts the right pattern for HDs. This is shown in (169).

(169) Assuming symmetric Local Contexts, we show HDs of the form $\mathbf{p}^+ \vee \mathbf{p}$ or $\mathbf{p} \vee \mathbf{p}^+$ are correctly predicted to be infelicitous.
Take $\pi = \mathbf{p}^+$ ((159a)'s 1st disjunct / (159b)'s 2nd disjunct).
LC$(\pi, c) = c \cap (\neg\mathbf{p})$, contradiction.

Assuming instead that LCs are computed *asymmetrically* for disjunctions (see (146b)), (145) incorrectly predicts strong-to-weak HDs to be fine. This is shown in (170).

(170) Assuming asymmetric Local Contexts, we show Hurford Disjunctions of the form $\mathbf{p}^+ \vee \mathbf{p}$ are incorrectly predicted to be felicitous.

Take π = (149a)'s 1st disjunct = $\mathbf{p}^+$.
LC(π, c) = c, consistent.
Take π = (149a)'s 2nd disjunct = $\mathbf{p}$.
LC(π, c) = $c \cap (\neg \mathbf{p}^+)$, consistent.

Therefore, accounting for HDs *via* NON-TRIVIALITY requires symmetric Local Contexts. However, Chapter 4 introduced data that we argued could *only* be captured assuming *asymmetric* Local Contexts (in particular, felicitous sentences of the form $p \vee (\neg p \rightarrow q)$).[7] In other words, (145) cannot capture the data presented in Chapter 4 along with HDs, based on unified underlying assumptions.

Turning to LDHDs and CHDs, NON-TRIVIALITY can account for LDHDs assuming symmetric Local Contexts (see (171)).

(171) Assuming symmetric Local Contexts, we show LDHDs of the form $(\mathbf{p}^+ \vee \mathbf{r}) \vee \mathbf{p}$ or $\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r})$ are correctly predicted to be infelicitous.
Take $\pi = \mathbf{p}^+$ ((161a)'s / (161b)'s 1st inner disjunct).
LC(π, c) = $c \cap (\neg \mathbf{p})$, contradiction.

However, just like previously introduced REDUNDANCY-based accounts, NON-TRIVIALITY cannot deal with the case of CHDs. This is shown in (172) assuming symmetric Local Contexts. It is easy to see that assuming asymmetric Local Contexts does not help resolve the issue.

(172) Assuming symmetric Local Contexts, we show CHDs of the form $\mathbf{q} \vee \mathbf{p}$ or $\mathbf{p} \vee \mathbf{q}$ (with $\mathbf{p}$ and $\mathbf{q}$ logically compatible but non-entailing) are incorrectly predicted to be felicitous.

[7] (147c) and (148), repeated from Chapter 4 below, show that only asymmetric Local Context capture the felicity of sentences of the form $p \vee (\neg p \rightarrow q)$. This clashes with the assumptions needed to capture HDs under the NON-TRIVIALITY view.

(147c) We show (131c)=$\mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$ is not locally trivial.
Take π = (131a)'s 1st disjunct = $\mathbf{p}$. LC(π, c) = c, consistent.
Take π = (131a)'s antecedent = $\neg\mathbf{p}$. LC(π, c) = LC($\neg\mathbf{p}$, c) = $\neg\mathbf{p}$ (negation of 1st disjunct), consistent.
Take π = (131a)'s consequent = $\mathbf{q}$. LC(π, c) = LC($\mathbf{q}$, c) = $\neg\mathbf{p}$ (negation of 1st disjunct / antecedent), consistent.

(148) Assuming symmetric Local Contexts, we show (131c)=$\mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$ is locally trivial.
Take π = (131a)'s 1st disjunct = $\mathbf{p}$.
LC(π, c) = $c \cap \neg(\neg\mathbf{p} \rightarrow \mathbf{q}) = c \cap (\neg\mathbf{p} \wedge \neg\mathbf{q})$, contradiction with $\mathbf{p}$.

Take $\pi = \mathbf{q}$ ((160a)'s 1st disjunct / (160b)'s 2nd disjunct).
LC$(\pi, c) = c \cap \mathbf{p}$, consistent.
Take $\pi = \mathbf{p}$ ((160a)'s 2nd disjunct / (160b)'s 1st disjunct).
LC$(\pi, c) = c \cap \mathbf{q}$, consistent.

We have just reviewed four prominent accounts of pragmatic oddness, and showed that, even if then can all capture basic HDs, they cannot capture CHDs. Some, but not all accounts, managed to capture LDHDs. This calls for another approach to these Hurford Sentences, that would capture their oddness despite structural opacity (in the case of CHDs), or structural "distance" between occurrences of redundant material (in the case of LDHDs). The next Section explores the predictions of our QuD-driven Non-Redundancy constraint introduced in Chapter 4 – dubbed Q-Non-Redundancy. We show that these predictions appear unsatisfying at first blush, but suggest a possible update of Q-Non-Redundancy that could cover the cases at stake, while crucially retaining the good results obtained for the data in Chapter 4.

5.3 Q-Non-Redundancy, and its shortcomings

5.3.1 Summary of the current account

In Chapter 4, we introduced a new Non-Redundancy constraint on LF-Qtree pairs, and showed that this constraint, unlike Global/Local/Super Redundancy, could capture a challenging dataset based on logical variations of $p \vee p \vee q$. This constraint, dubbed Q-Non-Redundancy and repeated in (153a), can be summarized as follows. If a Qtree Q is evoked by a sentence S and also by one of the sentence's formal simplifications S', then Q is deemed Q-Redundant given S. This definition is based on the concept of structural simplicity, repeated in (134) below.

(153a) **Q-Non-Redundancy** (to be revised). Let X be a LF and let $Qtrees(X)$ be the set of Qtrees evoked by X. For any $T \in Qtrees(X)$, T is deemed Q-Redundant given X (and thus, odd given X) iff there exists a formal simplification of X, X', and $T' \in Qtrees(X')$, such that $T = T'$.

(153b) **Qtree equality**. $T = T'$ iff T and T' have same structure and same verifying nodes.

(134) **Structural simplicity** (Katzir, 2007). S' is a simplification of S if S' can be derived from S by replacing nodes in S with their subconstituents.

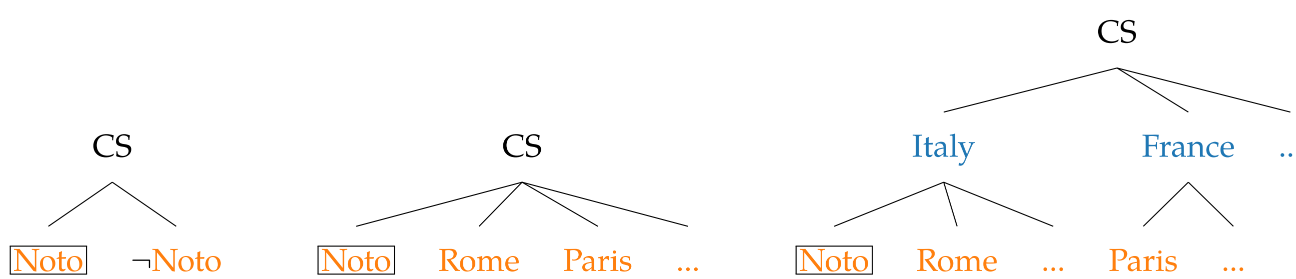
Additionally, a sentence that is compatible with no Qtree is deemed odd – see (76).

(76) **Oddness of a sentence.** A sentence S is odd if any Qtree T it evokes is odd given S.

5.3.2 An issue with the current account

Q-Non-Redundancy, as defined in (153a), unfortunately cannot even account for HDs. To see this, let us compute the Qtrees compatible with S_p = *SuB29 will take place in Italy*, S_{p^+} = *SuB29 will take place in Noto*, as well the Qtree(s) compatible with their two possible disjunctions (159a) and (159b).

Chapter 2 extensively discussed how to derive Qtrees from simplex sentences like S_p and S_{p^+}. Here, it is enough to say that such sentences may evoke three kinds of Qtrees: "polar" ones, splitting the Context Set (henceforth **CS**) into p and $\neg p$ worlds; "*wh*" ones, splitting the CS according to the Hamblin partition generated by same-granularity alternatives to the prejacent; and "*wh*-articulated" ones, whereby each layer corresponds to a Hamblin partition of increasing granularity from the top down, the last layer matching the granularity of the prejacent. In each case, leaves entailed by the prejacent are $\boxed{\text{flagged}}$ as "verifying", and as such keep track of at-issue content. In this Chapter, we will only consider two levels of granularity for S_p and S_{p^+}: by-city and by-country. This gives rise to the Qtrees in Figure A (for S_{p^+}) and Figure B (for S_p).



(1) "Polar". (2) "*Wh*". (3) "*Wh*-articulated".

Figure A: Qtrees evoked by S_{p^+} = *SuB29 will take place in Noto*.

(1) “Polar”.

(2) “*Wh*”.

Figure B: Qtrees evoked by S_{p} = *SuB29 will take place in Italy*.

Looking at Figures A and B, we can observe that the Qtrees in A1, A2, and B1, introduce orthogonal partitionings of the CS, as defined in (109), repeated below.

(109) **Orthogonal partitions**. Let $T = (\mathcal{N}, \mathcal{E}, R)$ and $T' = (\mathcal{N}', \mathcal{E}, R)$ be two depth-1 Qtrees sharing the same root R (equivalently, two partitions of the same CS). T and T' are orthogonal iff they involves two nodes that are strictly overlapping, i.e. $\exists(N, N') \in \mathcal{N} \times \mathcal{N}'.\ N \cap N' \neq \emptyset \wedge N \neq N'$. T and T' are orthogonal iff T and T' exhibit a partition clash.

By contrast, Figures A3 and B2, introduce consistent partitionings: Figure A3 can be in fact be seen as a refinement of Figure B2, as per (72), repeated below.

(72) **Qtree refinement**. Let T and T' be Qtrees. T is a refinement of T' (or: T is finer-grained than T'), iff T' can be obtained from T by removing a subset $\mathcal{T}$ of T's subtrees, s.t., if $\mathcal{T}$ contains a subtree rooted in N, then, for each node N' that is a sibling of N in T, the subtree of T rooted in N', is also in $\mathcal{T}$.

Now turning to the HDs (159a) and (159b), the Qtree(s) evoked by the disjunction of S_p and S_{p^+}, correspond to the well-formed union(s) of Qtrees evoked by S_p and S_{p^+}. Verifying nodes are also unioned. Chapter 2 in fact showed that depth-1 Qtrees corresponding to orthogonal partitions, are never properly disjoinable, in the sense that the output of the Qtree union operation, cannot be a Qtree. Relatedly, this Chapter showed, using analog sentences, that the HDs (159a) and (159b) are only compatible with one well-formed Qtree, namely, the Qtree obtained by unioning Figure A3 (for S_{p^+}), and Figure B2 (for S_p), which, as we have just argued, stand in a refinement relation. The result of this union, depicted in Figure C, is evoked by both (159a) and (159b). It is structurally equal to the input Qtree for p^+ (Figure A3), but flags two verifying nodes (*Italy* and *Noto*) instead of one.

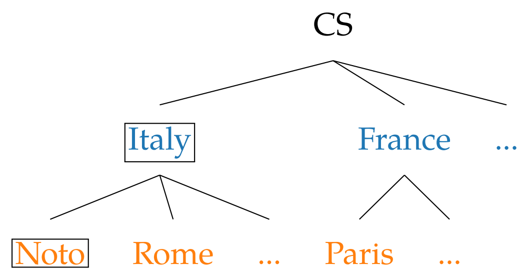


Figure C: Only Qtree evoked by (159a) = $S_{\mathrm{p}^+} \vee S_{\mathrm{p}}$ or (159b) = $S_{\mathrm{p}} \vee S_{\mathrm{p}^+}$.

Is this Qtree ill-formed, according to previously introduced constraints? First, this Qtree flags a non-empty set of nodes. Therefore, it does not trigger the Empty Labeling condition, repeated below.

(75) **Empty labeling.** If a sentence S evokes a Qtree T but does not flag any node as verifying in T, then T is deemed odd given S.

Is the Qtree in Figure C Q-Redundant given (159a)/(159b) then? Our current definition of Q-Non-Redundancy, calls for structural equality *and* equality of verifying nodes, between the Qtree in Figure C, and some Qtree evoked by a simplification of (159a)/(159b), i.e. S_p or S_{p^+}. Such Qtrees are precisely the ones depicted in Figures A and B. Are any of these Qtrees equal to the one in Figure C, in terms of structure and verifying nodes?

Focusing on verifying nodes, it is quite easy to see that none of the Qtrees in Figure A or B, have same verifying nodes as the Qtree in Figure C. This is because the Qtree in Figure C, has two verifying nodes (*Noto*, and *Italy*), while those in Figures Figure A or B, only have one (*either Noto*, or *Italy*). Therefore, the HDs (159a)/(159b) evoke a Qtree that is not equal to *any* Qtree evoked by simplifications of (159a)/(159b), and, as a result, this Qtree cannot be deemed Q-Redundant given (159a)/(159b).

This implies that there is no constraint in our current toolkit that predicts the HDs in (159a)/(159b) to be odd: they remain compatible with one well-formed Qtree, namely the one in Figure C. Moreover, it can be shown that Q-Non-Redundancy struggles with LDHDs, essentially for the same reason. We will further discuss this case, along with the case of CHDs, after proposing an update to Q-Non-Redundancy. But let us first confirm that updating Q-Non-Redundancy is the way to go – as opposed to updating other core components of the current machinery.

5.3.3 Should we update the rule for disjunctive Qtrees?

We have just seen that our current version of Q-Non-Redundancy could not even capture HDs. The natural conclusion is that Q-Non-Redundancy should be updated to cover such cases. However, a perhaps less intuitive solution, would be to keep Q-Non-Redundancy as it is, and instead, update the rule used to build disjunctive Qtrees. The hope is that updating such a rule would bock the derivation of well-formed Qtree for HDs. This Section shows that choosing this path is probably a bad idea, and that updating Q-Non-Redundancy should be the preferred way to go. It also better delineates what Q-Non-Redundancy should *rule-in*.

The argument is based on the disjunctive sentences in (173), featuring incompatible disjuncts.[8] Such sentences appear close to HDs, in the sense that they are disjunctive, and also feature disjuncts of different granularities: *Noto* is a city-level alternative, while *France* and *not Italy* are country-level. Yet, both (173a) and (173b) sound significantly more acceptable than HDs. This suggests that such disjunctions give rise to well-formed Qtrees. We can in turn conclude that the process deriving disjunctive Qtrees should not in principle ban Qtrees evoking different levels of specificity (i.e. Qtrees standing in a refinement relation), from being unioned together.

(173) a. SuB29 will take place in Noto or will not take place in Italy. $\mathbf{p}^{+} \vee \neg \mathbf{p}$

b. SuB29 will take place in Noto or in France. $\mathbf{p}^{+} \vee \mathbf{q}$

To get a better intuition as to how Q-Non-Redundancy should be updated, let us see what disjunctive Qtrees for (173a) and (173b) look like. Because (173a) and (173b) are felicitous, at least some of these Qtrees should be well-formed – providing information about what Q-Non-Redundancy should *rule-in*. Qtrees for S_q = *SuB29 will take place in France*, and $\neg S_p$ = *SuB29 will not take place in Italy*, are given in Figures D and E, respectively. Such Qtrees are similar to those evoked by S_p = *SuB29 will take place in Italy*, except that, in the case of *France*, the roles of the *Italy* and *France* nodes are swapped; and, in the case of *not Italy*, verifying nodes are flipped by negation.

(1) "Polar". (2) "*Wh*".

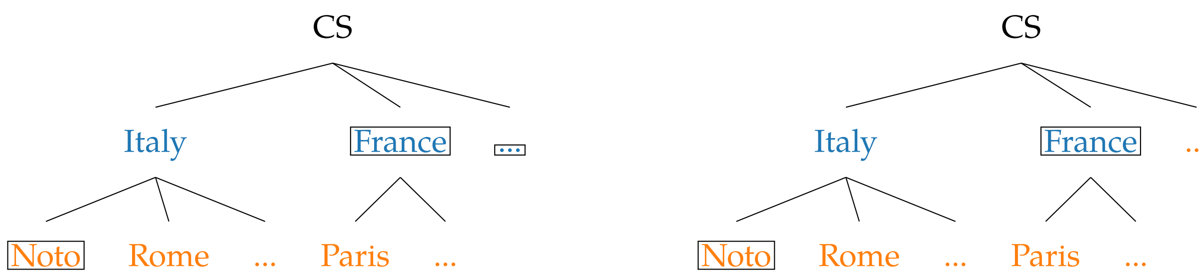
Figure D: Qtrees evoked by $S_{\mathbf{q}}$ = *SuB29 will take place in France*.

[8] I thank Amir Anvari for mentioning these examples to me.

(1) "Polar". (2) "*Wh*".

Figure E: Qtrees evoked by $\neg S_{\mathbf{p}}$ = *SuB29 will not take place in Italy*.

Such Qtrees can be disjoined with Qtrees evoked by *Noto* pretty much similarly to HDs. This is done in Figure F.



(1) Qtree evoked by (173a), resulting from (A3) ∨ (E2). (2) Qtree evoked by (173b), resulting from (A3) ∨ (D2).

Figure F: Qtrees for felicitous disjunctions involving disjuncts with different granularities.

These two examples and their Qtrees suggest what the issue might be with the infelicitous HDs (159a-159b) and their Qtree in Figure C: in this Qtree, the path connecting the root to the verifying node *Noto*, properly contains the path connecting the root to the verifying node *Italy*. Entertaining these two paths – or strategies of inquiry – appears suboptimal: why inquire about a country like *Italy*, if one already inquires about a city like *Noto*? The Qtrees in Figure F1 and F2 do not exhibit similar contained paths. We will further formalize these intuitions and relate them to pragmatic competition, when updating Q-Non-Redundancy in the next Section.

This Section showed that the current way to derive disjunctive Qtrees, although it allows to generate *a priori* well-formed Qtrees for HDs, turns out useful to generate Qtrees for closely related yet felicitous sentences such as (173a-173b). This motivates an update of Q-Non-Redundancy ruling out HDs and ruling in (173a-173b), as opposed to an update of the whole recipe deriving disjunctive Qtrees. We now show that updating Q-Non-Redundancy to capture HDs, is pretty straightforward, and moreover, can be easily shown to preserve the good results obtained in Chapter 4 (fast forward to Section 5.5.1).

5.3.4 Updating Q-Non-Redundancy

The predictions of Q-Non-Redundancy appear unsatisfying at first blush, but deriving Qtrees for the HDs in (159a)/(159b) and their simplifications, suggests a way forward. Let us further precisify the intuition. In Figure C, the two verifying nodes, *Noto* and *Italy*, appear on the same path originating from the CS root, and reaching the *Noto*-leaf. Recall that Chapter 2, Section 2.2.3, identified paths from the CS-root to any node with "strategies of inquiry", i.e. sequences of conditional questions of increasing specificity. In Figure C, *Noto* and *Italy* are thus part of the same strategy of inquiry – the path from the CS-root to *Noto*. Additionally, one can argue that the verifying status of *Noto* and *Italy*, gives this particular strategy of inquiry a special status, as well: it is *the* strategy of inquiry that converges to at-issue nodes.

Now, let us turn to Figure A3, the "*wh*-articulated" Qtree evoked by S_{p^+}, a simplification of the HDs in (159a)/(159b). As noted previously, this Qtree is almost the same as the one evoked by HDs, in Figure C. The only difference is that Figure A3 only flags the *Noto* node as verifying, while Figure C flags both *Noto* and *Italy*. What about strategies of inquiry? The strategy of inquiry made at-issue by *Noto* in Figure A3, happens to be exactly the same as the strategy of inquiry made at-issue by both *Noto* and *Italy* in Figure C. This is because, any path from the CS to both *Noto* and *Italy*, is a path to *Noto*.

In other words, if only structure and at-issue strategies of inquiry are considered when comparing Qtrees, the Qtree in Figure C (evoked by HDs), appears identical to the one in Figure A3 (evoked by a simplification of HDs). We use this observation to update Q-Non-Redundancy. (174) states that a Qtree evoked by a sentence is redundant if it is *equivalent* (instead of *equal*) to a Qtree evoked by one the sentence's formal simplifications.

(174) **Q-Non-Redundancy** (final version). Let X be a LF and let $Qtrees(X)$ be the set of Qtrees evoked by X. For any $T \in Qtrees(X)$, T is deemed Q-Redundant given X, iff there exists a formal simplification of X, X', and $T' \in Qtrees(X')$, such that $T \equiv T'$.

(175) defines equivalence between Qtrees in terms of both tree structure and optimal strategies of inquiry: two Qtrees are equivalent, if their structures and optimal strategies of inquiry are the same. Optimal strategies of inquiry, are defined as the smallest set of paths from the CS-root to each existing verifying node. This is unpacked in (176).

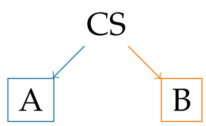
(175) **Qtree equivalence relation**. T and T' are equivalent ($T \equiv T'$) iff T and T' have same structure and same set of minimal verifying paths.

(176) a. **Verifying Paths**. The set of verifying paths $\mathbb{P}(T)$ of a Qtree T is the set of paths starting from the root of T, and such that each path finishes in some $N \in \mathbb{N}^+(T)$, and each $N \in \mathbb{N}^+(T)$ belongs to some path.

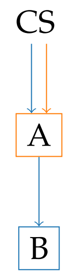
b. **Path containment**. Two paths p_1 and p_2 are in a containment relation ($p_1 \subseteq_{\mathbb{P}} p_2$) if p_1 (seen as an ordered list, i.e. a string, of nodes) is a prefix of p_2.

c. **Set of minimal verifying paths**. The set of minimal verifying paths $\mathbb{P}^*(T)$ of a Qtree T is the set of minimal elements of $\mathbb{P}(T)$ w.r.t. the path containment relation.

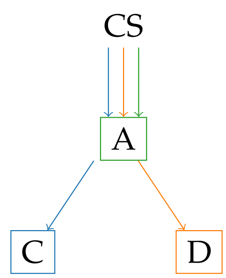
Figure G provides a few examples illustrating how minimal sets of verifying paths are computed. In these Figures, paths are identified by their color. Each path leads to a verifying node, whose "box" features the same color as the path leading to it. Figure G1 illustrate the case in which the two paths at stake are entirely disjoint. As a result, the set of minimal verifying paths, is the same as the initial set of paths. Figure G2 illustrate the case in which the two paths are in a containment relation: the orange path from the CS to A, is a prefix of the blue path from the CS to A to B. Therefore, the orange path is contained in the blue path as per (176b), and the set of minimal verifying paths only contains the blue path, from the CS, to A, to B. Lastly, Figure G2 illustrates the case in which the three paths at stake exhibit some mixed overlap: the blue and orange paths both go from the CS to A, but diverge after A. Therefore, they do not stand in any kind of containment relation. The green path, is a prefix of both the blue and the orange path, and so is contained in both paths. As a result, the set of minimal verifying paths, only contains the blue and the orange paths.



(1) Fully disjoint paths.
$\mathbb{P} = \{[CS; A], [CS; B]\}$
[CS;A] $\not\subseteq_{\mathbb{P}}$ [CS;B]
[CS;B] $\not\subseteq_{\mathbb{P}}$ [CS;A]
$\mathbb{P}^* = \{[CS; A], [CS; B]\}$



(2) Contained paths.
$\mathbb{P} = \{[CS; A; B], [CS; A]\}$
[CS;A] $\subseteq_{\mathbb{P}}$ [CS;A;B],
$\mathbb{P}^* = \{[CS; A; B]\}$



(3) Contained and disjoint paths.
$\mathbb{P} = \{[CS; A; C], [CS; A; D], [CS; A]\}$
[CS;A;C] $\not\subseteq_{\mathbb{P}}$ [CS;A;D]
[CS;A;D] $\not\subseteq_{\mathbb{P}}$ [CS;A;C]
[CS;A] $\subseteq_{\mathbb{P}}$[CS;A;C], [CS;A;D]
$\mathbb{P} = \{[CS; A; C], [CS; A; D]\}$

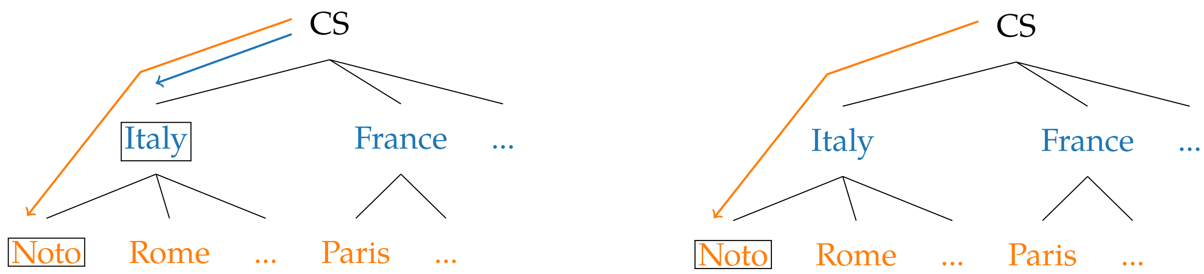
Figure G: Illustrations of the computation of minimal sets of verifying paths.

In the Next Section, we show that our updated version of Q-Non-Redundancy captures HDs, as well as LDHDs. We also come back to the case of CHDs, and show that our approach gets them "for free", in fact independently of Q-Non-Redundancy.

5.4 Capturing three varieties of Hurford Disjunctions

5.4.1 Hurford Disjunctions

As foreshadowed in the previous Section, our updated version of Q-Non-Redundancy explains why the only Qtree compatible with the HDs in (159a) and (159b), repeated in Figure H1, is redundant: it turns out to be equivalent (in terms of structure and set of minimal verifying paths) to the "*wh*-articulated" Qtree evoked by the simplification S_{p^+} = *Sub29 will take place in Noto*, repeated in Figure H2. First, the structure of the two Qtrees in Figure H1 and H2 is obviously the same. Second, both Qtrees are characterized by a set of minimal verifying paths containing only one such path, namely, the path from the CS root to Noto, *via* Italy. This is further justified in (177).



(1) Only Qtree evoked by (159a) = $S_{\mathbf{p}^+} \vee S_{\mathbf{p}}$ or (159b) = $S_{\mathbf{p}} \vee S_{\mathbf{p}^+}$.

(2) Qtree evoked by $S_{\mathbf{p}^+}$ = *SuB29 will take place in Noto.*

Figure H: Showing that the HDs (159a-159b) are Q-Redundant.

(177) $\mathbb{P}(\text{H1}) = \{[\text{CS, Italy, Noto}], [CS, Italy]\}$
$\mathbb{P}^*(\text{H1}) = \{[\text{CS, Italy, Noto}]\}$, because $[\text{CS, Italy}] \subseteq_{\mathbb{P}} [\text{CS, Italy, Noto}]$
$\mathbb{P}(\text{H2}) = \{[\text{CS, Italy, Noto}]\} = \mathbb{P}^*(\text{H2}) = \mathbb{P}^*(\text{H1})$

We have just seen that the pair formed by HDs and their unique Qtree is Q-Redundant (and thus more generally odd), due that Qtree being equivalent to a Qtree evoked by the HDs' stronger disjunct. We now turn to LDHDs and show that our Qtree model, complemented with our updated version of Q-Non-Redundancy straightforwardly captures them, as well.

5.4.2 Long-Distance Hurford Disjunctions

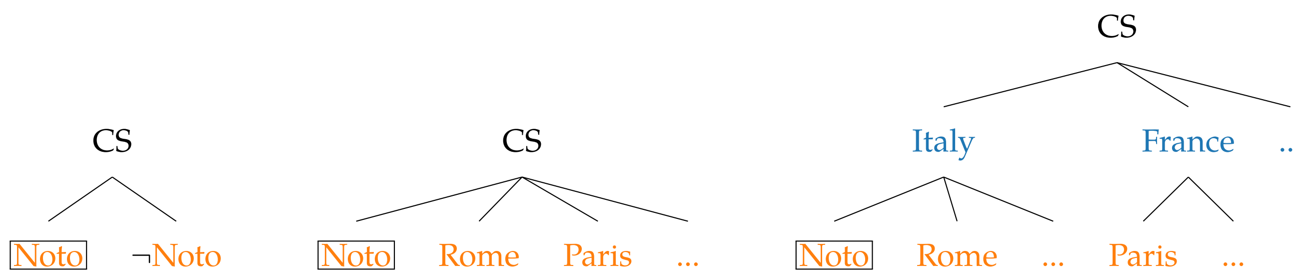
LDHDs, repeated in (161), are infelicitous, despite the fact that none of their disjuncts are in an entailment relation – thus falling outside Hurford's original constraint. As pointed out in the introduction, the infelicity of these constructions seems to stem from the fact that they feature some redundant material, but at different "levels": in one, "high" disjunct (p), and in one "low" disjunct (p^+). The *dependency* between these two "long-distance" occurrences, appeared challenging to capture for many previous accounts of oddness.

(161) a. # Either SuB29 will take place in Noto or Paris, or it will take place in Italy.
$(\mathbf{p}^+ \vee \mathbf{r}) \vee \mathbf{p}$ $\qquad \mathbf{p}^+ \vDash \mathbf{p}; (\mathbf{p}^+ \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

b. # Either SuB29 will take place in Italy, or it will take place in Noto or Paris.
$\mathbf{p} \vee (\mathbf{p}^+ \vee \mathbf{r})$ $\qquad \mathbf{p}^+ \vDash \mathbf{p}; (\mathbf{p}^+ \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

We will now show that our model of implicit QuDs, complemented with our new version of Q-Non-Redundancy, captures LDHDs, essentially because Qtrees keep track of verifying nodes in a compositional way, and Q-Non-Redundancy is (indirectly) sensitive to how these verifying nodes are arranged, in particular when it comes to dominance relations.

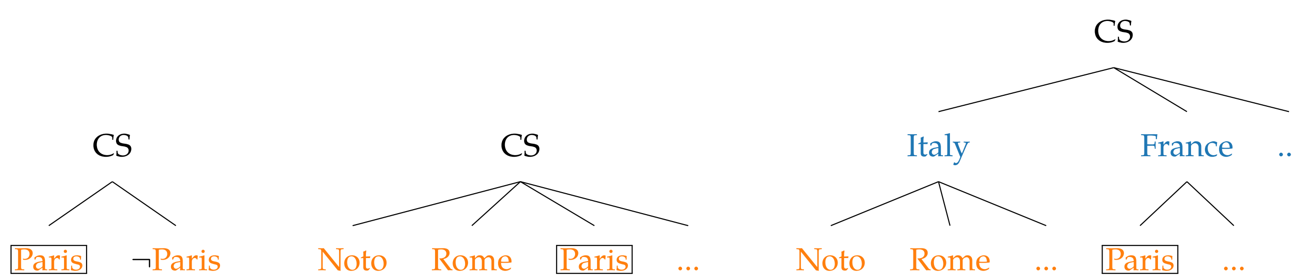
Let us then compute the Qtrees evoked by the LDHDs in (161). We start with the inner disjunction $S_{p^+} \vee S_r$ = *SuB29 will take place in Noto or Paris*. Qtrees evoked by S_{p^+} = *SuB29 will take place in Noto* are repeated in Figure I.



(1) "Polar". (2) "*Wh*". (3) "*Wh*-articulated".

Figure I: Qtrees evoked by S_{p^+} = *SuB29 will take place in Noto*.

Because *Paris* can be seen as a city-level alternative to *Noto*, the Qtrees evoked by S_r = *SuB29 will take place in Paris*, are analog to those in Figure I, swapping the roles of *Paris* and *Noto*. This is shown in Figure J

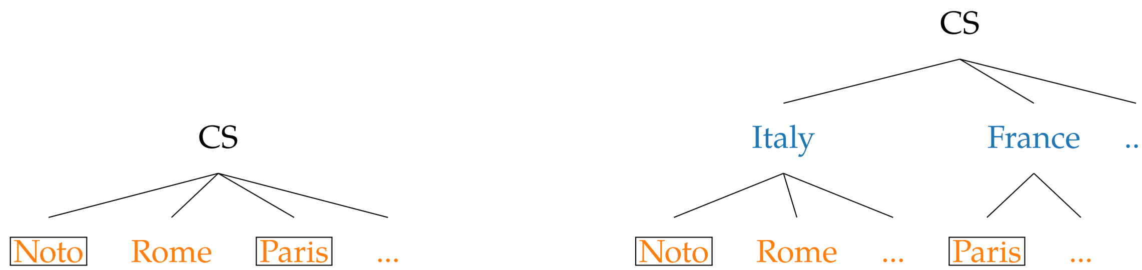


(1) "Polar". (2) "*Wh*". (3) "*Wh*-articulated".

Figure J: Qtrees evoked by S_r = *SuB29 will take place in Paris.*

Looking at Figures I and J, we can observe that the Qtrees in I1 and J1, introduce orthogonal partitionings of the CS. By contrast, Figures I2 and J2, are structurally identical, i.e. introduce consistent partitionings. The same holds for Figures I3 and J3.

Now, recall that the Qtree(s) evoked by a disjunction correspond to the well-formed union(s) of Qtrees evoked by the disjuncts. Qtrees which introduce orthogonal partitionings of the CS, cannot be properly disjoined. Therefore, deriving Qtrees for $S_{p^+} \vee S_r$ produces two Qtrees: one from the union of Figures I2 and J2 (see Figure K1), the other, from the union of Figures I3 and J3 (see Figure K2). These Qtrees are structurally identical to the Qtrees used to form them, but flag two nodes as verifying (*Noto* and *Paris*), instead of just one.



(1) "*Wh*". (2) "*Wh*-articulated".

Figure K: Qtrees evoked by $S_{\mathrm{p}^+} \vee S_{\mathrm{r}}$ = *SuB29 will take place in Noto or Paris.*

We can now turn to the outer disjunction, and form Qtrees for $(S_{p^+} \vee S_r) \vee S_p$ = (161a), and $S_p \vee (S_{p^+} \vee S_r)$ = (161b). Because the Qtree disjunction operation is symmetric, both variants will evoke the same Qtrees(s). Qtrees evoked by S_p = *SuB29 will take place in Italy*, are repeated in Figure L. These Qtrees now have to be properly disjoined with those evoked by $S_{p^+} \vee S_r$, represented in Figure K.

(1) "Polar". (2) "*Wh*".

Figure L: Qtrees evoked by S_{p} = *SuB29 will take place in Italy.*

Looking at Figures K and L, we can observe that the Qtrees in K1 and L1, introduce orthogonal partitionings of the CS. By contrast, Figures K2 and L2, stand in a refinement relation, i.e. introduce consistent partitionings of the CS. Is it therefore possible to union Figures K2 and B2, to form the only possible disjunctive Qtree evoked by (161a)/(161b), depicted in Figure M.

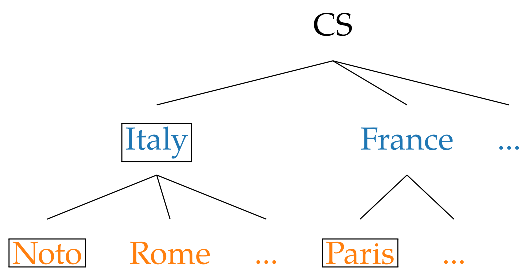


Figure M: Only Qtree evoked by (161a) = $(S_{\mathbf{p}^+} \vee S_{\mathbf{r}}) \vee S_{\mathbf{p}}$ or (161b) = $S_{\mathbf{p}} \vee (S_{\mathbf{p}^+} \vee S_{\mathbf{r}})$.

The Qtree in Figure M, turns out to be Q-Redundant given (161a)/(161b), because it features the same structure, and the same set of minimal verifying paths, as the Qtree in Figure K2, which is evoked by the simplification of (161a)/(161b) of the form $S_{p^+} \vee S_r$ = *SuB29 will take place in Noto or Paris*. The structural equality between the Qtrees in Figures M and K2, is obvious. The equality between their minimal sets of verifying paths, is justified in (178). Roughly, because the path from the CS root to the *Italy*-node in Qtree M, is contained within the path from the CS root to *Noto*, it gets excluded from the set of minimal verifying paths associated with this Qtree. The set of minimal verifying paths for the Qtree in Figure M is thus made of two (diverging) paths: one from the CS root to *Noto via Italy*, the other, from the CS root to *Paris via France*. This set then turns out identical to the set of minimal verifying paths in Qtree K2, which also features a path from the CS root to *Noto*, and from the CS root to *Paris*.

(178) $\mathbb{P}(\mathrm{M}) = \{[\mathrm{CS, Italy, Noto}], [\mathrm{CS, Italy}], [\mathrm{CS, France, Paris}]\}$
$\mathbb{P}^*(\mathrm{M}) = \{[\mathrm{CS, Italy, Noto}], [\mathrm{CS, France, Paris}]\}$
because $[\mathrm{CS, Italy}] \subseteq_{\mathbb{P}} [\mathrm{CS, Italy, Noto}]$
$\mathbb{P}(\mathrm{K2}) = \{[\mathrm{CS, Italy, Noto}], [\mathrm{CS, France, Paris}]\} = \mathbb{P}^*(\mathrm{K2}) = \mathbb{P}^*(\mathrm{M})$

We have just shown that our model of Qtrees, which keeps track of the at-issue propositions and how they are logically related to each other (in terms of dominance relations within Qtrees), supplemented with our new Non-Q-Redundancy constraint, captures LD-HDs. In the next Section, we discuss the case of CHDs, and argue that their infelicity simply follows from how the Qtree model structurally incorporates the concept of granularity.

5.4.3 Compatible Hurford Disjunctions

Interestingly, our model happens to capture CHDs, repeated in (160), independently of Q-Non-Redundancy.[9]. We will see that the infelicity of CHDs follows from the observa-

[9]A more thorough investigation of the possible Qtrees evoked by CHDs and their disjuncts, shows that CHDs can be captured, but *modulo* an extra independently motivated constraint on the formation of recursive

tion that their (logically compatible) disjuncts, evoke irreconcilable degrees of granularity. This will lead to Qtrees characterized by orthogonal partitionings of the CS, which will not be properly disjoinable.

(160) a. # SuB29 will take place in the Basque country or France. $\mathbf{q} \vee \mathbf{p}$; $\mathbf{q} \wedge \mathbf{p} \nvDash \bot$

b. # SuB29 will take place in France or in the Basque country. $\mathbf{p} \vee \mathbf{q}$; $\mathbf{p} \wedge \mathbf{q} \nvDash \bot$

To show this, we will proceed in two steps:[10] First, we will show that using the most intuitive depth-1 Qtrees for S_p and S_q predicts infelicity for CHDs, due to orthogonal partitionings. Second, we will show that considering "*wh*-articulated" Qtrees, does not help either, essentially due to the lack of entailment between p and q.

First, let us intuitively see where the problem lies. In (160), the q-disjunct suggests a by-region partition of the CS, such that the Basque country represents a (verifying) leaf; while the p-disjunct suggests a by-country partition, such that France represents a (verifying) leaf. The relevant Qtrees are depicted in Figures N and O. We omit the more complex "*wh*-articulated" Qtrees for now – we will later see that they cannot help resolve the underlying issue.

(1) "Polar". (2) "*Wh*".

Figure N: Qtrees for $S_{\mathbf{q}}$ = *SuB29 will take place in the Basque country*.

(1) "Polar". (2) "*Wh*".

Figure O: Qtrees evoked by $S_{\mathbf{p}}$ = *SuB29 will take place in France*.

Looking at Figures N and O, a now familiar pattern emerges. We can observe that there is no pair of Qtrees from Figure N and O, that introduce consistent partitionings of

partitions (i.e. Qtrees). See Appendix 5.7

[10] Appendix 5.7 adds a third step to this argumentation, by discussing one additional "pathological" Qtree generable by the framework presented in Chapter 2.

the CS. All pairs of partitions taken from these two Figures, appear orthogonal. This is a direct consequence of the fact that p and q, are compatible, but non-entailing propositions.

Therefore, there is simply no way to properly disjoin the Qtrees evoked by S_q in Figure N, with the ones evoked by S_p, in Figure O. Consequently, under these assumptions, the CHDs in (160) cannot evoke any well-formed Qtree, and should be deemed odd.

The only remaining ways to disjoin Qtrees associated with S_p and S_q, would be to generate a "*wh*-articulated" Qtree for S_q = *SuB29 will take place in the Basque country*, involving an intermediate by-country layer, or, to create a "*wh*-articulated" Qtree for S_p = *SuB29 will take place in France*, involving an intermediate by-region layer. In either case, the putative "*wh*-articulated" Qtree generated for S_q (resp. S_p) may be properly disjoined with the depth-1 *wh* Qtree for S_p (resp. S_q), since the former Qtree would constitute a refinement of the latter. These two options are symmetric, so let us focus on the former one.

A problem quickly arises in the formation of the desired Qtree. To see this, let us further summarize how Chapter 2 defined "*wh*-articulated" Qtrees. Such Qtrees are built following a "spine", or p-chain, which corresponds to an ordered sequence of alternatives to the prejacent, entailed by the prejacent p. Each proposition p' of the p-chain, is then used to define a set of same-granularity alternatives to p', which, in turn, allows to generate entire layers of the Qtree. The more verbose version of this definition, retrieved from Chapter 2, is given in (179).

(179) *Tiered Qtrees for simplex LFs (to be further generalized in Chapter 7).* Let X be a simplex LF denoting p, not settled in the CS. Let $\mathcal{A}_{p,X}$ be the set X's propositional alternatives. For any $q \in \mathcal{A}_{p,X}$, let $\mathcal{A}^q_{p,X} \subseteq \mathcal{A}_{p,X}$ be the set of alternatives from $\mathcal{A}_{p,X}$ sharing same granularity with q. We assume for simplicity that for any q, $\mathcal{A}^q_{p,X}$ partitions the CS. A "*wh*-articulated" Qtree for X is a depth-k Qtree ($k > 1$) constructed in the following way:

- Formation of a "p-chain" $p_0 = p \subset p_1 \subset ... \subset p_n$ where $p_0, ..., p_n$ are all in $\mathcal{A}_{p,X}$ but belong to different granularity tiers in $\mathcal{A}^{\sim_g}_{p,X}$: $\mathcal{A}^{p_0}_{p,X} \neq \mathcal{A}^{p_1}_{p,X} \neq ... \neq \mathcal{A}^{p_n}_{p,X}$.
- Generation of the "layers" of the Qtree, based on the partitions induced by the granularity tiers corresponding to each element of the p-chain: $\left\{\mathfrak{P}_{\mathcal{A}^{p_i}_{p,X},CS} \mid i \in [0;n]\right\}$.
- Determination of the edges between nodes (cells) of adjacent layers (and between the highest layer and the root), based on the subset relation.[11]

[11] This may not always create well-formed Qtrees. Chapter 7 will explore such cases update (93) in con-

$\boxed{\text{verifying nodes}}$ are defined as the set of leaves entailing p.

Given that q, the proposition that *SuB29 will take place in the Basque country*, does not entail that SuB29 will take place in any specific country (it could take place in either France or Spain), it is impossible to create a q-chain containing a country-level alternative to q. Consequently, no Qtree generated from *SuB29 will take place in the Basque country* using (179), will contain a by-country layer. As a result, no well-formed Qtree can be generated for (160a) or (160b) based, on the "*wh*-articulated" strategy.

To summarize, we predict the sentences in (160) to be odd because they feature disjuncts conveying incomparable degrees of granularity, and which cannot lead to any well-formed disjunctive Qtree. Interestingly, this approach to CHDs does not rely on Q-Non-Redundancy, and instead builds on the core model of Qtrees introduced in Chapter 2. Granted a reasonable model of conjunction, this kind of prediction could extend to "conjunctive" HDs (Zhang, 2022; Zhang, to appear), exemplified in (180). We however leave the specifics of this analysis for future work.

(180) a. # SuB29 will take place in Noto, or it will take place in Italy and it will be amazing.
$\mathbf{p}^{+} \vee (\mathbf{p} \wedge \mathbf{q})$ $\qquad$ $\mathbf{p}^{+} \vDash \mathbf{p}$; $\mathbf{q} \nvDash \mathbf{p}$; $\mathbf{p} \nvDash \mathbf{q}$

b. # SuB29 will take place in Italy and it will be a amazing, or it will take place in Noto.
$(\mathbf{p} \wedge \mathbf{q}) \vee \mathbf{p}^{+}$ $\qquad$ $\mathbf{p}^{+} \vDash \mathbf{p}$; $\mathbf{q} \nvDash \mathbf{p}$; $\mathbf{p} \nvDash \mathbf{q}$

5.5 Taking stock

5.5.1 A "conservative" extension

In this Chapter, we have proposed to update our definition of Non-Q-Redundancy to account for two kinds of Hurford Disjunctions: standard ones, and "long-distance" ones. Our revised version of Non-Q-Redundancy, still relies on the core notion of competition with simpler alternatives: an LF-Qtree pair is odd, if some simplification of the LF evokes an *equivalent* Qtree. The only difference between our revised version of Non-Q-Redundancy, and the earlier version introduced in Chapter 4, lies in what it means for two Qtrees to be equivalent. In Chapter 4, we considered the strictest notion of equivalence between Qtrees, in the form of structural equality *and* equality between sets of verifying nodes. The current Chapter proposed to weaken this notion of equivalence, replacing it

sequence.

by structural equality and, crucially, equality between *minimal* sets of verifying *paths*. At a more conceptual level, this revision suggests that optimal verifying strategies of inquiry, and not verifying nodes, constitute the right metric, when it comes to evaluating the contribution of implicit questions in a conversation. In other words, *how to optimally reach all maximal true answer*, seems to matter more, than knowing what exactly these maximal true answers are.

The revision of Non-Q-Redundancy we proposed in this Chapter, also raises the question of whether the results obtained in Chapter 4, under the earlier version of the constraint, are preserved under the revised version. The answer to this question, is yes, essentially because all the critical Qtrees derived for the sentences at stake in Chapter 4, either had one verifying node, or had two such nodes, but always located on diverging paths. As a result, all the critical Qtrees studied throughout Chapter 4, were such that their sets of verifying paths were identical to their *minimal* sets of verifying paths. Additionally, it can be shown that, when considering two Qtrees such that their sets of verifying paths are identical to their *minimal* sets of verifying paths, Qtree equality amounts to Qtree equivalence. This is spelled out in (181), and proved in (182).

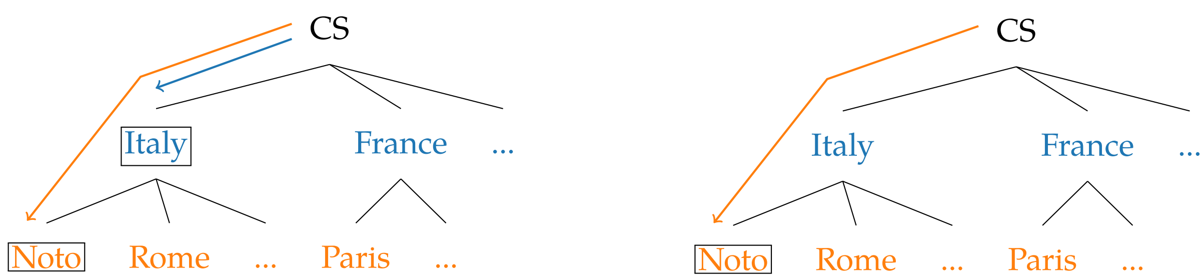
(181) *Link between Qtree equality and Qtree equivalence.* Let T and T' be two Qtrees s.t. $\mathbb{P}(T) = \mathbb{P}^*(T)$ and $\mathbb{P}(T') = \mathbb{P}^*(T')$. Then $T = T'$ iff $T \equiv T'$.

(182) *Proof of (181).* Let T and T' be two Qtrees s.t. $\mathbb{P}(T) = \mathbb{P}^*(T)$ and $\mathbb{P}(T') = \mathbb{P}^*(T')$.
The implication from $T = T'$ to $T \equiv T'$ is trivial.
Let us now assume $T \equiv T'$ and show $T = T'$. $T \equiv T'$ means T and T' have same structure and same minimal sets of verifying paths (see definition (175)). Given that $\mathbb{P}(T) = \mathbb{P}^*(T)$ and $\mathbb{P}(T') = \mathbb{P}^*(T')$, this is equivalent to saying that T and T' have same structure and same sets of verifying paths. Given that the verifying nodes of a Qtree can be retrieved by collecting the destinations of all its verifying paths (as per (176a)), T and T' having same sets of verifying paths implies they have the same sets of verifying nodes. Therefore $T = T'$.

Therefore, the results obtained in Chapter 4 when considering Qtree equality in the context of Q-Non-Redundancy, are preserved when Qtree equivalence is considered instead.

5.5.2 Comparison with previous Redundancy-based approaches to Hurford Disjunctions

At a certain level of approximation, all previous Redundancy-based accounts deemed HDs redundant by showing that such structures somehow turn out equivalent to their *weaker* disjunct. It is interesting to note that our implementation of Q-Non-Redundancy does the opposite: a LF-Qtree pair is typically Q-Redundant because the Qtree turns out to be equivalent to that of a logically *stronger* competitor. For instance, the only Qtree compatible with the HDs in (159a-159b), repeated in Figure P1, turns out equivalent to a Qtree (repeated in Figure P2) evoked by the simplification S_{p^+} = *SuB29 will take place in Noto*, which corresponds to (159a-159b)'s *stronger* disjunct.



(1) Only Qtree evoked by (159a) = $S_{p^+} \vee S_p$ or (159b) = $S_p \vee S_{p^+}$.

(2) Qtree evoked by S_{p^+} = *SuB29 will take place in Noto*.

Figure P: Showing that the HDs (159a-159b) are Q-Redundant.

This reversal might seem counterintuitive at first blush, but is in fact line with an idea of "inquisitive" (as opposed to "logical") redundancy. Namely, the idea that figuring out the answer to a more specific question (i.e. resolving a strategy of inquiry leading to a *stronger* answer), automatically answers any less specific question (i.e. resolves a strategy of inquiry leading to a *weaker* answer). Based on our example, inquiring about cities (as evoked by the stronger disjunct *Noto*), makes it useless to further inquire about countries (as evoked by the weaker disjunct *Italy*).

5.5.3 Comparison with previous tree-based accounts of HDs

Lastly, let us briefly discuss how the current approach relates to earlier accounts of HDs that were also based on tree-like structures.

First, Ippolito (2019), proposed that sentences evoke Structured Sets of Alternatives (henceforth **SSA**s), which are in effect very close to our Qtrees. SSAs were assumed to be subject to a Specificity Constraint, spelled out in (183).

(183) **Specificity Condition** (Ippolito, 2019). A sequence $\Sigma =< [_{S_i} ...\alpha_F ...], [_{S_j} ...\beta_F ...] >$, s.t. both S_i and S_j are answers to the same QuD and β is in the structured set of alternatives evoked by α ($T_{\mathcal{A}_\alpha}$), is felicitous if either:

(i) α and β are dominated by the same number of nodes in $T_{\mathcal{A}_\alpha}$ or

(ii) α or β is the only node on its branch in $T_{\mathcal{A}_\alpha}$.

Rephrased within our framework, (183) states that, for a Qtree to be well-formed, verifying nodes should be same-level (i), except if one of them is directly connected to the root (ii).[12] The same-level condition (i) is reminiscent of our discussion of what it means for two alternatives to have same granularity. However, unlike our approach, it ends up banning disjunctions with incompatible, different-granularity alternatives, like (173a) and (173b), precisely because such disjunctions flag verifying nodes at different levels (e.g. *Noto* and *France*). The other condition, even if it builds on the intuition that "short" branches are "as specific as possible", still appears like an exception, and as such, seems to miss some kind of deeper generalization. Thus, even if (183) may sound like a very reasonable *descriptive* generalization,[13] it faces the challenge of explanatoriness, essentially because it stipulates that specific structural configurations between (roughly speaking) verifying nodes, should lead to Qtree ill-formedness. But many other configurations could have been assumed to cause ill-formedness, as well. For instance, having α and β in a relation of dominance in $T_{\mathcal{A}_\alpha}$, could have constituted another sensible generalization. Although our current model draws from very similar core intuitions (i.e. same-granularity alternatives organized within trees), it has the advantage over the Specificity Condition that it does not need to posit that verifying nodes be in specific structural configurations. Instead, our Q-Non-Redundancy constraint recycles familiar pragmatic principles based on competition with simpler alternatives, to *derive* that some structural relations between verifying nodes, should be ill-formed. Crucially also, our approach predicts the characterization of ill-formed configurations to *depend* on which simpler alternatives are available, for any given sentence. Should a critical competitor be missing, Qtrees featuring descriptively ill-formed configurations (e.g. verifying nodes

[12]This rephrasing is not exactly accurate, given that in our framework, one sentence may evoke various Qtrees, and verifying nodes are compositionally derived and so do not always fully coincide with the propositions that are being uttered. Still, we will show that, regardless of how it is rephrased, the Specificity Condition runs into issues when it comes to explanatoriness.

[13]*modulo* data like (173a) and (173b).

in a dominance relation), would end up being ruled-in.[14] Having a fixed, descriptive constraint on SSAs/Qtrees, does not allow that kind of flexibility, or perhaps does, but at the cost of positing very complex subconditions.

A more recent approach (Zhang, 2022) proposes to cover HDs, LDHDs, as well as "conjunctive" HDs (see (180)), but raises the same conceptual concerns as Ippolito (2019). Under Zhang's view, QuD-trees must obey two conditions, Uniformity (building on Simons, 2001) and Distinctness, spelled out in (184) and (185), respectively. In that framework, QuD-trees are identified with Büring's d-trees, which typically alternate question-nodes with answer-nodes.

(184) **Uniformity**. A disjunction's disjuncts must evoke the same strategy of inquiry with respect to the QuD answered by the whole disjunction. In practice, this means that the QuD-tree of a disjunction, branches into two subtrees, whose roots are labeled by the same question, but whose leaves denote potentially different answers.

(185) **Distinctness**. Answers to the same question must be distinct in terms of non-entailment.

Figure (Q) (adapted from Zhang, 2022) depicts the d-tree evoked by HDs like (159a) and (159b). In this tree, the two disjuncts introduce the same question, namely *Where will SuB29 take place?* This satisfies Uniformity (184). But the answers provided as leaves to these two identical questions, stand in an entailment relation, thus violating Distinctness (185).

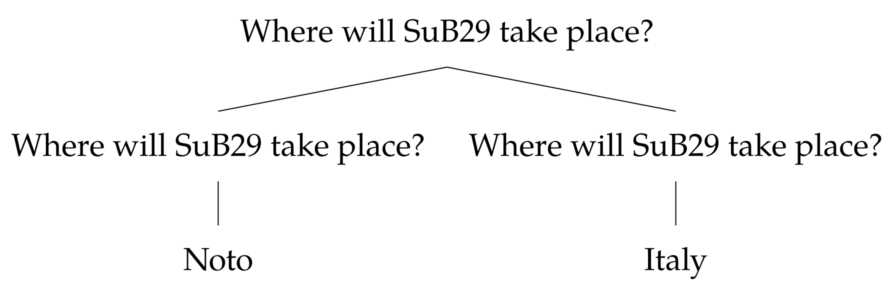


Figure Q: A d-tree for (159a) or (159b).

Though Uniformity and Distinctness, combined with the d-tree model, are characterized by a wider empirical coverage than Hurford's original descriptive constraint, the way Distinctness is phrased still remains quite descriptive, in the sense that one could

[14] Hénot-Mortier (pearc) discusses cases relevant to that discussion, in the context of Qtrees modified by *at least*.

wonder what are the pragmatic reasons for dispreferring compatible answers to a given question. Pu in another way, nothing would in principle prevent Distinctiveness to only disfavor *entailing* answers.

This Section reviewed two previous accounts of HDs (Ippolito, 2019; Zhang, 2022) and observed that they share many interesting insights with the current account – in particular, that oddness arises from specific structural relations between nodes in QuD-trees. However, one key difference between the current account and earlier ones, is that Q-Non-Redundancy directly builds on the concept of pragmatic competition, and as such allows to *derive* ill-formed configurations involving verifying nodes in Qtrees. In that sense, Q-Non-Redundancy appears conceptually closer to Zhang's more recent approach to "conjunctive" HDs, rooted in Inquisitive Semantics and also proposing a QuD-mediated, competition-based notion of Non-Redundancy (Zhang, to appear).

5.6 Conclusion

This Chapter proposed a conservative update of the Q-Non-Redundancy constraint introduced in Chapter 4, which was shown to account for two varieties of HDs: standard ones (featuring entailing disjuncts), and "long distance" ones (featuring a logical dependency between a weaker, high disjunct, and a stronger, low disjunct). This update was also shown to preserve the results established in Chapter 4. Beyond the data at stake, this Chapter suggests that the key criterion used to evaluate Redundancy, is less about the meanings conveyed by a sentence and its competitors (i.e. verifying nodes), than about what it takes to get to these meanings, in terms of optimal strategies of inquiry.

Our approach was shown to generalize to "compatible" HDs whose disjuncts are logically consistent, but non-entailing. Such constructions have been very challenging for earlier approaches to oddness, mainly due to their structural opacity: their two disjuncts are *not* structurally decomposed into inner disjunctions, meaning, their intuitively redundant components cannot be accessed by the grammar. We showed that our model of simplex and disjunctive Qtrees, was in fact enough to capture the oddness of CHDs,[15] because the Qtree machinery is precisely sensitive to the degree of granularity conveyed by propositions, and encodes granularity in objects made accessible to the grammar (namely, Qtrees). This prediction, which does not depend on Q-Non-Redundancy, is interesting,

[15]The Appendix following this Conclusion further discusses CHDs, and addresses one loose end of Section 5.4.3. But the overall conclusion and implications remain the same.

because it seems consistent with the intuition that CHDs exhibit a different flavor of oddness, as compared to HDs or LDHDs.[16]

The next Chapter extends the debate to "conditional" variants of HDs (Mandelkern and Romoli, 2018; Kalomoiros, 2024), obtained *via* the *or*-to-*if* tautology, in a way that will be reminiscent of Chapter 4. This will lead us to propose another constraint on Qtree formation, dubbed Q-Relevance.

5.7 Appendix: a more thorough analysis of CHDs

This Appendix constitutes a more in-depth analysis of Compatible Hurford Disjunctions, repeated in (160).

(160) a. # SuB29 will take place in the Basque country or France. $\mathbf{q} \vee \mathbf{p}$; $\mathbf{q} \wedge \mathbf{p} \not\models \bot$

b. # SuB29 will take place in France or in the Basque country. $\mathbf{p} \vee \mathbf{q}$; $\mathbf{p} \wedge \mathbf{q} \not\models \bot$

Section 5.4.3 assumed that S_q = *SuB29 will take place in the Basque country* and S_p = *SuB29 will take place in France*, "intuitively" evoked a by-region and a by-country partition of the CS, respectively. But could the model of Qtrees presented in Chapter 2 produce other Qtrees that could be shared by both S_q and S_p. Here, we show that the model presented in Chapter 2 predicts a less intuitive depth-1 Qtree to be possible for both S_p and S_q, and that this Qtree happens to rescue CHDs from infelicity. However, we will suggest that this Qtree should be blocked by an independently motivated constraint on question-partition matching (Fox, 2018), that we generalize to recursive partition, i.e. Qtrees.

The "pathological" Qtree that we will discuss in this Appendix is depicted in Figure R. The only layer of this Qtree corresponds to the Hamblin partition induced by the set of all country-alternatives, plus the *Basque country*-alternative. We now show that this Qtree can in principle be evoked by both S_p and S_q; as a result, (160a) and (160b) can in turn evoke a well-formed disjunctive Qtree, and escape oddness.

[16] I thank Nina for bringing this up, and I am also grateful for subsequent, more in-in depth discussions on that matter.

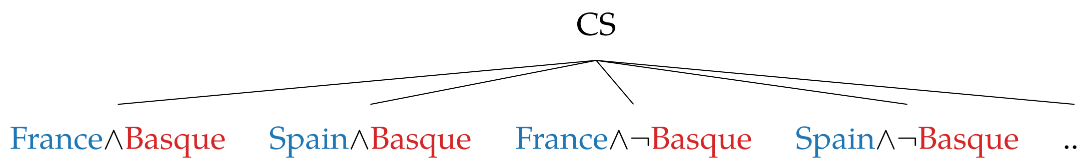


Figure R: A Qtree which, if evoked by both S_{p} and S_{q}, would give rise to a well-formed Qtree for the CHDs in (160).

Chapter 2 extensively discussed how Qtrees evoked by simplex sentences, encapsulate the concept of QuD granularity in their structure. In a nutshell, this was achieved by assuming that (non-polar) Qtrees must be such that each of their layers is generated based on *same-granularity* alternatives. Same-granularity alternatives to a proposition, were defined based on the Hasse diagram induced by $\vDash$ on the set of all alternatives to that proposition. Specifically, two alternatives were said to have same-granularity, if in this diagram, they were connected to a parent node (i.e. a proposition entailed by both alternatives), *via* the same number of intermediate nodes. This is exemplified in Figure S.

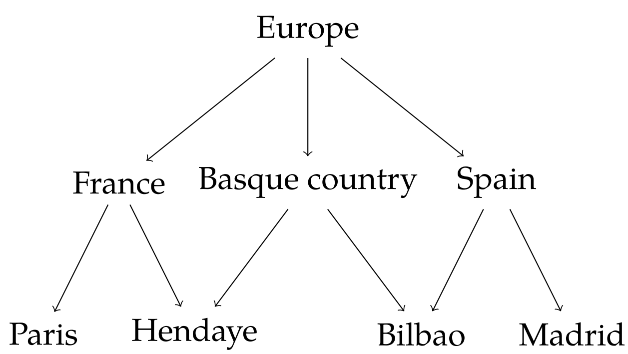


Figure S: Hasse diagram for
$\{Europe, France, Spain, Basque\ country, Paris, Hendaye, Bilbao, Madrid\}$.

This Figure suggests *Spain* and *France* have same granularity, because they both entail *Europe*, and, at a certain level of approximation, are both directly connected to Europe. Therefore, *France* and *Spain* can be used to generate a Qtree layer – as done in Figure O2, for instance. But under this definition, *the Basque country* and *France* also have same granularity, again, because they both entail *Europe*, and, at a certain level of approximation, are both directly connected to Europe. Therefore, a Qtree like the one in Figure R, whose layer corresponds to the Hamblin partition induced by the set of all country-alternatives, plus the *Basque country*-alternative, *could* be evoked by both S_p and S_q.

There remains one way to rule out that kind of Qtree, based on the independently motivated principle of Question-Cell Matching (Fox, 2018), given in (186). This principle states that there must be a bijection between the set of pointwise exhaustified alternatives that are part of a question's denotation, and the Hamblin partition induced by such alternatives on the CS. The main purpose of this principle, is to derive the presupposition that embedded questions must have a maximal true answer (Dayal, 1996).

(186) **Question-Cell Matching**. Let Q be a question (a set of potentially compatible alternatives) and let $\mathfrak{P}_{Q,CS}$ be the partition induced by Q on the CS. The following two conditions must hold:

- Cell Identification: $\forall c \in \mathfrak{P}_{Q,CS}.\ \exists p \in Q.\ CS \cap \text{exh}(Q,p) = c$;
- Non-Vacuity: $\forall p \in Q.\ \exists c \in \mathfrak{P}_{Q,CS}.\ CS \cap \text{exh}(Q,p) = c$

(186) appeals to the notion of exhaustification, which is defined in (187). Very roughly, the exhaustification operator exh can be understood as a covert variant *only*. More precisely, exh strengthens its prejacent with the negation non-weaker alternatives, in a non-arbitrary way (Innocent Exclusion, see (187a)), and also, asserts any remaining alternatives consistent with this strengthened meaning, again, in a non-arbitrary way (Innocent Inclusion, see (187b)).

(187) **Exhaustification** (Fox, 2007; Bar-Lev and Fox, 2017). Let p be a proposition and let Q be a set of relevant alternatives to p that are at most as complex as p, in the sense of Katzir (2007).
The exhaustification of p (prejacent) given Q, corresponds to p, conjoined with (i) the negation of all Innocently Excludable alternatives, and (ii) all Innocently Includable alternatives. In other words, $\text{exh}(Q,p) = p \wedge \bigwedge_{p' \in IE(Q,p)} \neg p' \wedge \bigwedge_{p' \in II(Q,p)} p'$.

a. **Innocent Exclusion**. p' is Innocently Excludable given Q and p ($p' \in IE(Q,p)$), iff p' belongs to the intersection of the maximal subsets of Q whose grand negation is consistent with p. In other words, $p' \in IE(Q,p) \iff p' \in \bigcap MaxExcl(Q,p)$, where $MaxExcl(Q,p) = Max_{\subseteq}(\{Q' \subset Q.\ p \wedge \bigwedge_{p' \in Q'} \neg p' \not\models \bot\})$.

b. **Innocent Inclusion**. p' is Innocently Includable given Q and p ($p' \in II(Q,p)$), iff p' belongs to the intersection of the maximal subsets of Q whose grand conjunction is consistent with p conjoined with the negation of Innocently Excludable alternatives. In other words, $p' \in II(Q,p) \iff p' \in \bigcap MaxIncl(Q,p)$, where $MaxIncl(Q,p) = Max_{\subseteq}(\{Q' \subset Q.\ p \wedge \bigwedge_{p' \in IE(Q,p)} \neg p' \wedge \bigwedge_{p' \in Q'} p' \not\models \bot\})$.

(186) was posited based on the standard semantics and pragmatics of questions, whereby questions denote sets of alternative propositions, which, at the pragmatic level, induce a partition of the CS (i.e. a depth-1 Qtree). But it can be naturally extended to recursive partitions, i.e. Qtrees. In Qtrees evoked by simplex sentence, each layer corresponds to the Hamblin partition induced by a set of same-granularity alternatives. So, we can reasonably assume that Question-Cell Matching, should apply to each layer of such Qtrees.

Until now, the absence or presence of this principle did not have any effect, because we only considered Qtrees generated from sets of same-granularity alternatives that were already exclusive, i.e. such that the mapping between same-granularity alternatives and partitions/Qtree layers, was always guaranteed. However, this kind of mapping is not guaranteed in the case of Figure R. We now show that assuming (186) applies to each layer of Qtrees evoked by simplex LFs, allows to rule out the problematic Qtree in Figure R.

As previously observed, the unique layer of this Qtree, is generated from the set of same-granularity alternatives containing all country-alternatives, plus the *Basque country* alternative. The goal is then to check if the pointwise exhaustifications of such alternatives, are in a bijective relation with the leaves of the Qtree in Figure R. (188) computes the exhaustified counterparts of country alternatives, supplemented by the *Basque country* alternative. First, let us notice that all these alternatives are logically independent; in particular, no alternative in that set is weaker than another alternative. Given this, there are three subcases to discuss: (i) the case of *France*/*Spain*; (ii) the case of other country alternatives like *Italy*; and (iii) the case of the *Basque country* alternatives. Starting with *France* (or alternatively *Spain*), it can be non-arbitrarily strengthened with the negation of all other alternatives. Because *France* already entails the negation of all other countries, the only meaningful strengthening is the negation of *the Basque country*. No alternative can be additionally asserted without contradicting this meaning, so Innocent Inclusion is vacuous. Thus, $\text{exh}(Q, \textit{France}) = \textit{France} \wedge \neg \textit{Basque}$ (see 188a). The same holds *mutatis mutandis* for the *Spain* alternative: $\text{exh}(Q, \textit{Spain}) = \textit{Spain} \wedge \neg \textit{Basque}$ (see (188b)). Let us now consider the case of other country alternatives, like *Italy*. *Italy* can be strengthened with the negation of all other alternatives in the set, but this strengthening is vacuous, because country alternatives are exclusive, and the Basque country is not in Italy. No alternative can be additionally asserted without contradicting this meaning, so Innocent Inclusion is also vacuous. Thus, $\text{exh}(Q, \textit{Italy}) = \textit{Italy}$ (see (188c)), and this holds for all other country alternatives different from *France* and *Spain*. Lastly, turning to the case of *the Basque country*, this alternative could in principle be non-vacuously strengthened with the negation of *France* (to mean *the Spanish Basque country*), or, with the negation of *Spain* (to mean *the French Basque country*). But negating *both* alternatives would lead to a contradictory meaning, and negating only one of the two, would be arbitrary. Therefore, Innocent Exclusion is vacuous. The same issue arises when considering Innocent Inclusion: *the Basque country* could be non-vacuously strengthened with the assertion of *France* (to mean *the French Basque country*), or, with the assertion of *Spain* (to mean *the Spanish Basque country*). But asserting *both* alternatives would lead to a contradictory meaning, and asserting only one of the two, would be arbitrary. Therefore, Innocent Inclusion is also vacuous, and $\text{exh}(Q, \textit{Basque}) = \textit{Basque}$ (see 188d).

(188) Pointwise exhaustification of $Q = \{\textit{France}, \textit{Spain}, ..., \textit{Italy}, \textit{the Basque country}\}$

a. $\text{exh}(Q, \textit{France}) = \textit{France} \wedge \neg\textit{Basque}$

b. $\text{exh}(Q, \textit{Spain}) = \textit{Spain} \wedge \neg\textit{Basque}$

c. $\text{exh}(Q, \textit{Italy}) = \textit{Italy}$

d. $\text{exh}(Q, \textit{Basque}) = \underline{\textit{Basque}}$

Now that the relevant pointwise exhaustified same-granularity alternatives have been computed, they must be compared to the leaves of the Qtree in Figure R, which correspond to the cells of the Hamblin partition induced by country alternatives, plus the *Basque country* alternative. This partition is summarized in (189). We then observe that there are three "mismatches" between the partition in (189), and the set of pointwise exhaustifications computed in (188): first, the partition contains a *French Basque country* and a *Spanish Basque country* cell (underlined), which correspond to none of the pointwise exhaustifications in (188). So the Cell Identification component of the Question-Cell Matching condition in (186), is violated. Moreover, the pointwise exhaustification of *the Basque country* in (188), yields *the Basque country* (underlined), which does not correspond to any cell in the partition in (188). So, the Non-Vacuity component of (186), is also violated.

(189) Partition induced by $Q = \{\textit{France}, \textit{Spain}, ..., \textit{Italy}, \textit{the Basque country}\}$ on a "complete" CS (equivalent to the problematic Qtree in Figure R):
$\mathfrak{P}_{Q,CS} = \{\underline{CS \wedge F \wedge B}, \underline{CS \wedge S \wedge B}, CS \wedge F \wedge \neg B, CS \wedge S \wedge \neg B, CS \wedge I, ...\}$

Therefore, the Question-Cell Matching condition in (186), is *not* verified by the set of alternatives under consideration, assuming the most general CS. What about a CS which would exclude the problematic cells (underlined), i.e. would presuppose that *SuB29 will not take place in the Basque country*? Such a CS would give rise to the restricted Hamblin partition in (190), for which each cell corresponds to some exhaustified alternative in (188).

(190) Partition induced by $Q = \{\textit{France}, \textit{Spain}, ..., \textit{Italy}, \textit{the Basque country}\}$ on a CS entailing $\neg B$:
$\mathfrak{P}_{Q,CS} = \{CS \wedge F \wedge \neg B, CS \wedge S \wedge \neg B, CS \wedge I, ...\}$

Still, an issue would remain when assuming such a CS, due to the presence of the *Basque country* alternative in the alternative set. This alternative would still be (vacuously) exhaustified, however its intersection with the CS would then be empty, and by definition would not correspond to any cell of the partition in (190). Therefore, the Non-Vacuity component of (186), would still be violated.

This overall suggests that the set of all country alternatives, supplemented with the *Basque country* alternative, induces a partition of the CS violating the Question-Cell Matching condition, no matter the CS. On that basis, and assuming that the same principle applies to the formation of layers in Qtrees evoked by simplex sentences, the Qtree depicted in Figure R should *not* be evoked by S_p or S_q, and, consequently, the disjunctions of S_p and S_q, should *not* evoke any well-formed Qtree. This Appendix showed that, even under a strict interpretation of what it means for two alternatives to be same-granularity, the result established in Section 5.4.3 is preserved – assuming that the independently motivated Question-Cell Matching condition applies to Qtree formation.

Chapter 6

Crossing countries: oddness in non-scalar Hurford Conditionals[1]

This Chapter extends the empirical landscape introduced in Chapter 5, which was mainly interested in Hurford Disjunctions, of the form $p \vee p^+$ or $p^+ \vee p$, with $p^+ \vDash p$. This Chapter is an investigation of Hurford *Conditionals* (Mandelkern and Romoli, 2018; Kalomoiros, 2024), of the form *If* p *then* $\neg p^+$ or *If* $\neg p^+$ *then* p, with $p^+ \vDash p$. Such conditionals are related to Hurford Disjunctions by the *or*-to-*if* tautology; but (arguably) unlike their disjunctive counterparts, they exhibit a crisp asymmetry: variants in which the negated stronger proposition is in the antecedent (*If* $\neg p^+$ *then* p) are degraded, while variants in which the negated stronger proposition is in the consequent (*If* p *then* $\neg p^+$), are fine. This Chapter explains this asymmetry, by building once again on the implicit QuD framework introduced in Chapter 2, and by proposing a new constraint on Qtree derivation, dubbed INCREMENTAL Q-RELEVANCE, building on Lewis's and Roberts's approaches to propositional RELEVANCE. This will predict that the antecedent and consequent of Hurford Conditionals should be ordered in terms of their conveyed degree of granularity; and will be shown to successfully extend to variants of these sentences. More broadly, this Chapter suggests that Hurford Disjunctions and Conditionals, display different "flavors of oddness".

[1]This Chapter builds on Hénot-Mortier (to appear), but develops a different (hopefully clearer and more principled) view on Hurford Conditionals. A number of similar intuitions will be exploited, including intuitions about granularity. I would like to thank the audiences and reviewers of the 2024 BerlinBrnoVienna Workshop, SuB29, and the 2024 Amsterdam Colloquium for relevant questions, datapoints and suggestions regarding this project and adjacent data. I want to give specials thanks to Amir, who first advised me to read Lewis (1988) almost two years ago, and Viola, who very wisely advised me to take another look at it this Spring.

6.1 Introducing Hurford Conditionals

Let us start by reviewing familiar data. As discussed in Chapter 5, Hurford Disjunctions (henceforth **HDs**), exemplified in (191), feature contextually entailing disjuncts ($p^+ \vDash_c p$), and are generally odd regardless of the order of their disjuncts (Hurford, 1974).[2]

(191) a. # SuB29 will take place in Noto or in Italy. $\mathbf{p^+} \vee \mathbf{p}$

b. # SuB29 will take place in Italy or in Noto. $\mathbf{p} \vee \neg\mathbf{p^+}$

Mandelkern and Romoli (2018) observed an intriguing, crisp contrast in so-called Hurford *Conditionals* (henceforth **HCs**), exemplified in (192): (192a) is odd while (192b) is fine. As a side note, $\rightarrow$ will be used as a shorthand for *if... then...*, throughout this Chapter (just like we did in Chapter 4), i.e. $\rightarrow$ will not imply that the conditionals under consideration are necessarily considered material.

(192) a. # If SuB29 will not take place in Noto, it will take place in Italy. $\neg\mathbf{p^+} \rightarrow \mathbf{p}$

b. If SuB29 will take place in Italy, it will not take place in Noto. $\mathbf{p} \rightarrow \neg\mathbf{p^+}$

Why is the contrast in (192) intriguing? Granted conditionals are material, the HC in (192a) is equivalent to the HD in (191a), and can also be made structurally equivalent to it *modulo* double-$\neg$ elimination. This is shown in (193). Therefore, it is perhaps unsurprising that (192a) appears degraded – it can be understood as an HD "in disguise".

(193) (192a) $= \neg\mathbf{p^+} \rightarrow \mathbf{p}$

$\equiv \neg(\neg\mathbf{p^+}) \vee \mathbf{p}$ (*or*-to-*if* tautology)

$\equiv \mathbf{p^+} \vee \mathbf{p}$ (double-$\neg$ elimination)

$=$ (191a)

The surprise comes from (192b). Just like (192a), (192b) can be seen as a conditional of the form $\neg q^+ \rightarrow q$, with $q^+ \vDash q$, by taking q^+ to be the proposition that *SuB29 will not take place in Italy* (so $q^+ = \neg p$), and q to be the proposition that *SuB29 will not take place in Noto* (so $q = \neg p^+$). Again, we have $q^+ \vDash q$, because $p^+ \vDash p$, and negation reverses entailments. This is all detailed in (194).

(194) (192b) $= \mathbf{p} \rightarrow \neg\mathbf{p^+}$

$\equiv \neg(\neg\mathbf{p}) \rightarrow (\neg\mathbf{p^+})$ (double-$\neg$ introduction)

$\equiv \neg\mathbf{q^+} \rightarrow \mathbf{q}$ ($\mathbf{q^+} := \neg\mathbf{p}$; $\mathbf{q} := \neg\mathbf{p^+}$; s.t. $\mathbf{q^+} \vDash \mathbf{q}$)

$\approxeq$ (192a)

[2] When the two disjuncts are the same modulo scalar expressions (e.g. $\langle$*some*, *all*$\rangle$) HDs *may* be rescued from infelicity Gazdar, 1979; Singh, 2008a; Fox, 2018; Hénot-Mortier, 2023 i.a.). Chapter **??** provides an overview of the challenges raised by "scalar" Hurford Sentences, and proposes an account elaborating on the framework introduced here.

(192b) is thus structurally similar to (192a), though not logically equivalent to it. So, one could say (192a) and (192b) are "isomorphic", in the sense that they have same logical structure, and can be derived from each other *via* a variable change preserving logical relations (see (195) for a formal definition).

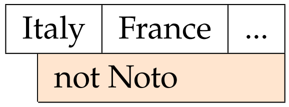
(195) **Isomorphy**. Let X and Y be two LFs. X and Y are isomorphic ($X \approxeq Y$) iff there is a substitution operation $\mathcal{S}$ targeting atomic propositions and preserving the logical relations between the elements in its domain ($aRb \iff \mathcal{S}(a)R\mathcal{S}(b)$, where R denotes entailment, contradiction, or independence) s.t. X and $\mathcal{S}(Y)$ have same parse.

This isomorphy between the infelicitous HC (192a) and the felicitous HC (192b) is problematic, because if (192a) is indeed an HD "in disguise", then so should (192b). Yet, this variant is felicitous. It appears that oddness in HCs is "asymmetric"; descriptively, the weaker item must be the antecedent, while the negated stronger item must be the consequent.

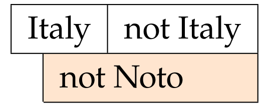
Kalomoiros (2024) proposed the first solution to both the HDs (191) and the HCs (192) (as well as other related datapoints). The approach was based on the idea that overt negation has a special status when it comes to evaluating if a sentence is redundant.

This Chapter argues for an alternative view, building on the idea that the questions evoked by an assertion must match the degree of granularity is conveys. The source of pragmatic oddness in HCs will then be tied to "granularity" violations, operationalized in terms of an incremental Relevance constraint. Roughly, this constraint will state that, whenever the question evoked by an assertion gets "restricted" to a certain domain of the Context Set, the domain must be Relevant to the question in a relatively standard sense, blending aspects of Lewis's and Roberts's view on Relevance. The directionality of this constraint will be directly tied to the computation of conditional Qtrees, which Chapter 2 defined as a kind of question-restriction.

According to this constraint, the HC in (192a) will be deemed deviant, essentially because its antecedent (*not Noto*) does not rule out any cell from any question evoked by its consequent (*Italy*). This is sketched in Figure A1.



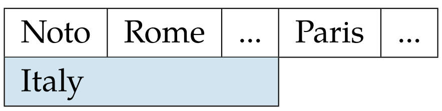
(1) (192a)'s consequent is not Relevant to the "*wh*"-partition evoked by (192a)'s antecedent.



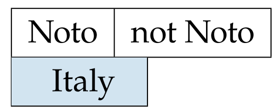
(2) (192a)'s consequent is not Relevant to the "polar"-partition evoked by (192a)'s antecedent.

Figure A: How (192a)'s antecedent interacts with (192a)'s consequent's evoked questions.

The HC in (192b) on the other hand, will be predicted to be fine, because its antecedent (*Italy*), interacts with the by-city partition evoked by its consequent (*not Noto*) in the following way: first, it rules out some cells – namely, all non-Italian city-cells – and second, it rules in some cells – namely, all Italian cities. This is sketched in Figure B1.



(1) (192b)'s antecedent is Relevant to the "*wh*"-partition evoked by (192b)'s consequent.



(2) (192b)'s antecedent is not Relevant to the "polar"-partition evoked by (192b)'s consequent.

Figure B: How (192b)'s antecedent interacts with (192b)'s consequent's evoked questions.

The contrast between the two HCs will therefore arise from an interaction between our novel Relevance constraint, and the maximal level of granularity evoked by propositions, meaning, how fine-grained the leaves of their Qtrees can be.

This Chapter is structured as follows. Section 6.2 provides an overview of Kalomoiros's approach to HCs, and outlines some of its limitations. Section 6.3 uses the machinery introduced in Chapter 2 to derive questions evoked by HCs. Section 6.4 motivates and defines Incremental Q-Relevance on LF-QuD pairs and shows how this constraint captures the HCs in (191) and (192). Section 6.5 explores more complex variants of HCs and shows how Incremental Q-Relevance captures them, sometimes in conjunction with Q-Non-Redundancy. Section 6.6 concludes and outlines remaining issues and questions.

6.2 Existing account

Mandelkern and Romoli (2018) show that HCs are problematic for virtually all accounts of HDs and their variants proposed before Kalomoiros (2024). Therefore, we will not review these approaches here, and directly jump to Kalomoiros's recent proposal.

6.2.1 Super-Redundancy

Kalomoiros' SUPER-REDUNDANCY, repeated in (142) from Chapter 4, states that a sentence S is super-redundant if it features a binary operation taking a constituent C as argument, and moreover there is no way of strengthening C to C^+ that would make the resulting sentence S^+ non-redundant (i.e., non-equivalent to its counterpart where C^+ got deleted).

(142) **SUPER-REDUNDANCY** (Kalomoiros, 2024). A sentence S is infelicitous if it contains $C * C'$ or $C' * C$, with $*$ a binary operation, s.t. $(S)^-_C$ is defined and for all D, $(S)^-_C \equiv S_{Str(C,D)}$. In this definition:

- $(S)^-_C$ refers to S where C got deleted;
- $Str(C, D)$ refers to a strengthening of C with D, defined inductively and whose key property is that it commutes with negation $(Str(\neg\alpha, D) = \neg(Str(\alpha, D)))$, as well as with binary operators $(Str(O(\alpha, \beta), D) = O(Str(\alpha, D), Str(\beta, D)))$;
- $S_{Str(C,D)}$ refers to S where C is replaced by $Str(C, D)$.

As already shown in Chapter 5, this constraint can capture HDs (191). The proof, adapted from Kalomoiros (2024), is repeated in (165).

(165) HDs are Super Redundant (SR).
We show (159a)=$\mathbf{p}^+ \vee \mathbf{p}$ and (159b)=$\mathbf{p} \vee \mathbf{p}^+$ are SR.
In either case, take C = $\mathbf{p}^+$.
We then have $(159a)^-_C = (159b)^-_C = \mathbf{p}$

$$\begin{aligned} \forall D.\ (159a)_{Str(C,D)} = (159b)_{Str(C,D)} &= (\mathbf{p}^+ \wedge D) \vee \mathbf{p} \\ &\equiv (\mathbf{p}^+ \vee \mathbf{p}) \wedge (D \vee \mathbf{p}) \\ &\equiv \mathbf{p} \wedge (D \vee \mathbf{p}) \\ &\equiv (\mathbf{p} \wedge D) \vee \mathbf{p} \\ &\equiv \mathbf{p} = (159a)^-_C = (159b)^-_C \end{aligned}$$

More interestingly perhaps, (142) also captures HCs, whether conditionals are assumed to be material, or strict. The proofs assuming material conditionals, are given in (196) for (192a) and (197) for (192b). In both cases, it is crucial that the local strengthening of $C = p^+$ be conjunctive *under* negation (and thus, disjunctive after applying De Morgan's law). In (196), this allows to remove p^+ from the chain of logical equivalences, and eventually derive Super-Redundancy. In the second part of (197) when $C = \neg p^+$, this ensures that the strengthening D *can* be disregarded, and that the equivalence does *not* obtain – eventually deriving a failure of SUPER-REDUNDANCY. Kalomoiros (2024) also shows that this account extends to strict (yet not variably strict) conditionals. We omit the proof here for brevity.

(196) Assuming implications are material, "strong-to-weak" HCs like (192a) are Super Redundant (SR).
We show (192a)=$\neg\mathbf{p}^+ \to \mathbf{p}$ is SR.
Take C = $\neg\mathbf{p}^+$.
We then have $(192a)^-_C = \mathbf{p}$.

$$\begin{aligned}\forall D.\ (192a)_{Str(C,D)} &= \neg(\mathbf{p}^+ \wedge D) \to \mathbf{p}\\ &\equiv (\mathbf{p}^+ \wedge D) \vee \mathbf{p}\\ &\equiv (\mathbf{p}^+ \vee \mathbf{p}) \wedge (D \vee \mathbf{p})\\ &\equiv \mathbf{p} \wedge (D \vee \mathbf{p})\\ &\equiv \mathbf{p} \wedge (D \vee \mathbf{p})\\ &\equiv \mathbf{p} = (192a)^-_C\end{aligned}$$

(197) Assuming implications are material, "weak-to-strong" HCs like (192b) are not Super Redundant (SR).
We show (192b)=$\mathbf{p} \to \neg\mathbf{p}^+$ is not SR.
Take C = (192b)'s antecedent = $\mathbf{p}$.
We then have $(192b)^-_C = \neg\mathbf{p}^+$.
Take $D = \bot$.

$$\begin{aligned}(192b)_{Str(C,D)} &= (\mathbf{p} \wedge D) \to (\neg\mathbf{p}^+)\\ &\equiv (\mathbf{p} \wedge \bot) \to (\neg\mathbf{p}^+)\\ &\equiv \bot \to (\neg\mathbf{p}^+)\\ &\equiv \top\\ &\not\equiv \neg\mathbf{p}^+ = (192b)^-_C\end{aligned}$$

Take C = (192b)'s consequent = $\neg\mathbf{p}^+$.
We then have $(192b)^-_C = \mathbf{p}$.
Take $D = \top$.

$$\begin{aligned}(192b)_{Str(C,D)} &= \mathbf{p} \to (\neg(\mathbf{p}^+ \wedge D))\\ &\equiv \mathbf{p} \to (\neg(\mathbf{p}^+ \wedge \top))\\ &\equiv \mathbf{p} \to (\neg\mathbf{p}^+)\\ &\equiv (\neg\mathbf{p}) \vee (\neg\mathbf{p}^+)\\ &\equiv \neg\mathbf{p}^+\\ &\not\equiv \mathbf{p} = (192b)^-_C\end{aligned}$$

This approach is compelling regarding its empirical coverage, but raises one conceptual interrogation. While earlier approaches to Redundancy (Meyer, 2013; Katzir and Singh, 2014; Mayr and Romoli, 2016 i.a.) link it to the concept of Brevity in the sense of Grice (1975), it remains unclear, under the Super-Redundancy view, why the notion of local strengthening is defined the way it is (in particular when it comes to its commuting with negation), and why it should be so central in deriving oddness. The next Section adds to this an empirical concern, by presenting data suggesting that overt negation may not be the only source of the contrast in (192).

6.2.2 Is overt negation really the culprit in HCs?

Super-Redundancy was originally motivated by the observation that negated HDs, like (198), appear felicitous. We will dub such sentences "disjunctwise" negated HDs, or **DNHD**s for short, to avoid confusion with expressions of the form $\neg(p^+ \vee p)$, where negation takes wide scope.

(198) Context (taken from Kalomoiros, 2024): we go into John's office and see a full pack of Marlboros in the dustbin. We are entertaining hypotheses about what's going on.
John either doesn't smoke or he doesn't smoke Marlboros. $(\neg \mathbf{p}) \vee (\neg \mathbf{p}^+)$

DNHDs are structurally identical to felicitous HCs, granted conditionals are material. This is shown in (199). Super-Redundancy predicts both (198) and (192b) to be fine.

(199) $$\begin{aligned}(192b) &= \mathbf{p} \rightarrow \neg \mathbf{p}^+ \\ &\equiv (\neg \mathbf{p}) \vee (\neg \mathbf{p}^+) && (\textit{or}\text{-to-}\textit{if}\ \text{tautology}) \\ &= (198)\end{aligned}$$

In this Section, we show that slight variations of (198) display unexpected downgrades in felicity. Moreover, such downgrades can in turn be mitigated by certain operators or expressions. We suggest that the entire paradigm may be better explained by assuming that (198) should in principle be deemed odd, but also that additional pragmatic processes, should be able to rescue it and its variants, only under certain conditions.

First, let us double-check that (142) predicts (198) to be fine. This is done in (200).

(200) DNHDs are not Super Redundant (SR).
We show (198)=$(\neg \mathbf{p}) \vee (\neg \mathbf{p}^+)$ is SR.
Take C = $\neg \mathbf{p}$.

We then have $(198)^-_C = \neg\mathbf{p}^+$.

Take $D = \bot$.

$$\begin{aligned}(198)_{Str(C,D)} &= (\neg(\mathbf{p} \wedge D)) \vee (\neg\mathbf{p}^+)\\ &\equiv (\neg(\mathbf{p} \wedge \bot)) \vee (\neg\mathbf{p}^+)\\ &\equiv (\neg\bot) \vee (\neg\mathbf{p}^+)\\ &\equiv \top \vee (\neg\mathbf{p}^+)\\ &\equiv \top\\ &\not\equiv \neg\mathbf{p}^+ = (198)^-_C\end{aligned}$$

Take C $= \neg\mathbf{p}^+$.

We then have $(198)^-_C = \neg\mathbf{p}$.

Take $D = \top$.

$$\begin{aligned}(198)_{Str(C,D)} &= (\neg\mathbf{p}) \vee (\neg(\mathbf{p}^+ \wedge D))\\ &\equiv (\neg\mathbf{p}) \vee (\neg(\mathbf{p}^+ \wedge \top))\\ &\equiv (\neg\mathbf{p}) \vee (\neg\mathbf{p}^+)\\ &\not\equiv \neg\mathbf{p} = (198)^-_C\end{aligned}$$

This fact should be unsurprising given that the HD in (198) is equivalent to the HC (192b) granted to *or*-to-*if* tautology, and that we already showed in (197) that (192b) is not Super-Redundant assuming implications are material.

We now show that slight variations of (198) displays felicity downgrades that can be mitigated by certain operators or expressions. First, (198) becomes degraded if its two disjuncts get swapped, as done in (201a). This is *not* expected under SUPER-REDUNDANCY, whose predictions are insensitive to the order of the disjuncts. Interestingly, adding *at all* to the second disjunct of (201a), recovers felicity, as shown in (201b). *At all* also seems to suggest that the first disjunct, *John doesn't smoke Marlboros*, implied *John smokes cigarettes*.

(201) a. # John either doesn't smoke Marlboros or he doesn't smoke.

b. John either doesn't smoke Marlboros or he doesn't smoke at all.

Although we do intend not provide a full-fledged account of the effect of *at all* here,[3] we believe the paradigm formed by (198) and (201), indicates that some additional incremental pragmatic mechanism is at play in DNHDs – meaning, (198) may be deemed deviant *a priori*, but may end up being rescued by some extra pragmatic mechanism made

[3]Here is an intuition however. *At all* seems to make the question whether John smokes (p vs. $\neg p$) more salient, and as such may force the $\neg p$ alternative to be considered when attempting exhaustification on the first disjunct $\neg p^+$. This would eventually make the two disjuncts contradictory, and rescue (201a) from oddness. See footnote 4 for a more in-depth discussion of the role of covert exhaustification in the sentence at stake, specifically in the subsequent focused variant (202).

unavailable in e.g. (201a). This in turn, suggests that the felicity of (198) should not necessarily be accounted for by a Non-Redundancy constraint.

Another observation in line with this hypothesis, is that (198) is significantly improved by focus, as shown in (202).

(202) (Either) John doesn't smoke or doesn't smoke MARLBOROS.

Our pre-theoretical understanding of the effect of focus in (202), is that not smoking MALRBOROS implies smoking cigarettes different from Marlboros, i.e. smoking still.[4] If this is indeed the case, (202) would end up meaning $\neg p \vee q$ with $q \equiv \neg p^+ \wedge p$. Descriptively, this disjunction features incompatible disjuncts, so does not violate Hurford's original condition. It is also predicted by most if not all accounts of oddness to be fine. Assuming that whatever focus achieves in (202), may also be achieved covertly and without explicit focus in (198), would explain (198)'s felicity independently of Super-Redundancy. We believe the pattern described here gets even crisper if disjuncts are picked to be more parallel[5]

Second, we note that (198) is made worse by removing *either*; see (203a). (203b) shows that adding *at all* to the stronger disjunct ($\neg p$) restores felicity.

(203) a. ?? John doesn't smoke or doesn't smoke Marlboros.
b. John doesn't smoke at all or doesn't smoke Marlboros.

Again, we do not wish to propose a full-fledged account of the effect of *either* in (198). But let us just observe that removing *either* in other sentences leads to the same kind of

[4]This may be backed by the theory, as well, assuming that focus forces covert exhaustification *via* the operator *exh* (Fox, 2007; Chierchia et al., 2009), and that $\neg p$ (*John does not smoke*) is a salient alternative to $\neg p^+$ (*John does not smoke Marlboros*) in (202). In that case, the enriched meaning of $\neg p^+$ would end up being $\neg p^+ \wedge \neg\neg p \equiv \neg p^+ \wedge p$, i.e. that *John does not smoke Marlboros, but does smoke*. Interestingly, this kind of inference licensed by *exh*, should be unavailable in (201a) – even when *Marlboros* gets focused – in order to capture the infelicity of this sentence. This could be ensured by assuming that relevant alternatives are somehow incrementally computed, and that $\neg p$ is not a Relevant alternative to $\neg p^+$ "out-of-the-blue" i.e. if $\neg p^+$ is not preceded by $\neg p$ (and, e.g. appears in the first disjunct of a disjunction).

[5]For instance, instead of having V and V+NP as disjuncts, we can have V+NP and V+NP$^+$, with $[\![\text{NP}^+]\!] \subset [\![\text{NP}]\!]$).

(i) Analogs of (198) (original sentence), (201a) (swapped disjuncts), and (201b) (swapped disjuncts, plus *at all*), respectively.
a. ? John either doesn't own a dog or he doesn't own a lab.
b. # John either doesn't own a lab or he doesn't own a dog.
c. John either doesn't own a lab or he doesn't own a dog at all.

(ii) Analog of (202) (focused p^+).
a. John either doesn't own a dog or doesn't own a LAB.

degradation. Such sentences, dubbed *bathroom sentences* (Evans, 1977) and attributed to Barbara Partee, are exemplified in (204a).

(204) a. Either there is no bathroom or it's upstairs.
b. ?? There is no bathroom or it's upstairs.
c. Either there is no bathroom or there is a bathroom and it's upstairs.

Roughly, (204a) requires its second disjunct to be interpreted given the negation of its first disjunct to be felicitous. This is because the pronoun *it* in (204a)'s second disjunct, requires an antecedent, which is not overtly introduced in (204a), but could be provided by an existential statement of the form *there is a bathroom*, which correspond to the negation of (204a)'s first disjunct. So, very roughly, (204a) could be felicitous, if understood as (204c), which has the form $\neg p \vee (p \wedge q_p)$, where q_p means that q presupposes p. This is quite similar to the possible pragmatic strengthening of (198)'s second disjunct with p, which we argued made this sentence felicitous. Removing *either* in both DNHDs and bathroom sentences, could be argued to prevent this rescue mechanism[6] – leading to a degradation, as shown in (203a) and (204b) respectively.

Let us now take stock and review the implications for HCs. We have just seen that DNHDs like (198), which constitute the basis of the argument supporting the Super-Redundancy approach, may be felicitous for reasons independent of Non-Redundancy. Specifically, we provided additional data suggesting that independent pragmatic mechanism(s) may force the weaker disjunct of (198) to contradict the stronger one, and that such mechanisms may be blocked or forced, when considering specific variants of (198). In fact, even ignoring such variants, we can observe that the felicity of (198) seems only guaranteed when a precise context is set up; but doing so may force a specific kind of QuD, and such a move was shown to improve other, non-negated HDs just as well (Haslinger, 2023). Besides, we can note that Kalomoiros (2024)'s Super-Redundancy is challenged by disjunctwise negated *Long-Distance* HDs (see Section 6.5.2), and, outside the domain of Hurford Sentences, by other varieties of redundant sentences obtained from the structure $p \vee p \vee q$ *via* the *or-to-if* tautology (see Chapter 4).

While the tentative explanations laid out here do not fully explain the complex patterns reported, we believe the overall data to be more in line with an analysis which, unlike Super-Redundancy, would not assign a key role to overt negation in HDs and HCs,

[6] This in fact would be in line with the idea that *either* somehow forces exclusivity between disjuncts (Nicolae et al., 2025 i.a.).

but instead, would interact with pragmatic processes themselves influenced by negation and incrementality. In our alternative proposal, we will in fact suggest that *granularity* differences (e.g., *Paris* being finer-grained than *France*, *smoking Marlboros* being more fine-grained than *smoking*), drive the contrast in (192).

6.3 QuDs evoked by Hurford Conditionals

To clarify the challenges posed by HCs in our framework, let us first derive Qtrees for these sentences, building on the model presented in Chapter 2. The two sentences under consideration are repeated below.

(192) a. # If SuB29 will not take place in Noto, it will take place in Italy. $\neg \mathbf{p}^{+} \rightarrow \mathbf{p}$

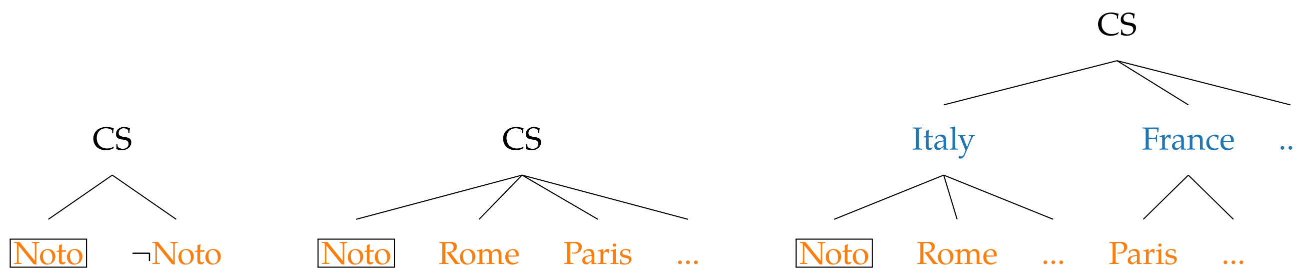
b. If SuB29 will take place in Italy, it will not take place in Noto. $\mathbf{p} \rightarrow \neg \mathbf{p}^{+}$

Crucial for this Section will be the idea that conditionals evoke Qtrees which assign asymmetric roles to antecedent and consequent. Namely, a conditional Qtree is a Qtree for the antecedent whose verifying nodes are *replaced* by their intersection with a Qtree for the consequent. We will see towards the end of this Chapter that this asymmetry may be exploited to derive the following generalization: a conditional whose consequent evokes at least one Qtree that is finer-grained than some Qtree evoked by its antecedent, is felicitous.

6.3.1 Qtrees for the antecedent and consequent of HCs

We first compute the Qtrees compatible with S_p = *SuB29 will take place in Italy*, and S_{p^+} = *SuB29 will take place in Noto*. The Qtrees for $\neg S_{p^+}$ = *SuB29 will not take place in Noto* will be subsequently derived from those evoked by S_{p^+}.

Chapter 2 extensively discussed how to derive Qtrees from simplex sentences like S_p and S_{p^+}. And Chapter 5 in fact discussed the Qtrees evoked by these exact sentences. Here, it is enough to say that such sentences may evoke three kinds of Qtrees: "polar" ones, splitting the Context Set (henceforth **CS**) into p and $\neg p$ worlds; "*wh*" ones, splitting the CS according to the Hamblin partition generated by same-granularity alternatives to the prejacent; and "*wh*-articulated" ones, whereby each layer corresponds to a Hamblin partition of increasing granularity from the top down, the last layer matching the granularity of the prejacent. In each case, leaves entailed by the prejacent are $\boxed{\text{flagged}}$ as "verifying", and keep track of at-issue content. In this Chapter, and just like in Chapter 5, we will only consider two levels of granularity for S_p and S_{p^+}: by-city and by-country. This gives rise to the Qtrees in Figure C (for S_{p^+}) and Figure D (for S_p).



(1) "Polar". (2) "*Wh*". (3) "*Wh*-articulated".

Figure C: Qtrees evoked by S_{p^+} = *SuB29 will take place in Noto*.

(1) "Polar". (2) "*Wh*".

Figure D: Qtrees evoked by S_{p} = *SuB29 will take place in Italy*.

We can already note that Figures C3 and D2, introduce consistent partitionings: Figure C3 can be in fact be seen as a refinement of Figure D2, as per (72), repeated below.

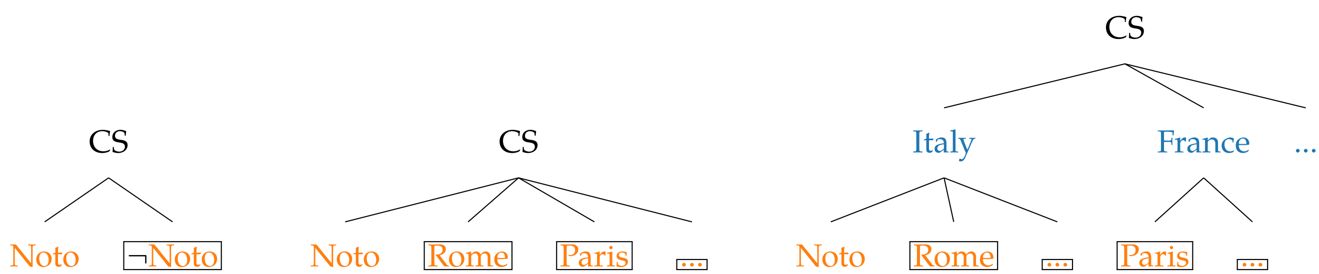
(72) **Qtree refinement**. Let T and T' be Qtrees. T is a refinement of T' (or: T is finer-grained than T'), iff T' can be obtained from T by removing a subset $\mathcal{T}$ of T's subtrees, s.t., if $\mathcal{T}$ contains a subtree rooted in N, then, for each node N' that is a sibling of N in T, the subtree of T rooted in N', is also in $\mathcal{T}$.

More precisely, C3 constitutes a *strict* refinement of Figure D2, as per (205). It may not be obvious at this point, but this stronger characterization will be the most Relevant to our subsequent predictions.

(205) **Strict Qtree refinement**. Let T and T' be Qtrees. T is a strict refinement of T' (or: T is strictly finer-grained than T'), if T is a refinement of T' and $\mathcal{L}(T) \cap \mathcal{L}(T') = \emptyset$.

Figures A3 and B2 thus structurally capture the intuition that S_{p^+} answers a finer-grained question than S_p. More generally, Figures C and D show that some Qtree obtained for S_{p^+} (namely Figure A3), (strictly) refines some Qtree for S_p (namely, Figure B2); while no Qtree obtained for S_p refines a Qtree for S_{p^+}.

Before computing the conditional Qtrees corresponding to (192a) and (192b), we need to compute the Qtree corresponding to the negation of S_{p^+}, namely $\neg S_{p^+}$ = *SuB29 will not take place in Noto*, which constitutes the antecedent of (192a) and the consequent of (192b). As discussed in Chapter 2, a negated LF evokes the same kind of question structure as its positive counterpart, but flags a disjoint set of verifying nodes. More specifically, given an LF X, evoking a Qtree T, a Qtree T' for $\neg X$ is obtained by retaining T''s structure (nodes and edges), and "swapping" T''s verifying nodes, by replacing any set of same-level verifying nodes in T by the set of non-verifying nodes at the same level in T. If the verifying nodes are all leaves, this operation simply corresponds to set complementation in the domain of leaves. This is done for $\neg S_{p^+}$ in Figure E.



(1) "Polar". (2) "*Wh*". (3) "*Wh*-articulated".

Figure E: Qtrees evoked by $\neg S_{\mathbf{p}^+}$ = *SuB29 will not take place in Noto*.

Because negation preserves Qtree structure and only affects verifying nodes, Figure E3, just like Figure C3, constitutes a strict refinement of Figure D2. More broadly, our observation about S_p and S_{p^+} extends to S_p and $\neg S_{p^+}$: some Qtree obtained for $\neg S_{p^+}$ (namely Figure E3), (strictly) refines some Qtree for S_p (namely, Figure D2); while no Qtree obtained for S_p refines a Qtree for $\neg S_{p^+}$.

This double observation will be crucial for our approach to HCs: felicitous HCs like (192b) are the ones whose antecedent evokes a question that is coarser-grained than that of their consequent (i.e. s.t. the antecedent Qtree *can* be strictly refined by a consequent Qtree); odd HCs like (192a) are the ones whose antecedent evokes a question that is finer-grained than that of their consequent (i.e. s.t. the antecedent Qtree *cannot* be strictly refined by any consequent Qtree).

6.3.2 Conditional Qtrees, and one useful result

Let us now turn to the Qtrees evoked by the HCs (192a) = $\neg S_{p^+} \rightarrow S_p$ and (192b) = $S_p \rightarrow \neg S_{p^+}$. Following Chapters 2 and 4, we assume that the "inquisitive" contribution of *if ... then ...* (glossed $\rightarrow$) is *not* material, meaning, a conditional Qtree is not derived by disjoining the negation of its antecedent Qtrees, with its consequent Qtrees.

Chapter 2 instead proposed that conditionals evoke questions pertaining to their consequent, set in the domain(s) of the CS where the antecedent holds. This was modeled by assuming that conditional Qtrees are derived by "plugging" a consequent Qtree T_C into the verifying nodes of antecedent Qtrees T_A. More concretely, for each verifying node N of T_A, N gets replaced by $T_C \cap N$, where $\cap$ refers to tree-node intersection. This operation is repeated in (66).

(66) **Tree-node intersection**. Let $T = (\mathcal{N}, \mathcal{E}, R)$ be a Qtree. Let p be a proposition. The tree-node intersection between T and p, noted $T \cap p$, is defined iff $R \cap p \neq \emptyset$ and, if so, is the Qtree $T' = (\mathcal{N}', \mathcal{E}', R')$ s.t.:

- $\mathcal{N}' = \{p \cap N \mid N \in \mathcal{N} \wedge p \cap N \neq \emptyset\}$
- $\mathcal{E}' = \{\{N_1 \cap p, N_2 \cap p\} \mid \{N_1, N_2\} \in \mathcal{E} \wedge (N_1 \cap p) \neq (N_2 \cap p) \wedge N_1 \cap p \neq \emptyset \wedge N_2 \cap p \neq \emptyset\}$
- $R' = R \cap p$

From an algorithmic perspective, the formation of a conditional Qtree based on T_A and T_C, amounts to (i) replacing every verifying node of T_A by its intersection with T_C; (ii) removing resulting empty nodes; (iii) removing resulting dangling and unary edges. Additionally, Chapter 2 assumed that only the consequent of a conditional contributes verifying nodes in the resulting conditional Qtree. In particular, nodes falsifying the antecedent are not considered verifying in the resulting conditional Qtree. The core idea behind this operation is that conditionals introduce a hierarchy between antecedent (backgrounded) and consequent (at-issue): the consequent Qtree gets *restricted* by the antecedent Qtree. (114), repeated below, summarizes these assumptions.

(114) *Qtrees for conditional LFs.* A Qtree T for $X \rightarrow Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

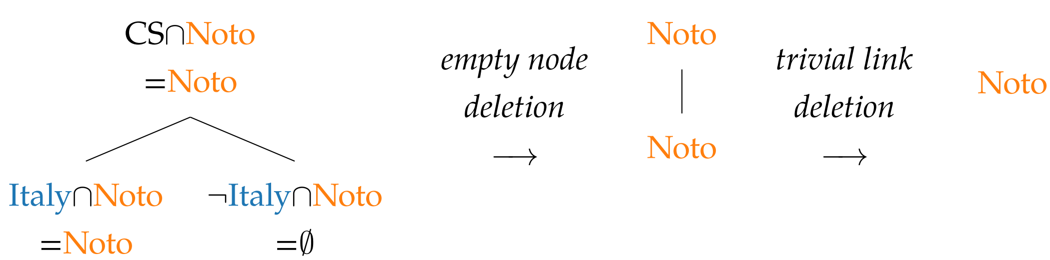
- replacing each node N of T_X that is in $\mathcal{N}^+(T_X)$ with $T_Y \cap N$ (see (**??**));
- returning the result only if it is a Qtree.

In other words, $Qtrees(X \rightarrow Y) = \{T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)}(T_Y \cap N) | (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y) \wedge T_X \cup \bigcup_{N \in \mathcal{N}^+(T_X)}(T_Y \cap N)$verifies (52)$\}$, and $\mathcal{N}^+(T_X \rightarrow T_Y) = \{N \cap N' | (N, N') \in \mathcal{N}^+(T_X) \times \mathcal{N}^+(T_Y) \wedge N \cap N' \neq \emptyset\}$.

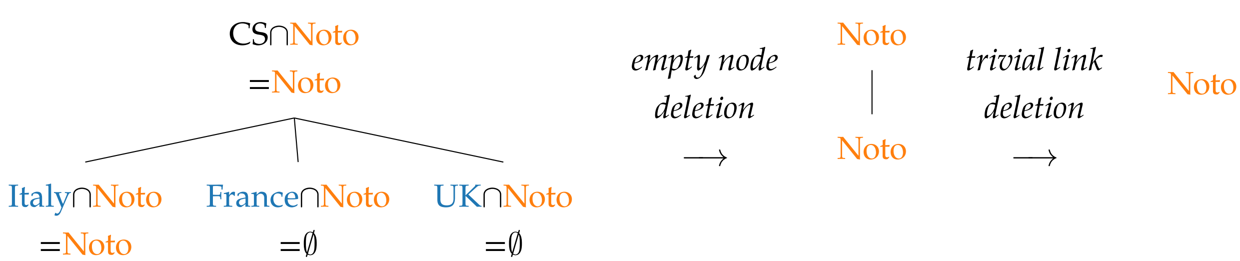
(66) comes with one useful prediction when it comes to HCs, namely that intersecting a city-level node with a country-level Qtree does not have any effect. This is consistent with the intuition that answering a question about cities automatically answers question about countries, and corresponds to the generalization in (116), repeated below.

(116) **Vacuous tree-node intersection**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). $T \cap N = N$ iff $\exists N' \in \mathcal{L}(T).\ N \vDash N'$.

Figure F illustrates this result, considering two possible Qtrees for S_p = *SuB29 will take place in Italy*, and their intersection with a city-level node like *Noto*.



(1) Derivation of *Noto*∩Tree D1=the *Noto*-node.



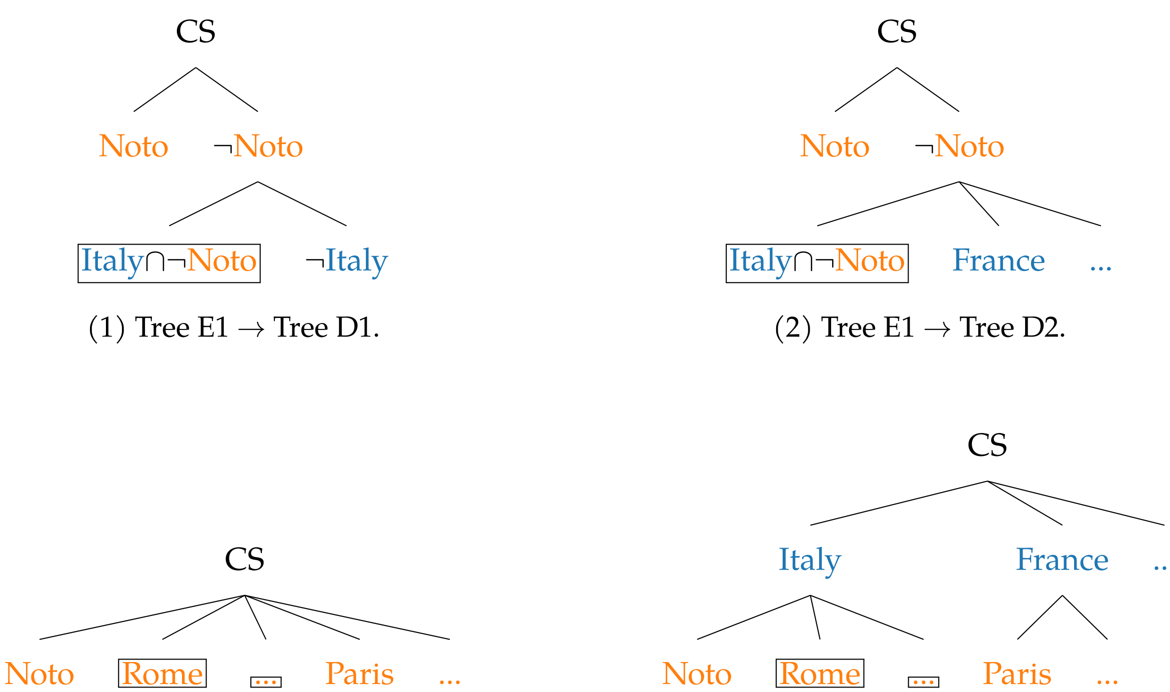
(2) Derivation of *Noto*∩Tree D2=the *Noto*-node.

Figure F: Intersecting a city-level node and a country-level tree yields the input city-level node.

A consequence of this result, given the definition of conditional Qtrees in (114), is the following: if an antecedent Qtree T_X and a consequent Qtree T_Y are such that each verifying node of T_X entails some leaf in T_Y, the conditional Qtree resulting from their composition, will have the same structure as T_X, and its verifying nodes will be exactly the verifying nodes in T_X that entail some verifying node in T_Y. This is the case, if T_X is a strict refinement of T_Y, whose verifying nodes are all leaves. We will use this observation to justify the derivation of conditional Qtrees for HCs in the next two Sections, and later when evaluating their well-formedness.

6.3.3 Qtrees for the HCs in (192)

We can now use the rule (114) to compute Qtrees for HCs. We start with the candidate Qtrees for the infelicitous HC (192a) = $\neg S_{p^+} \rightarrow S_p$. Applying (114) to this LF, using the Qtrees for $\neg S_{p^+}$ from Figure E as antecedent Qtrees, and the Qtrees for S_p from Figure D as consequent Qtrees, leads to the conditional Qtrees in Figure G.



(1) Tree E1 → Tree D1. (2) Tree E1 → Tree D2.

(3) Tree E2 → Tree D1/D2. (4) Tree E3 → Tree D1/D2.

Figure G: Qtrees for (192a)=#*If SuB29 will not take place in Noto, it will take place in Italy*.

The Qtrees in Figures G1 and G2 are obtained by replacing the verifying *not Noto* node of the "polar" antecedent Qtree from Figure E1, with the intersection between this node and a Qtree for *Italy* (either from Figure D1, or from Figure D2). Because *not Noto*, does not entail any leaf in the consequent Qtrees for *Italy* (it does not entail any particular city), the whole operation is *not* structurally vacuous, and the output Qtrees are of depth 2. Verifying nodes are inherited from the consequent Qtree after intersection, i.e. correspond to *Italy but not Noto*.

The Qtrees in Figures G3 and Figure G4 are obtained by replacing each leaf different from *Noto* in the non-"polar" antecedent Qtrees (from Figures E2 and E3 respectively), with the intersection between this leaf, and a Qtree for *Italy* (from Figure D1 or D2). Because each node different from *Noto*, is a city-node, it will entail some leaf in the Qtrees evoked by *Italy*. Therefore, the formation of a conditional Qtree based on these inputs, will be structurally vacuous. This explains why the Qtrees in Figures G3 and Figure G4 appear structurally similar to the antecedent Qtrees used to form them, in Figure E2 and Figure E3 respectively. The only difference between these inputs, and the outputs, lies in the verifying nodes, which are inherited from the consequent Qtrees, and correspond to Italian cities different from Noto. Figure H further details the derivation of the Qtree in Figure G3.

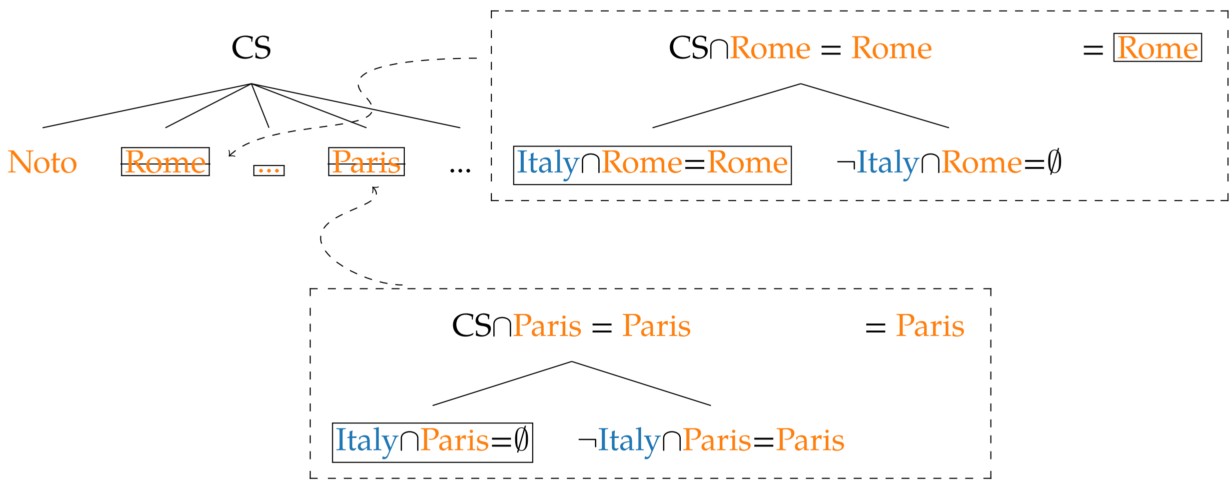


Figure H: Breakdown of the derivation of Figure G3, assuming Figure D1 is the consequent Qtree. The end result is unchanged if Figure D2 is considered instead.

Let us now turn to the candidate Qtrees for the felicitous HC (192b) = $S_p \to \neg S_{p^+}$. Applying (114) to this LF, using now the Qtrees for S_p from Figure D as antecedent Qtrees, and the Qtrees for $\neg S_{p^+}$ from Figure E as consequent Qtrees, leads to the conditional Qtrees in Figure I.

(1) Tree D1 → Tree E1.

(2) Tree D1 → Tree E2/E3.

(3) Tree D2 → Tree E1. (4) Tree D2 → Tree E2/E3.

Figure I: Qtrees for (192b)=*If SuB29 will take place in Italy, it will not take place in Noto.*

All the Qtrees in Figure I are obtained by replacing the verifying *Italy* node of the antecedent Qtree (from Figure E1 or E2), with the intersection between this node and a Qtree for *not Noto*. Because *Italy*, does not entail any leaf in the consequent Qtrees for *not Noto* (it entails neither *not Noto*, nor any specific city), the whole operation is never structurally vacuous, and the output Qtrees are all of depth 2. Verifying nodes are inherited from the consequent Qtree after intersection, i.e. correspond to *Italy but not Noto*.

At this point, it seems that many Qtrees are available, for both the felicitous variant (192b) and the odd variant (192a). But it appears that the two sets of Qtrees are different from each other, which comes from the fact that Qtrees for S_p and S_{p^+} differ in terms of granularity, and that the recipe for conditional Qtrees in (114), is asymmetric in nature. What is the key difference between these two sets of Qtrees then? It appears that *some* Qtrees compatible with (192b), namely those in Figure I2 and I4, still feature a by-city partition (as conveyed by the consequent) at their lowest level, defined on the *Italy*-subset of the CS.

By contrast, none of the Qtrees evoked by (192a) feature a properly restricted by-country partitions at their lowest level, i.e. a partition which both (i) contains some country nodes (as introduced by the consequent) and (ii) does not contain *all* country nodes. Instead, such Qtrees either feature by-city partitions at the leaf level (Figures G3 and G4), or partitions where no country node is fully missing (Figure G1 and G2).

We will see in the next Section, that these observed differences between the Qtrees evoked by the felicitous HC (192b) and those evoked by the infelicitous HC (192a), translate into the following generalization: the antecedent of (192b), *Italy can* be taken to be "Relevant" to the question evoked by (192b)'s consequent; while the antecedent of (192a), *not Noto*, *cannot* be taken to be "Relevant" to the question evoked by (192a)'s consequent.

We will then further justify this description in the form of a new incremental RELEVANCE constraint targeting Qtree derivation, and specifically tree-node intersection.

6.4 Hurford Conditionals and RELEVANCE

6.4.1 Do we actually need an extra constraint?

We have just seen that both HCs in (192) are in principle compatible with various Qtrees. First, let us double-check that the previous constraints on Qtrees (and LFs) defined in Chapters 2 and 5, are insufficient to capture the contrast between the two HCs in (192). The first constraint to check is the EMPTY LABELING constraint, which states that well-formed Qtrees should flag at least one node as verifying; see (75).

(75) **EMPTY LABELING**. If a sentence S evokes a Qtree T but does not flag any node as verifying in T, then T is deemed odd given S.

It is easy to see that none of the Qtrees in Figure G (corresponding to the infelicitous HC (192a)) or Figure I (corresponding to the felicitous HC (192b)), violate (75): all these Qtrees flag at least one node. So the EMPTY LABELING does not help at all in deriving the desired contrast.

The second constraint to check, is Q-NON-REDUNDANCY, which states that a Qtree evoked by an LF should not be equivalent to a Qtree evoked by some simplification of that LF (see (174) and (175)).

(174) **Q-NON-REDUNDANCY** (final version). Let X be a LF and let $Qtrees(X)$ be the set of Qtrees evoked by X. For any $T \in Qtrees(X)$, T is deemed Q-REDUNDANT given X, iff there exists a formal simplification of X, X', and $T' \in Qtrees(X')$, such that $T \equiv T'$.

(175) **QTREE EQUIVALENCE RELATION**. T and T' are equivalent ($T \equiv T'$) iff T and T' have same structure and same set of minimal verifying paths.

We can show that none of the Qtrees in Figures G and I violate (174). To see this, we need to review the Qtrees associated with the simplifications of (192a) and (192b). Let us use p and p^+ as shorthands for S_p = *SuB29 will take place in Italy*, and S_{p^+} = *SuB29 will take place in Noto*. The possible simplifications of (192a) and (192b) are summarized in Table 6.1.

Sentence	Simplifications
(192a)=$\neg\mathbf{p}^+ \to \mathbf{p}$	$\mathbf{p}$, $\mathbf{p}^+$, $\neg\mathbf{p}^+$, $\mathbf{p}^+ \to \mathbf{p}$
(192b)=$\mathbf{p} \to \neg\mathbf{p}^+$	$\mathbf{p}$, $\mathbf{p}^+$,$\neg\mathbf{p}^+$, $\mathbf{p} \to \mathbf{p}^+$

Table 6.1: Gathering the formal simplifications of (192a) and (192b).

Let us start by evaluating the simplifications of the infelicitous variant (192a). The Qtrees for (192a) are given in Figure G. For (192a) to be deemed deviant, all these Qtrees must be equivalent to *some* Qtree evoked by *some* simplification of (192a). For clarity, we proceed simplification-by-simplification.

First, Qtrees for the simplification p, shown in Figure D, have a different structure altogether from all the Qtrees in Figure G. So there is no way Qtree equivalence holds between some Qtree for p and some Qtree for (192a).

Second, among the Qtrees for the simplification p^+ shown in Figure C, two Qtrees are structurally identical to two Qtrees from Figure G. The first pair is made of the Qtree in Figure C2 and the one in Figure G3. These two Qtrees, though structurally identical, are not equivalent, because they each flag different sets of leaves as verifying; so their minimal sets of verifying paths cannot be the same. The second pair is made of the Qtree in Figure C3 and the one in Figure G4. Again, these two Qtrees, though structurally identical, are not equivalent, because they each flag different sets of leaves as verifying.

Third, the reasoning about the simplification p^+, extends to the simplification $\neg p^+$: there are two Qtrees evoked by $\neg p^+$ (in Figures E2 and E3) that are structurally identical to two Qtrees in Figure G, but these pairs, though structurally identical, are not equivalent, because they flag different sets of leaves as verifying.

Lastly, Qtrees for the simplification $p^+ \to p$ are the same as the Qtrees evoked by p^+. This is because, the only verifying node of the antecedent Qtree, *Noto*, always entails a leaf of the consequent Qtree (namely, *Italy*); therefore, the intersection operation performed to create conditional Qtrees is vacuous (as per (116)), and the resulting conditional Qtrees are just the same as the antecedent Qtrees used to form them. Since we have already shown that none of the Qtrees evoked by p^+ make the Qtrees in Figure G Q-Redundant, none of the Qtree evoked by $p^+ \to p$ make the Qtrees in Figure G Q-Redundant, either.

We have just gone through all the possible simplifications of the infelicitous HC (192a), and shown that none of these simplification evoke Qtrees triggering Q-Non-Redundancy. Therefore, Q-Non-Redundancy does not rule out (192a). This already motivates the introduction of a new constraint deriving (192a)'s oddness.

Let us now turn to the simplifications of the felicitous HC (192b). The Qtrees for (192b) are given in Figure I. For (192b) to be fine, some Qtree in Figure I should not be equivalent to *any* Qtree evoked by *any* simplification of (192b). Again, we proceed simplification-by-simplification and show something stronger, namely that no simplification evokes a Qtree equivalent to any Qtree in Figure G.

First, Qtrees for the simplification p, shown in Figure D, have a different structure altogether from all the Qtrees in Figure I. So there is no way Qtree equivalence holds between some Qtree for p and some Qtree for (192b).

Second, Qtrees for the simplification p^+, shown in Figure C, also have a different structure altogether from all the Qtrees in Figure I. So there is no way Qtree equivalence holds between some Qtree for p^+ and some Qtree for (192b). This extends to the simplification $\neg p^+$, whose Qtrees are structurally identical to those evoked by p^+.

Lastly, Qtrees for the simplification $p \rightarrow p^+$ are pairwise structurally identical to the Qtrees in Figure I (evoked by $p \rightarrow \neg p^+$). This is because Qtrees for $p \rightarrow p^+$ and $p \rightarrow \neg p^+$ are built using Qtrees for p as antecedent Qtrees, and Qtrees for $(\neg)p^+$ as consequent Qtrees, and negation does not affect Qtree structure. Still, the Qtrees evoked by the simplification $p \rightarrow p^+$ are *not* pairwise equivalent to the Qtrees in Figure I. This is roughly because, Qtrees for $p \rightarrow p^+$ will flag nodes verifying p^+ (consequent), while Qtrees for $p \rightarrow \neg p^+$ in Figure I, will flag nodes verifying $\neg p^+$. More precisely, the Qtrees evoked by $p \rightarrow p^+$, flag nodes that are Italian cities, and in fact Noto; while the Qtrees in Figure I, flag nodes that are Italian cities different from Noto. Thus, pairs of equivalent Qtrees from these two sets, end up flagging disjoint sets of verifying nodes, which in turn implies that there is no way Qtree equivalence holds between some Qtree for $p \rightarrow p^+$ and some Qtree for (192b).

We have just gone through all the possible simplifications of the felicitous HC (192b), and shown that none of these simplifications evokes Qtrees triggering Q-Non-Redundancy. Therefore, Q-Non-Redundancy does not incorrectly rule out (192b) – which is good news, and means we do not need to amend Q-Non-Redundancy to rule in (192b).

In brief, this Section confirmed that the constraints on Qtrees and LFs posited so far (Empty Labeling, Q-Non-Redundancy), if they do not incorrectly rule out felicitous HCs like (192b), also cannot rule out *in*felicitous HCs like (192a). In other words, both HCs are so far predicted to be felicitous. To account for the infelicity of (192a), while retaining the felicity of (192b), we will appeal to an updated definition of Relevance. The next Section will first motivate the use of a new Relevance constraint, by outlining some limitations of earlier approaches to Relevance.

6.4.2 Can earlier notions of Relevance help?

We have previously suggested that the contrast between felicitous and infelicitous HCs may be a matter of Relevance. Chapter 1 already defined ways in which a proposition could be understood as Relevant to a question, seen as a partition of the CS. Adapting insights from Lewis (1988) to the QuD framework, we stated that a proposition is Lewis-Relevant to a QuD, if it coincides with a (potentially empty) union of cells. This is repeated in (31).

(31) **Lewis's Relevance** (rephrased in the QuD framework). Let $\mathcal{C}$ be a conversation, Q a QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is Lewis-Relevant to Q, iff $\exists C \subseteq Q.\, p \cap CS(\mathcal{C}) = C$.

A corollary of this definition is given in (206). It says that p is Lewis-Relevant to a question, iff p does not introduce any truth-conditional distinction in any cell of that question. In other words, all the cells must either entail, or be imcompatible with, p. This view will be useful when we relate our novel approach to Relevance to Lewis's approach.

(206) **Lewis's Relevance** (corollary). Let $\mathcal{C}$ be a conversation, Q a QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is Lewis-Relevant to Q, iff $\forall c \in Q.\, \forall (w, w') \in c.\, p(w) = p(w')$.

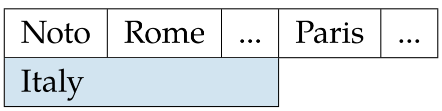
We also mentioned the view from Roberts (2012), according to which a proposition is Relevant if it rules out a cell; see (32).

(32) **Roberts's Relevance** (Roberts, 2012). Let $\mathcal{C}$ be a conversation, Q a (non-trivial) QuD defined as a partition of $CS(\mathcal{C})$. Let p be a proposition. p is Roberts-Relevant to Q, if $\exists c \in Q.\, p \cap c = \emptyset$.

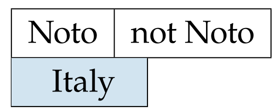
Ideally, we would like to reuse either (31) or (32) in the context of compositional Qtrees, and derive, for instance, that two LFs X and Y can form a conditional $X \rightarrow Y$, only if the proposition denoted by Y, is Relevant to a Qtree evoked by X. Note that this would be the most intuitive direction, because X, as antecedent, would be understood as "setting" the QuD, and Y, would be understood as some Relevant answer to it. This (stipulative) idea is summarized in (207).

(207) **Incremental Relevance** (naive version). Let X and Y be two LFs. $X \rightarrow Y$ is deviant if none of the questions X evokes (seen as partitions of the CS formed by the leaves of X's Qtrees), make the proposition Y denotes Relevant. Relevance may be understood as (31) or (32).

This however, would not quite work on the HCs at stake. In particular, (207) predicts an infelicitous HC like (192a) to be fine. Indeed, (192a) has its antecedent evoke Qtrees whose leaves either partition the CS into cities, or partition the CS into *Noto* vs. *not Noto*-worlds. These partitions are represented in Figures J1 and J2. Additionally, (192a)'s consequent denotes *Italy*. Is *Italy* Relevant to any of the partitions evoked by (192a)'s antecedent? Figure J1 shows that *Italy* is in fact both Lewis- and Roberts-Relevant to the antecedent's implicit "*wh*" question: it corresponds to a collection of Italian cities (hence Lewis-Relevant), and rules out the non-Italian cities (hence Roberts-Relevant). According to (207), this is enough to predict that (192a) should be fine. Note however that, if a polar partition is considered itself for the antecedent, the consequent is neither Lewis- not Roberts-Relevant (see Figure J2).



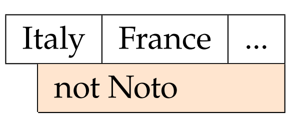
(1) (192a)'s consequent is both Lewis- and Roberts-Relevant to the "*wh*"-partition evoked by (192a)'s antecedent.



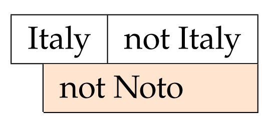
(2) (192a)'s consequent is neither Lewis- nor Roberts-Relevant to the "polar"-partition evoked by (192a)'s antecedent.

Figure J: How (192a)'s consequent interacts with (192a)'s antecedent's evoked questions.

Additionally, we can show that (207) predicts the felicitous HC in (192b), to be deviant. Indeed, (192b) has its antecedent evoke Qtrees whose leaves either partition the CS into countries, or partition the CS into *Italy* vs. *not Italy*-worlds. These partitions are represented in Figures K1 and K2. Additionally, (192b)'s consequent denotes *not Noto*. Is *not Noto* Relevant to any of the partitions evoked by (192a)'s antecedent? Figure K1 shows that *not Noto* is neither Lewis- nor Roberts-Relevant to the antecedent's implicit "*wh*" question: it does not correspond to a collection of countries (hence not Lewis-Relevant), and does not rule out any country (hence not Roberts-Relevant). The same holds for Figure K1.



(1) (192b)'s consequent is neither Lewis- nor Roberts-Relevant to the "*wh*"-partition evoked by (192b)'s antecedent.



(2) (192b)'s consequent is neither Lewis- nor Roberts-Relevant to the "polar"-partition evoked by (192b)'s antecedent.

Figure K: How (192b)'s consequent interacts with (192b)'s antecedent's evoked questions.

In fact, reversing the directionality of the principle in (207), i.e. stating that the antecedent should be Relevant to one of the consequent's implicit questions (see (208)), would in turn reverse the above predictions, and capture HCs.

(208) **Incremental Relevance** (reversed version). Let X and Y be two LFs. $X \to Y$ is deviant if none of the questions Y evokes (seen as partitions of the CS formed by the leaves of Y's Qtrees), make the proposition X denotes Relevant. Relevance may be understood as (31) or (32).

This principle, though less intuitive in terms of its directionality, is in effect close to a result derived by Lewis (1988), who showed that, under a (relatively weak) definition of "inquisitive" Relevance between two *questions*, and assuming conditionals are strict, the antecedent of a conditional *must* be "inquisitively" Relevant to the consequent. However, the Appendix shows that this result cannot account for the contrast observed in HCs, because the inquisitive take on Relevance assigns symmetric roles to the two questions it evaluates, i.e. cannot distinguish between $p \to q$ and $q \to p$.

In any event, the challenge is now to derive a less stipulative version of (208). In other words, the goal is to state a constraint making the same kind of prediction, but which would avoid stipulating the respective roles of X (antecedent) and Y (consequent) which, as we have seen, can so far be reversed. In the next Section, we propose to build a variant of (208) as a constraint on tree-node intersection, which is already an asymmetric operation, whose directionality was motivated by the data presented in Chapter 4. Tying Relevance to tree-node intersection (and more generally perhaps, to *restriction* operations), will give us the directionality stipulated in (208) "for free".

6.4.3 Incremental Q-Relevance

We have just seen that HCs could be captured assuming that LFs give rise to Qtrees matching their degree of granularity, and that, in a conditional, the proposition denoted by the antecedent, should be Relevant to some question evoked by the consequent, where a question is defined as the partition formed by the set of leaves of a Qtree (see (208)). Here, we modify this constraint to avoid stipulating the roles of the antecedent (so far assumed to provide a Relevant proposition) and consequent (so far assumed to provide the question). To this end, we rephrase (208) as a constraint on tree-node intersection, which itself assigns asymmetric roles to its "restrictor" node argument (contributed by the consequent) and tree argument (contributed by the consequent). Under that view, Relevance amounts to some notion of non-vacuity constraining tree-node intersection. Specifically, it is assumed that tree-node intersection should eliminate a leaf of the input Qtree, but also, retain at least one other leaf. This is capturing the idea that tree-node intersection should eliminate some relevant information, but not too much of it – crucially, it should retain at least one relevant "distinction" (cell/leaf) established by the input Qtree. This is all summarized in (209).

(209) **Incremental Q-Relevance**. Let N be a node and T be a Qtree. The tree-node intersection of T and N, noted $T \cap N$, is well formed only if Relevant. It is Relevant iff $T \cap N$'s leaves comprise at least one of T's leaves, and exclude at least one of T's leaves. A leaf of T is excluded, if it intersects with no leaf in $T \cap N$.

(210) **Incremental Q-Relevance** (corollary). Let N be a node and T be a Qtree. The tree-node intersection of T and N, noted $T \cap N$, is Relevant iff N is a superset of at least one leaf in T, and is disjoint from at least one leaf in T.

(211) *Proof of corollary (210).* Let N be a node and T be a Qtree. We assume $T \cap N$'s leaves comprise at least one of T's leaves noted L, and excludes at least one of T's leaves, noted L'. By definition, L is in both T and $T \cap N$, so $L = L \cap N$ i.e. $L \subseteq N$. By definition, L' is in T but not $T \cap N$, so $L' \cap N = \emptyset$.
We now assume N is a superset of at least one leaf in T, noted L, and is disjoint from at least one leaf in T, noted L'. Because $N \supseteq L$, $L \cap N = L$ and L is in $T \cap N$ and is a leaf. Because $L' \cap N = \emptyset$, L' cannot be in $T \cap N$. Moreover, given that the leaves of a Qtree are disjoint, no other leaf in $T \cap N$ intersects with L'.

First, let us quickly note that Incremental Q-Relevance does not jeopardize the results we previously established for HDs and their variants back in Chapter 5. This is simply because Incremental Q-Relevance is checked only when tree-node intersection is performed, i.e. only when conditional Qtrees are computed; and none of the sentences in Chapter 5 involved conditionals. Moreover, Incremental Q-Relevance does not lead to mispredictions in the context of the sentences analyzed in Chapter 4, even those involving conditionals. To show this, it is enough to focus on the only felicitous variant analyzed in that Chapter, repeated in (127b), and show that at least one of the Qtrees compatible with that structure after evaluating Q-Non-Redundancy, is ruled in by Incremental Q-Relevance. Figure L below repeats the two Non-Q-Redundant Qtrees evoked by (127b).

(127b) Either Jo is at SuB or if he is not at SuB then he is in Cambridge. $\mathbf{p} \vee (\neg \mathbf{p} \to \mathbf{q})$

(1) Tree A1 ∨ Tree G1
Q-IRRELEVANT

(2) Tree A1 ∨ Tree G2
Q-RELEVANT

Figure L: NON-Q-REDUNDANT Qtrees for (131c) = $\mathbf{p} \vee (\neg \mathbf{p} \rightarrow \mathbf{q})$

These two Qtrees were derived by intersecting "polar" and "*wh*" consequent Qtrees evoked by q, with $\neg p$, and then disjoining the result with a "polar" Qtree for p. The area of interest where tree-node intersection took place, is circled. In both cases, tree-node intersection fully retained the q-leaf from the consequent Qtree. Moreover, in the case of Figure L2, intersection fully ruled out the p leaf from the consequent Qtree, inducing a partition of the $\neg p$-domain of the form $\{q, r, ...\}$. Therefore the Qtree in Figure L2, in addition to being NON-Q-REDUNDANT, is Q-RELEVANT. And (127b)'s felicity is preserved, even when assuming INCREMENTAL Q-RELEVANCE.

Now that these initial concerns are addressed, let us go back to the definition of INCREMENTAL Q-RELEVANCE in (209). The concept of RELEVANCE used in (209) is a hybrid between Lewis's RELEVANCE and Roberts's RELEVANCE. This is perhaps made more obvious by the corollary in (210). The fact that the restrictor node N must comprise at least one cell, relates to the non-pathological instances of Lewis's RELEVANCE;[7] the fact that it must rule out one cell, relates to Roberts's RELEVANCE. Note however, that our principle does not constitute a proper conjunction of Lewis's and Roberts's RELEVANCE. Like Lewis's RELEVANCE, it allows N to be a proper union of cells. Unlike Lewis's RELEVANCE, it disallows this union to be maximal or empty; and allows cells to be partially covered, as soon as one full cell is. Like Roberts's RELEVANCE, it allows N to *not* be a proper union of cells (as soon as one full cell is covered). Unlike Roberts's RELEVANCE, it disallows strictly overinformative configurations whereby no full cell is covered. These properties are illustrated by the configurations in Figure M.

[7]We dub "pathological" the case in which p is a contextual contradiction: in that case, the intersection between p and the CS does not include *any* cell of the question (it is empty!), yet p is still identified as LEWIS-RELEVANT.

(1) Lewis-Relevant Roberts-Relevant Q-Relevant.

(2) Lewis-Irrelevant Roberts-Relevant Q-Irrelevant.

(3) Lewis-Irrelevant Roberts-Relevant Q-Irrelevant.

(4) Lewis-Irrelevant Roberts-Relevant Q-Relevant.

Figure M: Various QuD-proposition configurations (proposition/restrictor node defined by the gray area).

We will see that this "hybrid" definition will be crucial to capture what we will call "Compatible" HCs (see Section 6.5.1). Before showing how (209) captures HCs, let us establish three useful results that will significantly simplify the argument. The first result, has to do with what we previously called Vacuous tree-node intersection, repeated below.

(116) **Vacuous tree-node intersection**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). $T \cap N = N$ iff $\exists N' \in \mathcal{L}(T).\ N \vDash N'$.

Let us consider a specific subcase of the condition stated in (116), namely, the case in which the restrictor node at stake *strictly* entails some leaf in the Qtree it gets intersected with. In that case, tree-node intersection is vacuous, but also, Irrelevant. This is because it results in a single node, that is a strict subset of the leaf it entails. In other words, the final result does not preserve any leaf from the input Qtree. This is repeated in (212), and will prove very handy when dealing with HCs.

(212) *Irrelevance by* **Single Strict Entailment**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). If $\exists N' \in \mathcal{L}(T).\ N \vDash N' \wedge N \not\equiv N'$, then $T \cap N$ is Irrelevant.

This subcase has an interesting generalization, that will prove useful when analyzing "Compatible" HCs in Section 6.5.1. Suppose now the restrictor node at stake can be partitioned into a set of propositions, s.t. each of them strictly entails a leaf in the Qtree the node gets intersected with. In that case, tree-node intersection will *not* be vacuous, because the node does not entail a single leaf. It will be Irrelevant however. This is because, if the node N in question, can be partitioned into a set $\{N_1, N_2, ..., N_k\}$ of propositions, each of which strictly entails the leaves $\{L_1, L_2, ..., L_k\}$ in T, respectively, then, the intersection between N and T, will simply be the Qtree whose leaves are $\{N_1, N_2, ..., N_k\}$. Since none of these nodes fully coincides with a leaf in T, due to the assumption of strict entailment, the tree-node intersection operation fails to be Relevant. This is summarized in (213).

(213) *Irrelevance by* **Multiple Strict Entailment**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). If $\exists\{N_1, N_2, ..., N_k\}$ a partition

of N and $\exists\{L_1, L_2, ..., L_k\} \subset \mathcal{L}(T)$ s.t. $\forall i \in [1; k]$. $N_i \vDash L_i \wedge N_i \not\equiv L_i$, then $T \cap N$ is IRRELEVANT.

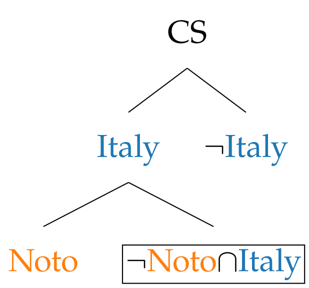
Lastly, let us consider one more case that will turn out useful when analyzing HCs. We now assume that the restrictor node at stake in tree-node intersection, is compatible with all the leaves in the Qtree is gets intersected with. In that case, tree-node intersection may shrink some leaves of the input Qtree, but, all leaves in the original Qtree, will still intersect with some leaf in the output Qtree, due to the assumption of compatibility. In other words, the intersection operation will not exclude any leaf. It will thus be deemed IRRELEVANT. This is summarized in (214).

(214) *Irrelevance by* **HOLISTIC COMPATIBILITY**. Let T be a Qtree whose leaves are $\mathcal{L}(T)$, and N a (non-empty) node (set of worlds). If $\forall L \in \mathcal{L}(T)$. $L \wedge N \not\vDash \bot$, then $T \cap N$ is IRRELEVANT.

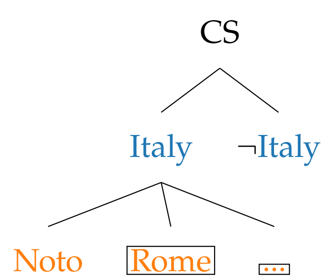
We are now equipped with the tools and definitions to smoothly deal with HCs.

6.4.4 Capturing the contrast in Hurford Conditionals

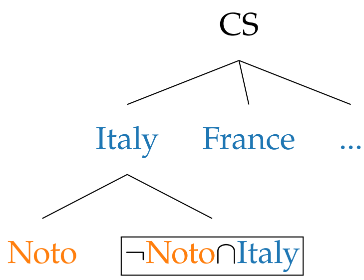
We now explain the oddness pattern of the HCs in (192). We start with the felicitous HC (192b), whose Qtrees are repeated in Figure N below. In such Qtrees, the depth-2 layer, was obtained by intersecting a Qtree for the consequent (*not Noto*), with the *Italy*-node verifying the antecedent.



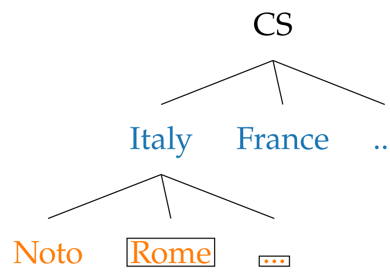
(1) Tree D1 → Tree E1.
Q-IRRELEVANT.



(2) Tree D1 → Tree E2/E3.
Q-RELEVANT.



(3) Tree D2 → Tree E1.
Q-Irrelevant.



(4) Tree D2 → Tree E2/E3.
Q-Relevant.

Figure N: Qtrees for (192b)=*If SuB29 will take place in Italy, it will not take place in Noto*.

Let us now review each Qtree and see whether the tree-node intersection operation used to form it, is Relevant. Note that it is enough to find one well-formed Qtree for (192b) to be correctly predicted to be felicitous.

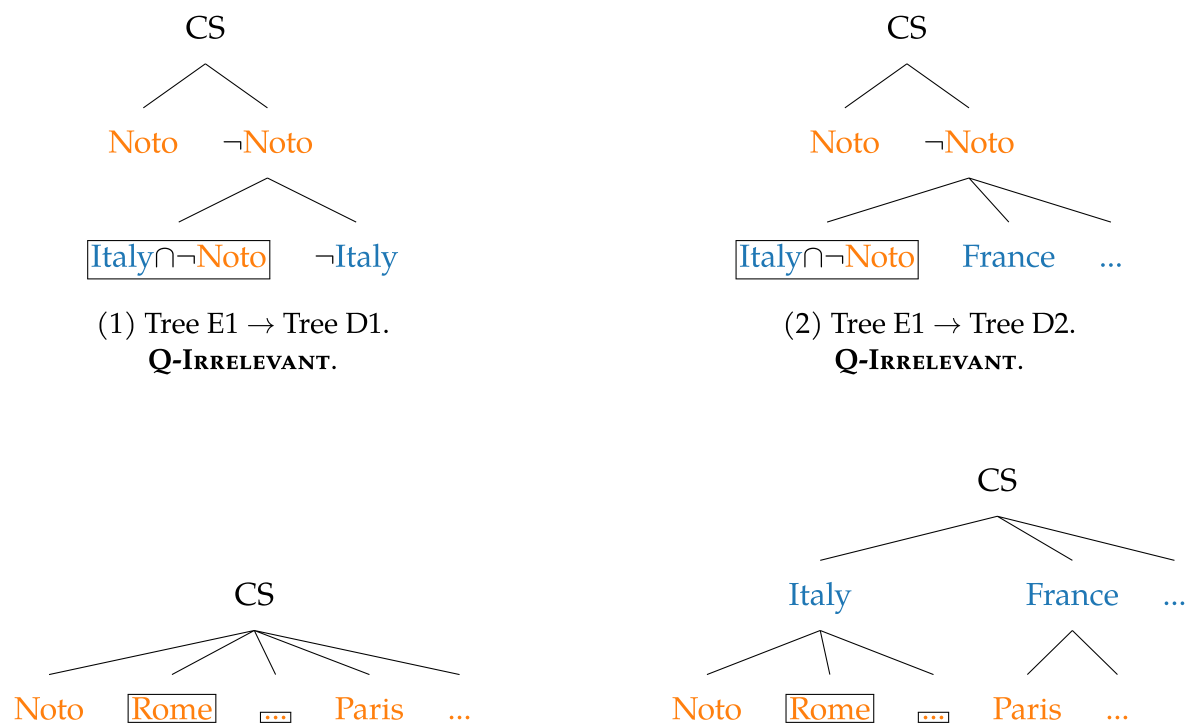
Starting with the Qtree in Figure N1, this Qtree was obtained by intersecting the "polar" Qtree for *not Noto*, with *Italy*. Because *Italy* is compatible with both *Noto* and *not Noto*, the Holistic Compatibility property (214) predicts the intersection operation to be Irrelevant. So this Qtree is odd given (192b).

Now considering the Qtree in Figure N2; this Qtree was obtained by intersecting the "*wh*" (articulated or not) Qtree for *not Noto*, with *Italy*. The resulting set of leaves, is the partition of *Italy* made of Italian cities. This set of leaves, makes the intersection operation Relevant: all Italian cities were leaves in the original "*wh*" (articulated or not) Qtree for *not Noto*, and additionally, the original "*wh*" Qtree for *not Noto*, contained non-Italian city leaves, that were properly excluded by the intersection operation. Therefore, the Qtree in Figure N2, does not violate Incremental Q-Relevance, and (192b) is correctly predicted not to be felicitous.

We could stop here, but let us review the Qtrees in Figures N3 and N4 for completeness. Turning to the Qtree in Figure N3; it was obtained by intersecting the "polar" Qtree for *not Noto*, with *Italy* – just like the Qtree in Figure N1. The intersection operation is therefore predicted to be Irrelevant too.

Lastly, the Qtrees in Figure N4 was obtained by intersecting the "*wh*" (articulated or not) Qtree for *not Noto*, with *Italy* – just like the Qtree in Figure N2. The intersection operation is therefore predicted to be Relevant, too.

We now proceed to analyzing the infelicitous HC (192a), whose Qtrees are repeated in Figure O below. In the Qtrees in Figure O1 and O2, the depth-2 layer, was obtained by intersecting a Qtree for the consequent (*Italy*), with the *not Noto*-node verifying the antecedent. In the Qtrees in Figures O3 and O4, all layers are contributed by the antecedent Qtree, because the intersection operation between city-level leaves and the country-level Qtree contributed by the consequent, was shown to be vacuous.



(1) Tree E1 → Tree D1. **Q-Irrelevant.**

(2) Tree E1 → Tree D2. **Q-Irrelevant.**

(3) Tree E2 → Tree D1/D2. **Q-Irrelevant.**

(4) Tree E3 → Tree D1/D2. **Q-Irrelevant.**

Figure O: Qtrees for (192a)=#*If SuB29 will not take place in Noto, it will take place in Italy*.

Let us now review each Qtree and show that, in each case, the tree-node intersection operation used to form it, was Irrelevant.

Starting with the Qtree in Figure O1, this Qtree was obtained by intersecting the "polar" Qtree for *Italy*, with *not Noto*. Because *not Noto* is compatible with both *Italy* and *not Italy*, the Holistic Compatibility property (214) predicts the intersection operation to be Irrelevant. So this Qtree is ill-formed.

Now considering the Qtree in Figure O2; this Qtree was obtained by intersecting the "*wh*" Qtree for *Italy*, with *not Noto*. Again, because *not Noto* is compatible with any country-level node, the Holistic Compatibility property (214) predicts the intersection operation to be Irrelevant. So this Qtree is ill-formed as well.

Turning to the Qtree in Figure O3; this Qtree was obtained by intersecting any Qtree for *Italy* (polar or "*wh*"), with city-leaves that are not *not Noto*. Let us start by considering the intersection between a city-leaf, and a polar Qtree for *Italy*, whose leaves are *Italy* and *not Italy*. Because any city, strictly entails *Italy* or strictly entails *not Italy*, the SINGLE STRICT ENTAILMENT property (212) predicts the intersection operation to be IRRELEVANT. Now consider the intersection between a city-leaf, and a "*wh*" Qtree for *Italy*, whose leaves are country-level. Because any city, strictly entails some country, the SINGLE STRICT ENTAILMENT property (212), again predicts the intersection operation to be IRRELEVANT. So, no matter how it gets derived, the Qtree in Figure O3, is derived *via* an IRRELEVANT tree-node intersection operation. So this Qtree is ill-formed.

Lastly, the Qtree in Figure O4 was obtained by intersecting any Qtree for *Italy* (polar or "*wh*"), with city-leaves that are not *not Noto*. The intersections operations are thus exactly similar to those performed in Figure O3, and which were just shown to be IRRELEVANT as per the SINGLE STRICT ENTAILMENT property (212). So, no matter how it gets derived, the Qtree in Figure O4, is derived *via* an IRRELEVANT tree-node intersection operation. So this Qtree is ill-formed.

Therefore all Qtrees derived for the infelicitous HC (192a), were derived *via* an IRRELEVANT tree-node intersection operation. As a result, (192a) is not compatible with any well-formed Qtree, and as such should be deemed odd. The contrast observed in HCs is captured.

6.4.5 Taking stock

In this Section, we have shown that HCs could not be accounted for by previously posited constraints (EMPTY LABELING; Q-NON-REDUNDANCY). We have then discussed how earlier notions or RELEVANCE could help, *modulo* stipulative assumptions. We then proposed a new notion of RELEVANCE, INCREMENTAL Q-RELEVANCE, that we framed as a constraint on the tree-node intersection operation, an operation recruited during the formation of conditional Qtrees. This way, we got the specific directionality of RELEVANCE, for free. We then showed how this new view could capture the contrast in HCs. Specifically, we showed that some Qtrees evoked by the felicitous HC (192b), were derived *via* an intersection operation that was properly "shrinking" the consequent Qtree – retaining at least one leaf; excluding at least one leaf. And we showed that no Qtree evoked by the infelicitous HC (192a), could be derived in a similar fashion.

Zooming out, the asymmetry we derived in HC can be traced back to how Qtrees for *Italy* (p) and (*not*) *Noto* ($(\neg)p^+$) were defined: crucially, we observed beck in Section 6.3.3, that at least one Qtree for *not Noto* formed a strict refinement of a Qtree for *Italy* – eventually leading to a Relevant tree-node intersection operation. This intuition is generalized in (215), which roughly says that two Qtrees can be "conditionalized" if all the verifying nodes of the antecedent Qtree, are further subdivided in the consequent Qtree.

(215) Qtree Refinements and Incremental Q-Relevance. Let T be a Qtree whose verifying leaves are $\mathcal{N}^+(T)$, and all have depth at least 1 (i.e. the root is not verifying). Let T' be a Qtree whose root is the same as T, whose nodes include $\mathcal{N}^+(T)$, and are s.t. $\mathcal{L}(T') \cap \mathcal{N}^+(T) = \emptyset$, i.e. any node in T' that is verifying in T, is further subdivided in T'. Then, a conditional Qtree can be formed out of T (as antecedent) and T' (as consequent).

(216) *Proof of (215).* Let T be a Qtree whose verifying nodes are $\mathcal{N}^+(T)$, and all have depth at least 1. Let T' be a Qtree whose root is the same as T, whose nodes include $\mathcal{N}^+(T)$, and are s.t. $\mathcal{L}(T') \cap \mathcal{N}^+(T) = \emptyset$. To show that a conditional Qtree can be formed out of T (as antecedent) and T' (as consequent), we must show that the intersection between each verifying leaf of T and T', is Relevant. Because all verifying nodes in T have the same characteristics in T', it is enough to show that the intersection between an arbitrary verifying node in T, and T', is Relevant. Let N be such a node. By assumption, N is in T' and further subdivided in T'. So $T' \cap N$ is exactly the subtree of T' rooted in N. Let us call this subtree T''. T'' contains at least a leaf from T' (in fact, all its leaves, are leaves from T'). Additionally, T'' does not contain all leaves from T'. This is because all leaves from T' partition the root of T'. Because N is by assumption different from the root (i.e. a subset of the root), $T'' = T \cap N$'s leaves, partition a subset of the root. So $T'' = T \cap N$'s leaves, form a subset of T''s leaves. Therefore, $T \cap N$ is Relevant, and the conditional Qtree formed out of T (as antecedent) and T' (as consequent), is well-formed.

A special case of this condition, is when the verifying nodes of the antecedent and consequent Qtree are all leaves, and when the consequent strictly refines the antecedent. In that case, the resulting conditional Qtree is defined. This is exactly the kind of configuration that felicitous HCs like (192b) give rise to: we observed that some Qtree for *not Noto* formed a strict refinement of some Qtree for *Italy*, and additionally, both Qtrees only had verifying leaves.

By contrast, when the verifying nodes of the antecedent and consequent Qtree are all leaves, and when the antecedent strictly refines the consequent, the resulting conditional Qtree is *not* defined. This is exactly the kind of configuration that infelicitous HCs like (192a) give rise to: we observed that no Qtree for *Italy* formed a (strict) refinement of a Qtree for *not Noto* – leading to Irrelevant tree-node intersection operations across the board. These general properties are spelled out in (217) and proved in (218).

(217) Strict Qtree Refinements and Incremental Q-Relevance. Let T and T' be Qtrees that are more than just one root, whose verifying nodes are leaves and s.t. T' strictly refines T. Then:

(i) no conditional Qtree can be formed out of T' (as antecedent) and T (as consequent);

(ii) while a conditional Qtree can be formed out of T (as antecedent) and T' (as consequent). This is a special case of (215).

(218) *Proof of (217).* Let T and T' be Qtrees that are more than just one root, whose verifying nodes are leaves and s.t. T' strictly refines T.

a. *Proof of (217i).* Let L' be verifying in T'. Because T' strictly refines T, L' cannot be a leaf in T, and must strictly entail a leaf in T. Therefore, $T \cap L'$ is Irrelevant as per the Single Strict Entailment property (212). Therefore, the conditional Qtree $T' \rightarrow T$ is not defined.

b. *Proof of (217ii).* Let L be verifying in T. Because T' strictly refines T, L also belongs to T', and has at least two children in T'. Therefore, $T' \cap L$ is the subtree of T' rooted in L, that is more than just one root. The leaves of this subtree are all leaves of T', and additionally, are not *all* leaves of T', otherwise L would have been T/T''s root, contrary to assumptions. Therefore, $T' \cap L$ is Relevant and the conditional Qtree $T \rightarrow T'$ is defined.

In summary, if the verifying, "restrictor" nodes provided by the antecedent Qtree, are too fine-grained for the consequent Qtree, the tree-node intersection operations performed when building a conditional Qtree, will be Irrelevant, and the entire derivation will crash. This leads us to mention a couple more important observations about how Incremental Q-Relevance operates. First, even if Incremental Q-Relevance restricts tree-node intersection, where the tree is contributed by the consequent and the restrictor node (a proposition) is contributed by the antecedent, it is indirectly sensitive to the structure of the antecedent Qtree, essentially because the verifying "restrictor" nodes passed to tree-node intersection, are determined by how fine-grained the antecedent's Qtree is. If the antecedent is fined-grained like (*not*) *Noto*, the verifying nodes of its Qtrees will be fine-grained as well, meaning, the restrictor nodes passed to tree-node intersection, will be fine-grained. This in turn will make tree-node intersection less likely to achieve Relevance. In that sense, Incremental Q-Relevance is conceptually different from Lewis's and Roberts (2012)'s approaches to "assertive" RELEVANCE,[8] which treated a RELEVANT or IRRELEVANT proposition simply as a set of worlds, and not as a set of nodes (i.e. a set of sets of worlds).

A second, yet observation, is that INCREMENTAL Q-RELEVANCE being a constraint on tree-node intersection, it will have to be checked for every single tree-node intersection operation performed as part of the formation of a conditional Qtree. If one of these operations fails to be RELEVANT, then the derivation of the entire conditional Qtree, will be expected to fail. There will be as many such operations, as there are verifying, "restrictor" nodes in the antecedent Qtree. In particular, increasing the complexity of the antecedent, may increase the number of verifying nodes. In that respect, an interesting subcase is that of a disjunctive antecedent mixing different levels of granularity. In such cases, the finer-grained component will determine what the finest-grained restrictor nodes are, and as such, will constitute the bottleneck for INCREMENTAL Q-RELEVANCE. We will see a few examples instantiating that observation in the next Section.

6.5 Extensions

In this Section, we explore the predictions of INCREMENTAL Q-RELEVANCE on both familiar and new data, that the other prominent account of HCs, SUPER-REDUNDANCY, is shown to struggle with.

6.5.1 "Compatible" Hurford Conditionals

In Chapter 5, we introduced "Compatible" Hurford Disjunctions (henceforth **CHD**s), repeated below. Such disjunctions feature merely compatible disjuncts, and still feel odd. Chapter 5 predicted the sentences in (160) to be odd, due to them featuring disjuncts conveying incomparable degrees of granularity, which in turn made it impossible to derive well-formed disjunctive Qtrees for these sentences.

(160) a. # SuB29 will take place in the Basque country or France. $\mathbf{q} \vee \mathbf{p}$; $\mathbf{q} \wedge \mathbf{p} \nvDash \bot$

b. # SuB29 will take place in France or in the Basque country. $\mathbf{p} \vee \mathbf{q}$; $\mathbf{p} \wedge \mathbf{q} \nvDash \bot$

What about conditional variants of the sentences in (160), obtained *via* the *or*-to-*if* tautology? Because we assumed that the formation of conditional Qtrees is distinct from that of disjunctive Qtrees, changing the disjunctions in (160) into conditionals, may lead to

[8] Both Lewis and Roberts (2012) however proposed RELEVANCE constraints between question-types. For Roberts for instance, a follow-up question is RELEVANT to a QuD, if all the alternatives in the denotation of that follow-up question, are ROBERTS-RELEVANT (as defined in (32)). This brings us a bit closer to what was done here. Lewis's approach is further described and analyzed in the Appendix.

different predictions. To get a more complete picture, let us first generate "disjunctwise" negated counterparts of (160). This is done in (219). The sentences in (219) still meet the description of CHDs, because, if p and q are merely compatible, so do $\neg p$ and $\neg q$.

(219) a. # SuB29 won't take place in the Basque country or won't take place in France.
$(\neg \mathbf{q}) \vee (\neg \mathbf{p})$; $(\neg \mathbf{q}) \wedge (\neg \mathbf{p}) \nvDash \bot$

b. ?? SuB29 won't take place in France or won't take place in the Basque country.
$(\neg \mathbf{p}) \vee (\neg \mathbf{q})$; $(\neg \mathbf{p}) \wedge (\neg \mathbf{q}) \nvDash \bot$

As a side note, we predict the sentences in (219) to be just as bad as those in (160), for the same reasons: their two disjuncts convey incomparable degrees of granularity – inherited from their unnegated counterparts. Kalomoiros's SUPER-REDUNDANCY on the other hand, predicts both sentences to be fine. This is detailed in the Appendix.

We can now apply the *or*-to-*if* tautology to the sentences in (160) and (219), to create four different kinds of "Compatible" Hurford Conditionals (henceforth **CHC**s). Note that we can generate four variants instead of just two (as it was the case for simple HCs), because the two disjuncts in CHDs and disjunctwise negated CHDs, are not in any kind of entailment relation, and therefore play interchangeable roles. In other words, either disjunct can appear as an antecedent or consequent in the derived conditionals. This is all done in (220).

(220) a. Derived from (160), using $\mathbf{q}$ as antecedent.
If SuB29 will not take place in the Basque country, it will take place in France.
$\neg \mathbf{q} \to \mathbf{p}$

b. Derived from (160), using $\mathbf{p}$ as antecedent.
?If SuB29 will not take place in France, it will take place in the Basque country.
$\neg \mathbf{p} \to \mathbf{q}$

c. Derived from (219) and double negation elimination, using $\neg \mathbf{q}$ as antecedent.
If SuB29 will take place in the Basque country, it will not take place in France.
$\mathbf{q} \to \neg \mathbf{p}$

d. Derived from (219) and double negation elimination, using $\neg \mathbf{p}$ as antecedent.
SuB29 will take place in France, it will not take place in the Basque country.
$\mathbf{p} \to \neg \mathbf{q}$

Interestingly, the four CHCs in (220), seem to contrast in terms of felicity. Although the judgments may be subtle, it appears that the variants featuring *France* in their antecedent ((220b) and (220d)), are less degraded than the variants featuring *the Basque country* in their antecedent ((220a) and (220c)). The felicitous variants (220b) and (220d), seem to be understandable as (221a) and (221b), respectively.

(221) a. If SuB29 will not take place in France, it will take place in the Spanish Basque country. $\neg \mathbf{p} \rightarrow (\neg \mathbf{p} \wedge \mathbf{q})$

b. If SuB29 will take place in France, it will not take place in the French Basque country. $\mathbf{p} \rightarrow \neg(\mathbf{p} \wedge \mathbf{q})$

Under the material implication hypothesis, Super-Redundancy predicts all variants in (220) to be fine, simply because the CHDs they are derived from, are already mispredicted by this account to be fine.

From our perspective, this pattern may also look surprising, because we just established that Relevance in conditionals was associated with "granularity" violations (specifically, cases in which the antecedent was finer-grained than the consequent), and moreover, Chapter 5 established that *SuB29 will take place in France*, and *SuB29 will take place in the Basque country*, conveyed *orthogonal* degrees of granularity, that could not be reconciled. So, at first blush, we may have expected all the conditionals in (220) to pattern the same.

We will now see that our model of evoked Qtrees, complemented with Incremental Q-Relevance *actually* captures the pattern in (220). This will boil down to the fact that, although *SuB29 will take place in France*, and *SuB29 will take place in the Basque country* do not evoke Qtrees in any kind of refinement relation, *SuB29 will take place in France* can be seen as "coarser-grained" than *SuB29 will take place in the Basque country*, in the following, weaker sense: some regions like the Basque country, can be included in the union of two countries, but it is harder to think of a country that would be included in the union of two (or more) regions.[9] We will see that this observation can be related to the property of Multiple Strict Entailment in (213), that we showed caused Irrelevance in tree-node intersection. This will be enough to derive that the tree-node intersection operations performed when deriving Qtrees for (220b) and (220d), are Relevant, while those performed in when deriving Qtrees for (220a) and (220c), are not.[10]

[9]Note that this difference may be even more obvious when considering similar configurations, but at different levels of granularity, e.g. by adopting Singh's original examples involving countries like Russia, and continents like Asia. Some countries are included in the union of two continents, but no continent is included in the union of two (or more) countries. Here, we just kept France and the Basque country to stay in the theme.

[10]This prediction will follow from the concept of Relevance defined here, but does not follow from the previous version of this principle proposed in Hénot-Mortier (peara).

To this end, let us first repeat the Qtrees for S_p = *SuB29 will take place in France* and S_q = *SuB29 will take place in the Basque country*, already derived in Chapter 5. Such Qtrees are given in Figure P and Q respectively.

(1) "Polar". (2) "*Wh*".

Figure P: Qtrees evoked by S_p = *SuB29 will take place in France*.

(1) "Polar". (2) "*Wh*".

Figure Q: Qtrees for S_q = *SuB29 will take place in the Basque country*.

Qtrees the negations of S_p and S_q, are given in Figures R and S respectively.

(1) "Polar". (2) "*Wh*".

Figure R: Qtrees evoked by $\neg S_p$ = *SuB29 won't take place in France*.

(1) "Polar". (2) "*Wh*".

Figure S: Qtrees for $\neg S_q$ = *SuB29 won't take place in the Basque country*.

We can now evaluate which sentences in (220) have their conditional Qtrees violate Incremental Q-Relevance. We start with the infelicitous variant (220a). (220a)'s Qtrees should be derived by composing antecedent Qtrees for *not Basque* (in Figure S) with consequent Qtrees for *France* (in Figure P).

First, let us consider Figure S1 as antecedent and Figure P1 as consequent. In that case, the *not Basque* node acts as the only restrictor node, and gets intersected with the polar partition *France* vs. *not France*. Because the set of *not Basque* worlds is compatible with both *France* and *not France*, the intersection operation is Irrelevant, as per the Holistic Compatibility property (214).

Second, let us consider Figure S1 as antecedent and Figure P2 as consequent. In that case, the *not Basque* node again acts as the only restrictor node, and gets intersected with a by-country partition. Because the set of *not Basque* worlds is compatible with any single country (including *France* and *Spain*), the intersection operation is Irrelevant, again as per the Holistic Compatibility property (214).

Third, let us consider Figure S2 as antecedent and Figure P1 as consequent. In that case, the region-level nodes different from the Basque country act as restrictor nodes, and each gets intersected with the polar partition *France* vs. *not France*. For the intersection operation to be Relevant, any region different from the Basque country, should strictly coincide with either *France* or *not France*. This would be the only way keep one cell, and exclude one other cell, from the consequent's partition and satisfy Incremental Q-Relevance. But obviously, this cannot hold of all regions – in fact, this hold of none of them. Therefore, the intersection operation is Irrelevant.

Fourth and lastly, let us consider Figure S2 as antecedent and Figure P2. In that case, the region-level nodes different from the Basque country act as restrictor nodes, and each gets intersected with a by-country partition. For the intersection operation to be Relevant, any region different from the Basque country, should contain at least one country, and exclude at least one country. This would be the only way keep one cell, and exclude one other cell, from the consequent's partition and satisfy Incremental Q-Relevance. But obviously, this cannot hold of all regions: many regions are strictly contained in one single country. Therefore, the intersection operation is Irrelevant. We have just shown that there is no way to derive a conditional Qtree for (220a) *via* well-formed (i.e. Relevant) tree-node intersection operations. Thus, (220a) is correctly predicted to be odd.

Now turning to the other infelicitous variant (220c). (220c)'s Qtrees should be derived by composing antecedent Qtrees for *Basque* (in Figure Q) with consequent Qtrees for *not France* (in Figure R).

First, let us consider Figure Q1 as antecedent and Figure R1 as consequent. In that case, the *Basque* node acts as the only restrictor node, and gets intersected with the polar partition *France* vs. *not France*. Because the set of *Basque* worlds is compatible with both *France* and *not France*, the intersection operation is Irrelevant, as per the Holistic Compatibility property (214).

Second, let us consider Figure Q1 as antecedent and Figure R2 as consequent. In that case, the *Basque* node again acts as the only restrictor node, and gets intersected with a by-country partition. Crucially here, because the set of *Basque* worlds can be partitioned into two subsets, namely, *the French Basque country* and *the Spanish Basque country*, each of which strictly entails a leaf in the consequent's Qtree (namely, *France* and *Spain*), the intersection operation is Irrelevant, as per the Multiple Strict Entailment property (213).

Third, let us consider Figure Q2 as antecedent and Figure R1 as consequent. In that case, the *Basque* node again acts as the only restrictor node, and gets intersected with the polar partition *France* vs. *not France*. We have already established that this is Irrelevant.

Fourth and lastly, let us consider Figure Q2 as antecedent and Figure R2. In that case, the *Basque* node again acts as the only restrictor node, and gets intersected with a by-country partition. We have already established that this is Irrelevant. We have just shown that there is no way to derive a conditional Qtree for (220c) *via* well-formed (i.e. Relevant) tree-node intersection operations. Thus, (220c) is correctly predicted to be odd.

Let us now show that the felicitous variants (220b) and (220d) *can* evoke conditional Qtrees derived *via* Relevant tree-node intersection operations. In the case of (220b), let us consider the polar Qtree for *not France* in Figure R1 as antecedent Qtree, and the "*wh*" Qtree for *the Basque country* in Figure Q2, as consequent Qtree. The resulting conditional Qtree is represented in Figure T.

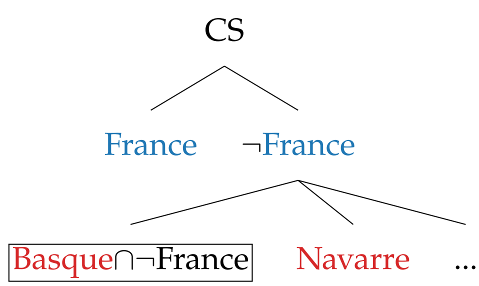


Figure T: Qtree evoked by ?(220b)=*If SuB29 will not take place in France, it will take place in the Basque country.*

In that case, the *not France* node acts as the only restrictor node, and gets intersected with a by-region partition. The result of this intersection, fully preserves all regions that are disjoint from France (e.g. *Navarre*) and fully rules out regions that are included in France (e.g. *Midi*). Regions partially not in France (e.g. *the Basque country*), are partially preserved. In any case, this intersection fully preserves one region from the original partition (e.g. *Navarre*), and fully excludes one (e.g. *Midi*). It is thus Relevant, and the resulting conditional Qtree in Figure T, is predicted to be well-formed. As a result, (220b) evokes at least one well-formed Qtree and is correctly predicted to be felicitous.

In the case of (220d), let us consider the polar Qtree for *France* in Figure P1 as antecedent Qtree, and the "*wh*" Qtree for *not Basque* in Figure S2, as consequent Qtree. The resulting conditional Qtree is represented in Figure U.

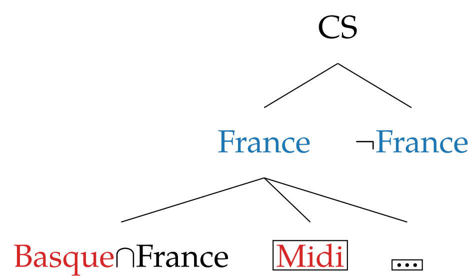


Figure U: Qtree evoked by (220d)=*If SuB29 will take place in France, it will not take place in the Basque country.*

In that case, the *France* node acts as the only restrictor node, and gets intersected with a by-region partition. The result of this intersection, fully preserves all regions included in France (e.g. *Midi*) and fully rules out regions completely out of France (e.g. *Navarre*). Regions partially in France (e.g. *the Basque country*), are partially preserved. In any case, this intersection fully preserves one region from the original partition (e.e. *Midi*), and fully excludes one (e.g. *Navarre*). It is thus Relevant, and the resulting conditional Qtree in Figure U, is predicted to be well-formed. As a result, (220d) evokes at least one well-formed Qtree and is correctly predicted to be felicitous.

In this Section, we have explored an interesting and relatively unexpected prediction of Incremental Q-Relevance, when it comes to "Compatible" HCs. We have shown that Incremental Q-Relevance, combined with our model of conveyed granularity, accounts for challenging contrasts affecting such CHCs. Interestingly, the intuitive readings of the felicitous variants (220b) and (220d), given in (221a) and (221b) respectively, are consistent with the well-formed Qtrees derived from these sentences, given in Figures T and U respectively. In these Figures, the verifying nodes are the ones contributed by the consequent, but *restricted* to the domain where the antecedent holds. For (220b), we end up with the non-French Basque country, i.e. the Spanish Basque country; for (220d), we end up with any French region that is not the French Basque countr. These verifying nodes correspond to how the antecedent gets understood in (220b) and (220d).

More broadly, this result suggests that conditionals whose antecedent and consequent evoke orthogonal questions, may not always be degraded.[11] It also predicts that INCREMENTAL Q-RELEVANCE may filter out some interpretations of these conditionals, in terms of the possible questions they evoke. [12]The next Section turns to another case of "non-entailing" HCs, derived from familiar variants of HDs.

6.5.2 "Long-Distance" HCs derived from LDHDs

In Chapter 5, we showed that "Long Distance" HDs (henceforth **LDHDs**, Marty and Romoli, 2022), exemplified in (222), were Q-REDUNDANT. Such constructions are obtained from standard HDs of the form $p \vee p^+$ by further disjoining p^+ (that we will call "strong" disjunct) with a proposition r, which is s.t. $p^+ \vee r$ is merely compatible with p (that we will call "weak" disjunct). This can be done by choosing r to contradict p.[13] In (222) for instance, NELS55 taking place in Göttingen, which is located in Germany, is incompatible with NELS55 taking place in the US (and *a fortiori*, with NELS55 taking place in Connecticut).

(222) a. # NELS55 will take place in the US, or will take place in Connecticut or in Göttingen.
$\mathbf{p} \vee (\mathbf{p^+} \vee \mathbf{r})$ $\qquad$ $\mathbf{p^+} \vDash \mathbf{p}$; $(\mathbf{p^+} \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

b. # NELS55 will take place in Connecticut or in Göttingen, or will take place in

[11]I thank Ivano Ciardelli for pointing out this issue to me. I was happy to realize that the concept of RELEVANCE introduced in this dissertation, unlike its previous version spelled out in Hénot-Mortier (peara) (which raised the initial concerns), *can* rule in such "orthogonal" conditionals, at least under certain conditions.

[12]A conditional like (i) for instance, can be felicitous granted that the proposition that *Jo gets into SuB29* includes at least all the worlds in which Jo feels a certain emotion, and excludes all the worlds in which Jo feels another emotion. Also note that the question raised by the consequent of this sentence can only be polar (i.e. be about whether Jo is happy or not), if Jo getting into SuB29 strictly coincides with Jo being happy or Jo being not happy. This would lead to derive conditional perfection for (i). We leave a more systematic analysis of these observations for future work.

(i) If Jo gets into SuB29, they'll be very happy.

[13]In fact, choosing r to be compatible with $\neg p$ would be enough to achieve the desired relation between the main disjuncts of an LDHD.

the US.
$(\mathbf{p}^+ \vee \mathbf{r}) \vee \mathbf{p}$ $\qquad$ $\mathbf{p}^+ \vDash \mathbf{p}$; $(\mathbf{p}^+ \vee \mathbf{r}) \wedge \mathbf{p} \nvDash \bot$

Here, we are interested in conditionals derived from LDHDs *via* the *or*-to-*if* tautology – in a way similar to how (C)HCs were derived earlier in that Chapter. To build such conditionals, we need two kinds of LDHDs: the ones in (222), and their "disjunctwise" negated counterparts. These negated variants are shown in (223). Here is how such variants are constructed. In both (223a) and (223b) the weak disjunct, *not Connecticut*, corresponds to the negation of the stronger disjunct of (222a)/(222b). Additionally, (223a) and (223b)'s stronger disjunct, *not the US*, corresponds to the negation of the weak disjunct of (222a)/(222b). Again, just like in (222), the extra proposition disjoined with it (*New Haven*), is chosen to be incompatible with the weaker disjunct, which is now *not Connecticut*.

(223) a. # NELS55 won't take place in Connecticut, or, won't take place in the US or will take place in New Haven.
$(\neg\mathbf{p}^+) \vee (\neg\mathbf{p} \vee \mathbf{r}) = \mathbf{q} \vee (\mathbf{q}^+ \vee \mathbf{s})$
With $\mathbf{q} := \neg\mathbf{p}^+$; $\mathbf{q}^+ := \neg\mathbf{p}$ s.t. $\mathbf{q}^+ \vDash \mathbf{q}$; $(\mathbf{q}^+ \vee \mathbf{s}) \wedge \mathbf{q} \nvDash \bot$

b. # NELS55 won't take place in the US or will take place in New Haven, or, won't take place in Connecticut.
$(\neg\mathbf{p} \vee \mathbf{r}) \vee (\neg\mathbf{p}^+) = (\mathbf{q}^+ \vee \mathbf{s}) \vee \mathbf{q}$
With $\mathbf{q} := \neg\mathbf{p}^+$; $\mathbf{q}^+ := \neg\mathbf{p}$ s.t. $\mathbf{q}^+ \vDash \mathbf{q}$; $(\mathbf{q}^+ \vee \mathbf{s}) \wedge \mathbf{q} \nvDash \bot$

The sentences in (223), appear extremely degraded, which intuitively seems to come the observation that they directly combine propositions conveying very different degrees of granularity. But despite this intuition, at the logical-structural level, these disjunctwise negated LDHDs are to LDHDs exactly what disjunctwise negated HDs are to HDs. In particular, they are identical to LDHDs both in terms of their structure and in terms of the logical relations between their constitutive parts. This implies that the sentences in (223) are isomorphic with the sentences in (222) – just like HDs are isomorphic with their disjunctwise negated counterparts.

Before "converting" the sentences in (222) and (223) to conditionals, let us highlight that the very degraded sentences in (223) are both predicted by Super-Redundancy to be fine, for the same reasons as DNHDs. Under our view, such LDHDs are Q-Redundant due to their simplification $\neg p^+ \vee r$. This is all detailed in the Appendix.

Turning the LDHDs in (222) and (223) into conditionals *via* the *or*-to-*if* tautology, leads to the paradigm in (224). Similarly to the CHCs in Section 6.5.1, this paradigm is made of four different conditionals (instead of just two, as it was the case with simple HCs), because the main two disjuncts in (222) and (223), are not in any kind of entailment relation, i.e. play completely symmetric roles. As a result, it makes sense to apply the *or*-to-*if* tautology in either direction, i.e. to treat either disjunct's negation as the antecedent, and the remaining disjunct, as the consequent, of the resulting conditional.

(224) a. Derived from (222), using the simple ("weak") disjunct as antecedent.
If NELS55 will not take place in the US, it will take place in Connecticut or in Göttingen.
$\neg\mathbf{p} \rightarrow (\mathbf{p}^{+} \vee \mathbf{r})$

b. Derived from (222), using the complex disjunct as antecedent.
If NELS55 will not take place in Connecticut or in Göttingen, it will take place in the US.
$\neg(\mathbf{p}^{+} \vee \mathbf{r}) \rightarrow \mathbf{p}$

c. Derived from (223), using the simple ("weak") disjunct as antecedent.
If it's not the case that NELS55 won't take place in Connecticut, it won't take place in the US or will take place in New Haven.
$\neg(\neg\mathbf{p}^{+}) \rightarrow (\neg\mathbf{p} \vee \mathbf{s})$

d. Derived from (223), using the complex disjunct as antecedent.
If it's not the case that NELS55 won't take place in the US or will take place in New Haven, it will not take place in Connecticut.
$\neg(\neg\mathbf{p} \vee \mathbf{s}) \rightarrow (\neg\mathbf{p}^{+})$

All the conditionals in (224) appear infelicitous. Furthermore, it feels like these various conditionals, exhibit different "flavors" of oddness. Both (224a) and (224c) seem to convey contradictory information as part of their consequent (granted their antecedent). (224a)'s consequent entertains the idea that NELS55 will take place in Connecticut, while its antecedent said *not the US*). (224c)'s consequent entertains the idea that NELS55 won't take place in the US, while its antecedent said *Connecticut*. We will in fact show that both (224a) and (224c) are Q-Redundant, considering simplifications removing these locally contradictory pieces of information from their consequents. Turning to (224b), it just feels like its consequent does not make sense at at all, given its antecedent; we will in fact show that (224b) violates Incremental Q-Relevance. Lastly, (224d) is almost impossible to make sense of as-is. We will show (224d) violates Incremental Q-Relevance, as well.

This pattern is interesting, because, again, it is not predicted by Super-Redundancy, at least assuming conditionals are material. Indeed, under these (perhaps simplifying) assumptions, the prediction made by Super-Redundancy are insensitive to transformations like the *or*-to-*if* tautology, but *are* sensitive to variable changes of the form $q := \neg p$. Super-Redundancy thus predicts the LDHC in (224a) and (224b), derived from the LDHDs in (222), to be deviant (because the LDHDs in (222) were already predicted so). Similarly, it predicts the LDHCs in (224c) and (224d), derived from the LDHDs in (223), to be fine (because the LDHDs in (223) were already predicted so – see Appendix). In other words, the mispredictions of Super-Redundancy in the case of (223), spread to its conditional variants (224c) and (224d).

It is additionally worth noting that the conditionals in (224c) and (224d) may be simplified, by eliminating the double negation in the case of (224c), and by applying De Morgan's Law in the case of (224d). This is done in (225a) and (225b), respectively. In our current framework, double negation elimination, performed in (225a), is innocuous[14] It is also inovuous from the point of Super-Redundancy, whose misprediction for (224c) carries over to (225a). However, De Morgan's Law, performed in (225b), is *a priori* not innocuous, at least for our account, because it introduces a brand new operator, conjunction, for which we have not yet devised an inquisitive contribution. And in fact, this conjunctive variant appears significantly more felicitous than the variant it is derived from. We will leave this conjunctive variant for future work.

(225) a. Derived from (224c), *via* double negation elimination.
If NELS55 will take place in Connecticut, it won't take place in the US or will take place in New Haven.
$\mathbf{p}^{+} \rightarrow (\neg \mathbf{p} \vee \mathbf{r})$

b. Derived from (224d), *via* De Morgan's Law.
If NELS55 will take place in the US and not in New Haven, it will not take place in Connecticut.
$(\mathbf{p} \wedge \neg \mathbf{r}) \rightarrow (\neg \mathbf{p}^{+})$

We now proceed to show that all variants in (224) – even when reasonably simplified – are predicted by our framework to be odd, for various reasons. We first focus on the two Q-Redundant cases ((224a) and (224c)/(225a)), then turn to the two Q-Irrelevant cases ((224b) and (224d)).

[14]The way we defined negation makes it almost involutive, meaning, applying it twice very often leads to the same result as not applying it at all. One pathological case, is when an entire layer is flagged as verifying in the input Qtree. Applying negation to the input Qtree once, erases this flagging on the entire layer. Applying negation a second time, does not recover the flagged layer, because the flipping operation induced by negation only targets layers that have at least one verifying node. But sentences like (225a) do not belong to this pathological class.

First, we show that (224a), repeated below, is Q-Redundant, given its simplification that omits the *Connecticut* (p^+) disjunct within its consequent; see (226)

(224a) # If NELS55 will not take place in the US, it will take place in Connecticut or in Göttingen.
$\neg \mathbf{p} \rightarrow (\mathbf{p}^+ \vee \mathbf{r})$

(226) If NELS55 will not take place in the US, it will take place in Göttingen.
$\neg \mathbf{p} \rightarrow \mathbf{r}$

S_{p^+} = *NELS55 will take place in Connecticut* is coarser-grained than S_r = *NELS55 will take place in Gottingen*, therefore both sentences evoke Qtree that can stand in a refinement relation, whose disjunction gives rise to the Qtree for $S_{p^+} \vee S_r$ in Figure V.[15]

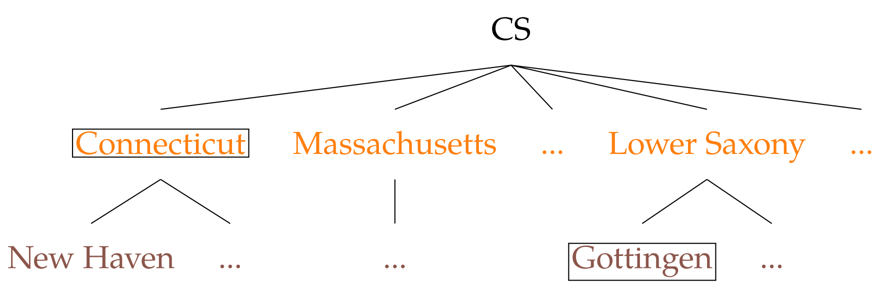
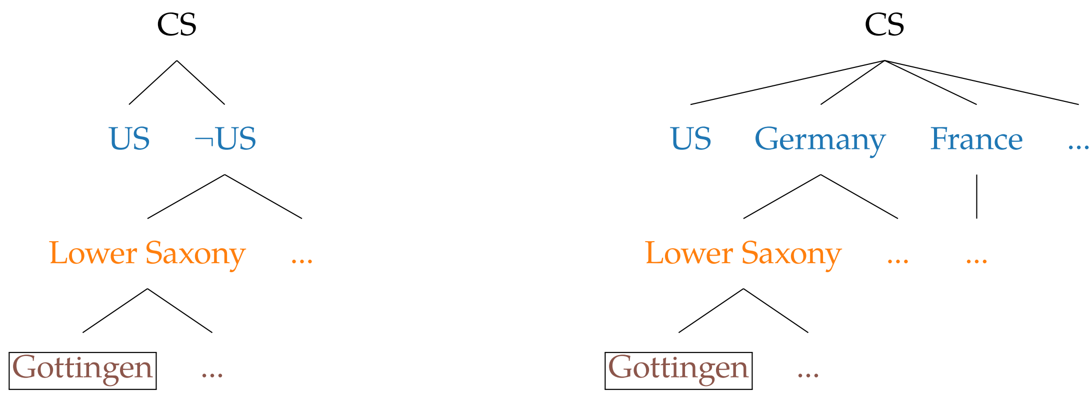


Figure V: Qtree evoked by $S_{\mathbf{p}^+} \vee S_{\mathbf{r}}$

Combining the Qtree in Figure V with an antecedent Qtree for $\neg S_p$ = *NELS55 will not take place in the US* (either "polar" or "*wh*"), yields the Qtrees in Figure W. In such Qtrees, the *Connecticut* nodes that was verifying in the consequent, got filtered by tree-node intersection. And the Qtrees in Figure W, are thus also evoked by the simplification $\neg S_p \rightarrow S_r$ in (226). As a result, (224a) is predicted to be odd due to violations of Q-Non-Redundancy.

[15] This Qtree could have involved more layers on top of the US state/region layer, but the presence/absence of such layers do not affect the final outcome.



(1) "Polar" antecedent Qtree $\rightarrow$ Tree V. (2) "*Wh*" antecedent Qtree $\rightarrow$ Tree V.

Figure W: Qtrees evoked by #(224a) = $(\neg S_{\mathbf{p}}) \rightarrow (S_{\mathbf{p}^+} \vee S_{\mathbf{r}})$.
Same as some Qtrees evoked by (226) = $\neg S_{\mathbf{p}} \rightarrow S_{\mathbf{r}}$, so Q-Redundant.

Secondly, let us show that (225a), derived from (224c) *via* double negation elimination,[16] is also Q-Redundant. This time, it is due to its simplification that omits the *not the US* ($\neg p$) disjunct within the consequent; see (227).

(225a) # If NELS55 will take place in Connecticut, it won't take place in the US or will take place in New Haven.
$\mathbf{p}^+ \rightarrow (\neg \mathbf{p} \vee \mathbf{s})$

(227) If NELS55 will take place in Connecticut, it will take place in New Haven.
$\mathbf{p}^+ \rightarrow \mathbf{s}$

$\neg S_p$ = *NELS55 won't take place in the US* is coarser-grained than S_s = *NELS55 will take place in New Haven*, therefore both sentences evoke Qtree that can stand in a refinement relation, whose disjunction gives rise to the Qtree for $\neg S_p \vee S_s$ in Figure X.[17]

[16] The case of (224c) is pretty obvious: it is Q-Redundant given (225a). That is why we focus on the simpler variant (225a).

[17] This Qtree may not have involved a US state/region layer, but the presence/absence of this layer (as well as the presence/absence of upper layers) does not affects the final outcome.

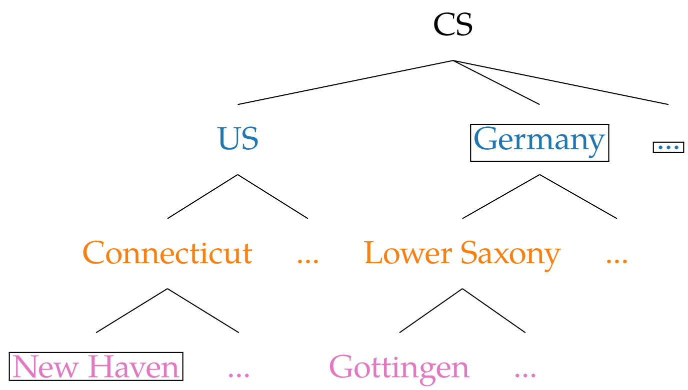
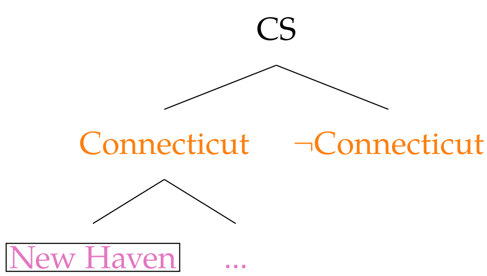
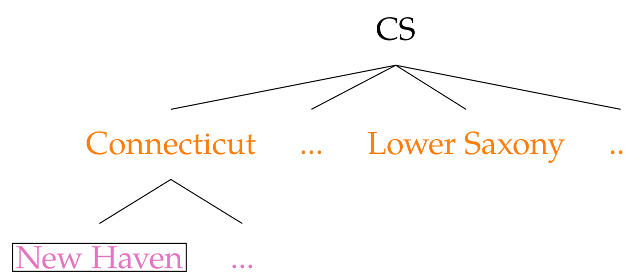


Figure X: Qtree evoked by $\neg S_{\mathbf{p}} \vee S_{\mathrm{s}}$

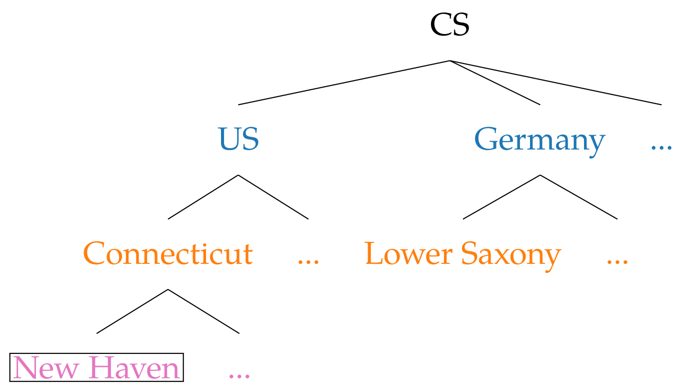
Combining the Qtree in Figure X with an antecedent Qtree for S_{p^+} = *NELS55 will take place in Connecticut* ("polar", "*wh*", or "*wh*-articulated"), yields the Qtrees in Figure Y. In such Qtrees, the non-US nodes that were verifying in the consequent, got filtered by tree-node intersection. And the Qtrees in Figure Y, are thus also evoked by the simplification $\neg S_{p^+} \rightarrow S_s$ in (227). As a result, (225a) is predicted to be odd due to violations of Q-Non-Redundancy.



(1) "Polar" antecedent Qtree $\rightarrow$ Tree X.



(2) "*Wh*" antecedent Qtree $\rightarrow$ Tree X.



(3) “*Wh*-articulated” antecedent Qtree $\rightarrow$
Tree X.

Figure Y: Qtrees evoked by #(225a) = $S_{\mathbf{p}^+} \rightarrow (\neg S_{\mathbf{p}} \vee S_{\mathbf{s}})$.
Same as some Qtrees evoked by (227) = $S_{\mathbf{p}^+} \rightarrow S_{\mathbf{s}}$, so Q-Redundant.

Thirdly, we show that (224b), repeated below, turns out incrementally Irrelevant, essentially because the finest degree of granularity conveyed by its disjunctive antecedent is by-city, and as such, is finer-grained than the granularity conveyed by the consequent – incurring a violation close to the one observed in infelicitous HCs.

(224b) # If NELS55 will not take place in Connecticut or in Göttingen, it will take place in the US.
$\neg(\mathbf{p}^+ \vee \mathbf{r}) \rightarrow \mathbf{p}$

Figure Z shows the disjunctive Qtree evoked by (224b)’s antecedent, $\neg(S_{\mathbf{p}^+} \vee S_{\mathbf{r}})$ = *NELS55 will not take place in Connecticut or in Gottingen*. It is directly derived from the Qtree in Figure V, by simply flipping its verifying nodes layer-by-layer. Qtrees for (224b)’s consequent S_p = *NELS55 will take place in the US*, are given in Figure AA.

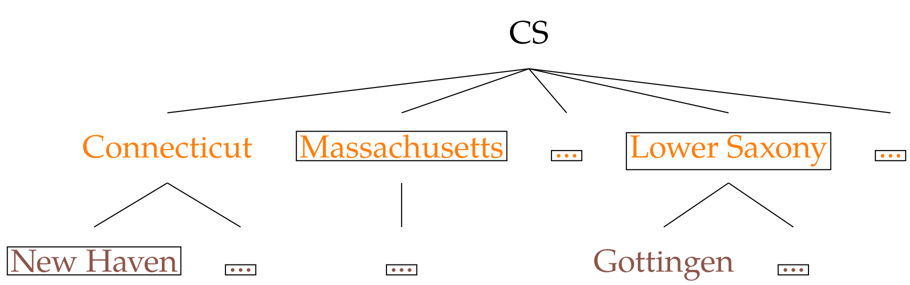


Figure Z: Qtree evoked by $\neg(S_{\mathbf{p}^+} \vee S_{\mathbf{r}})$

Figure AA: Qtrees evoked by $S_{\mathbf{p}}$.

It can be shown that combining the Qtree in Figure Z (as antecedent Qtree) with either Qtree in Figure AA (as consequent Qtree) *via* the conditional rule, violates Incremental Q-Relevance. Indeed, Incremental Q-Relevance imposes that intersecting either consequent Qtree in Figure AA with any verifying nodes from the Qtree in Figure Z, fully rules in a leaf and fully rules out another leaf. Let's consider the *New Haven* node, which is verifying in Figure Z. This node strictly entails *the US*, and so, intersecting the *New Haven* node with any Qtree from Figure AA, does not rule in any leaf, and so violates Incremental Q-Relevance. This is enough to predict that the formation of a conditional based on Figure Z as antecedent, and either Qtree in Figure AA, as consequent, will crash. As a result, (224b) is predicted to be odd due to violations of Incremental Q-Relevance.

Fourth, and lastly, we show that this last result extends to (224d), repeated below.

(224d) If it's not the case that NELS55 won't take place in the US or will take place in New Haven, it will not take place in Connecticut.
$\neg(\neg\mathbf{p} \vee \mathbf{s}) \rightarrow (\neg\mathbf{p}^{+})$

Figure AB shows the disjunctive Qtree evoked by (224d)'s antecedent, $\neg(\neg S_{\mathbf{p}} \vee S_{\mathbf{s}}) =$ *It's not the case that NELS55 won't take place in the US or will take place in New Haven*. It is directly derived from the Qtree in Figure X, by simply flipping its verifying nodes layer-by-layer. Qtrees for (224d)'s consequent $\neg S_{p^+}$ = *NELS55 will not take place in Connecticut*, are given in Figure AC.

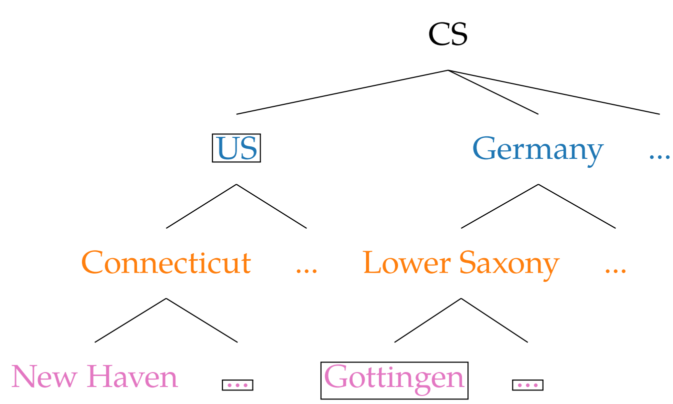


Figure AB: Qtree evoked by $\neg S_{\mathbf{p}} \vee S_{\mathbf{s}}$

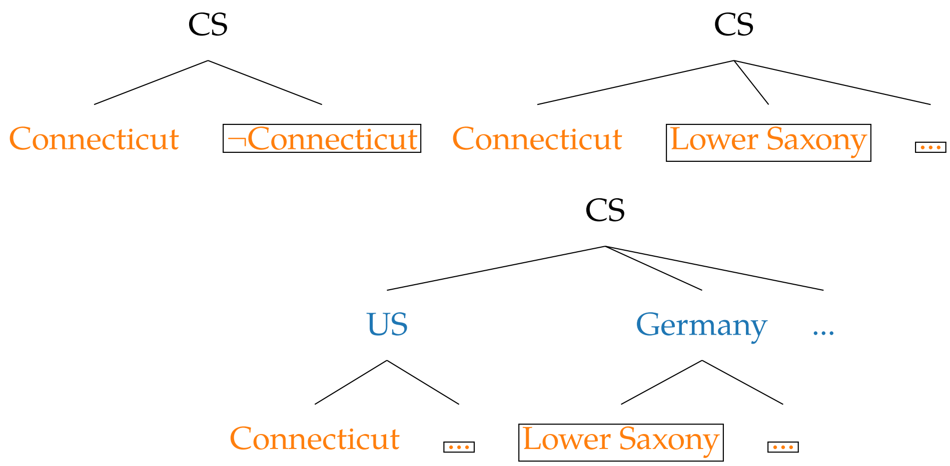


Figure AC: Qtrees evoked by $\neg S_{p^+}$.

Similarly to what was done for (224b), it can be shown that combining the Qtree in Figure AB (as antecedent Qtree) with any Qtree in Figure AC (as consequent Qtree) *via* the conditional rule, violates INCREMENTAL Q-RELEVANCE. Let's consider the a *Connecticut*-node that is not *New Haven*. This node is verifying in Figure AB and strictly entails *Connecticut*, and so, intersecting this node with any Qtree from Figure AC, does not rule in any leaf, and so violates INCREMENTAL Q-RELEVANCE. This is enough to predict that the formation of a conditional based on Figure AB as antecedent, and any Qtree in Figure AC, as consequent, will crash. As a result, (224d) is predicted to be odd due to violations of INCREMENTAL Q-RELEVANCE.

In this Section, we have investigated "Long-Distance" Hurford Conditionals, derived from LDHDs. Such conditionals involve complex antecedents or consequents, and mix degree of granularity in various ways, but all appear quite degraded. They were shown to be challenging for at least one account of oddness solely based on the concept of REDUNDANCY. Unlike this family of approach, our proposed framework talk about other ways to create long distance variants of HC by driectly disjoining the antecedent/conseunr of HCs.

6.6 Conclusion and outlook

TALK ABOUT PERFECTION In this Chapter, we captured the challenging contrasts displayed by Hurford Conditionals, using two main ingredients. The first, was that sentences evoke questions (Qtrees) matching their degree of granularity. The second ingredient, was that the core operation behind the formation of conditional Qtrees (tree-node intersection), which is asymmetric in nature, is constrained by a new concept of RELEVANCE. Drawing from both Lewis (1988) and Roberts (2012), this new concept of RELEVANCE was made asymmetrically sensitive to Qtree granularity. The contrast observed in HCs then boiled down to the (rough) intuitive idea that felicitous conditionals should display an antecedent that is coarser-grained than their consequent.

Beyond simple HCs, we explored more involved predictions of our RELEVANCE constraint, and showed that, surprisingly, conditionals whose antecedent and consequent evoke orthogonal questions, *can* be felicitous under certain conditions. Finally, we saw that certain paradigms could be explained by a combination of RELEVANCE and REDUNDANCY constraints – accounting for the intuition that different sentences give rise to different flavors of oddness.

One datapoint that the current framework cannot account for, is given in (228). (228) is obtained from the infelicitous HC (192a), by simply replacing *Italy* with *France* in the consequent. This leads to a sensible improvement of the judgment. Yet, out model of RELEVANCE, which is only sensitive to the degree of granularity evoked by the consequent, and not to what the consequent actually denotes, does not distinguish between (192a) and (228) – meaning, both sentences are predicted to be equally odd[18]

(228) If SuB29 will not take place in Noto, it will take place in France. $\neg \mathbf{p}^+ \rightarrow \mathbf{q}$

This Chapter, along with Chapter 5 constituted an extensive discussion of non-scalar odd constructions, whether disjunctive or conditional in nature. The next Chapter leaves aside countries and cities (at least!) to investigate what happens in "scalar" counterparts of HDs, HCs, and some of their variants.

[18]The account laid out in Hénot-Mortier (peara) captures this datapoint, because it assigns a central role to nodes flagged as verifying by the consequent, when it comes to evaluating RELEVANCE. The main idea, is that intersecting *Italy* with *not Noto* shrinks *Italy*, in turn causing infelicity, while intersecting *France* with *not Noto*, does not shrink *France*, thus preserving felicity. Hénot-Mortier's account however, has less empirical coverage regarding "orthogonal" conditionals, including CHCs.

6.7 Appendix: granularity-sensitivity in question-answer pairs

However, we have argued the contrast in HCs should be explained by a constraint sensitive to the level of granularity conveyed by the antecedent and consequent. But neither LEWIS's RELEVANCE nor ROBERTS's RELEVANCE are sensitive to how the proposition at stake packages information: the only relevant(!) factor is the set-theoretic relation between the proposition as a whole, and the QuD, seen as a partition of the CS. This implies that the level of granularity conveyed by the proposition is not directly taken into account when assessing its RELEVANCE to the question (although, of course, the strength of the proposition is). Looking beyond the case of HCs, this does not seem to match intuitions about what a RELEVANT answer to a question is.

The question-answer pairs in (229) for instance, show that the answer's conveyed granularity should be taken into account when assessing RELEVANCE. (229)'s QuD strongly suggests a by-country partition of the CS. This predicts both (229a) and (229b) to be RELEVANT under both views, because both sentences refer to a proper subset of all countries. Yet, (229b) does not appear felicitous without hedging (e.g., using *for all I know*). Its relative oddness in this context is however intuitive: *Europe* feels "coarser-grained" than, e.g. *France or Belgium*. This can be modeled by saying *Europe* can less straightforwardly be partitioned into country-level cells, than *France or Belgium*.

(229) In which country did Jo grow up?

a. – They grew up in France or Belgium. (31) ✓ (**??**) ✓

b. ?? – They grew up in Europe. (31) ✓ (**??**) ✓

c. ? – They grew up in Paris (or Brussels). (31) ✗ (**??**) ✓

d. ?? – They speak French natively (31) ✗ (**??**) ✗

In fact, this is exactly the kind of intuition that Qtrees incorporate:[19] a Qtree for (229a) ould feature a country-level terminal layer, properly coinciding with the QuD; while a Qtree for (229b) would "stop" at the continent-level. But no continent properly "fits" within a country-cell of the QuD. Making RELEVANCE sensitive to distinctions in evoked Qtrees could thus explain the contrast in (229a-229b).

Having a granularity-sensitive notion of RELEVANCE may also help explain why finer-grained, overinformative answers like (229c), are not so infelicitous:[20] they suggest a par-

[19]See also Benbaji-Elhadad and Doron (2024) for a "dynamic" view of RELEVANCE along the same lines.

[20]We however note that (32) achieves this, too.

tition of the CS whose cells (city-level) can all be mapped to a single (country-level) cell of the partition provided by the QuD. (229d), which is also overinformative and predicted to be Irrelevant, sounds more odd than (229c) and (**??**), because the partition it suggests (*What's Jo's level in French?*) cannot be properly mapped to the partition set by the QuD. For instance, Jo could very well be fluent in French, without having grown up in France. This suggests that granularity-sensitive notion of Relevance should state that a proposition is Relevant if it can be partitioned into more specific "sub-propositions" (=verifying nodes), s.t. each sub-proposition "fits" a cell of the question, i.e., entails it.

This may also capture the contrast between (229c) and (229d) – (229c) turns out not so odd, because it typically evokes a Qtree featuring a terminal city-layer, in which each node can be "fitted" within single country (i.e. is Relevant to the QuD according to (**??**-**??**)). (31) on the other hand, feels worse because the kind of question it evokes (*What's Jo's proficiency in French?*), features cells that cannot be properly mapped to the partition set by the QuD. These data overall suggest that a proposition is Relevant if it evokes a Qtree whose nodes all entail some cell of the overt QuD. This constitutes an extension (and a strengthening) of (**??**-**??**),

6.8 Appendix: Lewis's view of Relevance between questions

According to Lewis (1988), Relevance between to propositions, can be cashed out in terms of Relevance between their evoked questions. he notion of Relevance at stake amounts to inclusion (meaning, refinement) or connection between the questions evoked by the two propositions. This is shown by Lewis to be equivalent to connection, and at least one proposition being non-contingent. Two questions are said to be connected if one can find a pair of cells, one from each question, whose overlap is empty. Lewis additionally shows an interesting result, which is that, under a strict analysis of conditionals, *if p then q* holds iff p and q are Relevant to each other. This however, still fails to predict an asymmetry in HCs, because Relevance, defined as connection between evoked questions, and non-contingency, is a symmetric concept, that is also very weak.

We think there are still two issues with

6.9 Appendix: DNCHDs, DNLDHDs and Super-Redundancy

This Appendix contains proofs that Super-Redundancy predicts disjunctwise negated CHDs and LDHDs to be fine – contrary to intuitions.

(230) "Disjunctwise" negated CHDs are not Super Redundant (SR).
We show (219)=$(\neg\mathbf{p}) \vee (\neg\mathbf{q})$, with $(\neg\mathbf{p}) \wedge (\neg\mathbf{q}) \not\models \bot$, is SR.
Take C = $\neg\mathbf{p}$.
We then have $(198)^{-}_{C} = \neg\mathbf{q}$.
Take $D = \bot$.
$$\begin{aligned}(219)_{Str(C,D)} &= (\neg(\mathbf{p} \wedge D)) \vee (\neg\mathbf{q})\\ &\equiv (\neg(\mathbf{p} \wedge \bot)) \vee (\neg\mathbf{q})\\ &\equiv (\neg\bot) \vee (\neg\mathbf{q})\\ &\equiv \top \vee (\neg\mathbf{q})\\ &\equiv \top\\ &\not\equiv \neg\mathbf{q} = (219)^{-}_{C}\end{aligned}$$
Exact same reasoning when taking C = $\neg\mathbf{q}$, swapping the roles of $\mathbf{p}$ and $\mathbf{q}$.

(231) a. (222)=$\mathbf{p} \vee (\mathbf{p}^{+} \vee \mathbf{r})$ is SR.
C = $\mathbf{p}^{+}$. $\forall D.\ \mathbf{p} \vee ((\mathbf{p}^{+} \wedge D) \vee \mathbf{r}) \equiv \mathbf{p} \vee ((\mathbf{p}^{+} \vee \mathbf{r}) \wedge (D \vee \mathbf{r})) \equiv (\mathbf{p} \vee \mathbf{r}) \wedge (\mathbf{p} \vee D \vee \mathbf{r}) \equiv \mathbf{p} \vee \mathbf{r}$

b. (223) = $\neg\mathbf{p}^{+} \vee (\neg\mathbf{p} \vee \mathbf{r})$ is not SR.
C = $\neg\mathbf{p}^{+}$. Take $D = \top$.
$\neg(\mathbf{p}^{+} \wedge D) \vee (\neg\mathbf{p} \vee \mathbf{r}) \equiv \neg\mathbf{p}^{+} \vee (\neg\mathbf{p} \vee \mathbf{r}) \equiv \neg\mathbf{p}^{+} \vee \mathbf{r} \not\equiv \neg\mathbf{p} \vee \mathbf{r}$
C = $\neg\mathbf{p}$. Take $D = \bot$.
$\neg\mathbf{p}^{+} \vee (\neg(\mathbf{p} \wedge D) \vee \mathbf{r}) \equiv \neg\mathbf{p}^{+} \vee (\top \vee \mathbf{r}) \equiv \top \not\equiv \neg\mathbf{p}^{+} \vee \mathbf{r}$
C = $\mathbf{r}$. Take $D = \top$.
$\neg\mathbf{p}^{+} \vee (\neg\mathbf{p} \vee (\mathbf{r} \wedge D)) \equiv \neg\mathbf{p}^{+} \vee \mathbf{r} \not\equiv \neg\mathbf{p}^{+} \equiv \neg\mathbf{p}^{+} \vee \neg\mathbf{p}$
C = $(\neg\mathbf{p} \vee \mathbf{r})$. Take $D = \top$.
$\neg\mathbf{p}^{+} \vee ((\neg\mathbf{p} \wedge D) \vee (\mathbf{r} \wedge D)) \equiv \neg\mathbf{p}^{+} \vee (\neg\mathbf{p} \vee \mathbf{r}) \equiv \neg\mathbf{p}^{+} \vee \mathbf{r} \not\equiv \neg\mathbf{p}^{+}$

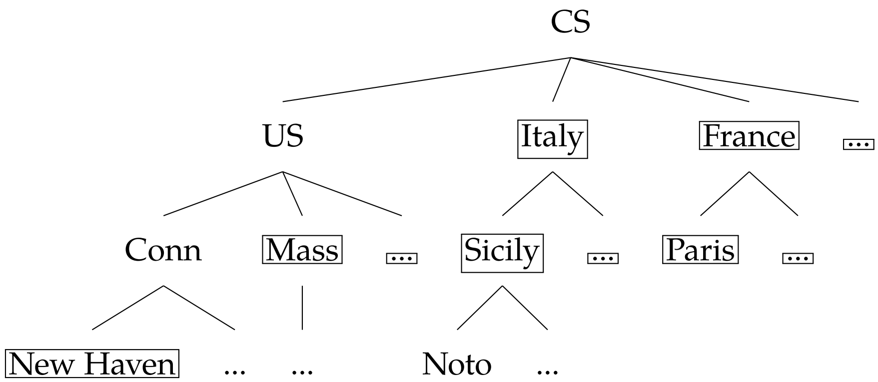
(232) (224b) is not SR

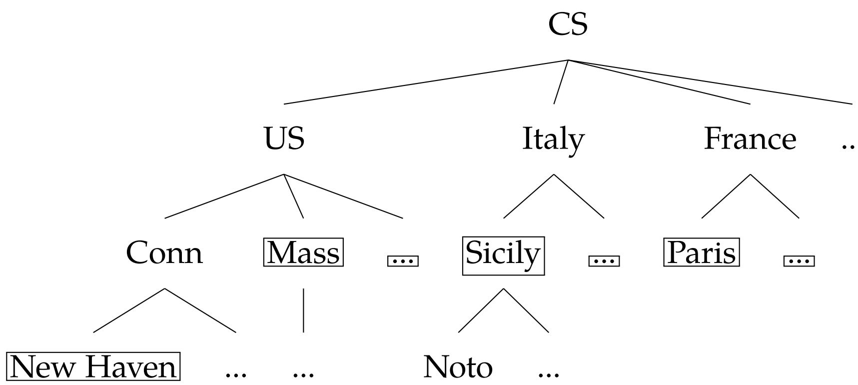
a. C = $\mathbf{p}^{+} \vee \mathbf{r}$. Take $D = \top$.
$\neg((\mathbf{p}^{+} \vee \mathbf{r}) \wedge D) \rightarrow \mathbf{p} \equiv \neg(\mathbf{p}^{+} \vee \mathbf{r}) \rightarrow \mathbf{p} \equiv \mathbf{r} \vee \mathbf{p} \not\equiv \mathbf{p}$

b. C = $\mathbf{p}$. Take $D = \bot$.
$\neg(\mathbf{p}^{+} \vee \mathbf{r}) \rightarrow (\mathbf{p} \wedge D) \equiv \neg(\mathbf{p}^{+} \vee \mathbf{r}) \rightarrow (\mathbf{p} \wedge \bot) \equiv \mathbf{p}^{+} \vee \mathbf{r} \vee \bot \equiv \mathbf{p}^{+} \vee \mathbf{r} \not\equiv \neg(\mathbf{p}^{+} \vee \mathbf{r})$

c. $\mathrm{C} = \mathbf{p}^{+}$. Take $D = \bot$.

$$\neg((\mathbf{p}^{+} \wedge D) \vee \mathbf{r}) \rightarrow \mathbf{p} \equiv \neg(\mathbf{p}^{+} \vee \mathbf{r}) \rightarrow \mathbf{p} \equiv \mathbf{p}^{+} \vee \mathbf{r} \vee \bot \equiv \mathbf{p}^{+} \vee \mathbf{r} \not\equiv \neg(\mathbf{p}^{+} \vee \mathbf{r})$$





Chapter 7

Some but not all redundant sentences escape infelicity: oddness and scalarity[1]

This Chapter focuses on Hurford Disjunctions and Conditionals featuring logically entailing *scalar* items, like *some* and *all* (Gazdar, 1979;Singh, 2008b;Singh, 2008a;Fox and Spector, 2018 i.a.). It will be divided into three clearly distinct components. First, we will introduce scalar Hurford Disjunctions, along with an experimental assessment of the ordering asymmetry they supposedly display. Second, we will propose a new account of the observed asymmetry, which unlike previous accounts, directly recycles independent assumptions about the nature of (c)overt exhaustification and constraints on question answering. Lastly, we will introduce Hurford Conditionals and show that they may receive a treatment solely based on Incremental Q-Relevance, as defined in Chapter 6.

7.1 Experimentally assessing asymmetries in scalar Hurford Disjunctions

7.1.1 The data

Recall that Hurford Disjunctions (henceforth **HD**s, Hurford (1974)), already introduced in Chapter 5, typically involve entailing disjuncts and appear infelicitous regardless of the linear order of the disjuncts. This is shown in (233).

[1]This Chapter constitutes a longer and hopefully more readable adaptation of Hénot-Mortier (to appear). I would like to thank the audience and reviewers of the Harvard Language & Cognition Talk Series, the 2024 HeimFest at MIT, the 2024 Amsterdam Colloquium and SALT35, in particular Jonathan Bobaljik, Ivano Ciardelli, Alexandre Cremers, Kate Davidson, Lisa Hofmann, Manfred Krifka, Jesse Snedecker, Benjamin Spector, for questions, datapoints and suggestions regarding earlier iterations of this project. I also thank my colleagues Omri Doron, Nina Haslinger, and Jad Wehbe.

(233) a. # SALT35 will take place in the United States or Massachusetts. $\mathbf{p} \vee \mathbf{p}^+$

b. # SALT35 will take place in Massachusetts or the United States. $\mathbf{p}^+ \vee \mathbf{p}$

However, Gazdar (1979) observed that HDs can become felicitous if the disjuncts are the same *modulo* scalemates, like $\langle s, s^+ \rangle = \langle \mathit{some}, \mathit{all} \rangle$. This is exemplified in (234).

(234) Jo read some or all of the books. $\mathbf{s} \vee \mathbf{s}^+$

Singh, (2008b, 2008a) later observed that this apparent obviation of Hurford's Constraint, is dependent on the order of the two disjuncts. If the order of the two disjuncts is reversed, as in (235b), infelicity tends to remain. We will call the two HDs in (235), **bare scalar HDs** (or simply scalar HDs). Descriptively, it seems that scalar HDs can be rescued from infelicity, only if the weaker disjunct precedes the stronger one.

(235) a. Jo read some or all of the books. $\mathbf{s} \vee \mathbf{s}^+$

b. ?? Jo read all or some of the books. $\mathbf{s}^+ \vee \mathbf{s}$

Additionally, Singh noticed that scalar HDs can be overtly rescued *via only*. Note that (236a) may sounds weirder, just because it appears equivalent to its variant without *only*, (235a), which simpler, ad felicitous.

(236) a. ? Jo read only some or all of the books. $O(\mathbf{s}) \vee \mathbf{s}^+$

b. Jo read all or only some of the books. $\mathbf{s}^+ \vee O(\mathbf{s})$

This dataset is challenging, because, first, one must come up with a story explaining why bare scalar HDs like those in (233) are asymmetrically rescued, in a completely covert way; and, second, why *only*, seen as an overt rescuer, is not influenced by liner order. Section 7.2 will actually present a novel solution to these two puzzles. For now let us review the mainstream approach to such data. The specifics of the analysis will not be relevant to the experiment subsequently presented in this Section. This experiment is only intended to clarify the initially murky empirical picture: is the observation that bare scalar HDs are asymmetrically rescued a robust fact tied to pragmatics? What about scalar HDs involving *only*?

7.1.2 Previous accounts

The asymmetry in (235) has received several accounts (Singh (2008b); Fox (2018); Tomioka (2021); Ippolito (2019); Hénot-Mortier (2023) i.a.). Most of these accounts specifically focused on the pair in (235) – leaving (236) aside (Singh, 2008a and Ippolito, 2019 being the two notable exceptions). All these accounts capitalize on the idea that

(235a) can be rescued *via* a local scalar implicature of the form *some* $\rightsquigarrow$ *some but not all*, targeting the first disjunct. This would allow the two disjuncts in (235a) to become incompatible. Therefore, (235a) would avoid violating Hurford's Constraint, or, for that matter, any implementation of Hurford's Constraint we reviewed in this dissertation.

Local scalar implicatures are permitted by the covert operator *exh*, which stands for *exhaustification* (Fox, 2007;Spector et al., 2009 i.a.). A definition of *exh* is given in (237).[2] This operator non-arbitrarily conjoins the proposition it attaches to, called prejacent, with the negation non-weaker alternatives, while making sure the resulting strengthened meaning is maximally informative and non-contradictory. Ensuring the final result is non-contradictory and maximally informative, amounts to computing the set $MaxExcl(Q, p)$ of maximal "candidate" sets of alternatives which can be negated and conjoined with the prejacent without a contradiction. Ensuring the final result is not obtained in an arbitrary way, amounts to actually negating only the alternatives that belong to *all* candidate sets in $MaxExcl(Q, p)$. There alternatives form the set of so-called Innocently Excludable alternatives $IE(Q, p)$. Ensuring non-arbitrariness in exhaustification appears crucial when it comes to sets of alternatives to a prejacent that properly partition it. This is known as the Symmetry Problem (Kroch, 1972; Fox, 2007) and will be briefly discussed at the end of this Section.

(237) *Exhaustification.* Let p be a proposition and let Q be a set of relevant alternatives to p that are at most as complex as p, in the sense of Katzir (2007).
The exhaustification of p (prejacent) given Q, corresponds to p, conjoined with the negation of all Innocently Excludable alternatives in Q. In other words, exh(Q, p) $= p \wedge \bigwedge_{p' \in IE(Q,p)} \neg p'$.

(238) *Innocent Exclusion.* p' is Innocently Excludable given Q and p ($p' \in IE(Q, p)$), iff p' belongs to the intersection of the maximal subsets of Q whose grand negation is consistent with p. In other words, $p' \in IE(Q, p) \iff p' \in \bigcap MaxExcl(Q, p)$, where $MaxExcl(Q, p) = Max_{\subseteq}(\{Q' \subset Q.\, p \wedge \bigwedge_{p' \in Q'} \neg p' \not\models \bot\})$.

Exh has an effect that is very close to that of overt *only*. When applied to the first disjunct of (235a) for instance, it typically leads to the strengthening *Jo read some but not all of the books*, which is synonymous with *Jo read only some of the books*. This is because, *some* typically only has one non-weaker alternative, *all*, which is trivially Innocently Excludable.

[2]This definition does not cover cases in which non-weaker alternatives are included (Bar-Lev and Fox, 2017), but is enough for our purposes here.

However, without additional assumptions this theory predicts that *exh* can be inserted in both (235a) and (235b). Both variants would in turn be predicted to be felicitous. This is illustrated in (239).

(239) a. Jo read *exh*(some) or all of the books. $exh(\mathbf{s}) \vee \mathbf{s}^+$
≡ Jo read some but not all or all of the books. $(\mathbf{s} \wedge \neg\mathbf{s}^+) \vee \mathbf{s}^+$
b. ?? Jo read all or *exh*(some) of the books. $\mathbf{s}^+ \vee exh(\mathbf{s})$
≡ Jo read all or some but not all of the books. $\mathbf{s}^+ \vee (\mathbf{s} \wedge \neg\mathbf{s}^+)$

Therefore, assuming covert and local exhaustification allows to correctly predict the felicity of (239a), but also mispredicts the felicity of (239b). The challenge then shifts to explaining why (235b) cannot be rescued by *exh* in the same way as (235a). Meaning, one must explain *exh* cannot be inserted (or at least do its job) in the second disjunct of (239b).

Although the implementations vary, the asymmetry between (235b) and (235a) in terms of covert exhaustification, ends up being modeled as an interaction between the meaning of the first disjunct, and the licensing/timing of *exh* in the second disjunct. One prominent account, due to Fox (2018), suggests *exh* should not be applied to an expression E if it turns out to be Incrementally Weakening (abbreviated **IW**). Very roughly, *exh* is IW in a sentence if it leads to an equivalent/weaker meaning no matter how the sentence is finished. The constraint is spelled out in (240); (241-244) unpack the definition.

(240) *Economy Condition on Exhaustification.* Let $exh(Q, p)$ be the exhaustification of p given a set of alternatives Q. $*S[exh(Q, p)]$, if $exh(Q, q)$ is incrementally weakening in S.

(241) *Incremental Weakening.* An occurrence of *exh* taking p as argument is incrementally weakening in S if it is globally weakening for every continuation of S at point p.

(242) *Global Weakening.* Let $IE(p, Q)$ be the set of Innocently Excludable alternatives to p that belong to Q (see (238)). An occurrence of $exh(Q, p)$ is globally weakening in a sentence S$[exh(Q, p)]$, if $S[p] \vDash S[exh(Q, p)]$.[3]

(243) S' is a continuation of S at point A if S' can be derived from S by replacement of constituents that follow A.

(244) Y follows A if all the terminals of Y are pronounced after those of A.

[3]The more complex constraint spelled out in Fox (2018), is: $\exists Q'$. $\mathrm{IE}(Q', p) \subset \mathrm{IE}(Q, p) \wedge S[exh(Q', p)] \vDash S[exh(Q, p)]$. If Q' can only be the empty set, the condition becomes $\mathrm{IE}(\emptyset, p) \subset \mathrm{IE}(Q, p) \wedge S[exh(\emptyset, p)] \vDash S[exh(Q, p)]$, i.e. $S[A] \vDash S[exh(Q, p)]$; as given in the main text here.

Given IW, the contrast in (235) then boils down to the fact *exh* is not IW in the first disjunct of (235a) (see (245a)), while it is in the second disjunct of (235b) (see (245b)).

(245) a. $exh(\{s, s^+\}, s) = s \wedge \neg s^+$ is not IW in the first disjunct of (235a).
We have $S[exh(\{s, s^+\}, s)] = exh(\{s, s^+\}, s) \vee s^+$, and $S[s] = s \vee s^+$.
Take S' to be $exh(\{s, s^+\}, s) \vee \bot$. S' is a continuation of S after $exh(\{s, s^+\}, s)$, because it can be derived from S by replacing its second disjunct with a contradiction. $exh(\{s, s^+\}, s)$ is not globally weakening in S':

$$\begin{aligned} exh(\{s, s^+\}, s) \vee \bot &\equiv exh(\{s, s^+\}, s) \\ &\equiv s \wedge \neg s^+ \\ &\not\Rightarrow s \\ &\not\Rightarrow s \wedge s^+ \\ &\not\Rightarrow S[s] \end{aligned}$$

Thus, $exh(\{s, s^+\}, s)$ is not incrementally weakening in S, and *exh* can be inserted in the first disjunct of (235a).

b. $exh(\{s, s^+\}, s) = s \wedge \neg s^+$ is IW in the second disjunct of (235b).
We have $S[exh(\{s, s^+\}, s)] = s^+ \vee exh(\{s, s^+\}, s)$, and $S[s] = s^+ \vee s$.
Let S' be a continuation of S after $exh(\{s, s^+\}, s)$. Because S' must result from the replacement of a constituent *following* $exh(\{s, s^+\}, s)$ in S, S' can only be S.
$exh(\{s, s^+\}, s)$ is globally weakening in $S' = S$:

$$\begin{aligned} s^+ \vee exh(\{s, s^+\}, s) &\equiv s^+ \vee exh(\{s, s^+\}, s) \\ &\equiv s^+ \vee (s \wedge \neg s^+) \\ &\equiv s^+ \vee s \\ &\equiv S[s] \end{aligned}$$

Thus, $exh(\{s, s^+\}, s)$ is incrementally weakening in S, and *exh* cannot be inserted in the second disjunct of (235b).

As a result, *exh* can be inserted in the first disjunct of (235a), which breaks the entailment between the two disjuncts. By contrast, *exh* cannot be applied to the second disjunct of (235b), and the problematic entailment between disjuncts remains. This is illustrated in (239).

(246) a. Jo read *exh*(some) or all of the books. $exh(s) \vee s^+$
≡ Jo read some but not all or all of the books. $(s \wedge \neg s^+) \vee s^+$

b. ?? Jo read all or **exh*(some) of the books. $s^+ \vee s$
≡ Jo read all or some of the books. $s^+ \vee s$

Lastly, note that this does not overgenerate in the case of non-scalar HDs like those in (233). In particular, (233a) cannot be rescued like (235a), either because *Massachusetts* is not a natural alternative to *the United States* out-of-the blue, or, because *Massachusetts* is not an Innocently Excludable alternative to *the United States*. Let us further decompose the second option. If *Massachusetts* can be considered a relevant alternative to *the United States*, all other US states most likely can, too. Such alternatives properly partition the prejacent; thus negating them all together, would create a contradiction with the prejacent. However, negating any strict subset of these alternative, would allow to maintain consistency with the prejacent. For instance, negating *Massachusetts* would lead to a strengthened meaning along the lines of *the United States, but not Massachusetts*. More drastically even, negating all US states but *Massachusetts*, would lead to assert *Massachusetts*. But notice that all of these options are arbitrary: negating any subset of the relevant alternatives, prevents us from negating other, equally legitimate alternatives. This is addressed by the concept of Innocent Exclusion, which forces Innocently Excludable alternatives to belong to *all* maximal candidate sets of excludable alternatives. In the case of (233a), and considering state alternatives to *the United States*, the maximal candidate sets of excludable alternatives, are made of all states, but one. So their intersection, which corresponds to the set of Innocently Excludable alternatives, is predicted to be empty. Therefore, exhaustification is vacuous in (233a) (and (233a), for similar reasons), and as a result, both HDs in (233) are still correctly predicted to be infelicitous.

In this Section, we have described one prominent account of the asymmetry in (235), However, the subtleness of the contrast in (235), casts doubts on whether such an elaborate approach is needed in the first place.[4]

7.1.3 Experiment

The experiment presented in this Section aims at answering two questions. First, is the contrast between (235b) and (235a) real and robust? Second, is it really based on pragmatic factors? The first question, originates from a small-scale corpus study performed by Fox, which showed that, although the contrast between (235a) and (235b) was clearly a trend, infelicitous instances of the form (235a), were anyway attested. The second question, is motivated by accounts of linear asymmetries in (conjoined) "binomials", like *salt and pepper* vs. *pepper and salt* (Benor and Levy, 2006). It was shown that crisp ordering

[4]It is still worth mentioning that the approach presented here comes with a range of good predictions, when it comes to more complex variants of (235)– however characterized by equally subtle judgments – but also beyond HDs. We do not cover all these predictions here, for reasons of space.

preferences in such binomials arise from a variety of extra-pragmatic factors, including metrical and frequency constraints. Is *some or all* preferable to *all or some* for similar reasons?[5] To answer these questions, we propose to assess the felicity of the sentences in (235) and (236), repeated below, along with their "long" variants, in (247) and (248).

(235) "Short" disjuncts, no *only*

a. Jo read some or all of the books. $\mathbf{s} \vee \mathbf{s}^+$

b. ?? Jo read all or some of the books. $\mathbf{s}^+ \vee \mathbf{s}$

(236) "Short" disjuncts, *only*

a. ? Jo read only some or all of the books. $O(\mathbf{s}) \vee \mathbf{s}^+$

b. Jo read all or only some of the books. $\mathbf{s}^+ \vee O(\mathbf{s})$

(247) "Long" disjuncts, no *only*

a. Jo read some of the books or all of them. $\mathbf{s} \vee \mathbf{s}^+$

b. ?? Jo read all of the books or some of them. $\mathbf{s}^+ \vee \mathbf{s}$

(248) "Long" disjuncts, *only*

a. ? Jo read only some of the books or all of them. $O(\mathbf{s}) \vee \mathbf{s}^+$

b. Jo read all of the books or only some of them. $\mathbf{s}^+ \vee O(\mathbf{s})$

As discussed in the previous Section, such sentences have been discussed extensively in the theoretical pragmatics literature, but the robustness of the judgments reported in the above was never systematically assessed in an experimental setting. The only notable exception is Chemla et al. (2013)[6], however, this study focused on the felicitous weak-to-strong ordering (247a), with the goal of better understanding the fined-grained processing

[5]There are *a priori* three arguments against this hypothesis. The first argument, is that there is no obvious metrical or frequency-based difference between *some* and *all*, so it is hard to see which order an analysis like Benor and Levy (2006) would predict to be the best. However, one could in turn argue that additional *semantic* factors (e.g., likelihood, informativity) are at play in such pairs. The second, perhaps stronger argument, is that under a multivariate analysis of *some or all* disjunctions *à la* Benor and Levy (2006), one might expect some cross-linguistic variation in the preferred ordering of *some* and *all*. But it does not seem to be the case (although, one could in turn argue that languages tend to assign *some* and *all* similar extra-pragmatic features, metrical, frequency, etc.). The third argument, is that the ordering asymmetry in (235) was argued to disappear when such disjunctions are embedded in certain environments, for instance, under universal modals, or universal quantifiers (Fox, 2018). This obviation of the asymmetry is unexpected under Benor and Levy (2006)'s analysis, because the features of the scalemates and their immediate environment, are not affected by embedding under universals. Of course, the robustness of the data, could also be questioned. Our experiment intends to bring more empirical arguments to the table, in order to better figure out the division of labor between the aforementioned pragmatic and extra-pragmatic factors.

[6]The full paper that came out of this presentation (Chemla et al., 2016), was focusing on "scalar" tautological sentences of the form *Jo read some or none of the books*, instead of scalar HDs. The methodology was however similar.

signature of covert exhaustification. Additionally, little emphasis has been put on potential differences between the “short”variants (235) and the “long” variants (247), and on the effect of *overt* exhasutification with *only* in (236) and (248). The study presented thus intends to fill these gaps, and specifically, to determine what kind of pragmatic theory is sufficient to account for the above data.

Hypothesis 1: If covert exhaustification (of the form "some" > "some but not all") is possible at the embedded level and the only way to rescue the above disjunctions from redundancy, a sentence like (1a) Jo read some or all of the books, should be felicitous. Felicity should be maintained even if the two scalemates are more linearly distant, as in (1a′) Jo read some of the books or all of them. Sentences like (2a) Jo read only some or all of the books, or (2a′) Jo read only some of the books or all of them, should be degraded by competition with the simpler (1a)/(1a′). Hypothesis 1.A: If covert exhaustification is moreover influenced by linear order, a sentence like (1b) Jo read all or some of the books, should be less felicitous than (1a). This contrast should be maintained when considering linearly "distant" scalemates, i.e. (1b′) Jo read all of the books or or some of them, vs. (1a′). Hypothesis 1.B: If covert exhaustification is *not* influenced by linear order, a sentence like (1b) Jo read all or some of the books, should be as felicitous as (1a). This should also hold when considering linearly "distant" scalemates, i.e. (1b′) Jo read all of the books or or some of them, vs. (1a′). Sentences like (2b) Jo read all or only some of the books, or (2b′) Jo read all of the books or only some of them, should be degraded by competition with the simpler (1b)/(1b′). PREDICTIONS ASSUMING EMBEDDED EXHAUSTIFICATION (Hypothesis 1): -If incremental: (2a) (1b) < (1a) (2b); (2a′) (1b′) < (1a′) (2b′). ==> Same interaction between order of the scalemates and presence of only, regardless of disjunct size. -If not incremental: (2a) (2b) < (1b) (1a); (2a′) (2b′) < (1b′) (1a′) ==> Effect of only, regardless of disjunct size. Hypothesis 2: Alternatively, if covert exhaustification is *not* possible at the embedded level, a sentence like (1a) Jo read some or all of the books, may still be felicitous because "some or all" can be interpreted as some kind of frozen expression determined by other, extra-semantic factors (like e.g. "salt and pepper" vs. "pepper and salt"). (2b) is expected to pattern like (1a) because its competitor (1b) is expected to be degraded. There is no clear prediction for (2a), given that the status of "frozen" expressions when it comes to pragmatic competition is a bit unclear. In any case felicity should *not* be maintained if the two scalemates are made more linearly distant ((1a′), (1b′)). This in turn predicts their variants with only ((2a′), (2b′)), to be better

PREDICTIONS ASSUMING NO EMBEDDED EXHAUSTIFICATION: (1b) < (1a) (2b); (1b′) (1a′) < (2a′) (2b′). ==> Interaction between order of the scalemates and presence of only with short disjuncts, effect of only with long disjuncts In effect, we may expect a mixture of Hypothesis 1 (purely pragmatic) and 2 (purely extra-pragmatic). PREDICTIONS ASSUMING EMBEDDED EXHAUSTIFICATION AND EXTRA-PRAGMATIC FACTORS: -If incremental: (2a) (1b) < (1a) (2b); (1b′) < (2a′) (1a′) < (2b′) ==> Interaction between order of the scalemates and presence of only, modulated by disjunct size -If not incremental: some contrast between the (1) and (2) sentences; directionality depends on the weight of pragmatic vs. extra-pragmatic factors

Participants will be presented with a short scenario involving 3 individuals, A, B, and C (real names used in the scenarios). A asks something about C to B when C is away/unavailable. B then answers to A a critical sentence that the participant has to read one word at a time in a self-paced fashion, and then has to rate on a scale from 0 to 100 (no precise label/score displayed on screen). The critical sentences follow a 2x2x2 design: "only"-factor: Presence/absence of only outscoping some "order"-factor: "some" before "all"/"all" before "some" "disjunct size"-factor: short (see e.g. (1a)) or long (see e.g. (1a′)) only is between group order and disjunct size are within-group Each participant is exposed to 4 randomized practice items: - the practice items involve critical sentences that are disjunctive but do not make use of scalemates. 2 practice items are redundant ("Hurford") disjunctions (e.g. "a book or a novel'), 2 practice items are non- redundant (e.g. "a cat or a rabbit"). Participants receive positive/negative feedback if they are

below/above score of 25 in the redundant case, and above/below a score of 75 in the non-redundant case. Each participant is then exposed to 4 blocks, each containing 8 items (4 targets, 4 fillers): - targets correspond to the two possible orders and the two possible disjunct sizes. - fillers are 2 non-redundant structures (all or the books / some of the books), and 2 sharply redundant structures (all or all of the books / some or some of the books). The sharply redundant structures give rise to the same kind of feedback as the one given during practice on the redundant disjunctions. Target items (2x2=4 treatments) and sentence frames/scenarios follow a Latin Square design, so each group (only/no only), is subdivided into 4 subgroups, s.t. each treatment gets paired with a given frame/scenario once across 4 subgroups. Filler are interspersed within each block, s.t.: Each block contains exactly 4 fillers, one of each type; How the fillers gets randomly inserted changes between block 1, 2, 3, and 4; How the fillers gets randomly inserted for a given block (e.g. block 1), does *not* change across groups (only/no only) or subgroups (as generated by the Latin Square design).

Participants will be recruited through Prolific. Participants will be paid TODO for agreeing to participate. SEE CONDITIONS WITH KATE

3 variables: -"only" (between-subjects): whether or not "only" occurs before "some" in the target sentences. -"order" (within-subjects): whether the "some" disjunct is presented

before the "all" disjunct, or vice- versa. -"disjunct size" (within-subjects): whether the disjunction has the form X Ved some/all or all/some of the Ys (short disjuncts), or X Ved some/all of the Ys or all/some of them (long disjuncts)

For the main analysis, we will analyze the score assigned by the participants to the target sentences. The score is between 0 and 100. Participants will not have access to the specific values of the scores ("blind" Likert scale). For exploratory analyses, reaction times (between the full display of the target sentence and the submission of a score), and well as reading times (as recorded during the self-paced reading stage), will be analyzed.

We will use a linear mixed effect linear regression model (R lmer/lmertest package), to evaluate if the felicity score assigned to a sentence depends on an interaction between the presence/absence of "only" and the order of the scalemates (some<all or all<some). We will include the maximum random effect structure supported by the data. No files selected Transformations No response Inference criteria Using the anova function in R (lme4 package), we will compare a model with an order*only interaction term, to models with only as main effect and a model with a three way interaction of the form

order*only*disjunct-size. We will use the p-values returned by anova, based on likelihood ratio test comparisons (chi-square). Data exclusion Participants who have failed at all 4 practice items (i.e., assigned a score higher than 25 to redundant disjunctions, and a score lower than 75 to non-redundant disjunctions), will be excluded. Participants who have failed at more than 4/16 fillers (i.e., assigned a score higher than 25 to redundant fillers, and a score lower than 75 to non-redundant fillers), will be excluded. Participants who at the end of the study reported that the native language is not English will be excluded. Missing data No response Exploratory analysis Exploratory analyses will include: -group-by-group analyses (only/no only): check the effect of order and disjunct size -group-by-group analyses (long/short disjuncts): check the effect of order and only -analyses of reaction times (time between the full completion of the self-paced reading step, and the submission of a score): check if lower ratings correlate with higher RTs; check if RTs can be predicted by LMER using the same formulas a the main analysis. -analyses of reading times (at the self-paced reading stage): does "some" take longer to read than "all"? Is the reading time for "some"/"all" differentially influenced by the ordering of the disjuncts/the presence of only? To do this, we check if the word reading times of scalar items can be predicted by LMER, using item type ("some" vs. "all"), disjunct position (1st vs. 2nd), presence of "only", and disjunct size as potential factors.

7.2 A novel account account of the asymmetries in scalar Hurford Disjunctions

7.2.1 Qtrees of simplex LFs: scalar vs. non-scalar case

Chapter 6 defined the set of possible Qtrees evoked by a simplex LF X denoting p. Roughly, we assumed that a Qtree for X may be a depth-1 Qtree whose leaves denote p and $\neg p$; a depth-1 Qtree whose leaves correspond to the Hamblin partition of the CS generated by p and same-granularity alternatives to p; or a "tiered" Qtrees whose layers are each generated from a set of same-granularity alternatives to an alternative to p entailed by p. We also assumed that in each case, Qtrees derived from simplex LFs get "flagged" by defining their verifying nodes as the set of nodes entailing p.

Defining the same-granularity relation

So far we took granularity as a primitive. We now submit that scalemates such as *some* and *all may* be seen as same granularity alternatives to each other, while non-scalemates, like *Paris* and *France*, *cannot* be considered being so, at least out-of-the blue. From this, it follows that *some* and *all* may answer the same QuD (partitioning the CS into the *none*, *some but not all*, and *all* worlds), while *Paris* and *France* never do.

At the intuitive level, the difference between *some*/*all* and *France*/*Paris* seems to be related to the symmetry problem (Kroch, 1972; Fox, 2007) that arises with the latter kind of alternatives. If *Paris* and all other cities are considered to be same-granularity alternatives, and if, on top of this, *Paris* and *France* are considered same-granularity, then by transitivity *France* and any city in France should be considered same-granularity. This appears counter-intuitive, given that at a certain level of abstraction, all French cities together cover France. This intuition leads us to define same-granularity alternatives as in (249). (250) clarifies some of the terms introduced in (249).

(249) *Set of same granularity alternatives to q.* Let X be a LF denoting p and $\mathcal{A}_{p,X}$ be the set of all possible alternatives to p, obtained by the replacement of focused material in X by relevant, same-complexity and same-type constituents. Let $\mathcal{H}(\mathcal{A}_{p,X})$ be the Hasse diagram generated by $\vDash$ on $\mathcal{A}_{p,X}$, directed from top (logically stronger) to bottom (logically weaker). For any $q \in \mathcal{A}_{p,X}$, a set of same-granularity alternatives to q ($\mathcal{A}^q_{p,X}$) is obtained by:

1. (obligatory) adding all same-level alternatives to q in $\mathcal{H}(\mathcal{A}_{p,X})$ to $\mathcal{A}^q_{p,X}$;

2. (optional) for each level higher than q's level in $\mathcal{H}(\mathcal{A}_{p,X})$ (from the lowest to the highest level), adding to $\mathcal{A}^q_{p,X}$ all the alternatives that are not yet covered by a subset of $\mathcal{A}^q_{p,X}$. A set $\mathcal{S}$ of sets covers another set s is $\bigcup \mathcal{S} = s$.
3. (optional) for each level lower than q's level in $\mathcal{H}(\mathcal{A}_{p,X})$ (from the highest to the lowest level), adding to $\mathcal{A}^q_{p,X}$ the grand intersection of the maximal sets of alternatives that together do not cover any subset of $\mathcal{A}^q_{p,X}$.[7]

The three steps are ordered, and steps (249.2-3) are optional.

(250) a. *Same-level nodes in a Hasse diagram.* p and q are same-level nodes in a Hasse diagram $\mathcal{H}$ if $\exists r \in \mathcal{H}.\ \exists n \in \mathbb{N}.\ p \rightarrow^n r \wedge q \rightarrow^n r$, where $\rightarrow^n$ represents n iterations of the accessibility relation in $\mathcal{H}$.

b. *Level in a Hasse diagram.* $\mathcal{L}$ is a level in a Hasse diagram $\mathcal{H}$ iff $\exists p \in \mathcal{H}.\ \mathcal{L} = \{q \in \mathcal{H} \mid p$ and q are same-level nodes in $\mathcal{H}\}$

c. *Level higher/lower than a node in a Hasse diagram.* A level $\mathcal{L}$ in a Hasse diagram $\mathcal{H}$ is higher than a node p if $\exists q \in \mathcal{L}.\ p \rightarrow^* q$. It is lower than a node p if $\exists q \in \mathcal{L}.\ q \rightarrow^* p$.

Let us now see how these definitions works when considering same-granularity alternatives to LFs containing *Paris*, *some*, and *all*.

Non-scalar items

Starting with *Paris*, one should consider $\mathcal{A}_{Paris}$ to be a set of locations organized in a Hasse diagram like Figure A.

[7]Note that this relates to Fox (2007)'s notion of Innocent Exclusion: the goal here is to non-arbitrarily *include* (rather than *exclude*) a subset of alternatives that together do not already cover a union of alternatives in $\mathcal{A}^q_{p,X}$. In particular, if the set of alternatives considered at a given level is symmetric w.r.t. some alternative already present in $\mathcal{A}^q_{p,X}$, then, none of these alternatives will be added to $\mathcal{A}^q_{p,X}$.

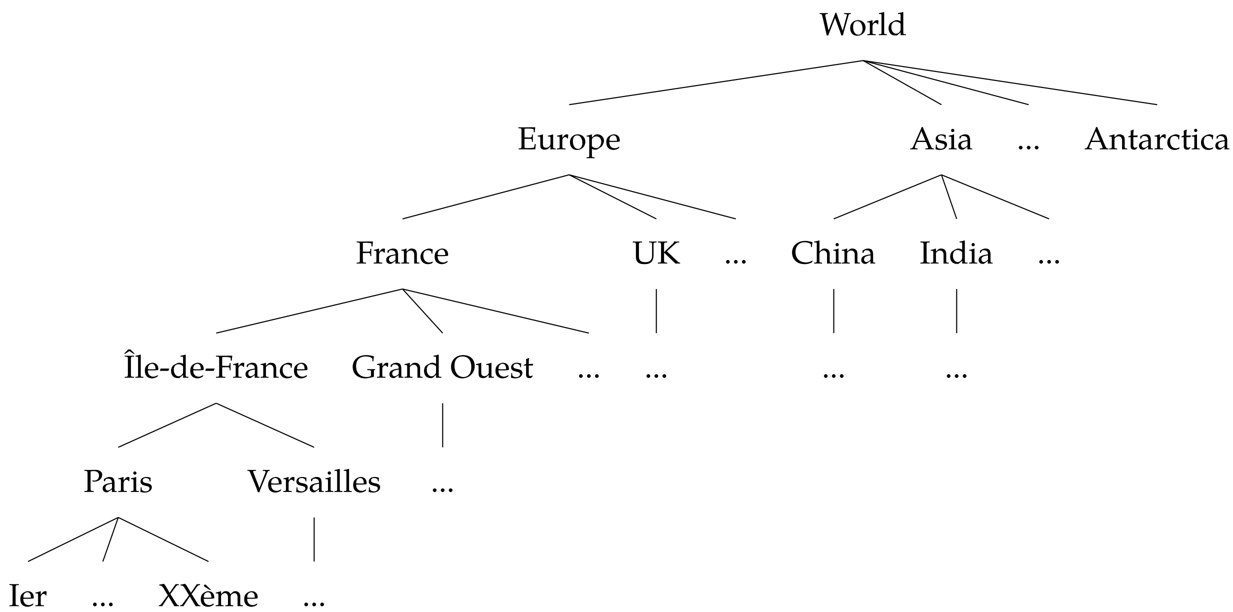


Figure A: Hasse diagram generated by alternatives to *Paris*. Entailment goes upward. Nodes that are vertically aligned are same-level.

In this diagram, *Paris* is at a level that typically involves other cities. Applying (249.1) then adds all those cities to the set of same-granularity alternatives to *Paris*; so at this point, $\mathcal{A}^{Paris}_{Paris} = \{Paris, Versailles, Lyon, London...\}$. Figure A shows that some higher-level locations, like *Antarctica*, may not be subdivided into cities. Applying (249.2) then adds *Antarctica* to $\mathcal{A}^{Paris}_{Paris}$, since no subset of cities that are already part of $\mathcal{A}^{Paris}_{Paris}$ covers it. Countries like *France* and *Germany* cannot be added to $\mathcal{A}^{Paris}_{Paris}$ in the same way, because they are covered by a subset of cities that are already part of $\mathcal{A}^{Paris}_{Paris}$. So at this point, $\mathcal{A}^{Paris}_{Paris} = \{Paris, Versailles, Lyon, London, ...Antarctica\}$. Assuming any city comes with a set of districts that fully partition it (i.e. districts are symmetric w.r.t their respective cities), (249.3) applies vacuously, since for no city already present in $\mathcal{A}^{Paris}_{Paris}$ is it possible to non-arbitrarily add to $\mathcal{A}^{Paris}_{Paris}$ a set of districts that does not fully cover the given city. In sum, we derive that same-granularity alternatives to *Paris* are typically cities, but may also involve intuitively coarser-grained alternatives that cannot be reasonably subdivided into cities (*e.g. Antarctica*). Crucially, no location that is subdivided into cities is part of this set.

This reasoning easily extends to an intuitively coarser-grained alternative to *Paris* like *France*. Same-granularity alternatives to *France* are typically countries, but may also involve intuitively coarser-grained alternatives that cannot be reasonably subdivided into countries (*e.g. Antarctica*). Just like districts (which tend to partition cities) could not be added to the set of same-granularity alternatives to *Paris*, cities (which tend to partition countries) cannot be added to the set of same-granularity alternatives to *France*. A consequence of this, is that non-scalemates like *Paris* and *France* have inherently distinct sets of same-granularity alternatives – and in turn, will be predicted to give rise to inherently distinct sets of Qtrees.

Scalar items

Now turning to *some* and *all*. We assume that the set of alternatives for such items is typically made of $\forall$ (*all*), $\exists$ (*some*), and $\neg\exists$ (*none*), but does not contain, e.g. $\neg\forall$ or $\exists \wedge \neg\forall$, because such logical meanings correspond to expressions (*not all*, *some but not all*) that are strictly more complex. So, $\mathcal{A}_{some} = \mathcal{A}_{all} = \{\neg\exists, \exists, \forall\}$. The resulting Hasse diagram for both *some* and *all* is given in Figure B.

$$\begin{array}{cc} \exists & \neg\exists \\ | & \\ \forall & \end{array}$$

Figure B: Hasse diagram generated by alternatives to *some*/*all*. Entailment goes upward. Each node belongs to a different level.

In this diagram, $\exists$ and $\forall$ are at different levels, since $\forall$ strictly entails $\exists$. To build a set of same-granularity alternatives to $\exists$ ($\mathcal{A}^{some}_{some}$=$\mathcal{A}^{some}_{all}$), we start by applying (249.1), which adds $\exists$ to $\mathcal{A}^{some}_{some}/\mathcal{A}^{some}_{all}$. Applying (249.2) is vacuous, since there is no higher level above $\exists$. Applying (249.3) then adds $\forall$ to $\mathcal{A}^{some}_{some}/\mathcal{A}^{some}_{all}$, because doing so is non-arbitrary (only possibility), and $\forall$ does not cover $\exists$ ($\forall$ is strictly contained in $\exists$). Note that the absence of $\exists \wedge \neg\forall$ from the Hasse diagram is crucial to derive this: had $\exists \wedge \neg\forall$ been present, $\exists \wedge \neg\forall$ and $\forall$ would have been symmetric w.r.t. $\exists$, and none of these alternatives could have been non-arbitrarily added to $\mathcal{A}^{some}_{some}/\mathcal{A}^{some}_{all}$. In sum, $\mathcal{A}^{some}_{some}$=$\mathcal{A}^{some}_{all} = \{\exists\}$ (by only applying step (249.1)) or $\{\exists, \forall\}$ (by applying all steps).

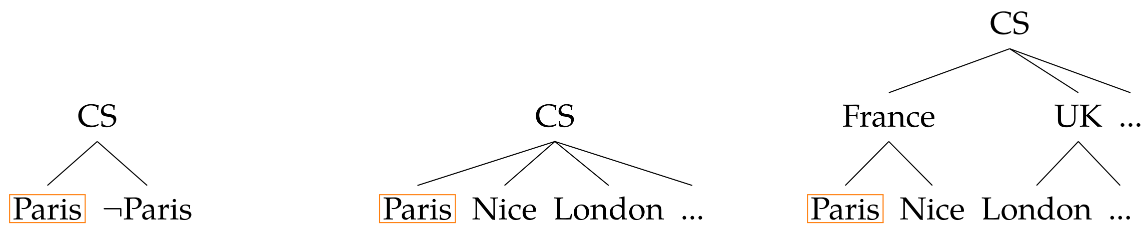
To build a set of same-granularity alternatives to $\forall$ ($\mathcal{A}^{all}_{some}$=$\mathcal{A}^{all}_{all}$), we start by applying (249.1), which adds $\forall$ to $\mathcal{A}^{all}_{some}/\mathcal{A}^{all}_{all}$. Applying (249.2) then adds $\exists$ to $\mathcal{A}^{all}_{some}/\mathcal{A}^{all}_{all}$, because $\forall$ does not cover $\exists$. Applying (249.3) is vacuous, because $\forall$ already forms the lowest level of the diagram. In sum, $\mathcal{A}^{all}_{some}$=$\mathcal{A}^{all}_{all} = \{\forall\}$ (by only applying step (249.1)) or $\{\exists, \forall\}$ (by applying all steps).

We therefore derive that *some* and *all* *may* give rise to the same set of same-granularity alternatives, namely, $\{\exists, \forall\}$. This will eventually predict that *some* and *all* may give rise to the same kind of Qtree, namely, a Qtree partitioning the CS into $\neg\exists$-, $(\exists \wedge \neg\forall)$-, and $\forall$-worlds. Note however that *some* and *all* may also give rise to distinct sets of same-granularity alternatives, respectively $\{\exists\}$ and $\{\forall\}$ – if we assume that only step (249.1) is applied. Zooming out, definition (249) allowed to model a crucial distinction between non-scalemates like *Paris* and *France*, and scalemates like *some* and *all*: the former will never give rise to the same sets of same-granularity alternatives, while the latter can. Because Qtrees for simplex LFs were defined layer-by-layer based on the notion of same-granularity alternatives back in Chapter 6, we derive that non-scalemates will never give rise to identical Qtrees, while scalemates may do so.

Deriving Qtrees for scalemates and non-scalemates

Now that same-granularity alternatives are defined for scalar and non-scalar items, we are in a position to apply the recipe (**??**) from Chapter 6 to derive Qtrees for sentences like *SALT35 will take place in Paris* or *Jo read some of the books*.

Starting with the LF X^+ = *SALT35 will take place in Paris*. Following principle (**??**.i), a "polar" Qtree can be built out of X^+ from the partition {*Paris*, ¬*Paris*}. This is done in Figure C1. Following principle (**??**.ii), one must generate a Hamblin partition out of a set of same-granularity alternatives to X^+. We just saw that same-granularity alternatives to X^+ form a set {*Paris*, *Nice*, *London*, ...} containing cities (and intuitively coarser-grained locations that are not partitioned by cities). This set happens to be equal to its Hamblin partition, given that the alternatives it contains are already mutually exclusive. Applying principle (**??**.ii) using this Hamblin partition then generates the Qtree in Figure C2. Lastly, according to principle (**??**.iii), X^+ can give rise to a chain of entailing propositions of the form *SALT35 will take place in Paris*, *SALT35 will take place in France*, *SALT35 will take place in Europe* etc. Restricting ourselves to the *Paris-France* chain, a "tiered" Qtree can be created by generating Hamblin partitions from same-granularity alternatives to *Paris* and *France*. We just saw that the Hamblin partition obtained for *SALT35 will take place in Paris* takes the form {*Paris*, *Nice*, *London*, ...}. Similarly for *SALT35 will take place in France*, the relevant Hamblin partition corresponds to country alternatives (and potentially intuitively coarser grained alternatives that are not subdivided by countries), i.e. {*France*, *UK*, ...}. Following principle (**??**.iii), a "tiered" Qtree for X^+ is then built by "stacking" the *Paris* and *France* partitions, as done in Figure C3. Of course, principle (**??**.iii) may generate more than one "tiered" Qtree, e.g., a Qtree with a continent tier on top of a country tier. We omit these extra Qtrees for simplicity.



(1) (2) (3)

Figure C: Qtrees for *SALT35 will take place in **Massachusetts***.

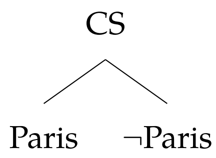
Constructing Qtrees for the LF X = *SALT35 will take place in France* follows a very similar line of reasoning. Following principle (**??**.i), a "polar" Qtree can be built out of X from the partition {*France*, ¬*France*}. This is done in Figure D1.

Following principle (**??**.ii), one must generate a Hamblin partition out of a set of same-granularity alternatives to X. We just saw that same-granularity alternatives to X form a set {*France*, *UK*, ...} containing countries (and intuitively coarser-grained locations that are not partitioned by countries); and that this set happens to be equal to its Hamblin partition. Applying principle (**??**.ii) using this Hamblin partition then generates the Qtree in Figure D2. Lastly, according to principle (**??**.iii), X can give rise to a chain of entailing propositions of the form *SALT35 will take place in France*, *SALT35 will take place in Europe*, etc. For simplicity, and to remain consistent with how we dealt with X^+=*SALT35 will take place in Paris*, we omit the tiered Qtrees generated from this kind of chain.

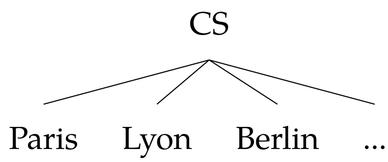
(1) (2)

Figure D: Qtrees for *SALT35 will take place in **the United States***.

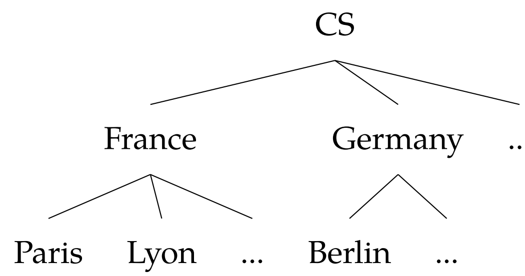
We can now turn to the scalar case, with Y^+=*Jo read all of the books*, and Y=*Jo read some of the books*.



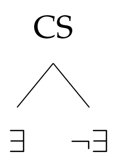
(1) If $\mathcal{A}_{Paris} = \{Paris\}$



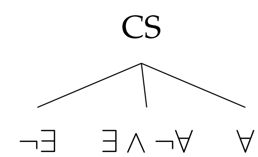
(2) If $\mathcal{A}_{Paris} = \{Paris, Lyon, Berlin, ...\}$



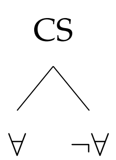
(3) If $\mathcal{A}_{Paris}^{Paris} = \{Paris\}$ and $\mathcal{A}_{Paris}^{France} = \{France, Germany, ...\}$



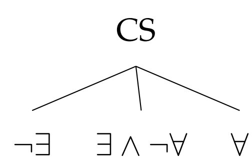
(1) If $\mathcal{A}_{some} = \{\exists\}$



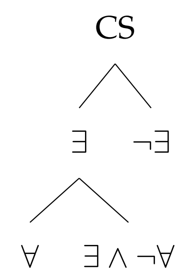
(2) If $\mathcal{A}_{some} = \{\exists, \forall\}$



(1) If $\mathcal{A}_{all} = \{\forall\}$



(2) If $\mathcal{A}_{all} = \{\exists, \forall\}$



(3) If $\mathcal{A}_{all}^{all} = \{\exists, \forall\}$ or $\{\forall\}$ and $\mathcal{A}_{all}^{some} = \{\exists\}$

Intuitively, this renders the intuition that non-scalemates like *Paris* and *France* will answer different kinds of questions – finer-grained *which city?* question (251), vs. coarser-grained *which country?* question (252) – while scalemates *may* answer similar questions – *e.g. how much/many?* (253).

In (251), a hedges like *all I know is that...* allows to shift the question and be less informative than originally expected (251c). Being more informative a

(251) In which city does Jo study?
  a. SALT35 will take place in Paris.
  b. # SALT35 will take place in France.
  c. All I know is that SALT35 will take place in France.

(252) In which country does Jo study?

a. SALT35 will take place in France.

b. ?? SALT35 will take place in Paris.

c. # All I know is that SALT35 will take place in Paris.

(253) How many students passed the class?

a. All passed.

b. Some passed.

c. All I know is that some passed.

One might argue that *Paris* and *France* may in fact answer the same, more general question: *where?*. We think this kind of question can be coerced by the answerer into a more specific question (e.g. *which city?*), depending on how informed they are. That kind of coercion does not seem to be needed in the case of *how much/many?* questions answered by *some* or *all*.

According to this definition, *Jo read all of the books* gets paired with a "polar" Qtree corresponding to whether or not she read all the books (see Fig. H); and a "wh" Qtree corresponding to whether she read none, only some, or all of the books (generated by $\mathrm{Alt}(\exists) = \{\exists, \forall\}$, see Fig. I). Same can be done for *Jo read some of the books*, except the "polar" Qtree is different (see Fig. J). For *SALT35 will take place in Paris* (resp. *France*), *wh*-Qtrees are generated by city (resp. country) alternatives, see Fig. C and K. Verifying nodes are boxed.

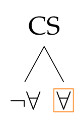


Figure H: "Polar" Qtree for *Jo read **all** of the books*

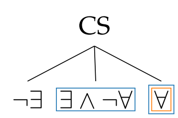


Figure I: "Wh" Qtree for *Jo read {**some**/**all**} of the books.*

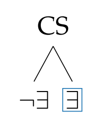


Figure J: "Polar" Qtree for *Jo read **some** of the books*

Figure K: Qtrees for *SALT35 will take place in **the United States**.*

7.2.2 Getting compositional

Just like the meanings of simple sentences are incrementally composed, their sets of candidate Qtrees get incrementally combined. The Qtrees compatible with a negated LF $\neg X$, are Qtrees for X in which the set of compatible nodes is "flipped" on a layer-by-layer basis. *Jo did not read all of the books* is thus linked to the Qtrees in Fig. L and *Jo didn't study in Paris* to those in Fig. M. The Qtrees compatible a disjunctive LF $X \vee Y$, are all the Qtrees that result from the union of a tree for X, and a tree for Y. The union operation – understood as union over sets of nodes, sets of edges, and sets of verifying nodes – ensures that the Qtree of a disjunction addresses the QuDs evoked by *both* disjuncts in parallel (Simons (2001); Zhang (pear)). *Jo read some or all of the books* is therefore only compatible with Tree I because all the other unions obtained from of Trees H, J and I fail to generate proper Qtrees. The HDs (233) are compatible with no Qtree, because the Qtrees for *Paris* and those for *France* always subdivide the CS differently.[8] The Qtrees compatible with a conditional LF $X \rightarrow Y$ are Qtrees for X, where each verifying node is replaced by its intersection with a Qtree for Y. Verifying nodes are inherited from the consequent Qtree (in line with the observations in (**??**)). (255a) is then compatible with the Tree in Fig. N; (255b), with Fig. O, (254a) with Fig. P and (254b) with Fig. Q. We proceed to show that both trees associated with (254b) violate some notion of relevance; while no trees associated with (255a), (255b), and (254a) do. Roughly, the issue is that none of the trees evoked by (254b) fully preserve the answer conveyed by its consequent (the *France*-node); while those evoked by (255a), (255b) and (254a) do.

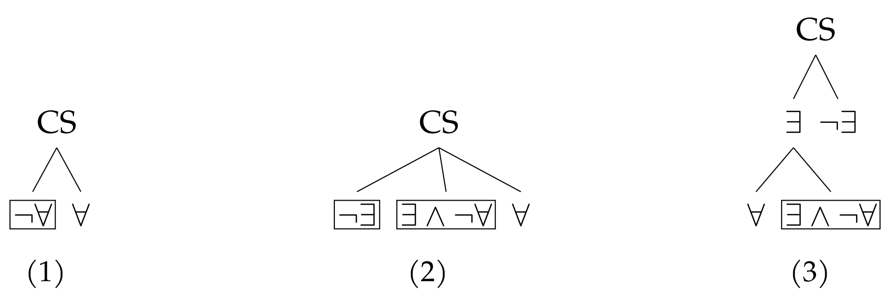


Figure L: Qtrees for*Jo did**n't** read* ***all*** *of the books*, derived from Fig. H&I

[8]Hénot-Mortier (peara,p) predict that (233a-**??**) do create proper Qtrees, but that such Qtrees (paired with their LFs) are Q-Redundant.

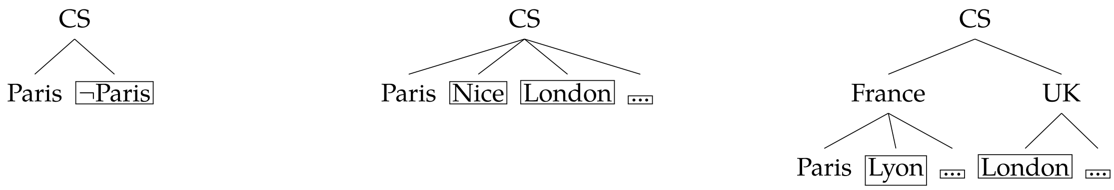


Figure M: Qtrees for *Jo* ***didn't*** *study in* ***Massachusetts***, derived from Fig. C.

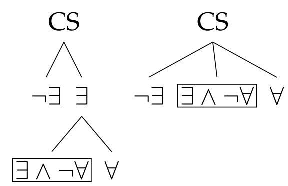


Figure N: Qtrees compatible with (255a) derived from Fig. J&L1/L2

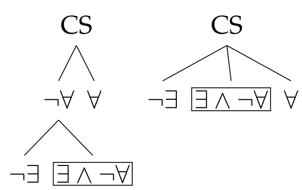


Figure O: Qtrees compatible with (255b) derived from Fig. L1&I

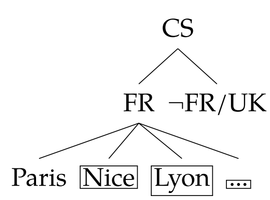


Figure P: Qtree for (254a), derived from Fig. K&M.

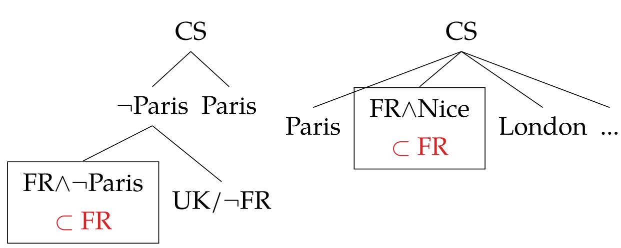


Figure Q: Qtree for (254b), derived from Fig. M&K.

7.3 The case of scalar Hurford Conditionals

Hurford Conditionals (HCs) involving scalemates appear felicitous, despite the fact that *exh* is not predicted to rescue such structures from redundancy constraints previously introduced in the literature. We show that Q-Relevance, as introduced in Chapter 6, can explain this pattern, modulo the intuitive assumption that scalar items can evoke fine-grained enough questions (generated by their scalemates) out-of-the-blue, while non-scalar items conveying different degrees of granularity cannot.

In Chapter 6 we investigated Hurford Conditionals such as # *If Jo did not study in Paris, she studied in France*. In this Chapter, we investigate Hurford Conditionals involving exhaustifiable scalemates

The problem

Does the pattern exhibited by scalar HDs in (235) extend to structures isomorphic to these HDs assuming material implication? Mandelkern and Romoli (2018) observed that an asymmetry arises in so-called Hurford Conditionals (henceforth **HCs**, see Chapter 6), when the antecedent and consequent are *not* natural scalemates, as in (254). Interestingly, we observe that the asymmetry *disappears*[9] in HCs involving scalemates, as shown in (255). We call such structures **scalar HCs**.

(254) a. If SALT35 will take place in the United States she did not study in Massachusetts. $\mathbf{p} \rightarrow \neg\mathbf{p}^{+}$

b. # If Jo did not study in Massachusetts she studied in the United States. $\neg\mathbf{p}^{+} \rightarrow \mathbf{p}$

(255) a. If Jo has read some of the books she hasn't read all. $\mathbf{s} \rightarrow \neg\mathbf{s}^{+}$

b. If Jo hasn't read all of the books she has read some. $\neg\mathbf{s}^{+} \rightarrow \mathbf{s}$

HDs and HCs therefore pattern differently, in both the scalar and the non-scalar case. Kalomoiros (2024) proposed a constraint called Super Redundancy accounting for (254), that we introduced in Chapter 6 and repeat here in (256).

(256) Super Redundancy. A sentence S is infelicitous if it contains a subconstituent C combining with a binary operator, such that $(S)^{-}_{C}$ is defined and for all D, $(S)^{-}_{C} \equiv S_{Str(C,D)}$. In this definition, $(S)^{-}_{C}$ designates S where C got deleted.

[9] Some speakers I consulted reported that (255a) was hard to make sense of in English (it is fine in my French). We discuss this *caveat* towards the end of this Chapter.

$Str(C,D)$ refers to a strengthening of C with D, which commutes with negation ($Str(\neg\alpha,D) = \neg(Str(\alpha,D))$) and with binary operators ($Str(O(\alpha,\beta),D) = O(Str(\alpha,D),Str(\beta,D))$). $S_{Str(C,D)}$ designates S where C is replaced by $Str(C,D)$.

Let us briefly summarize how (254) is captured by Super Redundancy. (254b) is Super Redundant (abbreviated **SR**), because any local strengthening of its antecedent (*not Paris*) yields a conditional expression equivalent to its consequent (*France*). This is proved in (257a). (254a) on the other hand, is not SR: its antecedent (resp. consequent), can be strenghtened in such a way that the entire conditional becomes logically non-equivalent to its consequent (resp. antecedent). This is shown in (257b). SR can also cover the HDs in (233), and, together with IW, the scalar HDs in (235).

(257) a. (254b) is SR.
$C = \neg\, \mathbf{p}^{+}.\ \forall D.\ \neg(\mathbf{p}^{+} \wedge D) \rightarrow \mathbf{p} \equiv (\mathbf{p}^{+} \wedge D) \vee \mathbf{p} \equiv \mathbf{p}$

b. (254a) is not SR.
$C = \neg\, \mathbf{p}^{+}$. Take $D = \top$. $\mathbf{p} \rightarrow \neg(\mathbf{p}^{+} \wedge D) \equiv \mathbf{p} \rightarrow \neg(\mathbf{p}^{+} \wedge \top) \equiv \mathbf{p} \rightarrow \neg\mathbf{p}^{+} \not\equiv \mathbf{p}$
$C = \mathbf{p}$. Take $D = \bot$. $(\mathbf{p} \wedge D) \rightarrow \neg\mathbf{p}^{+} \equiv (\mathbf{p} \wedge \bot) \rightarrow \neg\mathbf{p}^{+} \equiv \bot \rightarrow \neg\mathbf{p}^{+} \equiv \top \not\equiv \neg\mathbf{p}^{+}$

What about (255a) vs. (255b)? (258) shows that that *exh* is IW in the antecedent and the consequent of (255a), whether the conditional is seen as material or as strict. (255a) is therefore isomorphic to (254a), and so is correctly predicted to be non-SR, like (254a).

(258) a. *exh* is IW in the antecedent of (255a); material case.
$\forall\Gamma.\ \text{exh}(\mathbf{s}) \rightarrow \Gamma \equiv \neg(\mathbf{s} \wedge \neg\mathbf{s}^{+}) \vee \Gamma \equiv \neg\mathbf{s} \vee \mathbf{s}^{+} \vee \Gamma \Dashv \neg\mathbf{s} \vee \Gamma \equiv \mathbf{s} \rightarrow \Gamma$

b. *exh* is IW in the antecedent of (255a); non-material case.
$\forall\Gamma.\ \forall w : \text{exh}(\mathbf{s})(w).\ \Gamma \equiv \forall w : \mathbf{s}(w) \wedge \neg\mathbf{s}^{+}(w).\ \Gamma \Dashv \forall w : \mathbf{s}(w).\ \Gamma \equiv \mathbf{s} \rightarrow \Gamma$

c. *exh* is IW in the consequent of (255a); material case.
$\forall\Gamma.\ (\mathbf{s} \rightarrow \text{exh}(\neg\mathbf{s}^{+}))\ \Gamma \equiv (\neg\mathbf{s} \vee (\neg\mathbf{s}^{+} \wedge \mathbf{s}))\ \Gamma \equiv (\neg\mathbf{s} \vee \neg\mathbf{s}^{+})\ \Gamma \equiv (\mathbf{s} \rightarrow \neg\mathbf{s}^{+})\ \Gamma$

d. *exh* is IW in the consequent of (255a); non-material case.
$\forall\Gamma.\ \forall w : \mathbf{s}(w).\ \text{exh}(\neg\mathbf{s}^{+})(w) \equiv \forall w : \mathbf{s}(w).\ \neg\mathbf{s}^{+}(w) \wedge \mathbf{s}(w) \Dashv \forall w : \mathbf{s}(w).\ \mathbf{s}^{+}(w) \equiv \mathbf{s} \rightarrow \mathbf{s}^{+}$

(259) shows that this reasoning incorrectly extends to (255b): *exh* is IW in both the antecedent and the consequent of (255b), so SR incorrectly predicts (255b) to pattern like (254b), i.e. to be infelicitous.

(259) a. *exh* is IW in the consequent of (255b); material case.
$\forall\Gamma.\ (\neg\mathbf{s}^{+} \rightarrow \text{exh}(\mathbf{s}))\ \Gamma \equiv (\mathbf{s}^{+} \vee (\mathbf{s} \wedge \neg\mathbf{s}^{+}))\ \Gamma \equiv (\mathbf{s}^{+} \vee \mathbf{s})\ \Gamma \equiv (\neg\mathbf{s}^{+} \rightarrow \mathbf{s})\ \Gamma$

b. *exh* is IW in the consequent of (255b); non-material case.
$\forall\Gamma.\ \forall w : \neg\mathbf{s}^+(w).\ \text{exh}(\mathbf{s})(w) \equiv \forall w : \neg\mathbf{s}^+(w).\ \mathbf{s}(w) \wedge \neg\mathbf{s}^+(w) \equiv \forall w : \neg\mathbf{s}^+(w).\ \mathbf{s}(w) \equiv \neg\mathbf{s}^+ \rightarrow \mathbf{s}$

c. *exh* is IW in the antecedent of (255b); material case.
$\forall\Gamma.\ (\text{exh}(\neg\mathbf{s}^+) \rightarrow \mathbf{s})\ \Gamma \equiv (\neg(\neg\mathbf{s}^+ \wedge \mathbf{s}) \vee \mathbf{s})\ \Gamma \equiv (\mathbf{s}^+ \vee \neg\mathbf{s} \vee \mathbf{s})\ \Gamma \dashv (\neg\mathbf{s}^+ \rightarrow \mathbf{s})\ \Gamma$

d. *exh* is IW in the antecedent of (255b); non-material case.
$\forall\Gamma.\ \forall w : \text{exh}(\neg\mathbf{s}^+)(w).\ \mathbf{s}(w) \equiv \forall w : \neg\mathbf{s}^+(w) \wedge \mathbf{s}(w).\ \mathbf{s}(w) \equiv \top \dashv \mathbf{s}^+ \rightarrow \mathbf{s}$

Exploring a potential solution

At this point, one might want to revise IW, or SR. If SR is maintained and IW is assumed to be inactive in conditionals, then both HCs in (255) would be correctly predicted to be felicitous, due to *exh* being licensed in the consequent of (255b). This is shown in (260).

(260) (255b) with *exh* in the consequent is not SR.
C = $\neg\ \mathbf{s}^+$. Take D = $\top$. $\neg(\mathbf{s}^+ \wedge D) \rightarrow \text{exh}(\mathbf{s}) \equiv \mathbf{s}^+ \vee (\mathbf{s} \wedge \neg\mathbf{s}^+) \equiv \mathbf{s} \not\equiv \text{exh}(\mathbf{s})$
C = exh($\mathbf{s}$). Take D = $\bot$. $\neg\mathbf{s}^+ \rightarrow (\text{exh}(\mathbf{s}) \wedge D) \equiv \neg\mathbf{s}^+ \rightarrow \bot \equiv \mathbf{s}^+ \not\equiv \neg\mathbf{s}^+$

A possible argument against this view comes from a "Long-Distance", non-scalar variant of (255b). At the end of Chapter 6, we discussed two kinds of LDHDs derived from HDs by further disjoining the stronger disjunct with a proposition incompatible with the weaker one; and we observed that the infelicity of such LDHDs persists once the outer disjunction is changed into a conditional *via* the *or-to-if* tautology, as shown in (261).

This is problematic for the hypothesis that SR is the only constraint at stake in conditionals: under this view, and because *exh* is inactive in the sentences in (261), SR would be expected to rule them out beyond repair. But, if SR correctly rules out (261a), it incorrectly rules in (261b).

(261) a. # If Jo did not study in Europe, she studied in France or in New York.
$\neg\mathbf{p} \rightarrow (\mathbf{p}^+ \vee \mathbf{r})$

b. # If SALT35 will take place in the United States, she did not study in Europe or she studied in Paris.
$\neg\mathbf{q} \rightarrow (\mathbf{q}^+ \vee \mathbf{r})$ with: $\mathbf{q} := \neg\mathbf{p}^+$; $\mathbf{q}^+ := \neg\mathbf{p}$

This suggests that our original problem in the scalar HC (255b) cannot be easily alleviated by maintaining SR and relaxing IW, since in environments where IW plays no role, such as in (**??**) and (261b), SR alone makes unexpected predictions.

To capture the scalar HCs in (255) and (261) while retaining the right predictions for the HDs in (233), (235) and the non-scalar HC in (254), we thus suggest to maintain IW, and propose an alternative to SR based on three ideas:

- Questions under Discussion (**QuD**, **?**Roberts (1996)) are compositionally accommodated when processing out-of-the-blue declaratives (see previous chapters);
- QuD computation is constrained by Q-Relevance (see Chapter 6);
- scalemates may answer same-granularity QuDs, while non-scalemates with different levels of granularity cannot (new claim).

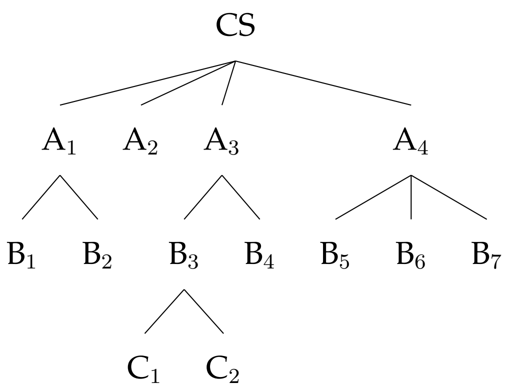
The scalar HCs in (255) can then escape a violation of Q-Relevance, because their consequent can evoke a question of the form *none, some but not all, or all?* that is fine-grained enough to "fit" a question introduced by their antecedent. In the non-scalar case, (254a) can do the same (*not Paris* evokes a proper subdivision of *France*), but crucially not (254b) (*France* cannot evoke a proper subdivision of *not Paris*).

7.4 Scalarity and accommodated QuDs

We use the two core ideas we entertained in the previous chapters of this thesis: that out-of-the-blue declaratives evoke the potential QuDs they may answer; and that the derivation of such implicit QuDs is compositional and (incrementally) constrained. In line with Katzir and Singh (2015)'s insights, we take that a sentence is odd if it is compatible with no reasonable implicit QuD. Chapter 6 already used this formalism to capture the non-scalar HD in (233) and the non-scalar HC in (254). We now focus on explaining the scalar HCs in (255). The core claim we introduce in this section in order to capture (255), is that scalemates *may* evkoe similar QuDs, while non-scalemates like *Paris* and *France* cannot. Chapter **??** will cover the case of scalar HDs (235) building on this assumption, while also presenting a possible alternative to IW.

7.4.1 Qtree recap

Let us briefly summarize the basis of the formalism presented in Chapter 6. Building on Büring (2003); Riester (2019); Onea (2016); Zhang (pear), we took QuDs to be trees (**Qtrees**), that have the Context Set (**CS**, Stalnaker (1974)) as their root, and are such that each intermediate node is a subset of the CS, partitioned by its children nodes. Thus, the set of leaves of a Qtree forms a partition of the CS, and correspond to the standard denotation of questions (Hamblin, 1958,Groenendijk, 1999). Any subtree rooted in N can be seen as a conditional question, granted N. A proposition answers a Qtree if it can be identified with the union of a strict subset of the Qtree's nodes.



The Tree on the left is a Qtree iff...

- $\{A_1, A_2, A_3, A_4\}$ partitions CS;
- $\{B_1, B_2\}$ partitions A_1;
- $\{B_3, B_4\}$ partitions A_3;
- $\{B_5, B_6, B_7\}$ partitions A_4;
- $\{C_1, C_2\}$ partitions B_3.

It follows from this that...

- $\{B_1, B_2, A_2, C_1, C_2, B_4, B_5, B_6, B_7\}$ (leaves) partitions CS;
- $\{B_1, B_2, A_2, B_3, B_4, B_4, B_5, B_6, B_7\}$ partitions CS (because $\{C_1, C_2\}$ partitions B_3);
- $\{B_1, B_2, A_2, C_1, C_2, B_4, A_4\}$ partitions CS (because $\{B_5, B_6, B_7\}$ partitions A_4);
- etc.

Figure R: Illustration of some Qtree properties.

Building on Katzir and Singh (2015); Hénot-Mortier (peara,p), we take that any out-of-the-blue declarative sentence denoting a proposition p gets paired with the set of salient Qtrees p may answer. Such Qtrees additionally carry information about how p answers them, in the form of specific nodes entailing p (**verifying nodes**). We refer to the structure formed by Qtrees, along with their verifying nodes, as "flagged Qtrees" (or sometimes just Qtrees). The pairing between LF and flagged Qtrees is compositional, meaning, the flagged Qtrees evoked by a complex LF, are derived from the flagged Qtrees derived from its parts, and from how these parts combine.

7.5 Capturing scalar HCs via Q-Relevance

Chapter 6 defined Q-Relevance as a constraint on Qtree computation: when combining Qtrees incrementally, none of the verifying nodes of the input Qtree should be cut across (i.e. be strictly entailed by some node) in the output Qtree. The constraint is repeated in (**??**).

(**??**) Q-RELEVANCE. Let X and Y be LFs and let $Qtrees(X)$ and $Qtrees(Y)$ be the sets of Qtrees compatible with X and Y. Let $\circ$ be a Qtree-level operation, e.g. $\neg$, $\vee$, or $\rightarrow$. Let C be a non-empty partial LF (incremental context). Two cases:

- $C = \circ$, with $\circ$ a unary operation. For any $T \in Qtrees(X)$, $\circ T$ is Q-RELEVANT with respect to $\circ X$ iff $\forall N \in \mathbb{N}^+(T).\ \neg\exists N' \in \mathbb{N}(\circ T).\ N' \subset N$.
- $C = X\circ$, with $\circ$ a binary operation. For any $T \in Qtrees(Y)$, $T_X \circ T_Y$ is Q-RELEVANT with respect to $X \circ Y$ iff $\forall N \in \mathbb{N}^+(T_Y).\ \neg\exists N' \in \mathbb{N}(T_x \circ T_Y).\ N' \subset N$.

(262) Q-RELEVANCE (*applied to conditionals*). Let X and Y be LFs and let $Qtrees(X)$ and $Qtrees(Y)$ be the sets of Qtrees compatible with X and Y. For any $T \in Qtrees(Y)$, $T_X \rightarrow T_Y$ is Q-RELEVANT with respect to $X \rightarrow Y$ iff $\forall N \in \mathbb{N}^+(T_Y).\ \neg\exists N' \in \mathbb{N}(T_x \rightarrow T_Y).\ N' \subset N$.

This allowed to account for the contrast in (254). Let us briefly summarize the argument. (254a) corresponds to the Qtree in Fig. P, which is obtained from a country-level antecedent Qtree and a city-level consequent Qtree; therefore, all verifying leaves of the consequent (city nodes different from Paris) are contained in some leaf of the antecedent Qtree, and can thus "fit" into the output Qtree without being cut across. Q-RELEVANCE is thus satisfied. (254b) corresponds to the Qtrees in Fig. Q, which are obtained from a city-level antecedent Qtree and a country-level consequent Qtree; in such trees, the *France* verifying leaves are always cut across, either by *not Paris*, or by individual city-nodes different from *Paris*. Q-RELEVANCE is thus violated.

The same kind of reasoning shows that the Qtrees corresponding to (255a) and (255b), in resp. Fig. N and O, verify Q-RELEVANCE. Starting with Qtree N: it can be built by incrementally combining Qtree J (antecedent Qtree), with Qtree L2 (consequent Qtree). Qtree L2 has $\neg\exists$ and $\exists \wedge \neg\forall$ as verifying nodes; in the output Qtree N, both nodes are fully preserved. So (255a) is compatible with a Qtree and is thus felicitous. As for (255b), its Qtree O can be built by incrementally combining Qtree H (antecedent Qtree), with Qtree L2 (consequent Qtree). Qtree L2 has $\neg\exists$ and $\exists \wedge \neg\forall$ as verifying nodes; in the output Qtree N, both nodes are fully preserved. So (255b) is compatible with a Qtree and is thus felicitous. In brief, (254b) and (254a) are both rescued by the fact their consequent can evoke a Qtree whose verifying nodes are fine-grained enough to properly "fit" the structure already introduced by the antecedent Qtree.

7.5.1 Interim conclusion

We proposed an account of (scalar) HCs exploiting the intuitive idea that conditionals evoke "restricted" questions whose composition is constrained by the new notion of relevance presented back in Chapter 6, Q-Relevance. The contrast between scalar and non-scalar HCs was thus captured, not *via exh per se*, but instead by appealing to how scalar vs. non-scalar pairs of items differ information-structurally. Specifically, it was assumed scalar items could evoke fine-grained enough questions (generated by their scalemates) out-of-the-blue, while non-scalar items with different granularities could not.

Before moving on to more complex cases in which scalarity and Q-Relevance also appear relevant(!), let us discuss the felicity profile of the scalar HCs in (255), repeated below.

(255) a. If Jo has read some of the books she hasn't read all. $\mathbf{s} \rightarrow \neg\mathbf{s}^{+}$

b. If Jo hasn't read all of the books she has read some. $\neg\mathbf{s}^{+} \rightarrow \mathbf{s}$

In consulting with various speakers, judgments for (255a) and (255b) varied quite a bit. In particular, some speakers reported that (255a) was hard to make sense of. This potential infelicity appears problematic for all accounts of Hurford Sentences – in particular the current account, and Kalomoiros (2024)'s SR. Here is however the sketch of a solution within the current framework. Recall that Q-Relevance imposes that some QuD evoked by the consequent of a conditional "fit" the information structure already introduced by the antecedent. One noticeable difference between (255a) and (255b), is that (255a), unlike (255b), features a *negated* scalemate within its consequent. So far, our model of accommodated QuDs was assumed to handle negation quite transparently; specifically, we made the assumption that negation preserves Qtree structure, and only affects verifying nodes. But this might be too simplistic, and does not account for the intuition that negated expressions (e.g. *not all*) may more saliently evoke "polar" QuDs (e.g. $\forall/\neg\forall$) as opposed to other QuDs (e.g. $\forall/\exists \wedge \neg\forall/\neg\exists$). If this is the case, then *not all* in (255a) may be less likely to evoke the kind of tripartite Qtree that rescued both scalar HCs in (255). When combined with an antecedent QuD for *some*, the polar QuD evoked by *not all* then ends up violating Q-Relevance. The subtleness of the subsequent infelicity may be explained by the fact that negated expression *preferentially* (but not always) evoke polar Qtrees.

This observation can be related to informativity: uttering $\neg p$ when the question is *whether p?*, is maximally informative, because it identifies one single cell – the $\neg p$-cell. Uttering $\neg p$ when the questions is e.g. *p, q, or r?*, is underinformative, because it does *not* identify a single cell. To account for this, one might want to say that Qtrees ar ranked according to how well they are addressed by the assertion evoking them – Qtree with smaller sets of verifying nodes should be preferred.

The last section of the Chapter focuses on extensions of the current account, and in particular, explores predictions of Q-Relevance together with the intuition that scalemates may answer the same QuD.

also talk about negated HDs... Jo did not read all of the book or she did not read some of them <=>Jo read some but not all or she read none ==> should be ok but is not

The case of Long-Distance scalar talk plans to dive into *Context: Cafeteria Xor's meal plan is all you can eat starter XOR main dish XOR desserts.*

(263) If Jo didn't have all starters or the main dish then she had some starters. $\neg(\mathbf{s}^{+}\vee\mathbf{r})\rightarrow\mathbf{s}$

(264) ?If Jo had some starters then she didn't have all starters or the main dish. $\mathbf{s}\rightarrow\neg(\mathbf{s}^{+}\vee\mathbf{r})$

if not all of the S or the main dish then some of the S fine if some of the S then not all of the S or the main dish sounds trivial but fine ==> exh vacuous there (at least under material implication... just use commutativity and the fact exh is vacuous under neg) ==> should pattern like 11 and 12... not the case! ==> kalomoiros predicts them correctly to be fine

(1) m has read some of the books if not all of them fine (2) m has not read all of the books, if she has *(even) read some of them badish should be fine if no exh

what do linear fs say (1) can be parsed as m has read sbna of the books if not all of them if not all then sbna not super redundant

(2) must be parsed as if some then not all analog to if france then not paris should be good

what do hierarchical fs say (1) can be parsed as m has read some of the books if not all of them if not all then some not super redundant

(2) must be parsed as if some then not all analog to if france then not paris should be good

m did not study in paris, if she studied in france fine m studied in france, if she did not study in p bad ==> with non scalar hc reversal did not affect judgment

(265) Jo did not study in Paris, if she studied in France.

(266) SALT35 will take place in France, if she did not study in Paris. still bad

(267) ?Jo has not read all of the books, if she has read some.

(268) Jo has read some of the books, if she has not read all.

5 - not paris then france (not paris or not D) then france (paris and D) or france === france 7, no exh - if not all then some (not all or not d) then some (all and d) or some === some 7, with exh - if not all then sbna (not all or not d) then sbna (all and d) or sbna =/= sbna ==> having exh makes 7 not super redundant

what about 6 with exh? some then (not all and some) (some and D) then (not all and some) not some or not D or (sbna) =/= sbna

(some) then (sbna and D) not some or (sbna and D) =?= not some => having exh makes 6 not super redundant too!

if we buy super redundancy, then we have to say something about exh-licensing 6 == not p or not p+ not p or (not p+ and p) not p or not p+ and not p or p not p or not p+ ==> exh vacuous 7 == p+ or p p+ or (p and not p+) (p+ or p) and (p+ or not p+) p+ or p ==> exh vacuous

All I have to do is update exh-licensing to make it ok in conditionals

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