“One tool to rule them all”? An integrated model of the QuD for Hurford sentences[1]

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Abstract. A recent line of research (Katzir and Singh (2015) a.o.) develops the idea that felicitous sentences should be possible answers to a “good” Question under Discussion (QuD, Roberts (1996); Van Kuppevelt (1995)).Yet, it remains unclear whether a QuD model is *needed* as an additional explanatory tool for pragmatics, partly because the formalization of QuD composition at the subsentential level remains understudied. In this paper, we develop a compositional machinery linking assertions to the implicit questions they evoke, and show that relocating a number of pragmatic principles previously associated to assertions, in the domain of their implicit questions, allows to solve puzzles pertaining to Hurford Disjunctions and variants thereof, in an intuitive way.

Keywords: redundancy, relevance, question under discussion

1. Introduction

Hurford Disjunctions (henceforth, HD, Hurford (1974)), such as (1a-1b), are disjunctions which typically feature entailing disjuncts. Such constructions, at least when they do not involve scalemates such as *some* and *all*, appear redundant regardless of the order of the weak (p) vs. strong (p^+) disjunct.

(1) a. # SuB29 will take place in Noto[2] or Italy. $p^+ \vee p$

b. # SuB29 will take place in Italy or Noto. $p \vee p^+$

Such constructions have been a long-standing puzzle for pragmatic theory, because it appears difficult to devise a single principle accounting for them, as well as all their variants (Marty and Romoli, 2022). Hurford Conditionals (henceforth HC, (Mandelkern and Romoli, 2018)) like (2a-2b) for instance, exhibit an asymmetry that is challenging for existing accounts of Hurford Sentences, due to the fact that (2a-2b) can be directly derived from (1a) *via* the *or-to-if* tautology and basic principles of classical logic (cf. 3).

(2) a. # If SuB29 will not take place in Noto, it will take place in Italy. $\neg p^+ \rightarrow p$

b. If SuB29 will take place in Italy, it will not take place in Noto. $p \rightarrow \neg p^+$

(3) *Equivalence between HDs and HCs*

a. (2a) $\equiv \neg p^+ \rightarrow p \overset{\clubsuit}{\equiv} \neg(\neg p^+) \vee p \overset{\spadesuit}{\equiv} p^+ \vee p \equiv$ (1a)

b. (2b) $\equiv p \rightarrow \neg p^+ \overset{\clubsuit}{\equiv} (\neg p) \vee (\neg p^+) \overset{\heartsuit}{\equiv} q^+ \vee q \equiv$ (1a)

$\clubsuit$: *or-to-if* tautology; $\spadesuit$: double-negation elimination; $\heartsuit$: variable change of the form $\neg p := q^+; \neg p^+ := q$, with $q^+ \Rightarrow q$.

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[2]Noto is located in Italy and is where the main session of SuB29 was actually organized.

An existing account of the above contrast (Kalomoiros, 2024) builds on the idea that overt negation has a special status when it comes to evaluating redundancy. In this paper, I want to argue for an alternative view that is perhaps more in line with basic intuitions on might have about HCs and HDs – namely the fact that disjunctions and conditionals package information differently, in terms of the potential questions they evoke, and as such are not equally sensitive to the "granularity" of their arguments.

2. Previous approaches

In this section I briefly present three existing accounts of Hurford Sentences: Local Redundancy Checking, Local Contexts, Super-Redundancy. I show how the first two fall short in explaining HCs, even if the conditional is understood as non-material. I then show how the last account captures the contrast between HDs and HCs.

2.1. Local Redundancy Checking

Katzir and Singh (2014) propose that the semantic computation evaluates, at certain nodes, whether the semantic composition principle that applies there is non-vacuous. This gives rise to the principle in (4).

(4) *Local Redundancy Checking.* S is deviant if S contains γ s.t. $[\![\gamma]\!] = [\![O(\alpha,\beta)]\!] \equiv_c [\![\zeta]\!]$, $\zeta \in \{\alpha,\beta\}$.

This predicts the HDs (1a-1b) to be deviant, because both are contextually equivalent to one of their disjuncts. But, assuming conditionals denote material implications, this also predicts *both* HCs (2a-2b) to be deviant. In those constructions, the only candidate for γ is the whole conditional, with arguments $\neg p^+$ and p. (2a) is contextually equivalent to p, and thus equivalent to its consequent; (2a) is contextually equivalent to $\neg p^+$, and thus equivalent to its consequent, as well. The issue persists if we adopt a non-material analysis of conditionals, because, in such cases, the whole conditional will never be contextually equivalent to its antecedent or consequent, regardless of what they denote. In other words, both HCs (2a-2b) would be predicted to be non-deviant.

2.2. Logical Integrity

Anvari (2018) proposed a principle forcing the logical relation between a sentence and its non-weaker alternatives to be preserved once contextual information is considered.

(5) *Logical Integrity.* Let S be a sentence and S' be one of its alternatives. S is infelicitous in a context c if S does not logically entail S', but S contextually entails S' in c.

This predicts the HDs (1a-1b) to be deviant, because both have their weak disjunct as alternative, do not logically entail it, but do so once the contextual information that Noto is in Italy is considered. But, assuming conditionals denote material implications, this also predicts *both* HCs (2a-2b) to be deviant. (2a) has its consequent (p) as alternative, does not logically entail it, but does so once the contextual information that Noto is in Italy is considered. (2b) has its consequent ($\neg p^+$) as alternative, does not logically entail it, but does so once the contextual information that Noto is in Italy (and thus, that not being in Italy means not being in Noto) is considered. The issue persists if we adopt a non-material analysis of conditionals, because, in such cases, it is unclear if the conditional even logically entails one of its alternatives, end even so, no contrast would be predicted between the two HCs.

2.3. Non-triviality

Another line of work, building on the notion of local contexts (Schlenker, 2009), associates redundancy with triviality (Stalnaker, 1999): a sentence should not contain a part that is trivially true or false when evaluated against its local context (Mayr and Romoli, 2016).

(6) *Non-triviality.* A sentence S cannot be used in a context c if some part π of S is entailed or contradicted by the local context of π in c.

Assuming disjunctive local contexts are symmetric, this predicts the HDs (1a-1b) to both be deviant, because both have their strong disjunct be trivially false when interpreted in the context of the negation of their weaker disjunct. But, assuming conditionals denote material implications, this also predicts *both* HCs (2a-2b) to be fine. In (2a), the consequent p, is informative in the context created by the antecedent ($\neg p^+$): not being in Noto neither entails nor contradicts being in Italy. In (2a), the consequent $\neg p^+$, is also informative in the context created by the antecedent (p): being in Italy neither entails nor contradicts not being in Noto.

nonmaterial conditionals TODO

2.4. Super-Redundancy

Kalomoiros (2024) proposes an adaptation of the REDUNDANCY view, based on the novel notion of SUPER-REDUNDANCY. Roughly, a sentence is super-redundant if there is no way of strengthening one of its subconstituents that that would make the resulting sentence non-redundant.

(7) *Super-redundancy.* A sentence S is infelicitous if it contains a subconstituent C s.t. $(S)^-_C$ is defined and for all D, $(S)^-_C \equiv S_{Str(C,D)}$.

Roughly, $(S)^-_C$ in the above definition designates S where C got deleted, while $Str(C,D)$ refers to a strengthening of C with D, defined inductively and whose key property is that it commutes with negation: $Str(\neg\alpha, D) = \neg(Str(\alpha, D))$ – as well as with binary operators $Str(O(\alpha,\beta),D) = O(Str(\alpha,D),Str(\beta,D))$. $S_{Str(C,D)}$ designates S where C is replaced by $Str(C,D)$

This predicts the HDs (1a-1b) to be deviant, because, given $C = p^+$, no matter what D is, $S_{Str(C,D)} = (p^+ \wedge D) \vee p$ is equivalent to $p = (S)^-_C$. And this predicts the HC (2a) to be deviant as well: given $C = \neg p^+$, no matter what D is, $S_{Str(C,D)} = \neg(p^+ \wedge D) \rightarrow p = (p^+ \wedge D) \vee p$ is equivalent to $p = (S)^-_C$. In that case, it was crucial that the local strengthening of $C = p^+$ remained conjunctive under negation. (2b) on the other hand, is predicted to be fine: given $C = \neg p^+$, and setting D to $\top$ we have, $S_{Str(C,D)} = p \rightarrow \neg(p^+ \wedge \top) \equiv \neg p \vee \neg p^+ \vee \bot$ which is *not* equivalent to $p = (S)^-_C$. Given $C = p$, and setting D to $\top$ we have, $S_{Str(C,D)} = (p \wedge \top) \rightarrow \neg p^+ \equiv \neg p \vee \bot \vee \neg p^+$ which is *not* equivalent to $\neg p^+ = (S)^-_C$. In both cases, it was again crucial that the local strengthening of $C = p^+$ was conjunctive under negation (and thus, disjunctive after applying De Morgan’s law). Kalomoiros (2024) also shows that this account extends to strict (yet not variably strict) conditionals.

This account is compelling and allows to only introduce minimal changes to the usual machinery. However, while traditional view on REDUNDANCY link it to pragmatic principles such as brevity, it remains unclear under the super-redundancy view, where the notion of local strengthening (Str) stems from, why it is defined the way it is, and why it should be so central in deriving redundancy. Also, the account was originally motivated by the observation that negated HDs, like (8) appear felicitous.

(8) John either doesn't smoke or he doesn't smoke Marlboros.

While we agree with the above judgment, we think something else might be at stake in (8), given that it is improved by focus (9a), but made worse by removing *either* (9b), or by swapping the negated disjuncts (9c). The last two cases can be repaired by adding *at all* to the stronger disjunct.

(9) a. John doesn't smoke or doesn't smoke MARLBOROS.
b. John doesn't smoke $^{?}$(at all) or doesn't smoke Marlboros.
c. John either doesn't smoke Marlboros or he doesn't smoke $^{\#}$(at all).

The same patterns can be amplified if the two disjuncts are made more parallel (i.e., instead of having V and $V+NP$, we have $V+NP$ and $V+NP^{+}$, with $[\![NP^{+}]\!] \subset [\![NP]\!]$). This is done in (10) whose variants (analog to (9a-9c)) are given in (11).

(10) ? John either doesn't own a dog or he doesn't own a lab.

(11) a. John doesn't own a dog or doesn't own a LAB.
b. John doesn't own a dog $^{??}$(at all) or doesn't own a lab.
c. John either doesn't own a lab or he doesn't own a dog $^{\#}$(at all).

This suggests that some pragmatic mechanism is at play in the weaker disjunct and makes it contradict the stronger one. In particular, *John does not smoke MARLBOROS* seems to imply John smokes cigarettes of a brand different from Marlboros, i.e. smokes. While this does not alone explain the complex pattern of repairs in (9) and (11), this appears consistent with an analysis of HDs which would not assign a special status to overt negation, but instead interacts with pragmatic principles which themselves, are influenced by negation.

3. Linking assertions to questions

To explain the contrast between HDs and HCs, I propose a compositional machinery linking Logical Forms of assertive sentences to the implicit questions they may raise. One sentence might be associated to multiple potential questions. This kind of machinery is independently motivated by the fact that sentences are never uttered in and of themselves; their purpose is to answer a question, overt or not, and to induce further questions. A pragmatic model of assertion therefore needs to integrate what sentences mean, but also what kind of information *structure* they evoke. I will start by defining questions evoked by simplex LFs, containing no operator, quantifier or connective. Once this is done, I will extend the model inductively, by assigning a semantics to negation, disjunction, and implication, in terms of how they manipulate questions and create more complex ones.

3.1. Background assumptions on question semantics

Let us start by reviewing the standard view on questions. Questions are usually seen as the set of their potential answers Hamblin (1973), i.e. as partitions of the Context Set (henceforth CS, Stalnaker (1974)). This is formalized in (12).

(12) *Standard semantics for questions*
Given a Context Set S, i.e. a set of worlds compatible with the premises of the conversation, a question on S is a partition of S, i.e. a set of subsets of S ("cells") $\{c_1,...,c_k\}$ s.t.:
- "No empty cell": $\forall i \in [1;k].\ c_i \neq \emptyset$
- "Full cover": $\bigcup_{i\in[1;k]} c_i = S$
- "Pairwise disjointness": $\forall (i,j) \in [1;k]^2.\ i \neq j \Rightarrow c_i \cap c_j = \emptyset$

Given a Context Set S, and a set of propositions $P = \{p_1,...,p_l\}$ a partition of S can be induced by grouping together the worlds of S which agree on all the propositions of P. This is formalized in (13).

(13) *Partition induced by a set of propositions.*
Given a Context Set S and a set of propositions $\{p_1,...,p_l\}$, one can define:
- an equivalence relation $\equiv_P$ s.t. $\forall (w,w') \in S.\ w \equiv_P w' \Leftrightarrow \forall p \in P.\ p(w) = p(w')$
- a partition of S induced by P as the set of equivalence classes of $\equiv_P$ on S, i.e. the set $\{\{w' | w' \in S \wedge w \equiv_P w'\} | w \in S\}$.

We call PARTITION(S, P) the partition on S induced by P.

We can then define the questions evoked by a proposition p as the partitions evoked either by p alone, or by p and relevant alternatives to p. If p is not settled in the CS, the former partition takes the form $\{p, \neg p\}$ and amounts to the question of *whether p*. If the set $\mathscr{A}_p$ of relevant alternatives to p contains mutually exclusive propositions covering the CS, then the latter partition is simply $\mathscr{A}_p$ and amounts to a *wh*-question for which p is a felicitous answer.

3.2. Questions evoked by simplex LFs

Let us now go one step further and adapt this definition to a more elaborate model of questions, which will eventually reflect the intuition that logically equivalent sentences can "package" information differently. Building on (Büring, 2003; Riester, 2019; Onea, 2016; Zhang, 2024), we take questions to denote *parse trees* of the CS, i.e. ways to hierarchically organize the worlds that are compatible with the premises of the conversation. Such trees ("Qtrees") have the structure defined in (14).

(14) *Structure of Question-trees (Qtrees)*
Qtrees are trees whose nodes are all subsets of the CS and s.t.:
- Their root generally[3] denotes the CS;
- Any intermediate node is partitioned by the set of its children.

[3]We assume this is the case in the absence of extra presuppositions. In this paper, we will focus on presupposition-less sentences, so all Qtrees will have the same CS as root. But it is reasonable to think that a sentence carrying a presupposition p introduces a questions whose root denotes the CS intersected with p.

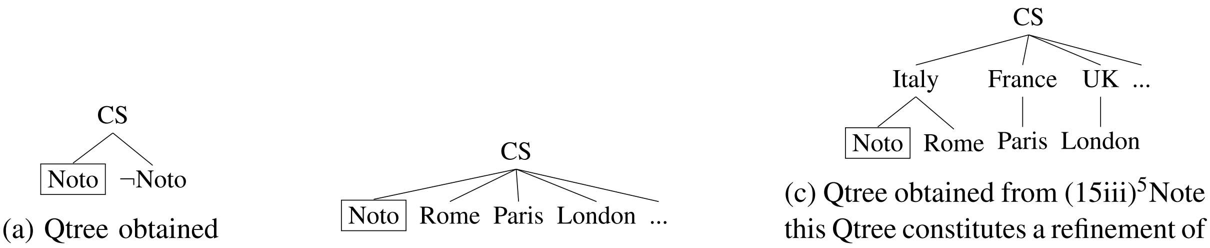
The nodes of such trees can be assigned the following interpretation. The root denotes a tautology over the CS, and any other node, a possible answer to the global question denoted by the tree. Intermediate nodes can generally be seen as non-maximal answers, while leaves can generally be seen as maximal answers.[4] By construction, the leaves of such trees form a partition of the CS, and allow to retrieve the previous notion of question-as-partition. In those trees, any subtree rooted in a node N can be understood as conditional question taking for granted the proposition denoted by N. Finally, a path from the root to any node N can be seen as a strategy of inquiry (or a sequence of conditional questions) leading to the answer denoted by N.

We now use this definition to define the possible Qtrees a simplex LF is compatible with. Before doing this, let us add one last ingredient to the current model, which is that, sentences also distinguish specific nodes (typically leaves) within the Qtrees they evoke, namely the nodes that verify the proposition denoted by the sentence (*prejacent*). In other words, a Qtrees associated with an assertion not only specifies which question the assertion addresses, but also how the assertion actually answers the question. We assume that if a Qtree evoked by a sentence ends up being associated with an empty set of verifying nodes at some point of the Qtree-derivation process, this Qtrees should be deemed ill-formed.

(15) *Qtrees for simplex LFs*
Let X be a simplex LF (no negation, no connective, no quantification) denoting p, not settled in the CS. Let $\mathscr{A}^g_{p,X}$ be a set of relevant alternatives to p, obtained from formal alternatives to X derived *via* the substitution of focused material by a same-granularity alternatives. We assume $\mathscr{A}^g_{p,X}$ partitions the CS. A Qtree for X is either:
   (i) A depth-1 Qtree whose leaves denote $\text{PARTITION}(\text{CS}, \{p\}) = \{p, \neg p\}$
   (ii) A depth-1 Qtree whose leaves denote $\text{PARTITION}(\text{CS}, \mathscr{A}^g_{p,X}) = \mathscr{A}^g_{p,X}$.
   (iii) A depth-k Qtree ($k > 1$), whose leaves denote $\mathscr{A}^g_{p,X}$, and such that removing those leaves yields a Qtree for an LF Y which is a formal alternative to X associated with a strictly coarser granularity.
In any case, the set of $\boxed{\text{verifying nodes}}$ is defined as the set of p-leaves.

Let us see how this applies to LFs such as X^+=*SuB29 will take place in Noto* and X=*SuB29 will take place in Italy*. The same-granularity alternatives to X^+ different from X^+ are of the form {*SuB29 will take place in Rome*, *SuB29 will take place in Paris* ... } where *Noto* is replaced by city-level alternatives. The same-granularity alternatives to X different from X are of the form {*SuB29 will take place in France*, *SuB29 will take place in the UK* ... } where *Italy* is replaced by country-level alternatives. Moreover, X can be seen as a coarser-grained alternative to X^+. This implies that X^+ and X are respectively compatible with the Qtrees in Figures 1 and 2. In such trees, we assume each node denotes the proposition it is labeled after, properly intersected with the CS. Boxed node represent verifying nodes, as induced by the prejacent proposition. Because X is coarser grained than X^+, the Qtrees obtained *via* principle (15iii) for X^+ will always be refinements of the Qtrees obtained for X *via* the same principle. The refinement relation is defined in (16).

[4]We say "generally" here because we think some operators like *at least* can actually influence the relevant level of granularity addressed by a Qtree, such that intermediate nodes can sometimes end up being seen as maximal answers.

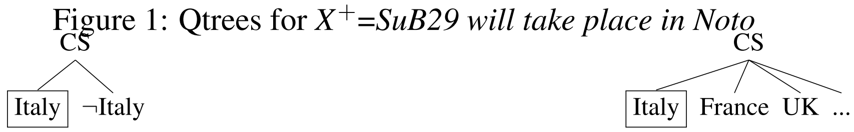


(a) Qtree obtained from (15i)

(b) Qtree obtained from (15ii)

(c) Qtree obtained from (15iii)[5]Note this Qtree constitutes a refinement of the Qtree in Figure 2b.

Figure 1: Qtrees for X^+=*SuB29 will take place in Noto*



(a) Qtree obtained from (15i)

(b) Qtree obtained from (15ii)

Figure 2: Qtrees for X=*SuB29 will take place in Italy*

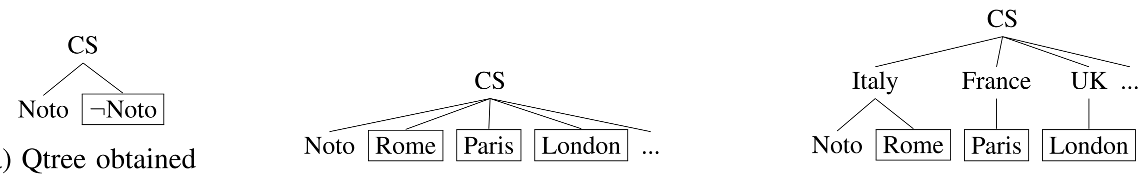
(16) *Qtree Refinement*. A Qtree T is a refinement of another Qtree T' iff T' can de obtained from T *via* some recursive trimming of T's leaves.

3.3. Questions evoked by negated LFs

We assume negated LFs evoke questions that are structurally similar to those evoked by their non-negated counterpart. The only difference resides in the set of verifying nodes, which is flipped by negation. This is formalized in (17).

(17) *Qtrees for negated LFs*
A Qtree T' for $\neg X$ is obtained from a Qtree T for X by:
- retaining T's structure;
- defining the set of T''s verifying nodes, $\mathbb{N}^+(T')$ as $\{N' | N' \notin \mathbb{N}^+(T) \wedge \exists N \in \mathbb{N}^+(T).\ d(N',T') = d(N,T)\}$, where $d(N,T)$ denotes the depth of a node N in a tree T.[6]

Qtrees corresponding to $\neg X^+$=*SuB29 will not take place in Noto* are given in Figure 3. They are derived by simply swapping verifying and non-verifying leaves in the Qtrees from Figure 1, corresponding to X^+=*SuB29 will take place in Noto*.



(a) Qtree obtained from (15i)

(b) Qtree obtained from (15ii)

(c) Qtree obtained from (15iii)[7]

Figure 3: Qtrees for $\neg X^+$=*SuB29 will not take place in Noto*

[5]Note that in principle more tiers can be added to that kind of Qtree, according to principle (15iii). For simplicity we only consider a city vs. country distinction here. The crucial point is that both X and X^+ are parametrized by the same tiers of same-granularity alternatives, whatever they are.

[6]Note that, if all verifying nodes are leaves, this definition is simplified: $\{N' | N' \notin \mathbb{N}^+(T) \wedge leaf(N')\}$. Moreover, because T and T' have same structure, the tree-argument is irrelevant to determine node depth in that particular case: $\forall N.\ d(N,T') = d(N,T)$. We keep it because, in the general case, node-depth depends on tree structure.

3.4. Questions evoked by disjunctive LFs

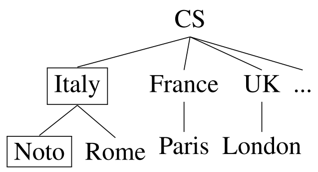
Building on Simons (2001); Zhang (2024), we assume disjunctive LFs raise questions pertaining to both disjuncts *in parallel*. In other words, disjuncts should mutually address each-other's questions. This is modeled by assuming that disjunctions return all possible unions of the Qtrees evoked by both disjuncts, filtering out the outputs that do not qualify at Qtrees.

(18) *Qtrees for disjunctive LFs*
A Qtree T for $X \vee Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:
- unioning the nodes, edges, and verifying nodes of T_X and T_Y;
- returning the output only if it is a Qtree.

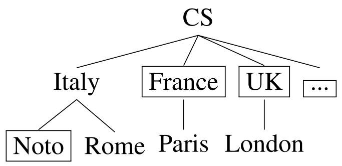
In other words, $Qtrees(X \vee Y) = \{T_X \cup T_Y | T_X \cup T_Y \text{verifies } (14) \wedge (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y)\}$

GIVE PROBLEMATIC CONFIGURATION

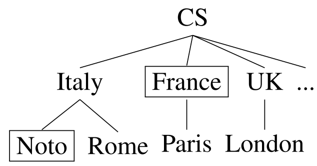
A prediction of this definition is that two Qtrees sharing the same CS can be properly disjoined only iff they appear structurally parallel up to a certain level, and any further partitionings they independently introduce do not "clash" with each other.[8] In our particular case, this predicts that two sentences evoking different levels of granularity (e.g., city-level vs. country level) can in principle be disjoined by picking Qtrees T and T' for resp. the finer-grained and coarser-grained disjunct, s.t. T, constitutes a refinement of T' as per (16). The only Qtree compatible with (1a-1b), obtained in this way, is given in Figure 4a.



(a) Qtree for the HDs (1a-1b), derived by disjoining Qtrees 1c and 2b



(b) Qtree for (19a).



(c) Qtree for (19b).

Figure 4: Qtrees for disjunctive sentences featuring disjuncts with different levels of granularity.

Why should we predict that Qtrees for odd HDs like (1a-1b) are derivable in the first place? We believe this kind of prediction is in fact useful, to derive Qtrees for other closely related

[8] We assume two Q-trees T and T' feature a bracketing clash iff there is $N \in T$ and $N' \in T'$ s.t. $N = N'$ but the sets of children of N and N' differ. We show that if T and T' exhibit such a clash, their disjunction is not a Q-tree. Let's call C and C' the sets of nodes of resp. T and T' that induce a bracketing clash; by assumption, C and C' are s.t. $C \neq C'$, and have mothers N and N' s.t. $N = N'$. Because $\vee$ achieves graph-union, $T \vee T'$ will have a node N with $C \cup C'$ as children, and because $C \neq C'$, $C \cup C' \supset C, C'$. Given that both C and C' are partitions of N, $C \cup C'$ cannot be a partition of N. Conversely, if two Q-trees T and T' sharing the same CS as root are s.t. their union $T \cup T'$ is not a Qtree, it must be because T and T' had a bracketing clash. Indeed, under those assumptions, $T \cup T'$ not being a Qtree means one node N in $T \cup T'$ is not partitioned by its children. Given N is in $T \cup T'$, N is also in T, T', or both. If N was only in, say, T, then it means N's children are also only in T, but then, T itself would have had a node not partitionned by its children, contrary to the assumption T is a Qtree. The same holds *mutatis mutandis* for T', so, N must come from *both* T and T'. Let us call C and C' the partitioning introduced by N in resp. T and T'. The fact C, C', but not $C \cup C'$ partition N entails $C \neq C'$, i.e. T and T' feature a bracketing clash.

disjunctive sentences such as (19a-19b), which feature disjuncts with different levels of granularity (just like (1a-1b)), but that are not in an entailment relation. Qtrees for (19a-19b) are given in Figures 4b and 4b.

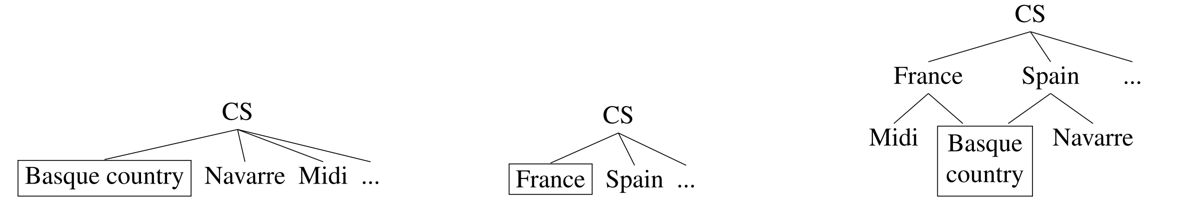
(19) a. SuB29 will take place in Noto or will not take place in Italy.
b. SuB29 will take place in Noto or in France.

These two additional examples and their Qtrees also suggest what the issue seems to be in the case of the HDs (1a-1b) and their Qtree in Figure 4a: a strategy of inquiry connecting the root to the verifying node *Noto*, properly contains a strategy of inquiry connecting the root to the verifying node *Italy*. In other words, inquiring about *Noto* amounts to inquiring about *Italy*. We will formalize this intuition in the form of an updated REDUNDANCY constraint in the next Section.

Before moving on to the conditional case, let us point out one case where disjunction fails to produce any Qtree, based on the sentence in (20), whose disjuncts are not in an entailment relation, but still appear mutually compatible Singh (2008).[9]

(20) # Sub29 will take place in the Basque country or France.

(20) has its first disjunct suggest a partitioning involving regions, and such that the Basque country represents a (verifying) leaf; and its second disjunct suggest a by-country partition, such that France represents a (verifying) leaf. This is exemplified in Figures 5a and 5b using principle (15ii) for simplex Qtree formation (the prediction is the same under principle (15i)). But such trees cannot be properly disjoined together, due to the fact that they introduce different, parallel partitionings. The only remaining way to disjoin Qtrees associated with each disjunct of (20), would be to create a tiered Qtree involving a by-country layer for *Sub will take place in the Basque country*, *via* principle (15iii). But a problem arises in the formation of such a Qtree, which is that building edges between the country-level tier and the region-level tier leads to a cycle, as shown in Figure 5c. In other words, the resulting "Qtree" cannot be a tree in the first place. To summarize, we predict a sentence like (20) to be odd because it cannot lead to any well-formed disjunctive Qtree.



(a) A simple Qtree for *Sub will take place in the Basque country*, given principle (15ii).

(b) A simple Qtree for *Sub will take place in France*, given principle (15ii)

(c) A tentative Qtree for *Sub will take place in the Basque country*, given principle (15iii)

Figure 5: Tentative Qtrees for the disjuncts of (20).

[9]Given tat the Basque country is made up of Northern Central Spain and southwestern France.

3.5. Questions evoked by conditional LFs

Building on insights from the psychology literature which revealed that subjects tend to massively overlook the eventualities falsifying the antecedent when verifying the truth conditions of conditionals Wason (1968), we assume conditional LFs preferentially raise questions pertaining to their consequent, *in the domain(s) of the CS where the antecedent holds*. This is modeled by assuming that implications return Qtrees evoked by their antecedent whose verifying nodes get replaced by their intersection with a Qtree evoked by the consequent. Similarly to disjunctions, this process is assumed to filter out the outputs that do not qualify at Qtrees.

(21) *Qtrees for conditional LFs*

A Qtree T for $X \to Y$ is obtained from a Qtree T_X for X and a Qtree T_Y for Y by:

- replacing each node N of T_X that is in $\mathbb{N}^+(T_X)$ by $N \cap T_Y$, where $N \cap T_Y$ (intersection between a node and a Qtree) is defined as T_Y, where each node gets intersected with N and empty nodes as well as trivial ("only child") links get removed; and where T_Y's verifying nodes are preserved;
- returning the result only if it is a Qtree.

In other words, $Qtrees(X \to Y) = \{T_X \cup \bigcup_{N \in \mathbb{N}^+(T_X)} (N \cap T_Y) | (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y) \land T_X \cup \bigcup_{N \in \mathbb{N}^+(T_X)} (N \cap T_Y) \text{verifies (14)}\}$, and $\mathbb{N}^+(T_X \to T_Y) = \{N \cap N' | (N, N') \in \mathbb{N}^+(T_X) \times \mathbb{N}^+(T_Y) \land N \cap N' \neq \emptyset\}$

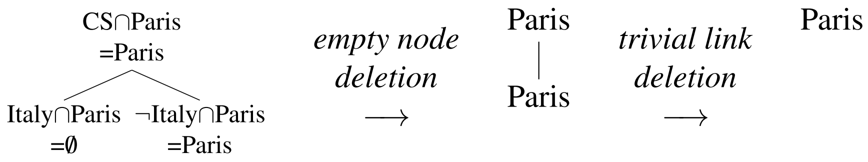
A general prediction of this definition is that, for an antecedent Qtree T_X and a consequent Qtree T_Y to be properly combined, the CS (root) of T_Y should be a superset of each verifying node of T_X. In other words, anything the antecedent asserts to be true should be part of the CS of the consequent.[10] Violations of this condition do not arise with the data at stake here, because we assume that antecedent and consequent Qtrees share the same CS. But this is to keep in mind for cases where the consequent is taken to introduce additional presuppositions further restricting the size of its "local" CS.[11]

A more targeted prediction of the above definition that directly applies to our case study, is that intersecting a city-level node with a country-level Qtree does not have any effect – consistent with the intuition that answering a question about cities automatically answers the a similar question at the country level. This is generalized in (22), and illustrated in Figure 6, using *Paris* as city-node and the trees from Figure 2 to represent country-level questions.

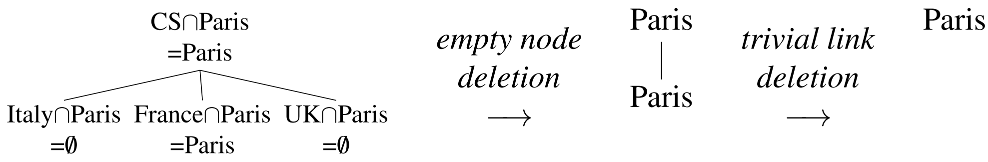
(22) If N is a node and T a Qtree with a leaf L entailed by N (i.e. s.t. $N \cap L = N$) then $N \cap T = N$

[10]Indeed, if T_X had a verifying node N that were a strict superset of the CS of T_Y, then the intersected tree $N \cap T_Y$ replacing N in T_X by the effect of the Qtree conditionalization operation, would have a strict subset of N as its root, which would entail a violation of the partition property on the resulting conditional Qtree (in particular, $N \cap CS_Y \subset N$ and its sisters would no longer fully cover the set denoted by their mother).

[11]More specifically, if we assume the consequent Qtree's root denotes $CS \cap p$, CS being the root of the antecedent Qtree, the condition becomes $\forall N \in \mathbb{N}^+(T_X).\ N \subseteq CS \cap p$ which entails $\forall N \in \mathbb{N}^+(T_X).\ N \subseteq p$. In other words, we expect presuppositions carried by the consequent to be entailed by the local context defined by the antecedent Qtree (in the form of $\mathbb{N}^+(T_X)$).



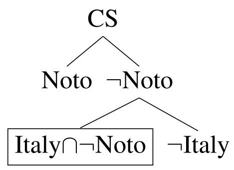
(a) Derivation of Paris∩Tree 2a=Paris



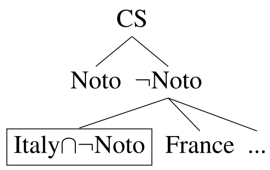
(b) Derivation of Paris∩Tree 2b=Paris

Figure 6: Intersecting a city-level node and a country-level tree yields the input city-level node.

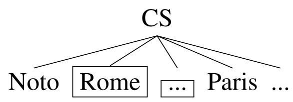
Let us now turn to the HCs (2a-2b) and their Qtrees, shown in Figures 7 and 8. Note that the Trees in Figures 7c and 7d appear structurally similar to the antecedent Qtrees used to form them, due to property (22).



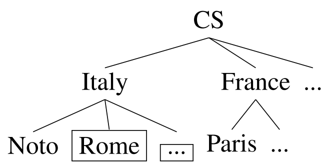
(a) Qtree obtained from Tree 3a and Tree 2a.



(b) Qtree obtained from Tree 3a and Tree 2b.

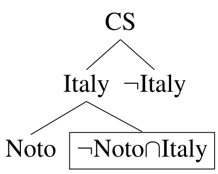


(c) Qtree obtained from Tree 3b and Tree 2b or 2a.

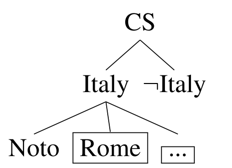


(d) Qtree obtained from Tree 3c and Tree 2b or 2a.

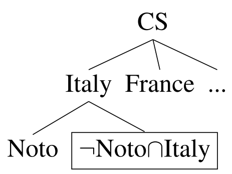
Figure 7: Qtrees for (2a)=#*If SuB29 will not take place in Noto, it will take place in Italy.*



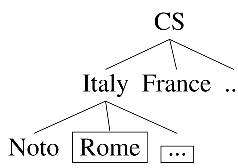
(a) Qtree obtained from Tree 2a and Tree 3a.



(b) Qtree obtained from Tree 2a and Tree 3b or 3c.



(c) Qtree obtained from Tree 2b and Tree 3a.



(d) Qtree obtained from Tree 2b and Tree 3b or 3c.

Figure 8: Qtrees for (2b)=*If SuB29 will take place in Italy, it will not take place in Noto.*

In any case, it seems that many Qtrees are available, for both the felicitous variant (2b) and the infelicitous variant (2a). What is the difference between these two sets of Qtrees? Intuitively, it seems that *some* Qtrees compatible with (2b), namely Trees 8b and 8d, *still* discuss (after conditionalization) a by-city distinction pertaining to their consequent; while *none* of the Qtrees compatible with (2a) still discuss a by-country distinction as introduced by their consequent. In other words, it seems the consequent of (2b) can be taken to be relevant to the global question raised by this sentence, while the consequent of (2a) *cannot*. We will formalize this intuition in the form of an updated RELEVANCE constraint in the next Section.

4. Updating Redundancy and Relevance

We now rephrase two constraints previously posited in the literature, REDUNDANCY and RELEVANCE Katzir and Singh (2014); Lewis (1988). in order to make them both sensitive to the *implicit question(s)* raised by sentences. The general enterprise is to make such constraints sensitive to the logical meaning of sentences, but also, to how sentences "package" this logical information, *via* their implicit QuDs. This idea, combined with the distinct semantics we assigned to disjunctions and conditionals at the inquisitive level, allows to account for the contrast between HDs and HCs.

4.1. Redundancy

REDUNDANCY-based approaches to Hurford Sentences state that a sentence contextually equivalent to one of its formal simplifications is redundant, and hence odd Katzir and Singh (2014); Meyer (2013); Mayr and Romoli (2016). The problem with such approaches is that they cannot discriminate between HDs and HCs, due to them having the same kind of logical structure and the same logical meaning. However, under our QuD-informed view, disjunctions and implications do have a different inquisitive contribution. We now show that making REDUNDANCY sensitive to such a difference in information structure allows to capture the existence of a contrast between HDs and HCs. We extend REDUNDANCY to the domain of accommodated questions by stating that a question that is evoked by a sentence and also one the sentence's formal simplification, is redundant, and as such should be ruled out from the set of possible Qtrees of the sentence. This is spelled out in (23a); (23b)-(23g) unpack the definition.

(23) a. *QuD-driven* REDUNDANCY. Let X be a LF and let $Qtrees(X)$ be the set of the Qtrees compatible with X. For any $T \in Qtrees(X)$, T is deemed Q-REDUNDANT with respect to X iff there exists a formal simplification of X, X', and $T' \in Qtrees(X')$, such that $\mathscr{R}(T) \equiv \mathscr{R}(T')$.

b. *Formal simplification.* X is a formal simplification of X if X' can be derived from X *via* a series of constituent-to-subconstituent substitutions.

c. Qtree equivalence relation $\equiv$. $T \equiv T'$ iff T and T' have same structure and same set of maximal verifying paths.

d. *Qtree reduction function* $\mathscr{R}$. $\mathscr{R}(T)$ is the tree obtained from T by removing all empty nodes and recursively replacing only children by their mother, percolating the "verifying" property as needed.[12]

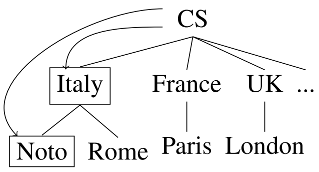
e. *Set of verifying paths* $\mathbb{P}(T)$ *of a Qtree* T. Set of paths starting from the root of T, and such that each path finishes in each $N \in \mathbb{N}^+(T)$.

f. *Path containment.* Two paths p_1 and p_2 are in a containment relation ($p_1 \subseteq_{\mathbb{P}} p_2$) if p_1 (seen as an ordered list, i.e. a string, of nodes) is a prefix of p_2.

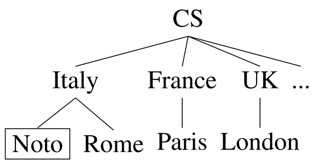
[12]This means that, if the only child deletion operation targets a mother node M and its only child node N, the output is M and is verifying iff M *or* N is verifying.

g. *Set of minimal verifying paths* $\mathbb{P}^*(T)$ *of a Qtree* T. Set of minimal elements of $\mathbb{P}(T)$ w.r.t. the path containment relation.

This provides an explanation as to why the only Qtree compatible with the HDs (1a-1b), repeated in Figure 9a, is redundant: it turns out to be equivalent (in terms of structure and maximal verifying paths) to a Qtree evoked by *Sub29 will take place in Noto*, repeated in Figure 9b. The equality between the sets of minimal verifying paths for those two trees is justified in (24).



(a) Qtree for the HDs (1a-1b).



(b) Qtree for *Sub29 will take place in Noto* (obtained from principle (15iii)).

Figure 9: Showing that the HDs (1a-1b) are Q-REDUNDANT

(24) $\mathbb{P}(9a) = \{[CS, Italy, Noto], [CS, Italy]\}$
$\mathbb{P}^*(9a) = \{[CS, Italy, Noto]\}$, because $[CS, Italy] \subseteq_{\mathbb{P}} [CS, Italy, Noto]$
$\mathbb{P}(9b) = \{[CS, Italy, Noto]\} = \mathbb{P}^*(9b) = \mathbb{P}^*(9a)$

Regarding the HCs (2a-2b), it can be shown that none of their Qtrees (given in Figures 7 and 8) violate Q-REDUNDANCY. To justify this, let us review the Qtrees associated to the simplifications of (2a) and (2b). First, Qtrees for the simplifications *Sub29 will take place in* {*Noto, Italy*}, and *Sub29 will not take place in Noto*, shown in Figures 1, 2 and 3, either have a different structure, or different maximal verifying paths as the Qtrees in Figures 7 and 8. This stems from the intuition that the HDs (2a-2b) package information pertaining to both *not Noto* and *Italy*. Regarding the simplification of (2a) *If SuB29 will take place in Noto, it will take place in Italy*, it is predicted, as per property (22), to give rise to the same Qtrees as *Sub29 will take place in Noto*, which, as said above, do not trigger Q-REDUNDANCY. Regarding the simplification of (2b) *If SuB29 will take place in Italy, it will take place in Noto*, it is predicted to give rise to Qtrees structurally similar to those in Figure 8, but whose verifying nodes support *Noto* (instead of *not Noto*), and as such cannot trigger Q-REDUNDANCY. To account for the infelicity of (2a), while retaining the felicity of (2b), we appeal to an updated definition of RELEVANCE, introduced in the next section.

4.2. Relevance

A view on RELEVANCE, due to Lewis (1988), is that a sentence is relevant given a QuD (seen as a partition of the Context Set), if the denotation of the sentence does not cut across cells.

(25) *Relevance*. A sentence S denoting p is relevant given a QuD $Q = \{c_1, ..., c_k\}$ iff $\exists Q' \subseteq Q.\ p = \bigcup Q'$.

We extend this principle to Qtrees by stating that Qtree composition should never cut across verifying nodes. In other words, the verifying nodes of the input Qtrees should be either fully preserved or fully ruled out by Qtree composition rules.

(26) *QuD-driven* RELEVANCE. Let X and Y be two LFs and let $Qtrees(Y)$ and $Qtrees(Y)$ be the set of the Qtrees compatible with X and Y respectively. We define RELEVANCE on a Qtree derived from a LF, Z, involving X, Y, or both. Two subcases:[13]

- Unary case: $Z = \circ X$ with $\circ$ a unary operation. $T_Z \in Qtrees(Z)$ will be relevant iff there is a Qtree $T_X \in Qtrees(X)$ s.t. T_Z can be derived from $\circ$ and T_X, and s.t. all verifying nodes of T_X are nodes of T_Z (verifying, or not): $\exists T_X \in Qtrees(X).\ T_Z = \circ T_X \wedge \forall N \in \mathbb{N}^+(T_X).\ N \in \mathbb{N}(T_Z)$.
- Binary case: $Z = X \circ Y$ with $\circ$ a binary operation. $T_Z \in Qtrees(Z)$ will be relevant iff there is a Qtree $T_X \in Qtrees(X)$ and a Qtree $T_Y \in Qtrees(Y)$ s.t. T_Z can be derived from $\circ$, T_X, and T_Y, and s.t. all verifying nodes of T_X and T_Y are nodes of T_Z (verifying, or not): $\exists (T_X, T_Y) \in Qtrees(X) \times Qtrees(Y).\ T_Z = T_X \circ T_Y \wedge \forall N \in (\mathbb{N}^+(T_X) \cup \mathbb{N}^+(T_Y)).\ N \in \mathbb{N}(T_Z)$.

The predictions of Q-RELEVANCE for our $\{\neg, \vee, \rightarrow\}$ are the following. First, because $\neg$ is structure preserving, we can be sure that all the input verifying nodes are part of the negated output Qtree, satisfying Q-RELEVANCE. Second, because $\vee$ forces well-formed unions of Qtrees (in terms of structure and verifying nodes), we can be sure that all the input verifying nodes are part of the disjunctive output Qtree, again, satisfying Q-RELEVANCE.
The interesting cases arise with $\rightarrow$, because this operation involves intersecting verifying nodes and Qtrees, i.e. performing operations of the form $N \cap T$, potentially affecting N and the verifying nodes of T. Let us consider the Qtree $T_X \rightarrow T_Y$, and within $T_X \rightarrow T_Y$, the subtree $N \cap T_Y$, where $N \in \mathbb{N}^+(T_X)$). Q-RELEVANCE imposes that N be part of $N \cap T_Y$, and that any verifying node in T_Y be part of $N \cap T_Y$, too. The former condition is verified, given that the root of T_Y is a superset of N (this superset condition is necessary for the whole conditional Qtree to be well-formed, cf. footnote 10), i.e. the root of $N \cap T_Y$ is simply N. The latter condition is less trivial, and entails $\forall N' \in \mathbb{N}^+(T_Y).\ N \cap N' = N' \vee N \cap N' = \emptyset$, i.e. $\forall N' \in \mathbb{N}^+(T_Y).\ N' \subseteq N$. Zooming out, this last condition states that each verifying node of

GIVE PROBLEMTIC CONFIGURATION casesin whic replacement kicksin non-terminal node: structure gets overwritten if Germany or Paris then

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[13]The two bullet points below can be generalized to n-ary operators in the following way:

- n-ary case: $Z = \circ(X_1, ...X_n)$ with $\circ$ a n-ary operation. $T_Z \in Qtrees(Z)$ will be relevant iff there is a family of Qtrees $(T_i)_{i \in [1;n]} \in \prod_{i=1}^{n} Qtrees(X_i)$ s.t. T_Z can be derived from $\circ$ and $(T_i)_{i \in [1;n]}$, and s.t. each T_i's verifying nodes are nodes of T_Z (verifying, or not): $\exists (T_i)_{i \in [1;n]} \in \prod_{i=1}^{n} Qtrees(X_i).\ T_Z = \circ(T_1, ...T_n) \wedge \forall i \in [1;n].\ \forall N \in \mathbb{N}^+(T_i).\ N \in \mathbb{N}(T_Z)$.

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