9.19 Computational Psycholinguistics

Basic semantics

November 15, 2023

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Introduction, and some technical background

From syntax to semantics

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  - The trees aimed to capture notions such as **constituency** (e.g. the fact that a transitive verb forms a "chunk" with its object, but not with its subject), and **thematic roles** assigned by a verb to its arguments (e.g. the Object, Theme, Goal...).
  - We also saw that some sentences were **structurally ambiguous** (*John saw the girl with binoculars*).
- Now, we'd like to define a way to systematically compute the logical meaning of a given sentence, given its syntax tree. In other words, we'd like to define a **mapping between trees and first-order logic**.
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The Principle of Compositionality

- To devise a consistent mapping between syntax and semantics, we exploit the following idea which dates back (at least) from **Gottlob Frege** (1884):

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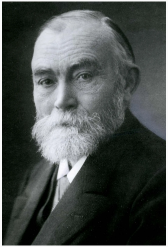
the meaning (=denotation) of a complex expression is determined by its **structure** and **the meanings of its constituents**.

- This idea was revived in 1960's by **Richard Montague**.
- Montague's thesis was that natural languages and formal languages (in particular programming languages) can be treated in the same way.



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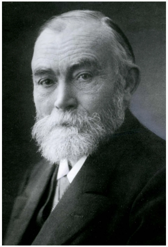

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- **Idiom chunks** (e.g. *kick the bucket*), **logical metonymy** (e.g. *John began the book*) [Pustejovsky, 1995]
- Context-sensitive elements, such as **indexicals** (*I*, *you*, *here*, *now*...), **gradable adjectives** (e.g. *tall*), **subjective predicates** (e.g. *yummy*).
- Proper names vs. definite descriptions under **belief verbs**:

(2) *Context: Ralph saw Ortcutt at the beach and believes the man he saw is a spy. But Ralph did not realize the man was actually Ortcutt.*
Ralph **believes** the man at the beach is a spy.
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Is language really compositional? continued

- **Bracketing paradoxes**: *unhappier* is parsed [un-[happi-er]] (-er cannot attach to a 2-syllable adjective!), yet means *more unhappy* [Allen, 1978].
- **Weakened/strengthened** modals/logical operators:

(5) Minimal Sufficiency readings [von Fintel and Iatridou, 2007]:
To get good cheese you **only have to** go to the North End.
$\rightsquigarrow$ You **don't have to** go to the North End (but it's the easisest option).

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A detour through λ-calculus and types

- To compose meanings together, we will need functions.
- λ-calculus can be seen as a compact way of writing and applying functions. λ-terms can take 3 forms (inductive definition):
  - a **variable** x;
  - a **function** $(\lambda x.\ M)$ where x is a bound variable and M is a term;
  - an **application** $M(N)$ where both M and N are terms.
- If x has type α (written "x : α") and M type β, then the term $(\lambda x.\ M)$ has type $\alpha \to \beta$. It's a function which, given an $x : \alpha$, returns a term $M : \beta$ that usually depends on x. Note that both x and M can be functions themselves.
- Lambda-terms terms can be "reduced" using the following operation (assuming the types are right):

$$(\lambda x.\ M)(y) = M[y/x]$$

- Meaning: applying the function $(\lambda x.\ M)$ to an input y amounts to substituting any occurrence of the bound variable x in M by the input y.

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A few examples of λ-terms

- The "add 10" function (=partial application of the "sum" function):

$$(\lambda x.\ \lambda y.\ x + y)(10) = (\lambda y.\ x + y)[10/x] = (\lambda y.\ 10 + y)$$

- Adding 10 to 5 (=total application of the sum function):

$$(\lambda y.\ 10 + y)(5) = (10 + y)[5/y] = 10 + 5 = 15$$

- Testing if 10 is prime (a Boolean function):

$$(\lambda x.\ \mathbf{isprime}(x))(10) = (\mathbf{isprime}(x))[10/x] = \mathbf{isprime}(10) = \bot$$

- Negating the "prime" function (notice that we renamed the bound variable in the input term into "y" to avoid variable capture):

$$\begin{aligned}(\lambda P.\ \lambda x.\ \neg P(x))(\lambda x.\ \mathbf{isprime}(x)) &= (\lambda x.\ \neg P(x))[(\lambda y.\ \mathbf{isprime}(y))/P] \\ &= (\lambda x.\ \neg(\lambda y.\ \mathbf{isprime}(y))(x)) \\ &= (\lambda x.\ \neg\mathbf{isprime}(y)[x/y])\end{aligned}$$

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Deriving the meaning of simple sentences

Denotation of sentences

- We assume that whole sentences are defined by the conditions under which they are true (**truth conditions**).
- Note that this is slightly different from a simple Boolean value (0 or 1). For instance, the meaning of *a cat is on the mat* is not always 0 or 1; rather, it will evaluate to 1 **iff** there exists something that's a cat that is located on the unique salient mat; and 0 otherwise.
- We call the type of sentences (i.e. elements with truth conditions) $\mathtt{t}$. We use the double-bracket notation ($[\![.]\!]$) to indicate the meaning (=denotation) of a given string.

$$[\![\textit{a cat is on the}^1\ \textit{mat}]\!] = \begin{cases} 1 & \text{if } \exists x.\ \mathbf{cat}(x) \wedge \exists! y.\ \mathbf{mat}(y) \wedge\ \mathbf{on}(x)(y) \\ 0 & \text{otherwise} \end{cases}$$

[1]The meaning of *the* is more complex than what we do here: *the* should *presuppose* the existence and the uniqueness of a mat. This is just to get a rough idea of a possible denotation of the sentence.

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Denotation of terminals

- How to properly *compute* the meaning of sentences like *a cat is on the mat*? Principle of Compositionality: **from the meaning of the terminals** and how they merge in the tree.
- We assume that each terminal of the tree can be mapped to a lexical "meaning". For instance:
  - **Proper names** refer to fixed entities ($\sim$ constants) belonging to a certain domain D. We call $\mathtt{e}$ the type of entities.
  - **Predicates** (*happy*, *teacher*...) or verbs (*like*, *jump*...) are functions mapping one or more entities (type $\mathtt{e}$) to truth values (type $\mathtt{t}$).

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  - Some special terminals ("traces"/pronouns) may denote bound variables or type $\mathtt{e}$.
  - We keep determiners for later.
- Now let's try to combine all those things together!

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    - We keep determiners for later.
- Now let's try to combine all those things together!

Denotation of terminals

- How to properly *compute* the meaning of sentences like *a cat is on the mat*? Principle of Compositionality: **from the meaning of the terminals** and how they merge in the tree.
- We assume that each terminal of the tree can be mapped to a lexical "meaning". For instance:
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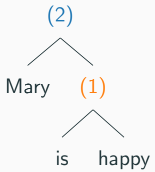
Functional Application



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- Compositionality, again: the meaning of *Mary is happy* depends on the meanings of *Mary*, *is*, and *happy*, and how they combine together.
- To combine 2 nodes together, we introduce the rule of **Functional Application (FA)**:

If $X : \alpha$ merges with $Y : \alpha \to \beta$, then $[\![\mathrm{Y\ X}]\!]^2 = \mathrm{Y(X)}$.

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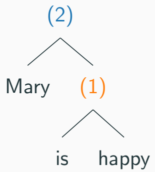

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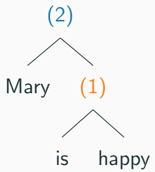

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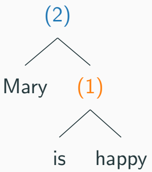

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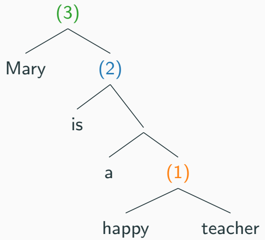

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- Both *happy* and *teacher* denote functions of type $\mathtt{e} \to \mathtt{t}$... we can't combine them with Functional Application!
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$$= 1 \text{ iff } \mathbf{happy}(\mathbf{M}) \wedge \mathbf{teacher}(\mathbf{M})^{3}$$

[3] Food for thought: does the sentence really mean that Mary is happy, and is a teacher? Or does it rather mean that Mary is happy, *for a teacher*?

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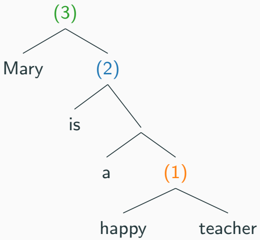

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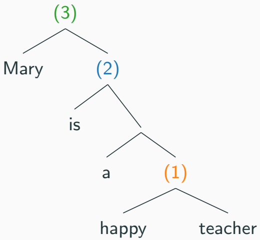

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Taking stock

- What you might think at this point: we started with a string saying "Mary is happy" and ended up with the meaning that "Mary is happy"...well that's not so impressive.
- First, we should keep in mind that the 2 "Mary is happy" are in different *languages*.
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Quantification

Generalized quantifiers

- Natural languages are endowed with various quantifiers: *every*, *some*, *most*, *few*...

(9) Every student smiled. $\rightsquigarrow \forall x.\ \textbf{student}(x) \implies \textbf{smiled}(x)$

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- Natural language quantifiers are *restricted*: they do not quantify over the whole set of possible entities, but rather on specific subsets denoted by predicates of type $\mathtt{e} \rightarrow \mathtt{t}$ such as **student** in (9) and **dogs** in (10). Those are called **restrictors**.
- Quantifiers moreover relate elements verifying the restrictor to another property, e.g. smiling in (9) or barking in (10). This property, also of type $\mathtt{e} \rightarrow \mathtt{t}$, is called the **(nuclear) scope** of the quantifier.
- In brief, a **generalized quantifier** says something about the relation between its restrictor (predicate of type $\mathtt{e} \rightarrow \mathtt{t}$) and its scope (also (predicate of type $\mathtt{e} \rightarrow \mathtt{t}$)). It is thus a function of type $(\mathtt{e} \rightarrow \mathtt{t}) \rightarrow (\mathtt{e} \rightarrow \mathtt{t}) \rightarrow \mathtt{t}$

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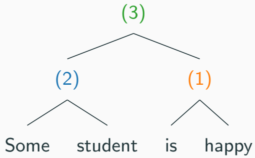
Denotation of a quantified sentence



$$[\![Some]\!] = \lambda P.\lambda Q.\ \exists x.\ P(x) \land Q(x)$$
$$[\![student]\!] = \lambda x.\ \mathbf{student}(x)$$
$$[\![is]\!] = \lambda P.\ P$$
$$[\![happy]\!] = \lambda x.\ \mathbf{happy}(x)$$

$$[\![(1)]\!] = [\![is\ happy]\!] = [\![happy]\!] = \lambda x.\ \mathbf{happy}(x)$$

$$\begin{aligned} [\![(2)]\!] &= [\![Some\ student]\!] \overset{FA}{=} [\![some]\!]([\![student]\!]) \\ &= (\lambda P.\ \lambda Q.\ \exists x.\ P(x) \land Q(x))(\lambda y.\ \mathbf{student}(y)) \\ &= \lambda Q.\ \exists x.\ (\lambda y.\ \mathbf{student}(y))(x) \land Q(x) \\ &= \lambda Q.\ \exists x.\ \mathbf{student}(x) \land Q(x) \end{aligned}$$

$$\begin{aligned} [\![(3)]\!] &= [\![Some\ student\ is\ happy]\!] \overset{FA}{=} [\![Some\ student]\!]([\![is\ happy]\!]) \\ &= (\lambda Q.\ \exists x.\ \mathbf{student}(x) \land Q(x))(\lambda y.\ \mathbf{happy}(y)) \\ &= \exists x.\ \mathbf{student}(x) \land (\lambda y.\ \mathbf{happy}(y))(x) \\ &= \exists x.\ \mathbf{student}(x) \land \mathbf{happy}(x) \end{aligned}$$ [4]

[4] Food for thought: this meaning is compatible with *all* ($\forall$) students being happy. Is this consistent with your intuitions about *some*? Should we then change the lexical entry of *some*?

Quantifier monotonicity

- An interesting property to study with quantifiers is **monotonicity**, i.e. how quantifiers influence entailment patterns verified by their arguments (restrictor, and scope).
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Quantifier monotonicity: the case of all ($\forall$)

- Let's consider $[\![\textit{all students}]\!] = \lambda P.\ \forall x.\ \textbf{student}(x) \Rightarrow P(x)$. It is the quantifier *all* partially applied to its restrictor (the set of students). Is it monotone w.r.t. its nuclear scope argument?
  - Let's consider $P_1 = \lambda x.\ \textbf{french}(x)$ and $P_2 = \lambda x.\ \textbf{european}(x)$. We have $P_1 \subseteq P_2$.
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  - **All is upward monotone w.r.t. is nuclear scope**.
- Let's consider $\lambda P.\ [\![\textit{all}]\!](P)([\![\textit{delicious}]\!]) = \lambda P.\ \forall x.\ P(x) \Rightarrow \textbf{delicious}(x)$. It is the quantifier *all* partially applied to its nuclear scope (the set of delicious things). Is it monotone w.r.t. its restrictor?
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  - **All is upward monotone w.r.t. is nuclear scope**.
- Let's consider $\lambda P.\ [\![\textit{all}]\!](P)([\![\textit{delicious}]\!]) = \lambda P.\ \forall x.\ P(x) \Rightarrow \textbf{delicious}(x)$. It is the quantifier *all* partially applied to its nuclear scope (the set of delicious things). Is it monotone w.r.t. its restrictor?
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  - **All is downward monotone w.r.t. its restrictor**.

Quantifier monotonicity: the case of all ($\forall$)

- Let's consider $[\![\textit{all students}]\!] = \lambda P.\ \forall x.\ \mathbf{student}(x) \Rightarrow P(x)$. It is the quantifier *all* partially applied to its restrictor (the set of students). Is it monotone w.r.t. its nuclear scope argument?
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Bonus: quantification in object position, and scope ambiguity

A case of semantic ambiguity

- The sentence:

  (17) Every boy admires some girl.

- Has 2 readings: one in which each boy admires a different girl ("$\forall > \exists$"), and one in which there is a single girl s.t. each boy admires her ("$\exists > \forall$"). How to derive those 2 readings?
- First problem: there is no obvious way of combining the quantified NP *some girl* in the object position to the 2-place predicate *admire*: **type-mismatch**!

- Ideally, we'd like something of type e in place of *some girl*...

A case of semantic ambiguity

- The sentence:

  (18) Every boy admires some girl.

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A case of semantic ambiguity

- The sentence:

  (19) Every boy admires some girl.

- Has 2 readings: one in which each boy admires a different girl ("$\forall > \exists$"), and one in which there is a single girl s.t. each boy admires her ("$\exists > \forall$"). How to derive those 2 readings?
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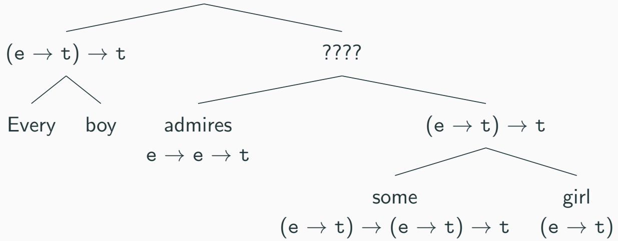

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A case of semantic ambiguity

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  (20) Every boy admires some girl.

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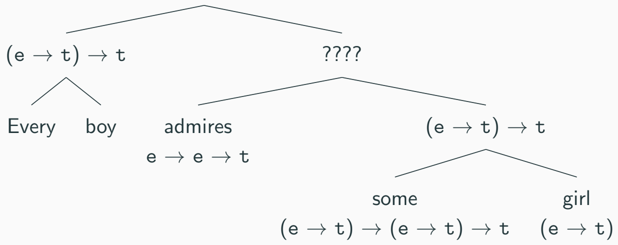
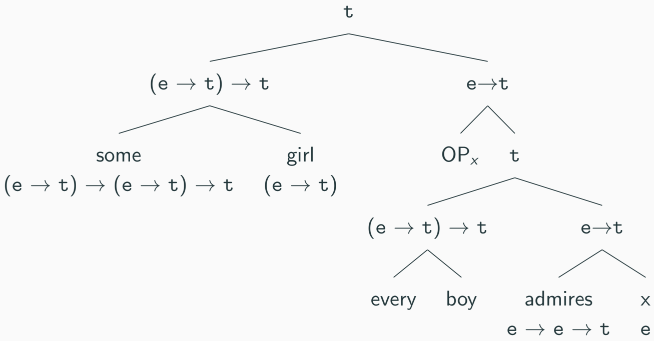


- Ideally, we'd like something of type e in place of *some girl*...

Resolving type-mismatch, and deriving the $\exists > \forall$ reading



- To resolve the type-mismatch, we moved the quantified NP *some girl* to the top of the tree, and replaced its "trace" by an e-type variable x. We also introduced a λ-abstractor OP$_x$ binding x and changing its input sentence back into a predicate (**type shifting**):[5]

$$[\![\mathrm{OP}_x]\!] = \lambda S.\ \lambda x.\ S$$

[5] This is a very simplified account of binding. A proper account would involve indices and assignment functions.

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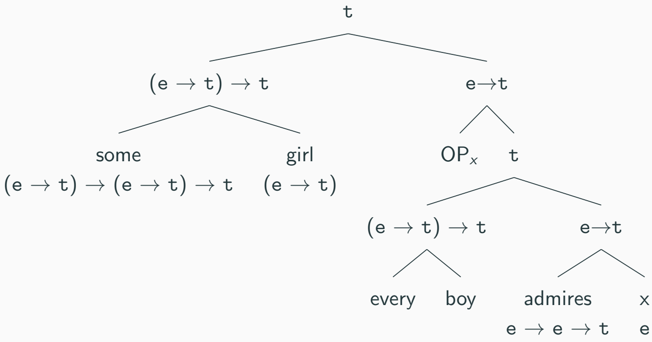

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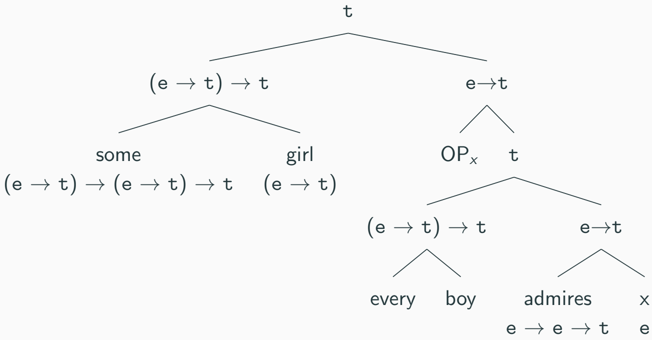


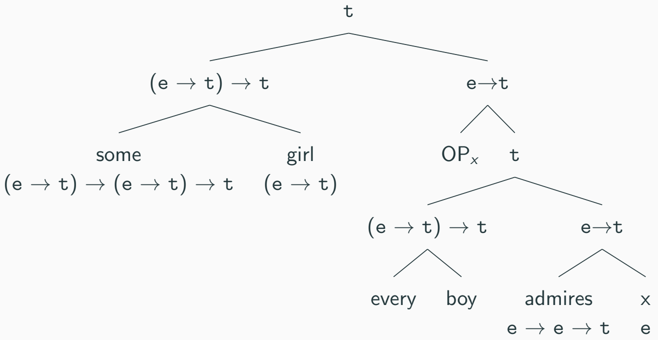
$$[\![\textit{ every boy admires x }]\!] = 1 \text{ iff } \forall y.\ \mathbf{boy}(y) \Rightarrow \mathbf{admire}(y)(x)$$

$$[\![\textit{ OP}_x \textit{ every boy admires x }]\!] = \lambda x.\ \forall y.\ \mathbf{boy}(y) \Rightarrow \mathbf{admire}(y)(x)$$

$$[\![\textit{ Some girl ... admires x }]\!] = 1 \text{ iff } \exists z.\ \mathbf{girl}(z) \wedge (\lambda x.\ \forall y.\ \mathbf{boy}(y) \Rightarrow \mathbf{admire}(y)(x))(z)$$
$$= 1 \text{ iff } \exists z.\ \mathbf{girl}(z) \wedge \forall y.\ \mathbf{boy}(y) \Rightarrow \mathbf{admire}(y)(z)$$

- That is the reading according to which there is one girl that every boy admires. To get the other reading, we need to do one more thing.

Resolving type-mismatch, and deriving the $\exists > \forall$ reading



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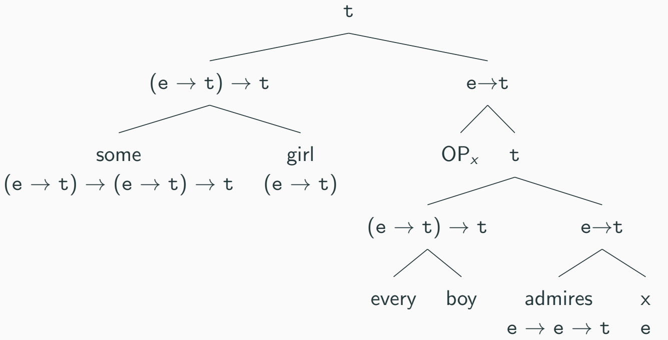

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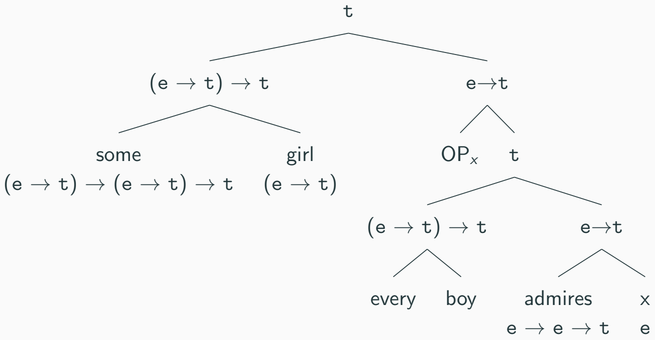

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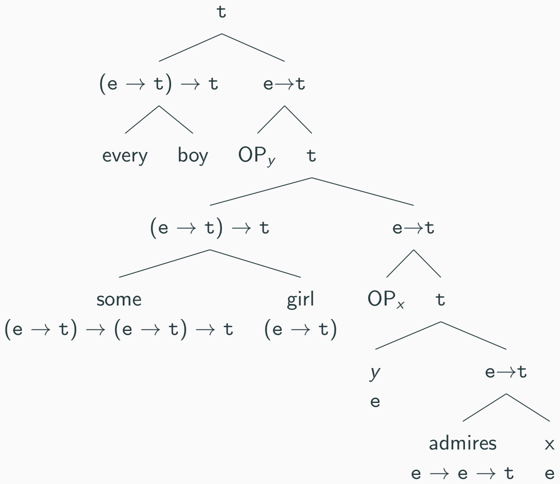
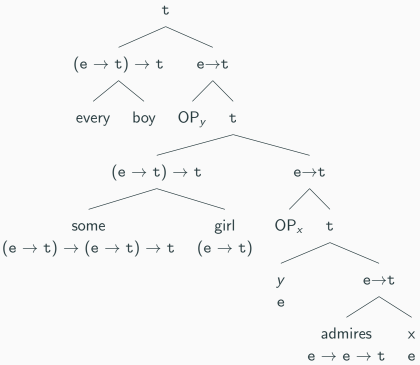
- That is the reading according to which there is one girl that every boy admires. To get the other reading, we need to do one more thing.

Resolving type-mismatch, and deriving the $\forall > \exists$ reading



- We have now moved *every boy* to the top of the tree (above *some girl*) and replaced its "trace" by an e-type variable y bound by an abstractor OP_y...

Quantification in object position: the $\forall > \exists$ reading



$$[\![y \ admires \ x]\!] = 1 \text{ iff } \mathbf{admires}(y)(x)$$

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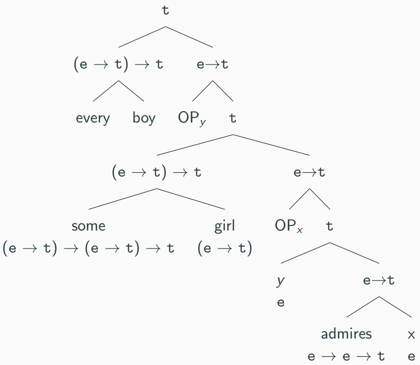

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Take away

- We derived the desired semantic scope ambiguity by moving the quantified NPs to the top of the tree. This is known as **quantifier raising** (QR). Semantic ambiguity was thus cashed out as some form of structural ambiguity in the tree.
- This might sound fishy, especially given that this kind of movement is not *audible*, and that the quantified NPs do not have the same type as their traces ($(\mathtt{e}\to\mathtt{t})\to\mathtt{t}$ vs. $\mathtt{e}$).
- However, recall QR was originally motivated by a type issue posed by the quantifier *some girl* interpreted in the object position.
- There might be other solutions to this puzzle, in particular solutions making use of covert type-shifting operators instead of movement. But the analysis we gave here is widely accepted and remains relatively tractable.

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