CSE 571

Problem Set 5

Q1. Joint Probability Distribution

Given the Joint probability distribution of random variables A, B and C, compute the probabilities given below:

A	B	C	Probability
0	0	0	0.18
0	0	1	0.15
0	1	0	0.05
0	1	1	0.10
1	0	0	0.21
1	0	1	0.10
1	1	0	0.03
1	1	1	0.18

Q1.1. $P(A = 0)$

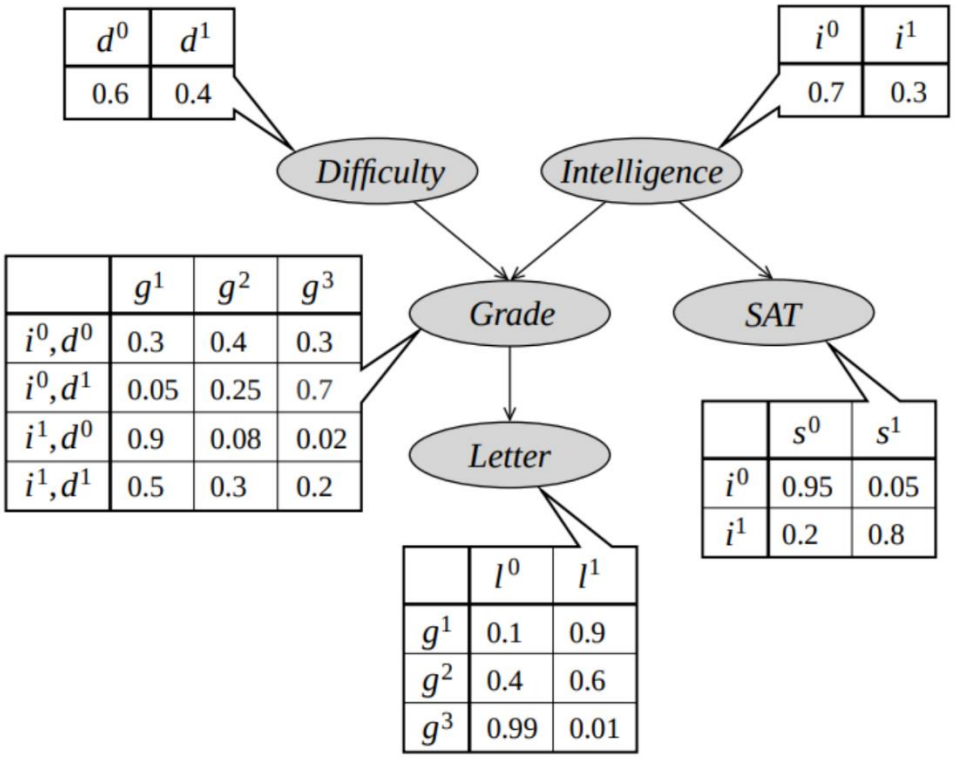
Q1.2. $P(B = 1)$

Q1.3. $P(B = 0, C = 1)$

Q1.4. $P(A = 1 \mid B = 1, C = 0)$

Q1.5. $P(B = 1|C = 0)$

Q2 Bayes Net



Given the Bayes net in the figure above with the conditional probability distribution beside each node, compute the following probabilities:

Q2.1. $P(i^1, d^0, g^2, s^1, l^0)$

Q2.2. $P(i^0, d^1, g^2, s^0, l^1)$

Q2.3. $P(i^0, d^1, g^3, s^0, l^0)$

Q2.4. $P(l^1 | i^0, d^0)$

Q2.5. $P(l^1 | i^0)$

Q2.6. $P(i^1 | g^2, d^1)$

Q3. Conditional Independence

Given the same Bayes net as in the previous question, let D, I, G, S, L represent the random variables Difficulty, Intelligence, Grade, SAT, and Letter respectively. Are the following true or not:

Q3.1. $(L||I)|G$

Q3.2. $(D||L)$

Q3.3. $(D||L)|S$

Q3.4. $(G||S)|I$

Q3.5. $(G||S)$

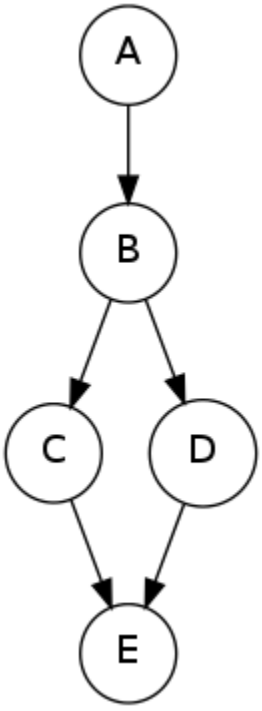
Q3.6. $(D||I)$

Q3.7. $(D||I)|G$

Q3.8. $(D||I)|L$

Q4. Rejection Sampling

We will now work with a Bayes Net of the following structure:



Q4.1. In this question, we will perform rejection sampling to estimate $P(C = 1 \mid B = 1, E = 1)$. Perform one round of rejection sampling, using the random samples given in the table below. Variables are sampled in the order A, B, C, D, E. Write down the value (0 or 1) that each variable gets assigned to. Note that the sampling attempt should stop as soon as you discover that the sample will be rejected. In that case mark the assignment of that variable and write none for the rest of the variables. For example, if C gets rejected, write *none* for D and E.

When generating random samples, use as many values as needed from the table below, which we generated independently and uniformly at random from $[0,1)$. Use numbers from left to right. To sample a binary variable W with probability $P(W = 0) = p$ and $P(W = 1) = 1 - p$ using a value a from the table, choose $W = 0$ if $a < p$ and $W = 1$ if $a \geq p$.

0.320	0.037	0.303	0.318	0.032	0.969	0.018	0.058	0.908	0.249

A	$P(A)$
0	0.200
1	0.800

B	A	$P(B\|A)$
0	0	0.800
1	0	0.200
0	1	0.400
1	1	0.600

C	B	$P(C\|B)$
0	0	0.600
1	0	0.400
0	1	0.400
1	1	0.600

D	B	$P(D\|B)$
0	0	0.800
1	0	0.200
0	1	0.600
1	1	0.400

E	C	D	$P(E\|C,D)$
0	0	0	0.800
1	0	0	0.200
0	1	0	0.600
1	1	0	0.400
0	0	1	0.400
1	0	1	0.600
0	1	1	0.400
1	1	1	0.600

Q4.2. Which variable will get rejected?

Q5. Estimating Probabilities from Samples

Below are a set of samples obtained by running rejection sampling for the Bayes' net from the previous question. Use them to estimate $P(C = 1 \mid B = 1, E = 1)$.

Sample 1

	0	1	rejected
A		x	
B	x		x
C			
D			
E			

Sample 2

	0	1	rejected
A	x		
B		x	
C	x		
D	x		
E		x	

Sample 3

	0	1	rejected
A		x	
B		x	
C		x	
D	x		
E		x	

Sample 4

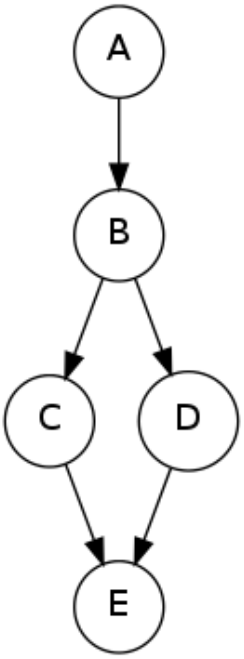
	0	1	rejected
A		x	
B	x		x
C			
D			
E			

Sample 5

	0	1	rejected
A	x		
B		x	
C		x	
D	x		
E	x		x

Q6. Likelihood Weighting

We will now work with a Bayes Net of the following structure:



Q6.1. In this question, we will perform likelihood weighting to estimate $P(C = 1 \mid B = 1, E = 1)$. Generate a sample and its weight, using the random samples given in the table below. Variables are sampled in the order A, B, C, D, E. Write down the assignments to the variables that you sampled.

When generating random samples, use as many values as needed from the table below, which we generated independently and uniformly at random from $[0,1)$. Use numbers from left to right. To sample a binary variable W with probability $P(W = 0) = p$ and $P(W = 1) = 1 - p$ using a value a from the table, choose $W = 0$ if $a < p$ and $W = 1$ if $a \geq p$.

0.249	0.052	0.299	0.773	0.715	0.550	0.703	0.105	0.236	0.153

A	$P(A)$
0	0.200
1	0.800

B	A	$P(B\|A)$
0	0	0.400
1	0	0.600
0	1	0.200
1	1	0.800

C	B	$P(C\|B)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

D	B	$P(D\|B)$
0	0	0.800
1	0	0.200
0	1	0.600
1	1	0.400

E	C	D	$P(E\|C,D)$
0	0	0	0.200
1	0	0	0.800
0	1	0	0.600
1	1	0	0.400
0	0	1	0.800
1	0	1	0.200
0	1	1	0.800
1	1	1	0.200

Q6.2. What is the weight for the sample you obtained above?

Q7. Estimating Probabilities from Weighted Samples

Below are a set of weighted samples obtained by running likelihood weighting for the Bayes' net from the previous question. Use them to estimate $P(C = 1 \mid B = 1, E = 1)$.

(use only the structure of the bayes net from the previous question, the probabilities and the samples can vary.)

Sample 1

	0	1
A		x
B		x
C		x
D		x
E		x

Weight = 0.64

Sample 2

	0	1
A		x
B		x
C		x
D		x
E		x

Weight = 0.64

Sample 3

	0	1
A		x
B		x
C	x	
D		x
E		x

Weight = 0.32

Sample 4

	0	1
A		x
B		x
C	x	
D	x	
E		x

Weight = 0.16

Sample 5

	0	1
A	x	
B		x
C		x
D		x
E		x

Weight = 0.48

Q8. Parameters

Say you are given a set X of 5 Boolean random variables. What is the minimum and maximum number of probability values that could be needed to specify a Bayes net over X. (Recall that a Bayes net is specified by a topology of nodes, and each node is assigned a probability of being True given its parent nodes' values).

Q9. True/False

Note: You don't need to provide a reason, just the answer would suffice.

Q9.1. Bayes net can reduce the number of parameters needed to represent a joint distribution of random variables.

Q9.2. In a Bayes net, the probabilities present in the conditional probability distribution of any node always add up to 1.

Q9.3. Bayes net's topology encodes conditional independence assertions between the random variables over which it is defined.

Q9.4. In a Bayes net, an arrow from a random variable A to another random variable B necessarily implies that A causes B in the domain.

Q9.5. When filling the CPTs of a bayes network, if the probabilities in the various entries are not set just right, the distribution might become undefined.