

Problem Set 5

$$1.1) P(A=0) = P(A=0 / B=0, C=0) + P(A=0 / B=0, C=1) \quad (F)$$

$$+ P(A=0 / B=1, C=0) + P(A=0 / B=1, C=1) \quad (NF)$$

$$= 0.18 + 0.15 + 0.05 + 0.10 = 0.48$$

$$1.2) P(B=1) = P(B=1 / C=0, A=0) + P(B=1 / C=1, A=0)$$

$$+ P(B=1 / C=0, A=1) + P(B=1 / C=1, A=1)$$

$$= 0.05 + 0.10 + 0.03 + 0.18 = 0.36$$

$$1.3) P(B=0, C=1) = P(B=0, C=1 / A=0) + P(B=0, C=1 / A=1)$$

$$= 0.15 + 0.10 = 0.25$$

$$1.4) P(A=1 / B=1, C=0) = P(A=1 / B=1, C=0) / P(A=1 / B=1, C=0) + P(A=0 / B=1, C=0)$$

$$= 0.03 / 0.03 + 0.05 = 3/8 = 0.375$$

$$1.5) P(B=1 / C=0) = P(B=1, C=0) / [P(B=0, C=0) + P(B=1, C=0)]$$

$$= (0.05 + 0.03) / ((0.05 + 0.03) + (0.18 + 0.1))$$

$$= 0.08 / (0.08 + 0.31) = 8/47 = 0.1702$$

$$2.1) P(i^1, d^0, g^2, s^1, l^0) = P(i^1) P(d^0) P(g^2 / i^1, d^0) P(s^1 / i^1)$$

$$P(l^0 / g^2)$$

$$= 0.3 \times 0.6 \times 0.08 \times 0.8 \times 0.4 = 0.004608$$

$$2.2) P(i^0, d^1, g^2, s^0, l^1) = 0.7 \times 0.4 \times 0.25 \times 0.95 \times 0.6 = 0.0399$$

$$2.3) P(i^0, d^1, g^3, s^0, l^0) = 0.7 \times 0.4 \times 0.7 \times 0.95 \times 0.99 = 0.184388$$

$$2.4) P(l^1 / i^0, d^0) = P(i^0) P(d^0) [P(l^1 / g_1) + P(l^1 / g_2) + P(l^1 / g_3)]$$

$$[P(l_1 / g_1) P(g_1 / i^0, d^0) + P(l_1 / g_2) P(g_2 / i^0, d^0)]$$

$$+ P(l_1 / g_3) P(g_3 / i^0, d^0)$$

$$\approx 0.513 [0.9 \times 0.3 + 0.6 \times 0.4 + 0.01 \times 0.3]$$

$$= 0.513$$

$$2.5) P(l' | i^0) = \frac{P(e', i^0)}{P(l', i^0) + P(e^0, i^0)}$$

$$= 0.9 \times 0.3 \times 0.6 + 0.9 \times 0.25 \times 0.4$$

$$+ 0.6 \times 0.4 \times 0.6 + 0.6 \times 0.25 \times 0.4$$

$$+ 0.01 \times 0.3 \times 0.6 + 0.01 \times 0.7 \times 0.4$$

$$= \cancel{0.000000000000000} 0.3886.$$

$$2.6) P(i' | g^2, d') = \frac{P(i^0, g^2, d')}{P(g^2, d')}$$

$$= \frac{0.3 \times 0.3 \times 0.4}{0.3 \times 0.3 \times 0.4 + 0.25 \times 0.7 \times 0.4}$$

$$= \frac{0.036}{0.09 + 0.036} = \frac{36}{106} = 0.3396$$

- 3.1) True 3.2) False 3.3) False 3.4) ~~False~~ True
 3.5) ~~True~~ False 3.6) True 3.7) False 3.8) False

$$P(A=1) = P(B=0) + P(B=1) \times P(A=1|B=1) = 0.2 + 0.8 \times 0.1 = 0.28$$

- 4.1) $A=1, B=0$ None $A=1, B=1$ None
 4.2) $B=0$ will get rejected because we need
 $B=1$

- 5) # of samples where $B=1, E=1$ is 2 (sample 2, 3)
 # of samples in above samples where $C=1$ is 1
 Probability = $1/2 = 0.5$

6) 6.1) $A=1, B=1, C=0, D=0, E=1$
 $0.249, 0.052, 0.299$

6.2) weight = $0.8 \times 0.8 = 0.64$

- 7) Samples with Sum of weights where $B=1, E=1 \wedge C=1$
Sum of weights where $B=1, E=1$

$$\begin{aligned} 0.01 &= \frac{0.64 + 0.64 + 0.48}{0.64 + 0.64 + 0.32 + 0.16 + 0.48} = \frac{176}{224} \\ &= 0.7857 \end{aligned}$$

- 8) Minimum is when everything is independent
so we will need $x=5$ probabilities.

Max is when each node is connected to all other nodes which is $2^n - 1$

$$n=5 \quad \therefore 2^n - 1 = 2^5 - 1 = 32 - 1 = 31$$

- a.1) True a.2) False a.3) True
a.4) False a.5) False