Problem Set 4 Solutions

1.

1.1 Remove implications and simplify

$p \Rightarrow q \lor r$ becomes $\neg p \lor q \lor r$

$p \land q \Rightarrow r$ becomes $\neg p \lor \neg q \lor r$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg p \lor q \lor r$	$\neg p \lor \neg q \lor r$
T	T	F	F	F	T	T	F

Hence the two are not equivalent

1.2 Remove implications

$p \Rightarrow q \lor r$ becomes $\neg p \lor q \lor r$

$p \land \neg q \Rightarrow r$ becomes $\neg(p \land \neg q) \lor r$ which becomes $\neg p \lor q \lor r$

We get the same expression hence they are equivalent

1.3 Remove implications

$p \Rightarrow q \lor r$ becomes $\neg p \lor q \lor r$

$\neg p \land q \Rightarrow r$ becomes $\neg(\neg p \land q) \lor r$ which becomes $p \lor \neg q \lor r$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg p \lor q \lor r$	$p \lor \neg q \lor r$
T	F	F	F	T	T	F	T

Hence the two are not equivalent

1.4 Remove implications

$p \Rightarrow q \lor r$ becomes $\neg p \lor q \lor r$

$\neg r \land p \Rightarrow q$ becomes $r \lor \neg p \lor q$

Both are equivalent

1.5 Remove implications

$(p \Rightarrow (q \Rightarrow r))$ becomes $p \Rightarrow (\neg q \lor r)$ which becomes $\neg p \lor \neg q \lor r$

$p \land q \Rightarrow r$ becomes $\neg(p \land q) \lor r$ which becomes $\neg p \lor \neg q \lor r$

Both are equivalent

1.6

$((p \Rightarrow q) \lor (q \Rightarrow r))$ becomes $(\neg p \lor q) \lor (\neg q \lor r)$ which is always **True**

$(p \lor \neg p)$ is always **True**

Hence both are equivalent

Q2
2.1
{p ⇒ q ∨ r} ⊨ **(p ⇒ r)**
{p ⇒ q ∨ r} becomes **¬ p ∨ q ∨ r**
(p ⇒ r) becomes **¬ p ∨ r**

p	q	r	¬ p	¬ q	¬ r	¬ p ∨ q ∨ r	¬ p ∨ r
T	T	T	F	F	F	T	T
T	T	F	F	F	T	T	F
T	F	T	F	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	T	F	F	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	T	T	T

{p ⇒ q ∨ r} ⊨ **(p ⇒ r)** is false as there is a case when **{p ⇒ q ∨ r}** is True, **(p ⇒ r)** is False

2.2
{p ⇒ r} ⊨ **(p ⇒ q ∨ r)**
Truth table same as above
{p ⇒ r} ⊨ **(p ⇒ q ∨ r)** is True as everytime **{p ⇒ r}** is True, **(p ⇒ q ∨ r)** is also True

2.3
{p ⇒ q ∨ r, q ⇒ r} ⊨ **(p ⇒ r)**

p	q	r	¬ p	¬ q	¬ r	¬ p ∨ q ∨ r	¬ q ∨ r	¬ p ∨ r
T	T	T	F	F	F	T	T	T
T	T	F	F	F	T	T	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	F	T	T	T

F	T	F	T	F	T	T	F	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	T	T	T	T

Wherever both **p ⇒ q** ∨ **r** and **q ⇒ r** are True **(p ⇒ r)** is True
Hence, **{p ⇒ q** ∨ **r, q ⇒ r}** ⊨ **(p ⇒ r)** is True

Q3.
3.1 CNF of **(A ⇒ B) ⇒ C** is **(A** ∨ **C)** ∧ **(¬ B** ∨ **C)**
3.2 CNF of **A ⇒ (B ⇒ C)** is (**¬ A** ∨ **¬ B** ∨ **C)**
3.3 CNF of **(¬P ⇒ (P ⇒ Q))** is **True**

Q4 Given KB.
1. B ∧ C → A becomes ¬B ∨ ¬C ∨ A
2. B
3. D ∧ E → C becomes ¬D ∨ ¬E ∨ C
4. E ∨ F
5. D ∧ ¬F

Since we want to entail A add ¬A
6. ¬A

Resolving (1) and (2) we get (¬C ∨ A)
Resolving (¬C ∨ A) and (3) we get (¬D ∨ ¬E ∨ A)
Resolving (¬D ∨ ¬E ∨ A) and (4) we get (¬D ∨ A ∨ F)
Resolving (¬D ∨ A ∨ F) and (5) we get (A)
Resolving (A) and (6) we get empty
Hence the knowledge base can entail A

Q5 Given KB
1. A
2. B
3. D
4. ¬A ∨ ¬B ∨ C
5. ¬C ∨ ¬D ∨ E

Since we want to entail E add ¬E
6. ¬E

Resolving (4) and (1) we get (¬B ∨ C)
Resolving (¬B ∨ C) and (2) we get (C)
Resolving (C) and (5) we get (¬D ∨ E)
Resolving (¬D ∨ E) and (3) we get (E)
Resolving (E) and (6) we get empty
Hence the knowledge base can entail E

Q6.
State variables: S = {a, b, c, d}
Initial State: I = {a, b}
Goal State: G = {b, d}
Actions (format: <parameters, precondition, effect>):
O1: < ∅, {a, b}, {¬b, c}>
O2: < ∅, {a, b}, {¬a, ¬b, d}>
O3: < ∅, {c}, {b, d}>

6.1. Yes we can reach the goal state by performing O1 and O3 in the same order.
6.2. With {a, b} O1 and O2 actions can be done.
When O1 is performed we get {¬b, c}
When O2 is performed we get {¬a, ¬b, d}
Both these states will be added to the search space when {a,b} is expanded by progression search.

Q7.
State variables: S = {a, b, c, d}
Initial State: I = {a, b}
Goal State: G = {b, d}
Actions (format: <parameters, precondition, effect>):
O1: < ∅, {a, b}, {¬b, c}>
O2: < ∅, {a, b}, {¬a, ¬b, d}>
O3: < ∅, {c}, {b, d}>

Q7.1. null or empty set or can't be regressed as effect has ¬b and goal has b
Q7.2. null or empty set or can't be regressed as effect has ¬b and goal has b
Q7.3. {c}