Problem Set 4

1.1) $P \Rightarrow q \vee r$

$\equiv -P \vee q \vee r$

$P \wedge q \Rightarrow r \equiv -(P \wedge q) \vee r$

$\equiv -P \vee -q \vee r$

They are not logically equivalent.

1.2) $P \Rightarrow q \vee r \equiv -P \vee q \vee r$

$P \wedge -q \Rightarrow r \equiv -(P \wedge -q) \vee r \equiv -P \vee q \vee r$

They both are logically equivalent.

1.3) $P \Rightarrow q \vee r \equiv -P \vee q \vee r$

$-P \wedge q \Rightarrow r \equiv -(-P \wedge q) \vee r \equiv P \vee -q \vee r$

They are not logically equivalent.

1.4) $P \Rightarrow q \vee r \equiv -P \vee q \vee r$

$-r \wedge P \Rightarrow q \equiv -(-r \wedge P) \vee q \equiv r \vee -P \vee q$

$\equiv -P \vee q \vee r$

They are logically equivalent.

1.5) $(P \Rightarrow (q \Rightarrow r)) \equiv P \Rightarrow (-q \vee r) \equiv -P \vee -q \vee r$

$P \wedge q \Rightarrow r \equiv -(P \wedge q) \vee r \equiv -P \vee -q \vee r$

They both are logically equivalent.

1.6b) $((P \Rightarrow q) \vee (q \Rightarrow r)) \equiv (\neg P \vee q \vee \neg q \vee r) \equiv$ ~~(¬P...)~~ True.

$P \vee \neg P \equiv$ True

Both are logically equivalent.

2.1) $P \Rightarrow q \vee r \equiv \neg P \vee q \vee r$ $\quad P \Rightarrow r \equiv \neg P \vee r$.

P	q	r	$\neg P \vee q \vee r$	$\neg P \vee r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	T	T	T
F	T	T	T	T
F	F	F	T	T

From the above truth table $P \Rightarrow r$ is not true at all times when $P \Rightarrow q \vee r$

Hence $P \Rightarrow q \vee r$ does not entail $P \Rightarrow r$

2.2) From the above truth table $P \Rightarrow q \vee r$ is always true when $P \Rightarrow r$. Hence

$P \Rightarrow r \models P \Rightarrow q \vee r$

2.3) $p \Rightarrow q \vee r \equiv -p \vee q \vee r$ $q \Rightarrow r \equiv -q \vee r$ $p \Rightarrow r \equiv -p \vee r$

p	q	r	$-p \vee q \vee r$	$-q \vee r$	$-p \vee r$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	F	T
F	T	T	T	T	T
T	F	F	F	T	F
T	F	T	T	T	T
T	T	F	T	F	F
T	T	T	T	T	T

$(-p \vee q \vee r) \wedge (-q \vee r)$
T
T
F
T
F
T
F
F

This entailment is logically correct because $-p \vee r$ is true where ever $(-p \vee q \vee r) \wedge (-q \vee r)$ is true.

3.1) $((A \rightarrow B) \rightarrow C) \equiv (-A \vee B) \rightarrow C$

$\equiv -(-A \vee B) \vee C$

$\equiv (A \wedge -B) \vee C$

$\equiv (A \vee C) \wedge (-B \vee C)$

3.2) $A \rightarrow (B \rightarrow C) \equiv A \rightarrow (-B \vee C)$

$\equiv -A \vee (-B \vee C)$

$\equiv (-A \vee -B) \vee C$

3.3) $(\neg P \rightarrow (P \rightarrow Q)) \equiv (-P \rightarrow (-P \vee Q))$

$P \vee (-P \vee Q) \equiv True \vee Q \equiv True$

4)
- $B \wedge C \rightarrow A \equiv -(B \wedge C) \vee A \equiv -B \vee -C \vee A$ — (1)
- B — (2)
- $D \wedge E \rightarrow C \equiv -(D \wedge E) \vee C \equiv -D \vee -E \vee C$ — (3)
- $E \vee F$ — (4)
- $D \wedge -F$ — (5)
- $-A$ (The one we have to entail) — (6)

Resolve (1) & (2) over B we get $-C \vee A$ — (7)

Resolve (3) & (7) over C we get $-D \vee -E \vee A$ — (8)

Resolve (8) and (4) over E we get $-D \vee F \vee A$ — (9)

Resolve (9) over D we get $F \vee A$ — (10)

Resolve (10) over F we get A — (11)

Resolve (11) and (6) over A $\rightarrow$ we get ϕ

5) A — (1)

B — (2)

D — (3)

$\neg A \vee \neg B \vee C$ — (4)

$\neg C \vee \neg D \vee E$ — (5)

$\neg E$ — (6) ($\neg E$ to pro E is entailed)

Resolve (1) and (4) over A to get $\neg B \vee C$ — (7)

Resolve (2) and (7) over B to get C — (8)

Resolve (5) and (8) over C to get $\neg D \vee E$ — (9)

Resolveve (3) and (9) over D to get E — (10)

Resolve (6) and (10) over E to get ϕ

6)

6.1) (I) $\{a,b\} \xrightarrow{O_1} \{c\} \xrightarrow{O_3} \{b,d\}$ (G)

The sequence of actions to be taken are O_1 and O_3

6.2)

$$
\begin{tikzcd}
 & \{a,b\} \arrow[dl, "O_1"'] \arrow[dr, "O_2"] & \\
\{c\} \arrow[d, "O_3"] & & \{d\} \\
\{b,d\} & &
\end{tikzcd}
$$

Hence the ~~following~~ the states added are above

7)

7.1) O_1 has a state of $\{-b, c\}$ as the effect. Since our goal node $\{b, d\}$ does not have $\{c\}$ as one of ~~[illegible]~~ its states, regression will not give any node.

7.2) O_2 has a state of $\{-a, -b, d\}$ as the effect. Our goal node has state variables $\{b, d\}$ which can not be produced with O_2. Hence regressing a over O_2 will not give us any node.

7.3) Regressing goal node over O_3 can lead to multiple states. The ~~[illegible]~~ only precondition being $\{c\}$. Hence all states satisfying this can be reached.

~~[illegible]~~, ~~[illegible]~~, $\{b, c, d\}$, ~~[illegible]~~, $\{b, c\}$, $\{c, d\}$, $\{c\}$

~~[illegible]~~