

A. Setup in word

To set in word: a. Go to the Insert tab and select Symbol



b. Click on 'More Symbols' and search for the desired symbol

c. Select symbol and click on 'AutoCorrect'

d. In the replace field write the name that you want to be replaced with the symbol

B. Intro

CNF – Conjunctive Normal Form

BDD – Binary Decision Diagram

1 - not

\vee - disjunctie (OR)

\wedge - conjunctie (AND)

\rightarrow - implicatie logica

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Reguli:

Comutativitate: $A \wedge G \equiv G \wedge A$
 $A \vee G \equiv G \vee A$

Asociativitate $(A \wedge G) \wedge H \equiv A \wedge (G \wedge H)$
 $(A \vee G) \vee H \equiv A \vee (G \vee H)$

Absorbtie: $A \wedge (A \vee G) \equiv A$

$$A \vee (A \wedge G) \equiv A$$

Distributivitate:

$$A \wedge (G \vee H) \equiv (A \wedge G) \vee (A \wedge H)$$

$$A \vee (G \wedge H) \equiv (A \vee G) \wedge (A \vee H)$$

Negatie dubla:

$$\neg\neg A \equiv A$$

De Morgan's:

$$\neg(A \wedge G) \equiv (\neg A \vee \neg G)$$

$$\neg(A \vee G) \equiv (\neg A \wedge \neg G)$$

Altele:

$$A \vee \neg A \equiv \text{true}$$

$$A \wedge \neg A \equiv \text{false}$$

$$A \vee \text{true} \equiv \text{true}$$

$$A \wedge \text{false} \equiv \text{false (Zero Laws)}$$

$$A \vee \text{false} \equiv A$$

$$A \wedge \text{true} \equiv A \text{ (Identity Laws)}$$

CNF = o conjunctie (\wedge) de disjunctii (\vee) de literali, clause

$$(x \vee \neg z) \wedge (\neg y) \wedge (y \vee z) - \text{CNF}$$

$$(x \vee \neg y \vee z) - \text{CNF}$$

$$x \vee \neg y \wedge z - \text{NOT CNF}$$

$$p \rightarrow q \Leftrightarrow \neg p \vee q$$

$$\neg(p \rightarrow q) \Leftrightarrow p \wedge \neg q$$

C. Exemple transformare in CNF

1. $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

x	y	z	$(x \vee y) \wedge (x \vee z)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

$$\begin{aligned}
2. (p \wedge q) \vee (p \wedge \neg q) &= ((p \wedge q) \vee p) \wedge ((p \wedge q) \vee \neg q) \\
&= ((p \vee p) \wedge (q \vee \neg q)) \wedge ((p \vee \neg q) \wedge (q \vee \neg q)) \\
&= p \wedge (q \vee \neg q) \wedge (p \vee \neg q) \\
&= p \wedge (p \vee \neg q) \\
&= p
\end{aligned}$$

$$\begin{aligned}
3. a \rightarrow (b \rightarrow c) &= a \rightarrow (\neg b \vee c) \\
&= \neg a \vee (\neg b \vee c) \text{ -- o clauza cu disjunctii}
\end{aligned}$$

a	b	c	$\neg a \vee \neg b \vee c$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

$$\begin{aligned}
4. \neg((a \wedge b) \vee ((a \rightarrow (b \wedge c)) \rightarrow c)) &= \neg(a \wedge b) \wedge \neg((a \rightarrow (b \wedge c)) \rightarrow c) \\
&= (\neg a \vee \neg b) \wedge \neg((\neg a \vee (b \wedge c)) \rightarrow c) \\
&= (\neg a \vee \neg b) \wedge \neg(\neg(\neg a \vee (b \wedge c)) \vee c) \\
&= (\neg a \vee \neg b) \wedge (\neg a \vee (b \wedge c)) \wedge \neg c \\
&= (\neg a \vee \neg b) \wedge (\neg a \vee b) \wedge (\neg a \vee c) \wedge \neg c \text{ (dam factor comun pe } \neg a) \\
&= \neg a \vee (\neg b \wedge b) \wedge (\neg a \vee c) \wedge \neg c \text{ ((} \neg b \wedge b) = F) \\
&= \neg a \wedge (\neg a \vee c) \wedge \neg c \text{ (abortie)} \\
&= \neg a \wedge \neg c
\end{aligned}$$

a	c	$\neg a \wedge \neg c$
T	T	F
T	F	F
F	T	F
F	F	T

$$\begin{aligned}
5. (a \rightarrow (b \vee c)) \rightarrow (a \wedge d) &= \neg (a \rightarrow (b \vee c)) \vee (a \wedge d) \\
&\equiv (a \wedge \neg (b \vee c)) \vee (a \wedge d) \\
&\equiv (a \wedge \neg b \wedge \neg c) \vee (a \wedge d) \\
&\equiv ((a \wedge \neg b \wedge \neg c) \vee a) \wedge ((a \wedge \neg b \wedge \neg c) \vee d) \text{ (absorbtie)} \\
&\equiv a \wedge ((a \wedge \neg b \wedge \neg c) \vee d) \\
&\equiv a \wedge (a \vee d) \wedge (\neg b \vee d) \wedge (\neg c \vee d) \text{ (absorbtie)} \\
&\equiv a \wedge (\neg b \vee d) \wedge (\neg c \vee d) \text{ (CNF)}.
\end{aligned}$$

<https://www.wolframalpha.com/input?i=CNF%28a+and+b%29>

D. Exemple transformare BDD

$$f = (a \wedge b) \vee (c \wedge d)$$

$$f \mid a=T \Leftrightarrow b \vee (c \wedge d)$$

$$f \mid b=T \Leftrightarrow T \vee (c \wedge d) = T$$

$$f \mid b=F \Leftrightarrow c \wedge d$$

$$f \mid c=T \Leftrightarrow d$$

$$f \mid d=T \Leftrightarrow T$$

$$f \mid d=F \Leftrightarrow F$$

$$f \mid c=F \Leftrightarrow F$$

$$f \mid a=F \Leftrightarrow c \wedge d$$

$$f \mid c=T \Leftrightarrow d$$

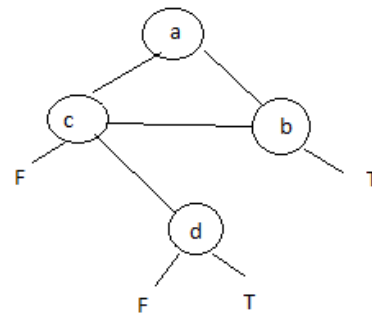
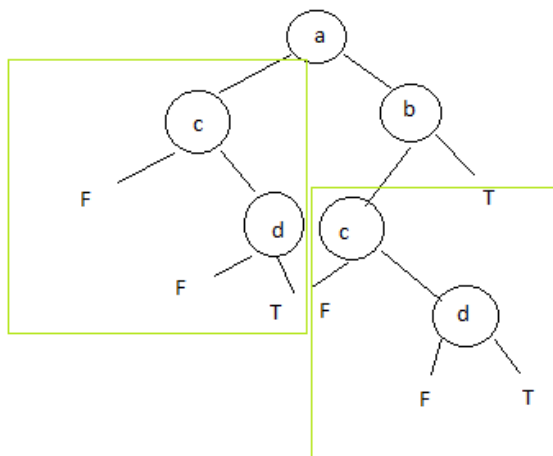
$$f \mid d=T \Leftrightarrow T$$

$$f \mid d=F \Leftrightarrow F$$

$$f \mid c=F \Leftrightarrow F$$

stanga – false

dreapta - true



Tema:

1. transformati in CNF: $(p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (r \rightarrow q))$
2. transformati in CNF: $(p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))$
3. BDD pentru: $((x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2))$