CPE 312

2023/2024

1a. Errors are variations in the values obtained from the same measurement. Error is also the difference between the measured value and the true or actual value of a quantity.

Types of Error,

⇒ Determinate
- Constant error
- Systematic Error (Instrumental, Operative, Personal, Methodical errors)

⇒. Inderminate errors

1b. Given, $I = 100mA$ (full scale)

$I_m = 1mA$ (meter current)

$R_m = 100\Omega$ (meter resistance)

Shunt Resistance, $R_s = \frac{I_m \times R_m}{I - I_m} = \frac{1 \times 100}{100 - 1} = \frac{100}{99}$

$= 1.01\Omega$

1c i Environmental factors (temperature, humidity)

ii Instrument calibration errors.

iii Parallax or observational errors.

iv Electric noise or electromagnetic interference.

2a. Given, $I = 10A \pm 0.1A$

$R = 100\Omega \pm 2\Omega$

$P = I^2R = (10)^2 \times 100 = 10000W$

$$\frac{\Delta P}{P} = 2\left(\frac{\Delta I}{I}\right) + \left(\frac{\Delta R}{R}\right)$$

$$= 2\left(\frac{0.1}{10}\right) + \left(\frac{2}{100}\right)$$

$$= 0.02 + 0.02$$

$$\frac{\Delta P}{P} = 0.04$$

$$\Delta P = 0.04 \times 10000$$

Probable error $= 400W$

$= \pm 400W$

2b. An instrument is a device used to measure, display or record physical ~~or electrical~~ quantities.

Categories (electronic instrument)

i Absolute instruments : Gives the value of the quantity to be measured in terms of constants of the instrument and their deflection e.g a Tangent galvanometer

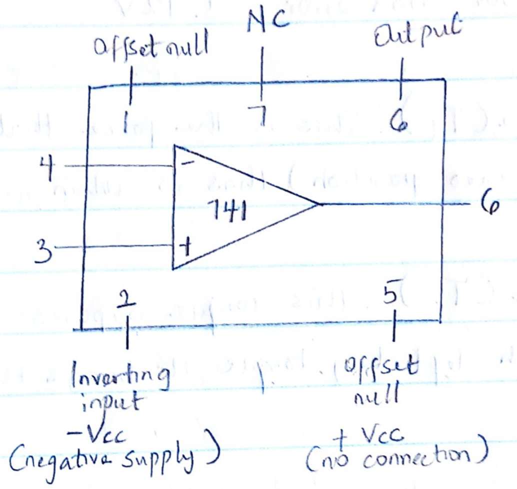
ii Secondary Instruments : Needs to be pre-calibrated by comparison with an absolute instrument.

2c $$R_{sh} = \frac{I_m \cdot R_m}{I - I_m}$$

3a. It's an inverting amplifier. where A_v = Differential voltage gain.

$$A_v = \frac{-R_f}{R_{in}}$$

3b.



It is an 8-pin IC commonly used for analog signal amplification. Each pin has a specific function

1 – used to eliminate offset voltage
2 – Signal applied here gets inverted at the output
3 – Signal applied here appears in phase at the output
4 – Connect to negative power supply
5 – Same as pin 1
6 – Amplified signal output.
7 – Connect to positive power supply
8 – NC (no connection) – Internally not connected to anything

3c. Measurement is the act of quantitative comparison between a predefined standard and an unknown magnitude.

4a. Turns = 100

Width = 10 mm = 0.01 m.

Depth = 30 mm = 0.03 m.

$B = 0.1\ Wb/m^2$.

$I = 10\,mA = 0.01\,A$.

Deflecting Torque

$T = B \cdot I \cdot N \cdot A$

$= 0.1 \times 0.01 \times 100 \times (0.01 \times 0.03)$

$= 0.00003\ Nm$

4b. Limiting error $= \pm 2\%$ of 600

$= \pm 0.02 \times 600$

$= \pm 12V$

So, even at 250V, max error $= \pm 12V$

4ci Deflecting Torque (T_d): This is the force that deflects the pointer from its rest (zero position) that is, when not being used for measurement.

ii Controlling Torque (T_c): This torque opposes the deflecting torque and increases with deflecting torque, it brings the pointer to rest at the correct position

5a. Meter movement = 1mA

Internal resistance = 100Ω

$$R_{sh} = \frac{I_m \times R_m}{I - I_m}$$

for 10mA range

$$R_{sh} = \frac{1 \times 100}{10-1} = \frac{100}{9} = 11.11\Omega$$

for 20mA range

$$R_{sh} = \frac{1 \times 100}{20-1} = \frac{100}{19} \approx 5.26\Omega$$

for 50mA range

$$R_{sh} = \frac{1 \times 100}{50-1} = \frac{100}{49} \approx 2.04\Omega$$

5b Full-scale deflection current $= 50\ \mu A$

Internal resistance $= 500\Omega$

Voltmeter range $= 0-10V$

$$R_{mult} = \frac{V}{I} - R_m$$

$$= \frac{10}{50\times10^{-6}} - 500$$

$$= 200000 - 500$$

$$= 199500\Omega$$

5ci <u>Measurand</u>: The physical quantity or property being measured.

ii <u>Transducer</u>: A device that converts one form of energy into another (e.g pressure to electrical signal).

iii <u>Signal conditioner</u>: A device that modifies a signal to make it suitable for processing (e.g, amplification, filtering)

6ai ~~I = V~~ given

Shunt $= 0.02\Omega$, coil resistance $= 1K\Omega$

voltage for full deflection $= 0.5V$

i $I = \frac{V}{R} = \frac{0.5}{1000} = 0.0005A = 0.5mA$

ii For 10A

$$R_{sh} = \frac{I_m \times R_m}{I - I_m} = \frac{0.0005 \times 1000}{10 - 0.0005} = \frac{0.5}{9.9995} \simeq 0.050\Omega$$

$$I_{shunt} = 10 - 0.0005 = 9.995A$$

For 75 A

$$R_{sh} = \frac{0.5}{75 - 0.0005} \approx 0.00667\Omega$$

$$I_{shunt} = (75 - 0.0005) = 74.9995\,A$$

6b Done (2b)

6c Done (4c)

7a. $T \propto I^2$ $\quad\quad I_1 = 10A,\ \theta_1 = 90^\circ$

θ (Deflection) $\propto T$ $\quad\quad I_2 = 3A$

For Spring-controlled

$\cancel{\theta_1 = 90^\circ}$ $\theta \propto I^2$

$$\frac{\theta_1}{\theta_2} = \frac{I_1^2}{I_2^2}$$

$\theta_1 = 90^\circ,\ I_1 = 10\,A$

$I_2 = 3A,\ \theta_2 = ?$

$$\frac{90}{\theta_2} = \frac{10^2}{3^2}$$

$$\theta_2 = \frac{90 \times 9}{100}$$

$$\theta_2 = 8.1^\circ$$

For Gravity Cotrolled

$T \propto I^2$

$I^2 \propto \sin\theta$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{I_1^2}{I_2^2}$$

$$\frac{\sin 90}{\sin\theta_2} = \frac{10^2}{3^2}$$

$$\frac{1}{\sin\theta_2} = \frac{100}{9}$$

$$100\sin\theta_2 = 9$$

$$\sin\theta_2 = \frac{9}{100}$$

$$\sin\theta_2 = 0.09$$

$$\theta_2 = \sin^{-1}(0.09)$$

$$\theta_2 = 5.16^\circ$$

7b Moving Iron Instruments

-> Attraction type.

-> Repulsion type.

The operation of the attration type depends on the attraction of a single piece of soft iron into a magnetic field.

The repulsion type depends on the repulsion of two adjacent pieces of iron magnetized by the same magnetic field.

7c. Secondary instrument are instruments that require calibration using a standard to give accurate measurements (e.g voltmeters, ammetas)

2020/2021

1a. Calibration is the art and science of determining the uncertainty associated with measurement, and, if possible, reducing that uncertainty.

Three parts

-> Verifying that the current measurement capability of the measuring device is within specification.

-> Adjusting the device to reduce its measurement uncertainty.

-> Verifying the measurement capability of the device to check if the measurement uncertainty has been reduced.

1bi Accuracy: The closeness of the measured value to the true value.

ii Precision: The degree of consistency among repeated measurements under the same conditions. It doesnot imply accuracy.

1c. AC Bridge Problem.

Arm AB: Capacitor $C_1 = 0.8\,\mu F$ in parallel with $1\,K\Omega$

Arm BC: $3\,K\Omega$

Arm CD: Unknown capacitor C_x in parallel with unknown Resistor R_x

Arm DA: Capacitor of $0.4\,\mu F$.

$f = 1\,KHz = 1000\,Hz$.

For an AC bridge.

$$\frac{Z_{AB}}{Z_{CD}} = \frac{Z_{BC}}{Z_{DA}}$$

$$Z_{AB} \cdot Z_{DA} = Z_{CD} \cdot Z_{BC}$$

Express Known impedance.

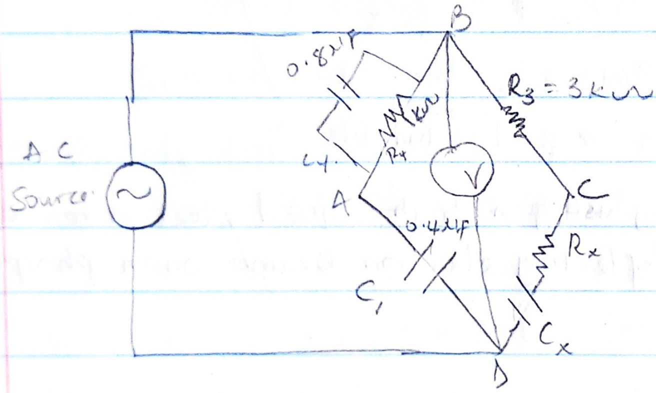
$R = 1000\Omega$

$C = 0.8\,\mu F$

$w = 2\pi f = 2\pi(1000) = 6283.2$ rad/s

$X_c = \frac{1}{wC} = \frac{1}{6283.2 \times 0.8 \times 10^{-6}} \approx 199\Omega$

$\frac{1}{Z_{AB}} =$



i) $C_x = C_1 \left(\frac{R_4}{R_3}\right)$

$= \frac{0.4 \times 10^{-6} \times 1 \times 10^3}{3 \times 10^3}$

$= 1.3 \times 10^{-7} F$ or $0.13\,\mu F$

ii) $R_x = R_3 \left(\frac{C_4}{C_1}\right)$

$= \frac{3 \times 10^3 \times 0.8 \times 10^{-6}}{0.4 \times 10^{-6}}$

$= 6\,k\Omega$

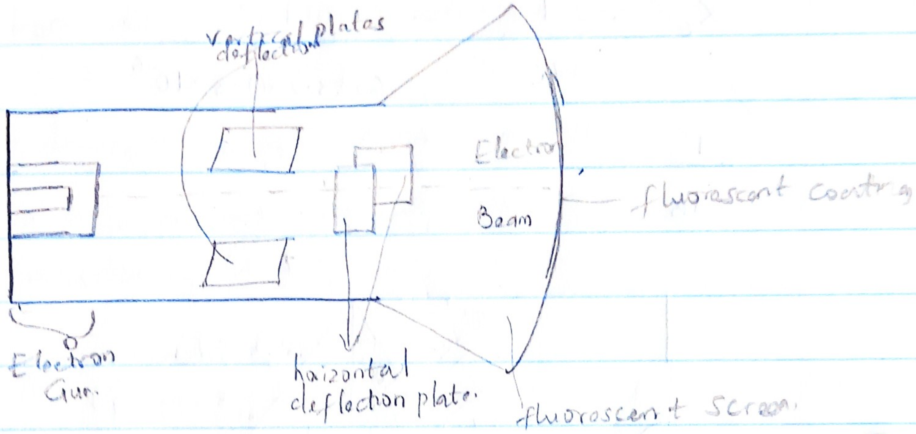
$$\text{Dissipation} = \omega C_x R_x$$

$$= 2\pi \times f \times C_x \times R_x$$

$$= 2\pi \times 1000 \times 0.13 \times 10^{-6} \times 6000$$

$$= 4.9$$

2a.



A CRO works by ~~using a high speed electron beam~~ accelerating and deflecting electron beams on a phosphorescent screen.

2b. Shunt (Ammeter) in parallel

$$S = \frac{I - I_m}{I_m} \cdot R_m$$

Multiplier (voltmeter) in series

$$R = \frac{V - V_m}{I_m} = \left(\frac{V}{V_m} - 1\right) R_m$$

Let I = total current measured

I_m = full-scale deflection current of the meter

R_m = internal resistance of the meter

S = shunt resistance

Since the ammeter and shunt are in parallel, the voltage across them is the same :

$$V_m = I_m R_m = V_s \, I_s S$$

But, $I_s = I - I_m$

$$I_m R_m = (I - I_m) S$$

$$\therefore S = \frac{I_m R_m}{I - I_m}$$

V = total voltage to be measured.

V_m = voltage across the meter (at full-scale deflection)

I_m = full-scale current of the meter.

R_m = internal resistance of the meter.

R = Series multiplier resistance

$$V = I_m (R_m + R) \quad [\text{Same Current}]$$

$$R = \frac{V}{I_m} - R_m.$$

or using $V_m = I_m R_m$.

$$R = \left(\frac{V}{V_m} - 1 \right) R_m.$$

*2c. full-scale deflection : 1Ω internal resistance

Shunt : $\frac{1}{4999} \Omega$

$$I = \frac{V}{R} = \frac{250}{4999 + 1} = 0.05A$$

$$= 50 \text{ mA}.$$

~~3a~~ 2c. $R = (4999 + 1)$ ohm.

$V = 250V$

$$I = \frac{V}{R} = \frac{250}{5000} = 0.05A$$

Current through the shunt, $I_{sh} = \frac{I_m R_m}{R_{sh}}$

$$= \frac{0.05 \times 1}{1/499}$$

$$= 25A$$

ii. Full-scale deflection current $= 50A$

$$R_{sh} = \frac{R_m}{\frac{I}{I_m} - 1}$$

$$= \frac{5000}{\frac{50}{0.05} - 1}$$

$$= \frac{5000}{999}$$

$$= 5.005\,\Omega$$

3ai. Two basic methods of measurement.

-> Direct Measurement - Using instruments directly e.g voltmeter

-> Indirect measurement :- Calculated via formulas using measured quantities.

ii. Maxwell's Bridge :

Advantages

- Suitable for wide range of inductances.
- Simple to balance.

Disadvantages.

- Requires a standard capacitor.
- Not suitable for low Q-factor inductors.

3b. $V_{out} = V_C - V_D = V_{R2} - V_{R4} = 0$ $\quad V_C = V_D$ and $V_{R2} = V_{R}$

$$R_L = \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_D = \frac{R_4}{R_3 + R_4}$$

At balance: $R_C = R_D$

$$\frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4}$$

$$R_2 (R_3 + R_4) = R_4 (R_1 + R_2)$$

$$R_2 R_3 + R_2 R_4 = R_4 R_1 + R_4 R_2$$

$$R_2 R_3 = R_4 R_1$$

$$\therefore R_4 = \frac{R_2 R_3}{R_1} = R_x$$

i.e. $R_x = \frac{R_2 R_3}{R_1}$ where resistors, R_1 and R_2 are known or present values.

3c. $$R_x = \frac{R_2 R_3}{R_1}$$

$$R_2 R_3 = R_1 R_x$$

$$R_3 = \frac{R_1 R_x}{R_2}$$

$$= \frac{500 \times 100}{10}$$

$$= 5000\Omega$$

4a) Attraction: Soft iron attracted to coil field (e.g moving iron).
Repulsion: Two similar poles repel each other for movement.

* ii Moving Iron Instrument:

4b. PMMC (Permanent Magnet Moving coil)

Advantage:
- High accuracy.
- Linear scale.

Disadvantages:
- Expensive
- Limited to DC.

4c *

For Current, $I = 1.7A$

At $1.6A$, $\theta = 61.5°$

At $1.8A$, $\theta = 74.5°$

At $1.7A$, $\theta = 61.5 + \frac{1.7 - 1.6}{1.8 - 1.6} \times (74.5 - 61.5)$

$= 68.0°$

To radians, $\theta = \frac{68 \times \pi}{180} = 1.1868$ radians.

From the table.

At $1.6A$: $T = K\theta$ -2.

$T \propto I^2$.

$T = K I^2$

$K = \frac{\text{Constant} \times I^2}{T}$

$$\frac{dL}{d\theta} = \frac{578.3 - 577.8}{68 - 61.5}$$

$= 0.077\mu H/\text{degree}$

$= 4.4 \mu H/\text{rad}$

Deflecting torque, $T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$

$= \frac{1}{2}(1.7)^2 \times 4.4 \times 10^{-6} = 6.36 \times 10^{-6}$ N.m

Spring constant $K = \frac{T_d}{\theta} = \frac{6.36 \times 10^{-6}}{1.1868}$

$= 5.36 \times 10^{-6}$ N-m/rad

5ai Importance of measurement.

- It enables comparison of physical quantities
- It ensures product quality and compliance with standards

ii Three benefits of calibration.

- Ensures accuracy of instruments.
- Minimizes errors in readings.
- Compliance with regulations and standards

5bi Instrumental Errors: Those arise from imperfections or faults in the measuring instrument itself e.g zero-error in voltmeter.

ii Environmental Errors: Caused by external environmental conditions affecting the measurement. e.g temperature changes iii **Observational Errors**: This results from human mistakes during measurement or reading e.g Parallax error

iv **Theoretical Errors**; Come from assumptions or simplications made in the theory or method of measurement.

5c.

$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{\sqrt{X_L}}$$

$$= \frac{1}{R} + \frac{j}{X_C} = \frac{1}{R_1} + jwC$$

$$\frac{1}{Z_1} = \frac{1 + JwCR_1}{R_1}$$

$$Z_1 = \frac{R_1}{1 + jwCR_1}$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_x + jwL_x$$

Balance condition is $Z_1 Z_4 = Z_2 Z_3$

$$\frac{R_1}{1+jwCR_1} (R_x + jwL_x) = R_2 R_3$$

$$R_1 R_x + jwL_x R_1 = R_2 R_3 \times jwCR_1 R_2 R_3$$

Separating the real and imaginary parts, we have.

$$R_3 = \frac{R_1 R_x}{R_2} \quad \text{and} \quad L_x = C_1 R_2 R_3.$$

6a i Uses of Wheatstone bridge:

- Measuring unknown resistance.
- Calibration of measuring instruments.

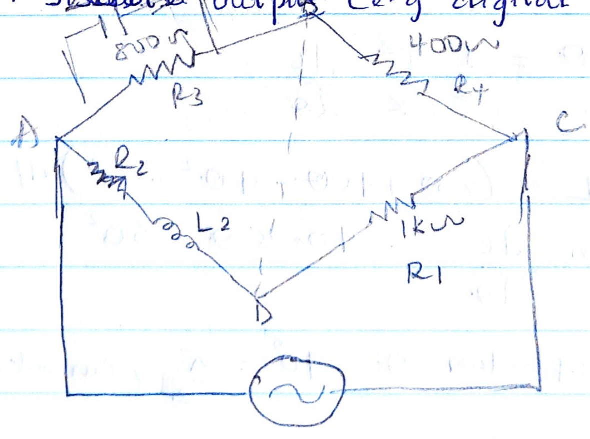
ii Three main steps in calibration (Done - 1a)

6b i Active instruments requires external power e.g electronic voltmeter. while; Passive do not require external power e.g Spring balance.

ii Null instruments: Balance condition measured e.g Potentiometer. Deflection: Quantity causes a physical movement e.g ammeter.

iii Analog: Continuous output e.g analog voltmeter. Digital: ~~Discrete~~ output (e.g digital multimeter)

6c



$R_1 R_3 = R_2 R_4$

$R_2 = \frac{R_1 R_3}{R_4} = \frac{1000 \times 800}{400}$

$= 2000\Omega$

$R_2 = 2000\Omega$

$L_2 = C R_1 R_3$ or $C R_2 R_4$

$= 2 \times 10^{-6} \times 1000 \times 800$

$= 1.6H$

7a i **Accuracy** :: How close the meter's reading is to the true value.

ii **Sensitivity**: Ability of the meter to detect small changes in the input quantity.

iii **Range**: The span of values a meter can measure accurately, min to max.

7a II → Controlling force restores pointer to zero after measurements, ensures stability

→ Damping force ~~allows~~ prevents oscillations, allows quick steady-state reading.

7b i $R_s = \dfrac{V}{I_s} = \dfrac{100 \times 10^{-3}}{100} = 1\,m\Omega$ or $0.001\,\Omega$

ii $R_m = \dfrac{V}{I_m} = \dfrac{1000}{10 \times 10^{-3}} = 100\,k\Omega$

7c. deflection $\theta = \dfrac{1}{2}\dfrac{I^2}{K}\dfrac{dL}{d\theta}$

$$L = (200 + 40\theta - 4\theta^2 - \theta^3)\,\mu H/\ldots$$

Then $\dfrac{dL}{d\theta} = 40 - 8\theta - 3\theta^2$

for a deflection $\theta = 90° = \dfrac{\pi}{2}$, current, $I = 1.5A$.

$$\frac{\pi}{2} = \frac{1}{2} \times \frac{1.5^2}{K} \times \left[40 - 8\left(\frac{\pi}{2}\right) - 3\left(\frac{\pi}{2}\right)^2\right]$$

$K = 14.348$ N-m/rad

Now for a current of 1A, the angular deflection is

$$\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$$

$$\theta = \frac{1}{2} \times \frac{1^2}{14.348} \times (40 - 8\theta - 3\theta^2)$$

$$40 - 8\theta - 3\theta^2 = 28.696\,\theta$$

$$3\theta^2 + 36.696\theta - 40 = 0$$

$$\theta = 1.00712\,\text{rad} = 57.7^\circ$$