Notation

available measurement — true measurement — measurement noise ← "random variable"

$$\tilde{y} = y + v$$

i.e. $v \sim \mathcal{N}(0, 0.05)$

normal (gaussian) — mean — std dev

$$\tilde{x} = \hat{x} + e$$

measured state — estimated state — residual error

We can represent time series as:

$$y(t) = \underline{y} = \mathbf{y} = \begin{bmatrix} y(t_0) \\ y(t_1) \\ \vdots \\ y(t_m) \end{bmatrix}$$

By default, vectors will always be columns.

Capital letters will always be matrices, ie A

mean residual error: $\mu = \frac{1}{m} \sum_{i=1}^{m} [\tilde{y}(t_i) - \hat{y}(t_i)]$

variance residual error: $\sigma^2 = \frac{1}{m-1} \sum_{i=1}^{m} \{[\tilde{y}(t_i) - \hat{y}(t_i)] - \mu\}$

std dev residual error: $\sigma = \sqrt{\sigma^2}$

Motivating Example: Static Parameter

Static Parameter Estimation [continuos time, discrete meas.]
Given noisy measurements $\tilde{y}(t)$, estimate static parameters $\underline{x}$.

$$\tilde{y}(t) = t + \sin(t) + 2\cos(2t) - \frac{0.4\, e^{t}}{10^{-4}} + v(t)$$

We can write 1.1 compactly as:

$$\underline{\tilde{y}} = H\,\underline{x} + \underline{v}$$

coefficients for basis functions

Measurements in time

$$\begin{bmatrix} \tilde{y}(t_0) \\ \tilde{y}(t_1) \\ \vdots \\ \tilde{y}(t_m) \end{bmatrix} = \begin{bmatrix} h_1(t_0) & h_2(t_0) & \cdots & h_n(t_0) \\ h_1(t_1) & h_2(t_1) & \cdots & h_n(t_0) \\ \vdots & & & \vdots \\ h_1(t_m) & \cdots & & h_n(t_m) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} v(t_0) \\ v(t_1) \\ \vdots \\ v(t_m) \end{bmatrix}$$

basis functions

noise

Specific example:

$$\tilde{y}(t) = t + \sin(t) + 2\cos(2t) - \frac{0.4e^{t}}{1\times 10^{4}} + v(t)$$

Compact form:

$$\tilde{\underline{y}}(t) = \begin{bmatrix} t & \sin(t) & \cos(2t) & e^{t} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -0.4\times 10^{-4} \end{bmatrix} + \underline{v}(t)$$

all rows are identical
1 row for each measurement

We will use this example for the first 2 weeks. Let's practice building this example in python & set standards for plotting.