ECON20210 (S25): The Elements of Economic Analysis III Honors

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1 Introduction

We will view the neoclassical growth model as a benchmark.

1.1 Measurement

GDP captures the total amount of production, incomes, or expenditures. More formally, GDP is the dollar amount of "final" goods and services produced per unit of time. It is a flow.

Heuristically, we have:

- GDP captures how well the local economy is doing.
- GNP captures how well the nationals in a country are doing.
- Nominal and real GDP:
  - Nominal GDP values goods and services at current prices. $Y_t^n = \sum_i P_{i,t} Q_{i,t}$.
  - Real GDP values goods and services at **constant** prices. $Y_t^r = \sum_i P_{i,0} Q_{i,t}$, where $P_{i,0}$ is the price of good i in the base year.
  - The GDP deflator is the ratio of nominal to real GDP. $P_t = Y_t^n / Y_t^r$. This is the Paasche index.

As an measurement of expenditure, we have

$$Y = C + I + G + EX - IM,$$

where

- C is consumption purchases by households ($\sim 70\%$),
- I is investment (purchases of new capital goods by businesses, $\sim 15\%$),
- G is government spending,
- $NX = EX - IM$ is net exports (what foreigners purchase net of what we buy from them, $\sim -5\%$).

As an measurement of income, we have

$$Y = wL + \pi + rK + T,$$

where

- wL is wage and compensations to workers $\sim 66\%$,
- π are corporate profits, rK are compensations to capital owners. ($\pi + rK$ take up $\sim 35\%$),
- T are taxes.

As an measurement of output, we may think

$$Y = f(A, K, L, X),$$

where

- A is technology, K is capital, L is labor, X are other factors.

1.2 Growth Rate

Discrete growth rate is

$$\gamma = \frac{Y_{t+1} - Y_t}{Y_t}, \quad Y_{t+1} = (1+\gamma)Y_t.$$

If Y is exponentially growing, we have by using the approximation $\log(1+\gamma) \approx \gamma$ that

$$\log Y_{t+1} - \log Y_t \approx \gamma.$$

1.3 Consumer Price Index

Fix basket in base year Q_0 and trace the cost of the basket in year t:

$$X_t \coloneqq \sum_i P_{i,t} Q_{i,0}.$$

The price index in year t is then defined as

$$P_t \coloneqq \frac{X_t}{X_0} = \sum_i \frac{\sum P_{i,0} Q_{i,0}}{\sum_j P_{j,0} Q_{j,0}} \frac{P_{i,t}}{P_{i,0}},$$

a weighted average of the individual inflation rates.

- This is called the Laspeyres index.
- The weights $\sum P_{i,0} Q_{i,0} / \sum_j P_{j,0} Q_{j,0}$ are the expenditure shares of the relevant goods in the base year basket.
- If we fix instead the quantities to that in the current year, we get the GDP deflator, or the Paasche index.
- We are ignoring the possibility of substitution between goods, or new goods being introduced, or old goods being made better. Thus:

Proposition 1.1. *The numerator of the Laspeyres index is a Taylor approximation to $C(u_0, P_t)$. The denominator of the Paasche index is a Taylor approximation to $C(u_t, P_0)$. The Laspeyres index overestimates the "true" inflation rate, while the Paasche index underestimates it.*

The **"true" inflation rate** should measure the cost of achieving the same level of utility as in the base year. Thus it is

$$X_t = \frac{C(u_0, P_t)}{C(u_0, P_0)},$$

where $C(u, P)$ is the Hicksian cost function. The Laspeyres index is then the Slutsky approximation to the true inflation rate, where $C(u_0, P_0)$ is approximated by $P_t \cdot x(u_0, P_0)$.

Proof. It is clear from the above that this is an over approximation of $C(u_0, P_t) = P_t \cdot x(u_0, P_t)$.

Alternative, note that C is concave in P, and $P_t \cdot x(u_0, P_0)$ is a first order Taylor approximation to C starting at P_0:

$$C(u_0, P_t)|_{P_t=P_0} \approx C(u_0, P_0) + \sum \left.\frac{\partial C}{\partial P_i}\right|_{P_t=P_0} (P_t - P_0).$$

By Shephard's lemma we have $\partial C(u_0, P_0)/\partial P_{i,0} = Q_{i,0}(u_0, P_0)$ and so

$$C(u_0, P_t) \approx C(u_0, P_0) + \sum Q_{i,0}(P_{i,t} - P_{i,0}) = \sum P_{i,t} Q_{i,0}.$$

$\square$

The Fisher ideal index is a middle ground:

Definition 1.2.

$$\left(\frac{\sum_i p_{i,t} q_{i,0}}{\sum_i p_{i,0} q_{i,0}}\right)^{\frac{1}{2}} \left(\frac{\sum_i p_{i,t} q_{i,t}}{\sum_i p_{i,0} q_{i,t}}\right)^{\frac{1}{2}}.$$

Note that this geometric average is equivalent to the arithmetic average of the net inflation rates. (To see this, take log on both sides).

Definition 1.3. The **chain price index** is calculated as

$$\frac{P_{t+k}}{P_t} = \frac{P_{t+1}}{P_t} \cdots \frac{P_{t+k}}{P_{t+k-1}}.$$

Definition 1.4.

$$\text{Unemployment rate} = \frac{\text{Unemployment}}{\text{Labor Force}}.$$

$$\text{Labor force participation rate} = \frac{\text{Labor Force}}{\text{Adult Population}}.$$

2 Economic Growth

Kaldor Facts:

(i) The share of labor incomes and the share of capital incomes in the national income have been roughly constant:

$$\frac{wN}{Y} \approx \frac{2}{3}; \quad \frac{r^k K}{Y} \approx \frac{1}{3}.$$

(ii) Capital per worker K/N and output per worker Y/N rise steadily at the same rate (around $2 \sim 2\%$).

(iii) Capital output ratio K/Y remains constant. (Implied by the previous point.)

(iv) The rate of return on capital r^k is roughly constant. (Implied by the points 1 and 3.)

Kotaro facts:

- “conditional catch-up”: poor countries grow faster than rich countries, conditional on institution (e.g., OECD countries, US states).