ECON20110 (W25): The Elements of Economic Analysis II Honors

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1 Review

1.1 Constrained Maximization

E.g.,

$$\max_{\mathbf{x}} U(\mathbf{x}, \boldsymbol{\theta}) \quad \text{s.t.} \quad G(\mathbf{x}, \boldsymbol{\theta}) \geq 0.$$

Solving a whole class of optimization problems parameterized by $\tilde{\theta}$ generates two functions:

- The solution function
- The Value function

Results like the envelope theorem relates these two functions.

1.2 The Kuhn-Tucker Theorem

Consider the maximization function $\max_x f(x)$. The first order condition gives $f'(x^*) = 0$. Now suppose that x_1 is such that $f'(x_1) > 0$. We may be temped to argue that x_1 is not a solution since we can increase f by increasing the value of x, but this assumes that x is in the interior of the domain. Thus the first order condition addresses only interior solutions. The Kuhn-Tucker theorem addresses this issue.

Theorem 1.1 (Kuhn-Tucker)**.** *The FOCs for the constrained optimization problem*

$$\max_{\mathbf{x}} U(\mathbf{x}, \boldsymbol{\theta}) \quad \textit{s.t.} \quad G(\mathbf{x}, \boldsymbol{\theta}) \geq 0.$$

are:

- *for each* $i = 1, \ldots, n$*:* $\partial \mathcal{L}/\partial x_i \leq 0$ *and* $x_i \geq 0$*, with complementary slackness;*
- $\partial \mathcal{L}/\partial \lambda \geq 0$ *and* $\lambda \geq 0$*, with complementary slackness.*