```
dec lt : num -> list num;
dec lst: num -> list num;
dec cabeca: list num -> num;
dec cauda : list num -> num;
dec cauda2 : list num -> list num;
dec ultimo : list num -> num;
dec arranjo: list num -> list num;
--- lst 0 <= [];
--- lst n <= n :: lst (n - 1);
--- cabeca [] <= error ("Lista vazia");
--- cabeca (x :: xs) <= x;
--- cauda [] <= error ("Lista Vazia");
--- cauda2 [] <= error ("Lista Vazia");
--- cauda2 x <= [];
--- cauda2 (x :: xs) <= xs;
--- ultimo [] <= error ("Lista vazia");
--- ultimo ([x]) <= x;
--- ultimo (x :: xs) <= ultimo xs;
--- arranjo [] <= error ("Lista vazia");
--- arranjo ([x]) <= [];
--- arranjo (x :: xs) <= x :: arranjo xs;
```

```
dec faixa: num # num # num -> list num;
dec comp: list num # (num -> truval) -> list num;
dec listapot : num # list num -> list num;
--- faixa (i, f, p) <= if i > f then [] else i :: faixa (i + p, f, p);
--- comp ([], qualificador) <= [];
--- comp (x :: xs, qualificador) <= if qualificador x then x :: comp (xs, qualificador)
else comp (xs, qualificador);
--- listapot (n, []) <= [];
--- listapot (n, x :: xs) \le pow(x, n) :: listapot(n, xs);
uses base1, listas;
dec multiplo : num # num -> truval;
dec divisor : num -> list num;
dec listamult : list num # num -> list num;
dec divisor2 : num -> list num;
dec checa_primo : num -> truval;
dec lprimos : num -> list num;
dec ehPrimoAux : num # num -> truval;
dec ehPrimo : num -> truval;
dec lprimos2 : num -> list num;
--- multiplo (n, m) \le if n \mod m = 0 then true else false;
--- divisor n \le comp (faixa (1, n, 1), lambda d = multiplo(n, d));
```

```
--- listamult ([], n) <= [];
--- listamult (x :: xs, n) \le if n \mod x = 0 then x :: listamult (xs, n) else listamult (xs,
n);
--- divisor2 n <= listamult (faixa (1, n, 1), n);
--- checa_primo 1 <= false;
--- checa_primo 2 <= true;
--- checa_primo n \leq if tamanho (listamult (faixa (2, n - 1, 1), n)) > 0 then false else
true;
--- lprimos n <= comp (faixa (1, n, 1), lambda x => checa_primo x);
--- ehPrimoAux (p, q) \leq if pow (q, 2) \leq p then true else if p mod q = 0 then false else
ehPrimoAux (p, q + 1);
--- ehPrimo p \le if p \le 2 then false else ehPrimoAux (p, 2);
--- lprimos2 n \le if n \le 2 then [] else if ehPrimo (n - 1) then lprimos2 (n - 1) \le [n - 1]
else lprimos2 (n - 1);
dec x_pi : num;
dec acirc: num -> num;
dec soma: num # num -> num;
--- x_pi <= 3.14159;
--- acirc r <= x_pi * pow (r, 2);
```

```
--- soma (x, y) \le x + y;
uses recursao;
dec fatbase: num # num -> num;
dec fat2 : num -> num;
dec fibo2 : num # num # num -> num;
dec fib2 : num -> num;
dec par : num -> truval;
dec base2_1 : num -> num;
dec base2x: num # num -> num;
dec base2_2 : num -> num;
dec quantasVezes : num # num # num -> num;
dec qquantasVezes1 : num # num -> num;
--- fatbase (0, x) \le x;
--- fatbase (n, x) \le fatbase (n - 1, n * x);
--- fat2 n <= fatbase (n, 1);
--- fibo2 (0, anter, atual) <= anter;
--- fibo2 (1, anter, atual) <= atual;
--- fibo2 (2, anter, atual) <= atual + anter;
--- fibo2 (n, anter, atual) <= fibo2 (n - 1, atual, anter + atual);
--- fib2 n <= fibo2 (n, 0, 1);
--- par 0 <= true;
--- par n \le not (par (n - 1));
```

```
--- base2_1 0 <= 1;dec
--- base2_1 n <= 2 * base2_1 (n - 1);
--- base2x (0, x) \le x;
--- base2x (n, x) \le base2x (n - 1, x * 2);
--- base2_2 n <= base2x (n, 1);
--- quantasVezes (0, k, x) \le 0;
--- quantasVezes (n, k, x) \leq if n = 0 then x else if n mod 10 = k then quantasVezes
(n, k \text{ div } 10, x + 1) \text{ else quantas Vezes } (n, k \text{ div } 10, x);
--- qquantasVezes1 (k, n) <= quantasVezes (k, n, 0);
uses base2;
dec membro: num # list num -> truval;
dec juncao: list num # list num -> list num;
dec unico: list num -> list num;
dec insira: num # list num -> list num;
dec classifica: list num -> list num;
dec uniao : list num # list num -> list num;
dec intersecção : list num # list num -> list num;
dec diferenca: list num # list num -> list num;
dec sub_lista : list num # list num -> truval;
dec igualdade : list num # list num -> truval;
```

--- membro (n, []) <= false;

```
--- membro (n, x :: xs) \le if n = x then true else membro (n, xs);
--- juncao ([], []) <= [];
--- juncao (a, []) <= a;
--- juncao ([], b) <= b;
--- juncao (a :: ax, b) <= a :: juncao (ax, b);
--- unico [] <= [];
--- unico (x :: xs) <= if membro (x, xs) then unico xs else x :: unico xs;
--- insira (n, []) <= [n];
--- insira (n, x :: xs) \le if n = x then n :: x :: xs else x :: insira <math>(n, xs);
--- classifica [] <= [];
--- classifica (x :: xs) <= insira (x, classifica xs);
--- uniao (a, b) <= classifica (unico (juncao (a, b)));
--- interseccao (a, []) <= [];
--- interseccao ([], b) <= [];
--- intersecção (a, x :: b) \leq if membro (x, a) then x :: intersecção (a, b) else
interseccao (a, b);
--- diferenca (a, []) <= a;
--- diferenca ([], b) <= [];
--- diferenca (a :: ax, b) <= if membro (a, b) then diferenca (ax, b) else a :: diferenca
(ax, b);
--- sub_lista ([], ys) <= true;
```

```
--- sub_lista (x :: xs, ys) <= if membro (x, ys) then sub_lista (xs, ys) else false;
--- igualdade (a, b) <= sub_lista (a, b) and sub_lista (b, a);
uses base2, conjuntos;
dec b: list num;
dec a: list num;
dec c: list num;
--- b <= [1, 3, 5, 6, 7];
--- a <= [1, 3, 5, 7];
--- c \le [1, 2, 5, 6, 7, 8];
uses base2;
dec inverte : list num -> list num;
dec divisores : num -> list num;
dec produtolst : list num -> num;
dec faixa_primos : num # num -> list num;
dec multlstn: num # list num -> list num;
dec tabuada : num -> list num;
dec somalst : list num -> num;
dec amigos : num # num -> truval;
dec abundante : num -> truval;
dec div84 : num -> truval;
dec div84': num -> truval;
dec multip_faixa : num # num -> num;
```

```
dec deficiente : num -> truval;
dec perfeito : num -> truval;
dec primeiros_enquanto : (num -> truval) # list num -> list num;
dec suspende_enquanto : (num -> truval) # list num -> list num;
dec primeiros : num # list num -> list num;
dec itera: (num -> truval) # num -> list num;
dec itera': (num -> num) # num -> list num;
--- inverte [] <= [];
--- inverte (x :: xs) <= inverte xs <> [x];
--- divisores 0 <= [];
--- divisores n \le inverte (comp (faixa (1, n + 1, 1), lambda x = multiplo (n, x));
--- produtolst [] <= 0;
--- produtolst ([x]) <= x;
--- produtolst (x :: xs) <= x * produtolst xs;
--- faixa_primos (0, 0) <= [];
--- faixa_primos (n, m) <= comp (faixa (n, m, 1), lambda x => checa_primo x);
--- multlstn (x, []) <= [];
--- multlstn (x, n :: ns) <= x * n :: multlstn (x, ns);
--- tabuada n <= multlstn (n, faixa (1, 10, 1));
--- somalst [] <= 0;
--- somalst (x :: xs) \le x + somalst xs;
```

```
--- amigos (x, y) \le somalst (divisores x) - x = y and somalst (divisores y) - y = x;
--- abundante n \le somalst (comp (faixa (1, n + 1, 1), lambda x => multiplo (n, x))) -
n > n;
--- div84 n <= if n mod 8 = 4 then true else false;
--- div84' n <= n mod 8 = 4;
--- multip_faixa (x, y) \le if x > y then 0 else if x = y then y else x * multip_faixa <math>(x + 1, y) \le if x > y
y);
--- deficiente n \leq somalst (comp (faixa (1, n + 1, 1), lambda x => multiplo (n, x))) - n
< n;
--- perfeito n \le somalst (comp (faixa (1, n + 1, 1), lambda x => multiplo (n, x))) - n =
n;
--- primeiros_enquanto (funcao, []) <= [];
--- primeiros_enquanto (funcao, x :: xs) <= if funcao x then x :: primeiros_enquanto
(funcao, xs) else [];
--- suspende_enquanto (funcao, []) <= [];
--- suspende_enquanto (funcao, x :: xs) <= if funcao x then suspende_enquanto
(funcao, xs) else x :: xs;
--- primeiros (0, _) <= [];
--- primeiros (_, []) <= [];
--- primeiros (n, x :: xs) <= x :: primeiros (n - 1, xs);
```

```
--- itera' (x, n) \le n :: itera' (x, x n);
dec primeiros : num # list num -> list num;
--- primeiros (0, []) <= [];
--- primeiros (n, []) <= [];
dec primeiros': num # list num -> list num;
--- primeiros' (0, []) <= [];
--- primeiros' (n, []) <= [];
--- primeiros' (n, x :: xs) <= x :: primeiros' (n - 1, xs);
dec penultimo : list num -> num;
--- penultimo [] <= error ("lista vazia");
--- penultimo ([x]) <= error ("lista com um elemento");
--- penultimo (x :: y :: xs) <= penultimo (y :: xs);
dec maisDeTres : list num -> num;
--- maisDeTres [] <= 0;
--- maisDeTres (x :: xs) \leq if x > 3 then x + maisDeTres xs else maisDeTres xs;
dec dolar: num # num -> num;
dec squad: num # num -> num;
--- dolar (real, valor) <= real / valor;
--- squad (a, b) <= a ^ 2 + b ^ 2;
```

```
uses base2;
dec inverte : list num -> list num;
dec divisores : num -> list num;
dec produtolist : list num -> num;
dec faixa_primo : num # num -> list num;
dec multlstn: num # list num -> list num;
dec tabuada : num -> list num;
dec tabuada': num -> list num;
dec somalst : list num -> num;
dec amigos : num # num -> truval;
dec abundante : num -> truval;
dec abundante': num -> truval;
dec div84 : num -> truval;
dec lstdiv84 : num -> list num;
dec multip_faixa : num # num -> num;
dec produto: list num -> num;
dec multipFaixa: num # num -> num;
dec deficiente : num -> truval;
dec deficiente' : num -> truval;
dec perfeito : num -> truval;
dec primeiros_enquanto : (num -> truval) # list num -> list num;
dec eh_par : num -> truval;
dec suspende_enquantos : (num -> truval) # list num -> list num;
dec suspende_enquanto': (num -> truval) # list num -> list num;
dec primeiros: num # list num -> list num;
dec itera: (num -> num) # num -> list num;
--- inverte [] <= [];
```

```
--- inverte (x :: xs) <= inverte xs <> [x];
--- divisores 0 <= [];
--- divisores n \le inverte (comp (faixa (1, n + 1, 1), lambda x = multiplo (n, x));
--- produtolist [] <= 0;
--- produtolist ([x]) \le x;
--- produtolist (x :: xs) <= x * produtolist xs;
--- faixa_primo (0, 0) <= [];
--- faixa_primo (0, 1) <= [1];
--- faixa_primo (n, m) <= comp (faixa (n, m, 1), lambda x => checa_primo x);
--- multlstn (0, [x]) <= [];
--- multlstn (n, x :: xs) <= x * n :: multlstn (x, xs);
--- tabuada n <= multlstn (n, []);
--- tabuada' n <= multlstn (n, faixa (1, 10, 1));
--- somalst [] <= 0;
--- somalst (x :: xs) <= x + somalst xs;
--- amigos (x, y) \le somalst (divisores x) - x = y and somalst (divisores y) - y = x;
--- abundante n <= somalst (divisores n) > n;
```

```
--- abundante' n \le somalst (comp (faixa (1, n + 1, 1), lambda x => multiplo (n, x))) -
n > n;
--- div84 n \leq if n mod 8 = 4 then true else false;
--- lstdiv84 n <= comp (faixa (1, n, 1), lambda x => div84 x);
--- produto [] <= 1;
--- produto (x :: xs) <= x * produto xs;
--- multipFaixa (x, y) \le if x > y then 0 else if x = y then y else x * multipFaixa (x + 1, y) \le if x > y
y);
--- deficiente n <= somalst (divisores n) < n;
--- deficiente' n \le somalst (comp (faixa (1, n + 1, 1), lambda x => multiplo (n, x))) -
n < n;
--- perfeito n \le somalst (comp (faixa (1, n + 1, 1), lambda x => multiplo (n, x))) - n =
n;
--- primeiros_enquanto (funcao, []) <= [];
--- primeiros_enquanto (funcao, x :: xs) <= if funcao x then x :: primeiros_enquanto
(funcao, xs) else primeiros_enquanto (funcao, xs);
--- eh_par x \le x \mod 2 = 0;
--- suspende_enquantos (funcao, []) <= [];
--- suspende_enquantos (funcao, x :: xs) <= if funcao x then suspende_enquantos
(funcao, xs) else x :: xs;
```

```
--- suspende_enquanto' (funcao, []) <= [];
--- suspende_enquanto' (funcao, x :: xs) <= if funcao x then suspende_enquanto'
(funcao, xs) else x :: xs;
--- primeiros (0, _) <= [];
--- primeiros (_, []) <= [];
--- primeiros (n, x :: xs) <= x :: primeiros (n - 1, xs);
--- itera (x, y) \le y :: itera (x, x y);
uses listas;
dec somalst: list num -> num;
dec mult2: list num -> list num;
dec duplique : list num -> list num;
dec intervalo : list num -> list num;
dec intervalo2 : num # num -> list num;
dec inverte : list num -> list num;
dec inverte': list num -> list num;
dec poe_final : num # list num;
dec poe_final2 : num # list num -> list num;
dec poe_finalCerto : num # list num -> list num;
dec mult: list num -> list num;
dec mult3 : list num -> num;
dec invertBas : list num # list num -> list num;
dec inverte": list num -> list num;
```

--- somalst [] <= 0;

```
--- somalst (x :: xs) <= x + somalst xs;
--- mult2 [] <= [];
--- mult2 (x :: xs) <= x * 2 :: mult2 xs;
--- duplique [] <= [];
--- duplique (x :: xs) <= x :: duplique xs;
--- duplique (x :: xs) <= x :: x :: duplique xs;
--- intervalo2 (m, n) \leq if m > n then [] else m :: intervalo2 (m + 1, n);
--- inverte [] <= [];
--- inverte (x :: xs) <= inverte xs <> [x];
--- inverte' [] <= [];
--- inverte' xs <= ultimo xs :: inverte' (arranjo xs);
--- poe_final2 (n, []) <= [];
--- poe_finalCerto (n, []) <= [n];
--- poe_finalCerto (n, x :: xs) <= x :: poe_finalCerto (n, xs);
--- mult [] <= [];
--- mult3 [] <= 1;
--- mult3 (x :: xs) <= x * mult3 xs;
--- invertBas ([], ys) <= ys;
```

```
--- invertBas (x :: xs, ys) <= invertBas (xs, x :: ys);
--- inverte" xs <= invertBas (xs, []);
uses basico;
dec soma_com_10: num -> num;
dec subtrai_2 : num -> num;
dec x_pi_em_5 : num -> num;
--- soma_com_10 x \le x + 10;
--- subtrai_2 x <= x - 2;
--- x_pi_em_5 x <= x * x_pi / 5;
uses array;
dec suspende : num # list num -> list num;
dec concatena: list num # list num -> list num;
dec remove : num # list num -> list num;
dec posicao: num # list num -> num;
dec rmvult : list num -> list num;
dec rmvult' : list num -> list num;
dec ate: num -> list num;
dec quadrados : num -> list num;
dec quadrados': num -> list num;
dec quads_inv : num -> list num;
--- suspende (0, [xs]) <= [xs];
```

```
--- suspende (_, []) <= [];
--- suspende (n, x :: xs) <= suspende (n - 1, xs);
--- concatena ([], lst) <= lst;
--- remove (n, []) <= [];
--- remove (n, x :: xs) \le if n = x then xs else x :: remove <math>(n, xs);
--- posicao (n, []) <= 0;
--- posicao (n, x :: xs) \leq if n = x then 1 else 1 + posicao (n, xs);
--- rmvult [] <= error ("Lista vazia");
--- rmvult (x :: xs) <= arranjo xs;
--- rmvult ([x]) <= [];
--- rmvult' [] <= error ("Lista vazia");
--- rmvult' ([x]) <= [];
--- ate 0 <= [];
--- ate n \le ate(n-1) \le [n-1];
--- quadrados 0 <= [];
--- quadrados n \le quadrados (n - 1) \le [pow (n - 1, 2)];
--- quadrados' 0 <= [];
--- quadrados' n <= quadrados' (n - 1) <> [pow (n, 2)];
--- quads_inv 0 <= [];
```

```
--- quads_inv n <= [pow (n, 2)] <> quads_inv (n - 1);
dec contaDigitos : num -> num;
dec SomaDigitos : num -> num;
dec mediaDigitos : num -> num;
dec collaz : num -> num;
dec calcN: num -> num;
dec exp: num # num -> num;
dec somaFaixa: num # num -> num;
dec dec2bin : num -> num;
dec sucessor : num -> num;
dec antecessor : num -> num;
dec minhaSoma: num # num -> num;
dec produto: num # num -> num;
dec bin2dec : num -> num;
dec binon: num # num -> num;
--- contaDigitos 0 <= 0;
--- contaDigitos n <= 1 + contaDigitos (n div 10);
--- SomaDigitos 0 <= 0;
--- SomaDigitos n <= n mod 10 + SomaDigitos (n div 10);
--- mediaDigitos n <= SomaDigitos n / contaDigitos n;
--- collaz 1 <= 1;
--- collaz n \leq if abs n mod 2 = 0 then collaz (abs n div 2) else collaz (abs n * 3 + 1);
```

```
--- calcN 0 <= 0;
--- calcN n \le 1.5 + calcN (n - 1) / 2;
--- \exp(1, 1) \le 1;
--- \exp(x, n) \le x * \exp(x, n - 1);
--- somaFaixa (m, n) \le if n \le m then 0 else if m = n then m else n + somaFaixa <math>(m, n)
- 1);
--- dec2bin 0 <= 0;
--- dec2bin n \le if n = 1 then 1 else n;
--- dec2bin n <= 10 * dec2bin (n div 2) + n mod 2;
--- sucessor n <= n + 1;
--- antecessor n \le n - 1;
--- minhaSoma (0, b) <= b;
--- minhaSoma (a, 0) <= a;
--- minhaSoma (0, 0) <= 0;
--- minhaSoma (a, b) <= minhaSoma (sucessor a, antecessor b);
--- produto (0, 0) \le 0;
--- produto (0, b) <= 0;
--- produto (a, 0) <= 0;
--- produto (1, b) <= b;
--- produto (a, 1) <= a;
--- produto (a, b) <= a + produto (a, b - 1);
```

```
--- bin2dec 0 <= 0;
--- bin2dec n <= 2 * bin2dec (n div 10) + n mod 10;
--- binon (n, 0) <= 1;
--- binon (n, k) \le (n - 1) / (k - 1) + (n - 1) / k;
--- binon (n, k) \le if n \le k then error ("n \le k") else if k = 0 then 1 else if k = n then 1
else binon (n - 1, k - 1) + binon (n - 1, k);
uses recursao;
dec prodIntervalo1 : num # num -> num;
dec resto: num # num -> num;
dec somaCubos : num -> num;
dec contaPares : num -> num;
dec somaDigitos : num -> num;
dec somaDigitos2 : num -> num;
dec primeiro Digito: num -> num;
dec primeiroDigito2: num -> num;
dec zeros : num -> num;
dec somaQuadrados: num -> num;
--- prodIntervalo1 (m, n) \leq if m > n then 1 else m * prodIntervalo1 (m + 1, n);
--- resto (p, q) <= p - q * (p / q);
--- somaCubos 0 <= 0;
--- somaCubos n \le pow(n, 3) + somaCubos(n - 1);
```

```
--- contaPares 0 <= 1;
--- contaPares 1 <= 1;
--- contaPares n \le if n \mod 2 = 0 then contaPares (n - 1) + 1 else contaPares (n - 1);
--- somaDigitos 0 <= 0;
--- somaDigitos n <= n mod 10 + somaDigitos (n div 10);
--- somaDigitos2 n <= if n = 0 then 0 else n mod 10 + somaDigitos2 (n div 10);
--- primeiroDigito n <= n mod 10;
--- primeiroDigito2 n <= n div 10;
--- primeiroDigito2 0 <= 0;
--- zeros 0 <= 1;
--- zeros n \leq if n \leq 10 then 0 else if n mod 10 = 0 then 1 + zeros (n div 10) else zeros
(n div 10);
--- somaQuadrados 0 <= 0;
--- somaQuadrados n <= n ^ 2 + somaQuadrados (n - 1);
dec ehPotenciaDeDois : num -> truval;
dec media Digitos: num -> num;
dec contaDigitos: num -> num;
dec somaDigitos : num -> num;
dec collatz : num -> num;
dec calc: num -> num;
dec exp: num # num -> num;
```

```
dec somaFaixa: num # num -> num;
dec dec2bin : num -> num;
dec sucessor : num -> num;
dec antecessor : num -> num;
dec minhaSoma: num # num -> num;
dec produto: num # num -> num;
dec bin2dec : num -> num;
dec binon: num # num -> num;
--- ehPotenciaDeDois 1 <= true;
--- ehPotenciaDeDois n \le if n > 1 and n \mod 2 = 0 then ehPotenciaDeDois (n div 2)
else false;
--- mediaDigitos n <= somaDigitos n / contaDigitos n;
--- contaDigitos 0 <= 0;
--- contaDigitos n <= 1 + contaDigitos (n div 10);
--- somaDigitos 0 <= 0;
--- somaDigitos n <= n mod 10 + somaDigitos (n div 10);
--- collatz 1 <= 1;
--- collatz n <= if abs n mod 2 = 0 then collatz (abs n div 2) else collatz (abs n * 3 +
1);
--- calc 0 <= 0;
--- calc n \le 1.5 + calc (n - 1) / 2;
--- calc n \le (1.5 + calc (n - 1)) / 2;
```

```
--- \exp(x, 0) \le 0;
--- \exp(x, n) \le x * \exp(x, n - 1);
--- somaFaixa (m, n) \le if n \le m then 0 else if n = m then m else n + somaFaixa <math>(m, n)
- 1);
--- dec2bin 0 <= 0;
--- dec2bin 1 <= 1;
--- dec2bin n <= 10 * dec2bin (n div 2) + n mod 2;
--- sucessor n \le n + 1;
--- antecessor n \le n - 1;
--- minhaSoma (0, b) <= b;
--- minhaSoma (a, 0) <= a;
--- minhaSoma (0, 0) <= 0;
--- minhaSoma (a, b) <= minhaSoma (sucessor a, antecessor b);
--- produto (x, 0) \le 0;
--- produto (0, y) <= 0;
--- produto (x, y) \le x + produto (x, y - 1);
--- bin2dec 0 <= 0;
--- bin2dec n <= 2 * bin2dec (n div 10) + n mod 10;
--- binon (n, 0) <= 1;
--- binon (n, k) \le if n \le k then error ("n \le k") else if k = 0 then 1 else if k = n then 1
else binon (n-1, k-1) + binon (n-1, k);
```

```
dec lstfib: num -> list num;
dec primeiros4 : num # list num -> list num;
--- fibo (n, m) \le n :: fibo (m, n + m);
--- fibo (n, m) \le n :: fibo (m, n + m);
--- lstfib n <= primeiros4 (n, fibo (0, 1));
--- primeiros4 (0, _) <= [];
--- primeiros4 (_, []) <= [];
--- primeiros4 (n, x :: xs) <= x :: primeiros4 (n - 1, xs);
uses basico;
dec par : num -> truval;
dec par': num -> truval;
dec impat : num -> truval;
dec impar": num -> truval;
dec valor: num -> num;
--- par n <= if n mod 2 = 0 then true else false;
--- par' n \le n \mod 2 = 0;
--- impat n \le n \mod 2 = 0;
--- valor 0 <= 0;
```

dec fibo: num # num -> list num;

```
--- valor 1 <= 2;
--- valor n \le if n > 1 and n \le 9 then n * 5 else n / 5;
uses fluxo;
dec negativo : num -> num;
dec classif: num # num -> num # num;
dec div84 : num -> truval;
dec extenso: num -> list char;
dec calculadora: num # num # num -> num;
--- negativo 0 <= 0;
--- negativo n \le if n \le 0 then n else 0 - n;
--- div84 n \le if n \mod 8 = 4 then true else false;
--- extenso 0 <= "zero";
--- extenso 1 <= "um";
--- extenso 2 <= "dois";
--- extenso 3 <= "tres";
--- extenso 4 <= "quatro";
--- extenso 5 <= "cinco";
--- extenso 6 <= "seis";
--- extenso 7 <= "sete";
--- extenso 8 <= "oito";
--- extenso 9 <= "nove";
--- extenso _ <= "Valor invalido";
--- calculadora (x, y, 0) \le x + y;
```

```
--- calculadora (x, y, 1) <= x - y;
--- calculadora (x, y, 10) <= x * y;
--- calculadora (x, y, 11) <= x / y;
--- calculadora (x, y, _) <= error ("valor invalido");
uses array;
dec elem2: list num -> num;
dec tamanho: list num -> num;
dec elempnt : list num -> num;
--- elem2 [] <= error ("Lista vazia");
--- elem2 ([x]) <= error ("Poucos elementos");
--- elem2 (a :: b :: bs) <= b;
--- tamanho [] <= 0;
--- tamanho (x :: xs) <= 1 + tamanho xs;
--- elempnt [] <= error ("Lista Vazia");
--- elempnt ([x]) <= error ("Poucos numeros");
--- elempnt (x :: xs) <= if tamanho xs = 1 then x else elempnt xs;
dec x_pi : num;
dec area_circ : num -> num;
dec qsoma: num # num -> num;
dec x_e: num;
dec x_pi_em_e : num -> num;
dec suc : num -> num;
dec c2f: num -> num;
```

```
dec f2c: num -> num;
```

$$suc x <= x + 1;$$

$$c2f c \le c * 9 / 5 + 32;$$

$$f2c f \le (f - 32) * 5 / 9;$$

$$k2c k \le k - 273.15$$
;

dec x_pi : num;

dec acirc: num -> num;

dec qsoma: num # num -> num;

dec x_e: num;

```
dec x_pi_em_x : num -> num;
dec suc : num -> num;
dec c2f: num -> num;
dec c2k: num -> num;
dec metro2centimetro: num -> num;
dec saoiguais : num # num -> truval;
dec ant : num -> num;
dec cubo: num -> num;
dec k2f: num -> num;
dec f2k: num -> num;
dec imc: num # num -> num;
dec produto: num # num -> num;
dec eq1grau: num # num -> num;
dec area_ret : num # num -> num;
--- x_pi <= 3.14159;
--- acirc r <= x_pi * pow (r, 2);
--- qsoma(x, y) \le pow(x, 2) + pow(y, 2);
--- x_e <= 2.71828;
--- x_pi_em_x a <= a * x_pi / x_e;
--- suc x <= x + 1;
--- c2f f \le (f - 32) * 5 / 9;
```

```
--- c2k c <= c + 273.15;
--- metro2centimetro c <= c * 100;
--- saoiguais (a, b) <= if a = b then true else false;
--- ant x \le x - 1;
--- cubo c <= c ^ 3;
--- k2f k \le (k - 273.15) * 9 / 5 + 32;
--- f2k f \le (f - 32) * 5 / 9 + 273.15;
--- imc (p, a) <= p / pow (a, 2);
--- produto (x, y) \le x * y;
--- eq1grau (a, b) <= b / a;
--- area_ret (lado1, lado2) <= lado1 * lado2;
uses man;
dec x_pi : num;
--- x_pi <= 3.14159;
```

dec acirc : num -> num;

```
--- acirc r <= x_pi * pow (r, 2);
dec soma: num # num -> num;
--- soma (x, y) \le x + y;
dec suc : num -> num;
--- suc res <= res+1;
dec ant : num -> num;
--- ant res <= res-1;
dec qsoma: num # num -> num;
--- qsoma (x,y) \le (x+y)^2;
dec cubo: num -> num;
--- cubo res <= pow(res,3);
dec produto: num # num -> num;
--- produto (x,y) \le x^*y;
dec imc : num # num -> num;
--- imc (p,a) \leq p/(a^2);
dec dolar: num # num -> num;
--- dolar (real,valor) <= real / valor;
dec squad: num # num -> num;
--- squad (a,b) \le (a^2)+(b^2);
```

```
dec par : num -> truval;
--- par n \le if n \mod 2 = 0 then true else false;
uses fluxo;
dec fat1: num -> num;
dec fib1 : num -> num;
dec potencia: num # num -> num;
--- fat1 0 <= 1;
--- fat1 n <= n * fat1 (n - 1);
--- fib1 0 <= 0;
--- fib1 1 <= 1;
--- fib1 n <= fib1 (n - 1) + fib1 (n - 2);
--- potencia (n, 0) <= 1;
--- potencia (b, e) <= b * potencia (b, e - 1);
uses recursao;
dec fbs: num # num # num -> num;
dec fib2 : num -> num;
dec par : num -> truval;
dec base2_1 : num -> num;
dec base2x : num # num -> num;
dec base2_2: num -> num;
dec quantasVezes : num # num -> num;
```

```
dec qvbase : num # num # num -> num;
 dec quantasVezes2 : num # num -> num;
--- fbs (0, anter, atual) <= anter;
--- fbs (1, anter, atual) <= atual;
--- fbs (2, anter, atual) <= atual + anter;
--- fbs (n, anter, atual) <= fbs (n - 1, atual, anter + atual);
--- fib2 n <= fbs (n, 0, 1);
--- par 0 <= true;
--- par n <= not (par (n - 1));
--- base2_1 0 <= 1;
--- base2_1 n <= 2 * base2_1 (n - 1);
--- base2x (0, x) \le x;
--- base2x (n, x) \le base2x (n - 1, x * 2);
--- base2_2 n <= base2x (n, 1);
 --- qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, n, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i) \le if n = 0 then i else if n mod 10 = k then qvbase (k, i
 else qvbase (k, n div 10, i);
--- quantasVezes2 (k, n) <= qvbase (k, n, 0);
fat1: num -> num;
fat1 0 <= 1;
```

```
fat1 n <= n * fat1(n-1);
fib1: num -> num;
fib1 0 <= 0;
fib1 1 <= 1;
fib1 n <= fib1 (n-1) + fib1(n-2);
uses base2, conjuntos, basico, fluxo;
dec mapa: list num # (num -> num) -> list num;
dec filtro: list num # (num -> truval) -> list num;
dec reducao : list num # (num # num -> num) # num -> num;
dec compacta: list num # list num -> list (num # num);
dec oposto: list num -> list num;
dec dcp_base: list num # list num # list (num # num) -> list num # list num;
dec descompacta: list (num # num) -> list num # list num;
--- mapa ([], funcao) <= [];
--- mapa (x :: xs, funcao) \le funcao x :: mapa (xs, funcao);
--- filtro ([], funcao) <= [];
--- filtro (x :: xs, funcao) <= if funcao x then x :: filtro (xs, funcao) else filtro (xs,
funcao);
--- reducao ([], funcao, n) <= n;
--- reducao (x :: xs, funcao, n) <= funcao (x, reducao (xs, funcao, n));
--- compacta ([], b) <= [];
--- compacta (a, []) <= [];
```

```
--- compacta (x :: xs, y :: ys) <= (x, y) :: compacta (xs, ys);
--- oposto [] <= [];
--- oposto (x :: xs) <= oposto xs <> [x];
--- dcp_base (xs, ys, []) <= (oposto xs, oposto ys);
--- dcp_base (xs, ys, (x, y) :: zs) <= dcp_base (x :: xs, y :: ys, zs);
--- descompacta [] <= ([], []);
--- descompacta xs <= dcp_base ([], [], xs);
uses array, listas;
dec intervalo: num # num -> list num;
--- intervalo (m, n) \le if m > n then [] else m :: intervalo <math>(m + 1, n);
dec mult2: list num -> list num;
--- mult2 [] <= [];
--- mult2 (x :: xs) <= x * 2 :: mult2 xs;
dec duplique : list num -> list num;
--- duplique [] <= [];
--- duplique (x :: xs) <= x :: x :: duplique xs;
dec somalista: list num -> num;
--- somalista [] <= error ("Lista Vazia");
--- somalista (x :: xs) <= x + somalista xs;
```

```
dec inverte: list alpha -> list alpha;
--- inverte [] <= [];
--- inverte (x :: xs) <= inverte xs <> [x];
dec invert': list num -> list num;
--- invert' [] <= [];
--- invert' xs <= ultimo xs :: invert' (arranjo xs);
dec poe_final : num # list num -> list num;
--- poe_final (n, []) <= [n];
--- poe_final (n, x :: xs) <= x :: poe_final (n, xs);
dec mult3: list num -> num;
--- mult3 [] <= 1;
--- mult3 (x :: xs) <= x * mult3 xs;
uses listas, array, recursao;
dec faixa: num # num # num -> list num;
--- faixa (i, f, p) <= if i > f then [] else i :: faixa (i + p, f, p);
dec comp: list num # (num -> truval) -> list num;
--- comp ([], qualificador) <= [];
--- comp (x :: xs, qualificador) <= if qualificador x then x :: comp (xs, qualificador)
else comp (xs, qualificador);
//Função anônima.
```

```
comp ([1,2,3,4], \ x => x \mod 2 = 0);
comp(faixa(1,7,1), \ x = \ par(x)); //Outro exemplo.
dec listapot : num # list num -> list num;
--- listapot (n, []) <= [];
--- listapot (n, x :: xs) \le pow(x, n) :: listapot(n, xs);
dec listapot : num # list num -> list num;
--- listapot (n, []) <= [];
--- listapot (n, x :: xs) \le pow(x, n) :: listapot(n, xs);
dec par : num -> truval;
dec impar : num -> truval;
dec valor : num -> num;
dec min: num # num -> num;
dec max: num # num -> num;
dec sinal: num # num -> num;
dec abc : num -> num;
--- par n \le if n \mod 2 = 0 then true else false;
--- impar n <= if n mod 2 /= 0 then true else false;
--- valor 0 <= 0;
```

```
--- valor 1 <= 2;
--- valor n \le if n > 1 and n \le 9 then n * 5 else n / 5;
--- min(a, b) \le if a \le b then a else b;
--- max(a, b) \le if a > b then a else b;
--- sinal (x, y) \le if x \le y then 0 - 1 else if x > y then 1 else 0;
--- abc 0 <= 1;
--- abc 1 <= 2;
--- abc 2 <= 4;
--- abc x <= if x > 2 then 9 else 0;
dec par : num -> truval;
dec impar : num -> truval;
dec valor : num -> num;
dec min: num # num -> num;
dec max: num # num -> num;
dec sinal: num # num -> num;
dec abc: num -> num;
--- par n <= if n mod 2 = 0 then true else false;
--- impar n <= if n mod 2 /= 0 then true else false;
--- valor 0 <= 0;
--- valor 1 <= 2;
```

```
--- valor n <= if n > 1 and n < 9 then n * 5 else n / 5;

--- min (a, b) <= if a < b then a else b;

--- max (a, b) <= if a > b then a else b;

--- sinal (x, y) <= if x < y then 0 - 1 else if x > y then 1 else 0;

--- abc 0 <= 1;

--- abc 1 <= 2;

--- abc 2 <= 4;
```

--- abc x <= if x > 2 then 9 else 0;