

Some Elementary Number Theory Problems

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1. What does the Fundamental Theorem of Arithmetic state?
2. Express the following as a product of power of primes:
 - (a) 4500
 - (b) 2349960
 - (c) 231
 - (d) 343
 - (e) 216
 - (f) 288
3. All integers are of the form $2x + 1$ for some integer x . True or False?
4. If $b \mid a$ and $a \mid c$, does this imply $b \mid c$?
5. For any natural number p , can 4^p end in zero?
6. If $t^x = 49$, $(t, x \in \mathbb{Z}^{\geq 0})$, find the value of

$$x^2t + xt^2 + 2tx$$

7. If $2^x \cdot 3^y \cdot 5^z \cdot 7^{(t^2+2t+1)} = 180$, find the value of

$$(t+1)(x^3 + y^3 + z^3)(xy + yz + xz)$$

8. If a, b, c are whole numbers that satisfy $3^a \cdot 5^b \cdot 7^c = 105$, then what is the value of

$$a - (b + c)$$

9. If $\alpha\beta^2\gamma = 12$, find

$$\alpha + \beta + \gamma$$

where $\alpha, \beta, \gamma \in \mathbb{N}$ and β is a prime such that $\beta \mid 4^p$ and $\alpha > \beta > \gamma$