Some Elementary Number Theory Problems*

Coolmanboi†

November 2020

1. What does the Fundamental Theorem of Arithmetic state?
2. Express the following as a product of power of primes:
   (a) 4500
   (b) 2349960
   (c) 231
   (d) 343
   (e) 216
   (f) 288
3. All integers are of the form $2x + 1$ for some integer x. True or False?
4. If $b \mid a$ and $a \mid c$, does this imply $b \mid c$?
5. For any natural number p, can 4^p end in zero?
6. If $t^x = 49$, $(t, x \in \mathbb{Z}^{\geq 0})$, find the largest value of
$$x^2t + xt^2 + 2tx$$
7. If $2^x \cdot 3^y \cdot 5^z \cdot 7^{(t^2+2t+1)} = 180$, find the value of
$$(t + 1)(x^3 + y^3 + z^3)(xy + yz + xz)$$
8. If a, b, c are whole numbers that satisfy $3^a \cdot 5^b \cdot 7^c = 105$, then what is the value of
$$a - (b + c)$$
9. If $\alpha\beta^2\gamma = 12$, find
$$\alpha + \beta + \gamma$$
where $\alpha, \beta, \gamma \in \mathbb{N}$ and β is a prime such that $\beta \mid 4^p$ and $\alpha > \beta > \gamma$

*Check Coolmanboi's website out
†All numbers in this handout are integers