A 附录

把方程17展开就有

$$\begin{aligned}\dot{\rho} = -i\mathcal{L}\rho - \frac{1}{2}\{(\Gamma_R a^\dagger a + \Gamma_L a a^\dagger)\rho + \rho(\Gamma_R a^\dagger a + \Gamma_L a a^\dagger) \\ - \Gamma_L a^\dagger \rho a - \Gamma_R a \rho a^\dagger - \Gamma_L a^\dagger \rho a - \Gamma_R a \rho a^\dagger\}\end{aligned} \tag{A.1}$$

令 $S = \Gamma_R a^\dagger a + \Gamma_L a a^\dagger$。且有 $S^\dagger = S$ 则方程A.1进一步简化为

$$\dot{\rho} = -i\mathcal{L}\rho - \frac{1}{2}[S, \rho]_+ + \Gamma_L a^\dagger \rho a + \Gamma_R a \rho a^\dagger \tag{A.2}$$

因为 Γ_L 和 Γ_R 为非负实数总可以写成 $\Gamma_L = \chi\chi^*$，$\Gamma_R = \eta\eta^*$ 带入到方程A.2重新定义算符 $G = \chi^* a^\dagger$，$R = \eta a$ 则

$$\dot{\rho} = -i\mathcal{L}\rho - \frac{1}{2}[S, \rho]_+ + G\rho G^\dagger + R\rho R^\dagger \tag{A.3}$$

定义非厄米哈密顿量 $H_{cond} = H_S - \frac{i}{2}S$, 则方程A.3可以表示成如下的等价形式

$$\rho(t + \Delta t) = \sum_{m=0}^{2} K_m \rho(t) K_m^\dagger \tag{A.4}$$

其中 $K_0 = \exp(-iH_{cond}\Delta t)$，$K_1 = G\sqrt{\Delta t}$，$K_2 = R\sqrt{\Delta t}$。这样就可以把方程A.1表示成一个类似于隐马尔科夫形式的方程。同样对于量子条件主方程18，可以得到如下形式

$$\dot{\rho}^{(n)} = -i\mathcal{L}\rho^{(n)} - \frac{1}{2}[S, \rho^{(n)}] + G\rho^{(n)}G^\dagger + R\rho^{(n-1)}R^\dagger \tag{A.5}$$

和A.4类似，

$$\rho^{(n)}(t + \Delta t) = K_0\rho^{(n)}(t)K_0^\dagger + K_1\rho^{(n)}(t)K_1^\dagger + K_2\rho^{(n-1)}(t)K_2^\dagger \tag{A.6}$$

对方程A.6按照上指标 n 求和就可以得到方程A.4。对更一般的情况，我们考虑方程15。假设 $A_\mu^{(-)}$ 和 $A_\mu^{(+)}$ 可以表示成如下形式：$A_\mu^{(-)} = C_\mu^{(-)} a_\mu$, $A_\mu^{(+)} = C_\mu^{(+)} a_\mu$。$C_\mu^{(\pm)}$ 是非负实数。把这样的假设带入到方程15中展开化简我们有

$$\dot{\rho} = -i\mathcal{L}\rho - \frac{1}{2}\sum_\mu [S_\mu, \rho]_+ + \sum_\mu (C_\mu^{(-)} a_\mu \rho a_\mu^\dagger + C_\mu^{(+)} a_\mu^\dagger \rho a_\mu) \tag{A.7}$$

令 $C_\mu^{(-)} = \chi_\mu \chi_\mu^*$, $C_\mu^{(+)} = \eta_\mu \eta_\mu^*$, 则方程A.7可以表示成

$$\dot{\rho} = -i\mathcal{L}\rho - \frac{1}{2}\sum_\mu [S_\mu, \rho]_+ + \sum_\mu (G_\mu \rho G_\mu^\dagger + R_\mu \rho R_\mu^\dagger) \tag{A.8}$$

其中 $G_\mu = \chi_\mu^* a_\mu^\dagger$, $R_\mu = \eta_\mu a_\mu$。定义非厄米哈密顿量 $H_{cond} = H_s - \frac{i}{2}\sum_\mu S_\mu$，定义 Kraus 算符

$$K_0 = \exp(-iH_{cond}\Delta t), K_{1,\mu} = G_\mu\sqrt{\Delta t}, K_{2,\mu} = R_\mu\sqrt{\Delta t} \tag{A.9}$$

用下标 m 对 Kraus 算符重新计数

$$\rho(t + \Delta t) = \sum_{m=0}^{2\mu} K_m \rho K_m^\dagger \tag{A.10}$$

对于量子条件主方程16，假设 $A_{L\mu}^{(\pm)} = C_{L\mu}^{(\pm)} a_\mu$，$A_{R\mu}^{(\pm)} = C_{R\mu}^{\pm} a_\mu$，

$$\dot{\rho}^{(n)} = -i\mathcal{L}\rho^{(n)} - \frac{1}{2}\sum_\mu [S_\mu, \rho^{(n)}]_+ + \sum_\mu (G_\mu \rho^{(n)} G_\mu^\dagger + R_\mu \rho^{(n)} R_\mu^\dagger + F_\mu \rho^{(n-1)} F_\mu^\dagger + T_\mu \rho^{(n+1)} T_\mu^\dagger) \tag{A.11}$$

其中 $S_\mu = \left(C_{L\mu}^{(-)} + C_{R\mu}^{(-)}\right) a_\mu^\dagger a_\mu + \left(C_{L\mu}^{(+)} + C_{R\mu}^{(+)}\right) a_\mu^\dagger a_\mu$，$G_\mu = \chi_\mu^* a_\mu^\dagger$，$C_{L\mu}^{(-)} = \chi_\mu \chi_\mu^*$；$R_\mu = \eta_\mu a_\mu$，$C_{L\mu}^{(+)} = \eta_\mu \eta_\mu^*$；$F_\mu = \kappa_\mu a_\mu$，$C_{R\mu}^{(-)} = \kappa_\mu \kappa_\mu^*$；$T_\mu = \zeta_\mu^* a_\mu^\dagger$，$C_{R\mu}^{(+)} = \zeta_\mu \zeta_\mu^*$。定义 Kraus 算符 $K_0 = \exp(-iH_{cond}\Delta t)$，$K_{1,\mu} = G_\mu\sqrt{\Delta t}$，

$K_{2,\mu} = R_\mu\sqrt{\Delta t}$，$K_{3,\mu} = F_\mu\sqrt{\Delta t}$，$K_{4,\mu} = T_\mu\sqrt{\Delta t}$。对下标重新计数得到

$$\rho^{(n)}(t+\Delta t) = \sum_{m=0}^{2\mu} K_m \rho^{(n)}(t) K_m^\dagger + \sum_{m=0}^{\mu} K'_m \rho^{(n-1)}(t) K'^\dagger_m + K''_m \rho^{(n+1)}(t) K''^\dagger_m \tag{A.12}$$