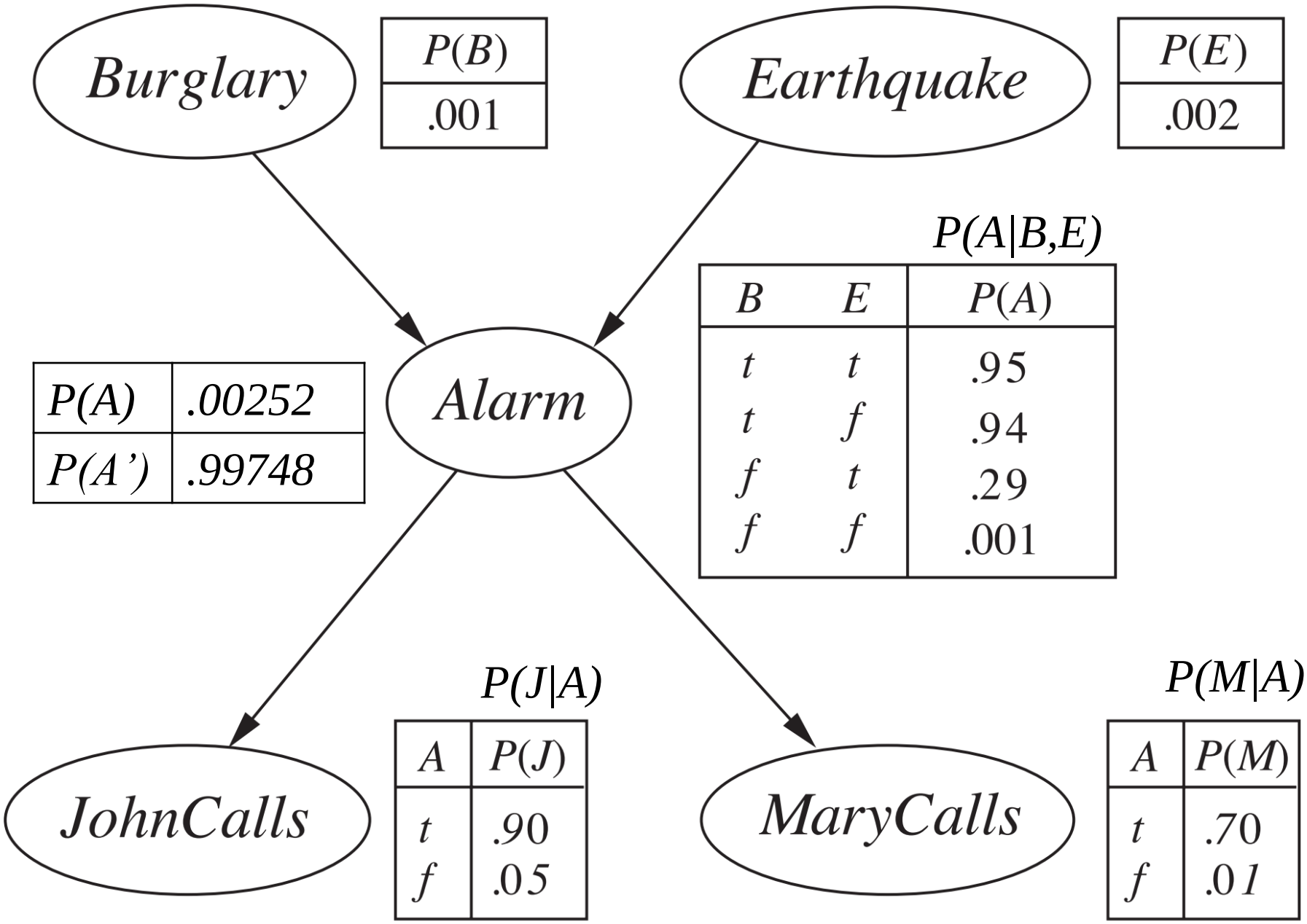
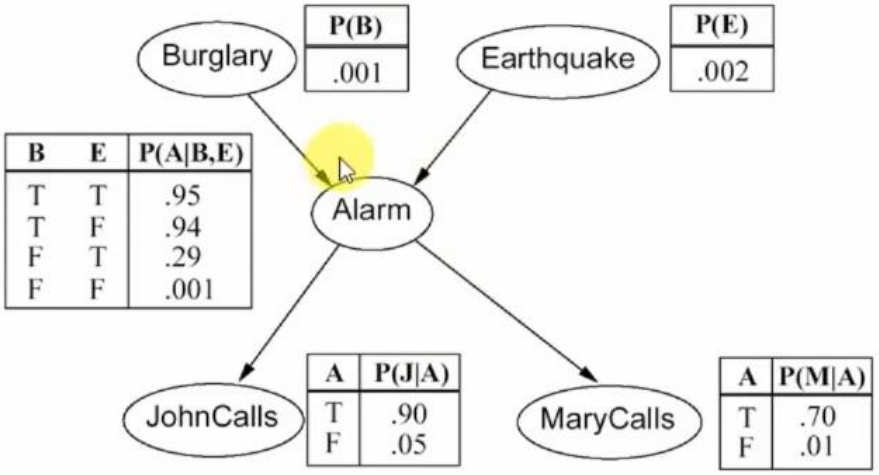
Bayesian Networks Example

- You have a new burglar alarm installed at home.
- It is fairly reliable at detecting burglary, but also sometimes responds to minor earthquakes.
- You have two neighbors, John and Merry , who promised to call you at work when they hear the alarm.
- John always calls when he hears the alarm, but sometimes confuses telephone ringing with the alarm and calls too.
- Merry likes loud music and sometimes misses the alarm.
- Given the evidence of who has or has not called, we would like to estimate the probability of a burglary.


Burglary
P(B)
.001
Earthquake
P(E)
.002
P(A|B,E)
B E P(A)
t t .95
t f .94
f t .29
f f .001
Alarm
P(A) .00252
P(A') .99748
P(J|A)
A P(J)
t .90
f .05
P(M|A)
A P(M)
t .70
f .01
JohnCalls
MaryCalls

1. What is the probability that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both John and Merry call?



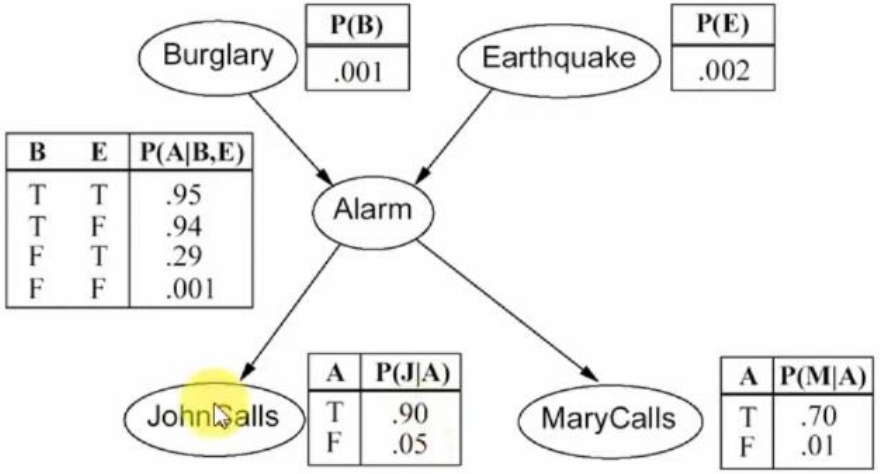
Solution:

$$P(j \wedge m \wedge a \wedge \neg b \wedge \neg e) = P(j \mid a)\, P(m \mid a)\, P(a \mid \neg b, \neg e)\, P(\neg b)\, P(\neg e)$$

$$= 0.90 \times 0.70 \times 0.001 \times 0.999 \times 0.998$$

$$= 0.00062$$

2. What is the probability that John call?



Solution:

$$P(J) = P(J|A)P(A) + P(J|A')P(A')$$
$$P(J) = 0.90 * P(A) + 0.05 * P(A')$$
$$P(J) = 0.90 * 0.00252 + 0.05 * 0.99748$$
$$P(J) = 0.0521$$

$$P(A) = 0.95 * 0.001 * 0.002 + 0.29 * 0.999 * 0.002 + 0.94 * 0.001 * 0.998 + 0.001 * 0.999 * 0.998$$
$$P(A) = 0.00252$$
$$P(A') = 0.99748$$

$$P(J) = P(J|A)P(A) + P(J|A')P(A')$$

$$P(A) = P(A|B,E) * P(B,E) + P(A|B',E) * P(B',E) + P(A|B,E') * P(B,E') + P(A|B',E') * P(B',E')$$
$$P(A) = P(A|B,E) * P(B) * P(E) + P(A|B',E) * P(B') * P(E) + P(A|B,E') * P(B) * P(E') + P(A|B',E') * P(B') * P(E')$$

$$P(A') = P(A'|B,E) * P(B,E) + P(A'|B',E) * P(B',E) + P(A'|B,E') * P(B,E') + P(A'|B',E') * P(B',E')$$
$$P(A') = P(A'|B,E) * P(B) * P(E) + P(A'|B',E) * P(B') * P(E) + P(A'|B,E') * P(B) * P(E') + P(A'|B',E') * P(B') * P(E')$$

Finding P (Burglary|John Calls)? Find P(B|J)?

$P(A'|B,E)$=0.05
$P(A'|B,E')$=0.06

$P(J|A) = 0.90$
$P(J|A') = 0.05$
$P(A|B,E)$=0.95
$P(A|B,E')$=0.94

$P(E')$=0.998
$P(E)$=0.002
$P(B)$=0.001
$P(J) = 0.0521$

$$P(B|J) = \frac{P(J,B)}{P(J)}$$

Now, need to find $P(J,B)$

$$P(J,B) = P(J|A)P(A|B) + P(J|A')P(A'|B)$$
$$P(A|B) = P(A|B,E)P(B)P(E) + P(A|B,E')P(B)P(E')$$
$$P(A'|B) = P(A'|B,E)P(B)P(E) + P(A'|B,E')P(B)P(E')$$

$P(A|B) = 0.95 * 0.001 * 0.002 + 0.94 * 0.001 * 0.998$
$P(A|B) = 0.00094$
$P(A'|B) = 0.05 * 0.001 * 0.002 + 0.06 * 0.001 * 0.998 = 0.00006$
$P(A'|B) = 0.00006$
$P(J,B) = 0.90 * 0.00094 + 0.05 * 0.00006 = 0.00085$
$P(B|J) = \frac{0.00085}{0.0521} = 0.0163$

Exercise

$P(M)=?$

$P(B|M)=?$

P(MarryCalls)=

$$P(M) = P(M|A)P(A) + P(M|A')P(A')$$
$$P(M) = 0.70 * P(A) + 0.01 * P(A')$$
$$P(M) = 0.70 * 0.00252 + 0.01 * 0.99748$$
$$P(M) = 0.01174$$

$$P(A) = 0.95 * 0.001 * 0.002 + 0.29 * 0.999 * 0.002 + 0.94 * 0.001 * 0.998 + 0.001 * 0.999 * 0.998$$
$$P(A) = 0.00252$$
$$P(A') = 0.99748$$

$$P(J) = P(J|A)P(A) + P(J|A')P(A')$$

$$P(A) = P(A|B,E) * P(B,E) + P(A|B',E) * P(B',E) + P(A|B,E') * P(B,E') + P(A|B',E') * P(B',E')$$
$$P(A) = P(A|B,E) * P(B) * P(E) + P(A|B',E) * P(B') * P(E) + P(A|B,E') * P(B) * P(E') + P(A|B',E') * P(B') * P(E')$$

$$P(A') = P(A'|B,E) * P(B,E) + P(A'|B',E) * P(B',E) + P(A'|B,E') * P(B,E') + P(A'|B',E') * P(B',E')$$
$$P(A') = P(A'|B,E) * P(B) * P(E) + P(A'|B',E) * P(B') * P(E) + P(A'|B,E') * P(B) * P(E') + P(A'|B',E') * P(B') * P(E')$$

Finding P (Burglary|Merry Calls)? Find P(B|M)?

$P(A'|B,E)$=0.05
$P(A'|B,E')$=0.06

$P(J|A) = 0.90$
$P(J|A') = 0.05$
$P(A|B,E)$=0.95
$P(A|B,E')$=0.94

$P(E')$=0.998
$P(E)$=0.002
$P(B)$=0.001
$P(J) = 0.0521$

$$P(B|M) = \frac{P(M,B)}{P(M)}$$

Now, need to find $P(M,B)$

$$P(M,B) = P(M|A)P(A|B) + P(M|A')P(A'|B)$$
$$P(A|B) = P(A|B,E)P(B)P(E) + P(A|B,E')P(B)P(E')$$
$$P(A'|B) = P(A'|B,E)P(B)P(E) + P(A'|B,E')P(B)P(E')$$

$P(A|B) = 0.95 * 0.001 * 0.002 + 0.94 * 0.001 * 0.998$
$P(A|B) = 0.00094$
$P(A'|B) = 0.05 * 0.001 * 0.002 + 0.06 * 0.001 * 0.998 = 0.00006$
$P(A'|B) = 0.00006$
$P(M,B) = 0.70 * 0.00094 + 0.01 * 0.00006 = 0.00066$
$P(B|M) = \frac{0.00066}{0.01174} = 0.05621$