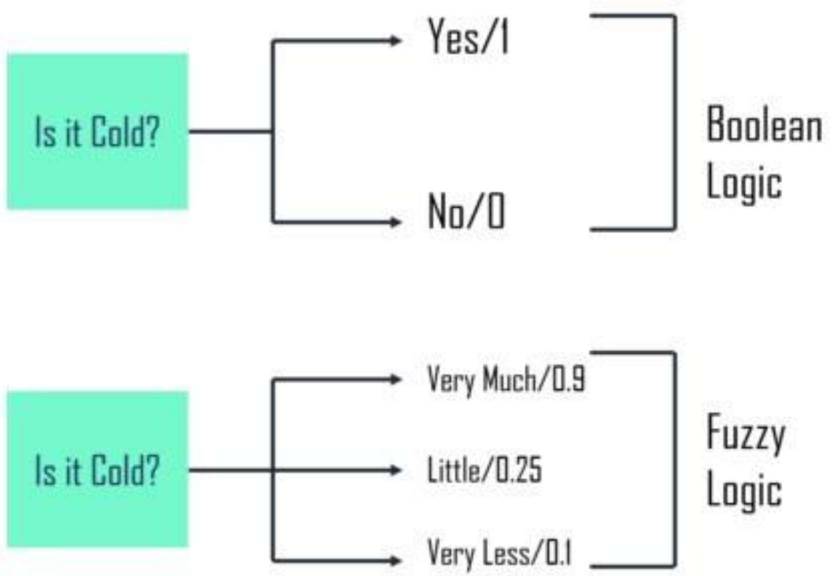
Fuzzy Logic

Introduction

- The word “fuzzy” means “vaguness (ambiguity)”.
- Fuzziness occurs when the boundary of a piece of information is not clear-cut.
- Fuzzy sets - 1965 Lotfi Zadeh as an extension of classical notation set.
- Classical set theory allows the membership of the elements in the set in **binary terms**.
- Fuzzy set theory permits membership function valued in the interval [0,1].

Fuzzy Logic?

Fuzzy Logic (FL) is a method of reasoning that resembles **human reasoning**. This approach is similar to how humans perform decision making. And it involves all intermediate possibilities between **YES** and **NO**.



Fuzzy Logic

Example:

Words like young, tall, good or high are fuzzy.

- There is no single quantitative value which defines the term young.
- For some people, age 25 is young, and for others, age 35 is young.
- The concept young has no clean boundary.
- Age 35 has some possibility of being young and usually depends on the context in which it is being considered.

Fuzzy set theory is an extension of classical set theory where elements have degree of membership.

Fuzzy Logic

- Fuzzy Logic (FL) is a method of reasoning that resembles human reasoning. The approach of FL imitates the way of decision making in humans.
- It is a mathematical language like
  - Relational Algebra
  - Boolean Algebra
  - Predicate Algebra
  - same we have fuzzy Algebra
- Fuzzy Logic deals with fuzzy set

Why Fuzzy Logic?

Generally, we use the fuzzy logic system for both commercial and practical purposes such as:

- It **controls machines** and **consumer products**
- If not accurate reasoning, it at least provides **acceptable reasoning**
- This helps in dealing with the **uncertainty in engineering**

So, now that you know about Fuzzy logic in AI and why do we actually use it, let's move on and understand the architecture of this logic.

Fuzzy Logic

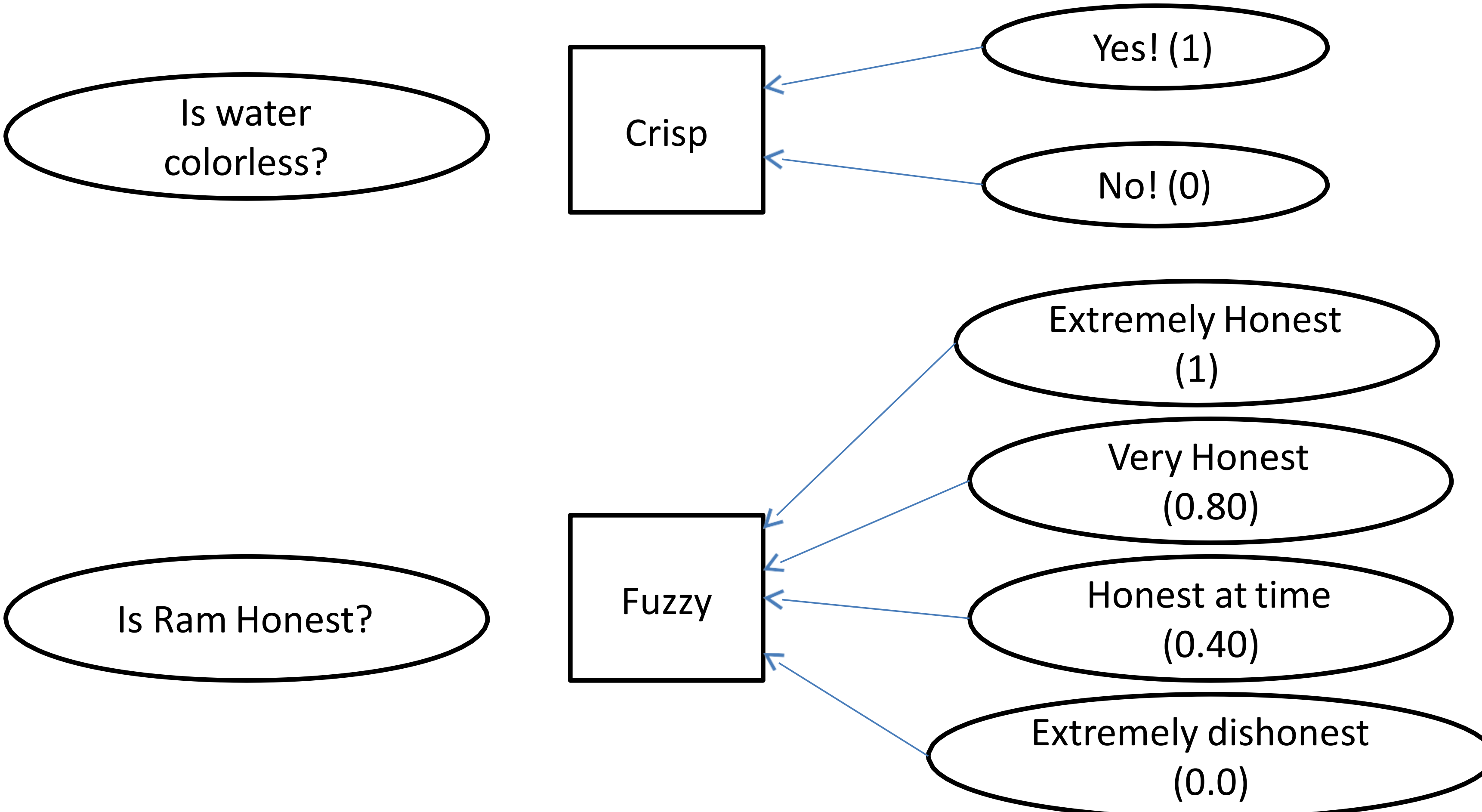
- In real world, there exist much fuzzy knowledge (i.e. vague, uncertain inexact etc).
- Human **thinking** and **reasoning** (analysis, logic, interpretation) frequently involved **fuzzy** information.
- Human can give satisfactory answers, which are probably true.
- Our systems are unable to answer many question because the systems are designed based upon classical set theory (Unreliable and incomplete).
- We want, our system should be able to cope with unreliable and incomplete information.
- Fuzzy system have been provide solution.

Fuzzy Logic

Classical set theory	Fuzzy set theory
• Classes of objects with sharp boundaries.	• Classes of objects with un-sharp boundaries.
• A classical set is defined by crisp(exact) boundaries, i.e., there is no uncertainty about the location of the set boundaries.	• A fuzzy set is defined by its ambiguous boundaries, i.e., there exists uncertainty about the location of the set boundaries.
• Widely used in digital system design	• Used in fuzzy controllers.

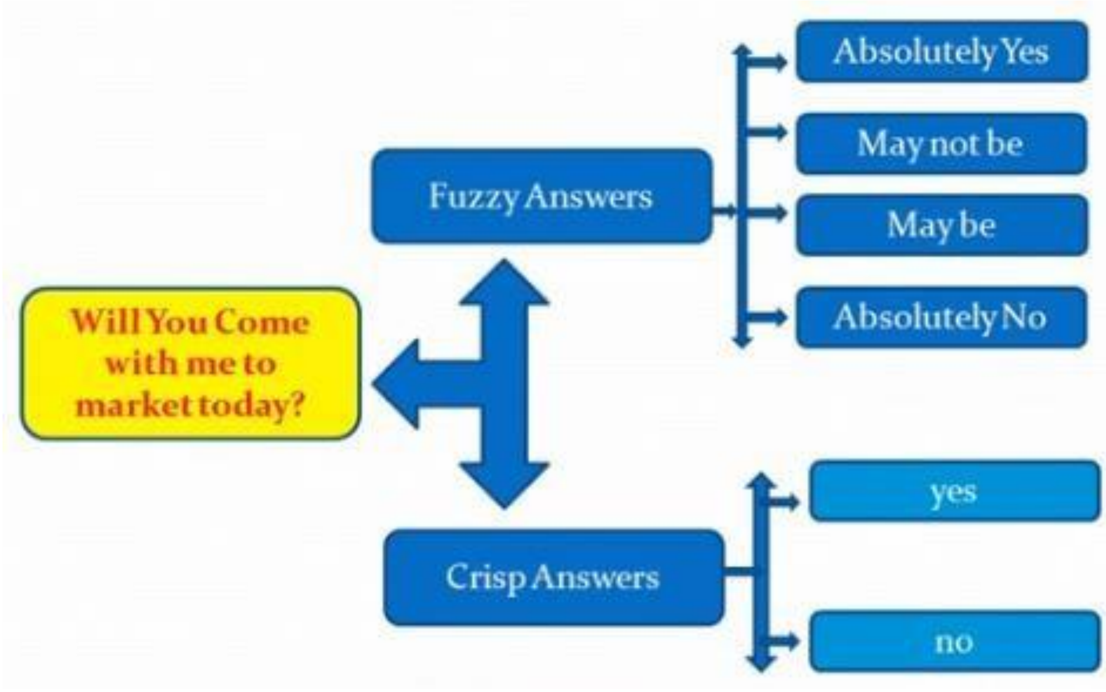
Fuzzy Logic (Continue)

Example



Fuzzy vs crips

Fuzzy Vs. Crisp

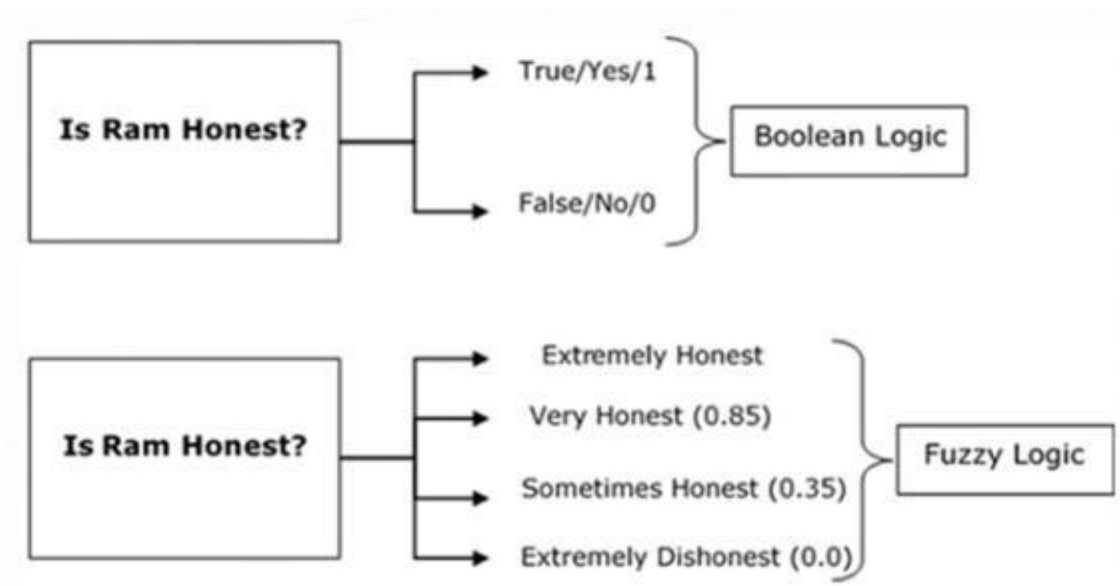


Crisp Vs Fuzzy

Crisp Vs Fuzzy Set

Crisp	Fuzzy
Suppose someone is running. Let's we takes the normalized value of running speed in range of 0 to 1. Suppose speed is greater than 0.5 value then let's it's consider as fast And if speed is less than 0.5 then its consider as slow Here value we define two crisp value that is slow and fast by defining a threshold value as 0.5	In case fuzzy logic. We can take intermediate value like slow, medium, fast, and very fast. Here we define 4 value instead of only 2 value. With fuZZY we can define more than what we defined here in this example For 0 to .25 ->slow For .25 to .5 ->medium For .5 to .75 -> fast For .75 to 1 -> very fast
It has strict Boundary Yes or No	Fuzzy boundary with a degree of membership
$S = \{s \mid s \in X\}$	$F = (s, \mu(s)) \mid s \in X$ and $\mu(s)$ is degree of s
Crisp set can be fuzzy	Fuzzy can not be crisp

Fuzzy Logic ?...



Is Ram Honest?
True/Yes/1
False/No/0
Boolean Logic
Is Ram Honest?
Extremely Honest
Very Honest (0.85)
Sometimes Honest (0.35)
Extremely Dishonest (0.0)
Fuzzy Logic

Classical set theory

- A Set is any well defined collection of objects.
- An object in a set is called an element or member of that set.
- Sets are defined by a simple statement,
- Describing whether a particular element having a certain property belongs to that particular set.

$$A = \{a_1, a_2, a_3, \ldots\ldots, a_n\}$$

- If the elements a_i ($i = 1,2,3,\ldots.,n$) of a set A are subset of universal set X, then set A can be represented for all elements $x \in X$ by its characteristics function

$$\mu_A(x) = 1 \text{ if } x \in X \text{ otherwise } 0$$

Operations on classical set theory

<u>Union:</u> the union of two sets A and B is given as

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

<u>Intersection:</u> the intersection of two sets A and B is given as

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

<u>Complement:</u> It is denoted by Ã and is defined as

$$\tilde{A} = \{ x \mid x \text{ does not belongs } A \text{ and } x \in X \}$$

Fuzzy Sets

- Fuzzy sets theory is an extension of classical set theory.
- Elements have varying degree of membership. A logic based on two truth values,
- ***True*** and ***False*** is sometimes insufficient when describing human reasoning.
- Fuzzy Logic uses the whole interval between 0 (false) and 1 (true) to describe human reasoning.
- A Fuzzy Set is any set that allows its members to have different degree of membership, called **membership function**, having interval [0,1].

FuZzy Set

- Rigid Boundaries

  BABY={age $\in$ AGE: 0 year $\leq$ age <1year},

  CHILD={age $\in$ AGE: 1 year $\leq$ age $\leq$10years},

  YOUNG= {age $\in$ AGE: 19 years $\leq$ age $\leq$ 40years},

  OLD= {age $\in$ AGE: 60 years $\leq$ age < 80years},

- and VERY OLD= {age $\in$ AGE: 80 years $\leq$ age < 120 years}.

Fuzzy Set

Not at all tall, if height < 5 feet

A little bit tall if his height = 5’ 2’’

Slightly tall if his height = 5’ 6’’

Reasonably tall if height = 5’ 9’’

Definitely tall if his height > 6 feet

Fuzzy Set

'Not at all tall', can be represented as 'tallness having value 0 '

'A little bit tall' can be represented as 'tallness having value 0.2 '

'Slightly tall' can be represented as 'tallness having value 0.5 '

'Reasonably tall' can be represented as 'tallness having value 0.7 '

'Definitely tall' can be represented as 'tallness having value 1 '

Fuzzy sets Example:

Tall = {Ram/.2; Sham/.7; Jodu/1; Modu/.5; Golu/0}

Fuzzy Sets

- **Fuzzy Logic** is derived from fuzzy set theory
- Many degree of membership (between 0 to 1) are allowed.
- Thus a membership function $\boldsymbol{\mu}_{A}^{(x)}$ is associated with a fuzzy sets Ã such that the function maps every element of universe of discourse X to the interval [0,1].
- The mapping is written as: $\mu_{\tilde{A}}(x)$: X → [0,1].

- Fuzzy Logic is capable of handing inherently imprecise (vague or inexact or rough or inaccurate) concepts

Fuzzy Sets

- **Fuzzy set** is defined as follows:

- If X is an universe of discourse and x is a particular element of X, then a fuzzy set A defined on X and can be written as a collection of ordered pairs

$$A = \{(x, \mu_{\tilde{A}}(x)), x \in X \}$$

Fuzzy Sets (Continue)

Example

- Let X = $\{g_1, g_2, g_3, g_4, g_5\}$ be the reference set of students.
- Let Ã be the fuzzy set of “smart” students, where “smart” is fuzzy term.

$$\tilde{A} = \{(g_1,0.4)(g_2,0.5)(g_3,1)(g_4,0.9)(g_5,0.8)\}$$

Here Ã indicates that the smartness of g_1 is 0.4 and so on

Fuzzy Sets (Continue)

Membership Function

- The membership function fully defines the fuzzy set
- A membership function provides a measure of *the degree of similarity* of an element to a fuzzy set

Membership functions can

- either be chosen by the user arbitrarily, based on the user's experience (MF chosen by two users could be different depending upon their experiences, perspectives, etc.)
- Or be designed using machine learning methods (e.g., artificial neural networks, genetic algorithms, etc.)

Fuzzy Membership Functions

- One of the key issues in all fuzzy sets is how to determine fuzzy membership functions
- The membership function fully defines the fuzzy set
- A membership function provides a measure of the degree of similarity of an element to a fuzzy set
- Membership functions can take any form, but there are some common examples that appear in real applications

- Membership functions can
  - either be chosen by the user arbitrarily, based on the user’s experience (MF chosen by two users could be different depending upon their experiences, perspectives, etc.)
  - Or be designed using machine learning methods (e.g., artificial neural networks, genetic algorithms, etc.)
- There are different shapes of membership functions; triangular, trapezoidal, piecewise-linear, Gaussian, bell-shaped, etc.

Fuzzy Sets (Continue)

There are different shapes of membership functions;

- **Triangular,**
- **Trapezoidal,**
- **Gaussian, etc**

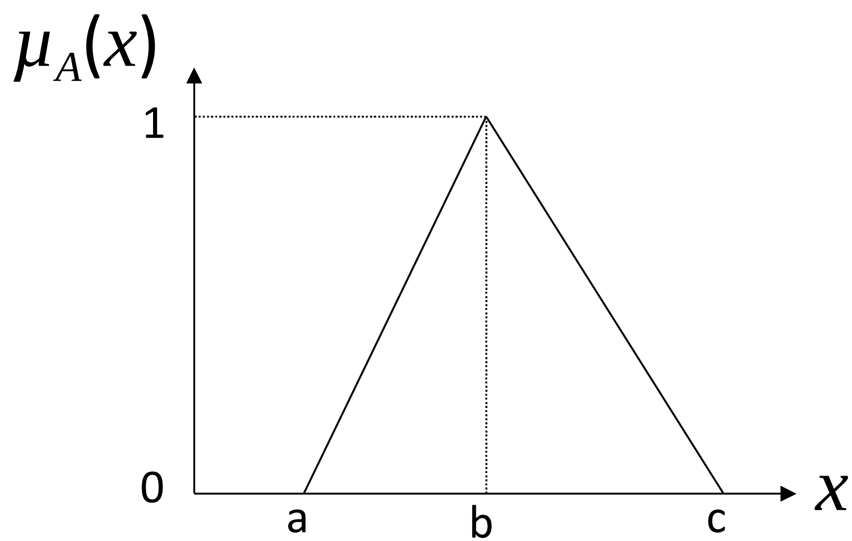
Fuzzy Sets (Continue)

- **Triangular membership function**

  A *triangular* membership function is specified by three parameters {a, b, c}

  a, b and c represent the x coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A (a: lower boundary and c: upper boundary where membership degree is zero, b: the centre where membership degree is 1)

$$\mu_A(x) = \begin{cases} 0 & if\ x \le a \\ \dfrac{x-a}{b-a} & if\ a \le x \le b \\ \dfrac{c-x}{c-b} & if\ b \le x \le c \\ 0 & if\ x \ge c \end{cases}$$



Fuzzy Sets (Continue)

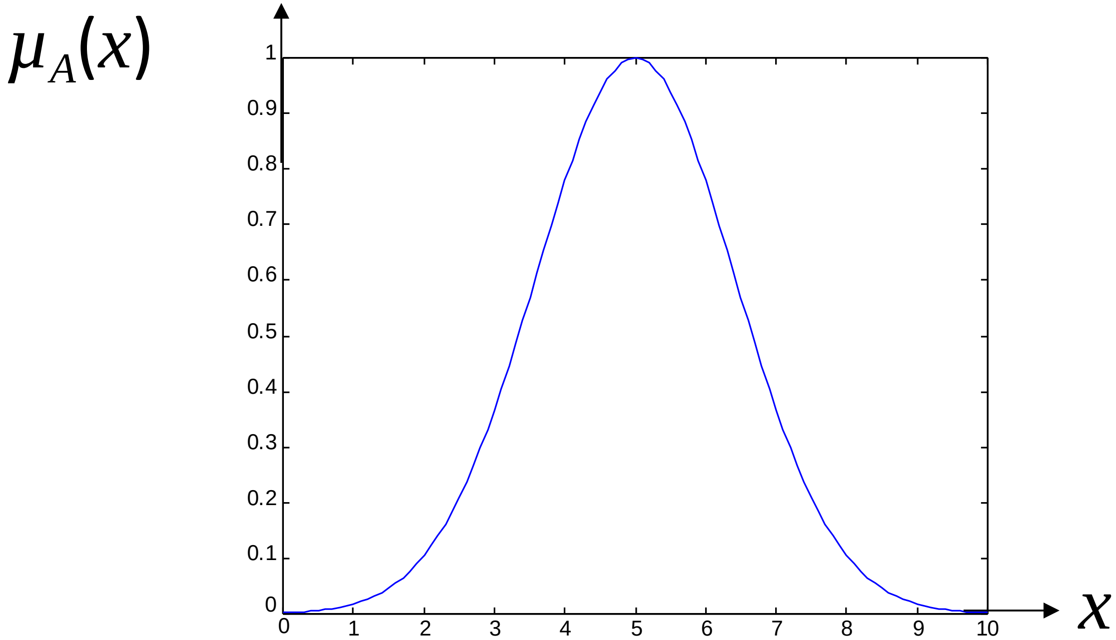
- **Trapezoid membership function**
- A *trapezoidal* membership function is specified by four parameters {a, b, c, d} as follows:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ 1 & \text{if } b \le x \le c \\ \dfrac{d-x}{d-c} & \text{if } c \le x \le d \\ 0 & \text{if } d \le x \end{cases}$$

- **Gaussian membership function**

$$\mu_A(x,c,s,m) = \exp\left[-\frac{1}{2}\left|\frac{x-c}{s}\right|^m\right]$$

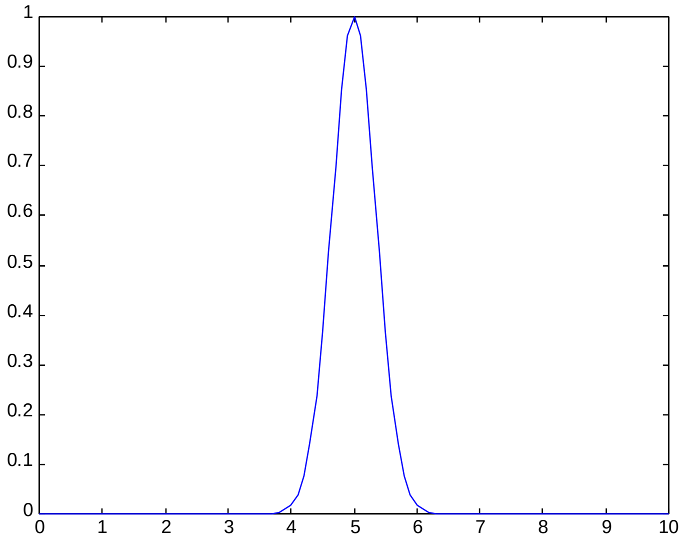
  - *c:* centre
  - *s:* width
  - *m:* fuzzification factor (e.g., *m*=2)



c=5

s=2

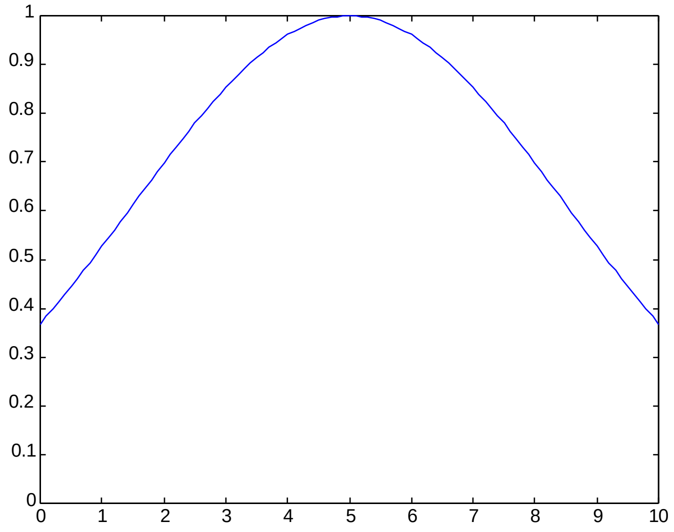
m=2



c=5

s=0.5

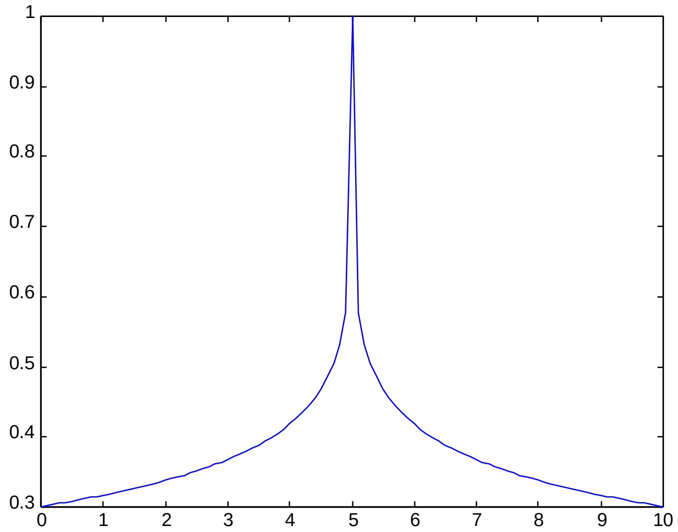
m=2



c=5

s=5

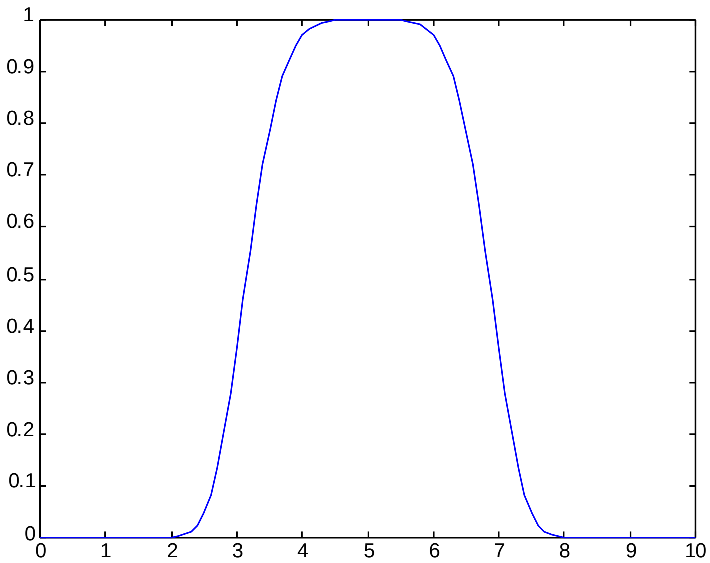
m=2



***c*=5**

***s*=2**

***m*=0.2**



$c=5$

$s=5$

$m=5$

Fuzzy Set Operation

Given X to be the universe of discourse and $\tilde{A}$ and $\dot{B}$ to be fuzzy sets with $\mu_A(x)$ and $\mu_B(x)$ are their respective membership function, the fuzzy set operations are as follows:

Union:

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

Intersection:

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Complement:

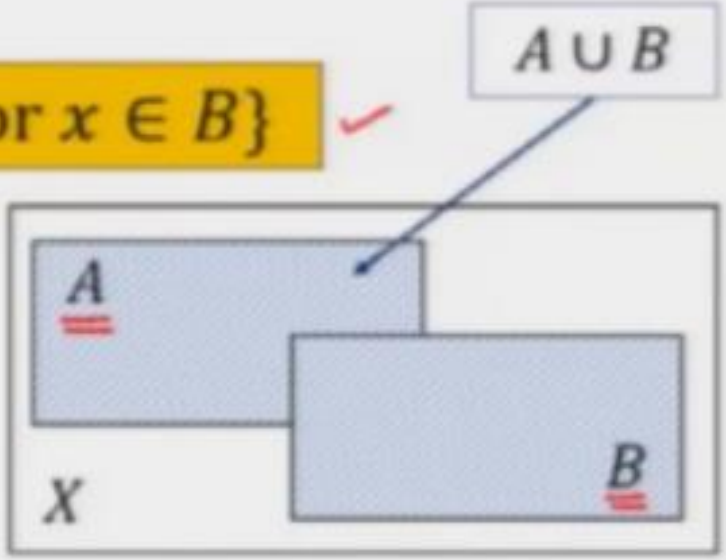
$$\mu_A(x) = 1 - \mu_A(x)$$

Union in classical Set

The union of two classical sets represents all elements in the universe of discourse X which belong to either the set A or the set B or both sets A and B. It is denoted by $A \cup B$.

It can be represented as:

$$\boxed{A \cup B = \{x | x \in A \text{ or } x \in B\}}$$

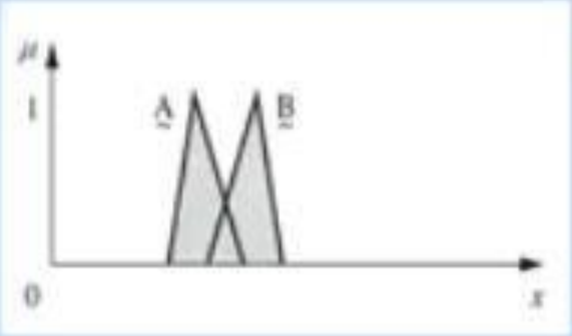


Venn Diagram

Union in Fuzzy Set

Union

$$\mu_{A \cup B}(x) = \mu_A(x) \cup \mu_B(x) \quad = \max(\mu_A(x), \mu_B(x))$$

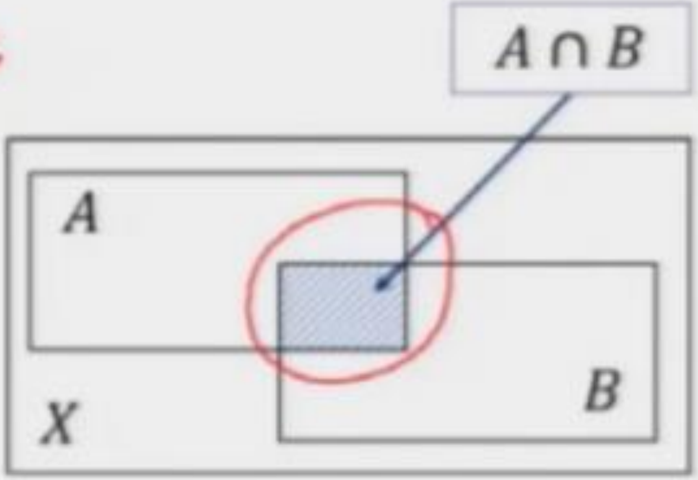


- E.g.
  - A = {1.0, 0.20, 0.75}
  - B = {0.2, 0.45, 0.50}
  - $A \cup B$ = {MAX(1.0, 0.2), MAX(0.20, 0.45), MAX(0.75, 0.50)}
    = {1.0, 0.45, 0.75}

Intersection in classical Set

The intersection of two sets A and B represents all elements in the universe of discourse X that simultaneously belong to both sets A and B. It is denoted by $A \cap B$ and can be represented as:

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

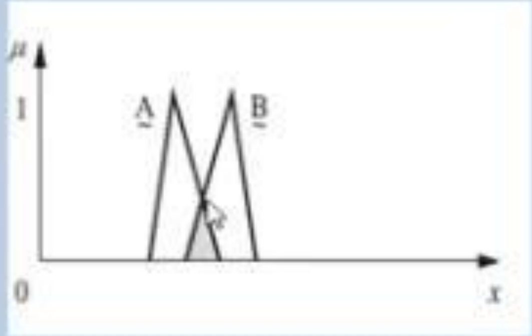


Venn Diagram

Intersection in Fuzzy Set

Intersection

- $\mu_{A \cap B}(x) = \mu_A(x) \cap \mu_B(x) = \min(\mu_A(x), \mu_B(x))$
- Example-:
  - A = {1.0, 0.20, 0.75}
  - B = {0.2, 0.45, 0.50}



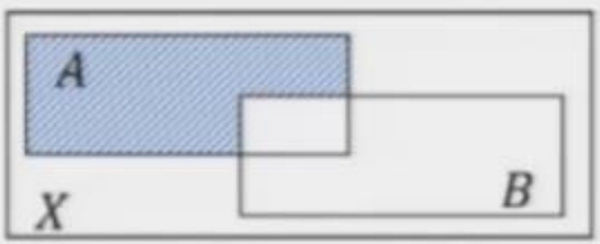
- $A \cap B$ = {MIN(1.0, 0.2), MIN(0.20, 0.45), MIN(0.75, 0.50)} = {0.2, 0.20, 0.50}

Difference in classical Set

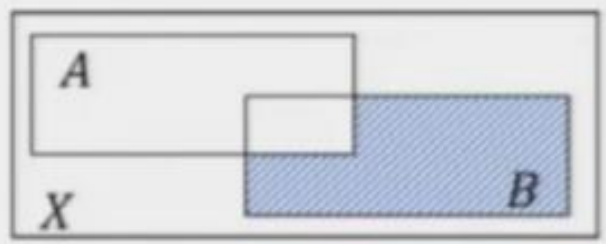
The difference of a set A with respect to B is defined as the collection of all elements in the universe of discourse X that belong to set A but does not belong to B. It is denoted by $A|B$ and can be represented as:

$$A|B = \{x | x \in A \text{ and } x \notin B\}$$

$$B|A = \{x | x \notin A \text{ and } x \in B\}$$



Venn Diagram of $A|B$



Venn Diagram of $B|A$

Difference in Fuzzy Set

For the given fuzzy sets A and B with the membership function values given as $\mu_A(x)$ and $\mu_B(x)$, respectively in the universe of discourse X, the fuzzy difference is given as:

$$\boxed{\mu_{A|B}(x) = \min[\mu_A(x), \mu_{\bar{B}}(x)] \ \forall x \in X}$$

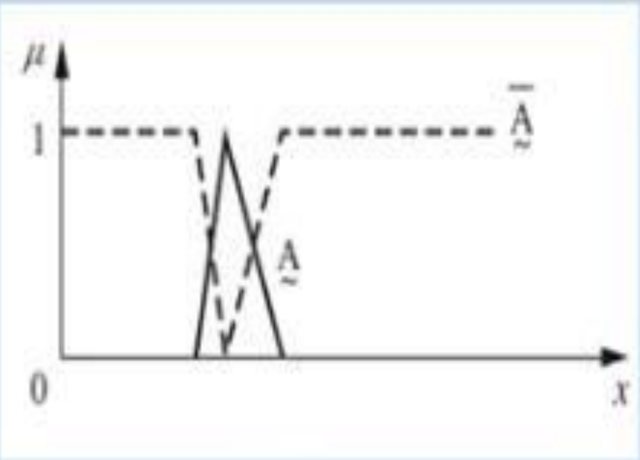
$$\boxed{\mu_{B|A}(x) = \min[\mu_B(x), \mu_{\bar{A}}(x)] \ \forall x \in X}$$

Complement in Fuzzy Set

Complement

- $\mu_{A'}(x) = 1 - \mu_A(x)$

  $A = \{1.0, 0.20, 0.75\}$



- $A' = \{ (1-1.0), (1-.20), (1-0.75)\}$
- $= \{0, .80, .25\}$

Fuzzy Set Operation (Continue)

Example:

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$ $B = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$

Union:

$$A \cup B = \{(x_1,0.8),(x_2,0.7),(x_3,1)\}$$

Because

$$\mu_{A\cup B}(x_1) = \max(\mu_A(x_1), \mu_B(x_1))$$

$$= \max(0.5,0.8)$$

$$= 0.8$$

$$\mu_{A\cup B}(x_2) = 0.7 \text{ and } \mu_{A\cup B}(x_3) = 1$$

Fuzzy Set Operation (Continue)

Example:

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$ $B = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$

<u>Intersection:</u>

$$A \cap B = \{(x_1,0.5),(x_2,0.2),(x_3,0)\}$$

Because

$$\mu_{A\cap B}(x_1) = \min(\mu_A(x_1),\ \mu_B(x_1))$$

$$= \max(0.5,0.8)$$

$$= 0.5$$

$$\mu_{A\cap B}(x_2) = 0.2 \text{ and } \mu_{A\cap B}(x_3) = 0$$

Fuzzy Set Operation (Continue)

Example:

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$

Complement:

$$A^c = \{(x_1,0.5),(x_2,0.3),(x_3,1)\}$$

Because

$$\mu_A(x_1) = 1 - \mu_A(x_1)$$

$$= 1 - 0.5$$

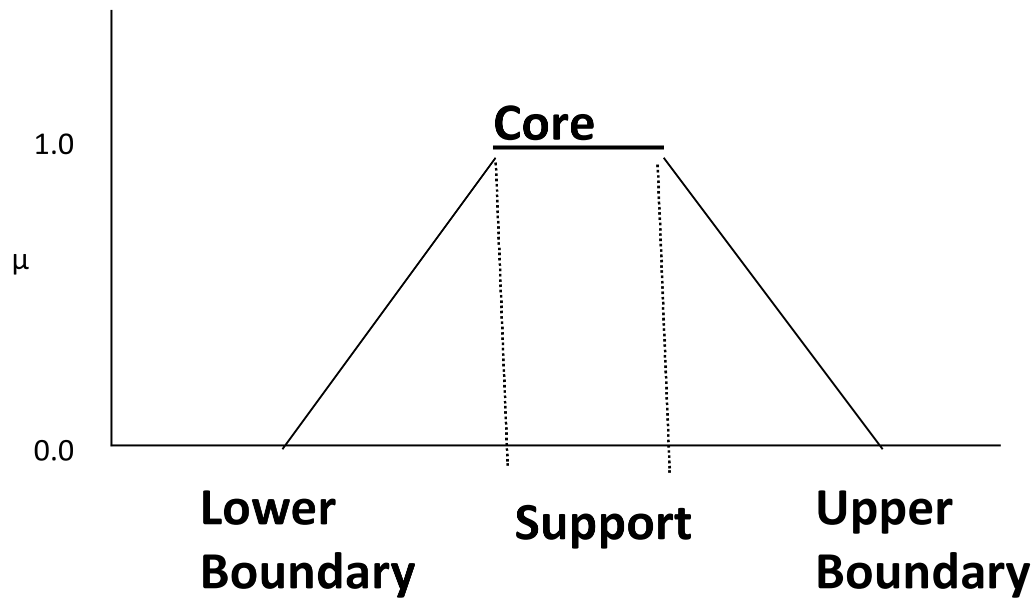
$$= 0.5$$

$$\mu_A(x_2) = 0.3 \text{ and } \mu_A(x_3) = 1$$

- **Support(A)** is set of all points x in X such that
  $\{(x| \mu_A(x) > 0 \}$

- **core(A)** is set of all points x in X such that
  $\{(x| \mu_A(x) = 1 \}$

- Fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called fuzzy singleton



Core
1.0
μ
0.0
Lower Boundary
Support
Upper Boundary

Example 1:

Suppose we have the following (discrete) fuzzy sets:

$$A = 0.4/1+0.6/2+0.7/3+0.8/4$$

$$B = 0.3/1+0.65/2+0.4/3+0.1/4$$

The union of the fuzzy sets *A* and *B*

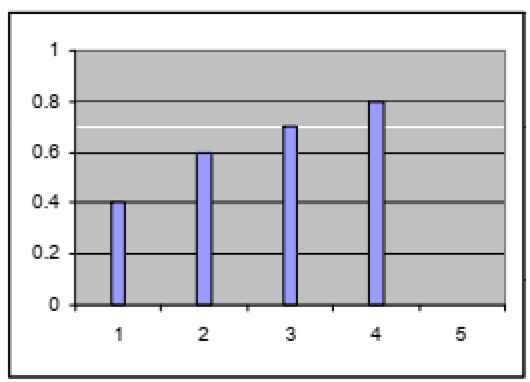
$= 0.4/1+0.65/2+0.7/3+0.8/4$

The intersection of the fuzzy sets *A* and *B*

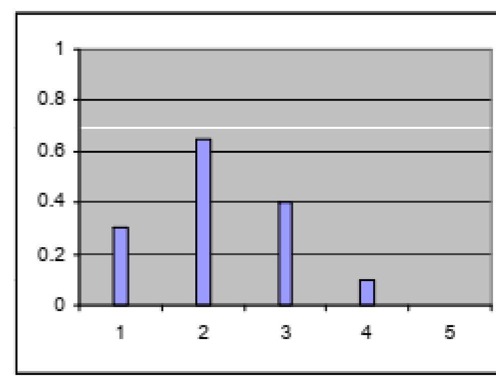
$= 0.3/1+0.6/2+0.4/3+0.1/4$

The complement of the fuzzy set A

$= 0.6/1+0.4/2+0.3/3+0.2/4$



A



B

Example 2

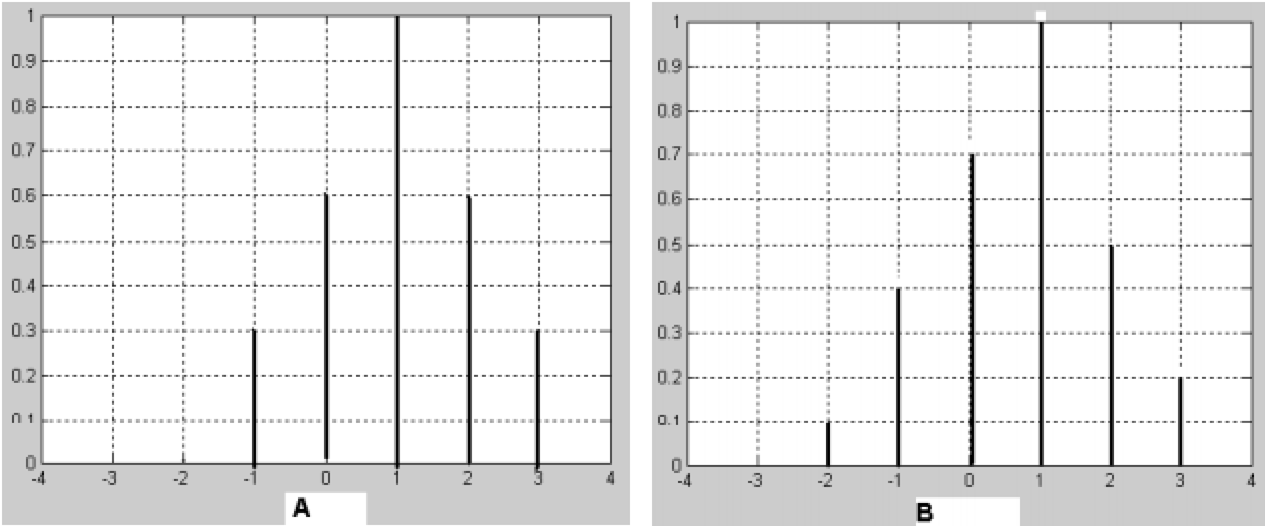
Given two fuzzy sets A and B

a. Represent A and B fuzzy sets graphically
b. Calculate the of union of the set A and set B
c. Calculate the intersection of the set A and set B
d. Calculate the complement of the union of A and B

$$A = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$$

$$B = 0.1/-2 + 0.4/-1 + 0.7/0 + 1.0/1 + 0.5/2 + 0.2/3 + 0.0/4$$

a



b

$$\text{Union} = \max(A, B) = 0.1/-2 + 0.4/-1 + 0.7/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$$

c

$$\text{Intersection} = \min(A, B) = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.5/2 + 0.2/3 + 0.0/4$$

d

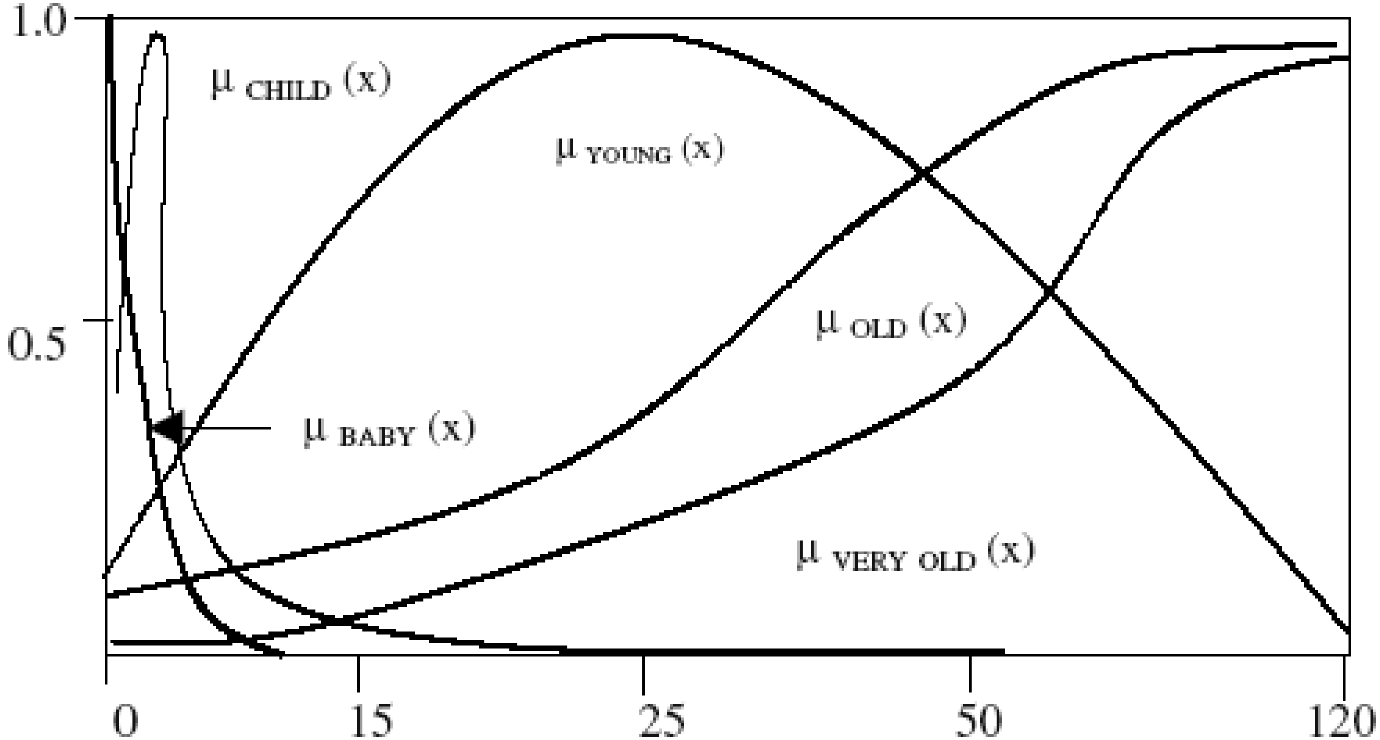
$$\text{Complement of (b)} = 1 - \max(A, B) = 0.9/-2 + 0.6/-1 + 0.3/0 + 0.0/1 + 0.4/2 + 0.7/3 + 1.0/4$$

Linguistic variable, linguistic term

- **Linguistic variable: A** *linguistic variable* is a variable whose values are sentences in a natural or artificial language.
- **For example**, the values of the fuzzy variable *height* could be *tall, very tall, very very tall, somewhat tall, not very tall, tall but not very tall, quite tall, more or less tall.*
- *Tall is a linguistic* value or primary term

- If ***age*** is a linguistic variable then its term set is
- *T(age) = { young, not young, very young, not very young,…… middle aged, not middle aged, … old, not old, very old, more or less old, not very old,…not very young and not very old,…}.*

Membership Function



Properties of Fuzzy Sets

Properties of Fuzzy Sets

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Commutativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

Associativity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Distributivity

$$A \cup A = A \qquad A \cap A = A$$
$$A \cup \varnothing = A \qquad A \cap \varnothing = \varnothing$$

Idempotence

If $A \subseteq B \subseteq C$, then $A \subseteq C$

Transitivity

$$(A \cap B)' = A' \cup B'$$
$$(A \cup B)' = A' \cap B'$$

De Morgans Law

Operations

- $1 - \max(\mu_A, \mu_B) = 1 + \min(-\mu_A, -\mu_B)$
- $1 - \max(\mu_A, \mu_B) = \min(1 - \mu_A, 1 - \mu_B)$
- $1 - \max(\mu_A, \mu_B) = 1 + (-1) \times \begin{cases} \mu_A: \mu_A \geq \mu_B \\ \mu_B: \mu_A < \mu_B \end{cases}$
- $1 - \max(\mu_A, \mu_B) = 1 + \begin{cases} -\mu_A: \mu_A \geq \mu_B \\ -\mu_B: \mu_A < \mu_B \end{cases}$
- $1 - \max(\mu_A, \mu_B) = 1 + \begin{cases} -\mu_A: -\mu_A \leq -\mu_B \\ -\mu_B: -\mu_A > -\mu_B \end{cases}$
- $1 - \max(\mu_A, \mu_B) = 1 + \min(-\mu_A, -\mu_B)$
- $1 - \max(\mu_A, \mu_B) = \begin{cases} 1 - \mu_A: -\mu_A \leq -\mu_B \\ 1 - \mu_B: -\mu_A > -\mu_B \end{cases}$
- $1 - \max(\mu_A, \mu_B) = \min(1 - \mu_A, 1 - \mu_B)$

De Morgan’s Law

- $\mu_{\overline{A \cup B}} = \mu_{\bar{A} \cap \bar{B}}$
- $\mu_{\overline{A \cap B}} = \mu_{\bar{A} \cup \bar{B}}$
- Proving
  - $\mu_{\overline{A \cup B}} = 1 - \max(\mu_A, \mu_B)$
  - $\mu_{\overline{A \cup B}} = \min(1 - \mu_A, 1 - \mu_B)$
  - $\mu_{\overline{A \cup B}} = \min(\mu_{\bar{A}}, \mu_{\bar{B}})$
  - $\mu_{\overline{A \cup B}} = \mu_{\bar{A} \cap \bar{B}}$
- Proving
  - $\mu_{\overline{A \cap B}} = 1 - min(\mu_A, \mu_B)$
  - $\mu_{\overline{A \cap B}} = max(1 - \mu_A, 1 - \mu_B)$
  - $\mu_{\overline{A \cap B}} = max(\mu_{\bar{A}}, \mu_{\bar{B}})$
  - $\mu_{\overline{A \cap B}} = \mu_{\bar{A} \cup \bar{B}}$

Fuzzy Logic vs Probability

Fuzzy Logic	Probability
In fuzzy logic, we basically try to capture the essential concept of vagueness.	Probability is associated with events and not facts, and those events will either occur or not occur
Fuzzy Logic captures the meaning of partial truth	Probability theory captures partial knowledge
Fuzzy logic takes truth degrees as a mathematical basis	Probability is a mathematical model of ignorance

Applications of Fuzzy Logic

The Fuzzy logic is used in various fields such as automotive systems, domestic goods, environment control, etc. Some of the common applications are:

- It is used in the **aerospace field** for **altitude control** of spacecraft and satellite.
- This controls the **speed and traffic** in the **automotive systems.**
- It is used for **decision making support systems** and personal evaluation in the large company business.
- It also controls the pH, drying, chemical distillation process in the **chemical industry**.
- Fuzzy logic is used in **Natural language processing** and various intensive applications in Artificial Intelligence.
- It is extensively used in **modern control systems** such as expert systems.
- Fuzzy Logic mimics how a person would make decisions, only much faster. Thus, you can use it with Neural Networks.

More Example

Consider the fuzzy sets A = damping ratio x considerably larger than 0.5, and B = damping ratio x approximately equal to 0.707. Note that damping ratio is a positive real number, i.e., its universe of discourse, X, is the positive real numbers $0 \le x \le 1$

Consequently, $A = \{(x, \mu_A(x)) | x \varepsilon X\}$ and $B = \{(x, \mu_B(x)) | x \varepsilon X\}$

where, for example, $\mu_A(x)$ and $\mu_B(x)$ are specified, as:

$$\mu_A(x) = \begin{cases} 0 & \text{if } 0 \le x \le 0.5 \\ \dfrac{1}{1+(x-0.5)^{-2}} & \text{if } 0.5 < x \le 1 \end{cases} \qquad \mu_B(x) = \frac{1}{1+(x-0.707)^4} \qquad 0 \le x \le 1$$

Figure (a): $\mu_A(x)$, $\mu_B(x)$ *Figure (b)*: $\mu_{A \cup B}(x)$

Figure (c): $\mu_{A \cap B}(x)$ *Figure (d)*: $\mu_B(x)$, $\mu_{\overline{B}}(x)$

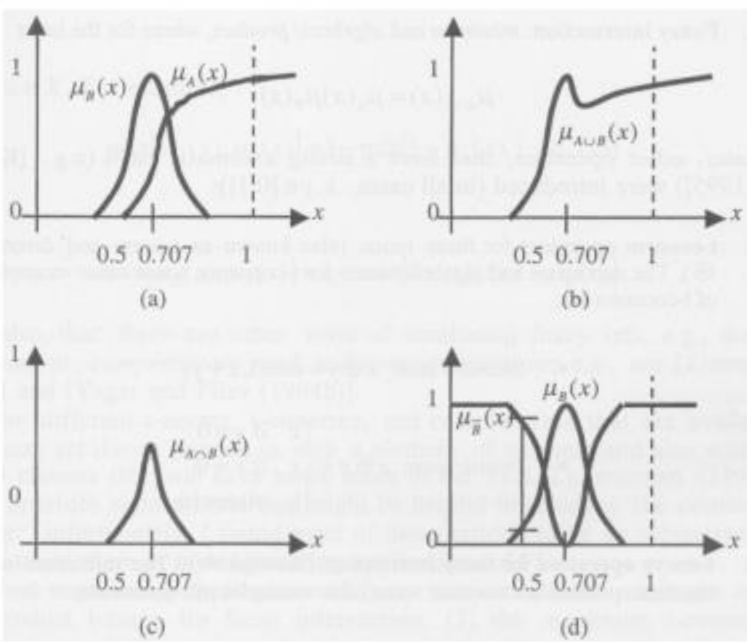


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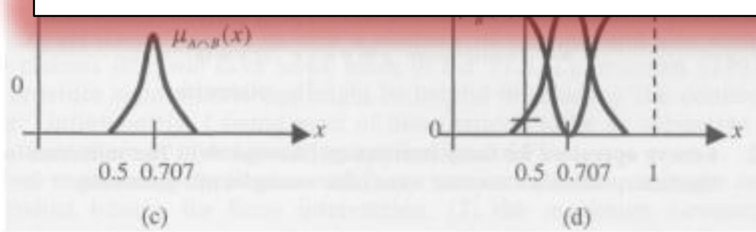
Figure (a): $\mu_A(x), \mu_B(x)$ *Figure (b):* $\mu_{A \cup B}(x)$

Figure (c): μ_{A}

- This example demonstrates that for fuzzy sets, the Law of Excluded Middle and Concentration are broken, i.e., *for fuzzy sets A and B*:

$$A \cup \bar{A} \neq X \text{ and } A \cap \bar{A} \neq \emptyset$$

- In fact, one of the ways to describe the difference between crisp set theory and fuzzy set theory is to explain that these two laws do not hold in fuzzy set theory



THANK YOU