APPLICATION OF FUZZY SET THEORY TO AUTONOMOUS MOBILE CONTROL

BY

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Introduction

In the physical world we do not always have clear distinction between some subject matters, that is, there exists much fuzzy knowledge. Instead we deal with degrees of truth, like temperature, height, distance, speed, beauty and so on. With this, we see that such degrees of truth play an important role in human thinking, especially in pattern recognition, communication of information and abstraction.

Fuzzy set theory provides a strict mathematical framework in which vague conceptual phenomenon can be precisely and rigorously studied.

Professor Lofti Asker Zadeh in 1950 founded the Fuzzy sets theory and his first publication was made in 1965.

The term Fuzzy refers to vagueness. The Fuzzy set theory attempts to model uncertainty as opposed to normal Boolean logic which has only two states, that is, either "yes or no" or "true or false" and nothing in between that is, they are crisp, deterministic and precise.

The story of fuzzy sets started way back and a few philosophers who laid the foundation are:

- Aristotle (put fort the so called "Laws of Thought")
- Plato(put up a third option besides true or false)
- Lukasiewicz(described 3-valued logic accompanied with mathematics and later multivalued logic)

Motivation

For systems that have a particular degree of vagueness or kind of uncertainty, we need some way to handle them. Hence, Fuzzy sets.

A classic example of vagueness can be gotten by referring to an old paradox, the Sorites.

Another example is defining the set of tall men where the criteria for being in the category depends on height.

If tall men are defined to be above 180cm and otherwise for below, crisp sets, that is, classical set theory asks the question: Is the man tall? While Fuzzy set asks the question: How tall is the man?

As classical sets cannot handle these sort of scenarios, Fuzzy set theory is then an extension of classical set theory.Therefore, a lot of classical (crisp) set theories can be extended to that of fuzzy.

Compactification is another important reason for Fuzzy sets. Due to limited capacity of technical systems or of short term human memory, storage of relevant data or presentation of massive amounts of data to someone in a manner that is easy to analyse is often not possible.

Technology developed via Fuzzy set theory have been used to reduce how complex data is to an acceptable degree either via linguistics variables or via fuzzy data analysis.

Basic definitions and theorems

What is a fuzzy set? According to Professor Zadeh, a fuzzy set is a class of objects with a continuum of grades of membership.

Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established.

DEFINITIONS:

1. Let X be a non-empty set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function

$$\mu_{\tilde{A}} : X \to [0, 1]$$

and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership(degree of truth) of element x in fuzzy set $\tilde{A}$ for each $x \in X$. The set $\tilde{A}$ can be written as a set of ordered pairs.

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}$$

$\mu_{\tilde{A}}$ is also called membership function.

2. (Continuous fuzzy sets) A fuzzy set is said to be continuous if its membership function is continuous.

3. (Singleton fuzzy sets)
A fuzzy set that has non-zero membership value for only one element of the universe of discourse is called a singleton fuzzy set.

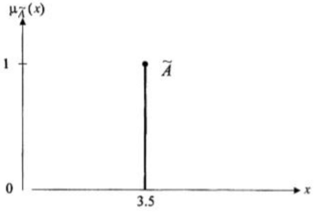


Figure: Singleton fuzzy set

4. (Support of a fuzzy set)
For a fuzzy set whose universe of discourse is X, all the elements in X that have non-zero membership values form the support of the fuzzy set. That is,

$$supp(\tilde{A}) = \left\{ x \in X | \tilde{A}(x) > 0 \right\}$$

5. (Height of a fuzzy set) The largest membership value of a fuzzy set is called the height of the fuzzy set.

6. (Fuzzy number)
A fuzzy number is a fuzzy set of the real line with a normal (that is, height of 1), convex and continuous membership function of bounded support.

7. (Triangular fuzzy number)
A fuzzy set $\tilde{A}$ is called triangular fuzzy number with centre a, left width $\alpha > 0$ and right width $\beta > 0$ if its membership function is of the following form;

$$\mu_{\tilde{A}}(t) = \begin{cases} 1 - \frac{a-t}{\alpha} & \text{if } a - \alpha \leq t \leq a \\ 1 - \frac{t-a}{\beta} & \text{if } a \leq t \leq a + \beta \\ 0 & \text{Otherwise} \end{cases}$$

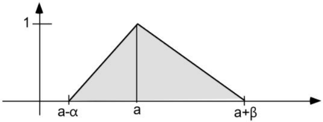


Figure: Graphical representation of triangular fuzzy number

8. (Trapezoidal fuzzy number)
A fuzzy set $\tilde{A}$ is called trapezoidal with tolerance interval [a,b], left width α and right width β if its membership function takes the form:

$$\mu_{\tilde{A}}(t) = \begin{cases} 1 - \frac{a-t}{\alpha} & if\ a - \alpha \leq t \leq a \\ 1 & if\ a \leq t \leq b \\ 1 - \frac{t-b}{\beta} & if\ a \leq t \leq b + \beta \\ 0 & Otherwise \end{cases}$$

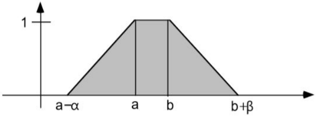


Figure: Graphical representation of trapezoidal fuzzy number

PROPERTIES

Since fuzzy set theory is an extension of the classical set theory we can have the following:

- Commutativity
- Associativity
- Distributivity
- Idempotency Property
- Involution
- Identity Property
- Transitivity
- De Morgan's principles

OPERATIONS

Some operations in fuzzy set are:

- **Intersection** The intersection,that is, the overlap of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is defined as;

$$\tilde{A} \cap \tilde{B} = \{x, \mu_{\tilde{A}\cap\tilde{B}}(x)\}$$

$$\mu_{\tilde{A}\cap\tilde{B}}(x) = min\left\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right\}, x \in X$$

- **Union** The union, that is, the gluing of two fuzzy $\tilde{A}$ and $\tilde{B}$ is defined as;

$$\tilde{A} \cup \tilde{B} = \{x, \mu_{\tilde{A}\cup\tilde{B}}(x)\}$$

$$\mu_{\tilde{A}\cup\tilde{B}}(x) = max\left\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right\}, x \in X$$

- **Complement** The complement of a fuzzy set $\tilde{A}$ is denoted as $\tilde{A}^c$ and defined as;

$$\tilde{A}^c = \{x, \mu_{\tilde{A}^c(x)\cup\tilde{B}}(x)\}$$

$$\mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x), x \in X$$

Main work

The main use of the fuzzy set theory as it applies to my project was in the use of fuzzy logic.

FUZZY LOGIC AND CONTROL

In trying to model human reasoning or decision making,the usual Boolean logic where we either have true or false does not always do justice.

This is because human reasoning ability is not crisp or definite.Hence,we need a form of logic to define degrees of truths.Then with the existence of fuzzy set theory,it is possible to construct a logic system which can give models close to human reasoning,that is, fuzzy logic.

Linguistic Variables

These are natural languages used as the values for variables instead of number. These are used since they are less specific than numerical ones. For example, "Temperature" is a linguistic variable with values like “very hot”, “hot”,"warm”, “cold”, "very cold" or “Age” with values like “very young”, “young”, “quite young”, “old”, “quite old”, “very old”.

Definition: A linguistic variable is defined by the quintuple $(x, T(x), U, G, \tilde{M})$ where x is the variable name, $T(x)$ is the collection of linguistic values, that is the term-set of x, U is the universe of discourse, G is the syntactic rule that generates the terms in $T(x)$ and $\tilde{M}$ is a semantic rule which associates each linguistic value, X, with its meaning, $\tilde{M}(X)$,where $\tilde{M}(X)$ is fuzzy subset of U.

An example of why linguistic variables are important is in the case of temperature control. Since the terms hot and cold are vague, linguistic variables serve as a way to give meaning to certain crisp variable.The membership function values of the crisp variables associated linguistic variables range from 0 to 1.

Fuzzy rules
A fuzzy rule can be defined as a conditional statement of the form;
IF x is $\tilde{A}$
Then y is B.

It comes in the form of an antecedent, that is, x is $\tilde{A}$ and a consequent, that is y is $\tilde{B}$, where x and y are linguistic variables; and $\tilde{A}$ and $\tilde{B}$ linguistic values determined by fuzzy sets on two universe of discourse X and Y respectively.

In traditional Boolean logic we may have
Rule: 1
IF speed is > 100
THEN *stopping_distance* is long

Rule: 2
IF speed is < 40
THEN *stopping_distance* is short

The variable speed can take any numerical value between 0 and $200km/hr$, but stopping distance which is a linguistic variable can take either short or long.

However in Fuzzy logic we can define the same rule as;
Rule: 1
IF speed is fast
THEN *stopping_distance* is long

Rule: 2
IF speed is slow
THEN *stopping_distance* is short

A fuzzy rule can have multiple antecedents. For example;

IF service is excellent
OR food is delicious
THEN tip is generous

A fuzzy rule can also have multiple consequents. For example;

IF temperature is hot
THEN *hot_water* is reduced;
cold_water is increased
Fuzzy rules are also usually represented in tabular form:

Temperature/target	Too-cold	Cold	Warm	Hot	Too-hot
Too-cold	No-change	Heat	Heat	Heat	Heat
Cold	Cool	No-change	Heat	Heat	Heat
Warm	Cool	Cool	No-change	Heat	Heat
Hot	Cool	Cool	Cool	No-change	Heat
Too-hot	Cool	Cool	Cool	Cool	No-change

Interpreted as:
IF(temperature is cold OR too cold)AND(target is warm)
THEN command is heat.

Fuzzy Inference System(Fuzzy Controller)

Definition : A way of using fuzzy control as a way to process fuzzy information is called a fuzzy controller.
The fuzzy controller works on a rule based system and it has to be able to function on fuzzy input data to create an output, usually also fuzzy,using the theories of fuzzy set.
There are two major types of fuzzy inference system:

- **Mamdani FIS**
- **Takagi-Sugeno FIS (TS)**

The difference between the two FIS lies in the fuzzy rule construction.

In the Mamdani, we have:
IF x_1 IS $\tilde{A}_1$ AND $\cdots$ AND x_M is $\tilde{A}_M$ THEN y_1 is $B_1, \ldots, y_P$ is B_P

In the Takagi Sugeno, we have:
IF x_1 is $\tilde{A}_1$ AND. . .AND x_M is $\tilde{A}_M$
THEN $y_1 = f_1(x_1, \ldots, x_M), \ldots, y_P = f_P(x_1, \ldots, x_M)$.

The fuzzy rule consequent is a function of the inputs in the TS-FIS while that of the mamdani is an output fuzzy set.

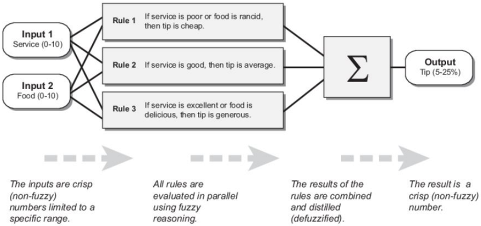
The steps considered in the creation of a fuzzy inference system are as follows:

- Fuzzification
- Fuzzy rule construction and evaluation
- Implication
- Aggregation
- Defuzzification

Example A popular example used in the explanation of fuzzy inference systems is the tipping problem.
Given a number between 1 and 10 that represents the quality of service and food of a restaurant(where 10 is excellent), what should the tip be?
This can be done rather easily with the fuzzy approach. First create fuzzy rules for which the input variables will be defined by. The rules can be

1. If service is poor or food is rancid, then tip is cheap
2. If service is good, then tip is average
3. If service is excellent or food is delicious, then tip is generous

These 3 rules form the core of the solution. Now we give meaning to the linguistic variables (like “average” tip, “delicious” food, “good” service and so on). This can be done quickly using the MATLAB fuzzy logic tool box. Given that we are considering two types of data, that is, service and food. Then this is a fuzzy inference system with two inputs and one output, that is, “tip”.



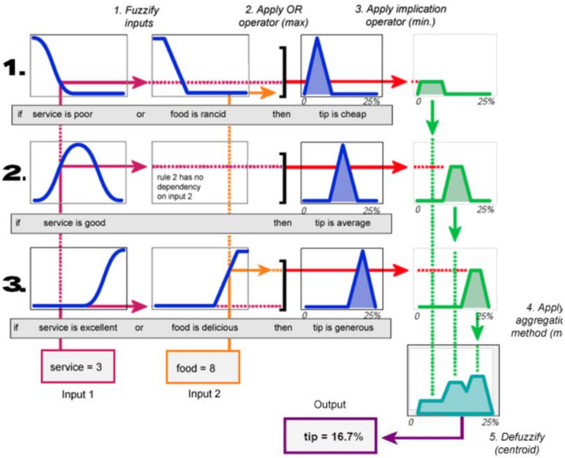


Figure: Fuzzy inference process

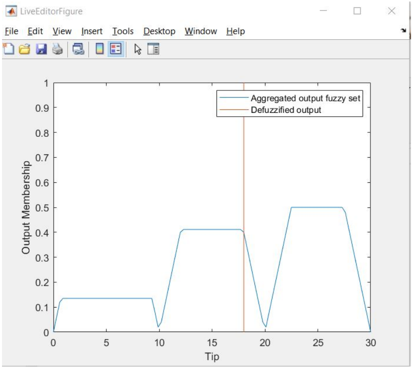


Figure: Defuzzification

This is carried out using the centroid (Center Of Area) method.It is given mathematically as,

$$x^* = \frac{\int \mu_{\tilde{A}}(x) \cdot x dx}{\int \mu_{\tilde{A}}(x) \cdot dx}$$

where x^* is the defuzzified output.
For the above example, the centroid can be calculated by breaking the shape into smaller shapes whose areas can be easily found.

	Sub-Area	Centroid of Area	Centroid of Area X Sub-Area
1	0.6	15	9
2	1.23836	5.025	6.2228
3	3.26824	14.925	48.778
4	3.4	24.975	84.915
Total	**8.5066**		**148.9158**

$$x^* = \frac{148.9158}{8.5066} = 17.506 \approx 18$$

Applications

Fuzzy set theory is applied in many areas besides in pure mathematics.

It is finding use in many aspects of the engineering, finance(for example,transportation models,maintenance models,inventory control models),computing world(for example,algorithms,mathematical programming)and various other real world problems.

In my application, I focused on a practical use of Fuzzy set theory. I worked on an obstacle avoidance robot using fuzzy logic using MATLAB's fuzzy toolbox, and Arduino Uno annd some other components and software I had available.

First was the creation of the Fuzzy Inference system in MATLAB. I used the TS process in my application as this is more computationally efficient.

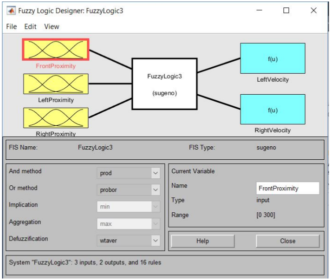


Figure: Fuzzy logic Designer

Next was the creation of the 3 input along with their membership functions defined.

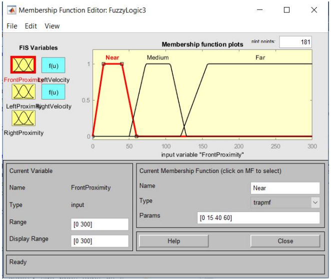


Figure: Front proximity

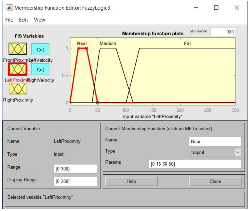


Figure: Left proximity

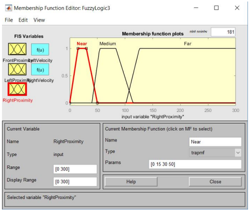


Figure: Right proximity

The three input variable each have three trapezoidal membership functions.These membership functions act on crisp input data gotten from each of the ultrasonic sensors defined for them namely:

- Near: I defined this to take values, [0 15 40 60] and are used in defining what the system perceives as nearness to an obstacle.
- Medium: I defined this to take values,[50 72.6 106.2 128] and are used in defining what the system perceives as medium distance to an obstacle.
- Far: I defined this to take values,[120.2 157 300 399] and are used in defining what the system perceives as far distance to an obstacle.

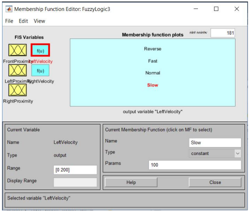


Figure: Left Velocity

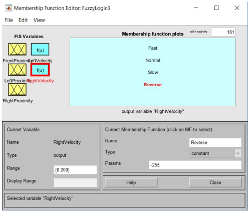


Figure: Right Velocity

The two output variables are functions of a combination of the inputs and are given by:

- Fast: This value is defined to have the value of 255.
- Slow: This value is defined to have the value of 100.
- Normal: This value is defined to have the value of 200.
- Reverse: This value is defined to have the value of -255

Front Proximity	Left Proximity	Right Proximity	Left Speed	Right Speed
N	N	N	r	n
N	N	F	f	s
M	M	F	n	s
M	N	F	f	s
M	M	M	s	s
M	N	M	f	s
F	F	F	f	f
N	M	F	s	f
N	N	0	s	f
0	N	0	f	s
M	M	N	n	r
M	F	N	s	f
N	M	N	f	s
N	0	N	s	n
0	0	N	r	n
N	0	0	r	n

Figure: My Inference Rules

The above table is read as;
IF FrontProximity is Near and LeftProximity is Near and RightProximity is Near
THEN LeftVelocity is reverse and RightVelocity is normal.

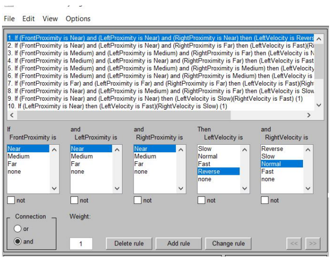


Figure: Rule Editor

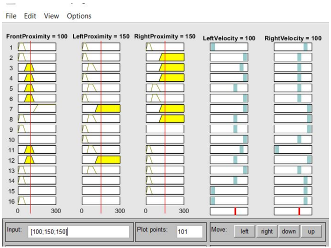


Figure: Rule Viewer

Conclusion

THANK YOU.