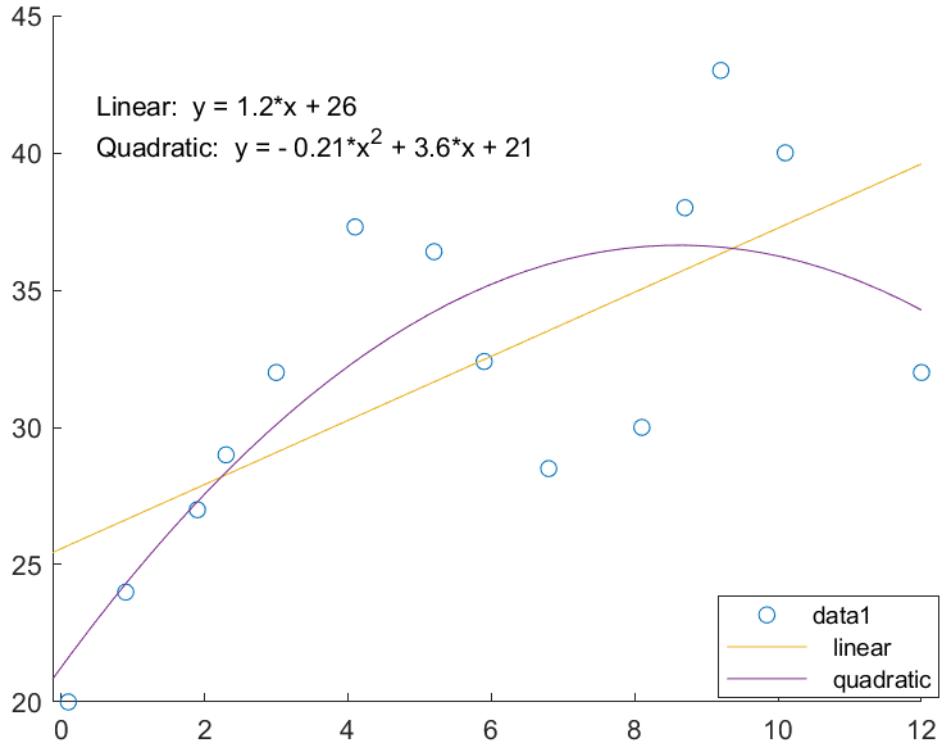
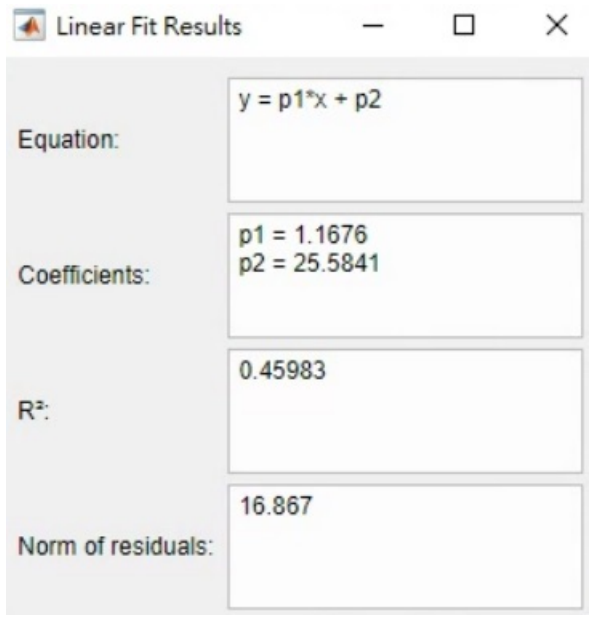
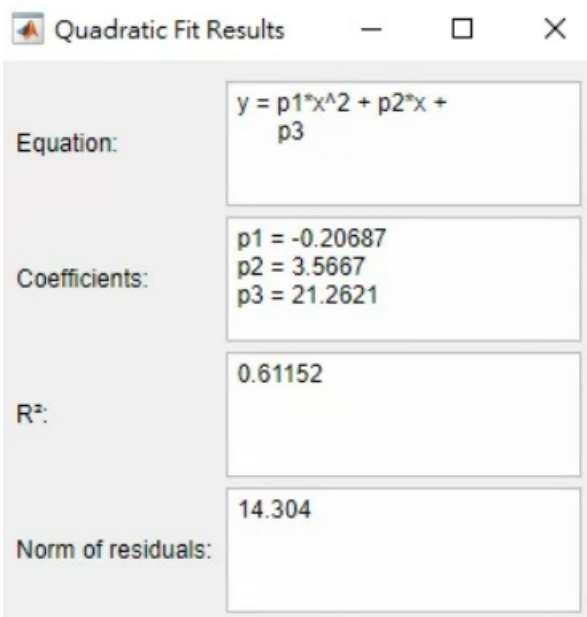
1

Use matlab to find (analytically) the coefficients.

Result for linear and quadratic regression：







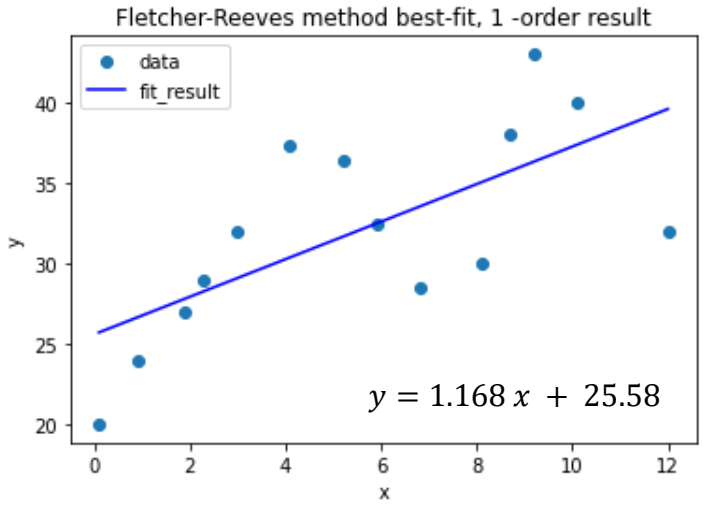
2.1

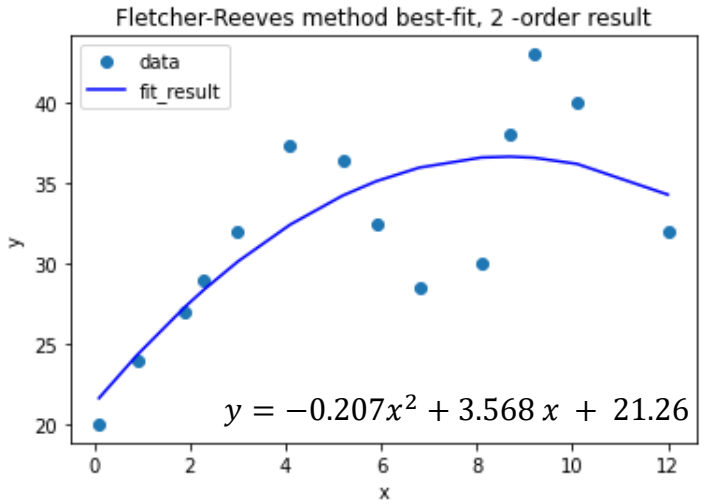
For Conjugate gradient method ： cg.py

Method：

$\mathbf{r}_0 := \mathbf{b} - \mathbf{A}\mathbf{x}_0$
if $\mathbf{r}_0$ is sufficiently small, then return $\mathbf{x}_0$ as the result
$\mathbf{p}_0 := \mathbf{r}_0$
$k := 0$
repeat
$\alpha_k := \frac{\mathbf{r}_k^\mathsf{T}\mathbf{r}_k}{\mathbf{p}_k^\mathsf{T}\mathbf{A}\mathbf{p}_k}$
$\mathbf{x}_{k+1} := \mathbf{x}_k + \alpha_k\mathbf{p}_k$
$\mathbf{r}_{k+1} := \mathbf{r}_k - \alpha_k\mathbf{A}\mathbf{p}_k$
if $\mathbf{r}_{k+1}$ is sufficiently small, then exit loop
$\beta_k := \frac{\mathbf{r}_{k+1}^\mathsf{T}\mathbf{r}_{k+1}}{\mathbf{r}_k^\mathsf{T}\mathbf{r}_k}$
$\mathbf{p}_{k+1} := \mathbf{r}_{k+1} + \beta_k\mathbf{p}_k$
$k := k + 1$
end repeat
return $\mathbf{x}_{k+1}$ as the result

Result for linear and quadratic regression：





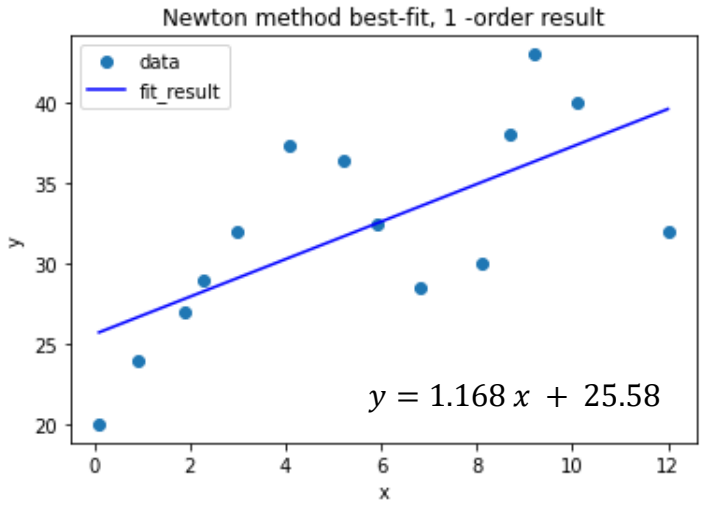
2.2

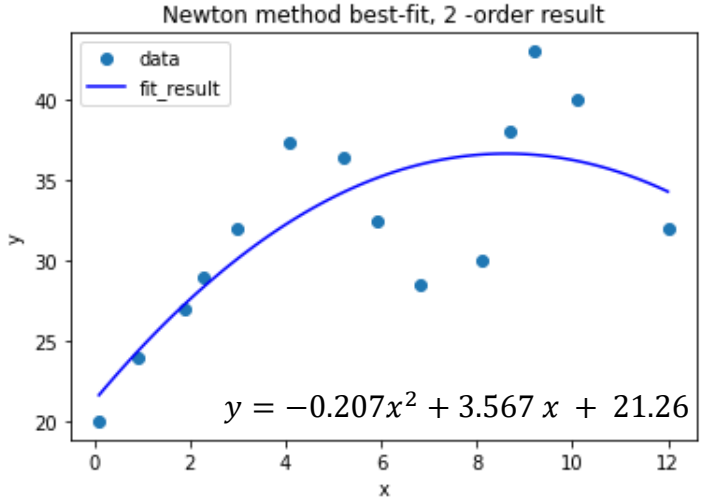
For Newton's method： nt.py

Method：

$$x^{(i+1)} = x^{(i)} - \frac{g(x^{(i+1)})}{g'(x^{(i)})}$$

Result for linear and quadratic regression：

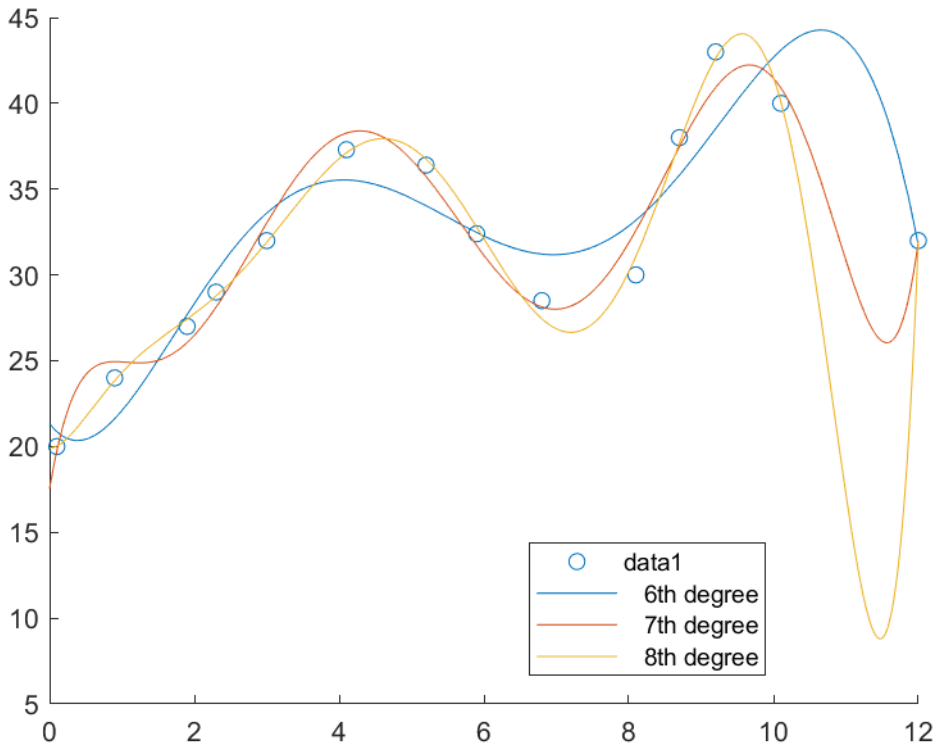




3

Use matlab to find (analytically) the coefficients.

Result：



Degree = 6

$y = 0.00015846x^6 - 0.013571x^5 + 0.2952x^4 - 2.5211x^3 + 8.4339x^2 - 5.4286x + 21.3294$

Degree = 7

$y = 0.00094533x^7 - 0.038215x^6 + 0.59816x^5 - 4.5504x^4 + 17.3089x^3 - 30.8375x^2 + 24.9024x + 17.5479$

Degree = 8

$y = 0.00022417x^8 - 0.0092746x^7 + 0.15099x^6 - 1.2287x^5 + 5.2657x^4 - 11.6072x^3 + 11.7821x^2 - 0.04113x + 19.9211$

當最高項次到 6 時，函數能剛好隔開每個點。

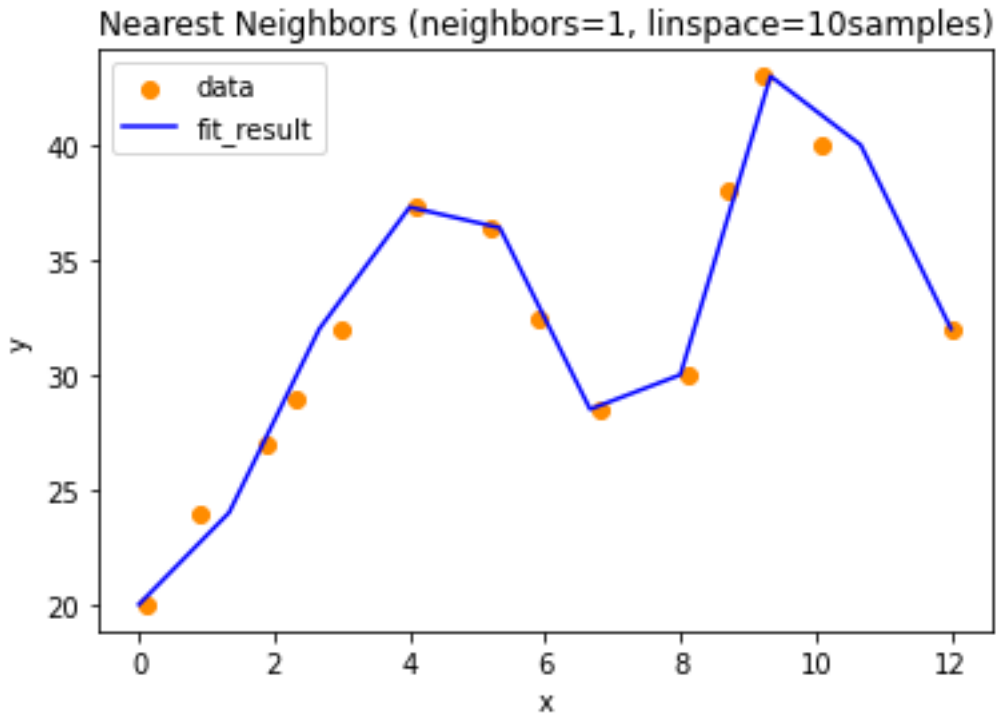
當最高項次到 8 時，函數能剛好通過每個點，但是線條有點overfitting。

如果把資料分成幾類，會比較容易看出有沒有overfitting。

4

Use Nearest Neighbors regression to fit the data：nn.py

Result：



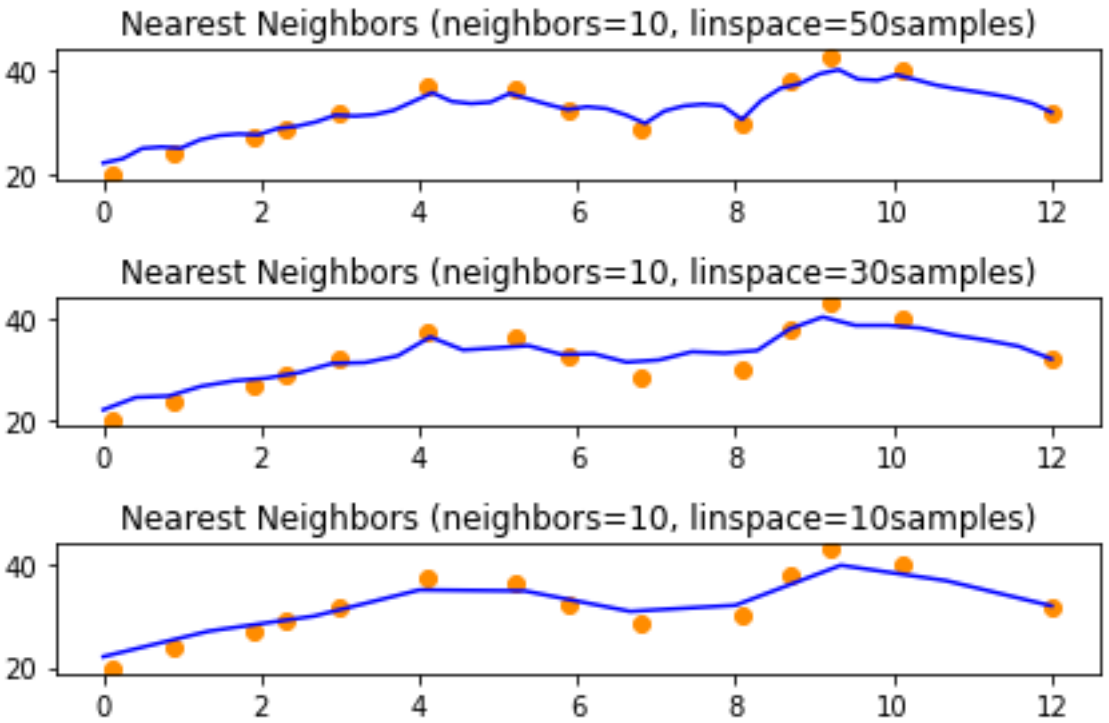
經過前一題的分析，在避免overfitting的前提下，適度的提高多項式次數，可以找到一個best fit方程式。
但是，如果增加或減少資料(點座標)，同一個best fit方程式就可能失效。
為了解決方程式會失效的問題，這裡使用Nearest Neighbors regression model。
透過持續搜索鄰近同類的方式，可以找到一條很好的best fit line，不因資料增加或減少而失效。

4

Best fit：14點皆為同一類，應該連在一起，而且不能overfitting

分析：減少鄰居及line space樣本數量，以達到best fit

左下圖
固定鄰居數量(=10)，減少line space的樣本數，曲線趨於平滑(降低overfitting)



右下圖
固定line space樣本數(=10)，減少鄰居數量，曲線趨於精準(14個點都是同一類)