Rules for reasoning about code

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Review:
Forward vs. backward reasoning

Forward reasoning is more intuitive for most people

- Helps you understand what will happen (simulates the code)
- Introduces facts that may be irrelevant to the goal
  - Set of current facts may get large
- Takes longer to realize that the task is hopeless

Backward reasoning is usually more helpful

- Helps you understand what should happen
- Given a specific goal, indicates how to achieve it
  - Given an error, gives a test case that exposes it

Reasoning about code statements

Goal: Convert assertions about programs into logic

Mechanical process; just follow rules:

- Rule for each type of statement
- Rule for combining/eliminating statements

There is a (forward and backward) rule for each statement in the programming language

- Loops have no rule: you have to *guess a loop invariant*

Hoare triples: A notation for properties about code


Sir Anthony Hoare

A Hoare triple: { P } **code** { Q }

P and Q are logical statements (about program values)

code is Java code

"{ P } **code** { Q }" means "if P is true and you execute **code**, then Q is true afterward"

"{ P } **code** { Q }" is a logical formula like "x + y = z"

Examples:

"1 + 2 = 3" is true

"2 + 2 = 5" is false

"{ x>0 } x++ {x>1}" is true

"{ x<0 } x++ {x<0}" is false

"{ x>0 } x++ {x>-5}" is true

Is this notation good for forward or for backward reasoning?

Backward reasoning: Assignment

```
// precondition: ??
x = e;
// postcondition: Q
```

Precondition = Q with all (free) occurrences of x replaced by e

Examples:

```
// assert:  ??
y = x + 1;
// assert y > 0
```

Precondition = (x+1) > 0

```
// assert:  ??
z = z + 1;
// assert z > 0
```

Precondition = (z+1) > 0

Notation: wp for "*weakest* precondition"

wp("`x=e;`", Q) = Q with x replaced by e

Weakest = most general

Strongest = most specific

Aside: weaker and stronger

Strength of an assertion corresponds to logical implication

- "x is stronger than y" corresponds to "$x \Rightarrow y$"
- "x is stronger than y" means x guarantees more than y
- "x is stronger than y" means fewer worlds satisfy x

Suppose that all of the following are true:

- $a \Rightarrow b$ $b \Rightarrow c$ $x \Rightarrow y$ $y \Rightarrow z$
- { b } **`mycode`** { y }

Then which of these are true?

- { a } **`mycode`** { y } True
- { c } **`mycode`** { y } False
- { b } **`mycode`** { x } False
- { b } **`mycode`** { z } True

Method calls

```
// precondition: ??
x = foo();
// postcondition: Q
```

If the method has no side effects: just like ordinary assignment

```
// precondition: ??
x = Math.sqrt(y);
// postcondition: x = 3
```

Precondition: (y = 9) and (x = anything)

```
// precondition: ??
x = Math.abs(y);
// postcondition: x = 22
```

Precondition: (y = 22 or y = -22)

If it has side effects: an assignment to every var method may <u>modify</u>

Use the method specification to determine the new value

```
// precondition: ??     z+1 = 22
incrementZ();      // spec: z_post = z_pre + 1
// postcondition: z = 22
```

Composition (statement sequences; blocks)

```
// precondition: ??
S1;          // some statement
S2;          // another statement
// postcondition:  Q
```

Work from back to front

Precondition = wp("`S1; S2;`", Q) = wp("`S1;`", wp("`S2;`", Q))

Example:

```
// precondition: ??
x = 0;
y = x+1;
// postcondition: y > 0
```

Think of this as:

```
// precondition: ??
x = 0;
// postcondition: ??
// precondition:  ??
y = x+1;
// postcondition: y > 0
```

Same condition

If statement example

// precondition: ??

```
if (x < 5) {
    x = x*x;
} else {
    x = x+1;
}
```

// postcondition: $x \geq 9$

If statements

// precondition: ??

```
if (b) S1 else S2
```

// postcondition: Q

Do case analysis:

wp("`if (b) S1 else S2`", Q)

= (b $\Rightarrow$ wp("`S1`", Q)

$\wedge$ $\neg$b $\Rightarrow$ wp("`S2`", Q))

= (b $\wedge$ wp("`S1`", Q)

$\vee$ $\neg$b $\wedge$ wp("`S2`", Q))

(Why is there no substitution in the condition?)

If statement example redux

```
// precondition: ??
if (x < 5) {
    x = x*x;
} else {
    x = x+1;
}
// postcondition:  x ≥ 9
```

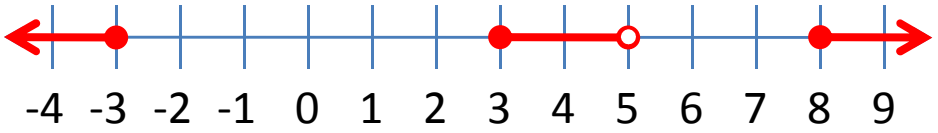
Precondition

= wp("`if (x<5) {x = x*x;} else {x = x+1}`", $x \geq 9$)

= $(x < 5 \wedge$ wp("`x=x*x`", $x \geq 9)) \vee (x \geq 5 \wedge$ wp("`x=x+1`", $x \geq 9))$

= $(x < 5 \wedge x*x \geq 9) \vee (x \geq 5 \wedge x+1 \geq 9)$

= $(x \leq -3) \vee (x \geq 3 \wedge x < 5) \vee (x \geq 8)$



If statements review

Forward reasoning

{P}
`if B`
  {P ∧ B}
  `S1`
  {Q1}
`else`
  {P ∧ !B}
  `S2`
  {Q2}
{Q1 ∨ Q2}

Backward reasoning

{ (B ∧ wp(S1, Q)) ∨ (¬B ∧ wp(S2, Q)) }
`if B`
  {wp(S1, Q)}
  `S1`
  {Q}
`else`
  {wp(S2, Q)}
  `S2`
  {Q}
{Q}

If statement with one branch empty

```
// precondition:  ??
if (x > y) {
    tmp = x;
    x = y;
    y = x;
}
// postcondition:  x < y
```