

Getting Started with Machine Learning

Raunak J

In the last video we learned Log Loss.

In this video we'll try to optimize that loss function.

Using Gradient Descent.

So Gradient Descent is basically a loss optimization algorithm, that derivates the loss function and reaches a global minima to shift the parameters.

So in our example of Logistic Regression Loss Optimization, parameters are constant and regression co-efficient.

So our loss function for Logistic Regression is Log Loss :

$$LL_{(y,\hat{y})} = \frac{1}{N} \sum_{i=1}^N -y_i \cdot \log(P(\hat{y}_i)) - (1 - y_i) \cdot \log(1 - P(\hat{y}_i))$$

This can be elaborated as :

$$LL_{(y,\sigma(Z))} = \frac{1}{N} \sum_{i=1}^N -y_i \cdot \log(\sigma(Z_i)) - (1 - y_i) \cdot \log(1 - \sigma(Z_i))$$

$$LL_{(y,\sigma(Z))} = \frac{1}{N} \sum_{i=1}^N -y_i \cdot \log(\sigma(Z_i)) - (1 - y_i) \cdot \log(1 - \sigma(Z_i))$$

$$LL_{(y,\sigma(Z))} = \frac{1}{N} \sum_{i=1}^N -y_i \cdot \log\left(\frac{1}{1 + e^{-Z_i}}\right) - (1 - y_i) \cdot \log\left(1 - \frac{1}{1 + e^{-Z_i}}\right)$$

$$LL_{(y,\sigma(Z))} = \frac{1}{N} \sum_{i=1}^N -y_i \cdot \log\left(\frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}}\right) - (1 - y_i) \cdot \log\left(1 - \frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}}\right)$$

So for performing Loss Optimization of Logistic Regression :

Step 1 : Derivate the Loss Function.

Step 2 : Converge the Loss Function with parameters iteratively.

$$LL_{(y,\sigma(Z))} = \frac{1}{N} \sum_{i=1}^N -y_i \cdot \log(\sigma(Z_i)) - (1 - y_i) \cdot \log(1 - \sigma(Z_i))$$

Following the step 1 :

$$\nabla_{\beta_i} = \sum_{i=1}^N -y_i \cdot \log\left(\frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}}\right) - (1 - y_i) \cdot \log\left(1 - \frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}}\right)$$

$$\nabla_{\beta_i} = \sum_{i=1}^N \left\{ -y_i \cdot \log\left(\frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}}\right) - (1 - y_i) \cdot \log\left(1 - \frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}}\right) \right\} \cdot x_i$$

$$\nabla_{\beta_i} = \sum_{i=1}^N \left\{ -y_i \cdot \log \left(\frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}} \right) - (1 - y_i) \cdot \log \left(1 - \frac{1}{1 + e^{-(\beta_0 + \beta_1 \cdot x_i)}} \right) \right\} \cdot x_i$$

$$\nabla_{\beta_i} = \sum_{i=1}^N LL_{\{y_i, \sigma(Z)\}} \cdot x_i$$

$$\nabla_{\beta_i} = \frac{1}{N} \sum_{i=1}^N LL_{\{y_i, \sigma(Z)\}} \cdot x_i$$

Step 2 :

$$\beta_i = \beta_i - \alpha \cdot \nabla_{\beta_i}$$

$$\beta_i = \beta_i - \alpha \cdot \frac{1}{N} \sum_{i=1}^N LL_{\{y_i, \sigma(z)\}} \cdot x_i$$

Where β_i are parameters and α is learning rate

Iterating {

$$\beta_i = \beta_i - \alpha \cdot \frac{1}{N} \sum_{i=1}^N LL_{\{y_i, \sigma(z)\}} \cdot x_i$$

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