ECS 204: SIGNALS AND SYSTEMS
PROGRAMMING ASSIGNMENT

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QUESTION 1.

Explanation/Theory

As obvious, the term consists of two words, 'Amplitude' which means the maximum value of a signal and 'Modulation' which means the process of varying the signal. So, Amplitude Modulation is the process of varying the amplitude of a signal. For example, in the case signal $y(t) = a_m(t)x(t)$ where $x(t) = A\sin(2\pi ft)$, the amplitude of the input signal is A and the frequency of the signal is f.
$a_m(t)$ is the modulating signal which is multiplied with the input signal to get the modulated signal. Thus, this Amplitute of the output signal is varied by the modulating signal.

MATLAB Code

```
% Define parameters
Am = 1;          % Amplitude of message signal
Ac = 1;          % Amplitude of carrier signal
fm = 2016/4.;    % Frequency of message signal (Hz)
fc = 2016/2.;    % Frequency of carrier signal (Hz)
fs = 2016;       % Sampling frequency (must satisfy Nyquist criterion)
T  = .01/fs;     % Sampling period
t  = 0:T:1;      % Time vector

% Define message signal m(t) and carrier signal c(t)
mt = Am*cos(2*pi*fm*t);
ct = Ac*cos(2*pi*fc*t);

% Calculate modulated signal s(t)
st = mt .* ct;

subplot(3,1,1);
plot(t, mt);
title('Message Signal m(t)');
xlabel('Time (s)');
ylabel('Amplitude');

subplot(3,1,2);
plot(t, ct);
title('Carrier Signal c(t)');
xlabel('Time (s)');
ylabel('Amplitude');

subplot(3,1,3);
plot(t, st);
```

```
title('Modulated Signal s(t)');
xlabel('Time (s)');
ylabel('Amplitude');

% Calculate and plot magnitude spectrum of modulated signal
figure;
L = length(st);
Y = fft(st);
P2 = abs(Y/L);
f = (0:L/190);
plot(f,P2(1:floor(L/190 + 1)));
title('Magnitude Spectrum of Modulated Signal');
xlabel('Frequency (Hz)');
ylabel('|S(f)|');
```

Output

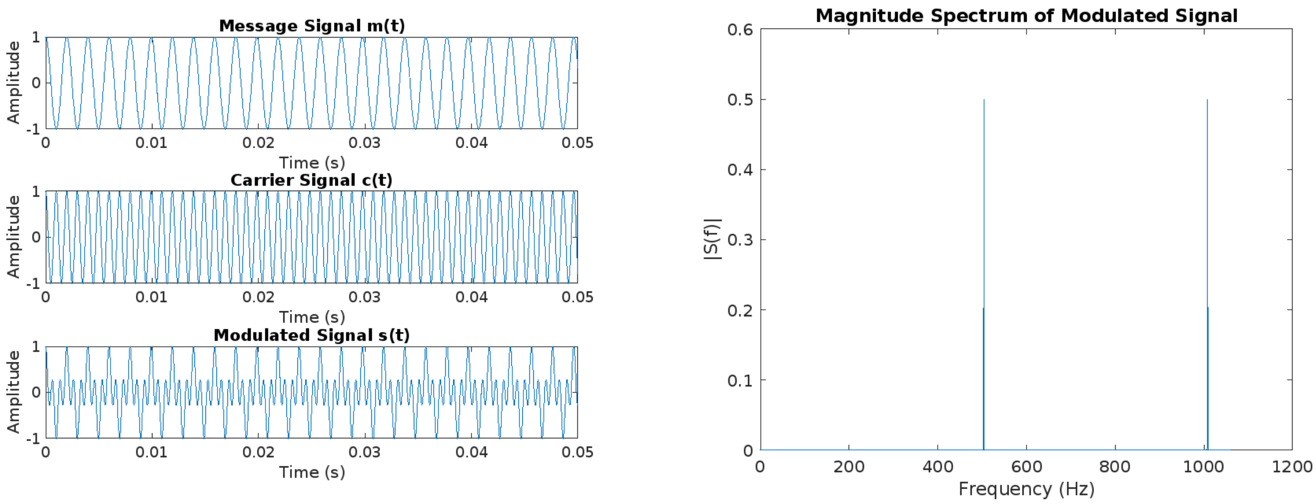


Figure 1: Individual Signals and Magnitude Spectrum

Inference

1. The amplitude of the output signal is varied by the modulating signal.
2. The phase of the output signal is same as the phase of the input signal.
3. The amplitude of the output signal is same as the max product of amplitude of the input signal and amplitude of the modulating signal.

QUESTION 2.

Explanation/Theory

$$x(t) = \cos\left(\frac{3\pi}{2}t\right) e^{-\frac{t^2}{2}}$$

(a) Evaluate $X(j\omega)$.

Given $x(t) = \cos\left(\frac{3\pi}{2}t\right)\cdot e^{-\frac{t^2}{2}}$

Let, $x_1 = \cos\left(\frac{3\pi}{2}t\right)$ & $x_2 = e^{-t^2/2}$

Now we know, $X(j\omega) = [X_1(j\omega) * X_2(j\omega)] \cdot 1/2\pi$

& $X_1(j\omega) = \int_{-\infty}^{\infty} \cos\left(\frac{3\pi}{2}t\right) e^{-j\omega t}\,dt$

$= \delta\left(j\omega - \frac{3\pi}{2}\right)$

& $X_2(j\omega) = \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} \cdot e^{-j\omega t}\,dt$

$= \int_{-\infty}^{\infty} e^{\left(\frac{j\omega}{\sqrt{2}}\right)^2} \cdot e^{-\left(\frac{t}{\sqrt{2}} + \frac{j\omega}{\sqrt{2}}\right)^2}\,dt$

$\left\{\text{Substitute } \frac{t}{\sqrt{2}} + \frac{j\omega}{\sqrt{2}} = u\right\}$

We get $X_2(j\omega) = \sqrt{2\pi} \cdot e^{-\frac{\omega^2}{2}}$

$X(j\omega) = 1/\sqrt{2\pi} \cdot \int_{-\infty}^{\infty} \delta\left(\omega - \frac{3\pi}{2}\right) \cdot e^{-\frac{(u-\omega)^2}{2}}\,du$

$= 1/\sqrt{2\pi}\; e^{-\left(\frac{3\pi}{2} - \omega\right)^2/2}$

Thus, $\mathcal{F}(x(t)) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\omega - \frac{3\pi}{2}\right)^2}{2}}$

Final answer: $\frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-\left(\omega-\frac{3\pi}{2}\right)^2}{2}}$

MATLAB Code

(b) Plot $\frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-\left(\omega-\frac{3\pi}{2}\right)^2}{2}}$

```
% Define the range of omega values
omega = linspace(-3*pi, 3*pi, 1000);

% Compute values of the function
X = (1/(2*pi)) * exp(-((omega - (3*pi/2)).^2)/2);

% Plot the function
plot(omega, X, 'LineWidth', 2);
title('$\frac{1}{2\pi} \cdot e^{\frac{- \left(\omega - \frac{3\pi}{2}\right)^2}{2}}$',
   'Interpreter', 'latex');
xlabel('\omega');
ylabel('f(\omega)');
grid on;
```

(c) Compare with the FT on sampled signal

```
% Function that computes the sum of the given function for each omega value
function sum_results = compute_sum(omegas, Ts)
    n = -25:25;
    x_n = cos(3*pi/2 * n .* Ts) .* exp(-((n.*Ts).^2)/2);

    sum_results = zeros(size(omegas));

    % Compute the sum for each omega value
    for i = 1:length(omegas)
        sum_results(i) = sum(x_n .* exp(-1j * n * omegas(i) * Ts));
    end
end

omega = linspace(-3 * pi, 3 * pi, 500);
fs = 2016/1000;
Ts = 1 / Ts;
X = compute_sum(omega, Ts);

subplot(2,1,1);
plot(omega, abs(X));
title('Ts = 1000/2016');
xlabel('\omega');
ylabel('X_\delta(\omega)');

Ts = Ts / 2.;
X = compute_sum(omega, Ts);

subplot(2,1,2);
plot(omega, abs(X));
title('Ts = 500/2016');
xlabel('\omega');
ylabel('X_\delta(\omega)');

grid on;
```

Output

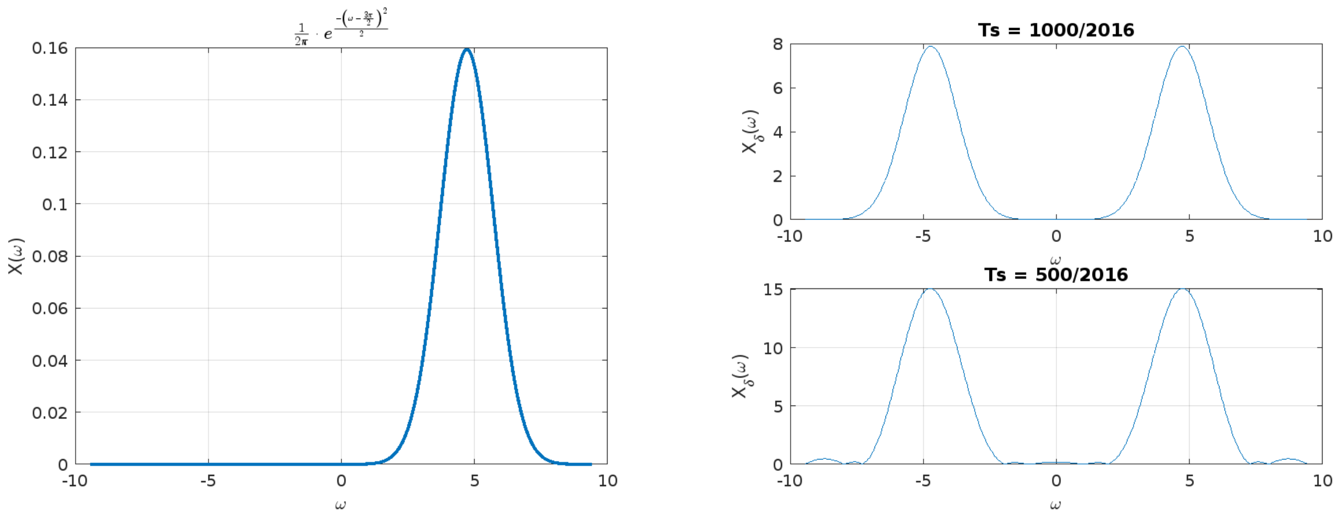


Figure 2: Plot of $X(j\omega)$ and $X_\delta(j\omega)$

Inference

- The plot of $X(j\omega)$ is a Gaussian function with a peak at $\omega = \frac{3\pi}{2}$.
- Looking at the graphs of $X_\delta(j\omega)$, one can very well approximate $X(j\omega)$ by sampling very few (in our case, 500) points and then computing the FT using summission formula.
- $X_\delta(j\omega)$ approximates $X(j\omega)$ very well at $T_s = \frac{2}{3\pi}$.
- There are multiple peaks at $X_\delta(j\omega)$ which is because, FT is periodic in case of sampled signals.

QUESTION 3.

Explanation/Theory

(a) Write the differential equation of the system in terms of ω_a and Q.

To write the differential equation of the system, we start with the standard second-order linear differential equation for a damped harmonic oscillator:

$$M\ddot{x} + D\dot{x} + Kx = 0$$

The above equations only states that sum of all forces in all the direction is zero.
To write the above equation in terms of ω_a and Q, we use the given facts:

$$\omega_a = \sqrt{\frac{K}{M}} \Rightarrow K = M\omega_a^2$$

$$\mathrm{Q} = \frac{\sqrt{KM}}{D} \Rightarrow D = \frac{\sqrt{KM}}{\mathrm{Q}} = \frac{M\omega_a}{\mathrm{Q}}$$

Substituting the above expressions in the differential equation, we get:

$$\ddot{x} + \frac{\omega_a}{\mathrm{Q}}\dot{x} + \omega_a^2 x = 0$$

(b) Find the frequency response of the system.

For a given input $x(t)$ let the response of the system be $y(t)$. Thus above differential equation can be written as:

$$\ddot{y} + \frac{\omega_a}{\mathrm{Q}}\dot{y} + \omega_a^2 y = x(t)$$

Now let CTFT of $x(t)$ and $y(t)$ be $X(j\omega)$ and $Y(j\omega)$ respectively. Then the above equation can be written in frequency domain as:

$$(j\omega)^2 Y(j\omega) + \frac{\omega_a}{\mathrm{Q}}(j\omega)Y(j\omega) + \omega_a^2 Y(j\omega) = X(j\omega)$$

$$\left[(j\omega)^2 + \frac{\omega_a}{\mathrm{Q}}(j\omega) + \omega_a^2\right] \cdot Y(j\omega) = X(j\omega)$$

Now to calculate the frequency response $H(j\omega)$:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{(j\omega)^2 + \frac{\omega_a}{\mathrm{Q}}(j\omega) + \omega_a^2}$$

This required frequency response is:

$$H(j\omega) = \frac{1}{(j\omega)^2 + \frac{\omega_a}{\mathrm{Q}}(j\omega) + \omega_a^2}$$

MATLAB Code

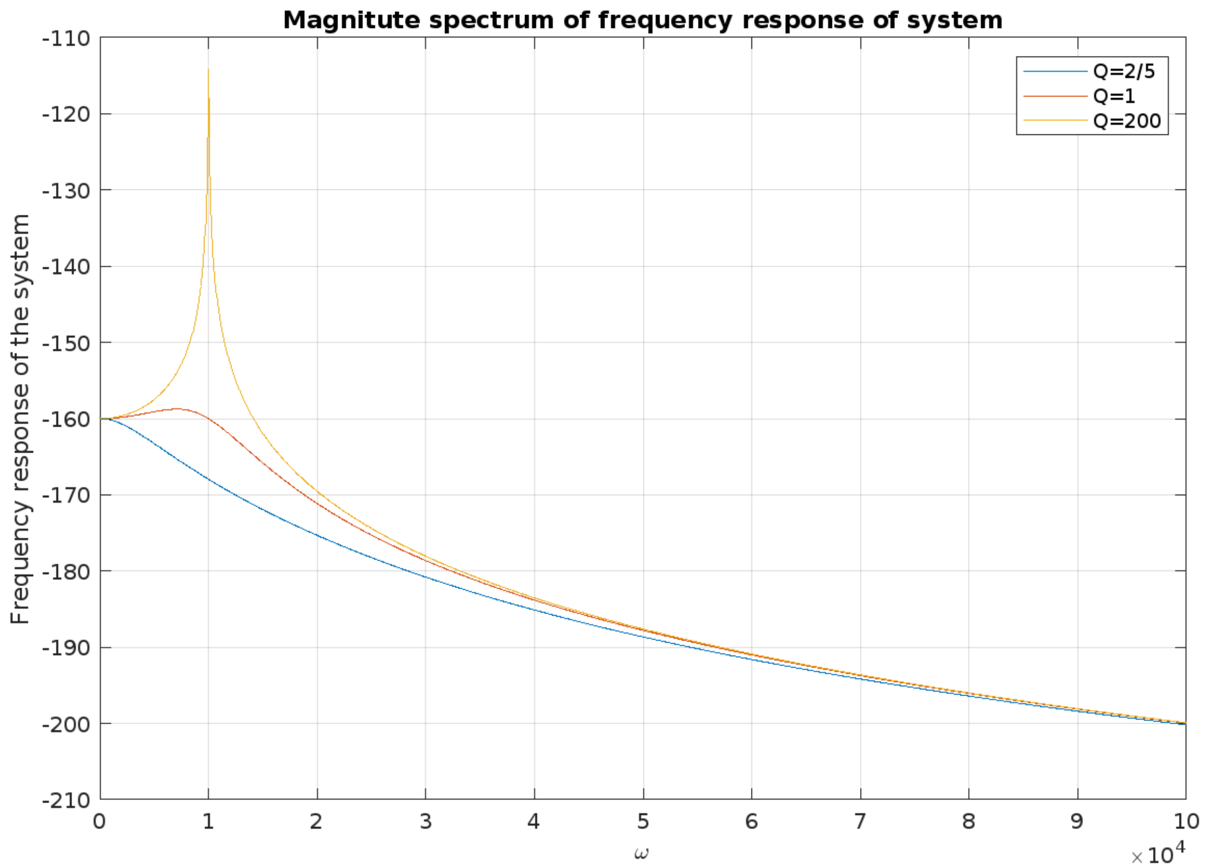
```
omega_a = 10000;
omega = linspace(0, 100000, 2000);

function resp = compute_response(Q, omega_a, omega)
    resp = (1i .* omega) .^2 + (omega_a / Q) .* (1i .* omega) + omega_a^2;
    resp = abs(1 ./ resp);
    resp = 20 .* log10(resp);
end

resp1 = compute_response(2/5, omega_a, omega);
resp2 = compute_response(  1, omega_a, omega);
resp3 = compute_response(200, omega_a, omega);

set(gcf, 'Units', 'inches', 'Position', [0, 0, 9, 6]);
plot(omega, resp1, omega, resp2, omega, resp3);
legend('Q=2/5', 'Q=1', 'Q=200');
xlabel('\omega');
ylabel('Frequency response of the system');
title('Magnitute spectrum of frequency response of system');
grid on;
```

Output



Inference

The frequency response of the system was plotted and we can infer the following:

1. The frequency response for $\mathrm{Q} = 2/5$ decreases monotonically as increase in frequency ω.
2. The frequency response for $\mathrm{Q} = 1$ remains more or less constant for ω in range $[0, 1e4]$. After that it decreases.
3. The frequency response for $\mathrm{Q} = 200$ has a sudden spike at $1e4$, before that it increases and after that it decreases.

The system is an Damped Harmonic Oscillator and the frequency response is as expected.