COMP 3761 Quiz 2 Solution

Name:

Useful formula:

$$\Sigma_{i=0}^{n} a^i = 1 + a + a^2 + \ldots + a^n = \frac{a^{n+1} - 1}{a - 1} \quad (a \neq 1).$$

Answer the following **two** questions:

1. Use backward substitution to solve the following recurrence relation (i.e., express $T(n)$ as a function of n). [5 marks]

$$T(n) = 2T(n-1) + 1, \text{ for } \quad n > 1, \quad T(1) = 1$$

Answer: $T(n) = 2^n - 1$, see Section 2.3 P.73 in the textbook.

2. Consider the following recursive algorithm:
Algorithm *MySecretAlg*($A[0..n-1]$)
if $n = 1$ return $A[0]$
else $temp \leftarrow$ *MySecretAlg*($A[0..n-2]$)
if $temp \leq A[n-1]$ return $temp$
else return $A[n-1]$

   a. What does this algorithm compute? [2 marks]
   It computes the value of the smallest element in a given array.

   b. Set up a recurrence relation for the algorithm's basic operation count. [2 marks]
   Recurrence relation: $T(n) = T(n-1) + 1 \quad$ for $n > 1$
   Initial condition: $T(1) = 0$.

   c. Solve the recurrence relation from (b). [1 marks]
   Solving (b) by backward substitution gives

$$T(n) = n - 1.$$