Review:

Propositional logic: Explaining how the validity of certain arguments is determined by "truth-functional" connectives, such as *and, or,* and *not*, together with the truth values of the "atomic" propositions

Non-propositional "and": When the truth of a proposition using "and" depends on more than just the truth of conjoined propositions

e.g., Lois believes that Clark Kent and Superman are different people.

Exclusive and inclusive "or"

Issues involving negation

Truth conditions for connectives: Truth tables can be used to show how the truth value (T/F) of compounded propositions depends on the truth values of the atomic propositions operated on by that connective

Truth conditions for arguments: Truth tables can be used to test the validity of arguments by revealing whether there are any truth-value assignments to the atomic propositions in the argument that will make the premises true and the conclusion false

1

Truth tables and truth-functional equivalence

Are "*p*" and "*p* & (*q* ∨ ~*q*)" truth-functionally equivalent?

p	*q*	~*q*	*q* ∨ ~*q*	***p***	***p* & (*q* ∨ ~*q*)**
T	T	F	T	***T***	***T***
T	F	T	T	***T***	***T***
F	T	F	T	***F***	***F***
F	F	T	T	***F***	***F***

CONDITIONALS: Propositions with the form, "If (antecedent), then (consequent)"

MOODY CONDITIONALS:

INDICATIVE: If I become a rich man, then I will buy a yacht"

SUBJUNCTIVE: If I were a rich man, then I would buy a yacht"

IMPERATIVE: If you become a rich man, then buy me a yacht!"

PROMISE: If I become a rich man, I will surely buy you a yacht"

Experts disagree about how logically to handle conditionals

We will consider only *propositional conditionals*: indicative conditionals whose antecedent and consequent are propositions (statements with truth values)

2

TRUTH TABLES FOR CONDITIONALS

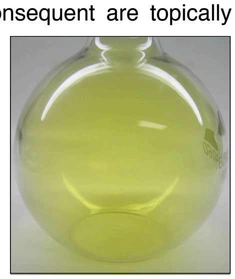
p	*q*	If *p*, then *q*	
T	T	?	
T	F	F	Obvious case
F	T	?	
F	F	?	

Other cases? Note that a *truth-functional* connective makes the compound depend *only* on the truth value of the component propositions

e.g., "If Toronto is in Ontario, then Gordon Campbell is BC's premier" fits the first line.

This may seem odd, since the antecedent and consequent are topically unrelated

But the more natural: "If Chlorine is a poisonous gas, then Chlorine will usually be handled carefully," also fits that pattern.



We can create a truth-functional "If p, then q" by working with the obvious truth table line.

The line says, in effect, that: If "If p, then q" is true then "It's not true that p is true and q is false"

3

In other words:

<u>If *p* then *q*</u>
∴ ~(*p* & ~*q*)

Are "If p, then q" and "~(*p* & ~*q*)" equivalent? That is, does one imply the other?

To see this, check whether "If p then q" follows from "~(*p* & ~*q*)"

i. First, we must suppose that "~(*p* & ~*q*)" is true (why?)

ii. Then, by the truth table for negation, we get that "*p* & ~*q*" is false.

iii. Now *if we suppose that "p" is true,* then, by the truth table for "&", we see that "~q" must be false, so "q" must be true.

In other words, assuming that "~(*p* & ~*q*)" is true, then if "p" is true, then "q" is true. Ta! da!

<u>~(*p* & ~*q*)</u>
∴ If *p* then *q*

So, "If p, then q" $=_{\text{def}}$ "~(*p* & ~*q*)"

or "*p* ⊃ *q*" $=_{\text{def}}$ "~(*p* & ~*q*)"

4

Which also means they have the same truth table:

p	*q*	~*q*	*p* & ~q	~p	~(*p* & ~q)	*p* ⊃ *q*	~p v q
T	T	F	F	F	T	T	T
T	F	T	T	F	F	F	F
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T

Material conditional (⊃) truth table

p	*q*	*p* ⊃ *q*
T	T	T
T	F	F
F	T	T
F	F	T

Check the validity of *modus ponens* **(MP)** using truth tables:

p ⊃ *q*
<u>*p*</u>
∴ *q*

p	*q*	*Pr* *p* ⊃ *q*	*Pr* *p*	CN q	
T	T	T	T	T	OK
T	F	F	T	F	
F	T	T	F	T	
F	F	T	F	F	

5

The validity of ***modus tollens*** **(MT)**?

p ⊃ *q*
<u>~*q*</u>
∴ ~*p*

p	*q*	*Pr* *p* ⊃ *q*	*Pr* ~*q*	CN ~*p*
T	T			
T	F			
F	T			
F	F			

The invalidity of *denying the antecedent*:

p ⊃ *q*
<u>~*p*</u>
∴ ~*q*

p	*q*	*Pr* *p* ⊃ *q*	*Pr* ~*p*	CN ~*q*
T	T			
T	F			
F	T			
F	F			

The invalidity of *affirming the consequent*:

p ⊃ *q*
<u>*q*</u>
∴ *p*

p	*q*	*Pr* *p* ⊃ *q*	*Pr* *q*	CN *p*
T	T			
T	F			
F	T			
F	F			

6

HYPOTHETICAL SYLLOGISM (HS)

$p \supset q$
$\underline{q \supset r}$
$\therefore p \supset r$

			Pr	Pr	CN
p	*q*	*r*	$p \supset q$	$q \supset r$	$p \supset r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

LOGICAL LANGUAGE AND EVERYDAY LANGUAGE

Everyday "and," "or" and conditionals often seem quite different from their truth-functional versions

E.g., "&" allows us to conjoin *any* true propositions, even if completely unrelated in content

In propositional logic, we represent "but," "however," "while," "although," "whereas," etc. by "&," and so lose their *discounting* function, as well as their asymmetry

Jane is unhappy, but she is rich (~H & R)

Jane is rich, but she is unhappy (R & ~H)

Still, despite their discounting differences, they are *logically* equivalent — one is true if and only if the other is true

Ordinary "or": In the vast majority of cases, we use "or" in such a way that it *seems to be* an exclusive disjunction [1]

[1]For a hypersophisticated analysis of "or" see: http://plato.stanford.edu/entries/disjunction/ by SFU prof Ray Jennings

The following argument is valid

(1) 1. Tim is an instructor
∴ 2. If Mars is the seventh planet, then Tim is an instructor.

(2) 1. Stephen Harper is not Prime Minister.
∴ 2. If Stephen Harper is Prime Minister, then Mars is the seventh planet.

(1) $\underline{p}$
$\therefore q \supset p$

(2) $\underline{\sim p}$
$\therefore p \supset q$

Check validity by short truth-tables

Some philosophers call these sort of arguments *paradoxical*, since (1) says a true proposition is implied by any proposition; while (2) says a false proposition implies every proposition

Two approaches:

(a) reject truth-functional logic as fully adequate for (most) conditionals (and possibly "and" and "or") and develop other logics that don't produce these results (e.g., relevance logics,[2] modal logics, etc).

http://plato.stanford.edu/entries/logic-relevance/
http://plato.stanford.edu/entries/logic-modal/

(b) **Grice's approach**: take truth-functional logic as determining ***propositional validity***, and explain the odd results by the rules associated with conversational implication

Thus, regarding "and," the rule of *relevance* requires that we not conjoin unconnected information, since it will almost certainly not be related to the conversational purpose

Still, that conversational irrelevancy does not affect the *validity* of simplification or conjunction

Similarly, Grice's view uses a simpler "inclusive or" and explains the exclusive case as a conversational restriction on a liberal logical base

The problem seems worse for conditionals, not merely because they allow us to recognize perhaps trivial validity, but because they seem to call valid some inferences that our instincts reject

Modal logic: "If p then q" ≠ "~(p &~q)"; rather, it means "Not *possibly* both *p* and not *q*"

or, $\sim\Diamond (p \,\&\sim q) = \Box \sim(p \,\&\sim q)$

By contrast, Grice argues that the "paradoxical" conditionals really are valid—since, it's impossible that the premises be true and the conclusion false

10

Grice pins the oddity on violating rules of Relevance: In "$p \therefore q \supset p$," "q" can be *any* proposition, and that includes ones we are not interested in

E.g., I say to you, "If you give me one million dollars, I'll let you live" the conditional is true, but violates the rule of Relevance, since it gives true information that is not relevant to the conclusion

The matter remains unsettled among logicians

OTHER CONDITIONALS IN ORDINARY LANGUAGE

q if *p* = If *p* then *q* $(p \supset q)$

p only if *q* = If *p* then *q* $(p \supset q)$

"Only if": "I'll do the dishes (D) *only if* you cook supper (C)"

Clearly, I will not do the dishes, if you don't cook supper.

That is, $(\sim C \supset \sim D) = (D \supset C)$

Sometimes the ordinary use of "only if" is thought to conversationally imply a *biconditional* (if and only if)

The above conditional $(D \supset C)$ is still true, if "D" is false; that is, by itself, it doesn't say that if I don't do the dishes, you won't cook supper

11

However, in the above case, this is the conversational implication—the rule of Quantity would be broken, since I would be allowing that I won't do the dishes, *whatever* you do, but without letting you know

In such cases, we can read "only if" as "if and only if"

That is, *for these cases*, "*p* only if *q*" should be interpreted as "*p* if and only if *q*" or

$[(p \supset q) \,\&\, (q \supset p)] = (p \equiv q)$

Even so, not all cases of "only if" contain this implication: e.g., "A person can become mayor of Vancouver only if they are a Canadian citizen"

"If and only if" or "just in case" are seldom used in day-to-day speech

"*p but* only if *q*" emphasizes the biconditional

"I will do the dishes, but only if you do the cooking"

Unless: "*p unless q*" or "Unless *p*, *q*"

"Mary will never become a doctor unless she stops fainting at the sight of blood"

Clearly, the above sentence is saying that, if Mary *doesn't* stop fainting at the sight of blood, she won't become a doctor.

12

F= Mary stops fainting at the sight of blood

D = Mary will become a doctor.

$\sim F \supset \sim D$

Does it imply that *if she stops fainting*, she'll become a doctor? $F \supset D$?

So, at the very least, "unless" means "if not"

However, sometimes when we use "unless," the conversational implication includes "if"

for example, "I won't do the dishes, unless you cook supper."

	Translates	***Sometimes* conversationally implies**
p if q	$q \supset p$	$(p \supset q)$ & $(q \supset p) = [p \equiv q]$
p only if q	$p \supset q$	$[(p \supset q)$ & $(q \supset p)] = [p \equiv q]$
p unless q	$\sim q \supset p$	$(p \supset \sim q)$ & $(\sim q \supset p)$

A) **Example: "**I eat if I'm hungry" = "If I'm hungry, I eat"

Logically, this just says that being hungry is one (of possibly many) condition under which I'll eat

Ordinarily, would this also be taken to conversationally imply: "If I eat, I'm hungry"?

13

What about: "I go to school, if it's Tuesday"? Does this conversationally imply: "If I go to school it's Tuesday"?

Possible analysis? If the context implies that there is only one condition (A) for the consequent (C) of the stated conditional, then the conversational implication favours the biconditional: $A \equiv C$

Otherwise, assume only the explicitly stated conditional: $A \supset C$

B) **Example:** "I'll buy a car only if I get a pay raise" = "If I buy a car, then I get a pay raise"

Would this ordinarily also be taken to imply "If I get a pay raise, I'll buy a car"?

Example: "I'll pay you only if you do the work"

Logically?

Conversationally?

C) **Example:** "You won't make the Olympics, unless you exercise"?

Logically?

Conversationally?

14

Example: "I won't do the work, unless you pay me"

Logically?

Conversationally?

Truth table for the biconditional: $(p \supset q)$ & $(q \supset p) = [p \equiv q]$

p	q	$p \supset q$	$q \supset p$	$(p \supset q)$ & $(q \supset p)$	$p \equiv q$
T	T				
T	F				
F	T				
F	F				

IN-CLASS: PRACTICE MIDTERM

HOMEWORK (FOR YOUR OWN PRACTICE):

XVIII: 1-4 (162)
XIX: 14,15 (162)
XXIV: 7, 9, 11 (169)
XXVII: 1 - 9 (175)
XXVIII: 4 - 9 (177)

15