Lesson 12 Comp 2121 Summer 08

July 21 - Assign 2 is due

The Addition Rule

Suppose a finite set A equals the union of k distinct, mutually-disjoint subsets

$A_1, A_2, A_3, \ldots, A_k$. Then:

$$N(A) = N(A_1) + N(A_2) + N(A_3) + \ldots + N(A_k)$$

eg A party with 5 females and 6 males

Total people = 5 + 6 = 11

OR

Consider the BCIT password example. Now assume you can choose up to 6 characters from the alphabet (26 letters) and the digits (10 of them).

Q: How many different pw can be made!

A: Of length 6: 36^6 +
5: 36^5 +
4: 36^4 +
3: 36^3 +
of length 2: 36^2 +
1: 36^1 +
0: $36^0 = 1$

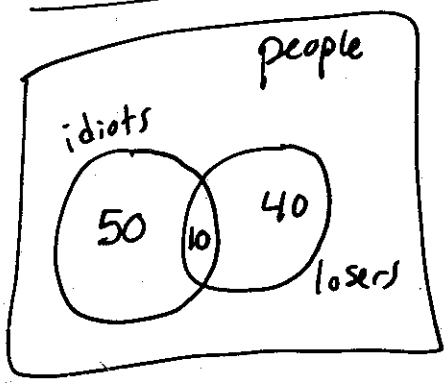
OR means ADD

2,238,976,117 pw

Q] One million people speak French.
One million people speak English.
How many people speak "French or English"?

A] Somewhere between 1 million and 2 million.

The Inclusion/Exclusion Rule



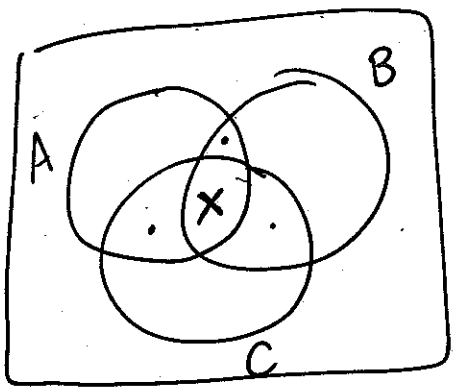
60 idiots
50 losers
100 people

If A, B, and C are finite sets, then

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

$N(A \cap B)$ got counted twice!

$$N(A \cup B \cup C) = N(A) + N(B) + N(C) - N(A \cap B) - N(B \cap C) - N(A \cap C) + N(A \cap B \cap C)$$



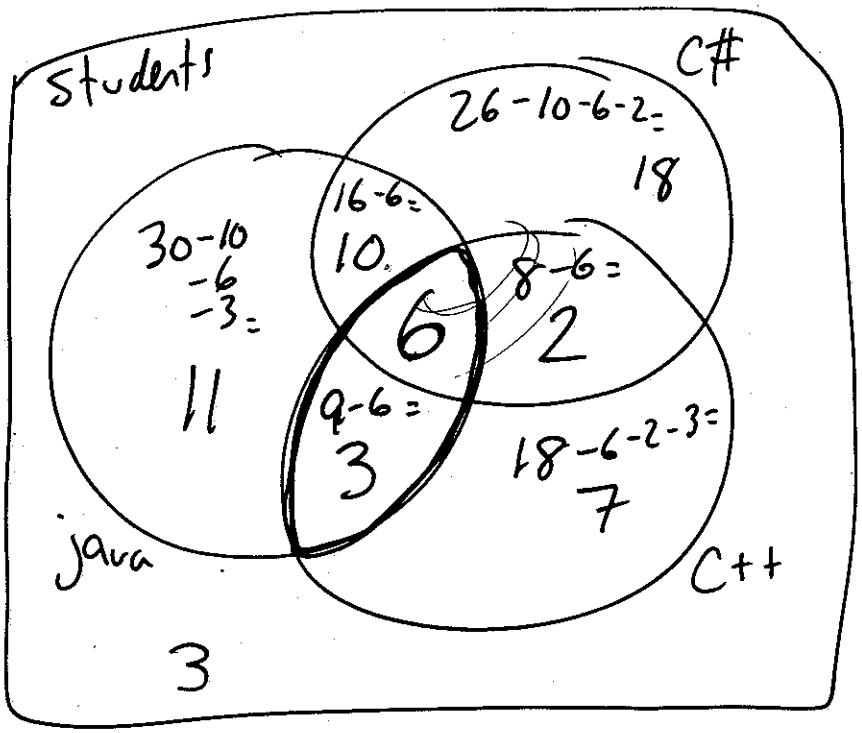
Q/ A class has 50 students

- 30 know java
- 18 know C++
- 26 know C#
- 9 know both java and C++
- 16 know both java and C#
- 8 know both C++ and C#
- 47 know at least one of the languages.

Q/ How many students know all 3 langoages? (SIX)

Q/ How many know java and ~~C++~~ C++ but not C#? (three)

Draw the complete Venn Diagram.



Use the I/E rule:

30+18+26-
9-16-8
+x=47
x=6

Q] How many integers from 500 to 2300 (inclusive) are multiples of 3 or 5?

A] Space $2300 - 500 + 1$

$= 1801$

$1800/3 = 600$ mult of 3

$/5 = 360$ mult of 5

$/15 = 120$ mult of 3 AND 5

$$N(A \cup B) = N(A) + N(B) - N(A \cap B)$$

$$= 600 + 360 - 120 + 1$$

$$= 841$$

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multiples of 3: 500 (501 ↓ 167×3) ... 2298 (↓ 766×3) } $766 - 167 + 1 = 600$

multiples of 5: 500 (↓ 100×5) ... 2300 (↓ 460×5) } $460 - 100 + 1 = 361$

multiples of 15: 510 (↓ 34×15) ... DOUBLE COUNTED ... 2295 (↓ 153×15) } $153 - 34 + 1 = 120$

$600 + 361 - 120 = 841$

Combinations

"subsets" "choices" dfn.

r Combinations of a set of n elements

$\binom{n}{r}$ "n choose r" ${}_nC_r$ nCr

Calculator

n, r are nonnegative integers with $r \leq n$

☆ Order does not matter

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

eg $\binom{49}{6} = \frac{49!}{6!\,43!} \doteq 14\,\text{M}$

Q) How many ways can 3 students be chosen from a class of 7?

A) $\binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\,4!} = 35$

Oops!

A B C D E F G

ABC	ACD	ADF	BCD	BDF	CDE	CFG
ABD	ACE	ADG	BCE	BDG	CDF	DEF
ABE	ACF	AEF	BCF	BEF	CDG	DEG
ABF	ACG	AEG	BCG	BEG	CEF	DFG
ABG	ADE	AFG	BDE	BFG	CEG	EFG

Use Set Notation

Note $\binom{7}{4} = \binom{7}{3}$

↘ pick 3 for your group

↘ pick 4 to leave out of group

Q) How many distinct groups of 5 can be chosen from a set of 20?

A) $\binom{20}{5} = \frac{20!}{5!\,15!} = 15504$

$$\frac{\overset{3}{\cancel{20}} \cdot 19 \cdot \overset{2}{\cancel{18}} \cdot 17 \cdot 16 \cdot \cancel{15!}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{15!}}$$

Note: $_nC_r = \frac{_nP_r}{r!}$

↳ there are $r!$ orderings of the r objects

Q How many distinct ways are there to choose 2 students from a group of 7? List them all.

A) {a,b}
{a,c} {b,c}
{a,d} {b,d} {c,d}
{a,e} {b,e} {c,e} {d,e}
{a,f} {b,f} {c,f} {d,f} {e,f}
{a,g} {b,g} {c,g} {d,g} {e,g} {f,g}

Q Let C = {A, K, Q, J}

(a) List all 3-combinations of C

(b) List all 3-permutations of C.

4 combo's:

No A: {J, Q, K}
No K: {J, Q, A}
No Q: {J, K, A}
No J: {Q, K, A}

24 perm's:

No A: JQK
JKQ
QJK
QKJ
KJQ
KQJ

etc...

Q) Consider a class with 13 people.
How many different (distinct) teams of 5 can be chosen? ← order is irrelevant

A) $\binom{13}{5} = \frac{13!}{5!\,8!} = 1287$ distinct teams.

Q) Same question, but the class contains a pair that refuses to be split up. Pick both or neither.

A) case 1: pick both: 1 way to do that AND $\binom{11}{3}$

OR case 2: ignore both: $\binom{11}{5}$

$$1\cdot\binom{11}{3} + \binom{11}{5} = 627$$

Q) Same question but the class contains a pair that refuses to work together. Pick one or the other or neither (not both).

A) case 1: pick A : 1 way to pick A, $\binom{11}{4}$ ways to pick the rest

OR case 2: pick B : 1 way to pick B, $\binom{11}{4}$ ways to pick the rest

OR case 3: pick neither $\binom{11}{5}$

$$1\cdot\binom{11}{4} + 1\cdot\binom{11}{4} + \binom{11}{5} = 1122$$

Q] How many 11-bit strings have exactly four 1's?

A] $\binom{11}{4} = \frac{11!}{4!7!} = 330 \quad \left(= \binom{11}{7}\right)$

Q] How many distinguishable orderings are there of the following words:

(a) MISSISSIPPI

(b) HULLABALLOO

(c) MAMMA MIA

A] (a) 11 letters

Step 1: Choose 4 places to put S: $\binom{11}{4}$

Step 2: Choose 4 places to put I: $\binom{7}{4}$

Step 3: Choose 2 places to put P: $\binom{3}{2}$

Step 4: choose 1 place to put M: $\binom{1}{1}$

$$\frac{11!}{4!4!2!1!} = 34650$$

Q/ How many of the $\binom{52}{5} = 2598960$ 5-card hands in poker contain 2 pairs.

-10-

-10-

JJ 55 K ✓

JJJ 55 X