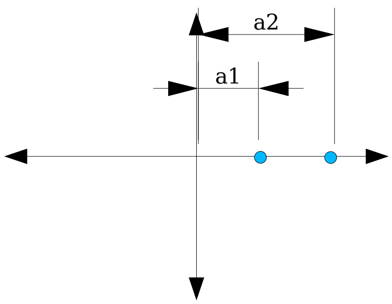
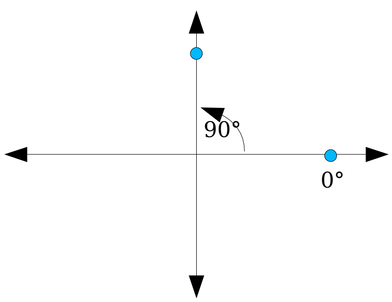
COMP3721 Week Four Lab Synopsis

Constellation Diagrams

- A modulation scheme is the set of rules (protocol) by which we vary a (simple) signal to encode information. For example, in last week's lab, we used a modulation scheme where phase was varied.
- Constellation diagrams (graphs) provide a concise way to express a modulation scheme
  - Shows amplitude shifts based on the length, or magnitude, of a point (vector) from the origin of the graph



  - Shows phase based on the angle of the vector relative to the



    positive x-axis

  - With this notation, we can compactly express modulation schemes using both amplitude and phase shifts
  - Does **not** allow obvious way to express frequency shifts

Bits and Baud

- Bits are a measure of information
  - expresses the number of possibilities
    - example – 3 bits <--> 8 possibilities
- A baud is a signal element – every time we modify our (simple) signal to encode information onto it, we introduce a baud
  - the baud rate is then the number of signal elements/second
- Two ways to transit data faster

1. send more signal elements per period of time – higher baud rate
   - **important concept:** there is a limit to how quickly we can send signal elements – see Nyquist below.
2. send more information with each signal element – more bits per baud
   - the number of bits/baud depends on the number of possible encodings for each signal element
     - for example, with only two phases shifts (last week's example), there is only 1 bit of information encoded in each baud
     - a more complex modulation scheme – more modulation points – provides more bits per baud
       - **important:** the number of modulation points is normally denoted by the variable *V*
       - **important:** the number of bits encoded per baud, *n*, is $\log_2 V$

$$V=2^n \qquad n=\log_{2V}$$

     - there is a limit to the complexity of the modulation scheme
       - noise means that the received signal, r(t), is never exactly the same as the signal sent, s(t).
       - this is equivalent to a movement of the transmitted point within the constellation diagram
       - the closer together the points are, the more susceptible to error
       - Shannon's theorem (covered below) addresses this limit

Digital Data Rate in an Analog Channel

- Nyquist established the maximum possible signaling (baud) rate
  - Maximum Baud Rate = 2H, where *H is the highest frequency* of the signal or channel

- Then the fastest we can send data in a channel is
  - Maximum Data Transfer Rate (DTR) = Max Baud Rate * Bits/Baud
    = 2H * $\log_2 V$
  - **Important:** Nyquist does not factor in noise – in a noiseless channel we could theoretically send as fast as we want by increasing the number of encoding levels (larger V).

- Shannon's theorem establishes the maximum data transfer rate in the presence of noise
  - Maximum DTR = H * $\log_2(1 + SNR_{raw})$, where
    - H is bandwidth of the signal or channel
    - SNR is the signal to noise ratio – the strength of the signal

relative to the noise

- SNR is often expressed in decibels, a logarithmic scale
- The Shannon equation requires the non-logarithmic, or raw, ratio

$$\frac{S}{N_{db}} = 10\log_{10}\left(\frac{S}{N_{raw}}\right) \qquad \frac{S}{N_{raw}} = 10^{\frac{S/N_{db}}{10}}$$

- The maximum DTR then depends on the following:
  - Bandwidth (H)
  - Noise – expresses as SNR
  - Modulation scheme – expresses as a number of points (V)

- Since all three factors are involved in most problems, we must use both Shannon and Nyquist to solve most problems.

Analog Data Rate in a Digital Channel

- We often want to capture, or transmit, analog data in digital form
  - for example, CDs capture analog sound in a digital form

- By Nyquist, we must sample the amplitude of the analog signal at a rate of *at least* 2H (where H is the highest signal frequency) to capture all signal elements
  - **important:** this is a *minimum* sampling rate because we must sample at least as fast as the *maximum* signaling rate.

- Samples are initially discrete in time (samples), but still analog in amplitude – this first step is called PAM
- To get a bit stream from this, each analog amplitude value must be mapped to a binary value
  - We divide the range of amplitudes into levels and map each sample to the closest level
  - Since the analog amplitude must be moved up or down to the nearest level, we no longer have the exact analog signal. The shift up or down is equivalent to noise in the system and is call quantization error or quantization noise.
  - Each level has a binary value associated with it – this is the output for each *quantized* sample

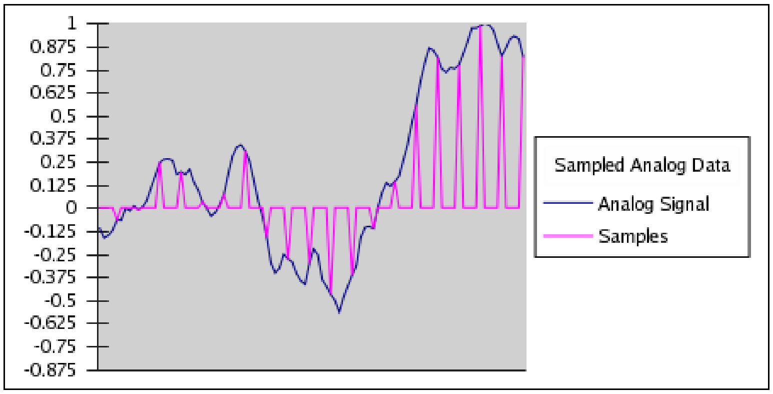


Figure i - 16 level quantization

- Using V to indicate the number of possible mapping levels:
  - bits/sample = $\log_2 V$
  - Digital data rate = sampling rate * bits/sample
    = $2H * \log_2 V$
  - Note that we are using the minimum sampling rate in the above equation – you could (and sometimes do) sample at a higher rate

- The more levels, the less quantization error in mapping the analog signal to a discrete value (the better the quality).
  - But also, the more levels, the higher the bit-rate generated and the greater the storage/transmission capacity required
  - the amount of quantization error is estimated by the following $\frac{S}{N}_{db} = 6n - \alpha$ where
    - n = $\log_2 V$ (the number of bits/sample)
    - $\alpha = 1.25$ for our purposes

    - Consider the figure above with 16 levels. Then there are 4 bits/sample (i.e. -0.875 --> 0000, -0.750 --> 0001, ..., 1.000 --> 1111).
    - The quantization noise due to using 4 bits/sample is 6(4)-1.25 = 22.75dB.
    - If we were to use more bits/sample (or equivalently, more levels), then the S/N due to quantization would increase. This may seem counter-intuitive, but a larger S/N is a good thing – it indicates that the signal power is larger in ratio to the quantization noise.

  - **important:** do not use Shannon to solve quantization problems (analog-->digital)

Understanding the Questions

- Most signal-related questions fall into one of two categories:

1. What digital data rate can be achieved through a given analog channel? (digital to analog)
   - Here we use the Shannon and Nyquist formulas.
   - Typically we have parameters for a channel, such as BW, the S/N ratio (of the channel) and/or a modulation scheme. Questions normally then revolve around the digital data rate given those parameters.
2. What digital data rate is generated in digitizing and analog signal? (analog to digital)
   - Here we need to know the sampling rate and number of quantization levels. Often it is necessary to determine the number of quantization levels – a given quality would be specified (i.e. minimum S/N due to quantization) and solved for with the $\frac{S}{N}_{db} = 6n - \alpha$ formula.
   - These types of questions are often best identified by recognizing the presence of an analog signal, i.e. voice, music, etc.

Follow-Up Questions

1. Assuming all other factors remain the same, how much would you have to increase the signal power in a system to affect a 10% increase in the maximum theoretical bit-rate? If the S/N is held constant, how much would the bandwidth need to be increased to achieve a 10% increase?
2. A 3dB increase in S/N represents how great an increase in signal power? Is increasing the signal power the only way to vary increase the S/N ratio? What does a negative S/N ratio represent – i.e. how do we interpret -3dB?
3. Determine the data rate generated by digitizing a stereo cassette tape, maintaining a minimum quantization S/N of 60dB.
4. Suggest a modulation scheme for a system where the channel frequency spectrum is 0 – 50MHz, the S/N = 90dB and the desired transfer rate is a 100Mbps.

Answers to Follow-up Questions

1. log2(1 + S/N_0) * 1.1 = $\log_2$(1 + S/N_1)
   (1 + S/N_0) * $2^{1.1}$ = 1 + S/N_1
   2.143546925 + 2.143546925 * S/N_0 = 1 + S/N_1
   $$\frac{S}{N_1} \approx 2.14 \frac{S}{N_0}$$
   $$H_1 = 1.1 H_0$$

2. 3db=10$\log_{10}$(S/N)
   S/N=$10^{0.3}$=2
   A 3dB increase in S/N represents a doubling of the signal power. Alternately it could represent a reduction in noise by a factor of two. A negative S/N represents a decrease in signal power relative to the noise (or an increase in noise relative to the signal power).

3. To achieve a minimum of 60dB S/N due to quantization, we need to quantize with 11 bits/sample, or 2048 levels.
   $$60dB \leq 6n - 1.25$$
   $$n \geq \frac{61.25}{6} = 10.21$$

   Since the audible spectrum ranges from 0 – 20kHz, we should sample at a minimum rate of 2 * 20kHz = 40kHz. Then the data rate is
   $$Digital\,data\,rate = 40000\,samples/sec * 11\,bits/sample = 440\,kbps$$

4. By Nyquist, the 100Mbps transfer rate can be achieved by using a 2-point (level) modulation scheme.
   $$100 \times 10^6 = 2 \times (50 \times 10^6) \log_{2V}$$
   $$\log_{2V} = 1$$
   $$V = 2^1 = 2$$

   While not required, we can verify that this (bit-rate) is within the Shannon limit:
   $$90dB = 10 log_{10}(S/N)$$
   $$S/N = 10^9$$
   $$100 \times 10^6 \leq 50 \times 10_2^{6log}(1 + 10^9) \approx 1.5 \times 10^9$$

   So it seems we're okay – well within the Shannon limit.