Suppose you have 3 cards:

A black card, black on both sides

A white card, white on both sides

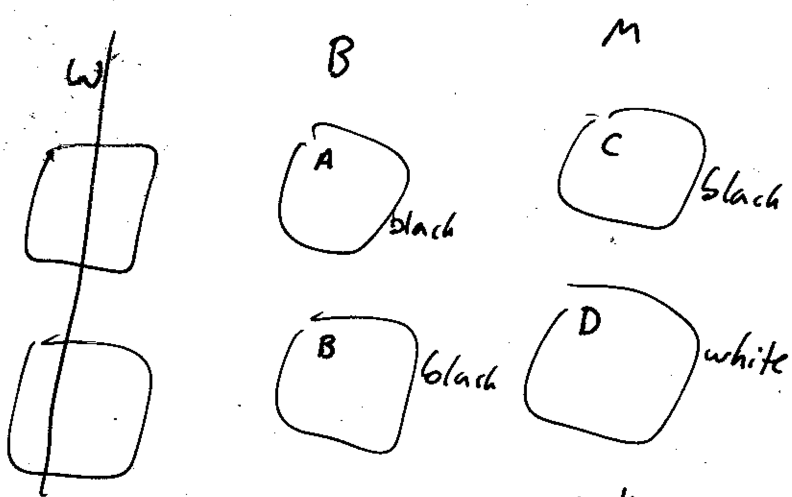
A mixed card, black on one side, white on other.

Put the cards in a hat.

Pull one out, onto a table.

The side facing up is black.

What are the odds the other side is black too?



have	other
A	✓
B	✓
C	✗

$\frac{2}{3}$

Today
Handouts: 1.2, 1.3 Conditional Statements

$p \to q$ "if p then q"
"p implies q"

Order of operations

1. parentheses
2. $\sim$ not
3. $\wedge \vee$
4. $\to$

$$\boxed{\boxed{p \wedge \boxed{\sim q}} \to p}$$

"if it is raining then the ground is wet."

☆ quiz

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

true by default

if today is Sunday then I am 20 feet tall.

hypothesis

conclusion

Two statements:

(A) Eat your dinner or you get no dessert

(B) If you do not eat your dinner then you will not get dessert.

Logically equivalent?

Let p = "eat dinner"
Let q = "get dessert"

p	q	$\sim q$	$p \vee \sim q$	$\sim p$	$\sim p \rightarrow \sim q$	(A)≡(B)?
T	T	F	T	F	T	✓
T	F	T	T	F	T	✓
F	T	F	F	T	F	✓
F	F	T	T	T	T	✓
			(A)		(B)	

LE.

quiz

ex: php

(1) open a file or stop the script

(2) if (file can be opened)

else stop the script

quiz

Negating conditional statements

$\sim(p \to q)$?

★ $\boxed{p \wedge \sim q}$

→ negation of a conditional is not another conditional.

recall:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \to q$		$\sim(p \to q)$	$\sim q$
T	T	T	$\neq$	F	F
T	F	F	$\neq$	T	T
F	T	T	$\neq$	F	F
F	F	T	$\neq$	F	T

$p \wedge \sim q$

$\underline{p \wedge \sim q}$

Negate: "if you study hard you will score an A+"

"you do study hard and will not get an A+"

(A) "If you eat your vegetables then you get dessert"

(B) "If you don't eat your vegetables then you do not get dessert"

Question: are these logically-equivalent statements? Prove it.

Let v = eat vegetables
Let d = get dessert

v	d	$v \to d$	$\sim v$	$\sim d$	$\sim v \to \sim d$	$A \equiv B$
T	T	T	F	F	T	✓
T	F	F	F	T	T	
F	T	T	T	F	F	
F	F	T	T	T	T	✓
		(A)			(B)	

Not LE.

kids like (A)

kids dislike (B)

Contrapositive

of $\boxed{p \to q}$ is $\boxed{\sim q \to \sim p}$

if you eat v.
you get d.

if you didn't get d.
you didn't eat v.

Contrapositive of a cond. statement is
L.E. to original cond. statement

v	d	v→d	~d	~v	~d→~v
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T
		Cond			Contra

$\underbrace{\text{Cond} \quad\quad \text{Contra}}_{\text{LE}}$

$a \to b$

negation: $a \wedge \sim b$

contrapositive: $\sim b \to \sim a$

Converse of a conditional statement.

Converse of $\boxed{p \to q}$ is $\boxed{q \to p}$

Convert p to q && q to p.

Not logically equivalent.

p	q	$p \to q$	$q \to p$	A = B ?
T	T	T	T	✓
T	F	F	T	×
F	T	T	F	×
F	F	T	T	✓
		A	B	

Not L.E.

if the canucks win, they scored

≠ if the canucks score, they win.

If a person is the killer
then they are 7' tall + red head + left handed.

Inverse

inverse of $p \to q$ is $\sim p \to \sim q$

L.E.?

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim p \to \sim q$	A = B?
T	T	T	F	F	T by default	✓
T	F	F	F	T	T by default	✗
F	T	T by default	T	F	F	✗
F	F	T by default	T	T	T	✓
		(A)			(B)	∴ Not LE

Converse ≡ Inverse but ≢ original conditional statement

is the contrapositive of the

"if I am talking then I am alive"

write the ~~negation~~ negation
- inverse
- converse
- contrapositive
- rewrite without if/then ("or"?)

Let t = "I am talking"
Let a = "I am alive"

~~English~~	Symbols	L.E. to ?
Negation	$t \wedge \sim a$	No
inverse	$\sim t \rightarrow \sim a$	No
Converse	$a \rightarrow t$	No
Contrapositive	$\sim a \rightarrow \sim t$	yes
no "$\rightarrow$" notation	$\sim t \vee a$	yes ☆

quiz

circuits use inverters, AND gates, OR gates
no "if" gates

study hard or fail.

$\equiv$

if you do not study hard
then you will fail

Only if

$\boxed{p \text{ only if } q \quad p \to q}$!

Q rewrite as conditional:

"the Canucks win only if they score"

A "if the Canucks win then they scored"

$\therefore$ if $\sim q$, then $\sim p$

$\sim q \to \sim p$ } contrapositive of $p \to q$

Questions:

What's the contrapositive of

$\boxed{\sim u \to v}$

Answer:

$\boxed{\sim v \to u}$

$\boxed{p \text{ if } q}$ Rewrite as conditional

answer: if q then p

$\boxed{q \to p}$

Biconditional

p if and only if q

$p \longleftrightarrow q$

p iff q

Order of operations:

(1) parentheses

(2) $\sim$

(3) $\wedge \ \vee$

(4) $\rightarrow \quad \leftrightarrow$

You win the game iff they outscore opponent

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \rightarrow q \wedge q \rightarrow p$ "only if" and "if"	$p \leftrightarrow q$
T	T	T	T	T	
T	F	F	T	F	
F	T	T	F	F	
F	F	T	T	T	

$p \longleftrightarrow q$

Write as a conditional:

"p is necessary for q"

What if $\sim p$?

Then $\sim q$ $\quad \sim p \to \sim q$ } contrapositive of: $\boxed{q \to p}$

"p is sufficient for q"

$\boxed{p \to q}$

"p is necessary and sufficient for q"

$\boxed{p \leftrightarrow q}$

$q \to p$ nec.

$\wedge\ p \to q$ suff

If Mickey Mouse is president of the USA...

then...

→ TRUE

Quiz Monday → from Handout

Not trivial

Lab from Handout: Pg 27 # 8, 13b, 18, 21 bc, 36

1.1 # 8bce, 30, 34, 44, 51 — handout answers:

✓

8b. $\sim w \wedge (h \wedge s)$

c. $\sim w \wedge \sim h \sim \wedge s$

e. $w \wedge \sim (h \wedge s)$ or $(w \wedge (\sim h \vee \sim s))$

30. Sam is not an orange belt
or
Kate is not a red belt.

34. The train is not late and my watch is not fast.

44. tautology

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

51. $(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q)$

$\equiv$	$(p \wedge (\sim(\sim p) \wedge \sim q)) \vee (p \wedge q)$	De Morgan's law
$\equiv$	$(p \wedge (p \wedge \sim q)) \vee (p \wedge q)$	double negative law
$\equiv$	$((p \wedge p) \wedge \sim q)) \vee (p \wedge q)$	associative law for $\wedge$
$\equiv$	$(p \wedge \sim q)) \vee (p \wedge q)$	idempotent law for $\wedge$
$\equiv$	$p \wedge (\sim q \vee q)$	distributive law
$\equiv$	$p \wedge (q \vee \sim q)$	commutative law for $\vee$
$\equiv$	$p \wedge \mathbf{t}$	negation law for $\vee$
$\equiv$	p	identity law for $\wedge$

HANDOUT SOLUTIONS

52. *b.* Yes.

p	q	r	$p \oplus q$	$q \oplus r$	$(p \oplus q) \oplus r$	$p \oplus (q \oplus r)$
T	T	T	F	F	T	T
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \oplus q) \oplus r$ and $p \oplus (q \oplus r)$ always have the same truth values. So they are logically equivalent.

c. Yes.

p	q	r	$p \oplus q$	$p \wedge r$	$q \wedge r$	$(p \oplus q) \wedge r$	$(p \wedge r) \oplus (q \wedge r)$
T	T	T	F	T	T	F	F
T	T	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \oplus q) \wedge r$ and $(p \wedge r) \oplus (q \wedge r)$ always have the same truth values. So they are logically equivalent.

54. The conditions are most easily symbolized as $p \vee (q \wedge \sim (r \wedge (s \wedge t)))$, but may also be written in a logically equivalent form.

Section 1.2

2. If I catch the 8:05 bus, then I am on time for work.

4. If you don't fix my ceiling, then I won't pay my rent.

6.

p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(p \vee q) \vee (\sim p \wedge q)$	$(p \vee q) \vee (\sim p \wedge q) \to q$
T	T	F	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	F	T	F	F	F	T

8.

p	q	r	$\sim p$	$\sim p \vee q$	$\sim p \vee q \rightarrow r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	F

10.

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \leftrightarrow (q \rightarrow r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

11.

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$p \wedge q \rightarrow r$	$(p \rightarrow (q \rightarrow r)) \leftrightarrow (p \wedge q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

13. *b.*

p	q	$\sim q$	$p \rightarrow q$	$\sim (p \rightarrow q)$	$p \wedge \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

same truth values

The truth table shows that $\sim (p \rightarrow q)$ and $p \wedge \sim q$ always have the same truth values. Hence they are logically equivalent.

14. *a.*

p	q	r	$\sim q$	$\sim r$	$q \vee r$	$p \wedge \sim q$	$p \wedge \sim r$	$p \to q \vee r$	$p \wedge \sim q \to r$	$p \wedge \sim r \to q$
T	T	T	F	F	T	F	F	T	T	T
T	T	F	F	T	T	F	T	T	T	T
T	F	T	T	F	T	T	F	T	T	T
T	F	F	T	T	F	T	T	F	F	F
F	T	T	F	F	T	F	F	T	T	T
F	T	F	F	T	T	F	F	T	T	T
F	F	T	T	F	T	F	F	T	T	T
F	F	F	T	T	F	F	F	T	T	T

same truth values

The truth table shows that the three statement forms $p \to q \vee r$, $p \wedge \sim q \to r$, and $p \wedge \sim r \to q$ always have the same truth values. Thus they are all logically equivalent.

b. If n is prime and n is not odd, then n is 2.

And: If n is prime and n is not 2, then n is odd.

15.

p	q	r	$q \to r$	$p \to q$	$p \to (q \to r)$	$(p \to q) \to r$	
T	T	T	T	T	T	T	
T	T	F	F	T	F	F	
T	F	T	T	F	T	T	
T	F	F	T	F	T	T	
F	T	T	T	T	T	T	
F	T	F	F	T	T	F	←
F	F	T	T	T	T	F	←
F	F	F	T	T	T	F	←

different truth values

The truth table shows that $p \to (q \to r)$ and $(p \to q) \to r$ do not always have the same truth values. (They differ for the combinations of truth values for p, q, and r shown in rows 6, 7, and 8.) Therefore they are not logically equivalent.

17. Let p represent "Rob is goalkeeper," q represent "Aaron plays forward," and r represent "Sam plays defense." The statement "If Rob is goalkeeper and Aaron plays forward, then Sam plays defense" has the form $p \wedge q \to r$. And the statement "Rob is not goalkeeper or Aaron does not play forward or Sam plays defense" has the form $\sim p \vee \sim q \vee r$.

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge q \to r$	$\sim p \vee \sim q \vee r$
T	T	T	F	T	T	T	T
T	T	F	F	T	T	F	F
T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	F	T	T

same truth values

The truth table shows that $p \wedge q \to r$ and $\sim p \vee \sim q \vee r$ always have the same truth values. Therefore they are logically equivalent.

18. *Part 1*: Let p represent "It walks like a duck," q represent "It talks like a duck," and r represent "It is a duck." The statement "If it walks like a duck and it talks like a duck, then it is a duck" has the form $p \wedge q \to r$. And the statement "Either it does not walk like a duck or it does not talk like a duck or it is a duck" has the form $\sim p \vee \sim q \vee r$.

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$p \wedge q \to r$	$(\sim p \vee \sim q) \vee r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	T

same truth values

The truth table shows that $p \wedge q \to r$ and $(\sim p \vee \sim q) \vee r$ always have the same truth values. Thus the following statements are logically equivalent: "If it walks like a duck and it talks like a duck, then it is a duck" and "Either it does not walk like a duck or it does not talk like a duck or it is a duck."

Part 2: The statement "If it does not walk like a duck and it does not talk like a duck then it is not a duck" has the form $\sim p \wedge \sim q \to \sim r$.

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \wedge q$	$\sim p \wedge \sim q$	$p \wedge q \to r$	$(\sim p \wedge \sim q) \to \sim r$	
T	T	T	F	F	F	T	F	T	T	
T	T	F	F	F	T	T	F	F	T	←
T	F	T	F	T	F	F	F	T	T	
T	F	F	F	T	T	F	F	T	T	
F	T	T	T	F	F	F	F	T	T	
F	T	F	T	F	T	F	F	T	T	
F	F	T	T	T	F	F	T	T	F	←
F	F	F	T	T	T	F	T	T	T	

different truth values

The truth table shows that $p \wedge q \to r$ and $(\sim p \wedge \sim q) \to \sim r$ do not always have the same truth values. (They differ for the combinations of truth values of p, q, and r shown in rows 2 and 7.) Thus they are not logically equivalent, and so the statement "If it walks like a duck and it talks like a duck, then it is a duck" is not logically equivalent to the statement "If it does not walk like a duck and it does not talk like a duck then it is not a duck." In addition, because of the logical equivalence shown in Part 1, we can also conclude that the following two statements are not logically equivalent: "Either it does not walk like a duck or it does not talk like a duck or it is a duck" and "If it does not walk like a duck and it does not talk like a duck then it is not a duck."

20. *b.* Today is New Year's Eve and tomorrow is not January.

c. The decimal expansion of r is terminating and r is not rational.

e. x is nonnegative and x is not positive and x is not 0.

Or: x is nonnegative but x is not positive and x is not 0.

Or: x is nonnegative and x is neither positive nor 0.

g. n is divisible by 6 and either n is not divisible by 2 or n is not divisible by 3.

21. By the truth table for $\rightarrow$, $p \rightarrow q$ is false if, and only if, p is true and q is false. Under these circumstances, (*b*) $p \vee q$ is true and (*c*) $q \rightarrow p$ is also true.

22. *b*. If tomorrow is not January, then today is not New Year's Eve.

    c. If r is not rational, then the decimal expansion of r is not terminating.

    e. If x is not positive and x is not 0, then x is not nonnegative.

    Or: If x is neither positive nor 0, then x is negative.

    g. If n is not divisible by 2 or n is not divisible by 3, then n is not divisible by 6.

23. *b*. *Converse*: If tomorrow is January, then today is New Year's Eve.

    Inverse: If today is not New Year's Eve, then tomorrow is not January.

    c. *Converse*: If r is rational then the decimal expansion of r is terminating.

    Inverse: If the decimal expansion of r is not terminating, then r is not rational.

    e. *Converse*: If x is positive or x is 0, then x is nonnegative.

    Inverse: If x is not nonnegative, then both x is not positive and x is not 0.

    Or: If x is negative, then x is neither positive nor 0.

25.

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \rightarrow \sim q$	
T	T	F	F	T	T	
T	F	F	T	F	T	$\leftarrow$
F	T	T	F	T	F	$\leftarrow$
F	F	T	T	T	T	

different truth values

The truth table shows that $p \rightarrow q$ *and* $\sim p \rightarrow \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent. Thus a conditional statement is not logically equivalent to its inverse.

27.

p	q	$\sim p$	$\sim q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

same truth values

The truth table shows that $q \rightarrow p$ *and* $\sim p \rightarrow \sim q$ always have the same truth values, so they are logically equivalent. Thus the converse and inverse of a conditional statement are logically equivalent to each other.

28. The if-then form of "I say what I mean" is "If I mean something, then I say it."

    The if-then form of "I mean what I say" is "If I say something, then I mean it."

    Thus "I mean what I say" is the converse of "I say what I mean." The two statements are not logically equivalent.

30. The corresponding tautology is $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

all T's

The truth table shows that $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$ is always true. Hence it is a tautology.

31. The corresponding tautology is $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$.

p	q	r	$p \to q$	$p \wedge q$	$p \to (q \to r)$	$(p \wedge q) \to r)$	$p \to (q \to r) \leftrightarrow (p \wedge q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

all T's

The truth table shows that $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$ is always true. Hence it is a tautology.

33. If Sam is not an expert sailor, then he will not be allowed on Signe's racing boat.

If Sam is allowed on Signe's racing boat, then he is an expert sailor.

34. The Personnel Director did not lie. By using the phrase "only if," the Personnel Director set forth conditions that were necessary but not sufficient for being hired: if you did not satisfy those conditions then you would not be hired. The Personnel Director's statement said nothing about what would happen if you did satisfy those conditions.

36. If it doesn't rain, then Ann will go.

37. *b.* If a security code is not entered, then the door will not open.

39. *a.*
$$\begin{aligned} p \vee \sim q \to r \vee q &\equiv\ \sim (p \vee \sim q) \vee (r \vee q) && \textit{[an acceptable answer]} \\ &\equiv (\sim p \wedge \sim (\sim q)) \vee (r \vee q) && \text{by De Morgan's law} \\ &&& \textit{[another acceptable answer]} \\ &\equiv (\sim p \wedge q) \vee (r \vee q) && \text{by the double negative law} \\ &&& \textit{[another acceptable answer]} \end{aligned}$$

b.
$$\begin{aligned} p \vee \sim q \to r \vee q &\equiv (\sim p \wedge q) \vee (r \vee q) && \text{by part (a)} \\ &\equiv\ \sim (\sim (\sim p \wedge q) \wedge \sim (r \vee q)) && \text{by De Morgan's law} \\ &\equiv\ \sim (\sim (\sim p \wedge q) \wedge (\sim r \wedge \sim q)) && \text{by De Morgan's law} \end{aligned}$$

The steps in the answer to part (b) would also be acceptable answers for part (a).