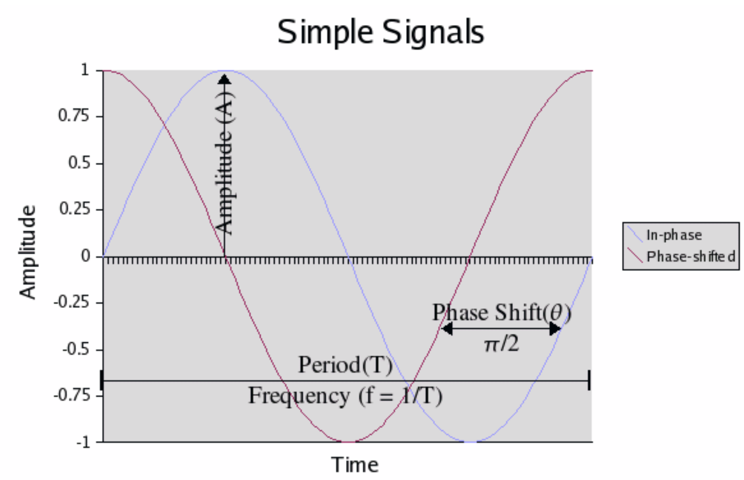
COMP3721 Week Three Lab Synopsis

Signals

- Signals can be divided into two categories
  - Simple signals: any signal that can be described as follows:
    $s(t) = A \cdot \sin(2\pi ft + \theta)$
    - sine is an arbitrary periodic function; we could alternately use cosine or the exponential function (with imaginary numbers)
    - A is the amplitude
    - f is the frequency (in Hz)
    - θ is the phase shift (in radians)



  - Complex signals: any signal that is not simple
    - **important concept:** all complex signals can be expressed as the sum (linear combination) of several/many simple signals
    - Example: a square pulse waveform can be expressed as the sum of a number of sinusoidal waves (shown later).
    - **Important concept:** this is not just a neat mathematical trick – the physical reality underlying a complex signal is a number of sinusoidal waves. For example, we may think we put a square pulse onto a cable by applying a voltage and then removing it, but in reality what happens is the creation of a number of (simple) sinusoidal waveforms (of differing frequencies) that added together form an (almost) square

On Frequencies and Radians

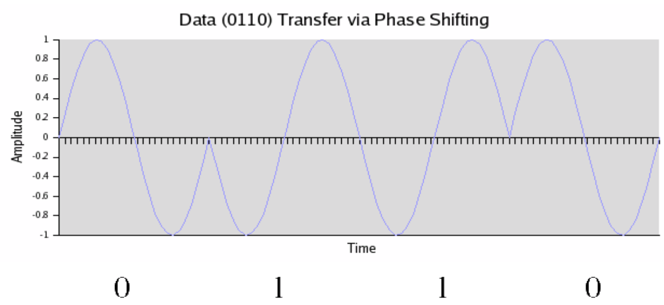
Mathematical functions typically expect their arguments to be expressed in radians. Since we generally work with frequencies in Hz, we need to multiply by 2π to get the appropriate input value in radians.

Sometimes, particularly in electronics, the frequency is expressed in radians/second rather than Hz – then the value 2π will be absent from the above formula, and the symbol ω is used rather than *f* to represent the frequency.

1 period (or cycle) = 360° = 2π radians

pulse.

- Signals are important as they underlie all data communication
  - information is transmitted by varying the fundamental characteristics of signals over time: amplitude, frequency, and phase
  - this information signal is typically expressed as a function of time, s(t)
    - Example: transferring data by shifting the phase of a signal
      - Logical 0 => $\theta = 0$ (no phase shift)
      - Logical 1 => $\theta = \pi$ (phase shift of one-half cycle)
      - Data: 0 1 1 0



    - Note that in this example, we varied a simple signal to transmit data. The resulting signal is no longer simple – we cannot provide values A, f and θ that can be applied to a periodic function to describe the waveform shown. In fact it is now a complex signal, composed of multiple simple signals added together.

Bandwidth

- Signal bandwidth – the range of frequencies present in a complex signal.
  - Different signals have different ranges of frequencies, or equivalently, different signal bandwidths.
- Channel bandwidth – the finite range of frequencies propagated by a channel.
- To propagate the signal s(t) from sender to receiver requires that all signal frequencies (all of the simple signals)
  - If the channel does not propagate all of the signal frequencies, then what is received is an approximation of the original signal.
  - Example: your voice has frequency components between 20 Hz and 10 kHz. A phone line only propagates frequency components between 0 Hz and 3.5 kHz. So your entire voice is not transmitted – instead only the portion of your voice within the range of 0 to 3.5 kHz – the receiver then hear an approximation of your voice. This is one reason that people's voices sound different over the phone (thin or tinny); also, the poor quality microphone and speaker in

most phones don't help.
  - **Important:** For data communications, we need most, if not all, of the frequency components to be transmitted to the receiver – that is, the reconstructed signal at the receiver must be very representative of the original signal, s(t). In general then, we will restrict ourselves to cases where the signal bandwidth is less than or equal to the channel bandwidth.
  - Test your understanding with this thought experiment: Person A is talking with Person B over the phone. At person A's location, a whistle outputs a loud, pure 5000 kHz tone. Person A complains about the tone, Person B doesn't hear anything. Why?
- There is a relationship between bandwidth and the ability to transfer data. Roughly stated, the more bandwidth available, the faster a system can transfer data. This is one reason you often hear the term bandwidth used to describe data transfer rate, i.e. high bandwidth ADSL meaning an ADSL line with a high data rate.
  - Both the Shannon and Nyquist formulas, to be discussed later, make this relationship explicit.

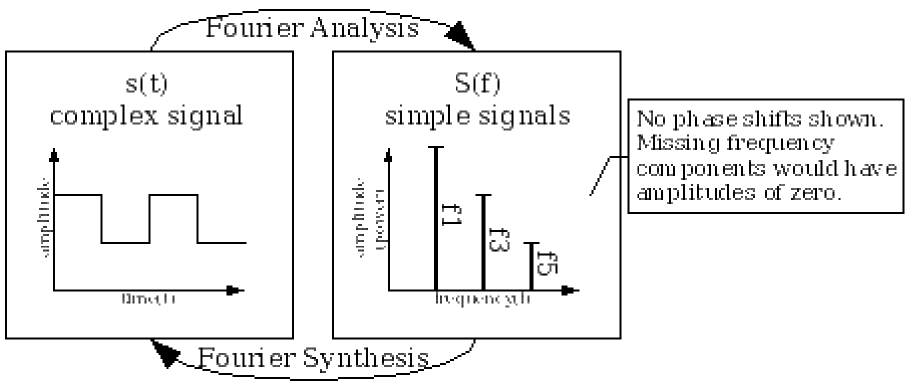
Complex Signals Expressed as the Sum of Simple Signals

- It has been stated that all complex signals are composed of the linear combination of simple signals. Another way of stating this is to consider that there are different views of the same signal:

  1. s(t) – a time-oriented view. This is the view we are most familiar with, where the value of the signal, generally amplitude, varies over time.
     - This is normally shown/graphed as amplitude varying over time.
     - Mathematically, we can express this as follows:
       $s(t)=s_1(t)+s_2(t)+\cdots+s_n(t)$
       where s(t) is the complex signal and $s_i(t)$ are each simple signals.
     - Each $s_i(t)$ signal is fully described by an amplitude (a_i), frequency (f_i) and phase shift (θ_i).
     - Fourier says the following:
       - the lowest frequency (f_1) present in the decomposition of the complex signal has the same period (or frequency) as the complex signal. This is called the fundamental frequency. For example, if the complex signal repeats 6 times/second (6 Hz), then f_1 = 6Hz.
       - all other frequencies are harmonics – integer multiples of the fundamental frequency; that is, $f_k=k\times f_1$. For example, the third harmonic for the above example is $f_3=3\times 6Hz=18Hz$.

2. S(f) - a frequency-oriented view
   - this is normally shown/graphed as a bar chart showing the amplitude (a_i) for each frequency (f_i) component. If phases shifts are also involved, then a second graph is necessary to show the phase shift (θ_i) at each frequency.

- To move from s(t) to S(f) requires *Fourier analysis.* This involves some integral calculus and you are not responsibly for doing this within this course, but you do have to understand the purpose:
  - we use Fourier analysis to understand the signal bandwidth of a time-oriented signal.
- The move from S(f) to s(t) is called *Fourier synthesis*. This is just the simple addition of the individual simple signals to form the complex signal.
  - See the associated square_pulse.xls spreadsheet for an example



Test Your Understanding

1. If the fundamental frequency of a signal is 10Hz, what is the frequency of the 6^{th} harmonic?
2. If a complex signal has a fundamental frequency of 8Hz the highest harmonic (with non-zero amplitude) is the 12^{th}, what is the signal frequency spectrum? What is the signal bandwidth?
3. For the square pulse show in lecture and lab, why were there no even harmonics? Why were there no phases shifts? Why was $a_k = \frac{1}{k}$ for all odd harmonics (k={1,3,5,7,...})?
4. For a given function s(t), why does it's associated frequency view, S(f) matter (in terms of data communication)?

Answers

1. $f_6 = 6 \times f_1 = 6 \times 10Hz = 60Hz$
2. 8Hz – 96Hz; Signal_{BW} = 96Hz – 8Hz = 88Hz
3. The specific amplitudes and phase shifts are obtained through Fourier analysis (not shown). We are simply using the known result. F*or this particular complex signal*, all even harmonics have an amplitude of zero, all harmonics are without phase-shift, and all odd harmonics have an amplitude $a_k = \frac{1}{k}$.
4. S(f) provides insight into the signal bandwidth. This information is necessary to understand whether the signal will properly propagate over a given channel.