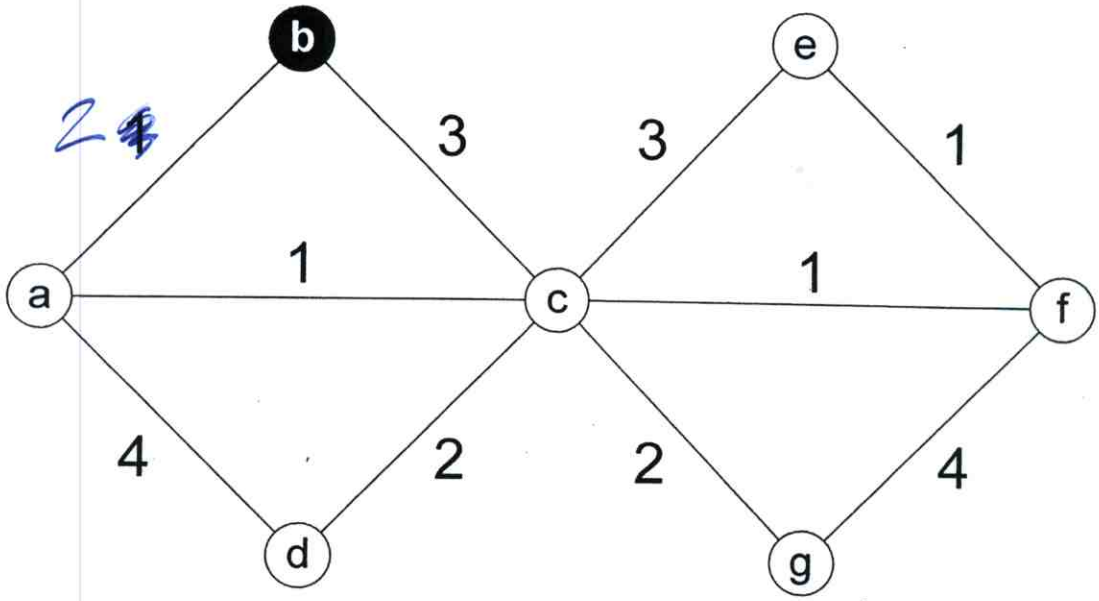
Shortest Path Problems

Problem: Single-Source Shortest Path SS-SP

- find the shortest path from one source vertex v to every other vertex in the graph
  - "source" means "starting vertex"



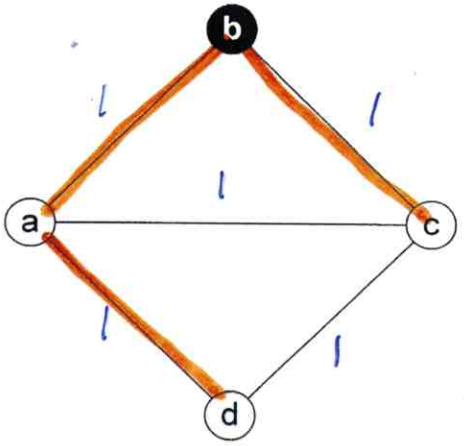
- b is source

→ SSSP from b → c is "2" (b-a-c)

" " b → d is "4" (b-a-c-d)

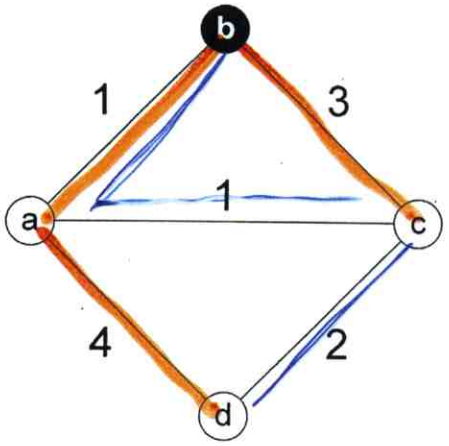
What about BFS?

- we know how to do this for an unweighted graph
  - BFS



SP B–a : 1
B–c : 1
B–D : 2

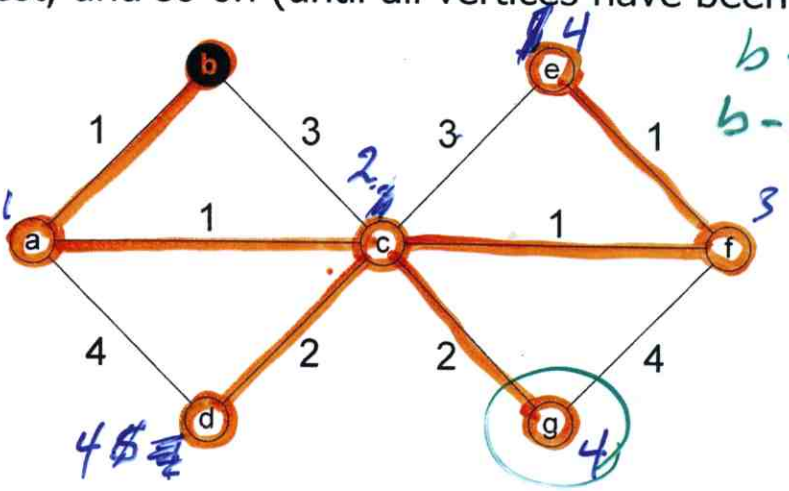
- but BFS doesn't work for weighted graphs, consider:



- *the algorithm to find shortest paths in weighted graphs needs to consider the weight on the edge before including it in the solution*

Dijkstra's Algorithm

- Greedy Algorithm
  - builds a tree of shortest paths rooted at the starting vertex
  - it is greedy because it adds the closest vertex, then the next closest, and so on (until all vertices have been added)



b → e

b-a-c-f-e

  - returns the *shortest distance to each vertex* in array `d[]`
  - returns the *parent of each vertex* in array `prev[]`

```
1. Initialise d and prev
2. Add all vertices to a PQ with distance
   from source as the key
3. While there are still vertices in PQ
4.      Get next vertex u from the PQ
5.      For each vertex v adjacent to u
6.          If v is still in PQ, relax v
```

a b c d e f g

d[1 0 2 4 4 3 4]

prev[b - a c f c c]

a b c d e f g

Relaxation

- Dijkstra always refers to "relaxing" a vertex
- this means "update the best known shortest path to v, and re-insert in the PQ"

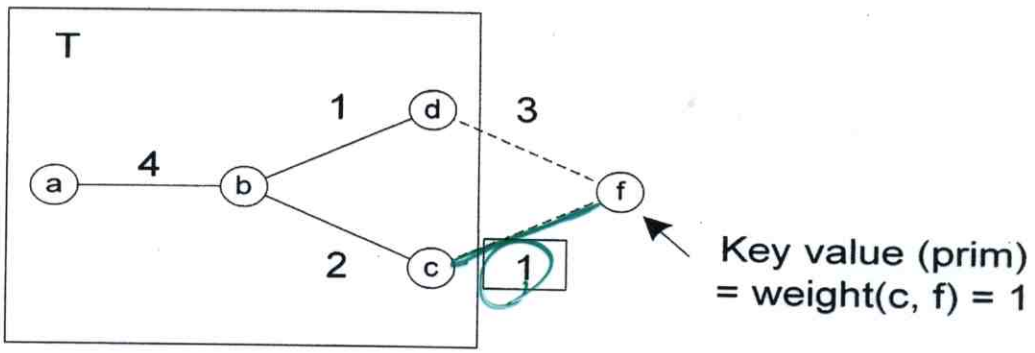
- the pseudocode for relaxation should read:

```
if d[u] + w(u,v) < d[v]
    d[v] ← d[u] + w(u,v)
    prev[v] ← u
    PQ.updateKey(d[v], v)
```

Similarity to Prim

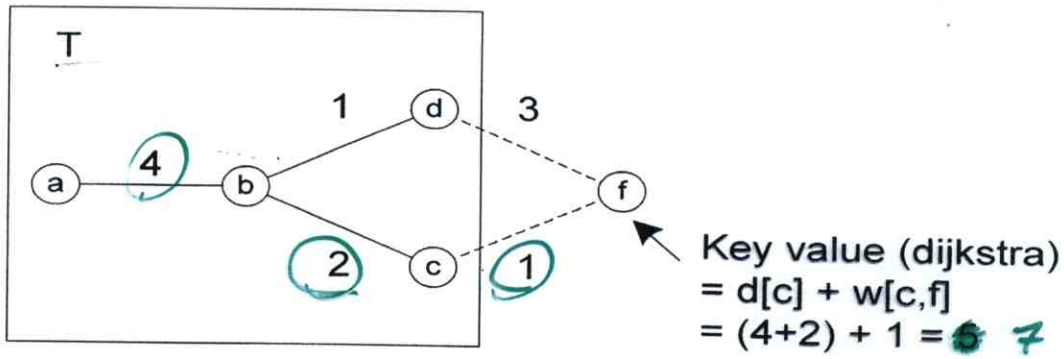
- algorithm is similar to Prim's algo
  - needs to select the minimum priority edge from the set of edges adjacent to the tree that has been built so far
  - in Prim's algo the "priority" of an edge *(u, v)* is defined by the weight of the edge

PRIM



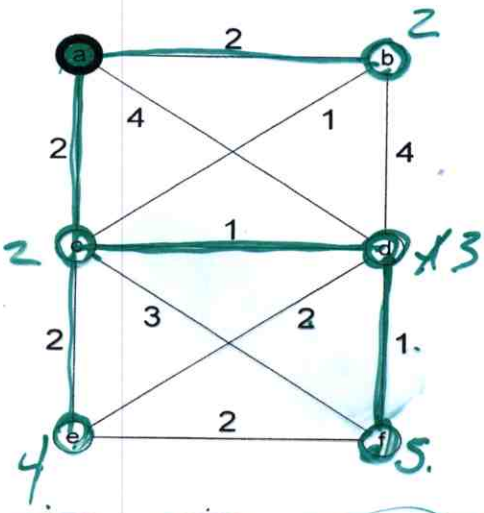
  - in Dijkstra the "priority" is given by the weight of the edge *(u, v)* plus the distance from the start to the parent of *v*

DIJKSTRA.



Some Dijkstra Examples

Key: distance from S
Value: vertex



d	prev
a 0	a –
b ~~∞~~ 2	b a
c ~~∞~~ 2	c a
d ~~∞~~ ~~4~~ 3	d ~~c~~ c
e ~~∞~~ 4	e ~~f~~ c
f ~~∞~~ ~~5~~ ~~4~~	f ~~c~~ d

PQ

~~a 0~~
~~2 b~~
~~2 c~~
~~3 4 d~~
~~4 e~~
~~4 5 f~~

START HERE !!!