COMP 3760

Algorithm Analysis and Design

Lesson 17: Spanning Tree Intro

Rob Neilson

rneilson@bcit.ca

Today's Agenda

- Prims

Reading for this week:

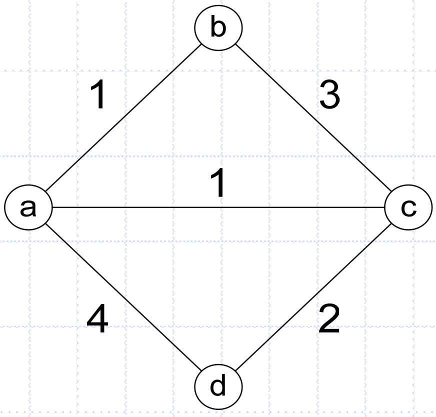
- Chapters 5.3
- questions 1 and 5

Minimum Spanning Trees

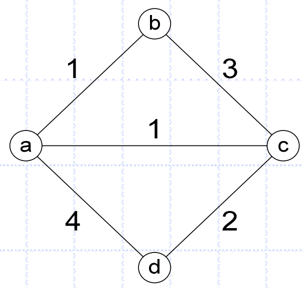
Definitions:

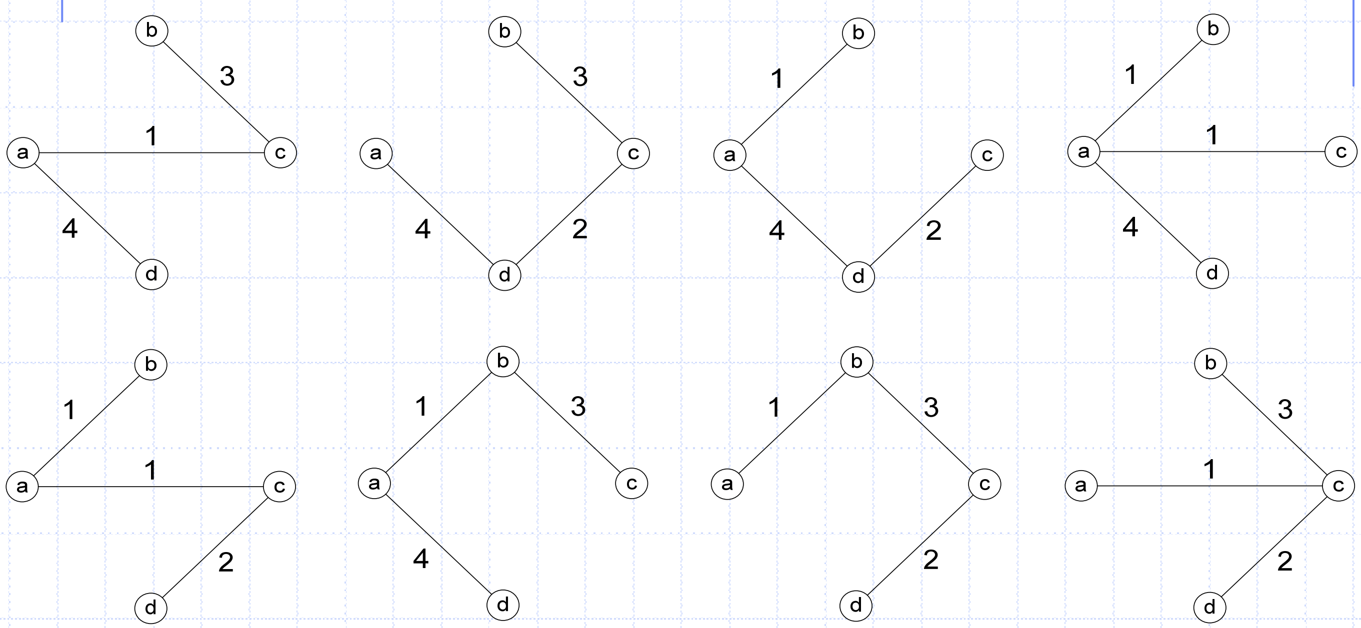
- *spanning tree* of a graph G: a connected acyclic subgraph (tree) of G that contains all the vertices
  - note: any graph with a cycle has more than one possible spanning tree
- *minimum spanning tree* (MST) T of a weighted graph G is the spanning tree with minimum weight (ie: the sum of the weights on all edges of T is lower than the sum of the weights for any other spanning tree of G)

What is the MST of the graph shown below?



MST's (Cont)





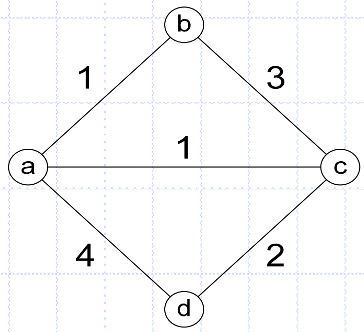
Prim

Prim's algo is a greedy approach to finding an MST

Prim (high level description):

```
Prim(G)  // return T, which is a MST of G
– add any vertex of G to the solution T
– let E be all edges of G that connect any vertex
  in T to any vertex not in T
– find an edge in E with minimal weight, and add
  it to T
– repeat the previous 2 steps until all vertices
  are in T
```

– from a greedy perspective we are continually adding edges such that we always add a minimum weight edge, as this is the edge that will get us closer to a solution at minimal cost



Prim (as written in your textbook)

```
Prim(G)
  V_T ← {v_0}
  E_T ← ∅
  for i ← 0 to |V|-1 do
      find a min-weight edge e from the set of edges
      {u,v} where v is in V_T and u is in V-V_T
      V_T ← V_T ∪ u
      E_T ← E_T ∪ e
  return E_T
```