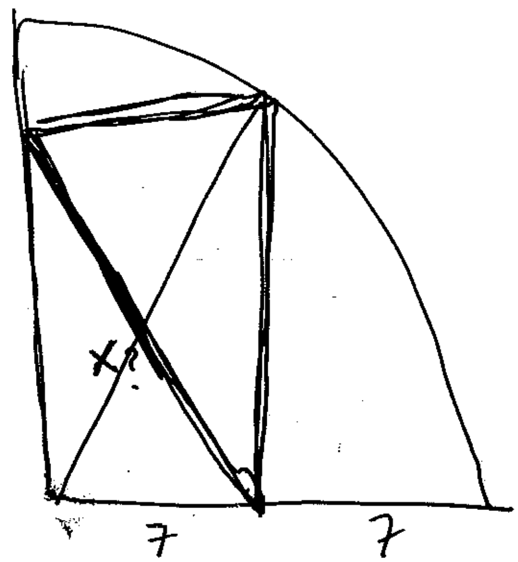
Lesson 10 (9 was Midterm)



1 Minute

x = units?

Chapter 3: Sets

We will not define set / element.

Sets contain elements.

Elements are contained in sets.

Notation: Sets are noted by UPPERCASE letters.

R Z

Z: infinite integers, infinite even integers.

R: infinite reals

Elements noted by lowercase letters

A set is completely defined by its elements.

The order of elements is irrelevant.
The multiplicity of elements is irrelevant. $\Big\}$ irrelevant in defining the set.

Jaymz: shoplifting, fighting, drugs
Aaron: drugs, fighting, drugs, shoplifting $\Big\}$ the same set of 3 elements

Sets are enclosed in {curly braces}

Elements in a set are separated by commas.

① $\{a, b, c\} = \{b, a, c\} = \{b, b, b, b, a, b, b, c, c\}$

② $\{a\} \neq a$

③ Sets can contain other sets

$\{a, \{b, c\}\}$

$\boxed{\text{Quiz}}$ <u>dfn</u>: subset

Set A is a subset of set B iff every element in A is also an element in B.

$$A \underbrace{\subseteq}_{\text{subset}} B \xleftrightarrow[\text{iff}]{} \forall x,\ x \in A \rightarrow x \in B$$

$$A \not\subseteq B \longleftrightarrow \exists x \mid x \in A \wedge x \notin B$$

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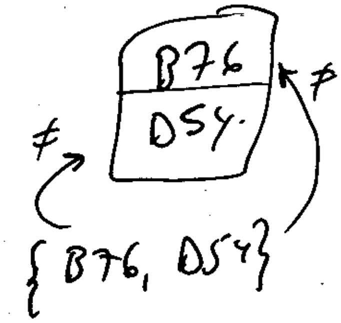
eg Equipment Models:

B76, XR3, D54, ES2, XLS

Let A = {B76, XR3, D54}

Let B = {B76, D54}

Let C = {ES2, XLS}



Q) Is $B \subseteq A$?

A) Yes <u>every</u> element in B is also an element in A

Q) Is $C \subseteq A$?

A) No there exists an element ES2 such that it's in C <u>and</u> <u>not</u> A

Q) Is $B \subseteq B$?

A) Yes every element in B is also an element in B.

Q) Is $B \underset{\text{element}}{\in} B$?

A) No

B76 ≠ {B76, D54}

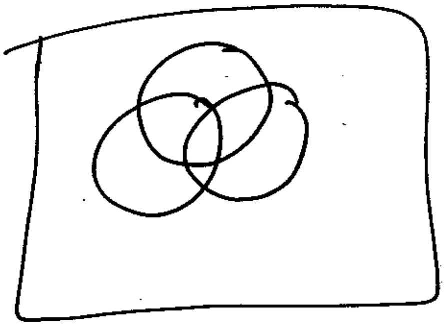
and

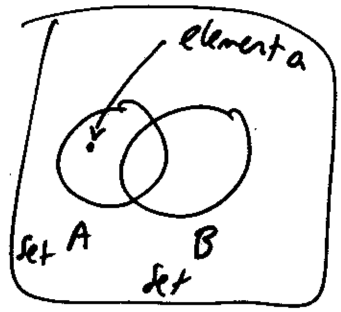
D54 ≠ {B76, D54}

ie there is no "{B76, D54}" in B

Venn Diagrams:

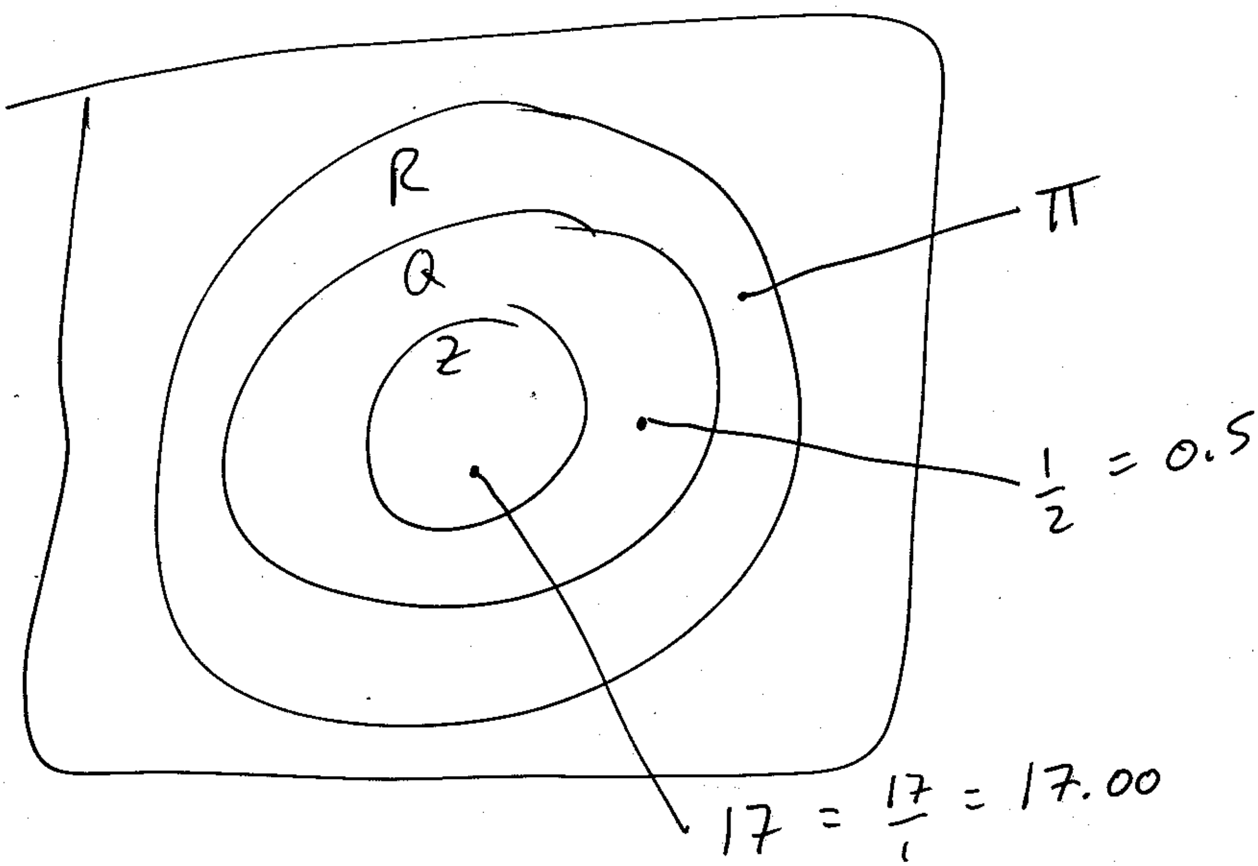
visually useful tools for explaining interactions among 2 or 3 sets.





Impossible to draw symmetric venn diagrams for any nonprime # of sets.

The relation among $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$



T or F !

a) $2 \in \{1, 2, 3\}$? True

b) $\{2\} \in \{1, 2, 3\}$? False
$1 \neq \{2\}$
$2 \neq \{2\}$
$3 \neq \{2\}$

c) $2 \subseteq \{1, 2, 3\}$? False ; 2 is not even a set!

d) $\{2\} \subseteq \{1, 2, 3\}$? True, every element in $\{2\}$ (ie. only "2", is also an element in $\{1, 2, 3\}$ right there

e) $\{2\} \subseteq \{\{1\}, \{2\}, \{3\}\}$? False, $\exists$ an element "2" in $\{2\}$ | "2" is not in

f) $\{2\} \in \{\{1\}, \{2\}, \{3\}\}$?

g) $\{\{2\}\} \subseteq \{\{2\}, \{1\}\}$? True

all the LHS's elements also in RHS

Picking sports teams winners

8 games : 8 winners

	2	4	8	16	32	64	128	256
	HV	HV	HV	HV	HV	HV	HV	HV
game	1	2	3	4	5	6	7	8

256 Combinations

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<u>dfn</u>: set equality

Given sets A and B, $A = B$ iff
every element in A is in B
and
every element in B is in A

$$A = B \leftrightarrow A \subseteq B \land B \subseteq A$$

Which sets are equal!

$A = \{a, b, c, d\}$

$B = \{d, e, a, c\}$

$C = \{d, b, a, c\}$

$D = \{a, a, d, e, c, e\}$

$E = \{B\}$

$A = C$: $A \subseteq C$ $\land C \subseteq A$

$B = D$: $B \subseteq D$ $\land D \subseteq B$

$A = A$

$B = B$

$C = C$

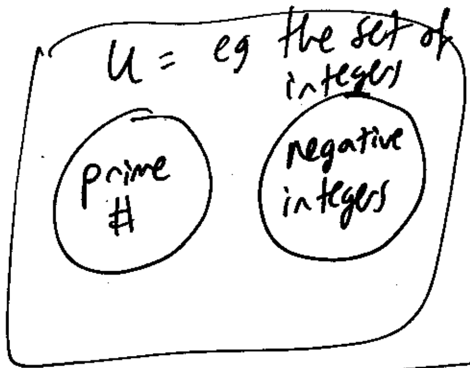
$D = D$

$E = E$

Is $4 = \{4\}$?

No, 4 is not even a set.

The context of a set is called the universal set U:



Kurt Gödel

This statement cannot be proved.

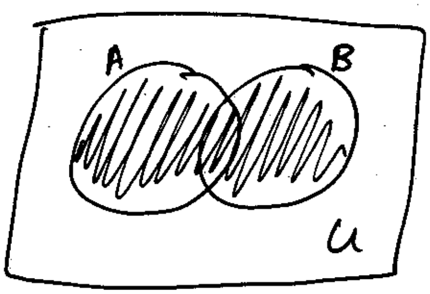
Symbols
producable in every single system

T ↓ true but unprovable

F ↓ contradictions

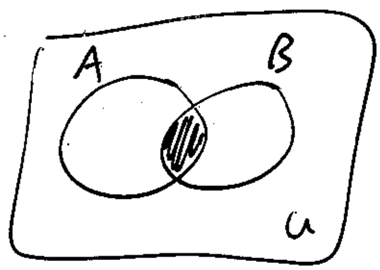
Dfns: Let A and B be subsets of a universal set U.

(1) The UNION of A and B, $A \cup B$, is the set of all elements x in U such that $x \in A$ or $x \in B$ (or both):



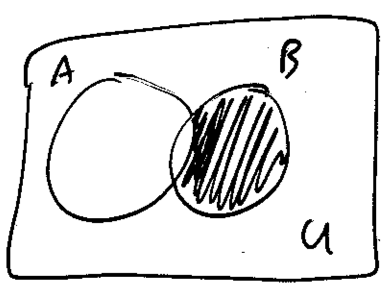
$A \cup B$ is the shaded area

(2) The INTERSECTION of A and B, $A \cap B$, is the set of all elements x in U such that $x \in A$ and $x \in B$.

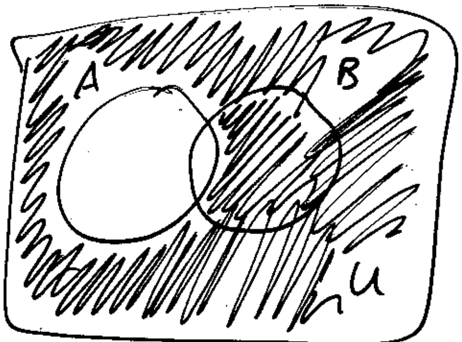


(3) The DIFFERENCE of "B minus A", $B - A$, is the set of all elements x in U such that x is in B and not in A.

ie "Unique to B"



(4) The COMPLEMENT of A, A^c is the set of all elements x in U such that x is not in A:



A^c

The empty set $\emptyset$ is the set with zero elements.
aka $\{\}$

★ There is only one empty set.

Let $U = \{a, b, c, d, e, f, g\}$

Let $A = \{a, c, e, g\}$

Let $B = \{d, e, f, g\}$

Find:

(a) $A \cup B = \{a, c, d, e, f, g\}$

(b) $B \cap A = \{e, g\}$

(c) $B - A = \{d, f\}$

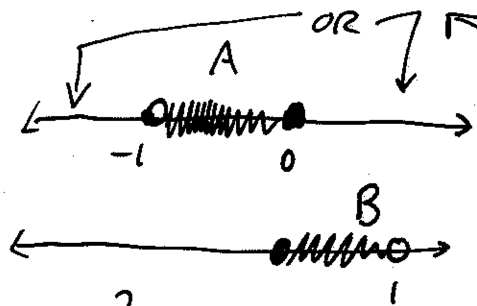
(d) $A - B = \{a, c\}$

(e) $B^c = \{a, b, c\}$

Let U = the set of all Reals

$A = \{x \in R \mid -1 < x \leq 0\}$

$B = \{x \in R \mid 0 \leq x < 1\}$



(a) $A \cup B$? $\{x \in R \mid -1 < x < 1\}$

(b) $A \cap B$? $\{0\}$

(c) A^c? $\{x \in R \mid x <= -1 \text{ or } x > 0\}$

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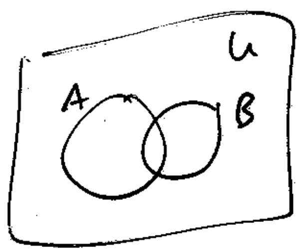
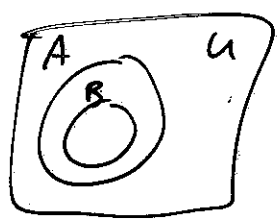
<u>dfn</u>: Two sets are called <u>disjoint</u> iff they have zero elements in common.

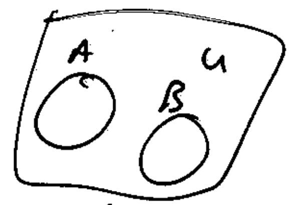
eg. $\left.\begin{array}{l}\text{the set of even numbers}\\ \text{the set of odd numbers}\end{array}\right\}$ disjoint

A and B are disjoint $\longleftrightarrow$ $A \cap B = \emptyset$

empty set {}

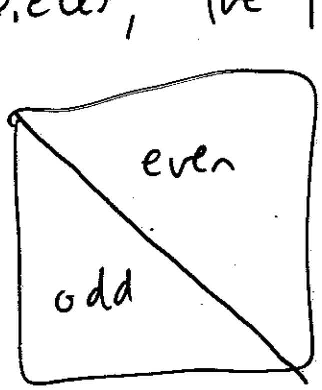




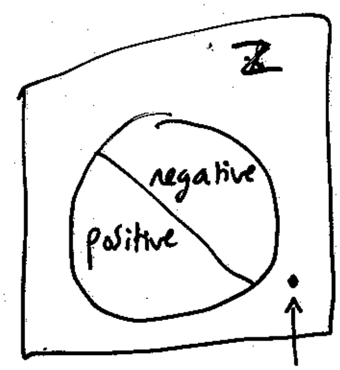


disjoint sets

When a set is <u>completely</u> <u>divided</u> into disjoint pieces, the pieces are called partitions

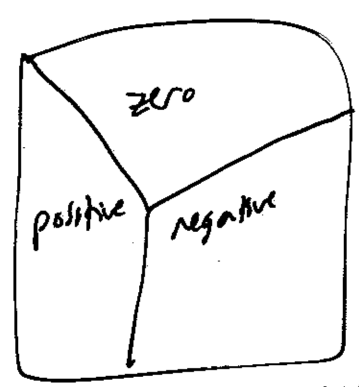


2 partitions
$U = \mathbb{Z}$



Not completely divided

zero



$U = \mathbb{Z}$ 3 partitions