

## CHAPTER 7: CATEGORICAL LOGIC

Though propositional logic can represent valid inferences among propositions, it cannot capture valid inferences involving categories

e.g.

*All bachelors are men.*

*All men are human beings.*

∴ *All bachelors are human beings.*

IMPORTANT: The above cannot be represented with conditionals (though they partially resemble conditionals)

$p \supset q$

$q \supset r$

∴  $p \supset r$  Is not the above argument's form

Why?

$p$ ,  $q$ , and  $r$  are propositional variables; but e.g., Neither “bachelors” nor “men” are propositions—they are categories

Further, the connective “if...then” is not in the original

Using propositional logic, the above can only be portrayed as:

$p$

$q$

∴  $r$

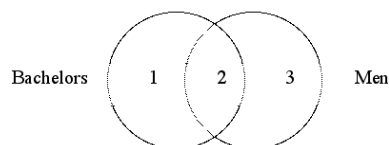
Which is (formally) invalid, though the original argument is clearly valid

The source of the argument's validity is relations among the categories *bachelors*, *men*, and *human beings*

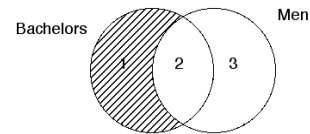
**Categorical propositions:** assert a “quantitative” relation between two categories

e.g., “**All** bachelors are men” says that 100% of bachelors are in the *men* category.

We can represent categories with circles, and categorical relations between two, overlapping circles. (Called *Venn diagrams*)

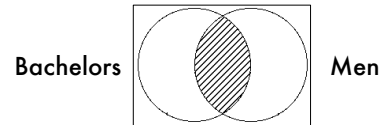


We can represent the fact that all bachelors are men, by shading region 1 to show it is empty



Region 1 contains all those bachelors that are not men =  $\emptyset$

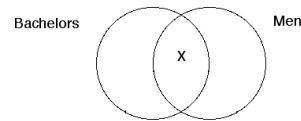
What about: “No bachelors are men”?



Region 2 contains all those bachelors that are men if there are any.

We shade this region to make the claim that there aren’t any male bachelors

“Some bachelors are men”?



The “X” (or “\*”) says that there is at least one bachelor that is a man.

Among logicians, “some” = at least one.

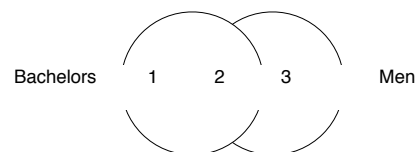
Granting this interpretation and that it is true that some bachelor are men, does this conflict with the claim that all bachelors are men?

e.g. is the following conditional true or false:

If at least one bachelor is a man, then not all bachelors are men?

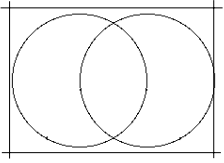
Logicians use the “inclusive some”: If some A are B, then it is possible that all A are B.

“Some bachelors are not men”?

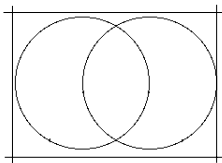




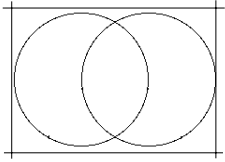
**A: All S is P**



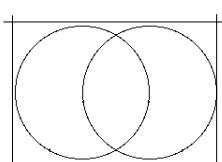
**E: No S is P**



**I: Some S is P**



**O: Some S is not P**



**TRANSLATION INTO BASIC FORMS:** People seldom use the basic categorical forms in everyday speech, though most statements can be translated into the basic form

**Examples:**

Monkeys are primates	
Monkeys can be found in the zoo	
A monkey is a mammal	
A monkey threw a banana at me	
Monkeys are all flea munchers ?	
Every conservative is cautious?	
Anything worth dying for is worth 10 dollars?	
Each person is beautiful in their own way?	
Nobody but a monkey could climb that tree?	
Only fools fall in love?	
The only thing worth living for is pizza?	

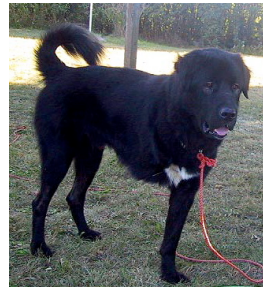
Not all men are mortal?	
Plato is Greek	
There are people without sense.	

**Don't use sentences of the form** "All S are not P," since they are ambiguous between:

- (a) No S are P
- (b) Not all S are P = Some S are not P

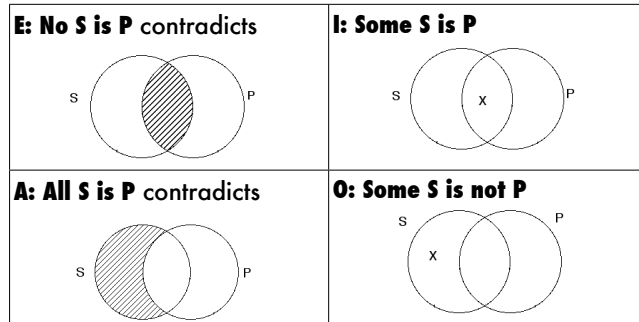
**Example:** All dogs are not four-legged

Does this mean: No dogs are four-legged? or  
Not all dogs are four-legged Some dogs are not  
four-legged?



**CONTRADICTIONS:** A pair of propositions are *contradictories* if and only if

- (a) they cannot both be true, and
- (b) they cannot both be false.



**CONTRARIES:** A pair of propositions are *contraries* if and only if

- (a) they cannot both be true, and
- (b) they can both be false

**EXISTENTIAL COMMITMENT:** If we say, "All lawyers are cunning people" are we implying that lawyers exist? That there are cunning people?

If we claim: "No child of mine will ever join the army" are we necessarily implying that I have a child, or that there is an army?

What about: "Santa is a right jolly elf"

"All things that are Santa are right jolly elves"?

Classical categorical logic assumed that A- and E-propositions had *existential commitment*—that in making them, one asserts the existence of things having both S and P features.



In Classical Categorical logic, E- and A-propositions are contraries, since a classical E-proposition asserts both that something is not an S and not a P; while a classical A-proposition also asserts that something is both an S and a P

So, they can't both be true and can both be false.

Modern categorical logic is based on denying existential commitment to A- and E-propositions

e.g. All A are B = If something is an A, then it's also a B =  $(\forall x) (Ax \supset Bx)$

No A are B = If something is an A, then it's not a B =  $(\forall x) (Ax \supset \sim Bx)$

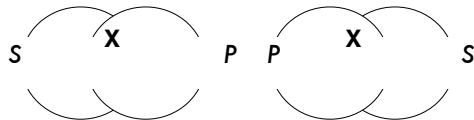
### VALIDITY AND CATEGORICAL PROPOSITIONS

Arguments using categorical propositions are valid if all the information diagrammed in the Venn diagram of the conclusion is already diagrammed in the Venn diagram for the premise e.g.,

Some posties are afraid of dogs.  $\therefore$  Some of those who are afraid of dogs are posties.

Premise: Some S is P

Conclusion: Some P is S



Generally, arguments of the form: Some S are P/ $\therefore$  Some P are S, are valid.

Caution: Though every argument that passes this test is valid, not every valid argument passes the test.

The categorical argument form is (categorically) invalid:

All S is P,  $\therefore$  All P is S.

e.g., All brothers are males,  $\therefore$  All males are brothers" is clearly invalid.

But "All brothers are male siblings,  $\therefore$  All male siblings are brothers" is valid, yet fails the test.

Or we might say that the argument is valid from the *meanings* of “male sibling” and “brother” but is not *categorially* valid

**IMMEDIATE INFERENCES:** are arguments that (a) have exactly one premise, and (b) use only A, E, I and O propositions

**Conversions:** A proposition is *converted* (into its *converse*) ... by reversing the subject and the predicate term of the original proposition

**On fully correct predicate terms:** Strictly, the predicate term should be a plural noun or a plural noun phrase, and not an adjective.

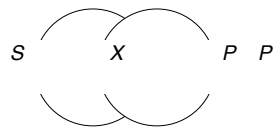
The reason shows up most clearly with conversion

“Some golfers are fit” converts to “Some fit are golfers”????

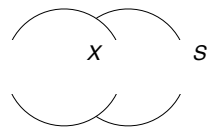
Rather, “Some golfers are fit people” converts to “Some fit people are golfers”

**I AND E PROPOSITIONS ALWAYS VALIDLY CONVERT**

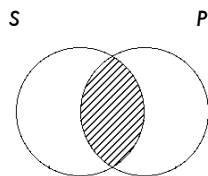
I: Some S is P



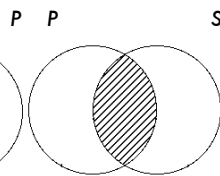
Converse: Some P is S



E: No S is P

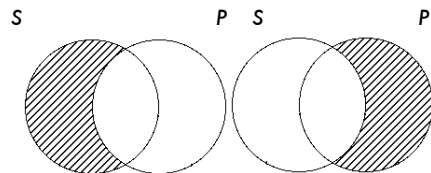


Converse: No P is S



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**Converse: All P is S**

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