COMP 2121

Quiz next ~~Wednesday~~ Monday

Midterm ~~[illegible]~~ ~~[illegible]~~ (~~[illegible]~~)
June 30 Monday

Arguments with Quantified Statements

The rule of "Universal Instantiation"

eg All men are mortal
Socrates is a man
∴ Socrates is mortal

If some property is true of everything in a domain then it is true of any particular thing in the domain.

$\forall x$ in D, $P(x) \equiv P(x_1) \wedge P(x_2) \wedge P(x_3) \ldots$

eg. for all real numbers a and b,
$ab = ba$
∴ By UI, $5 \cdot 4 = 4 \cdot 5$

Oct 6th 1972

Step 1: last 2 year digits

72

J 1	F 4	M 4
A 0	M 2	J 5
J 0	A 3	S 6
O 1	N 4	D 6

Step 2: Start a sum!

how many 12's in 72?

SUM → 6

Sum = 6

Step 3! how many leftover from 72 when I take away my 6 12's?

SUM → 0

Sum = 6

Step 4! how many 4's in that 0?

Sum → 0

Sum = 6

Sum Step 5! add the 6th →

Sum = 12

sum Step 6! add October #! 1

Sum = 13

Step 7! mod by 7

13 % 7 = 6

= Friday

Sa	Su	M	T	W	Th	F
0	1	2	3	4	5	6

Universal Modus Ponens:

$\forall x, \quad P(x) \rightarrow Q(x)$

$P(a)$ for a particular element a in X

$\therefore Q(a)$ (is true)

If a # is even then its square is even.

K is a particular number that is even

$\therefore k^2$ is even.

Universal Modus Tollens: used in proofs by contradiction

$\forall x, \quad P(x) \rightarrow Q(x)$

$\sim Q(a)$ for a particular a in X

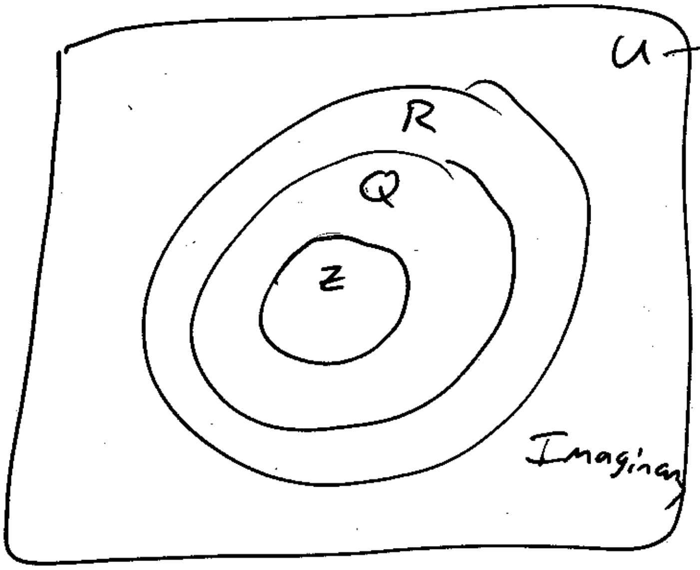
$\therefore$ by UMT, $\sim P(a)$

All humans are mortal

Zeus is not mortal

$\therefore$ Zeus is not human

Venn diagrams

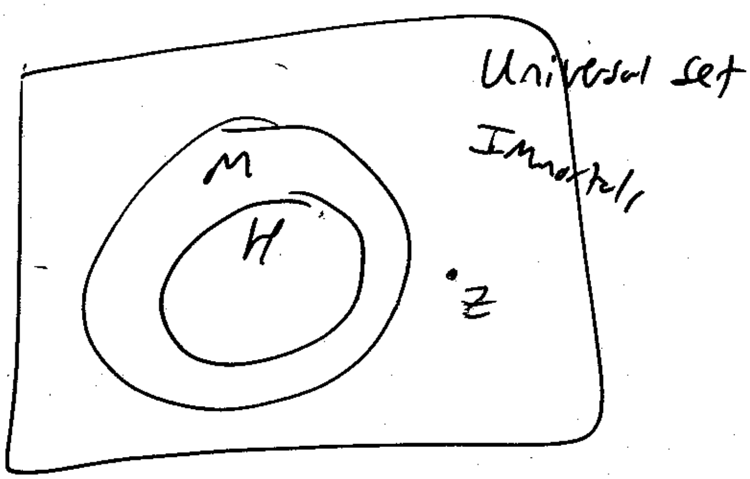


set of numbers

Draw : integers Z
reals R
rationals Q

Draw this argument. Is it valid.

All humans are mortal.
Zeus is not mortal
∴ Zeus is not human.

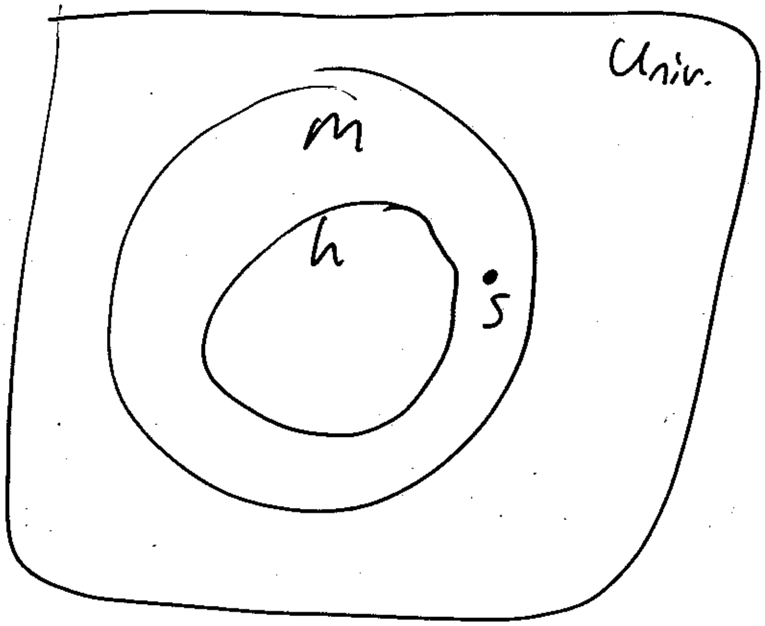


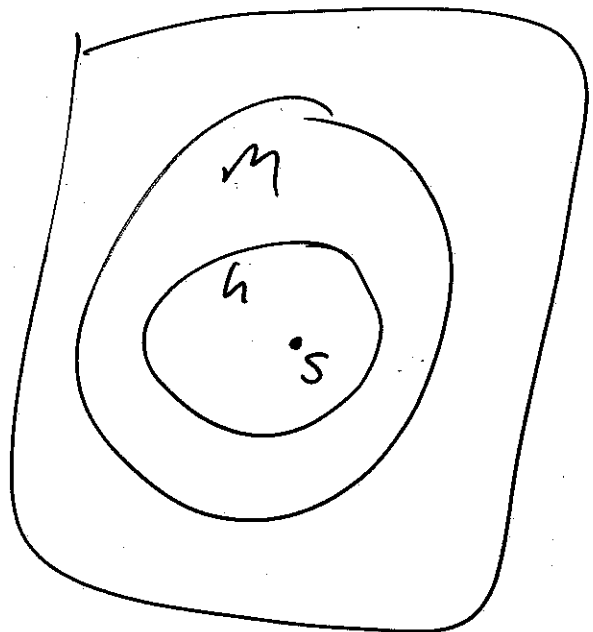
Draw. Valid!

All humans are mortal.

Shaun is mortal.

∴ Shaun is human





not human, maybe

human

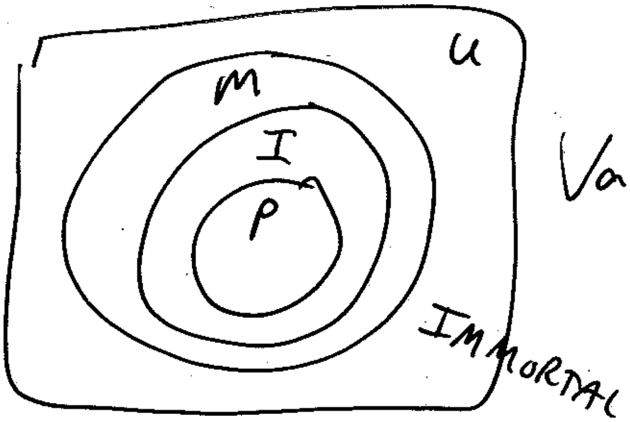
dunno. Invalid.

Converse Error

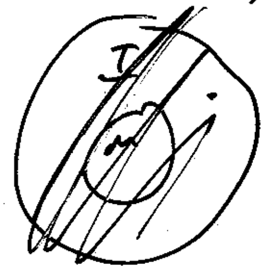
All people are idiots.
All idiots are mortal.
∴ All people are mortal

Draw. this argument, a Venn diagram and determine if it's valid

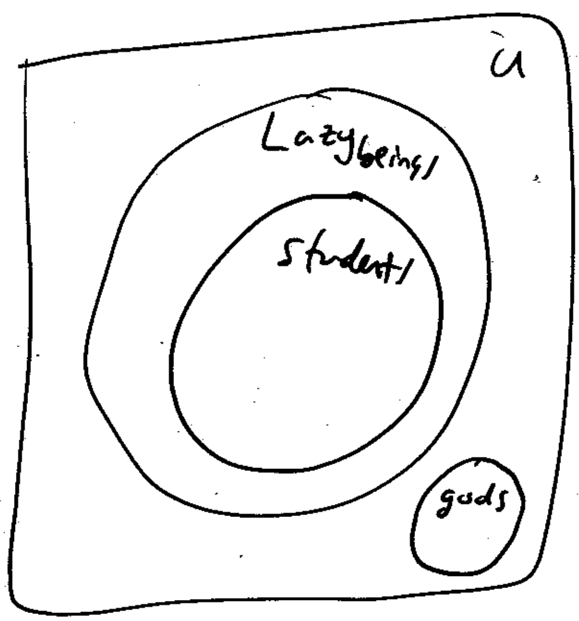
Valid?



Valid



All students are lazy
No gods are lazy
∴ No students are gods.



Valid

Lesson 6 of 16

dfn: Even

An integer n is even if and only if
n equals two times some integer.

$$n \text{ is even} \longleftrightarrow \exists \text{ an integer } K \mid n = 2K$$

Q] Is 18 even? Why/why not?
A] Yes, $\exists$ integer $9 \mid 18 = 2 \cdot 9$

Q] Is 1.8 even?
A] No not even an integer

Q] Is 0 even?
A] Yes, $\exists$ integer $0 \mid 0 = 2 \cdot 0$

Quiz

Q] Is -4 even?
A] Yes, $\exists$ integer $-2 \mid -4 = 2 \cdot -2$

Q] Is 7 even?
A] No, $\forall$ integers K, $2k \neq 7$

Q/ If x and y are integers,

Is $10x + 8y + 1$ even?

A/ No.

It has the form of $2\boxed{int} + 1$

and not even $2\boxed{int}$

☆ the product of integers is an integer

the sum of integers is an integer

the difference of integers is an integer

} rules

Dfn odd:

An integer n is odd if and only if

n equals two times some integer plus one.

n is odd $\longleftrightarrow$ $\exists$ an integer k | $n = 2k+1$

Q/ Is 63 odd?

A/ $\exists$ an integer 31 | $63 = 31 \cdot 2 + 1$

Q/ Is 40 odd?

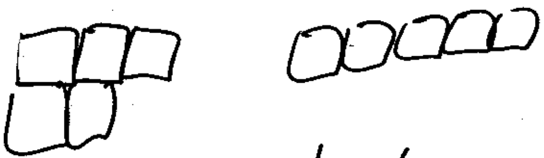
A/ If 40 is odd then $40 = 2k + 1$ (some int)

$2k = 39$

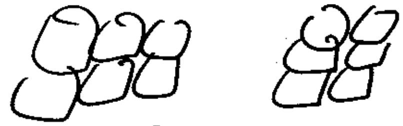
$k = 19.5$ ← Not an int.: not odd

<u>dfn</u>: prime

2, 3, 5, 7, 11, 13, ...



no rectangle = prime



Rectangle = not prime

An integer n is prime if and only if:

(1) $n > 1$ and

(2) for all positive integers r and s,
if $n = r \cdot s$
then $r = 1$ or $s = 1$

Q/ Is 5 prime?

A/ Yes!

(1) $5 > 1$

(2) $5 = 1 \cdot 5$ ✓

$5 = 5 \cdot 1$ ✓

$\therefore$ prime

Q/ Is 6 prime?

A/ No!

(1) $6 > 1$

(2) $6 = 6 \cdot 1$ ✓

$6 = 1 \cdot 6$ ✓

$6 = 2 \cdot 3$ ✗

Quiz! Know dfns.

1 is not prime. (by dfn)

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dfn: Composite

An integer n is composite if and only if

(1) $n > 1$

and

(2) $n = r \cdot s$ for some positive integers r and s, with $r \neq 1$ and $s \neq 1$

n is composite $\longleftrightarrow$ $\exists$ positive integers r and s | $n = r \cdot s$ and $r \neq 1$ and $s \neq 1$

Q| Is 45 composite?

A| Yes

$45 > 1$

and

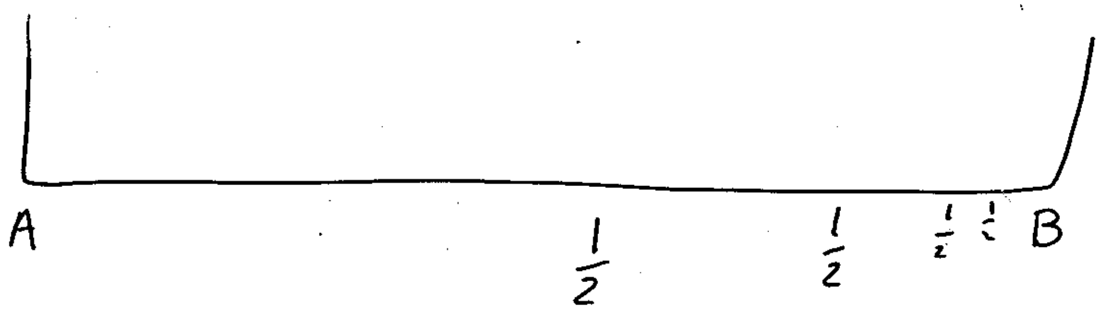
$45 = 9 \cdot 5$ | $9 \neq 1$ and $5 \neq 1$

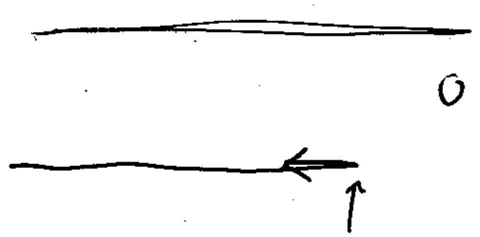
$x^4 + y^4 + z^4 = w^4$ No Solution

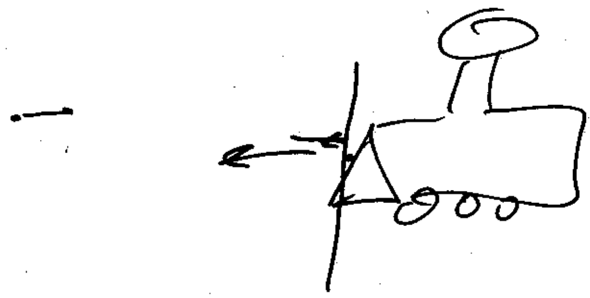
No Proof

$95800^4 + 217519^4 + 414560^4 = 422481^4$

Zeno's paradox







$$(1+1)-(1+1)-(1+1)-1\cdots-\infty$$

$$1+(1-1)+(1-1)+(1-1)\ldots=1$$

Proofs:

Prove existential statements.

$\exists x$ in $D \mid Q(x)$

~~Two~~ techniques:

"At least one swan is black"

1. Constructive proof:
   a - supply an x (that swan is black)
   b - supply an algorithm to find x

2. Nonconstructive proof:
   a - show that the existence of x is guaranteed by some ~~[illegible]~~ theorem.
   b - show that a contradiction arises if there is no solution.

Q) $\exists$ an even integer x that can be written in two different ways as a sum of two primes. Prove it.

A) $14 = 11 + 3$
and
$14 = 7 + 7$

Proving Universal Statements

Technique #1: for finite-sized sets, use the method of exhaustion.

eg $\forall n \in \mathbb{Z}$, if n is even and $4 \leq n \leq 12$ then n can be written as the sum of two primes.

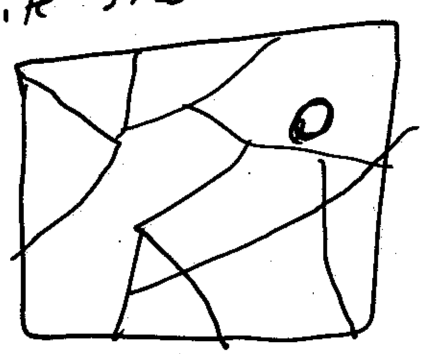
Universal Conditional Statement: the most important statement form in all of mathematics!

Proof
$$\begin{aligned} 4 &= 2+2 \\ 6 &= 3+3 \\ 8 &= 3+5 \\ 10 &= 5+5 \\ 12 &= 5+7 \end{aligned}$$

$\forall x$ in D, $P(x) \to Q(x)$

Consider infinite-sized set!

4 color problem



Quiz covers today's lesson only.

Multiply-quantified statements

All people enjoy at least one type of music.

Form: $\forall x$ in P, $\exists y$ in M | $E(x,y)$

x ↓ individual person; P ↓ people set; y ↓ a type of music; M ↓ music set; $E(x,y)$ — x enjoys y

Some people enjoy all types of music.

$\exists x$ in P | $\forall y$ in M, $E(x,y)$

Everyone hates someone.

Someone hates everyone.

$\exists x \in P \mid \forall y \in P,\ H(x,y)$

$\forall x \in P,\ \exists y \in P \mid H(x,y)$

use different variable letters

Symbols

All quizzable.