Midterm: Two weeks from today's class

The exam will cover everything up to and including Chapter 6 of Fogelin

PROPOSITIONAL LOGIC: Explaining validity

So far, we have defined a valid argument as one in which, if the premises are all true, the conclusion must also be true

> **Less formally**: A valid argument is one in which its conclusion follows from its premises

It's been fairly easy to recognize the validity of most of the valid arguments we've examined thus far

But we've had to rely on our "logical intuition" to guide us in determining validity

Logicians have come to recognize that validity (often) depends on *argument patterns:* The way in which propositions or categories are connected within the argument determines whether the argument is valid or invalid

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Example:

1. Either I'll buy the chesterfield or the fridge.
2. <u>I won't buy the chesterfield.</u>

∴ 3. ?

What follows validly from the premises?

Why?

Suppose we replace "I'll buy the chesterfield" and "I'll buy the fridge" with different sentences; will the argument still be valid?

P or Q.
<u>Not P.</u>
∴ ?

The reason seems to be that "or" and "not" have certain, general logical properties:

(i) They connect propositions (indicative sentences, statements) to produce *compound* propositions;

(ii) The truth of these compounds depends on the truth of the component propositions and the properties of the so-called "logical vocabulary"—e.g. "and" "or" "not" "if …, then ______"

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(iii) the original (uncompounded) propositions are called *atomic* propositions

Conjunction: We can compound (*conjoin*) two propositions using the (*propositional*) conjunction "and"

e.g., Yusuf and Yasmine are wise = Yusuf is wise and Yasmine is wise.

Truth conditions: The *truth conditions* for a logical connective, such as "and," "or," and "not," tell us when propositions compounded with that connective are true, given ***only*** the truth of the original, atomic propositions

Truth tables are often used to give *truth conditions*

Yusuf is wise	Yasmine is wise	Yusuf is wise and Yasmine is wise.
T	T	*T*
T	F	*F*
F	T	*F*
F	F	*F*

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When (this sort of) "and" is used to connect propositions, it doesn't matter which propositions we use, the truth table is the same

Propositional logic is *formal* when the logical relations among propositions are independent of the specific content of those propositions, but depend only on the properties of the logical connectives and the truth values of the component (atomic) propositions

We can make logical form clearer by replacing specific propositions with *propositional variables*: *p, q, r, s, …*

We can also replace "and" with the symbol "&"

The *propositional form* of a conjunction: *p* & *q*

Does "*p* & *q*" represent an actual proposition?

Substitution instances of propositional forms: We start with a propositional form and then replace the variables with propositions

Examples: propositional form: *p* & *q*

Substitution instances: Sid is vicious and Clark is mild-mannered.

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Jeff and Mutt are going to the park.

Marge is content and Marge is content.

Jenny and Jack are married.

Caution: "and" is not always used to connect two content-independent sentences

When "and" is not propositional, we can't symbolize "P and Q" by P & Q

The universal propositional variable: *p*

Any proposition (no matter how complex or compounded), is a substitution instance of *p*

Substitution rule: Different propositional variables may be replaced with the same proposition, but different propositions may not be replaced with the same variable.

e.g., Are any of the following propositions substitution instances of *p & p*?

John is a happy camper and George hates the outdoors.

George is of the jungle and George is of the jungle.

Marge likes transfats.

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Truth table for *p & q:* under each atomic proposition variable, we list all its truth possibilities, so that all possible combinations are covered:

p	*q*	*p & q*
T	T	T
T	F	F
F	T	F
F	F	F

"&" is like mathematical operators (+, -, ×, ÷) in that it can be used to conjoin any number of propositions: p & q & r & s & t & … & t_{25}

We use parentheses "(", ")", "[", "]", "{", "}" etc. to group propositions, so that it is clear which two propositions are connected by "&"

e.g., {[(p & q) & (r & s)] & t} **&** p_1 (*main connective* **bolded)**

Nonpropositional conjunctions: If a sentence using "and" doesn't express a conjunction of two independent propositions, the "and" is used *nonpropositionally*

1. A Catholic priest married John and Mary.
2. Fred had pie and ice cream for dessert.
3. Susan got married and had a child.
4. Sam and Samantha are friends.

Are there contexts above where the other "and" would make sense?

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VALIDITY FOR ARGUMENTS BASED ON CONJUNCTION

1. Chlorine is an element *and* water is a compound

∴ 2. Chlorine is an element

The form of this argument (called *Simplification*)

p & q

∴ *p*

How many substitution instances are there of this argument form?

Any argument with this form will be valid, because its validity depends only on “&”

Truth table for above argument

		premise	*conclusion*
p	*q*	*p & q*	*p*
T	T	T	*T*
T	F	F	*T*
F	T	F	*F*
F	F	F	*F*

Notice that the only time the premise is true, the conclusion is also true; that is, we never get a true premise and a false conclusion

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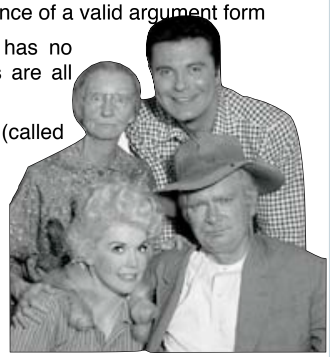
An *argument is valid* if it is a substitution instance of a valid argument form



An *argument form is valid* if and only if it has no substitution instances in which the premises are all true and the conclusion is false.

Another valid argument form using “&” (called “*Conjunction*”)

p

q .

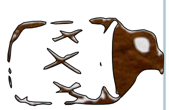
∴ *p* & *q*

Why is this form valid?

DISJUNCTION: Compound propositions using “or”

Jed or Granny drank the moonshine =

Jed drank the moonshine or Granny drank the moonshine.



Some truth conditions: Clearly if only one of these propositions are true, the whole thing is also true; and it’s also clear that if *neither* of the two propositions are true, then the combination is false.

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Suppose both disjuncts are true, is the disjunction itself true?

(a) You can have mashed potatoes or fries?

(b) If you ever want a good party, you could go to either Jim’s or Janet’s place?

Two theories of “or”:

(a) there are two meanings: an *inclusive* sense in which the compound (disjunction) is true when both components (disjuncts) are true; an *exclusive* sense, in which the disjunction is false when both disjuncts are true.

(b) there is only the inclusive sense, but sometimes context (e.g., a restaurant menu) “conversationally implies” that both options cannot be true

Truth tables for both kinds of “or”:

		Exclusive	**Inclusive**
p	*q*	*p* or *q*	*p* or *q*
T	T	F	T
T	F	T	T
F	T	T	T
F	F	F	F

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Our practice: we will use the inclusive "or," since logical tradition favours it (and Grice's explanation of the exclusive case is plausible)

Truth table for inclusive "or": *p* ∨ *q*

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

NEGATION: We can also create new propositions with the concept "not," which (on our usage) changes (negates) the truth value of the original

e.g., "John is married" negated becomes "It is not the case that John is married" or "John is not married"

Negating "Mary is not the Queen of Scots anymore" yields "It is not the case that Mary is not the Queen of Scots anymore" = "Mary is the Queen of Scots"

Truth table for negation (symbol "~")

p	$\sim p$	$\sim\sim p$
T	F	T
F	T	F

Translating sentences with "not" can be tricky, because poor placement of the "not" can make the meaning ambiguous

"Jack did not marry Jill because of her pail of water"

Did Jack marry Jill?

Some sentences don't have "not" in them, but are still negations of other propositions

e.g., "Nobody knows the trouble I've seen" is the negation of which proposition?

"Nothing works better than Anacin"

DISJUNCTIVE SYLLOGISM (PROCESS OF ELIMINATION)

Argumentative passage: Either Kelly Ellard murdered Reena Virk or Kelly Ellard is innocent of murder, but guilty of aggravated assault. (Choose your negation). So Kelly Ellard is …

The order of the disjuncts is irrelevant

$p \vee q$
$\underline{\sim p\quad}$
$\therefore q$ All arguments having this pattern (disjunctive syllogism) are valid

AFFIRMING A DISJUNCT:

$p \vee q$
$\underline{p\quad\ }$
$\therefore \sim q$

Why is this argument form *not* valid?

HOW TRUTH-FUNCTIONAL CONNECTIVES WORK

Truth-functional connectives & truth functions: The compound propositions we build using "and," "not," and "or" (i.e., "&," "~" and "v"), depend only on the *truth values* of the component propositions and *not* on the *meanings* of the particular propositions in the argument

We say that "&," "~" and "v" are *truth-functional connectives*, and the truth values of the compound propositions are *truth functions* of those of the original propositions

Examples: Suppose "A" and "B" represent true propositions, but "D" and "E" are false propositions; we should be able to determine the truth of any compound of these propositions, formed only by using "&," "~" and "v"

A = T, ***B*** = T, ***D*** = F, ***E*** = F

1. A ∨ D
2. E ∨ D
3. ~A & B
4. ~(A & B)
5. A & B
6. ~(E & D)
7. ~(A ∨ B)

DEMORGAN'S RULES: ~A & B ≠ ~(A & B)

~(A & B) means that it's not the case that both of A and B are true; in other words, at least one of A or B is false; that is,

DeMorgan's rule 1: ~(A & B) = ~A ∨ ~B

~(A ∨ B) means that neither A nor B are true

in other words, none of them are true, or both are false

DeMorgan's rule 2: ~(A ∨ B) = ~A & ~B

Analysing complex expressions: First assign truth values to the simplest (atomic) propositions, then their next simplest compound, and so on. (Recall: A = B = True; D = E = False)

~[~(A ∨ ~ B) & ~ (B ∨ ~ E)]

TESTING FOR VALIDITY

Truth tables are a general method for testing the validity of arguments

Here's an argument:

1. Michael Moore is either fair-minded or a Bush-hater
2. Michael Moore is neither a Bush-hater nor a Republican.

∴ 3. Michael Moore is fair-minded.

Abbreviations:

F = Michael Moore is fair-minded.
B = Michael Moore is a Bush hater.
R = Michael Moore is a Republican.

Argument symbolized:

F ∨ B	p ∨ q
~(B ∨ R)	~(q ∨ r)
∴ F	∴ p

			PR		PR	CN	
p	*q*	*r*	*p ∨ q*	*q ∨ r*	*~(q ∨ r)*	*p*	
T	T	T	T	T	F	T	
T	T	F	T	T	F	T	
T	F	T	T	T	F	T	
T	F	F	*T*	*F*	*T*	*T*	OK
F	T	T	T	T	F	F	
F	T	F	T	T	F	F	
F	F	T	F	T	F	F	
F	F	F	F	F	T	F	

We've shown the argument to be valid, because the only truth table line with all true premises, has a true conclusion

That is, there is no case where the premise are true, and the conclusion false. (Steps p. 159 of text)

Short truth-table method: Start by making the conclusion false, and see whether you can also make all the premises true.

PR	PR	CN
p v q	*~(q v r)*	*p*
FT ?	***T ?( )***	***F***

Another argument:

1. We're either going to finish this course or we won't graduate.
2. Either we'll graduate or we will live with our parents.

∴ 3. We're going to finish this course.

Abbreviations:

F = "We're going to finish this course"

G = "We will graduate"

P = "We will live with our parents"

Argument symbolized:

F v ~G　　　p v ~q
G v P　　　q v r
∴ F　　　∴ p

p	*q*	*r*	*~q*	PR *p v ~q*	PR *q v r*	CN *p*	
T	T	T	F	T	T	T	OK
T	T	F	F	T	T	T	OK
T	F	T	T	T	T	T	OK
T	F	F	T	T	F	T	
F	T	T	F	F	T	F	
F	T	F	F	F	T	F	
F	***F***	***T***	***T***	***T***	***T***	***F***	***invalid***
F	F	F	T	T	F	F	

We've shown the argument to be invalid (*row 7*)

Short method

PR	PR	CN
p v ~q	*q v r*	*p*
F T ??	***?T?***	***F***

TRUTH-FUNCTIONAL EQUIVALENCE: Two propositional forms are *truth-functionally equivalent* if they have the same truth value for every truth value assignment to their component variables

DeMorgan's Theorems:

~(p v q) ≡ ~p & ~q

p	*q*	*~p*	*~q*	*p v q*	~(p v q)	~p & ~q
T	T					
T	F					
F	T					
F	F					

~(p & q) ≡ ~p v ~q

p	*q*	*~p*	*~q*	*p & q*	~(p & q)	~p v ~q
T	T					
T	F					
F	T					
F	F					

Translating into propositional logic: e.g., "Not both *p* and *q*" can be equivalently translated as either, ~(p & q) or ~p v ~q

But some translations are more natural (or simpler) than others

e.g., "Either it's raining or it's not raining" is truth-functionally equivalent to "It's not the case that it's both not raining and raining"

i.e., (*p* v ~*p*) ≡ ~(~*p* & *p*)

What about the truth conditions for "Either it's raining or it's not raining"?

TESTING VALIDITY WITH TRUTH TABLES

$\underline{p \vee q}$
$\therefore\ (p \,\&\, r) \vee (q \,\&\, r)$

Standard Truth-table Method

			PR			CN	
p	*q*	*r*	*p* v *q*	*p* & *r*	*q* & *r*	(*p* & *r*) v (*q* & *r*)	
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

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Short Truth-table Method

PR	CN
p v *q*	(*p* & *r*) v (*q* & *r*)
T ***T*** T	T F F ***F*** T F F

EXERCISES

I	p146
III 1, 2	p146
IV ##5-9	p146-7
V ##4-6	p149
VIII ##5-8	p152
IX. ##5-7	p153
XII, ##8-11, 15	p156
XV, ## 6 - 9	p160

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