## LIBS 7008 Logic and Practical Reasoning Practice Final Exam

(1) Using the truth-functional connectives "&," "∨," "~," "⊃" or "≡," give a well-formed symbolization in propositional logic that ***best*** represents the literal meaning of each of the following statements. Bring out as much of the logical form as possible. Use the symbols given to you.

(i) [Jack wo***n't*** go up the hill = ~H], ***unless*** (= if it's not the case that) [Jill wants to fetch a pail of water = F], ***but*** (&) either [Jill does***n't*** want to fetch a pail of water = ~F] ***or*** (∨) [Jack has broken his crown = C].

H = Jack will go up the hill.
F = Jill wants to fetch a pail of water.
C = Jack has broken his crown.

**Answer:** (~F ⊃ ~H) & (~F ∨ C) or (~H ∨ F) & (~F ∨ C) or (H ⊃ F) & (F ⊃ C)

(ii) [You should do your tasks = Y] ***only if*** (⊃) [you do your tasks with all your might = P]; ***or*** **[**you should be a couch potato = D].

Y = You should do your tasks.
P = You do your tasks with all your might.
D = You should be a couch potato.

**Answer: Note:** "otherwise" means "unless" = "if not" = "or else" The "or" version is simplest.
(Y ⊃ P) ∨ D
Using "unless" as "if not": ~D ⊃ (Y ⊃ P) = ~D ⊃ (~Y ∨ P)

(2) Using the (long or short) truth-table method, determine whether each of the following is a valid argument. For your convenience, I've put in the long truth-table grid. Explain your answers.

(i) P ∨ (Q & R) .
∴ (P ∨ Q) & (P ∨ R)

| | | | | PR | | | CN | |
|---|---|---|---|---|---|---|---|---|
| P | Q | R | Q & R | P ∨ (Q & R) | P ∨ Q | P ∨ R | (P ∨ Q) & (P ∨ R) | |
| T | T | T | T | T | T | T | T | **OK** |
| T | T | F | F | T | T | T | T | **OK** |
| T | F | T | F | T | T | T | T | **OK** |
| T | F | F | F | T | T | T | T | **OK** |
| F | T | T | T | T | T | T | T | **OK** |
| F | T | F | F | F | T | F | F | |
| F | F | T | F | F | F | T | F | |
| F | F | F | F | F | F | F | F | |

The argument is *valid* because there is no case where the premise is true, but the conclusion is false. (Incidentally, the premise and conclusion are *logically equivalent*, since they have the same truth tables.)

(3) If the truth table for a propositional form is all "F"s, then the propositional form is that of a contradiction. You can easily show that the proposition "A & ~A" is a contradiction, while "~(A & ~A)" is not a contradiction. Is the following proposition a contradiction?

([P ∨ ( S & ~Q )] ⊃ R) & ~([P ∨ ( S & ~Q )] ⊃ R)

Circle one: Yes No

**(Bonus: Explain your choice):** The above proposition is a substitution instance of the contradictory propositional form: *p* & *~p*, where "([P ∨ ( S & ~Q )] ⊃ R)" substitutes into the propositional form, *p* and "~([P ∨ ( S & ~Q )] ⊃ R)" is an instance of the propositional form, *~p*.

(4) Express each of the following statements in standard (A, E, I, O) categorical form:

(i) More than 90 per cent of people are right handed.

| |
|---|
| **Answer:** Some people are right-handed beings. |

(ii) No one is a philosopher unless they love logic.

| |
|---|
| **Answer:** All philosophers are beings who love logic. |

(iii) Only the brave eat at Mickey D's.

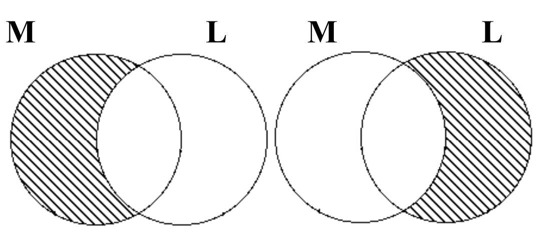
| |
|---|
| **Answer:** All those who eat at Mickey D's are brave creatures. |

(iv) Not all of my friends are homo sapiens.

| |
|---|
| **Answer:** Some friends of mine are ***not*** homo sapiens. |

(5) (i) Determine the converse of the following proposition. (ii) Then ***label*** and use the provided Venn diagrams to prove whether each is valid or invalid.

(i) The converse of "All monotremes (M) are mammals (L)" is: "All mammals (L) are monotremes (M)

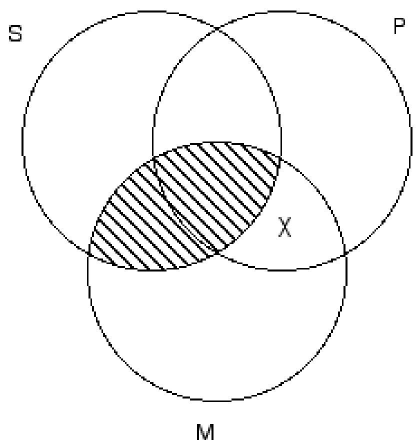


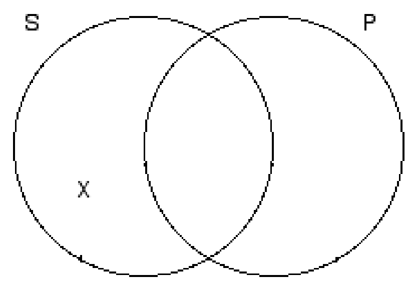
Is this inference valid or invalid (Why)? *The argument is **invalid**, since diagramming the information in the premise (first categorical proposition) **doesn't** diagram the information in the conclusion (converse of first proposition)*

(6) For the following arguments, construct a Venn diagram of the premises. Based on your Venn diagram, indicate whether the argument is valid or invalid.

(i). Some ***legumes*** (P) are ***peanuts*** (M).
No ***peanuts*** (M) are ***nuts*** (S).
∴ Some ***nuts*** (S) are not ***legumes*** (P).

**S** = nuts
**P** = legumes
**M** = peanuts




Valid or Invalid (Why)? ***Invalid****, because the information in the conclusion is not diagrammed by the premises.*

(7) Fill in the blanks with A, B, C, or D. Choose the one <u>best</u> answer.

A= Valid deductive arguments
B= Strong inductive arguments
C= Both of the above
D= Neither of the above

1. **<u>C (both)</u>** may have true conclusions.

2. **<u>C (both)</u>** may have false premises and a true conclusion.

3. **<u>B</u>** may be strengthened or weakened by the addition of new premises.

(8) Briefly ***explain two*** of the criteria for determining when one explanation is better than another as an inference to the best explanation. Then give a brief example of an IBE that fails both of these criteria.

| |
|---|
| On the exam, ***I will choose*** the two criteria (of seven); you will explain them, then provide some fact or set of facts that needs explaining, and then provide an explanation that violates both of the given criteria. You must also explain *how* your explanation violates the criteria. |

(i) (1st criterion) ______________________________

______________________________

(ii) (2nd) ______________________________

______________________________

(iii) (Example) ______________________________

______________________________

(9) Briefly explain ***two*** of the criteria for determining when an argument from analogy is stronger. Then give a brief example of an analogical argument that fails both of these criteria.

| |
|---|
| On the exam, ***I will choose*** two (of four) criteria; you will explain them, then provide an argument from analogy that violates both of the given criteria (***explain*** how your argument violates these criteria.) |

(i) (1st criterion)

______________________________

(ii) (2nd)

______________________________

(iii) (Example) ______________________________

______________________________

(10) In each of the following cases, put the appropriate letter in the blank to indicate whether:

A= X is the cause of Y.
B= Y is the cause of X.
C= There is a common cause of both X and Y.
D= The correlation is accidental.

***Then*** briefly explain your choice.

_B/C__ 1. During December, people's stress level gets very high (X) and people do their Christmas shopping (Y).

You might argue that Christmas shopping (especially for men) creates high stress levels (Y causes X); *probably better*: you might argue that the Christmas period is a third factor (common causes) which causes people to shop for presents and also increases people's stress levels.

_D__ 2. The world's population is increasing (X) and people in North America are getting fatter (Y).

The correlation is accidental, since there is no causal relation between X and Y. In fact, the correlation is *prima facie* puzzling, since you'd think that a greater population would mean less available food.

(11) Using the four rules for probability given in the text, calculate the probabilities for each of the following. Assume an ordinary deck with 52 cards that has been well-shuffled. (An ordinary deck has 13 different cards: Ace, Two, Three, Four, Five, Six, Seven, Eight, Nine, Ten, Jack, Queen, and King. Each card has four "suits": hearts, diamonds, spades, and clubs.) Show the setup for your calculation. Leave your answer in fractional terms (no need to convert to decimals).

(i). What is the probability of drawing at least one face card (Jack, Queen, or King) in a series of four consecutive draws, when the card drawn is not returned to the deck.

**Use Rule 4 (without independence)**: Pr(at least one J or Q or K on 4 draws) = 1 — Pr(No J, Q or K on 4 draws) = 1 — [40/52 × 39/51 × 38/50 × 37/49]

**Note:** Since there are 12 Jacks, Queens or Kings in a deck of 52 cards, there are 40/52 Non Jacks, Queens or Kings.

(ii). What is the probability of drawing a Jack of diamonds and then drawing a five of hearts, when the first card is not put back in the deck before the second draw?

Since the first draw affects the probability of the second draw, the events are not independent, and we need to use rule 2G: Pr(J♦ & 5♥) = Pr(J♦) × Pr(5♥/ J♦) = 1/52 × 1/51

(12) What is the name of the intuitive probability rule which determines the probability of an event by referring to an accessible set of known probabilities?

**Answer:** the *Availability heuristic*

(13)Put a check next to the right answer:

To see whether your continuing weight gain is associated with holiday eating, you compare your caloric intake in December to your daily bathroom scale readings. This is an example of using:

____ (A) The Necessary Condition Test
____ (B) The Sufficient Condition Test
_√_ (C) Concomitant Variation Test.

Recall that concomitant variation occurs when two quantities vary together or are ***correlated***, especially when both of them are always present in some degree. Here, *weight gain* and *caloric intake* are taken to vary together.

(14) Suppose that there are two little lotteries in town, each of which sells exactly 200 tickets.

(a) If each lottery has only one winning ticket, and you buy two tickets to the *same* lottery, what is the probability that you will have at least one winning ticket?

There are two ways to do this, since "at least one" is equivalent to "or" (the inclusive form). We can use "or" here, since there are only two draws. We can also use Rule 3 (not 3G), because both tickets can't win the *same* lottery.

**Rule 3:** $Pr(T_{1wins} \vee T_{2wins}) = Pr(T_{1wins}) + Pr(T_{2wins}) = 1/200 + 1/200 = 1/100$

**Rule 4:** $1- Pr(T_{1loses}\ \&\ T_{2loses}) = 1 - Pr(T_{1loses}) \times Pr(T_{2loses}/T_{1loses}) = 1 - (199/200 \times 198/199) =$

$1 - 39{,}402/39{,}800 = 39{,}800 - 39{,}402/39{,}800 = 398/39800 = 1/100$

(b) If each lottery has three winning tickets, and you buy one ticket to each of the two lotteries, what is the probability that you will have *one and only one* winning ticket?

**Answer:** Two permutations will give you exactly one winning ticket. You might win on the first lottery, but lose on the second. ***Or*** you might lose on the first, but win on the second lottery.

Use Rule 3, since each permutation is mutually exclusive:
$Pr[(W_{1st}\ \&\ {\sim}W_{2nd}) \vee ({\sim}W_{1st}\ \&\ W_{2nd}) = [Pr(W_{1st}) \times Pr({\sim}W_{2nd})] + [Pr({\sim}W_{1st}) \times Pr(W_{2nd})] =$
$[3/200 \times 197/200] + [197/200 \times 3/200] = 591/40{,}000 + 591/40{,}000 = 1182/40{,}000$

(15)You are presented with three bags: two contain a chicken-fat sandwich and one contains a cheese sandwich. You are asked to guess which bag contains the cheese sandwich. You do so, and the bag you have selected is set aside. (You obviously have one chance in three of guessing correctly.) From the two remaining bags, one containing a chicken-fat sandwich is then removed. You are now given the opportunity to switch your selection to the remaining bag. Will such a switch increase, decrease, or leave unaffected your chances of correctly selecting the bag with the cheese sandwich in it? Explain your answer.

***This question will be on your final exam.***

(16) **Bayes's Theorem Questions. Exercise VIII**, p 297-8

See the answers to the first five problems for Exercise Set #11. I'm also including the answer to question #7 of Exercise VI.

(7) Let $h$ = there is a high level of radon in your basement.
Let $e$ = the inexpensive test comes out positive.
Pr ($h$) = 0.2, so Pr ($\sim h$) = 0.8. Pr ($e|h$) = 0.8, and Pr ($e|\sim h$) = 0.1.
So:

$$\Pr(h|e) = \frac{(0.2 \text{ X } 0.8)}{(0.2 \text{ X } 0.8)+(0.8 \text{ X } 0.1)} = \frac{0.16}{0.16+0.08} = \frac{2}{3} = 0.666$$

| | High Radon | Not High Radon | Total |
|---|---|---|---|
| Tests Positive | 160 | 80 | 240 |
| Do Not Test Positive | 40 | 720 | 760 |
| Total | 200 | 800 | 1000 |

(17) **Revisit statistical generalization questions from Exercise II** (p224-5)

Please see my answers to the questions we did.

(18) **Revisit statistical application questions from Exercise IV (p. 228)**

Please see my answers to the questions we did.

## Appendix

**Probability Rules:**

**RULE 1:** The probability that an event will not occur is 1 minus the probability that it will occur; or, Pr (not $h$) = 1 - Pr($h$)

What's the probability of *not* tossing a "6", using one die?

**RULE 2:** given two *independent* events, the probability of their both occurring is the product of their individual probabilities.

$$\Pr(h_1 \ \& \ h_2) = \Pr(h_1) \times \Pr(h_2)$$

**RULE 2G.** Given two events, the probability of their both occurring is the probability of the first times the probability of the second, given that the first has occurred.

$$\Pr(h_1 \ \& \ h_2) = \Pr(h_1) \times \Pr(h_2/h_1)$$

**RULE 3**: The probability that at least one of two mutually exclusive events will occur is the sum of the probabilities that each of them will occur.

$$\Pr(h_1 \text{ or } h_2) = \Pr(h_1) + \Pr(h_2)$$

**RULE 4:** The probability that an event will occur at least once in a series of $n$ independent trials is simply 1 minus the probability that it will *not* occur in that number of trials.
$1 - \Pr(\text{not } h)^n$

$$\text{Bayes's Theorem}: \Pr(h/e) = \frac{\Pr(h) \times \Pr(e/h)}{[\Pr(h) \times \Pr(e/h)] + [\Pr(\neg h) \times \Pr(e/\neg h)]}$$