On your bike you came to a 5km long hill. You pedal up in one hour. How fast must you ride down so your average speed for the whole trip is 10 km/hour?

Summations again.

Q/ Write the following sequence using summation notation:

$$n + (n-1) + (n-2) + (n-3) + \ldots + 1.$$

A/ $\sum_{?}^{?} (n - ?)$

FIRST change all terms so they have the same form:

$$(n-0) + (n-1) + (n-2) + (n-3) + \ldots + (n-(n-1))$$

$$\boxed{\sum_{k=0}^{n-1} n-k}$$

last

first

$n - ? = 1$

$\boxed{n-1 = ?}$

☆ Separate off the final term:

$$\sum_{k=1}^{5} k = \boxed{\sum_{k=1}^{4} k} + 5$$

$$\sum_{k=0}^{5} (k^2-1) = \sum_{k=0}^{4} (k^2-1) + (5^2-1)$$

Q) Rewrite $\sum_{k=0}^{n-1} (n-k)$ by separating off the final term:

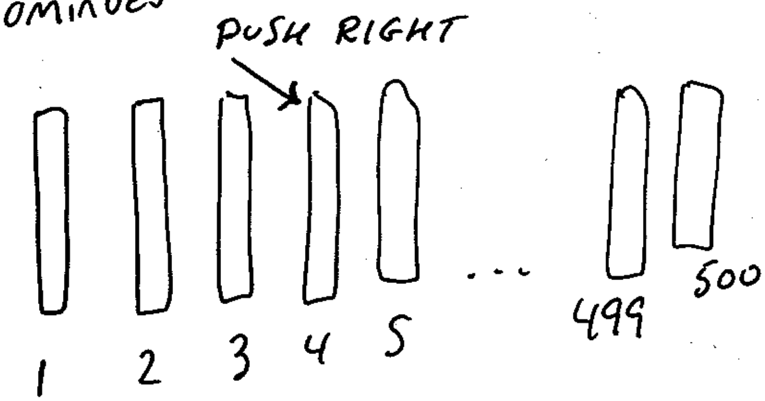
A) $\sum_{k=0}^{n-2} (n-k) + 1$

Mathematical Induction

Method of proof for UCS

Principle of MI:

dominoes



4 pushes over 5
5 pushes over 6
6 pushes over 7
⋮
499 pushes over 500

if ① SUPPOSE domino number K falls over
then ② SHOW K+1 will fall over (guaranteed)
③ SHOW that domino number 4 fell over.

Conclude dominoes 4 – 500 fell over.

Method of Proof by M.I.

To prove a statement of the form:

"$\forall$ integers $n \geq a$, a property $P(n)$ is true"

eg "four" — eg "Fell over"

our first case / minimum case

Step 1: Basis step / Base case
Show that the property is true for $n = a$

Step 2 Inductive Step.
Show that for all integers $k \geq a$,
(step 2a) IF the property is true for $n = k$ (if 11 fell over)
(step 2b) Then it is true for $n = k+1$. (then 12 fell over)

Inductive hypothesis:

$\boxed{2a}$ Suppose that the property is true for $n = k$ (k is a pbac integer $\geq a$)

$\boxed{2b}$ Show the property is true for $n = k+1$.

algebra

Q) Prove via MI that

$= P(n) \left\{ 1 + 2 + 3 + 4 + \ldots + n = \frac{n(n+1)}{2} \right.$ $\quad n \geq 1$, $n \in \mathbb{Z}$

A) Step 1: basis step

Show that $P(1)$ is true

(substitute 1 for n in p(n))

$1 = \frac{(1)(1+1)}{2}$

$1 = \frac{1(2)}{2}$ ✓ true ∴ continue

Step 2a: Suppose $p(k)$ is true $\longrightarrow$ pbac integer ≥ 1

(substitute k for n in p(n))

$1 + 2 + 3 + 4 + \ldots + k = \frac{k(k+1)}{2}$

Step 2b: show $P(k+1)$ is true

(substitute k+1 for n in p(n))

$1 + 2 + 3 + 4 + \ldots + (k+1) = \frac{(k+1)((k+1)+1)}{2}$

k is 2nd-last

$1 + 2 + 3 + 4 + \ldots + k + (k+1) = \frac{(k+1)(k+2)}{2}$

show 2nd last term

$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2}$

⋮

$\vdots$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

$$\frac{k^2+k+2k+2}{2} = \frac{k^2+3k+2}{2} \quad //$$

LHS = RHS

Step 2b yields a TRUE statement $\therefore$ $P(n)$ is true.

Q] Use MI to prove $\forall$ integers $n \geq 1$

$$\left.1 + 6 + 11 + 16 + 21 + \ldots + (5n-4) = \frac{n(5n-3)}{2}\right\} = P(n)$$

step 0

A] Step 1 - basis step

Plug in 1 for n into $P(n)$:

$$(5n-4) = \frac{n(5n-3)}{2}$$

$$(5 \cdot 1 - 4) = \frac{1(5 \cdot 1 - 3)}{2}$$

$$1 = \frac{2}{2}$$ ✓ true $\therefore$ continue

Step 2a Plug in k for n into $P(n)$

Suppose $P(k)$ is true, k is a pbac int ≥ 1

$$1 + 6 + 11 + 16 + 21 + \ldots + (5k-4) = \frac{k(5k-3)}{2}$$

Step 2b Plug in $(k+1)$ for n into $P(n)$

$$1 + 6 + 11 + 16 + 21 + \ldots + (5(k+1)-4) = \frac{(k+1)(5(k+1)-3)}{2}$$

Separate off final term:

$$\underbrace{1 + 6 + 11 + 16 + 21 + \ldots + (5k-4)} + (5(k+1)-4) = \frac{(k+1)(5k+5-3)}{2}$$

$$\frac{k(5k-3)}{2} + 5(k+1) - 4 = \frac{(k+1)(5k+2)}{2}$$

$$\frac{5k^2-3k}{2} + \frac{2 \cdot (5(k+1)-4)}{2} = \frac{(k+1)(5k+2)}{2}$$

$$\frac{5k^2-3k + 2(5k+5-4)}{2} = \frac{(k+1)(5k+2)}{2}$$

$$5k^2 - \frac{3k + 10k + 2}{2} = \frac{5k^2 + 7k + 2}{2}$$

LHS = RHS $\therefore$ $P(n)$ is true//

Q) Prove via MI!

$\forall$ integers $n \geq 0$, $\left.1 + 2 + 2^2 + 2^3 + 2^4 + \ldots + 2^n = 2^{n+1} - 1\right\} = P(n)$

A) Step 1: basis step
Show that $P(0)$ is true!

$$2^0 = 2^{0+1} - 1$$

$$1 = 2 - 1 \quad \text{true} \therefore \text{continue}$$

Step 2a: Suppose $P(k)$ is true ← a pbac integer ≥ 0

$$1 + 2 + 2^2 + 2^3 + 2^4 + \ldots + 2^k = 2^{k+1} - 1$$

Step 2b: Show $P(k+1)$ is true

$$\underbrace{1 + 2 + 2^2 + 2^3 + 2^4 + \ldots}_{=} + 2^{(k+1)} = 2^{(k+1)+1} - 1$$

$$2^{k+1} - 1 + 2^{k+1} = 2^{k+2} - 1$$

$$2^1 \cdot (2^{k+1}) - 1 = 2^{k+2} - 1$$

$$2^{k+1+1} - 1 = 2^{k+2} - 1$$

$$2^{k+2} - 1 = 2^{k+2} - 1$$

LHS = RHS $\therefore$ $P(n)$ is true //

Q] Prove via MI!

$$1 + 3 + 5 + \ldots + (2n-1) = n^2 \quad \forall \text{ Integers } n \geq 1 = P(n)$$

A] Show $P(1)$ is true

$$2(1) - 1 = 1^2$$

$$2 - 1 = 1 \quad \text{true} \therefore \text{Continue}$$

Suppose $P(k)$ is true! k is a pbac int ≥ 1

$$1 + 3 + 5 + \ldots + (2k-1) = k^2$$

Show $P(k+1)$ is true:

$$1 + 3 + 5 + \ldots + (2(k+1)-1) = (k+1)^2$$

$$k^2 + 2(k+1) - 1 = (k+1)^2$$

$$k^2 + 2k + 2 - 1 = (k+1)(k+1)$$

$$k^2 + 2k + 1 = k^2 + 2k + 1$$

LHS = RHS $\therefore$ $P(n)$ is true //

Q/ Use MI to prove:

$\forall$ integers $n \geq 1$,

$2 + 4 + 6 + 8 + 10 + \ldots + 2n = n^2 + n \Big\} = P(n)$

A/ Step 1: basis step:

<u>Show</u> P(1) is true:

$2(1) = (1)^2 + 1$

$2 = 1 + 1$ true $\therefore$ continue

<u>Suppose</u> P(k) is true: k is a pbac int ≥ 1

$2 + 4 + 6 + 8 + \ldots + 2k = k^2 + k$

<u>Show</u> P(k+1) is true:

$2 + 4 + 6 + 8 + \ldots + 2(k+1) = (k+1)^2 + (k+1)$

$k^2 + k + 2(k+1) = k^2 + 2k + 1 + k + 1$

$k^2 + k + 2k + 2 = k^2 + 3k + 2$

LHS = RHS $\therefore$ P(n) is true

Q) Use MI to prove

$$1+2+3+\ldots+n = \frac{n^2+n+2}{2} \quad \forall \text{ integer } n \geq 1 \quad = P(n)$$

-11-/-1

Step 1: basis step

Show $P(1)$ is true

$$1 = \frac{1^2+1+2}{2}$$

$$1 = \frac{4}{2}$$

$$1 = 2$$

false $\therefore$ $P(n)$ is false

$\exists$ an integer $1 \mid 1 \geq 1$ and $P(1)$ is false

//

Midterm:

2 hours

Includes every thing we've learned

bring a calculator - probably won't help.

Questions very similar to what we've done in class.

- 3 MI
- definitions