COMP 3761 Quiz 4 Solution

Student ID: Name:

1. Describe the basic steps of **Mergesort**. [**3 marks**]
   Refer to Lecture4 slides or textbook Section 4.1.

2. Name one difference between **Mergesort** and **Quicksort**. [**1 mark**]
   - Mergesort divides the input's elements based on their position, Quicksort divides its input's elements based on their value.
   - Mergesort uses a merging process to combine two sorted arrays into one sorted array; Quicksort uses a pivot to partition one array into two subarrays.
   - Mergesort has the worst-case time efficiency of $\Theta(n \log n)$; Quicksort has the worst-case time efficiency of $\Theta(n^2)$.
   - Mergesort requires extra space to store the new constructed array; Quicksort is in place.
   - Mergesort divides the array into two about same size of subarrays, where the subarray sizes of Quicksort depends on the input array.

3. What is the time efficiency class of **Quicksort** in the worst case? [**1 mark**]
   $$T_{worst}(n) \in \Theta(n^2)$$

4. a. Write a pseudocode for a divide-and-conquer algorithm to solve the exponential problem of computing a^n where $a > 0$ and n is a positive integer. [**3 marks**]
   The following divide-and-conquer algorithm for computing a^n is based on the formula $a^n = a^{\lfloor n/2 \rfloor} * a^{\lceil n/2 \rceil}$.
   Algorithm $DivConqPower(a, n)$
   //Input: A positive number a and a positive integer n
   //Output: The value of a^n
   if $n = 1$ return a
   else return $DivConqPower(a, \lfloor n/2 \rfloor) * DivConqPower(a, \lceil n/2 \rceil)$.

   b. Set up a recurrence relation for the number of multiplications made by your algorithm in a. [**2 marks**]

   $$M(n) = M(\lfloor n/2 \rfloor) + M(\lceil n/2 \rceil) + 1 \quad \text{for} \quad n > 1, \quad M(1) = 0.$$