Lesson 4 of 16 Comp 2121

Two bicycle tracks in the dirt.

Q/: Which was the bicycle headed?

Martingale system

Go to Vegas

Goal:
I want to
win $100

Bet $100 on roulette

If I win I get 2x my bet.

Scenario: bet 100
lose } lost 100 total

bet 200
lose } lost 300 total

win
+100

bet 400
lose } lost 700 total

win
+100

✡ Assignment 1 is due Monday June 23.

Logic of quantified statements

quantified → amount : some, none, one, all

All men are mortal
Socrates is a man
∴ Socrates is mortal

$\left.\begin{array}{l}\text{All men are mortal}\\ \text{Socrates is a man}\end{array}\right\}$ prove it

$\underbrace{\text{Aaron}}_{\text{subject}}\ \underbrace{\text{is a citizen of Canada}}_{\text{predicate (describes the subject)}}$

$\left.\begin{array}{l}\overset{}{C}(p)\end{array}\right\}$ person p is a citizen of Canada

C → "is a citizen of Canada"

p → person variable

More abstract:

$\underbrace{\text{Aaron}}_{\text{subject}}\ \underbrace{\text{is a citizen of}}_{\text{predicate}}\ \underbrace{\text{Canada}}_{\text{noun}}$

form: $C(x,y)$: x is a citizen of y

$C(\text{aaron}, \text{Canada})$ TRUE

$C(\text{aaron}, \text{china})$ FALSE

$P(x) = x^2 > x$ for all real numbers x

predicate variable

predicate

dfn: domain: contains the set of all values that can be substituted into predicate variables

$P(1)$ false $1^2 \not> 1$

$p(2)$ true $2^2 > 2$

Is $P(x)$ true? No $P(x)$ is not true since it's not true for all real numbers x.

☆ We must define the domain every time we state a predicate.

Q/ $Q(x) =$ "$x^3 = x$" for all real numbers. T/F?

A/ False because $2^3 \neq 2$

Q/ $Q(x) =$ "$x^3 = x$" domain of x is $\{-1, 0, 1\}$

A/ True because $(-1)^3 = -1$
and $0^3 = 0$
and $1^3 = 1$

Same statement can be T or F depending on the domain. Quantified statements are not complete unless the domain is stated.

Domain syntax uses set syntax.

Set names: use an uppercase letter

eg let H = the set of all Humans

Elements are contained in sets.

Element names use a lowercase letter

eg let h = Aaron

There are some reserved set letters:

R = - the set of all real numbers
- all #s on numberline
- π, 0, -2, $-\frac{3}{4}$, 5000000, 20.1, $\sqrt{2}$

Z = - the set of all integers
- whole #s
- 0, -2, 5000000, 20

Q = - the set of all rational numbers
- quotients of integers
- $\frac{1}{2}$, 0, 500, $\frac{17}{31}$, 0.333...

Notation: Superscript

Z^+ positive Integers $\{1,2,3,\dots\}$

Z^{nonneg} $\{0,1,2,3,\dots\}$

Z^- negative Integers $\{\dots, -2, -1\}$

T/F: All X are green. X is the set of all alligators.

False: albino alligators are white

Let $G(n)$: "n is a factor of 8" } domain of n is Z^+

$G(1)$ is true
$G(2)$ is true
$G(3)$ is false STOP

$G(n)$ is false for Z^+

dfn: truth set of a predicate

The set of all such elements that make the predicate true.

$G(n) =$ "n is a factor of 8" domain is $\mathbb{Z}^+$

Truth set $= \{1, 2, 4, 8\}$

such that

$\{ x \in D \mid P(x) \}$

P of x is true

set D

the set of

all variables x

- that are elements of
- in
- in the set
- in the domain
- that are members of

Q) Find truth set for $G(n) =$ "n is a factor of 8", $n \in \mathbb{Z}$

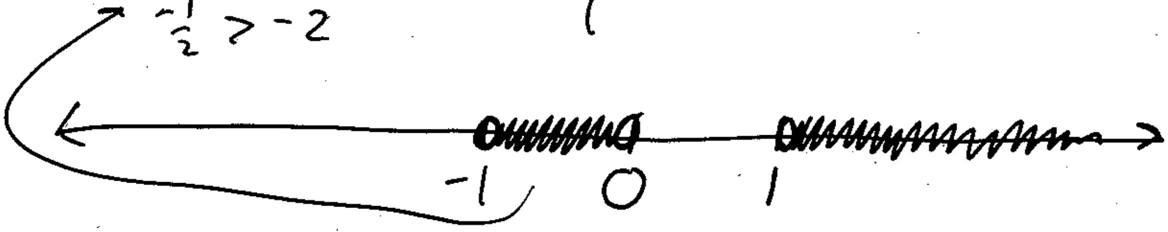
A) $\{-8, -4, -2, -1, 1, 2, 4, 8\}$

Q) Find truth set for $P(x) =$ "$x > \frac{1}{x}$" $x \in \mathbb{R}$

A) $-\frac{1}{2} > \frac{1}{(-\frac{1}{2})}$

$-\frac{1}{2} > -2$

$\{-1 < x < 0$ OR $x > 1\}$



The Universal Quantifier $\forall$

"for all"

Try these

Handouts:
Page 42
#36-40
are very
good
questions.

Review next day

$\forall x \in D, Q(x)$

For all x in D, Q of x is true.

For all particular swans in the set of all swans, that swan is white.

-or- All swans are white.

① Universal statements are true iff:
$Q(x)$ is true for every x in D.

② Universal statements are false iff:
$Q(x)$ is false for at least one x in D.

Negation of "all swans are white" is
"at least one swan is not white"

$\forall x \in H$, x is mortal $\equiv$ "all humans are mortal"

$\downarrow$

the set of humans

Notation: $\forall$ real numbers x and y, $x+y = y+x$

$\downarrow$

the "for all" applies to both x and y.

Q/: T/F?

$\forall x \in D,\ x^2 > x$ where $D = \{5, 4, 3, 2, 1\}$

$\uparrow$ Counterexample.

A/ False, $1^2 \not> 1$ STOP

Q/ T/F? $\forall x \in D,\ x^2 \geq x$

where $D = \{5, 4, 3, 2, 1\}$

A/ True, $5^2 > 5 \wedge 4^2 > 4 \wedge 3^2 > 3 \wedge 2^2 > 2 \wedge 1^2 >$

method of exhaustion

The Existential Quantifier $\exists$

"there exists"

$\exists x \in D \mid Q(x)$

true iff $Q(x)$ is true for at least 1 x in D.

"There exists a swan that is white"

False iff $Q(x)$ is false for every x in D

Q: "is white"

x: a particular swan

Q) T/F? Prove it: $\exists m \in \mathbb{Z} \mid m^2 = m$

$\mid$ such that

A) True: $1^2 = 1$ stop

Q) T/F? Prove it: $\exists m \in E \mid m^2 = m$ where $E = \{2, 3, 4, 5\}$

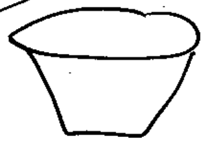
A) False:

$2^2 \neq 2$

$\wedge\ 3^2 \neq 3$

$\wedge\ 4^2 \neq 4$

$\wedge\ 5^2 \neq 5$



all the balls in the bowl are red.

TRUE,

to be false, $\exists$ a ball in the bowl $\mid$ it is not red.

Translate into English:

$\forall x \in R, x^2 \geq 0$

The squares of reals are ≥ 0

LAB: pg 86 - 10/10

☆ Try: #1, 4-8,
☆ 10, 12, 15-18
25, 26 ☆

Translate into symbols:

"No dogs have wings"

$\forall d \in D, W(d)$

D ↓ the set of all dogs

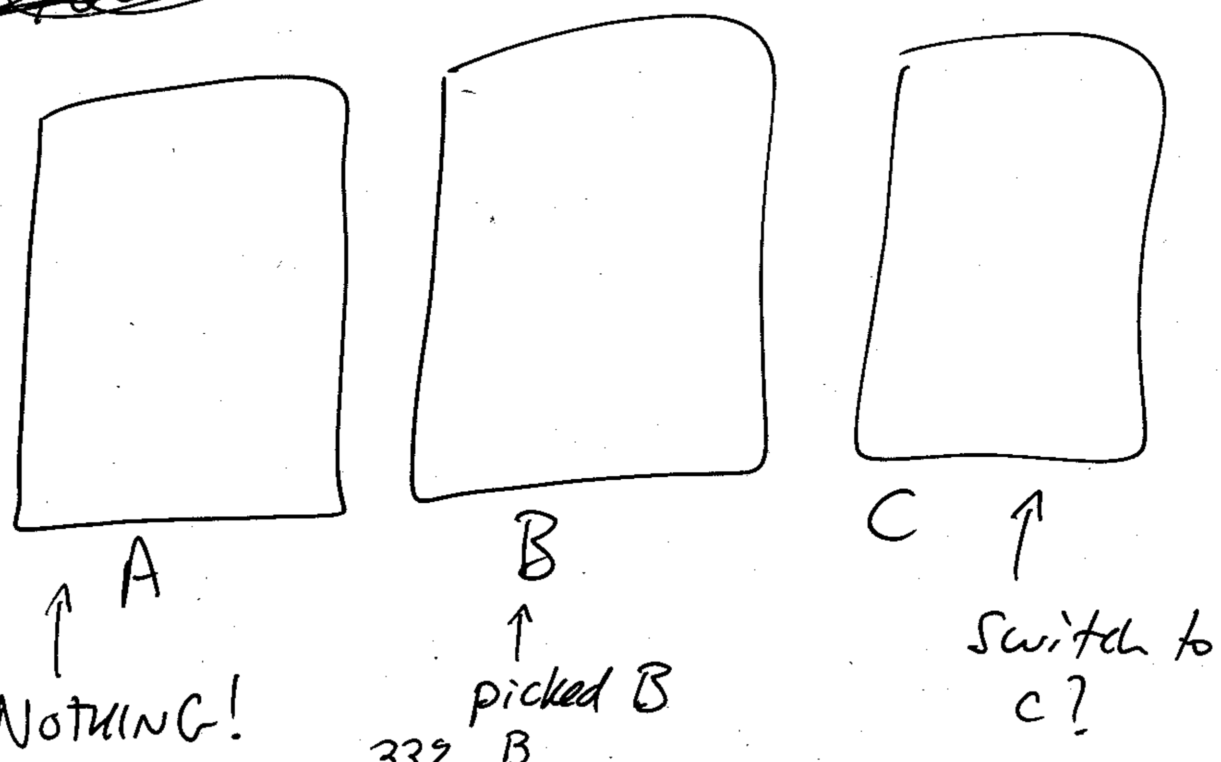
W(d) → "is wingless"

pg 85
blue: a c e g
black: b h j
gray: d f i k

[redacted]

Monty Hall Problem

1 prize
2 nothings



33% B
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