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# Topology optimization of heat exchangers: A review

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ABSTRACT

The critical importance of heat exchangers (HXs) on energy systems has been widely recognized, which can largely determine the overall efficiency. With the rise of the topology optimization (TO) and additive manufacturing techniques, it is anticipated that the TO may became a leading optimization technique in designing HX structures for the heat transfer intensification. Various algorithms for TO of HXs are dispersed in the literature, while a comprehensive and comparative review on their features, advantages, disadvantages, and limitations, is still lacking. Therefore, this paper aims at filling the literature gap by providing a comprehensive state-of-the art review on the TO for HXs over the past decades, so as to indicate the most promising technology roadmap. Each stage of the TO, i.e. the design parametrization, the heat transfer modeling, the optimization, and the final realization, is analyzed carefully in the corresponding section, highlighting the major pros, cons and challenges. Our statistics demonstrate that the current TO, though well-developed and fast improved, still have numerous limitations in handling the industrial HXs that hold the complicate structures and flow patterns. Eventually, three emerging schemes, i.e. machine learning, model order reduction, and moving morphable components, aimed to improve the efficiency of TO are also discussed.



**Contents**



*Abbreviations:* AI, Artificial intelligence.

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**Nomenclature**

| | |
|---|---|
| $\gamma$ | Densities (Design variables) |
| $\alpha$ | Inverse permeability |
| $\lambda$ | Physical properties |
| $\varnothing$ | Level Set function |
| $q$ | Penalization factor |
| $v$ | Velocity (m/s) |
| $T$ | Temperature (K) |
| $\rho_f$ | Fluid density (kg/m$^3$) |
| $\rho_s$ | Solid density (kg/m$^3$) |
| $C_{pf}$ | Specific heat capacity for fluid (J.kg$^{-1}$.K$^{-1}$) |
| $C_{ps}$ | Specific heat capacity for solid (J.kg$^{-1}$.K$^{-1}$) |
| $\mu_f$ | Fluid dynamic viscosity (Pa.s) |
| $P$ | Pressure (Pa) |
| $f$ | Fictious force |
| $Q$ | Heat transfer rate (W) |
| $U$ | Overall heat transfer coefficient (W.m$^{-2}$.K$^{-1}$) |
| $\Delta T_m$ | Mean temperature difference (K) |
| $t$ | Time (s) |
| $S$ | Source Term (W) |
| $k_s$ | Solid thermal conductivity (W.m$^{-1}$.K$^{-1}$) |
| $k_f$ | Fluid thermal conductivity (W.m$^{-1}$.K$^{-1}$) |
| $A$ | Heat transfer surface area (m$^2$) |
| $X$ | Position vector |

*Abbreviations*

| | |
|---|---|
| *TO* | Topology Optimization |
| *HX* | Heat Exchanger |
| *AM* | Additive Manufacturing |
| *RAMP* | Rational Approximation of Material Properties |
| *SIMP* | Solid Isotropic Material with Penalization |
| NS | Navier-Stokes |
| *CM* | Conventional Manufacturing |
| *LBM* | Lattice Boltzmann Method |
| *XFEM* | Extended Finite Element Method |
| *VOF* | Volume of Fluid method |
| *BCM* | Building Cube Method |
| *GA* | Genetic Algorithm |
| *RANS* | Reynolds Averaged Navier Stokes |
| *MMA* | Method of Moving Asymptotes |
| *GCMMA* | Globally Convergent MMA |
| *SQL* | Sequential Quadratic Programming |
| *MOSQL* | Multi-objective SQL |
| *NSA* | Null Space Algorithm |
| *SLP* | Sequential Linear Programming |
| *SLA* | Stereolithography |
| *DLP* | Digital Light Processing |
| *EDM* | Electrical Discharge Machining |
| *NSGA* | Nondominated Sorting Genetic Algorithm |
| *CFD* | Computational Fluid Dynamics |
| *CNC* | Computer Numerical Control |
| *CV* | Control Volume |
| *LES* | Large Eddy Simulation |
| *DES* | Detached Eddy Simulation |
| *BO* | Bayesian Optimization |
| *ML* | Machine Learning |
| *MMC* | Moving Morphable Component |
| *MOR* | Model Order Reduction |
| *S-D* | Steepest Descent Method |
| *H-J* | Hamilton-Jacobi equation |
| *R-D* | Reaction-Diffusion equation |
| *FVM* | Finite Volume Method |
| *FEM* | Finite Element Method |
| *LSF* | Level Set Function |
| *LSM* | Level Set Method |
| *EA* | Evolutionary Algorithm |
| *GP* | Gaussian Process |

## 1. Introduction

Energy, environment and sustainable development are closely related topics, while energy is at the center of the sustainable development paradigm. All energy conversion systems involve the heat transfer via fluid flows. More than two thirds of energy is lost in the energy conversion chain, from capture, conversion, transport, production, distribution, storage to end use. Increasing energy efficiency has been identified as one of the main challenges for energy systems and has attracted increasing attention from the academic and industrial communities [1–3].

Heat exchanger (HX) is a classical component [4–6] and the basic element not only for all systems and processes of energy conversion, production and use but also for many industries (food, cosmetics, medical, textile, chemical, metallurgical, materials, building, embedded systems, aeronautics, aerospace …). HXs are everywhere, indispensable, in different forms, to meet various needs, and are often subject to strong functional and operational constraints. It is a highly applied research topic that requires fundamental sciences such as thermodynamics, transport phenomena, fluid mechanics, materials, combined with high-performance numerical methods and optimization tools. The objective is to increase their overall performance. The key points are the intensification of heat transfer on the one hand, and the optimized management of fluid flows on the other hand, at each structural and temporary scale [6–9]. Therefore, how to improve the thermal performance of HXs has long been a hot topic in the research community of energy engineering.

Many theorems and methodologies have been developed for enhancing the heat transfer rate of HXs at the given pressure loss [10–14]. Starting with the basic heat transfer equation for HXs [10]:

$$Q = UA\Delta T_m \tag{1}$$

where $Q$ is the heat transfer rate (W), $U$ is the overall heat transfer coefficient (W.m$^{-2}$.K$^{-1}$), $A$ is the heat transfer surface area (m$^2$) and

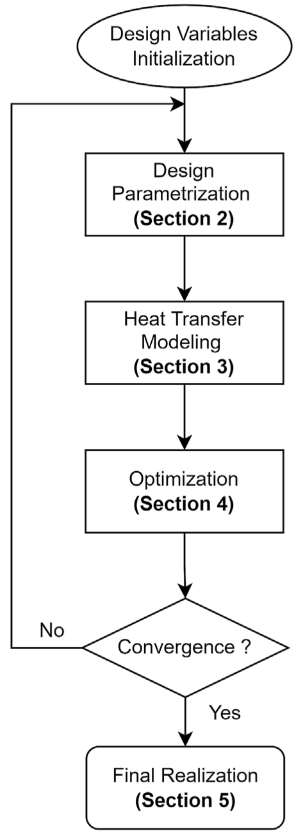



**Fig. 1.** The basic stages (the corresponding section) in the TO process.

$\Delta T_m$ is the mean temperature difference or the heat flux driving force (K). $U$ is composed of conduction and convection coefficients which are associated to the transport properties. Both heat transfer mechanisms (conduction and convection) could be magnified by enhancing the thermal properties of the HX material and by affecting the fluid flow pattern near to the heat transfer surfaces, respectively. Moreover, it is evident that increasing $A$ and better allocating the heat transfer driving force ($\Delta T_m$) will also intensify the heat transfer. For all these three aspects, a determinant factor is the shape/form/arrangement of the solid-fluid interface within the HX, on which the size/shape/topology optimization methods could be employed to play a critical role.

In general, the optimization of HXs can be classified into three types: the size optimization, the shape optimization and the topology optimization (TO). The size/shape optimization has been well developed for years [15–17], which refers to the design process that searches for the optimal size or shape in the given configuration or arrangement for a specific HX [18]. In practice, good performance improvement can still be achieved with the careful selection of the initial structures and optimization criteria [19]. Nevertheless, the size/shape optimization could not significantly change the prescribed configuration or arrangement of HXs set by designers, which may limit the optimization performance. Different from the size/shape optimization, the TO act directly on the topology of the (interface) geometry by spatially optimizing the distribution of fluid or solid phase and their connectivity, within a defined domain, which may attain any topology that minimize/maximize the optimization objective under some constraints. In theory, it holds the possibly maximum degrees of freedom in optimization, though in practice, the optimization objectives and constraints can also have a significant influence on the final results. In recent years, the TO has been regarded as a groundbreaking technique to obtain the innovative designs of HXs with greatly improved effectiveness, and has drawn more and more attention of researchers.

Fig. 1 shows a representative workflow of TO that includes four basic stages: (1) Design parametrization, (2) Heat transfer modeling, (3) Optimization process, and (4) Final realization. Compared to structural TO for mechanics, the issues that limit the TO's utilization for the HXs can emerge in each stage of the TO process. The HXs involve the conjugate flow and heat transfer [20]. Thus, the fluid problems should be solved during the iteration process of TO, leading to large computational expenses. This is actually the major obstacle for the practical utilization of TO for the real HXs of which the intermediate or final interface structures/topologies can be really complicated. Meanwhile, the mixing among different flows should be avoided by carefully designing the parametrization scheme, when updating the geometry of the solid phase that separates different fluids [21]. Additionally, maximizing the heat transfer rate is not always the only goal when designing HXs; the pressure loss should also be considered. To address this issue, the weighted-sum objective function [22] or a multiple-objective optimizer, such as NSGA-II [23], should be employed. Moreover, even if the rapid development of additive manufacturing (AM) techniques has greatly improved the ability to realize the optimized designs obtained by the TO, there are still some manufacturing constraints when applying a specific AM technique [24], and the proper post-treatments on the TO-derived structures are highly needed [25]. Actually, there are few researches that consider the fabricating constraints directly into the TO of HXs. Researchers have proposed some specific solutions for the issues mentioned above, which are dispersed among the literature. In the past years, several review articles were published to cover the literature of TO for microfluid devices [26], heat transfer systems [27], and fluid-based problems [28]. However, a comprehensive and comparative review on different TO stage's features, advantages, disadvantages, and limitations, is still lacking particularly for HXs.

Here, we will analyze and compare the researches on the TO for HXs in the most recent years with the main objectives of defining a research guideline for the development and improvement of TO for HXs. A brief understanding of different methods employed in all TO stages is intended to be provided which will help and clarify the implementation procedure for interested readers. However, there have been few TO researches handling such practical multi-flow HXs at the component level. In fact, most of the TO papers just deal with a specific element (such as a duct) within the whole HX structure. In order to extend the coverage of our review, the papers for single-flow heat sinks and fins that involve the physics of conjugate heat transfer are also included, while the pure heat conduction, the radiation, the phase change (evaporation, condensation), transient operations (thermal energy storage for example), and exothermic/endothermic reaction problems are excluded for clarity. According to this inclusion criterion, 91 studies published in the past fifteen years are covered in this review, which can well reflect the mostly-recent progress in the TO for HXs.

The present paper is organized following the procedure of TO, that is, each stage of TO will be discussed in the corresponding section. Those common approaches are presented, with emphasis in the issues that limit their utilization for the practical HXs. Afterwards, some new trends in this area aimed to improve the efficiency, like the integration of machine learning techniques, will also be covered in Section 6. A series of statistics, comparative tables and figures will be given in each section to demonstrate the features, advantages, disadvantages, and limitations of the developed schemes in the TO of HXs.

**Table 1**
Summary and comparison of design parametrization methods.

| Parametrization | Advantages | Disadvantages |
|---|---|---|
| Density-based | • Fixed mesh;<br>• Well developed in TO for years. | • No interface described;<br>• Numerical instabilities;<br>• Modified governing equations. |
| Level-set | • Crisp description of interface profile;<br>• No re-meshing in general. | • Slow convergence;<br>• Results dependent on initial configurations;<br>• Numerical artifacts. |
| Direct explicit | • Interface described explicitly;<br>• Straight-forward & relatively simple. | • Applicable only for simple geometries. |

## 2. Design parametrization

Design parametrization refers to the representation of optimization variables determining the design configurations that establish the relationship between the design variables (e.g., the density distribution describing the flow paths in the density-based TO problems) and the physical properties by the interpolation functions. Its sensitive representation strongly affects the TO's output results [29]. Furthermore, the design parametrization can vary from TO types, problem descriptions, and physical phenomena. It should be carefully chosen according to the problem's features, considering both efficiency and accuracy. As given in Table 1, there are three main types of parametrization methods: Density-based, Level set and Direct explicit.

### 2.1. Density-based method

The density-based method is the most popular means, which was first proposed by Bendsøe [30] in 1989. It is based on representing the design domain by densities or porosities to parametrize the fluid and solid phases. Researches started with the single-flow problems from Borrvall and Petersson [31]. Their representation of design variables (the density $\gamma$) consists of assigning $\gamma = 0$ for the solid phase or non-existing fluid phase, and $\gamma = 1$ for the fluid phase. The governing equation need be modified by introducing a fictitious force (the detailed equations will be given in Sec.3), which is determined by an inverse permeability ($\alpha$) for each element, with $\alpha_{min}$ corresponding to $\gamma = 1$ (fluid region) and $\alpha_{max}$ corresponding to $\gamma = 0$ (solid region). In the elements of solid with $\gamma = 0$, the fictitious force is maximum to block the flowing of fluid. During the iteration process, the inverse permeability ($\alpha$) is changing continuously in every element, which is determined by an interpolation function. Taking the widely-used SIMP (solid isotropic material with penalization) [30] as an example, it is given by,

$$\lambda(\rho) = \lambda_{max} + (\lambda_{min} - \lambda_{max})\gamma^q \tag{2}$$

where $q > 1$ is a penalization coefficient to minimize the presence of the gray elements, i.e. the ones of partial density from 0 to 1, $\lambda_{min}$, $\lambda_{max}$ are the minimum and maximum physical properties values (e.g., thermal conductivity), respectively. Note that the gray elements are usually regarded to hold no physical meaning and thus should be avoided by adjusting $q$. Thereafter, the above density-based representation with the different penalization functions, including SIMP and RAMP (rational approximation of material properties), etc., have been utilized in a wide range of single-flow HX problems [32–85]. Additionally, some other researchers [86–101] used an opposite representation of design parametrization by assigning $\gamma = 1$ for the solid phase or non-existing fluid phase and $\gamma = 0$ for the fluid phase. Note that no evidence demonstrates that such different representation of solid and liquid phases will significantly affect the solutions or efficiency of TO in the single-flow heat transfer problems. Currently, the first kind of representation accounts for the largest portion in the published articles, as shown in Fig. 2b.

Furthermore, the density-based method was extended to the multi-flow HX problems, which involve two or more fluids separated by one or more solid phases [21,101–105]. For instance, Kobayashi et al. [105] used one density ($\gamma$) to describe the multi-fluid problem by assigning $\gamma = 0$ for fluid 1, $\gamma = 1$ for fluid 2 and intermediate values ($0 < \gamma < 1$) for the solid phase. Tang et al. [101] divided a dual flow heat transfer problem into two independent one-flow and one-solid sub-problems, and thus one design variable (density) is used for both sub-problems.

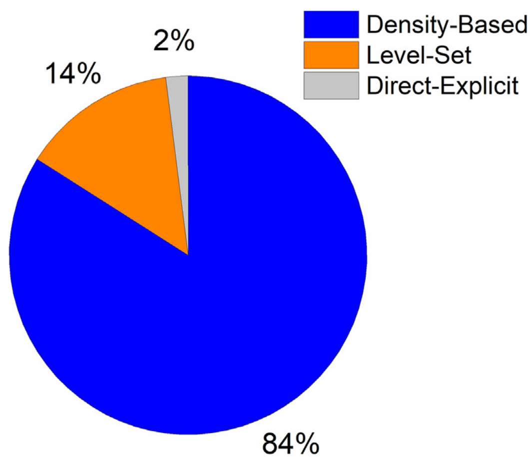
The density-based method has shown its high efficiency by avoiding the re-meshing process at each iteration. As for the interpolation functions, the RAMP has proved the ability to penalize the larger range of design variables compared to the SIMP function [106]. In fact, the majority of TO publications are based on the density-based method, as shown in Fig. 2a. However, some numerical issues are usually encountered, like the mesh-dependent results, the bad formation of solid/fluid cells in the optimized structure in which they are ordered similarly to the checkerboard configurations, and the intermediate densities values, etc. [107]. To remedy these instabilities, densities filters and projections should be implemented [107,108]. More importantly, it is not able to exactly describe the interface between different phases due to the element by element updating procedure, and thus not suitable for the problems where the interfacial profiles or the properties near the interfaces are important [109].

### 2.2. Level set method

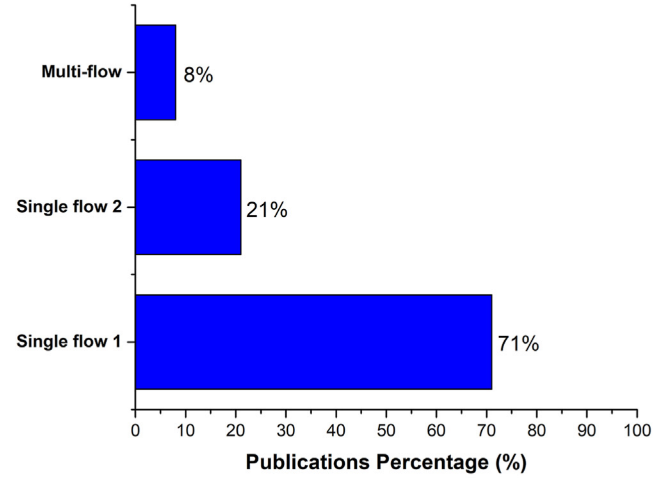
The level-set method (LSM) was first developed by Osher and Sethian [110], for the purpose of well defining the interface between phases. Most of the time, it implicitly describes the interface between multiple phases by a level-set function (LSF) [111–113], which allows a clear description of the interfaces and improves the accuracy of the responses captured at the boundaries. The design parametrization of LSM is given by,

$$\begin{cases} \varnothing(\boldsymbol{X}) > 0 \Leftrightarrow (\textit{Material Phase}) \\ \varnothing(\boldsymbol{X}) = 0 \Leftrightarrow (\text{Interface}) \\ \varnothing(\boldsymbol{X}) < 0 \Leftrightarrow (\textit{Void}) \end{cases} \tag{5}$$

where $\varnothing$ is the level-set function and $\boldsymbol{X}$ is the location vector of the design domain. The LSF is set to be zero at the interface, and the nodal values of LSF can be solved based on a governing equation or interpolated on the computational domain by a space function called "basis functions". The LSM has also been applied in the TO of HXs [96,104,114–123]. Feppon et al. [117] even used the LSM to deal with the 2D and 3D HXs involving two fluids. Furthermore, the LSF can be described in an explicit way [120,124]. Li et al. [124] suggested a component-based level-set parametrization to describe




(a) Publication percentage of different design parametrization methods




(b) Publication percentage of Density-based different design parametrization. Single flow 1 (γ=1 fluids, γ=0 solids). Single flow 2 (γ=0 fluids, γ=1 solids)

**Fig. 2.** Publications statistics for design parametrization methods in TO of heat exchangers (until 1-March-2022).

explicitly the solid/fluid interface for the TO of a micro-channel heat sink.

The clear and crisp description of interface in the LSM makes it a good option for the problems where the interfacial profiles or the properties near the interfaces really matter. Generally in the LSM, re-meshing is not needed, except in the case of conforming discretization (referring to the conforming discretization section in Ref. [125]). In this sense, the LSM can be well suitable for the HXs where heat transfer rate is largely determined by the flow velocity and temperature fields near the solid-liquid or liquid-solid-liquid interfaces. However, the dependence of output results on the initial configurations can significantly affect the accuracy and efficiency of LSM [125]. Another disadvantage is the slow convergence compared to the density-based method [125]. Moreover, similar to the density-based method, the regularization techniques are always necessary to avoid numerical artifacts and enhance the convergence rate in the LSM [125].

### 2.3. Direct explicit method

Direct explicit parametrization permits to describe interfaces in a direct way. One or several functions or arrays are used to describe the interfacial profiles explicitly. Among the literature, the direct explicit parametrization is very well established for shape optimization [126–128], but only applicable for some simple problems. As for TO of HXs, it is infrequently applied (see Fig. 2a), though it can eliminate the numerical artifacts encountered by the implicit representations [129]. For example, Mekki et al. [130] proposed an explicit voxel parametrization for optimizing the 2D fins of a HX: each voxel can represent either solid or liquid, and can be iteratively switched during the optimization process. Moreover, Shimoyama and Komiya [131] suggested a new explicit parametrization by representing the 3D lattice-structured heat sink using a point/edge system. However, up to date, there has been no published research that used the direct explicit method in the TO for complex HX problems.

## 3. Heat transfer modeling

The conjugate heat transfer in HXs is characterized by four equations, i.e., continuity eq. (4a), (b) momentum eq. (4b), and energy balance eq. (4c) for fluids, (4d) for solids,

$$
\begin{aligned}
&\frac{\partial \rho_f}{\partial t} + \nabla.\left(\rho_f v\right) = 0 \ (a)\\
&\rho_f\left(\frac{\partial v}{\partial t} + v.\nabla v\right) = -\nabla P + \mu_f.\nabla^2 v + f \quad (b)\\
&\rho_f C_{pf}\frac{\partial T}{\partial t} + \rho_f C_{pf} v.\nabla T = \nabla\,.\,\left(k_f \nabla T\right) + S \quad (c)\\
&\rho_s C_{ps}\frac{\partial T}{\partial t} = \nabla\,.\,(k_s \nabla T) + S \quad (d)
\end{aligned}
\tag{4}
$$

where $C_{pf}$ and $C_{ps}$ are the specific heat at constant pressure for fluid and solid phases respectively (J.kg$^{-1}$.K$^{-1}$), $k_s$ and $k_f$ are the thermal conductivities for solid and fluid phase respectively (W.m$^{-1}$.K$^{-1}$), $v$ the velocity (m.s$^{-1}$), $T$ the temperature (K), $P$ the pressure (Pa), $\mu_f$ the fluid dynamic viscosity (Pa.s), $\rho_f$ and $\rho_s$ are the fluid and solid phases densities (kg.m$^{-3}$), $t$ is the time (s), $S$ is the heat source term (W) and $f$ is the fictious force equal to -$\alpha v$, where $\alpha$ is the inverse permeability. This fictious force is an indispensable term specifically in density-based TO which represents the solid phase force on the fluid phase. Nevertheless, this fictious term is infrequently used in level set TO; in the direct explicit case, it is not needed. In the TO, the governing equations should be solved at each iteration to compute the objective function values. Apparently, the solver efficiency and accuracy will greatly affect the performance of TO. Furthermore, these numerical solvers encountered some

**Table 2**
Summary and comparison of the solvers in TO of HXs.

| Methods | Advantages | Disadvantages |
|---|---|---|
| FEM | • High availability in TO;<br>• Flexible with a wide range of physics. | • Not ensuring the conservation law locally;<br>• Numerical instabilities for convection. |
| FVM | • Ensuring the conservation law locally;<br>• Suitable for CFD problems. | • Relatively low availability in TO;<br>• Tough to design higher order schemes with high accuracy;<br>• High requirement of mesh quality especially for complex geometries. |
| XFEM | • Well capturing interfaces. | • Very low availability in TO;<br>• Not ensuring the conservation law locally. |
| LBM | • Ensuring the conservation law locally;<br>• Able to consider the size effects at microscale;<br>• Easy-meshing. | • Very low availability in TO;<br>• Difficulties in handling the multiphase flow, compressibility and 3D extension;<br>• Memory intensive. |

difficulties to correctly and efficiently simulate turbulent flows which are usually described by the velocity, pressure chaotic changes and unsteady eddies. In decades, several solvers have been developed to solve Eq. (4), as given in Table 2. In the majority of HXs applications, some acceptable simplifications and assumptions are made to simplify the numerical modeling e.g., steady-state, temperature independent thermophysical properties for fluid and solid phases, incompressible flows, etc. Additionally, some rare exceptional studies dealt with some more complicated conditions, e.g. temperature dependent thermophysical properties [60].

### 3.1. Finite element method (FEM)

The FEM is one of the well-developed techniques for solving partial differential equations. It was first proposed by Hrennikoff [132] and McHenry [133] on structural problems. FEM consists of discretizing the domains into small domains called “finite elements” to transform a continuous problem into a discrete one. Thereafter, the governing equations are integrated over each element by the weighted residual methods [134], e.g., Galerkin method. The elemental matrices are then formulated and assembled into the global discretized system of equations that enable to calculate the unknown variables at each node. FEM has proved its high flexibility of being applied to a wide range of physics with highly accurate results [135].

In the TO of HXs, the solvers based on the FEM have been extensively used for the steady-state laminar flow in both 2D and 3D cases [21,32–37,44,46,48–55,59–62,64–78,80,82–84,86,89,91,94–100,103–105,116–119,121,122,136–138]. As for the single-flow HXs, for instance, Dede et al. [46,89] used the FEM solver in the TO of a liquid cooled heat sink, and Matsumori et al. [32] optimized the channels of a HX using the FEM-integrated TO. In the multi-flow HX cases, the FEM was adopted in the TO by Papazoglou [21]. Sun et al. [36] executed the TO on a fin and tube HX using the FEM-based COMSOL Multiphysics software. Different from the preceding references, under laminar transient conditions, Zeng et al. [56] performed a TO on a 3D heat sink using a finite element solver. On the other hand, a few researches on the TO of HXs involving the turbulent flow have also been conducted mainly using the FEM to solve the RANS (Reynolds averaged Navier-Stokes) equations [40,61,85,87,124]. For example, Zhao et al. [87] adopted the Darcy-flow and RANS models for the TO of cooling channels problems under steady state conditions: the FEM-based commercial software (COMSOL Multiphysics) was employed to simulate the turbulent flow in the channels.

The combination of density-based method and FEM is the most convenient transfer from the TO of structural mechanics to that of conjugated heat transfer. In decades, a series of algorithms and codes have been developed, and recently the TO module has even be integrated in the FEM-based commercial software. Owing to such high availability, the FEM is currently the mostly-used solver in the TO of HXs, as illustrated in Fig. 3. However, in the FEM, the conservation law is not well guaranteed locally for each finite element [139]. This may lead to the numerical instabilities of the conjugate heat transfer problems [140], which can largely affect the performance of the TO of HXs.

### 3.2. Finite volume method (FVM)

The FVM is discretizing the design domain into a group of control-volumes (CVs) by directly integrating the governing equations over each CV, and the divergence theorem is applied to transform the CV integration into boundaries summation over each CV [141–143]. It has shown its robustness and stability in CFD (computational fluid dynamics) problems [144].

FVM was first used in the TO of heat conduction by Gersborg-Hansen et al. [145]. Then, it was implemented in a TO algorithm for the steady-state laminar flow HXs [38,43,45,58,63,81,90,101,102,130,131]. Tawk et al. [102] optimized both parallel and counter-flow HXs using the FVM-based TO for thermo-hydraulic enhancement purposes. Recently, the open source library OpenFoam based on FVM becomes popular to solve the flow problems. It has also been applied for TO of HXs [45,130] on a few cases.

As for the HXs involving turbulent flow, Kontoleontos et al. [88]

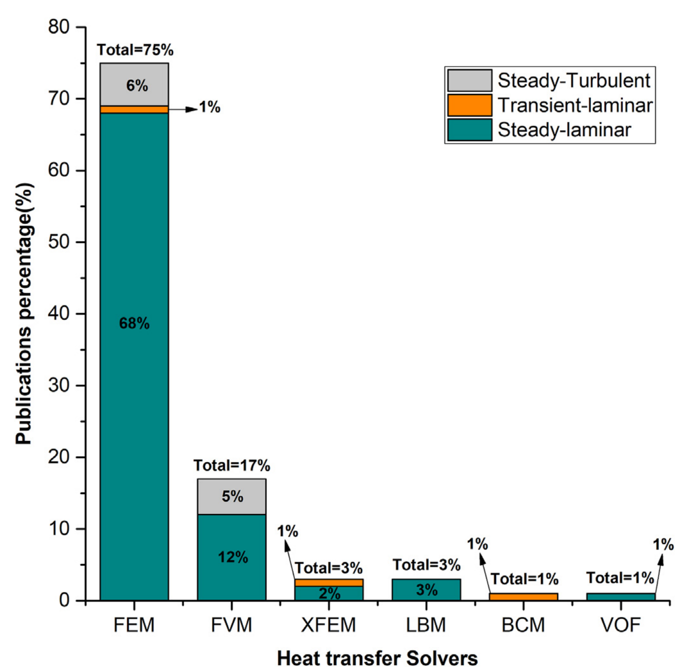



**Fig. 3.** Publications percentages for different solvers used in the TO of HXs under different conditions (until 1-March-2022).

used the FVM to solve the Spalart-Allmaras turbulence model in the TO of a thermal-fluid problem. In the same idea, Dilgen et al. [42] studied the turbulence effect on a heat sink using the *k-w* model at steady state conditions. With the intention of studying the turbulence effects inside a square tube HX, the FVM was applied by Pietropaoli et al. [93] to solve the RANS equations in the TO; then, they carried out a detached eddy simulation (DES) to evaluate the thermal performance of optimized structure. Ghosh et al. [79] used a FVM-based software (OpenFoam) to model the turbulent flow inside a cooling duct.

The FVM ensures the conservativeness over every CV [143], which makes it a good option for CFD problems. In fact, the majority of authors that used the FEM-based TO mentioned in Subsection 3.1 performed the CFD analysis using FVM solvers to evaluate the performance of the TO-derived structures, which underscores the advantages of the FVM over the FEM in CFD applications. However, the portion of the FVM-based TO of HXs to date happens to be rather small, as shown in Fig. 3. The low availability of the FVM-based TO programs may be the major reason for that. Moreover, the optimized results in the FVM-based TO can be mesh dependent, without integrating a proper filter [27]; it becomes difficult to design high order schemes that obtain a good accuracy using the FVM [146].

### 3.3. Extended finite element method (XFEM)

The XFEM extends the approach of the FEM by adding enrichment degrees of freedom on the nodes near the discontinuities to improve the description of discontinuities [147,148]. The XFEM was first used on the 2D cracks by Belytschko and Black [149] to study the crack propagation and interfaces.

As a very valuable attempt, Coffin and Maute [120] combined the XFEM and the LSM in the TO for the 2D and 3D, steady-state and transient single-flow heat transfer problems dominated by natural convection. Recently, Lin et al. [123] performed a topology optimization using the LSM-XFEM coupling to optimize the channel topology for a 2D heat sink under steady-state conditions. Thanks to the features of XFEM and LSM, the interface is well captured during the iterative optimization process, while the computational burden increases at the same time. In the XFEM-based TO, the description of the interface can be improved, however adding new degrees of freedom at the nodes near the interfaces induces a high algorithmic complexity that strongly increases the computational time. As demonstrated by Fig. 3, XFEM was scarcely used as a numerical solver for the TO of HXs. The same as the FEM, some instability problems are encountered due to the deficiency of conservative fluxes at each element [139]. Additionally, particularly for transient problems due to the rapid change of the physical properties (like temperature jumps near the interface), small time steps are required to capture it when using the XFEM [150].

### 3.4. Lattice Boltzmann method (LBM)

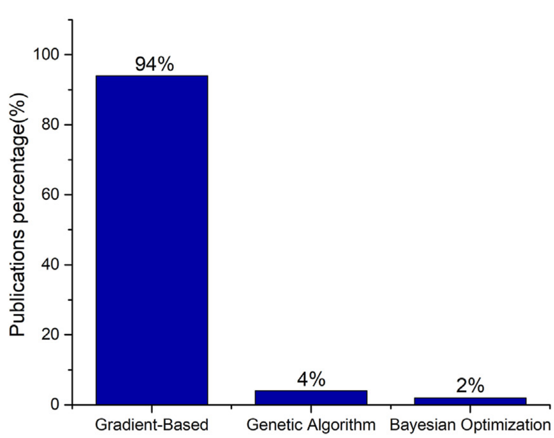
The LBM is a mesoscale method used for solving transport governing equations described at the macroscopic scale [151,152]. A set of Boltzmann transport equations are designed to correspond to the macroscopic governing equations, and then are solved in the representation of lattice gas.

The LBM is a relatively young technique compared to the FEM and the FVM. There are few researchers attempted to integrate the LBM in the TO for HX problems [39,57,115]. Yaji et al. [57] implemented a TO based on LBM to optimize the flow channels topology of a 2D thermo-fluid problem. In 2018, the LBM was adopted by Dugast et al. [115] for the TO of a 2D thermal fluid problem.

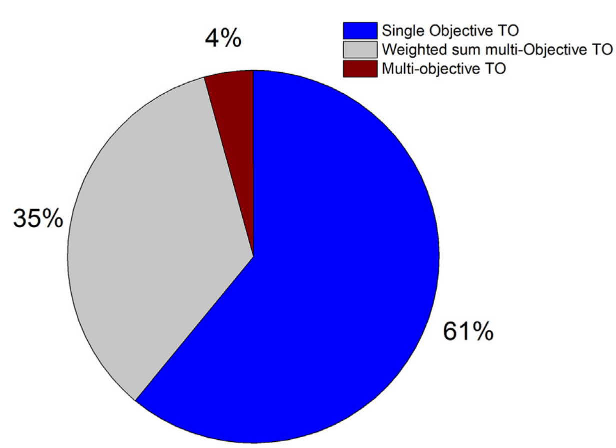
The LBM has showed its robustness and accuracy in the heat and mass transfer problems, particularly in the micro-scale cases where the size effects become significant [153]. It ensures the local conservation law, and has the advantages when dealing with the problems of complicated interfaces and size effects at the micro-scale. In this sense, the LBM-based TO may be promising for the multiscale HXs. However, the LBM has difficulties in handling with the multiphase flow, compressibility and 3D extension. Moreover, due to the iterative propagation step, the LBM is a memory-intensive method [154]. Importantly, the integration of LBM with TO is still at a starting stage, which is not enough mature for practical applications.

## 4. Optimization

After computing the objective function(s) using the heat transfer solvers, an optimization process is conducted to renew the design variables (defined in the parametrization stage), in order to minimize or maximize the objective function(s) under specific constraints. The objective functions serve as the optimization criteria and may influence the final topologies. Regarding the thermal performance, there have been at least 10 different




(a) Different optimization techniques




(b) Optimization types

**Fig. 4.** Publications percentages for different optimization techniques in the TO of heat exchangers (until 1-March-2022): (a) Different optimization techniques; (b) Optimization types.

**Table 3**
Summary and comparison of the optimizers in TO of HXs.

| Optimizers | Advantages | Disadvantages |
|---|---|---|
| Gradient-based | • Mostly efficient for the large-design-variable-number problems;<br>• High availability in TO. | • Deficiency in multi-objective problems;<br>• Local optima. |
| GA (Genetic Algorithm) | • Gradient-free;<br>• Global optima;<br>• Efficient in multi-objective problems. | • Slow convergence;<br>• Randomness. |
| Bayesian | • Efficient in big data problems. | • Expensive and complex computation;<br>• Scalability weakness with the number of objective function evaluations. |

objective functions among the literature of TO, including minimizing average temperature rise, minimizing thermal compliance, minimizing thermal resistance, maximizing exchanged heat, and maximizing recoverable thermal power. Optimization criteria of HXs where the subject of long discussions in the community of heat transfer [11,13,155]. For instance, in the view of thermodynamics, minimizing the exergy destruction can also be an objective function of heat transfer optimization. However, to our best knowledge, there has been no research that carefully investigates the influence of objective functions on the TO of HXs up to date. Additionally, the hydraulic performance of HXs can serve as either the constraint or one of the objective functions. Regarding the hydraulic performance, the choices of optimization criteria (or constraints) are not that diverse: minimizing the pressure drop and the energy dissipation (loss) of flow are the common ones. In addition, as illustrated by Fig. 4b, the majority of the researchers dealt with single objective functions. Some of these studies take an advantage to enhance the thermo-hydraulic performance simultaneously by dealing with a single objective function and setting the other objective as an optimization constraint. On the other hand, other researchers employed the weighted sum or the true multi-objective optimization for the same purpose of intensifying the heat transfer and improving the hydraulic performance concurrently of HXs. The optimizer, which is the core part of TO algorithm, determines the evolution of design domains and thus the final output result by the TO. Table 3 lists some commonly-used optimizers.

### 4.1. Gradient-based optimization

The gradient-based method also called “sensitivity analysis” is based on computing the gradients of the objective functions with respect to the design variables. These gradients represent the variation of the objective function with respect to the design variables at each iteration and are often solved using the adjoint method [156]. The adjoint method has shown its high efficiency in computing the objective function gradients [157]. The optimizer renews the design variables based on these gradient values.

As illustrated in Fig. 5, several gradient-based optimizers are utilized in the TO of HXs, including MMA (Method of Moving Asymptotes) [158](44%), GCMMA (Globally convergent MMA) [159](25%), SLP (Sequential linear programming) [160](6%), SQP (Sequential quadratic programming) [161](9%), Steepest descent [162](8%), Tosca [163](1%), Reaction-diffusion [164](4%), Hamilton-Jacobi [96](1%) optimizers, and Null Space algorithm [165](2%). These gradient-based optimizers hold the different mathematical natures and thus the distinct applications. The detailed explanation on their mathematical characteristics are beyond the scope of our review, and can be found in the relevant references.

The utilization of gradient-based optimizers is the mainstream in the TO not limited to the problems of HXs [21,32–105, 115–124,136,137]. According to Fig. 4a, more than 90% of papers on the TO of HXs utilize the gradient-based optimizers. This is mainly because of its good efficiency in handling problems involving such large number of design variables (usually equal to the number of nodes in the solver) [166]. Recently, some gradient-based multi-objective algorithms have been developed, like MOSQP (Multi-objective SQP) [167], which has been employed in the structural TO [99]; however, up to date there has been no published work on the TO of HXs using such algorithm. Additionally, the gradient-based optimization can converge to a local optimum when the objective function has several local optimums [134], and thus re-optimization is needed by setting different initial configurations.

### 4.2. Genetic algorithm (GA)

The GA is a stochastic evolutionary algorithm (EA) based on the biology of chromosomes and genes [168]. This evolutionary algorithm obtains the optimized solution(s) after several generations. Each generation starts by generating the initial population randomly to increase its diversity. Then, the fitness values are evaluated for each chromosome in the population using fitness function(s), i.e. objective function(s). The parent chromosomes are selected from the initial population using natural selection processes, e.g. roulette wheel [169]. The children are then obtained by the combination of two parents using crossover [170]. Thereafter, the mutation process based on randomness is applied on the children to mutate one or more of their genes before moving to the next generation. Finally, the elitism stage [171] moves one chromosome to the next generation without being edited by the crossover and mutation.

The GA has been developed by many researchers in different fields including heat transfer [172,173]. As for the TO of HXs, few researchers implemented the GA for generating optimized topologies [61,114,130,131]. Yaji et al. [61] proposed a multi-fidelity TO for a heat sink using EA main stages (selection, crossover,

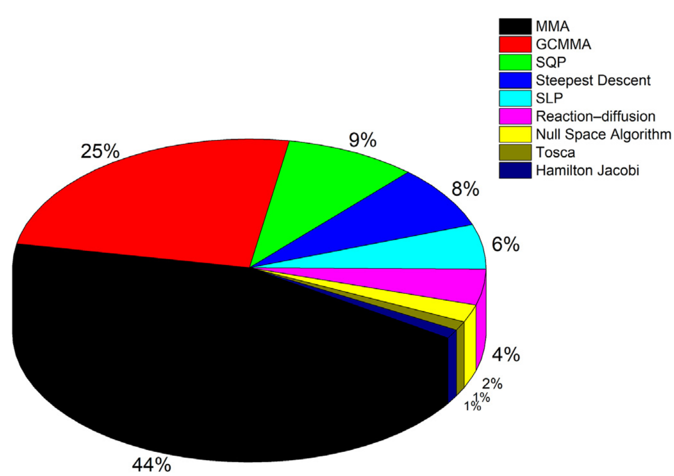



**Fig. 5.** Publications percentages of the optimizers used in gradient-based TO of heat exchangers (until 1-March-2022).

mutation). They first performed a low fidelity optimization problem based on Darcy flow model using $\epsilon$-constrained method [174]. According to low fidelity results, a high-fidelity evaluation was executed using Navier–Stokes equations. Then, a non-dominated sorting strategy (NSGA II) was employed to select the optimal pareto front.

The GA method avoids the gradient computation of the objective function (s) at each iteration. In theory, it will obtain the global optima, and screen the influence from the initial guess [172]. Moreover, the GA is a good option for the multi-objective problems, since it handles a group of candidates simultaneously, which is of advantage to derive the Pareto front [23]. Having similar stochastic behaviors, other evolutionary algorithms (e.g., Particle Swarm Optimization) could also be tested with the TO of HXs which may possess more efficiency than the GA in some cases. Despite the merits of GA, it has been rarely coupled with the TO of HXs, as shown in Fig. 4a. This is mainly attributed to the slow convergence of GA [172], which significantly increases the computational time of the TO.

### 4.3. Bayesian optimization (BO)

The BO is an optimization technique based on machine learning concept. It initially rose thanks to the work by Kushner [175], Zhilinskas [176] and Mockus [177]. Then it was popularized after the paper of Jones et al. [178]. The BO is composed of two main parts: statistical modeling and acquisition function. In the Bayesian statistical modeling section, a random set is initially generated. After that, the mean vector and the covariance matrix are calculated based on Gaussian process (GP) regression for the whole set. The acquisition function is then calculated and its optimum value is used to optimize the objective function for the next step (more details referring to section 4 in Ref. [179]).

The BO is a sequential optimization method that solves tasks in a sequence way. Due to its high data efficiency structure, the BO has shown its robustness in the big data applications [180]. Some structural TO problems have been studied by integrating the BO; for example, Lynch et al. [181] investigated a simple structural TO problem (i.e., minimizing the compliance of a 2D beam) to show the possibility of integrating BO in the TO for HXs. In fact, the concept of BO was also employed by Yoshimura et al. [114] and Shimoyama and Komiya [131] to handle HX problems. Both references built a Kriging surrogate model [182] to efficiently evaluate an approximated objective function which will emphatically diminishes the computational time. Apparently, the utilization of BO in the TO of HXs is very limited, and even less than that of GA up to date. The expensive and complex computation of the acquisition function optimization procedure at each iteration [183] may be a reason. Indeed, another disadvantage of BO is the scalability weakness which is represented by the asymptotically increase of the computational time when evaluating the objective function for a new sampling point or when computing the objective function derivatives [184].

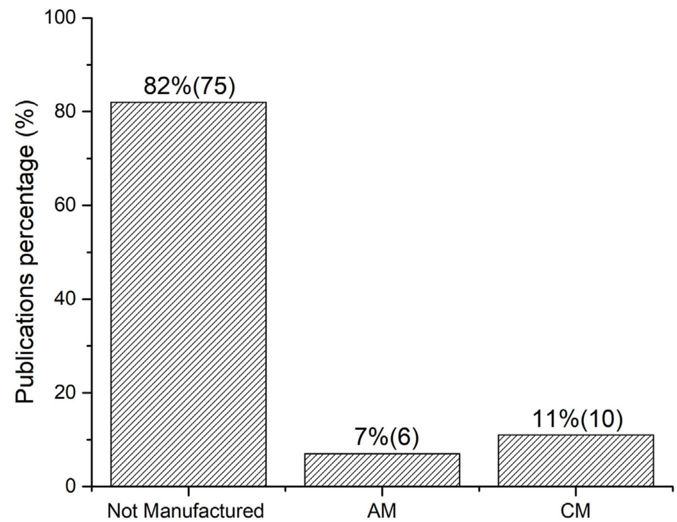



**Fig. 6.** Publication statistics for final realization statuses of optimized HXs (until 1-March-2022). Percentage (Numbers) of publications.

## 5. Final realization

The optimized complex structures obtained by the TO are generally difficult to be fabricated using the conventional manufacturing (CM) techniques. Therefore, additive manufacturing (AM) techniques also called as 3D printing have been applied to manufacture those very complex structures [25,185]. Table 4 compares CM and AM for fabricating the TO-derived structures of HXs. AM is an additive technique that build the structure by adding layers, while the CM techniques are subtractive, which remove material from the structure. Generally, the AM can remove the fabrication shackles of the CMs, but the equipment and materials of metallic AM are still very expensive currently [186]. Importantly, due to the restrictions of AM accuracy, some constraints, including length scale, connectivity, and overhang constraints, etc., should be subjected to the optimized structures obtained by the TO [24]. Those constraints are critically essential to eliminate the unmanufacturable features of the optimized structures. More details of those constraints can be found in Ref. [25].

As for the area of HXs optimization, referring to Fig. 6, only few researchers (about 18%) have manufactured the optimized structures obtained by the TO and tested them in practice. As a tradeoff, some researchers realized and tested the engineering simplified version of TO-resulted geometry due to the fabrication difficulty, the advantages of TO being partially or totally lost [58].

**Table 4**
Comparison between AM and CM for TO of HXs.

| Techniques | Advantages | Disadvantages |
|---|---|---|
| AM | • Manufacturing ability for complex geometries;<br>• High manufacturability efficiency for complex geometries. | • Relative limited understanding on the manufacturing constraints on the TO optimized structures;<br>• Expensive equipment and materials;<br>• Limited to prototype fabrication;<br>• Limited choices of materials. |
| CM | • High availability;<br>• High productivity;<br>• Cheap equipment compared to AM. | • Limited manufacturability for TO-derived structures;<br>• Slow fabrication and repairing process for complex TO-derived structures |

Using AM techniques, some researchers fabricated the TO-optimized HXs to validate their numerical results [41,54,68,83,99,137]. For example, Lei et al. [68] manufactured the optimized structure of a passive HX by the TO using 3D stereolithography (SLA) printing technique assisted with investment casting process. On the other hand, the CM methods [33,44,49,51,58–60,74,89,124], have also been utilized to fabricate some HXs obtained by the TO (mainly the 2D topologies, such as the 2D heat sinks). For instance, as referred by Koga et al. [33], the electrical discharge machining (EDM) was used to manufacture the optimized structure of a heat sink with the help of CNC (computer numerical control) milling. Fig. 7(a) and (b) illustrate the heat sinks fabricated by the AM [68] and the CM [51] techniques, respectively. Apparently, the AM method can attain more complex structures especially in the 3D case. Note that since the majority of current researches on the TO of HXs does not conduct the final realization of designed structures, there have been rare discussions on the fabricating constraints on the TO-optimized HXs, which should be improved in the further work. One recent paper mentions precisely the integration between AM and TO and the implementation of the overhang constraint in the TO for a fluidic problem [187].

Furthermore, in order to give a complete comparison among the existing literature, the papers on the TO of HXs analyzed in the sections above, i.e. Design parametrization, Heat transfer modeling, Optimization, and Final realization, are summarized in Table 5.

## 6. Some new trends

Here, we will move to discuss some emerging schemes aimed to improve the efficiency (i.e. reducing computational time and memory storage) of TO not limited to the area of HXs. Currently many of these novel schemes are designed for the structural TO problems; nevertheless, it is possible to transfer some of them to deal with the HXs problems.

### 6.1. Machine learning (ML)

With the rapid development of artificial intelligence (AI) in the recent years, the ML technique (a subset of AI) has become a powerful tool to handle various engineering problems. As for transport phenomena, the ML has exhibited the ability of predicting their solutions [189–192], due to its high potential of learning from existing data-sets. Using different strategies, the ML algorithms can be coupled with the density-based TO mainly for the structural problems at the present stage [193–196]. As for the TO of HXs, in a recent study, a data driven TO based on EA was suggested by Yaji et al. [61] for a heat sink under forced convection. In this research, a variational autoencoder [197] was implemented to perform the crossover operation by generating a new dataset. The ML proved the ability of increasing the TO efficiency by predicting the optimized structures for heat transfer problems with negligible time [198]. However, currently the research combining the ML and the TO for HXs is still rare, which may be attributed to the complexity of conjugating heat transfer (especially the fluid flow part) and the complicated structures of HXs. It requires more studies to clarify how to integrate the ML in the TO involving fluids and whether the ML can improve the efficiency.

### 6.2. Model order reduction (MOR)

The MOR is an approach aiming to decrease the complexity of models. It reduces the full original model into a reduced one by capturing the fundamental characteristics and neglecting the unimportant ones under certain accuracy [199]. The MOR methods

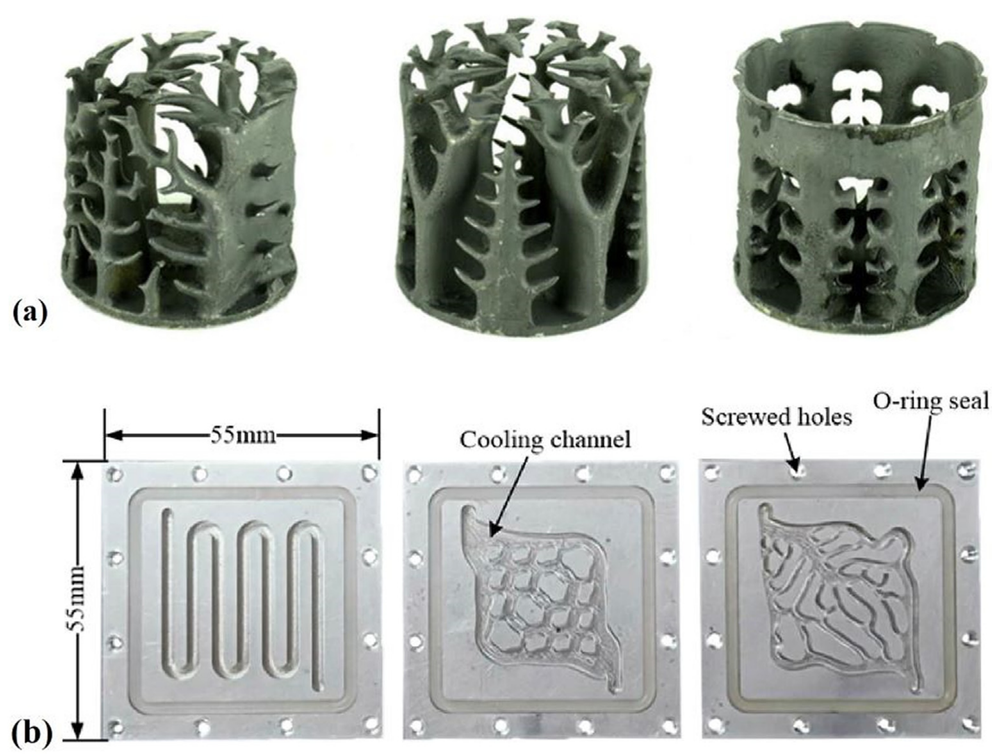



**Fig. 7.** (a) Heat sinks manufactured using the AM technique (the SLA printing assisted with investment casting) [68]. Reproduced from Lei et al., Investment casting and experimental testing of heat sinks designed by topology optimization. International Journal of Heat and Mass Transfer, 2018; 127: 396–412. Copyright © 2018 Elsevier. All rights reserved; (b) Heat sink channels manufactured using the CM technique (the CNC) [51]. Reproduced from Li et al., Experimental and numerical investigation of liquid-cooled heat sinks designed by topology optimization. International Journal of Thermal Sciences, 2019; 146:106065. Copyright © 2019 Elsevier Masson SAS. All rights reserved.

**Table 5**
Summary of researches on TO of HXs analyzed in the sections above.

| No. | Reference | Year | Parametrization | Solver | Objective Function | Optimizer | Final Realization |
|---|---|---|---|---|---|---|---|
| 1 | Dede [46] | 2009 | Density (SF1)[a] | FEM | Min (Mean Temperature & Energy Dissipation) | Gradient (MMA) | Not [d] |
| 2 | Yoon [95] | 2010 | Density (SF2)[b] | FEM | Min (Thermal Compliance) | Gradient (MMA) | Not |
| 3 | Dede [89] | 2012 | Density (SF2) | FEM | Min (Mean Temperature & Energy Dissipation) | Gradient (MMA) | CM (N/A) |
| 4 | Kontoleontos et al. [88] | 2012 | Density (SF2) | FVM | Min (Pressure Drop) & Max (Temperature Difference) | Gradient(S-D) | Not |
| 5 | Matsumori et al. [32] | 2013 | Density (SF1) | FEM | Max (Heat Generation) | Gradient (SQP) | Not |
| 6 | Marck et al. [63] | 2013 | Density (SF1) | FVM | Min (Pressure Drop) & Max (The Recoverable Thermal Power) | Gradient (MMA) | Not |
| 7 | Koga et al. [33] | 2013 | Density (SF1) | FEM | Min (Pressure Drop) & Max (Dissipated Heat) | Gradient (SLP) | CM (EDM, CNC) |
| 8 | Oevelen et al. [43] | 2014 | Density (SF1) | FVM | Min (Thermal Resistance) | Gradient (MMA) | Not |
| 9 | Alexandersen et al. [64] | 2014 | Density (SF1) | FEM | Min (Thermal Compliance) | Gradient (MMA) | Not |
| 10 | Yaji et al. [119] | 2015 | LSM | FEM | Max (Heat Generation) | Gradient (R-D) | Not |
| 11 | Papazoglou [21] | 2015 | Density (Multi-flow) | FEM | Max (Exchanged Heat) | Gradient (MMA) | Not |
| 12 | Yaji et al. [57] | 2015 | Density (SF1) | LBM | Min (Pressure Drop) & Max (Exchanged Heat) | Gradient (MMA) | Not |
| 13 | Coffin and Maute [120] | 2015 | LSM | XFEM | Min (Average Temperature) | Gradient (GCMMA) | Not |
| 14 | Łaniewski-Wołłk et al. [39] | 2016 | Density (SF1) | LBM | Max (Exchanged Heat) | Gradient (MMA) | Not |
| 15 | Qian and Dede [65] | 2016 | Density (SF1) | FEM | Min (Average Temperature & Dissipation Energy) | Gradient (MMA) | Not |
| 16 | Zhou et al. [136] | 2016 | Parametrization of [188] | FEM | Max (Reaction Flux) | Gradient (TOSCA) | Not |
| 17 | Alexendersen et al. [66] | 2016 | Density (SF1) | FEM | Min (Thermal Compliance) | Gradient (MMA) | Not |
| 18 | Li et al. [138] | 2016 | Density (N/A[c]) | FEM | Min (Heat Potential Capacity) | N/A | Not |
| 19 | Yoshimura et al. [114] | 2017 | LSM | BCM | Min (Pressure Drop) & Max (Bulk Mean Temperature) | GA & BO | Not |
| 20 | Haertel and Nellis [34] | 2017 | Density (SF1) | FEM | Max (Thermal Conductance) | Gradient (GCMMA) | Not |
| 21 | Zhao et al. [87] | 2017 | Density (SF2) | FEM | Min (Mean Temperature) | Gradient (MMA) | Not |
| 22 | Qian et al. [67] | 2017 | Density (SF1) | FEM | Min (RMS Temperature & Energy Dissipation) | Gradient (MMA) | Not |
| No. | Reference | Year | Parametrization | Solver | Objective Function | Optimizer | Final Realization |
| 23 | Sato et al. [121] | 2018 | LSM | FEM | Max (Heat Generation) & Min (Energy Dissipation) | Gradient (R-D) | Not |
| 24 | Haertel et al. [35] | 2018 | Density (SF1) | FEM | Min (Thermal Resistance) | Gradient (GCMMA) | Not |
| 25 | Zeng et al. [44] | 2018 | Density (SF1) | FEM | Min (Pressure Drop) | Gradient (GCMMA) | CM (CNC) |
| 26 | Dilgen et al. [42] | 2018 | Density (SF1) | FVM | Min (Average Temperature) | Gradient (MMA) | Not |
| 27 | Dugast et al. [115] | 2018 | LSM | LBM | Min (Mean Temperature) & Max (Exchanged Heat) | Gradient(S-D) | Not |
| 28 | Ramalingom et al. [90] | 2018 | Density (SF2) | FVM | Min (Pressure Drop) & Max (Recoverable Thermal Power) | Gradient(S-D) | Not |
| 29 | Santhanakrishnan et al. [96] | 2018 | Density (SF2), LSM | FEM | Min (Thermal Compliance) | Gradient (MMA, H-J) | Not |
| 30 | Lei et al. [68] | 2018 | Density (SF1) | FEM | Min (Thermal Compliance) | Gradient (MMA) | AM (SLA) |
| 31 | Sun et al. [36] | 2018 | Density (SF1) | FEM | Min (Pressure Drop) | Gradient (GCMMA) | Not |
| 32 | Lurie et al. [97] | 2018 | Density (SF2) | FEM | Min (Pressure Drop & Energy Dissipation) | Gradient (MMA) | Not |
| 33 | Saglietti et al. [37] | 2018 | Density (SF1) | FEM | Max (Exchanged Heat) | Gradient (MMA) | Not |
| 34 | Pietropaoli et al. [92] | 2018 | Density (SF2) | VOF | Min (Stagnation Pressure Drop) & Max (Temperature Gain) | Gradient(S-D) | Not |
| 35 | Makhija and Beran [86] | 2018 | Density (SF2) | FEM | Min (Average Temperature) | Gradient (GCMMA) | Not |
| 36 | Lv and Liu [47] | 2018 | Density (SF1) | N/A | Max (Heat Dissipation) & Min (Energy Dissipation) | Gradient (MMA) | Not |
| 37 | Subramaniam et al. [38] | 2019 | Density (SF1) | FVM | Min (Pressure Drop) Max (Recoverable Thermal Power) | Gradient (MMA) | Not |
| 38 | Saviers et al. [137] | 2019 | N/A | FEM | Max (Exchanged Heat) | Gradient (GCMMA) | AM (SLA) |
| 39 | Yu et al. [69] | 2019 | Density (SF1) | FEM | Min (Thermal Compliance & Energy Dissipation) | Gradient (MMA) | Not |
| 40 | Asmussen et al. [91] | 2019 | Density (SF2) | FEM | Min (Thermal Compliance) | Gradient (MMA) | Not |
| No. | Reference | Year | Parametrization | Solver | Objective Function | Optimizer | Final Realization |
| 41 | Zhang and Gao [52] | 2019 | Density (SF1) | FEM | Max (Heat Generation) | Gradient (MMA) | Not |
| 42 | Jahan et al. [98] | 2019 | Density (SF2) | FEM | Min (Thermal Compliance) | Gradient (MMA) | Not |
| 43 | Kobayashi et al. [48] | 2019 | Density (SF1) | FEM | Max (Heat Extraction) | Gradient (SLP) | Not |
| 44 | Tawk et al. [102] | 2019 | Density (Multi-flow) | FVM | Min (Pressure Drop) & Max (Exchanged Heat) | Gradient (MMA) | Not |
| 45 | Zeng and Lee [49] | 2019 | Density (SF1) | FEM | Min (Pressure Drop) | Gradient (GCMMA) | CM (CNC) |
| 46 | Yan et al. [50] | 2019 | Density (SF1) | FEM | Min (Maximum Temperature) | Gradient (MMA) | Not |
| 47 | Li et al. [59] | 2019 | Density (SF1) | FEM | Min (Pressure Drop) & Max (Exchanged Heat) | Gradient (SQP) | CM (CNC) |
| 48 | Li et al. [51] | 2019 | Density (SF1) | FEM | Min (Dissipation Energy) & Max (Exchanged Heat) | Gradient (SQP) | CM (CNC, Milling) |
| 49 | Dong and Liu [70] | 2019 | Density (SF1) | FEM | Min (Thermal Resistance & Pressure Drop & Energy Dissipation) | Gradient (SQP) | Not |
| 50 | Ghosh and Kapat [45] | 2019 | Density (SF1) | FVM | Min (Pressure Drop) & Max (Temperature Rise) | Gradient(S-D) | Not |
| 51 | Hu et al. [71] | 2019 | Density (SF1) | FEM | Min (Mean Temperature & Energy Dissipation) | Gradient (GCMMA) | Not |
| 52 | Kambampati et al. [116] | 2020 | LSM | FEM | Min (Thermal Compliance) | Gradient (SLP) | Not |
| 53 | Zhang et al. [53] | 2020 | Density (SF1) | FEM | Max (Exchanged Heat) | Gradient (GCMMA) | Not |
| 54 | Zeng et al. [56] | 2020 | Density (SF1) | FEM | Min (Average Temperature) | Gradient (GCMMA) | Not |
| 55 | Sun et al. [72] | 2020 | Density (SF1) | FEM | Min (Average Temperature) | Gradient (MMA) | Not |
| 56 | Zhang et al. [73] | 2020 | Density (SF1) | FEM | Min (Average Temperature) | Gradient (GCMMA) | Not |

(continued on next page)

**Table 5** (*continued*)

| No. | Reference | Year | Parametrization | Solver | Objective Function | Optimizer | Final Realization |
|---|---|---|---|---|---|---|---|
| 57 | Francisco et al. [99] | 2020 | Density (SF2) | FEM | Max (Thermal Conductivity) | Gradient (MMA, MOSQP) | AM (N/A) |
| 58 | Høghøj et al. [103] | 2020 | Density (Multi-flow) | FEM | Min (Enthalpy Difference) | Gradient (MMA) | Not |
| 59 | Troya et al. [104] | 2020 | Multi-flow, LSM | FEM | Max (Exchanged Heat) | Gradient (MMA, NSA) | Not |
| 60 | Lee et al. [100] | 2020 | Density (SF2) | FEM | Min (Thermal Resistance) | Gradient (MMA) | Not |
| 61 | Feppon et al. [117] | 2021 | LSM | FEM | Max (Exchanged Heat) | Gradient (NSA) | Not |
| 62 | Pietropaoli et al. [93] | 2021 | Density (SF2) | FVM | Min (Stagnation Pressure Drop) & Max (Temperature Rise) | Gradient (S-D) | Not |
| 63 | Dong and Liu [62] | 2021 | Density (SF1) | FEM | Min (Energy Dissipation) & Max (Recoverable Thermal Power) | Gradient (GCMMA) | Not |
| 64 | Kobayashi et al. [105] | 2021 | Density (Multi-flow) | FEM | Max (Exchanged Heat) | Gradient (SLP) | Not |
| 65 | Mekki et al. [130] | 2021 | Explicit | FVM | Max (Exchanged Heat) & Min (Pressure Drop) | GA | Not |
| 66 | Lee et al. [58] | 2021 | Density (SF1) | FVM | Min (Average Temperature & Dissipation Energy) | Gradient (GCMMA) | CM (Laser cutting) |
| 67 | Mario et al. [118] | 2021 | LSM | FEM | Max (Exchanged Heat) & Min (Energy Dissipation) | Gradient (R-D) | Not |
| 68 | Zhao et al. [76] | 2021 | Density (SF1) | FEM | Min (Average Temperature) | Gradient (GCMMA) | Not |
| 69 | Zhou et al. [74] | 2021 | Density (SF1) | FEM | Min (Temperature Difference, Average Temperature, Energy Dissipation) | Gradient (MMA) | CM (Machining) |
| 70 | Han et al. [54] | 2021 | Density (SF1) | FEM | Min (Temperature Difference & Energy Dissipation & Average Temperature) | Gradient (GCMMA) | AM (N/A) |
| 71 | Mo et al. [41] | 2021 | Density (SF1) | N/A | Min (Average Temperature & Energy Dissipation) | Gradient (MMA) | AM (N/A) |
| 72 | Ghasemi and Elham [94] | 2021 | Density (SF2) | FEM | Min (Thermal Resistance & Pressure Drop) | Gradient (GCMMA) | Not |
| 73 | Liu et al. [75] | 2021 | Density (SF1) | FEM | Min (Pumping Power, Mean and Standard Deviation of The Temperature) | Gradient (GCMMA) | Not |
| 74 | Liu et al. [55] | 2021 | Density (SF1) | FEM | Max (Exchanged Heat) & Min (Energy Dissipation) | Gradient (MMA) | Not |
| 75 | Zhao et al. [40] | 2021 | Density (SF1) | FEM | Min (Average Temperature Rise) | Gradient (MMA) | Not |
| 76 | Tang et al. [101] | 2021 | Density (SF2, Multi-flow) | FVM | Min (Mean Temperature & Pressure Drop) | Gradient (MMA) | Not |
| 77 | Chen et al. [77] | 2021 | Density (SF1) | FEM | Max (Heat Generation) | Gradient (SQP) | Not |
| 78 | Yaji et al. [61] | 2021 | Density (SF1) | FEM | Max (Exchanged Heat) & Min (Energy Dissipation) | Gradient (SLP), GA | Not |
| 79 | Li et al. [124] | 2021 | LSM | FEM | Min (Average Temperature) | Gradient (MMA) | CM (Milling) |
| 80 | Ghosh et al. [79] | 2021 | Density (SF1) | FVM | Max (Gained energy) & Min (Power lost) | Gradient (S-D) | Not |
| 81 | Qian et al. [60] | 2021 | Density (SF1) | FEM | Min (RMS[e] Temperature & Energy Dissipation) | Gradient (MMA) | CM (CNC) |
| 82 | Shimoyama and Komiya [131] | 2022 | Explicit | FVM | Max (Heat transfer rate) & Min (Material cost) | GA & BO | Not |
| 83 | Li et al. [122] | 2022 | LSM | FEM | Min (Thermal Compliance) | Gradient (R-D) | Not |
| 84 | Yu et al. [81] | 2022 | Density (SF1) | FVM | Min (Maximum Temperature) | Gradient (MMA) | Not |
| 85 | Zhou et al. [83] | 2022 | Density (SF1) | FEM | Min (Energy Dissipation, Average Temperature & Temperature Difference) | Gradient (GCMMA) | AM (N/A) |
| 86 | Zou et al. [82] | 2022 | Density (SF1) | FEM | Min (Average temperature & Pumping power) | Gradient (SQP) | Not |
| 87 | Marshall and Lee [84] | 2022 | Density (SF1) | FEM | Min (Pressure in fin area of the fluid) | Gradient (N/A) | Not |
| 88 | Yeranee et al. [85] | 2022 | Density (SF1) | FEM | Min (Pressure Drop) | Gradient (GCMMA) | Not |
| 89 | Lin et al. [123] | 2022 | LSM | XFEM | Min (Average Temperature) | Gradient (GCMMA) | Not |
| 90 | Xie et al. [78] | 2022 | Density (SF1) | FEM | Min (Pressure Drop) | Gradient (GCMMA) | Not |
| 91 | Huang et al. [80] | 2022 | Density (SF1) | FEM | Min (Average Temperature) | Gradient (SQP) | Not |

[a] SF1: single flow with $\gamma = 1$ fluids, $\gamma = 0$ solids.
[b] SF2: single flow with $\gamma = 0$ fluids, $\gamma = 1$ solids.
[c] N/A: Not Announced.
[d] Not: Not Manufactured.
[e] RMS: Root mean square.

were first proposed in 1980s [200–202]. Recently, Zhao et al. [87] proposed a poor man's approach to reduce the computational time for the TO of cooling channels: a simplified model was derived by imposing the Darcy flow model on NS equation and neglecting the effect of body force term. In the TO of a heat sink, Asmussen et al. [91] suggested a reduced order model by making some assumptions on the governing equations, which significantly reduce the number of degrees of freedom. Even with a high simplification, the reduced model obtains the subjectively analogous results with the full-based model [203]. Nevertheless, the simplifications are largely dependent on the knowledge of designers, and a poor simplification may degrade the accuracy and even cause severe computational issues represented mainly by unrealistic results and modeling problems. For instance, when the MOR technique is applied to NS equations under unsteady conditions or for turbulent flows, some computational problems frequently emerge [204].

### 6.3. Moving morphable components (MMC)

The MMC-based TO was originally proposed in 2014 by Guo et al. [205] then enhanced by Zhang et al. [206] for the 2D structural problems. Thereafter, the MMC method is extended to the 3D TO [207,208]. It represents the design domain by using several structural components. In the optimization process, the mathematical features described by center coordinates, width, length and inclination angles of these components are updated for achieving an optimized structure. In 2019, Yu et al. [69] proposed a density-based TO for a 2D heat transfer problem using the MMC. To take an advantage over the traditional MMC, Li et al. [124] utilized the quadratic Bézier curve permitting for more movement flexibility of the component. In a recent attempt, Yu et al. [81] suggested a component-based representation of the heat source distribution with various-intensities for the TO of a liquid cooled heat sink. The MMC-based TO can obtain optimized shapes in an explicit way, which can avoid the post-processing problems, like intermediate design variables, and checkboards, etc. [205]. Furthermore, the number of design variables is reduced compared to other implicit techniques, which may decrease the computational cost [206]. However, the design (including its geometry and initial distribution) of the domain components, that significant influences both the efficiency and the accuracy of the MMC-based TO, largely depends on the experience of designers. Moreover, as yet, it has been limited to some 2D HX problems, which may be attributed to the difficulties of explicit methods in describing the solid/fluid interface for 3D complex structures [129].

## 7. Conclusions and perspectives

The present paper provides a comprehensive review on the literature of the TO for HXs in the most recent years. Each stage of the TO is analyzed carefully with the statistical figures and comparison tables. Our review shows that the current TO methods are not powerful enough yet to handle the industrial HXs in energy systems: (a) the majority of the researches only deal with the 2D single-flow problems with single objective, limited to the relatively simple flow patterns (like the laminar flow and the turbulence described by the Reynolds-averaged models); (b) merely a small portion of work has manufactured and tested the TO-obtained structures in practice, and (c) few discussions have been conducted on the fabricating constraints of the TO-obtained HXs. Currently, a combination of the density-based method, the FEM, and the gradient-based optimization is the most popular TO method for HXs, since it is a straight-forward transfer from the structural mechanics to the conjugate heat transfer. However, the conjugate heat transfer holds some very different features compared to the structural mechanics, which may not be well addressed in the framework initially developed for the structural TO. Furthermore, three emerging schemes, i.e. ML, MOR, and MMC, aimed to improve the efficiency of TO are discussed. They are initially designed for the structural TO problems and show good performance in that area. Some of them have been extended to handle some simple heat transfer problems, and there has been limited evidence to prove that they are effective in improving the efficiency of the TO for conjugate heat transfer.

Apparently, future effort is still required for the TO of HXs, particularly to: (a) implement the suitable parametrization schemes that can well address the influence of interfaces in the conjugate heat transfer, (b) utilize solvers able to predict more complex flow problems, (c) develop optimizers able to handle multiple objectives, integrating the fabrication constraints, and (d) increase the realization of component level TO-optimized HXs, their experimental testing and validation.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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