Lock-in Amplifier with SEELab

Open lab report

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28-April-2023

ABSTRACT

A lock-in amplifier was constructed using SEELab as the data acquisition and measurement device, and the INA114 instrumentation amplifier. The lock-in detection algorithm was implemented in Python code. The lock-in amplifier was then used for measuring a small resistance and compared with the ideal value for checking its effectiveness. Further, we used it to measure the mutual inductance of a coil which also gave reasonable values. The results showed that despite some limitations such as noise and a slight phase shift, the SEELab device is a promising option for low-budget experiments and data acquisition in various scientific applications. The constructed lock-in amplifier proves to be feasible option for experiments in teaching laboratories.

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Chapter 1

Introduction

A lock-in amplifier is a widely used device for measuring small signals buried in noise. In our experiment, we utilized an all-in-one test and measurement tool called SEELab to construct a lock-in amplifier. The constructed amplifier was capable of sensing voltages in the range of μ volts with reasonable accuracy. To validate the effectiveness of the constructed lock-in amplifier, we measured a very low resistance using the setup and compared it with the value obtained from a commercial device. The mutual inductance of a coil was also measured using this setup. The results obtained were in good agreement with the expected values. Moreover, all the components used in the experiment were inexpensive and readily available, making this cost-effective method for lock-in detection suitable for teaching laboratories.

Chapter 2

Theory

2.1 Priciples of Lock-in Detection

A lock-in amplifier is a device used to measure small voltages in the presence of noise. The commercially available lock-in amplifiers achieve this through a technique called phase-sensitive detection. This technique involves the use of a reference signal and the noisy signal which is to be detected. The two signals are multiplied inside the device, and the resulting signal will have a maximum value only when the reference signal is in phase with the input signal. Here we also use a similar algorithm with minor modifications for lock-in detection.

Consider a signal of the form $V_{sig}\sin(\omega_{sig}t)$, which we want to measure. This signal was obtained as an output from a circuit as the response to an input signal given to the circuit. Suppose we want to measure the component in phase to our reference signal. For that, we construct a reference signal $2\sin(\omega_{ref}t)$, in phase to the input signal and take the product of the output signal with the reference signal, we obtain:

$$2V_{sig}\sin(\omega_{sig}t)\sin(\omega_{ref}t) = V_{sig}[\cos(\omega_{sig}-\omega_{ref})t - \cos(\omega_{sig}+\omega_{ref})t] \tag{2.1}$$

If we take $\omega_{sig} = \omega_{ref}$, then we will get a DC signal with the required amplitude V_{sig}, which can be extracted by performing a low pass filtering. This is how we can extract the in-phase component from the noisy signal. Similarly, suppose we want to extract the quadrature component for our experiment. In that case, we need to multiply with $2\cos(\omega_{ref}t)$, and performing low pass filtering will give the required value.

The total value of the amplitude of the signal can be obtained by:

$$V_{\text{total}} = \sqrt{{V_x}^2 + {V_y}^2} \tag{2.2}$$

V_x and V_y are the in-phase and quadrature components, respectively.

2.2 Measurement of Small Resistance

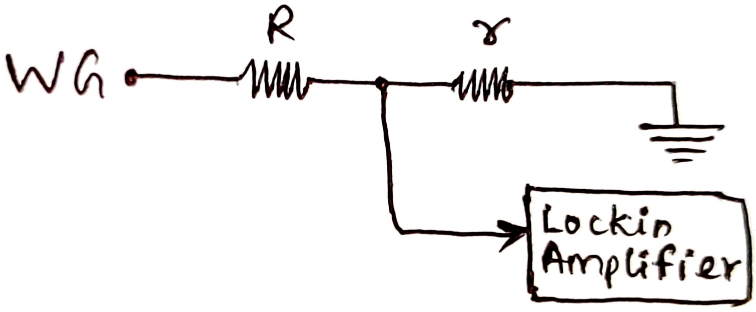


Figure 2.1: Circuit used for measuring small resistance

The circuit shown in Figure (2.1) is used to measure small resistance. Here r is the small resistance value we want to measure, and R is a known resistor whose value is much greater than r ($r \ll R$). The current through the circuit for a given applied voltage V is given by:

$$I = \frac{V}{R+r} \tag{2.3}$$

Since $r \ll R$ we can approximate $R + r = R$. Now the current becomes,

$$I = \frac{V}{R} \tag{2.4}$$

As the resistances are connected in series, the current through them will be the same. Therefore,

$$\frac{V_R}{R} = \frac{V_r}{r} = \frac{V}{R} \quad \implies r = \frac{V_r}{V} R \tag{2.5}$$

Here V_r is the voltage across the small resistance we want to measure. If α is the gain of the amplifier, then the voltage obtained through lock-in detection will be,

$$V_{\text{out}} = \alpha V_r \quad \implies V_r = \frac{V_{out}}{\alpha} \tag{2.6}$$

Substituting this in equation (2.5) we obtain the final working formula as:

$$r = R\frac{V_{out}}{\alpha V} \tag{2.7}$$

The amplification factor α can be obtained through calibration. The circuit for measuring the small resistance is also used for calibration. The only difference is that instead of connecting an unknown resistor in series with R, we connect a known resistor r_0. Using equation (2.7), we will obtain the value of α as:

$$\alpha = \frac{RV_{out}}{r_0 V} \tag{2.8}$$

In this experiment the signal applied from WG is taken as the reference signal.

2.3 Mutual Inductance of a Coil

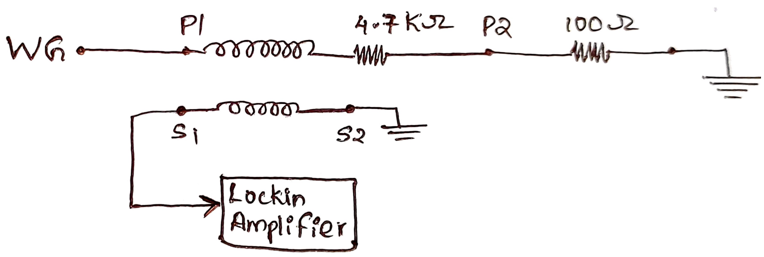


Figure 2.2: Circuit for measuring the mutual inductance of a coil

When two coils are placed close to each other, and a time-varying current is passed through one of them (primary coil), an emf is induced in the other (secondary coil) due to a change in magnetic flux associated with the coil. The amount of emf induced is proportional to the change in magnetic flux. The proportionality constant is called the mutual inductance of the coil. If the primary current varies as:

$$I = I_0 \sin(2\pi f t) = \frac{V}{R} \tag{2.9}$$

Here V is the voltage applied to the primary coil circuit, $R = 4.8$ kΩ is the total resistance in the circuit, and M is the mutual inductance. Then the emf induced in the secondary will be,

$$V_s = -M\frac{dI}{dt} = 2\pi M f \frac{V}{R} \tag{2.10}$$

In this experiment we use lock-in amplifier to measure the voltage across the secondary coil (V_s). If α is the gain of the lock-in amplifier and V_{out} is the voltage obtained after lock-in detection, then the voltage across secondary coil will be $V_s = \frac{V_{\text{out}}}{\alpha}$. Substituting this in equation (2.10) and rearranging we obtain the equation for M as:

$$M = \frac{V_{\text{out}} R}{V 2\pi f \alpha} \tag{2.11}$$

Amplification factor α can be obtained as explained above. In this experiment the signal applied from WG is taken as the reference signal.

Chapter 3

Experimental Setup

3.1 Signal Generation and Collection

In this experiment, we use an all-in-one test and measurement tool called SEELab, shown in Figure (3.1), for constructing a lock-in amplifier. The device can be programmed using Python to generate desired signals and collect the output signals. The input voltage to the external circuit is provided through the pin WG, which can produce signals in the range of 1 Hz-5000 kHz with amplitudes 80mV, 1V, or 3V. The pins A_1 and A_2 are used to collect the reference and output signals, respectively. These pins can detect signals up to 16V. In order to address anomalies in the signals collected during the initial acquisition process, we implemented a strategy of slicing the signal array into two separate arrays. Specifically, we selected the array that contains the signal acquired during the second half of the acquisition process in order to mitigate any potential anomalies associated with the initial part of the acquisition. The Python code for signal generation and collection is shown below.

```
########## IMPORTING ##########

from pylab import *
from scipy.signal import hilbert
from scipy.fft import fft
import eyes17.eyes
import matplotlib.pyplot as plt
import numpy as np
p = eyes17.eyes.open()

########## GENERATING AND CAPTURING A SINE WAVE ##########

amplitude = 2
frequency = 500
p.set_sine_amp(amplitude)
p.set_sine(frequency)
res = p.capture4(2000,10)
```

```
tA1 = res[0][len(res[0])//2::]
tA2 = res[2][len(res[2])//2::]
VA1 = res[1][len(res[1])//2::]
VA2 = res[3][len(res[3])//2::]
```

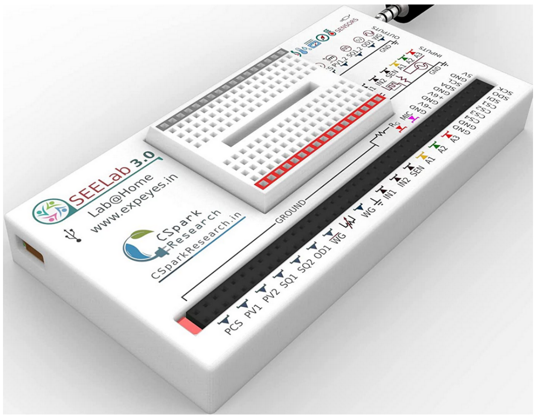


Figure 3.1: SEELab used in this experiment[1]

3.2 Lock-in Detection

Python code was utilized to perform lock-in detection after obtaining the signals. To generate the reference signal from the signal collected through A_1, the signal was scaled to an amplitude of 2. The next step was to produce a 90° ($\pi/2$ radians) phase shift in the signal to create the 90° reference signal. The Hilbert transform in the Scipy library of Python was used to achieve this task. The Hilbert transform is a popular tool used in signal processing, taking a real-valued function $f(t)$ as input and returning another function, denoted by $\mathrm{H}(f(t))$. The Hilbert transform has the property of being able to impart a -90° phase shift in every positive frequency component ($\omega > 0$) present in the signal. As a result, our desired 90° reference signal was easily produced from the collected signal $f(t)$ by executing the operation $-\mathrm{H}(f(t))$, which was subsequently scaled to an amplitude of 2. The below code is used for the creation of reference signals from the collected input. The reference signals obtained

[1] https://m.media-amazon.com/images/I/61PxgkoF13L._SL1500_.jpg

are shown in Figure (3.2).

```
########## CREATING 0-DEG AND 90-DEG REFERENCE SIGNALS #########

# Compute the Hilbert transform
v = VA1
t = tA1
x_hilbert = np.imag(hilbert(v))

# Introduce a 90 degree phase difference
x_90deg = np.real(v * np.exp(1j * np.pi / 2))
ref_0_deg_1 = v/max(v)*2
ref_90_deg = -2*x_hilbert/max(x_hilbert)
ref_0_deg = np.imag(hilbert(ref_90_deg))
plt.plot(t, ref_0_deg, label='0 Deg')
plt.plot(t, ref_90_deg, label='90 Deg')
plt.legend()
plt.show()
```

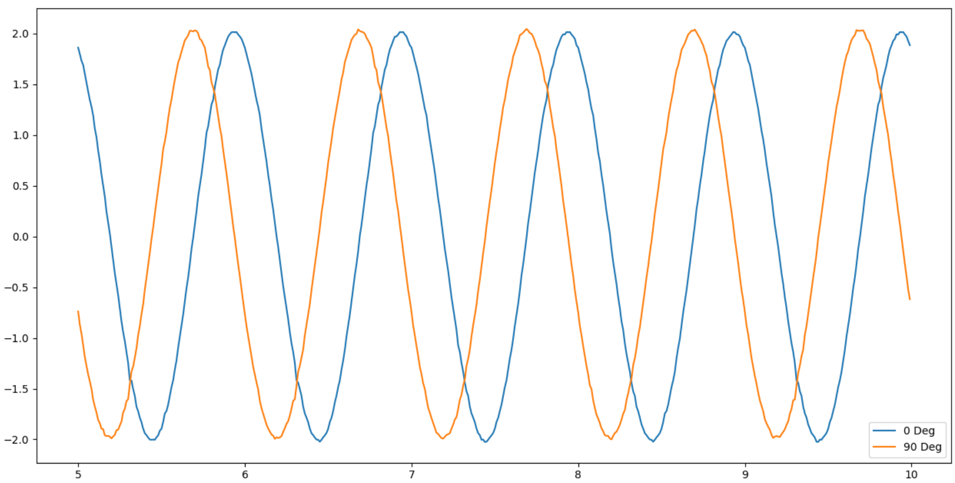


Figure 3.2: Reference signals obtained

The lock-in detection process was carried out as explained in section 2.1 using the generated reference signals. Instead of implementing a low pass filter, we used the Fourier transform of the product. We isolated the DC signal by selecting the element with a frequency of 0, the first element in the array obtained after performing the FFT, which is available in the Scipy library of python. The value obtained from this step was used to calculate V_{pp}. After performing lock-in detection, the in-phase and quadrature components of peak-to-peak voltage were extracted. The code used for lock-in detection is shown below.

```
########## THE LOCK-IN ALGORITHM ##########
v_sig = VA2
fft_input_0_Deg = []
fft_input_90_Deg = []
for i in range(len(v_sig)):
    fft_input_0_Deg.append(v_sig[i]*ref_0_deg[i])
for i in range(len(v_sig)):
    fft_input_90_Deg.append(v_sig[i]*ref_90_deg[i])
fft_output_0_Deg = fft(fft_input_0_Deg)
fft_output_90_Deg = fft(fft_input_90_Deg)

v_pp_0_deg = fft_output_0_Deg[0].real/(len(v_sig))*2
v_pp_90_deg = fft_output_90_Deg[0].real/(len(v_sig))*2
```

3.3 Need for Amplifier

The purpose of a lock-in amplifier is to detect small signals comparable in amplitude to the noise. However, the SEELab device alone cannot detect such small signals. Therefore, we need to amplify the signal before feeding it into the SEELab. Our initial attempt was to use the in-built amplifier of the SEELab, which could measure voltages up to 3.3V through pin $A3$. We could amplify the input signal by connecting a resistor from R_g to the ground. The manual of the SEELab provided the output voltage of the amplifier as follows:

$$V_{out} = V_{in}\left(1 + \frac{R_g}{10000}\right) \tag{3.1}$$

It took us a lot of time to figure out that the formula given in the manual is wrong and the possible correct formula will be:

$$V_{out} = V_{in}\left(1 + \frac{10000}{R_g}\right) \tag{3.2}$$

Moreover, the signals obtained through this amplifier were not appropriately measured by the SEELab, which prompted us to try another more suitable amplification circuit. Subsequently, we attempted to build a non-inverting amplifier with TL082 op-amp IC, intending to achieve a high gain of approximately 10^5, which required a very high feedback resistance. But the signals obtained were not as expected, leading us to explore an alternative solution - an instrumentation amplifier.

We first tried to make an instrumentation amplifier from scratch using TL082 IC. We observed that the output signal from the amplifier circuit exhibited an enormous DC offset of roughly 7V, which proved challenging to remove via a high-pass filter. We resolved the issue to a great extent by using INA114 instrumentation amplifier IC. The gain of the instrumentation amplifier can be calculated using the following equation:

$$G = 1 + \frac{50000}{R_G} \tag{3.3}$$

The internal circuit of the INA114 IC is shown in Figure (3.3). Here in the place of R_G, a 20k potentiometer was connected.

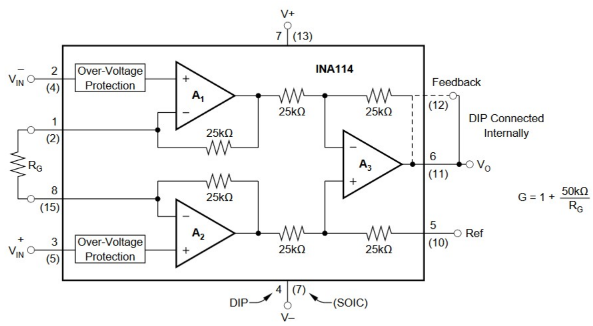


Figure 3.3: Internal circuit of INA114[2]

This amplifier could amplify the signal effectively, so that the SEELab could measure the signal correctly. Here we faced two issues.

1. There was a huge phase shift in the output signal when the gain was changed through the potentiometer. The phase shift was observed to increase as we increased the gain of the amplifier.

2. There was a DC offset of about 1V added to the output signal. This offset can be removed using an offset trimming circuit.

[2] https://doi.org/10.1016/j.progpolymsci.2013.05.001

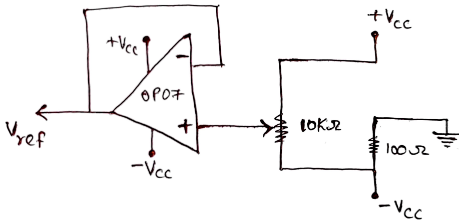


Figure 3.4: Circuit used for removing DC offset

The circuit in Figure (3.4) was used to remove the DC offset. By adjusting the potentiometer, we can change the DC offset.

The phase shift we observed was an inherent characteristic of the amplifier. We had only two options: constructing an external phase shifter or trying a different amplifier circuit. However, since the observed phase shift did not significantly impact our measurements, we decided not to use an external phase shifter. The following section provides data to support this conclusion. The final circuit is shown in Figure (3.5).

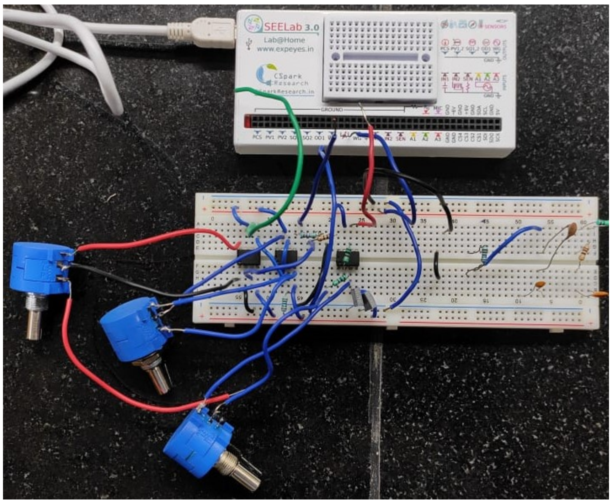


Figure 3.5: Final circuit constructed

Chapter 4

Observations and Calculations

4.1 Verification of working of lock-in Amplifier

The following measurements were carried out to verify the working of the lock-in detection algorithm. Lock-in detection was carried in various signals with same amplitude but differed in phase with respect to the reference signal. The phase shift was introduced using a phase shifter circuit with unit gain. The circuit used is shown in Figure (4.1).

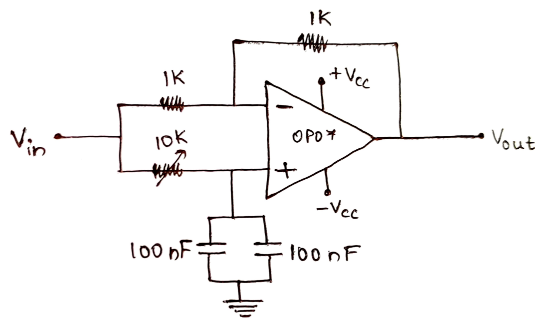
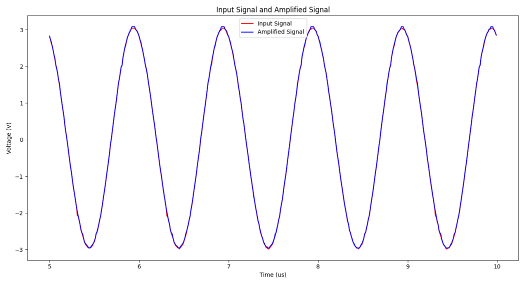
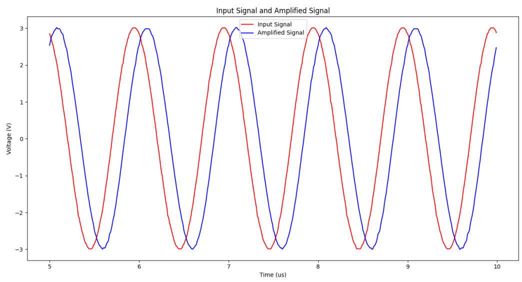


Figure 4.1: Circuit diagram of phase shifter used

The signal generated through WG is collected by A_1 as the reference signal and the same signal with a phase difference introduced through the phase shifter circuit is collected through pin A_2. The phase shifter circuit has unit gain so that the amplitude of the signal is not varied much with a change in gain. The signals obtained are shown in Figure (4.2). The table containing the values is shown in Table (??). The resultant voltage is not varying much even if the phase is changed. This ensures that the lock-in algorithm is working properly.



(a) Signal obtained with $0°$ phase difference



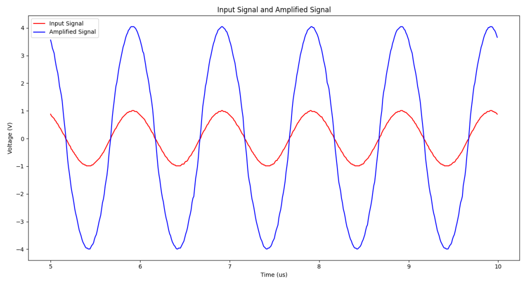
(b) Signal obtained with $50°$ phase difference

Figure 4.2: Signals obtained without the amplifier circuit

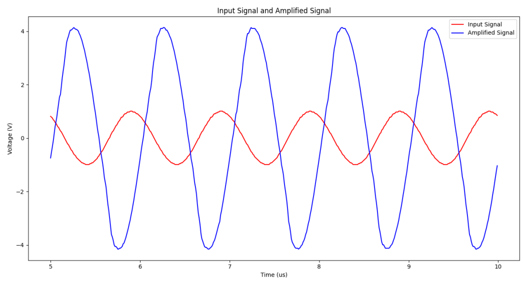
Phase (°)	V_x (V)	V_y (V)	V_{total} (V)
0	6.010	-0.003	6.010
25	5.428	-2.578	6.031
50	3.790	-4.669	6.014
90	-0.137	-5.991	5.993
135	-4.424	-4.060	6.005

Table 4.1: Data obtained in circuit without INA114 amplifier

The same experiment was then carried out by introducing the amplifier circuit. The output from WG is given to both the input of the amplifier circuit and pin A_1. The output from the amplifier circuit is collected through pin A_2. The signals obtained are shown in Figure (4.3). The table containing the values is shown in Table (4.2). As explained above, we had a phase shift in the signal from the amplifier circuit. The data given in Table (4.2) shows that this phase shift is not affecting our measurements to a great extent.



(a) Signal obtained with 0° phase difference



(b) Signal obtained with 135° phase difference

Figure 4.3: Signals obtained with the amplifier circuit

Phase (°)	V_x (V)	V_y (V)	V_{total} (V)
0	8.253	0.057	8.254
25	7.326	-3.827	8.266
50	4.935	-6.622	8.258
90	-0.360	-8.175	8.183
135	-5.889	-5.735	8.216

Table 4.2: Data obtained in circuit with INA114 amplifier

4.2 Measuring a Small Resistance

For calculating the value of small resistance we need the gain of the amplifier. For measuring the gain of the lock-in amplifier, calibration is carried out. The circuit used for calibration is given in Figure (2.1). The values of the resistors used are as follows:

- $R = 21.61\ \text{k}\Omega$
- $r_0 = 9.802\ \Omega$

The gain of the amplifier can be calculated using equation (2.8). The values obtained are shown in Table (4.3).

Frequency (Hz)	V_{out} (V)	Avg(V_{out}) (V)	α
500	2.972 3.008 2.973	2.985	1066.439
1000	2.556 2.549 2.556	2.554	912.539
1500	2.040 2.044 2.044	2.042	729.800
2000	1.734 1.719 1.729	1.727	617.213

Table 4.3: Data for Calibration

The same process is carried out for measuring small resistance after replacing r_0 with r. By substituting α in equation (2.7), the value of resistance can be calculated. The data taken and the values obtained are shown in Table (4.4). The average value of the small resistance is obtained as $1.098\ \Omega$.

4.3 Measuring Mutual Inductance of a Coil

After setting the appropriate gain in which the signal is obtained, calibration is carried out. The values of resistances used for calibration are $R = 21.88\ \text{k}\Omega$ and $r = 10.3\ \Omega$. The data obtained is shown in Table (4.5).

Frequency (Hz)	V_{out} (V)	Avg(V_{out}) (V)	r (Ω)
500	0.343 0.339 0.343	0.342	1.123
1000	0.289 0.291 0.291	0.291	1.115
1500	0.213 0.218 0.220	0.217	1.042
2000	0.198 0.197 0.192	0.196	1.111

Table 4.4: Data used for measuring small resistance

Frequency (Hz)	V_x (V)	V_y (V)	V_{res} (V)	V_{in} (V)	α	α_{avg}
1000	0.885 0.883 0.885	-0.183 -0.183 -0.183	0.904 0.902 0.904	5.962 5.962 5.962	322.183 321.244 322.155	321.861
500	0.900 0.904 0.907	-0.092 -0.093 -0.093	0.904 0.908 0.912	5.962 5.994 5.994	322.162 321.982 323.369	322.504

Table 4.5: Data for calibration

Using the value of α, mutual inductance is then calculated using equation (2.2). The values obtained are shown in Table (4.6). From the table, the average value of M is obtained as $M = 128.358\ \mu H$.

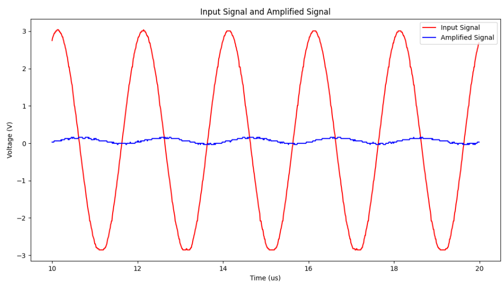


Figure 4.4: Signal collected from primary and secondary coils

Frequency (Hz)	V_x (V)	V_y (V)	V_{res} (V)	V_{in} (V)	M (μH)
1000	-0.060	-0.316	0.322	5.897	129.447
	-0.057	-0.312	0.317	5.865	128.458
	-0.058	-0.313	0.318	5.865	128.738
500	-0.008	-0.159	0.159	5.865	128.644
	-0.007	-0.156	0.156	5.897	125.482
	-0.010	-0.159	0.160	5.865	129.318

Table 4.6: Calculation of mutual inductance

4.4 Error Analysis

The propagation error for any quantity $f(x, y)$ is calculated using the equation:

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2} \tag{4.1}$$

where δx and δy are error in measuring x and y.

4.4.1 Error in r

Using equation (4.1) and equation (2.7), we obtain the error in r as:

$$\sigma_r = \sqrt{\left(\frac{V'_{\text{out}}}{\alpha V}\delta R\right)^2 + \left(\frac{RV'_{\text{out}}}{\alpha^2 V}\sigma_\alpha\right)^2} \tag{4.2}$$

Where $\delta R = 0.01$ kΩ and σ_α is the error in α which can be calculated in a similar way using equations (4.1) and (2.8). The error in α is given by the equation:

$$\sigma_\alpha = \sqrt{\left(\frac{V_{\text{out}}}{r_0 V}\delta R\right)^2 + \left(\frac{V_{\text{out}}R}{r_0^2 V}\delta r_0\right)} \tag{4.3}$$

In the equations V'_{out} and V_{out} are the voltages collected by A_2 during measurement of small resistance and calibration respectively. By substituting the values in equation (4.3) and equation (4.2) the error values can be calculated. The obtained values are shown in Table (4.7). From the table, we obtain σ_r to be 0.001 Ω.

The value of r measured using a commercial mutimeter is 0.993 Ω. Using this value the % error in r is obtained as 10%.

Frequency (Hz)	V_{out} (V)	α	σ_α	V'_{out} (V)	σ_r
500	2.985	1066.439	0.505	0.342	0.001
1000	2.554	912.539	0.432	0.291	0.001
1500	2.042	729.800	0.346	0.217	0.001
2000	1.727	617.213	0.292	0.196	0.001

Table 4.7: Table showing error in α and r

4.4.2 Error in M

Using equation (4.1) and equation (2.7), we obtain the error in M as:

$$\sigma_M = \sqrt{\left(\frac{V'_{\text{out}}}{V 2\pi f \alpha}\delta R\right)^2 + \left(\frac{V'_{\text{out}} R}{V 2\pi f \alpha^2}\sigma_\alpha\right)^2} \tag{4.4}$$

The error in alpha can be calculated using equation (4.3). On substituting the values in equation (4.4) the error in M can be obtained. Table (4.8) shows the values obtained after calculation. Here V'_{out} is the output voltage obtained while measuring M and V_{out} is the output voltage obtained while calibration.

Frequency (Hz)	V_{out} (V)	V'_{out} (V)	σ_α	σ_M (μH)
1000	0.9042	0.322	3.009	1.21E-06
	0.9016	0.317	3.000	1.20E-06
	0.9041	0.318	3.008	1.20E-06
500	0.9041	0.159	3.008	6.01E-07
	0.9085	0.156	3.007	5.86E-07
	0.9124	0.160	3.020	6.04E-07

Table 4.8: Table for error in M

Chapter 5

Results and Discussions

5.1 Results

A lock-in amplifier was successfully constructed using SEELab and INA114 IC. Using the amplifier a small resistance and mutual inductance was measured.

1. Small resistance:
   - $r = 1.098 \pm 0.001\ \Omega$
2. Mutual inductance:
   - $\alpha = 322.182 \pm 3.009$
   - $M = 128.348 \pm 1.210\ \mu H$

5.2 Discussions

- The high DC offset voltages observed with TL082 IC may be due to using a very high feedback resistance in the circuit. A high feedback resistance can cause a high voltage drop due to the input bias current. The TL082 IC has a high input base current of 20 nA. This voltage drop leads to a DC offset in the output. When multiple ICs are connected in a network, each might contribute to the DC offset, and hence we will get a substantial offset voltage. The INA114 IC has a very low input bias current of about 2 nA. Consequently, the DC offset in output obtained from the IC was also low.
- Although the exact cause of the phase shift observed in the instrumentation amplifier output was not determined, it is suspected that capacitive or inductive effects within the amplifier circuit may be responsible. These effects can

introduce a time delay between the input and output signals, resulting in a phase shift. Additionally, increasing the amplifier gain can cause a larger overall signal level at the output, increasing the capacitive and inductive effects and leading to a more significant time delay and phase shift between the input and output signals.

- After verifying the working of lock-in amplifier we used it to measure a small resistance whose ideal value measured through a multimeter was 0.999 Ω. When measured using the lock-in amplifier we obtained a value of with a percentage error of 10%. This demonstrated the effectiveness of the lock-in amplifier. Further, we used it to measure the mutual inductance of a coil.

- While measuring the mutual inductance of coil, the voltage across the secondary coil is expected to have a 90° phase shift. Even though it is not clearly visible in the signal, we can infer this fact by looking at the in-phase and quadrature component of voltages.

5.2.1 Precautions and Possible Sources of Error

- Make sure the connections are made properly and the components are not overheated.

- Set the total number of samples to be collected and the time difference between samples such that multiples cycles of the sine wave are collected.

5.2.2 Future Prospects

- The amplifier can be further improved by incorporating an external circuit to eliminate the phase shift.

- The lock-in algorithm can be made more computationally efficient by modifying the python code.

- Other innovative experiments, such as CV profiling, can be carried out after making necessary modifications.

Chapter 6

Conclusion

Using the INA114 instrumentation amplifier and SEELab as the data acquisition and measurement device, we successfully constructed a lock-in amplifier with python code implementing the lock-in detection algorithm. Despite some limitations, such as noise and a slight phase shift, we could measure small resistance values that closely matched ideal values. The affordability and versatility of the SEELab device make it a promising option for low-budget experiments and data acquisition in various scientific applications. The experiment highlights the potential of SEELab as a helpful tool for basic electronics experimentation and data acquisition.

Chapter 7

References

1. INA114 datasheet
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3. https://csparkresearch.in/seelab3
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