CHAOS IN NON-LINEAR CIRCUITS

A project submitted for the evaluation of

INTEGRATED PHYSICS LABORATORY in SEM-VIII

by

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ABSTRACT

Implementation of non-linear circuits is an interesting way of solving the problem of non-linear differential equations. In this study, we will be investigating the chaotic behavior in non-linear circuits using the Lorenz attractor and the Jerk circuit. Using the numerical methods, we will solve the same equations and compare the results. We also study the emergence of chaos across a range of parameters using their bifurcation diagram.

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Chapter 1

Introduction

Non-linear Circuits are circuit models that follow a set of non-linear differential equations which, in turn, simulate the complex dynamics of the related system. Hence, these non-linear circuits can be used to study those system's evolution through time in a simpler setup. The Lorenz circuit and the Jerk circuit are examples of the same and will be used in our experiments. The required components are of low cost and are commonly available, which further motivates the study of non-linear circuits.

We conduct our experiment with the following objectives:

1. To replicate and study the Lorenz circuit and the Jerk circuit.

2. To solve the same set of equations using the numerical methods and compare results with the experiment.

3. To produce a numerical simulation of bifurcation diagrams and verify experimentally.

Chapter 2

Theory

2.1 Non-linear Circuits

Non-linear differential equations contain terms of non-linearity, like variables with degrees greater than one or those with exponents. Non-linear dynamics deals with such systems that do not respond proportionally to the input. A complicated dynamical system can be explained using a set of non-linear differential equations.

In physics, Chaos theory focuses on systems that, in theory, should be predictable because they follow specific rules. However, in practice, these systems seem predictable for a while, and then they 'seem' to continue randomly, essentially affecting the long-term behavior. Non-linear circuits tend to show such behavior due to the perturbations being increased and carried into long term evolution, especially by the non-linear terms.

Such chaotic systems show properties like aperiodicity, boundedness, sensitivity to parameters, etc. These behaviors are often described by strange attractors that depict the system's long-term behavior, and bifurcation diagrams (explained in section 2.2). Lorenz circuit and Jerk circuit, two examples of non-linear circuits are explained in sections 2.1.1 and 2.1.2 respectively.

2.1.1 Lorenz Circuit

The Lorenz system models the atmospheric convection using a set of three differential equations. They were first determined by Edward Lorenz [3] and are given by:

$$\dot{x} = \alpha(y - x) \tag{2.1}$$

$$\dot{y} = \rho x - xz - y \tag{2.2}$$

$$\dot{z} = xy - \beta z \tag{2.3}$$

where α, ρ, and β are parameters determining the behavior of the system. x, y, and z are the state variables defining the state of the system at any given time. To obtain an experimental solution in reliable range (since the Voltage supply commonly available for the circuit components are 9 V), the variables are scaled as:

$$X = x/10 \tag{2.4}$$

$$Y = y/10 \tag{2.5}$$

$$Z = z/30 \tag{2.6}$$

Hence, the modified equations become:

$$\dot{X} = \alpha(Y - X) \tag{2.7}$$

$$\dot{Y} = \rho X - 30XZ - Y \tag{2.8}$$

$$\dot{Z} = \frac{100}{30}XY - \beta Z \tag{2.9}$$

This system of equations are modeled using the circuit given in Figure 2.1 following the equations:

$$C_1R_5\frac{dX}{d\tau} = -\left(1 + \frac{R_4}{R_1} + \frac{R_4}{R_2}\right)X + \frac{R_4}{R_3}Y \tag{2.10}$$

$$C_2R_{11}\frac{dY}{d\tau} = \frac{R_{10}}{R_8}X - \frac{R_{10}}{R_7}XZ - Y \tag{2.11}$$

$$C_3R_{17}\frac{dZ}{d\tau} = \frac{R_{16}}{R_{14}}XY - \left(1 + \frac{R_{16}}{R_{13}}\right)Z \tag{2.12}$$

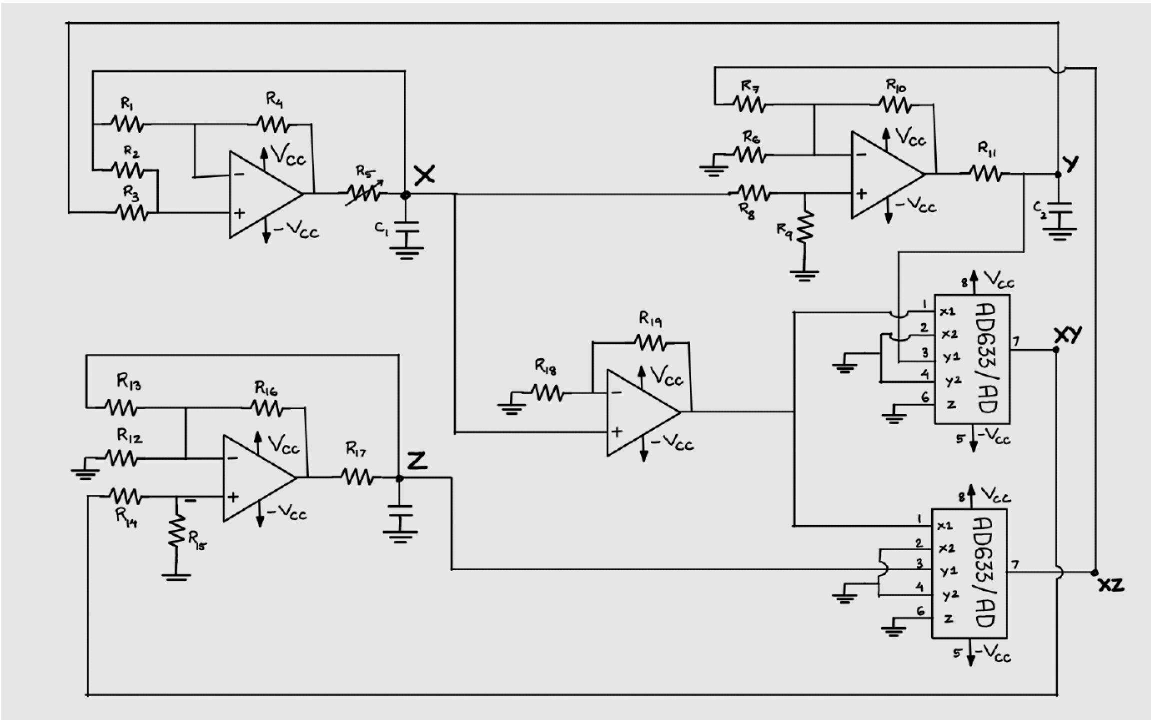

Figure 2.1: Lorenz circuit diagram

The value of the circuit components are chosen to roughly match the parameters ($\alpha = 10, \rho = 28, \beta = 8/3$) which shows chaotic behavior [1]. The temporal rescaling factor is fixed as $\kappa = \frac{1}{C_1 R_5} = \frac{1}{C_2 R_{11}} = \frac{1}{C_3 R_{17}} = 5000$.

The components are chosen as $R_1 = 10k\Omega, R_2 = 100k\Omega, R_3 = 10k\Omega, R_4 = 100k\Omega, R_5 = 1k\Omega, R_6 = 5.6k\Omega, R_7 = 3.3k\Omega, R_8 = 3.6k\Omega, R_9 = 3.19k\Omega, R_{10} = 100k\Omega, R_{11} = 1k\Omega, R_{12} = 3.3k\Omega, R_{13} = 37.5k\Omega, R_{14} = 3.3k\Omega, R_{15} = 3.74k\Omega, R_{16} = 100k\Omega, R_{17} = 1k\Omega, R_{18} = 1k\Omega, R_{19} = 9k\Omega, C_1 = 200nF, C_2 = 200nF, C_3 = 200nF$.

2.1.2 Jerk Circuit

Jerk circuit models after a *jerk systems*, a class of dynamical system in physics described by the non-linear differential equation of the form:

$$\dddot{x} = g(\ddot{x}, \dot{x}, x) \tag{2.13}$$

This equation is commonly referred to as the *jerk equation*. For our experiment, we will be using the particular form [2]:

$$\dddot{x} + A\ddot{x} + f(\dot{x}) + x = 0 \tag{2.14}$$

where $f(\dot{x})$ is given by $IR(e^{\dot{x}/\alpha} - 1)$. The parameter values ($A = 1$, $I = 10^{-12}$, $R = 10^3$, $\alpha = 0.026$) gives the chaotic trajectory solution. When written in state-space form, the equations become:

$$\dot{x} = y \tag{2.15}$$

$$\dot{y} = z \tag{2.16}$$

$$\dot{z} = z - Ax - 10^{-9}(e^{\dot{x}/\alpha} - 1) \tag{2.17}$$

Since it was found that no scaling was needed to fit the voltage limits, the circuit was implemented with the equations:

$$C_1R_1\frac{dX}{d\tau} = y \tag{2.18}$$

$$C_2R_2\frac{dY}{d\tau} = z \tag{2.19}$$

$$C_3R_5\frac{dZ}{d\tau} = -z - \frac{R_5}{R_6}x - R_5h \tag{2.20}$$

where $h = 10^{-9}\left(e^{y/\alpha} - 1\right)$ is implemented using a diode. The corresponding circuit is shown in FIgure 2.2.The temporal rescaling factor is fixed as $\kappa = \frac{1}{C_1R_1} = \frac{1}{C_2R_2} = \frac{1}{C_3R_5} = 1000$.

The components are chosen as $D = $ 1N4148 Diode, $R_1 = 1k\Omega,\ R_2 = 1k\Omega,\ R_3 = 1k\Omega,\ R_4 = 1k\Omega,\ R_5 = 1k\Omega,\ R_6 = 1k\Omega,\ C_1 = 1\mu F,\ C_2 = 1\mu F,\ C_3 = 1\mu F.$

2.2 Chaos and Bifurcations

Chaos in physical systems can be identified through numerous methods. Some of them are:

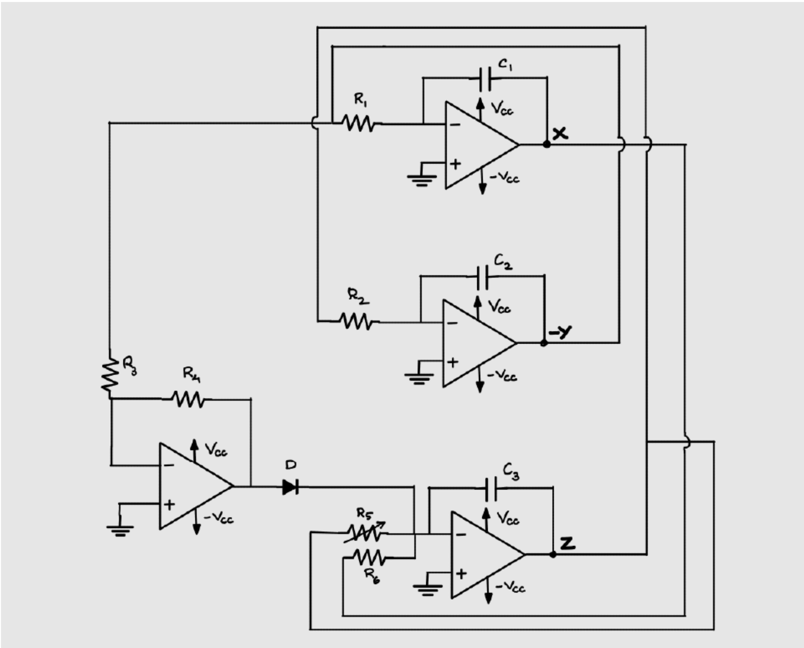

Figure 2.2: Jerk circuit diagram

- *Time series*: Aperiodicity being a character of chaos, we can examine the pattern exhibited in time series.

- *Fourier spectrum*: Aperiodicity can also lead to a continuum of frequencies in the signal which will become evident in Fourier spectrum, but need to be tracked along the change of system leading to chaos.

- *Phase portraits*: By tracking the graphical representation of state variables across time, we can check for closed loops (depicting periodic systems) or multiple distinct loops (depicting aperiodicity) for identifying chaos.

- *Lyapunov exponent*: By checking this mathematical variable that describes the rate of separation of two initially close trajectories in a non-linear dynamic systems, we can identifying chaotic behaviour (depicted by positive Lyapunov exponent).

- ***Bifurcation diagrams***

For our experiments, we will be using bifurcations diagrams to study the chaos in non-linear systems.

Bifurcation diagrams depict the state of the system under observation across a range of parameters. For each parameter value, the bifurcation diagram records the system response. For complicated systems, the range of values can be quite intensive, hence we can track the *equilibrium points* in the system. The equilibrium points are those in which the state variable stays constant for a brief time. In other words, we can consider the solution trajectory to be oscillating between a certain number of equilibrium points which can be tracked across parameters to check for stability.[1] The bifurcation diagram of a simple logistic map equation is shown in Figure 2.3.

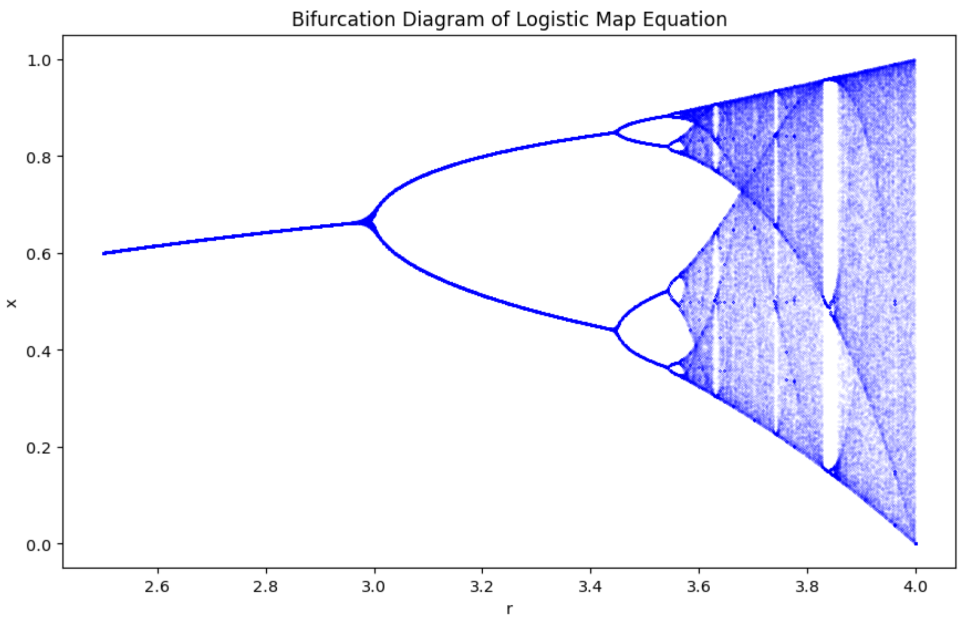

Figure 2.3: Bifurcation diagram for the logistic map equation

[1]This is justified as a regular bifurcation diagram obtained numerically are densely packed points in the regional limit of trajectories. In our approach, we are using just the stable points to indicate the regional limit of the trajectories.

The logistic map equation is given by:

$$x_{n+1} = rx_n(1 - X_n) \tag{2.21}$$

As you can see in Figure 2.3, after a certain value of the parameter r, the stable trajectories bifurcates and again at later values of r. This process is the result of system having new stable points with the variation of the parameter.

This bifurcation process seems to continue and at the end seems to have infinitely many stable trajectories. Oscillating between so many stable points, the trajectory becomes completely aperiodic. This is the point at which the system is termed chaotic.

Chapter 3

Experimental Setup and Methods

3.1 Numerical simulations of circuit

- The circuit equations were used to simulate the system numerically. i.e. Equations (2.10)-(2.12) for Lorenz circuit and equations (2.18)-(2.20) for Jerk circuit were used for simulations.
- The method `scipy.integrate.LSODA` was used to solve the differential equations.

Code implemented for Lorenz circuit simulation is given below:

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import LSODA
from tqdm import tqdm

#--------- Lorenz Attractor -----------#


# Circuit parameters (resistances in Ohms, capacitances in
   Farads, voltages in Volts)

R1 = 10e3
R2 = 100e3
R3 = 10e3
R4 = 100e3
R5 = 1e3 #variable (potentiometer)
R6 = 5.6e3
R7 = 3.3e3
R8 = 3.6e3
R9 = 3.19e3
R10 = 100e3
R11 = 1e3
```

```
R12 = 3.3e3
R13 = 37.5e3
R14 = 3.3e3
R15 = 3.74e3
R16 = 100e3
R17 = 1e3
R18 = 1e3
R19 = 9e3
C1 = 200e-9
C2 = 200e-9
C3 = 200e-9

# Initial conditions
state_0 = [0.1, 0.1, 0.9]
t_0 = 0.0
t_f = 0.04
max_step = 0.005

def equations(t, state):
    X, Y, Z = state

    # Non-linear circuit equations
    dxdt = (- X - ((R4/R1) * X) + ((R4/R2) * X) + (R4/R3) *
        Y) / (C1 * R5)
    dydt = (- Y - ((R10/R7) *X * Z) + ((R10/R8) * X)) / (C2 *
        R11)
    dzdt = (- Z - ((R16/R13) * Z) + ((R16/R14) * X * Y)) /
        (C3 * R17)

    return [dxdt, dydt, dzdt]

# Solve the ODE system using LSODA method
sol = LSODA(equations, t_0, state_0, t_f, vectorized=True)

# initialize the solution array
X = [sol.y[0]]
Y = [sol.y[1]]
Z = [sol.y[2]]
t = [sol.t]

while sol.status != 'finished':
    sol.step()
    t.append(sol.t)
    X.append(sol.y[0])
    Y.append(sol.y[1])
```

```
    Z.append(sol.y[2])

# creating figure
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot(X, Y, Z, lw=0.5,alpha=1)
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Lorenz Attractor')
plt.show()
```

Code implemented for the Jerk circuit simulation is given below:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import LSODA
from tqdm import tqdm

#-------- Jerk Attractor ----------#

# Circuit parameters (resistances in Ohms, capacitances in
   Farads, voltages in Volts)
# FROM Msc thesis

R1 = 1e3
R2 = 1e3
R3 = 1e3
R4 = 1e3
R5 = 1e3 #(potentiometer)
R6 = 1e3
alpha = 0.026
C1 = 1e-6
C2 = 1e-6
C3 = 1e-6

# Initial conditions
state_0 = np.array([0.0, 0.3, 0.0])
t_0 = 0.0
t_f = 0.5
max_step = 0.005

def equations(t, state):
    X, Y, Z = state

```

```
    # Non-linear circuit equations
    dxdt = Y / (C1 * R1)
    dydt = Z / (C2 * R2)
    dzdt = (- Z - ((R5/R6) * X) - (R5 * 1e-9 * (np.exp(Y /
       alpha) - 1))) / (C3 * R5)

    return [dxdt, dydt, dzdt]

# Solve the ODE system using LSODA method
sol = LSODA(equations,  t_0, state_0, t_f, vectorized=True)

# initialize the solution array
X = [ sol.y[0] ]
Y = [ sol.y[1] ]
Z = [ sol.y[2] ]
t = [ sol.t ]

while sol.status != 'finished':
    sol.step()
    t.append(sol.t)
    X.append(sol.y[0])
    Y.append(sol.y[1])
    Z.append(sol.y[2])

# creating figure
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot(X, Y, Z, lw=0.5)
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Jerk Attractor')
plt.show()
```

3.2 Experimental circuits

- The circuit diagrams for Lorentz system and the Jerk system is given in Figures 2.1 and 2.2 respectively.

- The values of the components chosen as mentioned in the theory (Sections 2.1.1 and 2.1.2). The Op-Amps used in the circuits are either TL082 or TL084 powered by a $V_{cc} = 9V$. The AD633/AD multiplier used in Lorenz circuit also used $V_{cc} = 9V$.

- Despite given a particular set of circuit component values, it is advised to replace an appropriate resistor with potentiometer to account for losses in the circuit while obtaining the chaotic solution of the system. We varied R_6 for Lorenz circuit and R_5 for Jerk circuit.

- The points X, Y and Z marked in the respective circuit diagrams were connected to the oscilloscope channels in pairs and viewed in the `XY mode`.

The circuit implementations done in lab are given in Figures 3.1 and 3.2.

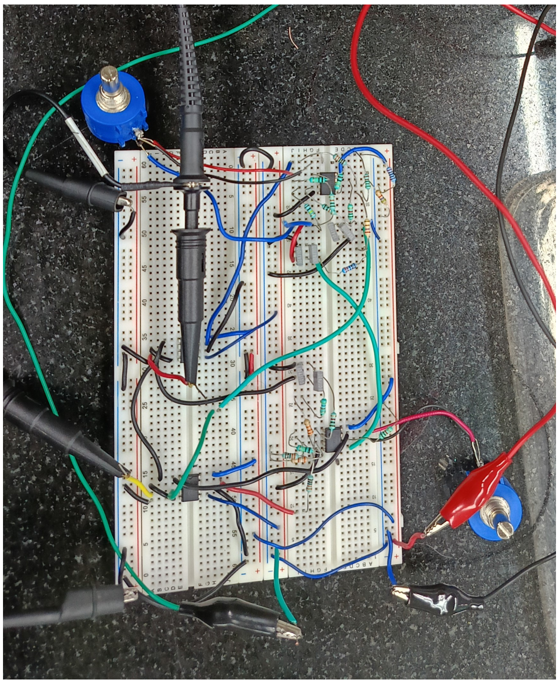

Figure 3.1: Lorenz circuit done in lab

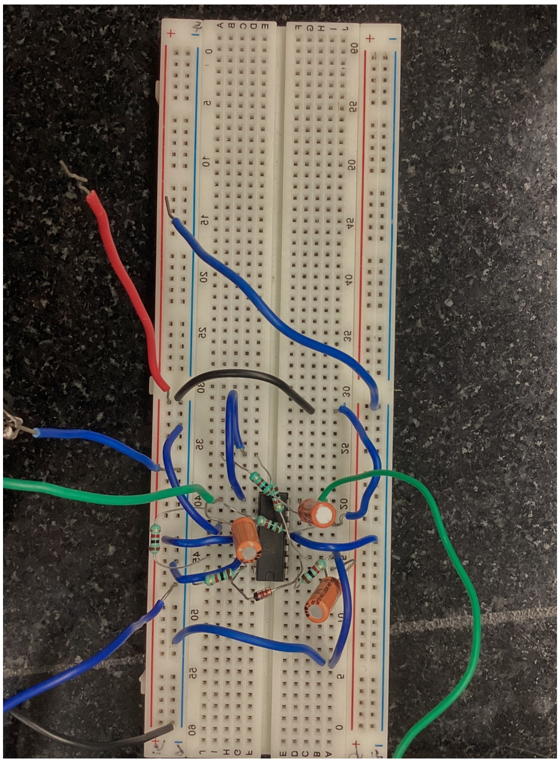

Figure 3.2: Jerk circuit done in lab

3.3 Numerical Bifurcations

- For obtain bifurcation diagrams numerically, the chosen resistor value is varied between a specified range in small steps.
- At each value of the parameter, the circuit simulation is completed and the equilibrium points of each state variable is stored separately along with the corresponding parameter value.
- By plotting the equilibrium points across the parameter, we obtain the bifurcation diagram.

A code implementation of bifurcation diagram of the Jerk circuit is given below (uses the previously defined variables and functions from numerical simulation of circuit):

```
def solve_for_R5(curr_R5):
    R5 = curr_R5
    sol = LSODA(equations,  t_0, state_0, t_f,
       vectorized=True)
    X = [ sol.y[0] ]
    Y = [ sol.y[1] ]
    Z = [ sol.y[2] ]
    t = [ sol.t ]
    while sol.status != 'finished':
        sol.step()
        t.append(sol.t)
        X.append(sol.y[0])
        Y.append(sol.y[1])
        Z.append(sol.y[2])
    return t, X, Y, Z

# R5  ranges from 0.3 kohm to 1.5 kohm, 500 points
R5_values = np.linspace(300, 1500, 500)

# initialize the bifurcation array
X2_max = []
R5_X2_max = []
X2_min = []
R5_X2_min = []
```

```

for R5 in tqdm(R5_values):
    t_, X_, Y_, Z_ = solve_for_R5(R5)

    for i in range(1,len(t_)-1):
        if ((X_[i] >= X_[i-1]) and (X_[i] >= X_[i+1])) and
           not ((X_[i] == X_[i-1]) and (X_[i] == X_[i+1])):
            X2_max.append(X_[i])
            R5_X2_max.append(R5)
        if ((X_[i] <= X_[i-1]) and (X_[i] <= X_[i+1])) and
           not ((X_[i] == X_[i-1]) and (X_[i] == X_[i+1])):
            X2_min.append(X_[i])
            R5_X2_min.append(R5)
# the above can be extended to other state variables

# plot the bifurcation diagram
fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(R5_X2_max, X2_max, 'r.', markersize=0.3,
    label='Maxima')
ax.plot(R5_X2_min, X2_min, 'b.', markersize=0.3,
    label='Minima')
ax.set_xlabel('R5')
ax.set_ylabel('X')
ax.set_title('Bifurcation diagram of X')
ax.legend(markerscale=10.)
plt.savefig(figpath+'\Jerk_sim_bifurcation4_X.png')
plt.close()
```

3.4 Experimental bifurcations

- For the experimental part, the potentiometer placed instead of an appropriate resistor will varied in a specified range.
- The oscilloscope is set to `Roll mode` which takes the input like a time series graph.
- Depending upon the time range being recorded, the most persistent/stable points during that brief time will be registered on the oscilloscope.

- By varying the potentiometer slowly and continuously in the same direction across the value range, these stable points are recorded across the parameter value (varied almost uniformly with time).

Chapter 4

Observations

4.1 Lorenz circuit

Following the experimental methods, we simulated the Lorenz circuit numerically as well as experimentally. The numerically obtained Lorenz attractor in 3-D form is given in Figure 4.1. The different projection of the numerical solution and the experimental solution are given Figures 4.2-4.7 for comparison. This value is very close to the original component settings.

The experimental observations were taken with a setting of $R_6 = 0.821k\Omega$. This value is very close to the original component settings.

As you can see in the following figures, our experimental results agree with the numerical simulations well.

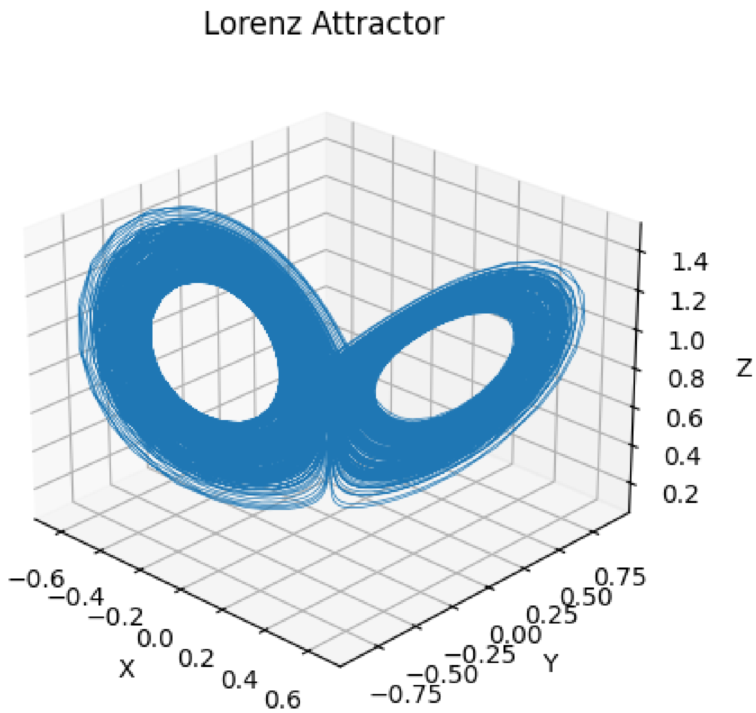

Figure 4.1: Numerical simulation of Lorenz attractor

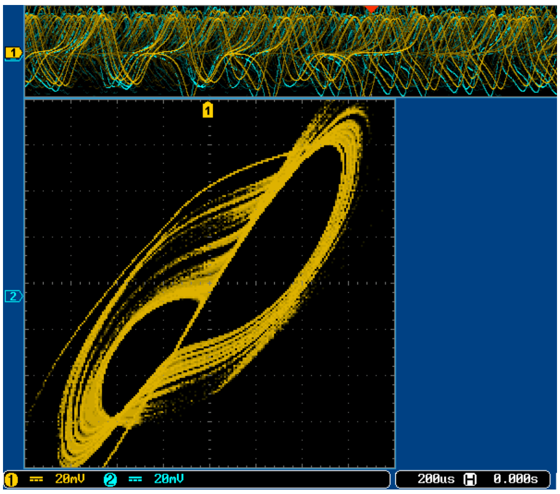

Figure 4.2: XY projection of Lorenz attractor - experimental

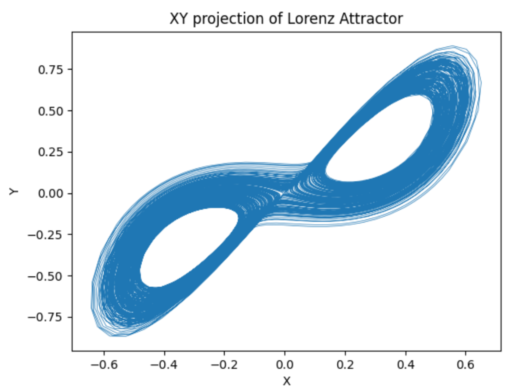

Figure 4.3: XY projection of Lorenz attractor - numerical

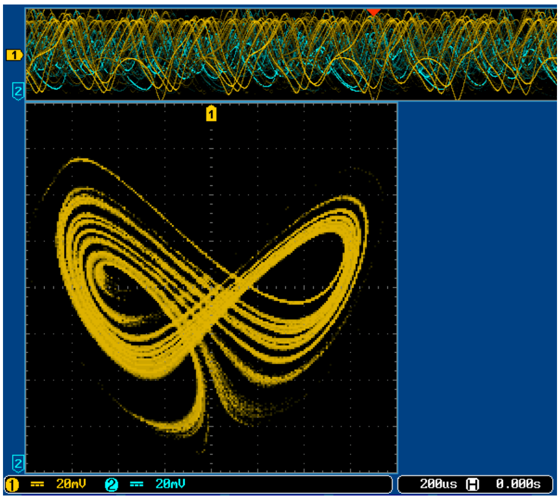

Figure 4.4: XZ projection of Lorenz attractor - experimental

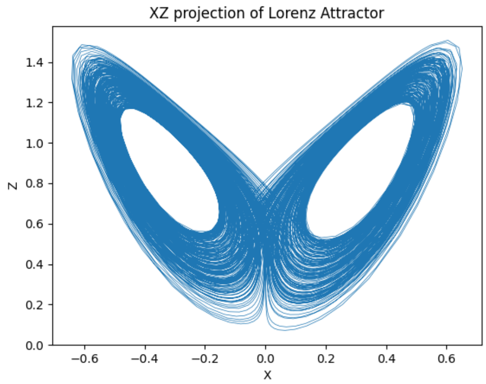

Figure 4.5: XZ projection of Lorenz attractor - numerical

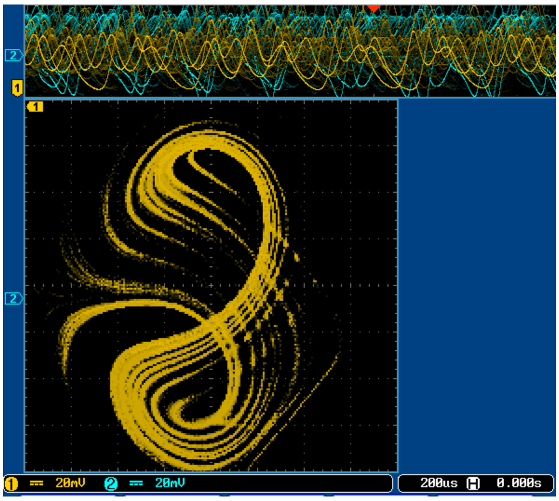

Figure 4.6: YZ projection of Lorenz attractor - experimental

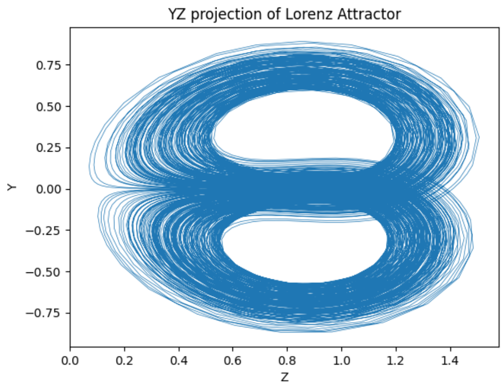

Figure 4.7: YZ projection of Lorenz attractor - numerical

4.2 Jerk circuit

Following the experimental methods, we simulated the Jerk circuit numerically as well as experimentally. The numerically obtained Jerk attractor in 3-D form is given in Figure 4.8. The different projection of the numerical solution and the experimental solution are given Figures 4.9-4.14 for comparison. The experimental observations were taken with a setting of $R_5 = 1.167k\Omega$. This value is very close to the original component settings. As you can see in the following figures, our experimental results agree with the numerical simulations well.

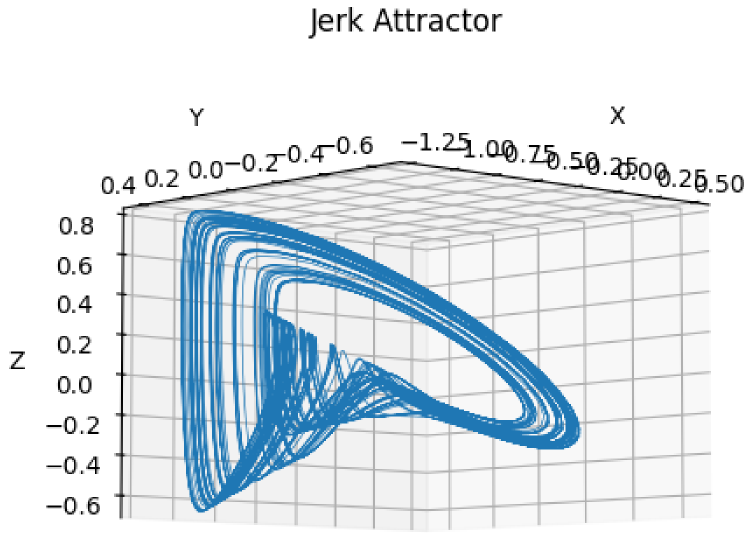

Figure 4.8: Numerical simulation of Jerk attractor

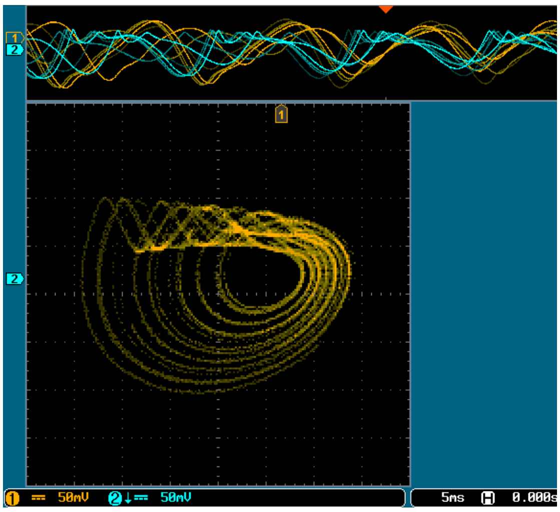

Figure 4.9: XY projection of Jerk attractor - experimental

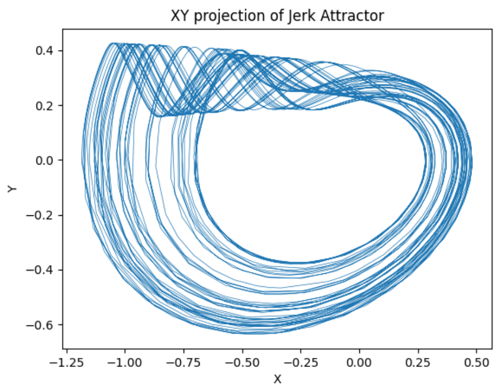

Figure 4.10: XY projection of Jerk attractor - numerical

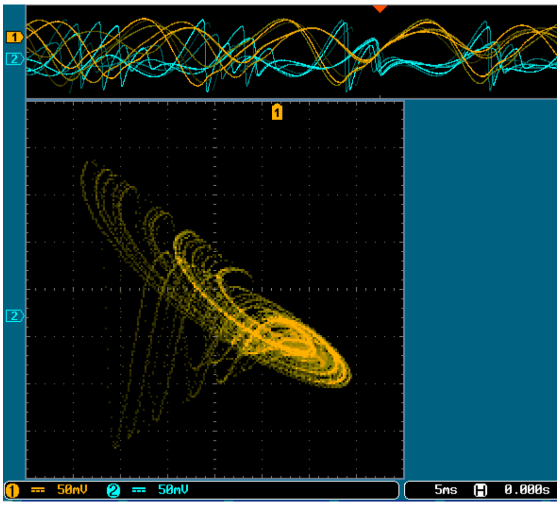

Figure 4.11: XZ projection of Jerk attractor - experimental

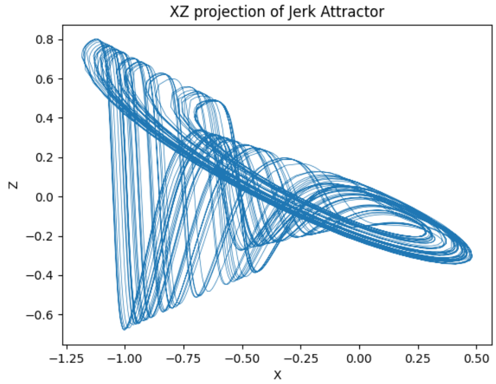

Figure 4.12: XZ projection of Jerk attractor - numerical

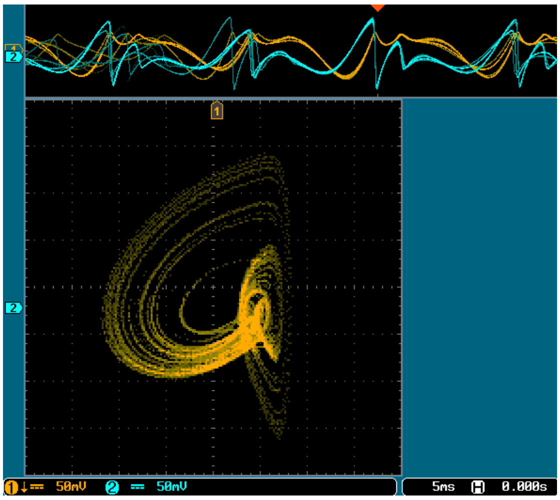

Figure 4.13: YZ projection of Jerk attractor - experimental

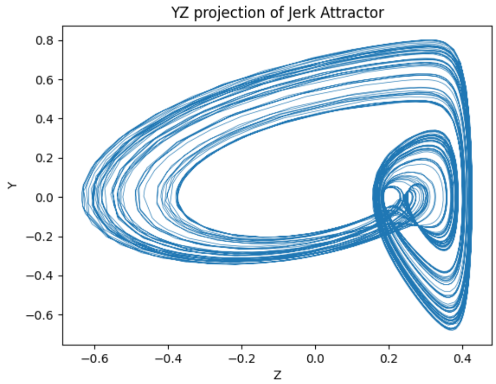

Figure 4.14: YZ projection of Jerk attractor - numerical

4.3 Bifurcation Diagram for Jerk circuit

As per the steps mentioned in experimental methods, we simulated the numerical simulation of bifurcation method and obtained the experimental observation from the circuit. These are given in Figures 4.15 and 4.16 respectively.

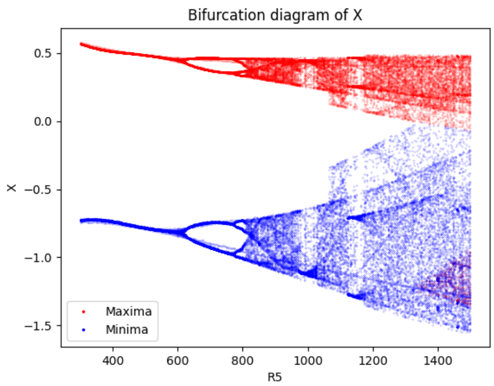

Figure 4.15: Numerical bifurcation diagram for Jerk circuit

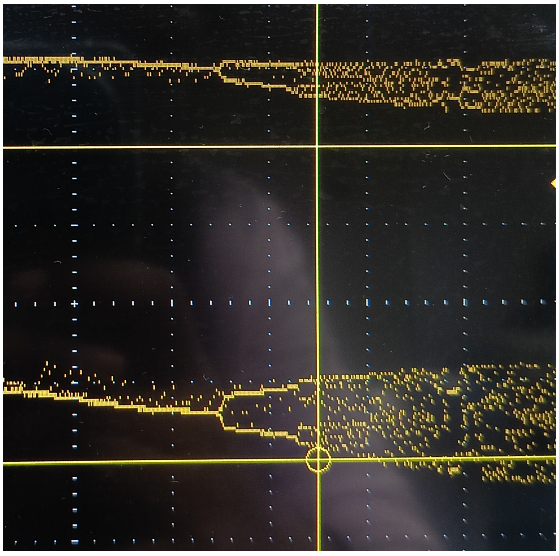

Figure 4.16: Experimental bifurcation diagram for Jerk circuit

As you can see, the occurrence of bifurcation and high number of stable points at the end of parameter range are visible in both figures. The experimental bifurcation diagram and its numerical simulation seem to to be in good agreement as well.

Chapter 5

Discussions

Why choose the circuit equations over original set of equations for numerical analysis?

For both circuits, the equations we acquired for the circuits with same values for components were used to simulate the system. This was done for a fair comparison between the simulation and the experiment, especially when the potentiometer is involved in bifurcation experiments.

A change in the resistor value will the corresponding parameter too. As mentioned before, the numerical simulations also vary the same resistor in bifurcation simulations, making comparison easier.

Why choose `LSODA` for solving the equations numerically?

The method `scipy.integrate.LSODA` (Livermore Solver for Ordinary Differential Equations with Automatic Method Switching) known for employing a combination of stiff and non-stiff solvers, automatically switching between them as needed based on the characteristics of the problem being solved. Stiff ODEs are those where there are very different time scales in the system, and traditional non-stiff solvers can struggle to efficiently handle them. It has wide applications ranging between chemical reactions, population dynamics, and many physical systems.

How is our experimental bifurcation method justified without uniform change in parameter?

AS discussed before, the experimental bifurcation is done by putting the oscilloscope in `Roll mode` which simulates a sort of live feed of the signal. With a sufficient long time division set, we can change the resistance value through potentiometer even slowly and small gaps in between the rotation of the knob would not affect the results due to long time division set.

What about the initial conditions used in the numerical simulations?

Each simulation is ran sufficiently long time to populate the solution in the state space. Usual algorithms in such chaotic systems leave the first few iterations of solution for this purpose. Since we are only taking the equilibrium points, this issue doesn't affect our results.

Further, the initial conditions are kept same for the bifurcation simulations.

Why are there two separate lines in Numerical bifurcations?

In Figure 4.16, the red points depicts the equilibrium points which are maximum (peak) whereas blue points depicts the equilibrium points which are minimum (trough). In terms of equilibrium, both have the same priority and appears in the experimental results as well.

What about the bifurcations in Lorenz system?

Lorenz system is much more complicated than the jerk system. Varying 'any' resistor will not result in a good bifurcation diagram which visually confirms the results. Especially in the experimental part, due to the limited range of bifurcation points, the results in oscilloscope were rather non-conclusive.

Varying the resistor R_6 would not result in proper bifurcations since the effect it has on the system is low (This is a case where the constant of proportionality between parameter and resistor was low, contradicting to out experimental method). On further reading, varying R_10 in a wider range of 200kΩ could be a potential candidate for bifurcations. Then, the maintaining a uniform rate of change of parameter value would be difficult as the required time division will be even larger than before. With a mechanism to change the resistor value rapidly and uniformly even in a larger range could make this possible.

5.1 Sources of error

The possible sources of error are:

- Use of incorrect or less precise circuit components.
- The practical experimental setup (long wires, inefficient connections) causing power dissipation. Overheating of components causing additional losses in circuit.
- Minor irregularities in uniform rate of change in parameter due to human errors.
- Even with small errors in the circuit, the sensitivity of the circuit to the noise (being a chaotic system) can bring about undesirable changes to output.
- The oscilloscope might not register all equilibrium points depending on the time division, sampling mechanism and the sample rate set in the instrument.

5.2 Precautions

Several precautions were taken during the experiment, including:

- Components with values closest to the specified ones are chosen with care. Additionally, the the potentiometer is set in one of the resistors to adjust for the practical errors.
- Components are individually checked and circuit is kept compact as much as possible using efficient connections. The circuit is turned off and kept at a moderate temperature and condition for observations.

- Securely fix all components to breadboard and checkfor short circuits within the compact circuit.
- To make the change in parameter during experimental bifurcation as smooth as possible, we set appropriate time division and rate of change of parameter.
- Adjusting the time division, and balancing it with rate of change of parameter to get better results.

Chapter 6

Conclusions

In summary, this experiment explored the complex dynamics of the Lorenz system and Jerk system through a comprehensive blend of experimental and theoretical methodologies. The investigation covered a spectrum of experiments, examining chaotic solutions, bifurcations as well as the numerical simulations of each.

From the observation made from each non-linear circuit, we obtained the chaotic trajectory with parameters close to the theoretical one. The observation is these attractors itself remain a proof of the chaotic system working well both experimentally and numerically.

The occurrence of bifurcations in Jerk system was confirmed both numerically and experimentally. The parameter values eventually led to a seemingly infinite number of stable points which is the same as having no stable points at all. This can be seen as the manifestation of chaos in Non-linear circuits.

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