Special topics : Radioactivity

Radioactive decay is a classic examples on use of Monte Carlo simulation.

First consider the radioactive decay $A \to B$.

Assume at time $t = 0$ there are $N_A(0)$ nuclei of radioactive atom A of half-life τ_A. Initially $N_B(0) = 0$.

At time $t > 0$ there are $N_A(t)$ nuclei of A remain, while $N_B(t)$ daughter nuclei have risen.

$$A \to B \ : \ \frac{dN_A}{N_A} = -\lambda_A \, dt \ \Rightarrow \ N_A(t) = N_A(0) \exp(-\lambda_A t) \text{ where } \lambda_A = \frac{\ln(2)}{\tau_A}$$

The probability that dN_A nuclei decay over time dt at any instant of time t obviously is dN_A/N_A *i.e.* $\lambda_A \, dt$.

Such radioactive decay process of nuclei can be simulated through Monte Carlo sampling. The process, at its most basic level, is downright simple –

Over a duration of dt, whether a given nuclei of A has decayed or not is determined by

$$\texttt{myrand()} <= \lambda_A \, dt \quad \text{if true} \ \ n_A = n_A - 1, \ n_B = n_B + 1$$

Next consider the radioactive decay $A \rightarrow B \rightarrow C$

Radioactive nuclei A and B have half-life τ_A and τ_B respectively.

At $t = 0$ the number of nuclei are $N_A(0),\ N_B(0) = N_C(0) = 0$. The process is described by coupled equations

$$\frac{dN_A(t)}{dt} = -\lambda_A\, N_A(t), \quad \text{and} \quad \frac{dN_B(t)}{dt} = -\lambda_B\, N_B(t) + \lambda_A\, N_A(t)$$

Monte Carlo procedure is similar to the previous case. In the duration dt, whether nuclei have decayed or not is determined by

$A \rightarrow B \quad : \quad$ `myrand()` $<= \lambda_A\, dt$ if true $n_A = n_A - 1,\ n_B = n_B + 1$

$B \rightarrow C \quad : \quad$ `myrand()` $<= \lambda_B\, dt$ if true $n_B = n_B - 1,\ n_C = n_C + 1$

Hence, the algorithm in both cases involve testing each surviving nuclei is decaying or not in the time interval dt.

In both the cases, `myrand()` returns pseudo-random numbers between 0 and 1.

The probability of observing R decays follow Poisson distribution

$$\mathcal{P}(R) = \frac{\mu^R \, e^{-\mu}}{R!}$$

where $\mu = Np$ is the mean number of decays. In experiments, however, the knowledge of N and p are never known.

In experiments, the count rate *i.e.* number of decays in a time interval is what can be measured. Hence, $\mu = \lambda \, dt$