Assignment-4

1. Use Lagrange interpolation and find $\log_{10} 656$ from the following table:

	x_0	x_1	x_2	x_3
x	654	658	659	661
$\log_{10} x$	2.8156	2.8182	2.8189	2.8202
	y_0	y_1	y_2	y_3

Soln

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} * y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} * y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} * y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} * y_3$$

for $x = 656$,

$$f(656) = \frac{(656-658)(656-659)(656-661)}{\cancel{(2.8156-2.8182}(654-658)(654-659)(654-661)} * 2.8156 + \frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} * 2.8182 + \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} * 2.8189 +$$

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$$\frac{(656-654)(656-658)(656-659)}{(661-654)(661-658)(661-659)} * 2.8202$$

$= 2.8168$ #

(2.) Apply Lagrange's formula to find the polynomial f(x) which passes through the following points and hence find f(3) and slope of the curve at x=2.

~~Soln~~

	x_0	x_1	x_2	x_3
x	0	1	2	5
y	2	3	12	147
	y_0	y_1	y_2	y_3

Soln

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} * y_0 \cancel{(x-x_0)} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} * y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} * y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} * y_3.$$

- for f(3);

$$\frac{(3-1)(3-2)(3-5)}{(0-1)(0-2)(0-5)} * 2 + \frac{(3-0)(3-2)(3-5)}{(1-0)(1-2)(1-5)} * 3 + \frac{(3-0)(3-1)(3-5)}{(2-0)(2-1)(2-5)} * 12 + \frac{(3-0)(3-1)(3-2)}{(5-0)(5-1)(5-2)} * 147$$

$= 35$

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$$f(x) = \frac{(x-1)(x-2)(x-3)}{-10} \times 2 + \frac{x(x-2)(x-5) \times 3}{4} +$$

$$\frac{x(x-1)(x-5)}{-6} * 12 + \frac{x(x-1)(x-2)}{60} * 147$$

$$= \frac{x^3 - 8x^2 + 17x - 10}{-5} + \frac{3x^3 - 21x^2 + 30x}{4} +$$

$$\frac{2x^3 - 12x^2 + 10x}{-1} + \frac{49x^3 - 147x^2 + 98x}{20}$$

$$= \left(\frac{-x^3}{5} + \frac{3x^3}{4} - 2x^3 + \frac{49x^3}{20}\right) + \left(\frac{8x^2}{5} - \frac{21x^2}{4} + 12x^2 - \frac{147}{20}x^2\right) +$$

$$\left(-\frac{17x}{5} + \frac{15x}{2} - 10x + \frac{49}{10}x\right) + \left(\frac{-10}{-5}\right) = x^3 + x^2 - x + 2$$

$\therefore f'(x) = 3x^2 + 2x - 1$

At $x = 2$, $f'(x) = 3 \times 2^2 + 2 \times 2 - 1 = 15$ #

(3) Evaluate $f(1.732)$ from the following set of data using Newton's divided difference interpolation.

x	-2	-1	0	1	2	3
$f(x)$	64	-5.5	-10	-9.5	56	3663.5

Soln Newton's divided difference table; ~~it is because there is~~

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-2	64	-69.5	32.5	-10	5	27.475
-1	-5.5	-4.5	2.5	10	142.375	
0	-10	0.5	32.5	579.5		
1	-9.5	65.5	1771			
2	56	3607.5				
3	3663.3					

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Now,

Newton's divided difference polynomial is given as:

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + (x-x_0)(x-x_1)(x-x_3)\Delta^4 y_0 + (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)\Delta^5 y_0$$

for $x = 1.732$

$$f(1.732) = 64 + (1.732-(-2)) * (-69.5) + (1.732+2)(1.732+1) * 32.5 + (1.732+2)(1.732+1)(1.732-0) * (10) + (1.732+2)(1.732+1)(1.732-0)(1.732-1) * 5 + (1.732+2)(1.732+1)(1.732-6)(1.732-1)(1.732-2) * 27.475$$

$$= -71.1506$$

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(4.) Evaluate $f(1.85)$ and $f(2.4)$ using Newton's forward and backward interpolation formula:

x	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$y=e^x$	5.747	6.050	6.686	7.389	8.106	9.025	9.974

Soln

Here,

Newton's forward difference table ~~is given~~ from given data points:

x	$y=e^x$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.7	5.747	0.303	0.333	-0.266	0.273	-0.272	0.271
1.8	6.050	0.636	0.067	0.007	0.001	-0.001	—
1.9	6.686	0.703	0.074	0.008	0	—	—
2.0	7.389	0.777	0.082	0.008	—	—	—
2.1	8.106	0.859	0.09	—	—	—	—
2.2	9.025	0.949	—	—	—	—	—
2.3	9.974	—	—	—	—	—	—

Now,

Using Newton's forward interpolation formula;

$$f(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0 + \frac{P(P-1)(P-2)(P-3)(P-4)}{5!}\Delta^5 y_0 + \frac{P(P-1)(P-2)(P-3)(P-4)(P-5)(P-6)}{6!}\Delta^6 y_0$$

where, $P = \frac{x - x_0}{h}$

for $x = 1.85$, $x_0 = 1.7$ and $h = 1.8 - 1.7 = 0.1$

$$P = \frac{1.85 - 1.7}{0.1} = 1.5$$

$\therefore f(1.85) = 5.747 + 1.5 * 0.303 + \frac{1.5(1.5-1)}{2!} \times 0.333 +$

$\frac{1.5(1.5-1)(1.5-2)}{3!} \times (-0.266) + \frac{1.5(1.5-1)(1.5-2)(1.5-3) * 0.273}{4!}$

$+ \frac{1.5(1.5-1)(1.5-2) * (1.5 \times 3)(1.5 \times 4) * (-0.272)}{5!} +$

$\frac{1.5(1.5-1)(1.5 \times 2)(1.5 \times 3)(1.5 \times 4)(1.5 \times 5) * 0.271}{6!}$

$= 5.747 + 0.4545 + 0.124875 + 0.016625 + 0.0063984 +$
$0.0031875 + 0.0018525$

$= 6.3544$

for $x = 2.4$, $P = \frac{2.4 - 1.7}{0.1} = 7$

$\therefore f(2.4) = 5.747 + 7 \times 0.303 + \frac{7(7-1)}{2!} \cancel{(7-2)} * 0.333 +$

$\frac{7(7-1)(7-2) * (-0.266)}{3!} + \frac{7(7-1)(7-2)(7-3) \times 0.273}{4!}$

$$+ \frac{7(7-1)(7-2)(7-3)(7-4)}{5!} \times (-0.272) + \frac{7(7-1)(7-2)(7-3)(7-4)(7-5)}{6!} * 0.271$$

$$= 5.747 + 2.121 + 6.993 - 9.31 + 9.555 + (-5.712) + 1.897$$

$$= 11.291$$

Now,

Newton's backward difference table from given ~~tat~~ data points:-

x	$y=e^x$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
1.7	5.747	—	—	—	—	—	—
1.8	6.050	0.303	—	—	—	—	—
1.9	6.686	0.636	0.333	—	—	—	—
2.0	7.389	0.703	0.067	0.266	—	—	—
2.1	8.166	0.777	0.674	0.007	0.273	—	—
2.2	9.025	0.089	~~0.608~~ 0.089	0.608	0.001	-0.272	—
2.3	9.974	0.949	0.09	0.008	0	-0.001	0.271

Now,

Using Newton's backward interpolation formula;

$$f(x) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + \frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_n + \frac{P(P+1)(P+2)(P+3)(P+4)}{5!}\nabla^5 y_n +$$

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$$\frac{P(P+1)(P+2)(P+3)(P+4)(P+5)\nabla^6 y_n}{6!}$$

where,

$$P = \frac{x - x_n}{h} \text{ and } h = x_n - x_{n-1}$$

for $x = 1.85$, $x_n = 2.3$ and $h = 0.1$

$$P = \frac{1.85 - 2.3}{0.1} = -4.5$$

$$\therefore f(1.85) = 9.974 + (-4.5)\times 0.949 + \frac{-4.5\times(-4.5+1)\times 0.09}{2!}$$

$$+ \frac{(-4.5)\times(-4.5+1)\times(-4.5+2)\times 0.008}{3!} +$$

$$\frac{(-4.5)\times(-4.5+1)(-4.5+2)(-4.5+3)\times 0}{4!} +$$

$$\frac{(-4.5)\times(-4.5+1)(-4.5+2)(-4.5+3)(-4.5+4)(-0.001)}{5!}$$

$$+ \frac{(-4.5)\times(-4.5+1)(-4.5+2)(-4.5+3)(-4.5+4)(-4.5+5) * (0.271)}{6!}$$

$$= 9.974 + (-4.2705) + 0.70875 - 0.0525 + 0 + 0.000246$$
$$- 0.0055576$$

$$= 6.3544$$

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for $x = 2.4$, $x_n = 2.3$, $h = 0.1$ & $P = \frac{2.4 - 2.3}{0.1}$

$= 1.$

Using Newton's backward interpolation formula;

$$f(x) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n +$$

$$\frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_n + \frac{P(P+1)(P+2)(P+3)(P+4)}{5!}\nabla^5 y_n +$$

$$\frac{P(P+1)(P+2)(P+3)(P+4)(P+5)}{6!}\nabla^6 y_n$$

$$= 9.974 + (1 \times 0.949) + \frac{1(1+1)}{2!} * 0.09 + \frac{1(1+1)(1+2)}{3!} \times 0.008$$

$$+ \frac{1(1+1)(1+2)(1+3)}{4!} \times 0 + \frac{1(1+1)(1+2)(1+3)(1+4)}{5!} \times 0.001$$

$$+ \frac{1(1+1)(1+2)(1+3)(1+4)(1+5)}{6!} \times 0.271 \times 0.271$$

$= 11.291$

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(5.) Use approximate method of interpolation to get f(3.5) from the given table

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

Soln

Newton's forward difference table from given data points.

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1	7	12	6	0
2	8	19	18	6	0
3	27	37	24	6	0
4	64	61	30	6	0
5	125	91	36	6	—
6	216	127	42	—	—
7	343	169	—	—	—
8	512	—	—	—	—

Now,

Using Newton's forward interpolation method;

$$f(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!} \times \Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!} \times \Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!} \Delta^4 y_0$$

where, $P = \frac{x - x_0}{h}$

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for $x = 3.5$, $x_0 = 1$, $h = 2 - 1 = 1$ &

$$p = \frac{3.5 - 1}{1} = 2.5$$

$$\therefore f(3.5) = 1 + 2.5 \times 7 + \frac{2.5(2.5-1)}{2} \times 12 + \frac{2.5(2.5-1)(2.5-2)}{3!} \times 6 + \frac{2.5(2.5-1)(2.5-2)(2.5-3)}{4!} \times 0$$

$= 42.875$ #

6. Find y at $x = 8$ from the following data using Natural cubic spline interpolation

x	3	5	7	9
y	3	2	3	1

Soln

$h_1 = x_1 - x_0 = 2$ $\qquad f_0 = 3$

$h_2 = 2$ $\qquad f_1 = 2$

$h_3 = 3$ $\qquad f_2 = 3$

$\qquad f_3 = 1$

Using formula 1,

$$h_i a_{i-1} + 2a_i(h_i + h_{i+1}) + h_{i+1} \cdot a_{i+1} = 6\left[\frac{f_{i+1} - f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}\right]$$

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for $i=1$; $\cancel{h_1 a_0}^{0} + 2a_1(h_1+h_2) + h_2 \cdot a_2 =$

$$6\left[\frac{f_2-f_1}{h_2} - \frac{f_1-f_0}{h_1}\right]$$

Since $a_0 = 0$;

$$2a_1(2+2)+2a_2 = 6\left[\frac{3-2}{2} - \frac{2-3}{2}\right]$$

or, $8a_1 + 2a_2 = 6$ —— (i.)

Again,

for $i=2$.

$$h_2 a_1 + 2a_2(h_2+h_3) + \cancel{h_3 a_3}^{0} = 6\left[\frac{f_3-f_2}{h_3} - \frac{f_2-f_1}{h_2}\right]$$

Since $a_3 = 0$;

$$2a_1 + 2a_2(2+2) = 6\left[\frac{1-3}{2} - \frac{3-2}{2}\right]$$

or, $2a_1 + 8a_2 = -9$ —— (ii.)

Solving (i.) and (ii.) we get;

$a_1 = 1.1$

$a_2 = -1.4$

Now,

To Evaluate $f(8)$, we use formula 2 with $i=3$ [Since 8 lies in 3rd interval]

$$P_i(x) = \frac{a_{i-1}}{6h_i}\left[h_i^2 u_i - u_i^3\right] + \frac{a_i}{6h_i}\left[u_{i-1}^3 - h_i^2 u_{i-1}\right] + \frac{1}{h_i}\left[f_i u_{i-1} - f_{i-1} u_i\right]$$

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Here,

$U_2 = x - x_2 = 8 - 7 = 1$
$U_1 = x - x_1 = 8 - 5 = 3$
$U_3 = x - x_3 = 8 - 9 = -1$

Now,

for $i = 3$

$\cancel{P(3)} = P_3(8) = \frac{a_2}{6h_3}\left[h_3^2 U_3 - U_3^3\right] + \frac{a_3}{6h_3}\left[U_2^3 - h_3^2 U_2\right]$
$+ \frac{1}{h_3}\left[f_3 U_2 - f_2 U_3\right]$

$= \frac{-1.4}{6\times2}\left[4\times-1 - (-1)^3\right] + \frac{0}{6\times2}\left[1 - 4\times1\right] +$
$\frac{1}{2}\left[1\times1 - 3\times(-1)\right)$

$= 0.35 + 2$
$= 2.35$ #