Assignment-4.

(1.) What is 3D transformation? Explain the issue in 3D that makes it more complex than 2D.

Ans.

3D transformation is the process of manipulating the view of a 3D object with respect to its original position by modifying its physical attributes through various method of transformations like Translation, Scaling, Rotation, Shear, etc

Properties of 3-D Transformation:

- Lines are presented.
- Parallelism is preserved.
- Proportional distances are preserved.

These are various types of 3D tranformation:

1. Translation.
2. Scaling
3. Rotation.
4. Shear
5. Reflection.

3D transformation using homogenous coordinat are typically represented as 4x4 matrix, known as transformation matrix.

When we model and display a three dimensional scene, there are many more considerations we must take into account besides just including coordinate values as 2D, some of them are:

- Relatively more coordinates points are necessary
- Object boundaries can be constructed with various combination of plane and curved surface.
- Consideration of projection (dimension change with distance) and transparency.
- Many consideration on visible surface detection are remove the hidden surfaces.

(2) Explain 3D rotation and Translation in detail.

Ans.

3D rotation:

Rotation involves changing the orientation of an object around one or more axes. The rotation occurs along an axis. It also includes the angle of rotation that determine the extent to which the object will be turned about that axis. If θ is positive, the rotation will be counter clockwise.

Suppose (x, y, z) is a point in 3D shape, and $(x', y', z', 1)$ is a transformed point in 3D shape, then the rotation along the x-axis would be,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The rotation along the y axis would be,

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The rotation along the z axis would be;

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Here θ represents angle of rotation.

- 3D Translation:

It is the movement ~~moment~~ of an object from one position to another position. Translation is done using translation vertex. There are three ~~vert~~ vector in 3D instead of two. These vectors are in x, y and z directions. Translation in x-direction is represented using

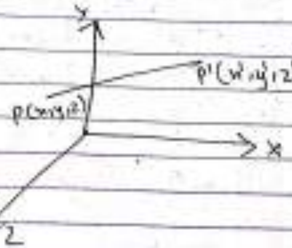
t_x, y direction is represented using t_y and z direction is represented using t_z

A point is translated from position $P(x,y,z)$ to position $P'(x',y',z')$ with the matrix operation as:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

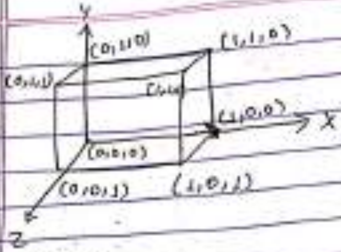
The matrix representation is equivalent to three equations:

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$



(3) Find the new coordinates of a unit cube 90 degree rotated about an axis defined by its end point A(2,1,0) and B(3,3,1).

Soln



Now,
Translating the point (A) to the origin,

$$T = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,
Rotate the A'B' about x axis by angle α until vector $\vec{u}$ lies on xz plane. Where,

$$\vec{v} = \vec{B} - \vec{A}$$
$$= (3,3,1) - (2,1,0)$$
$$= (1,2,1)$$

Unit vector along $\vec{v}$, $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{(1,2,1)}{\sqrt{6}}$

$$= \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$= (a, b, c) \text{ say}$$

and, $d = \sqrt{b^2+c^2}$

$$= \sqrt{\frac{5}{6}}$$

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$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Again,

Rotating A'B' about y axis by angle β until it coincides with axis

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{5/6} & 0 & -1/\sqrt{6} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{6} & 0 & \sqrt{5/6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotating the unit cube 90 degress without z axis

$$R_z(90^\circ) = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The combined transformation rotation matrix about the arbitrary axis becomes.

$$R(\theta) = T^{-1} R_x^{-1}(\alpha) \cdot R_y^{-1}(\beta)(90^\circ) \cdot R_y(\beta) \cdot R_x(\alpha) \cdot T$$

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & -2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{5}/6 & 0 & 1/\sqrt{6} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{6} & 0 & \sqrt{5}/6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{5}/6 & 0 & -1/\sqrt{6} & 0 \\ 0 & 1 & 0 & 0 \\ 1/\sqrt{6} & 0 & \sqrt{5/6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore R(\theta) = \begin{bmatrix} 0.116 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.075 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,
Multiplying $R(\theta)$ by the matrix of original unit cube.

$$P' = R(\theta)\, P$$

$$= \begin{bmatrix} 0.116 & -0.075 & 0.983 & 1.742 \\ 0.742 & 0.667 & 0.073 & -1.151 \\ -0.650 & 0.741 & 0.167 & 0.560 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.650 & 1.687 & 1.834 & 2.816 & 2.7525 & 1.742 & 1.909 & 2.891 \\ -0.558 & -0.484 & 0.256 & 0.184 & -1.225 & -1.152 & -0.409 & -0.483 \\ 1.467 & 1.301 & 0.650 & 0.817 & 0.726 & 0.586 & -0.091 & 0.076 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(4) Explain 3D ~~two~~ reflection, shearing, and scaling with matrix representation.

Ans-

• 3D reflection:

Reflection is also called a mirror image of an object. For the reflection axis and reflection of plane is selected. Three-dimensional reflection are similar to two dimensions. Reflection is 180° about the given axis.

In 3D reflection the reflection takes place about a plane.

(a) About xy-plane (z-axis)

→ The transformation changes the sign of the z-coordinate, leaving the x and y coordinate value unchanged.

$$Rf_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) About xz-plane (y-axis)

→ The transformation changes the sign of the y-coordinate leaving the x and z coordinate unchanged.

$$Rf_{xz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) About yz-plane (x-axis)

→ The transformation changes the sign of the x coordinate leaving the y and z coordinate unchanged.

$$Rf_{yz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 3D shearing:

Shearing is the process of slanting an object in 3D shape either in x, y or z direction. Shearing changes the shape of the object.

(a) z-axis shearing:

→ This transformation alter x and y coordinates values by amount that is proportional to the z value while leaving z-coordinate unchanged.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$x' = x + sh_x z$

$y' = y + sh_y z$

$z' = z$

(b) x-axis shearing:

→ This transformation alter y and z coordinates value by amount that is proportional to the x-value while leaving x-unchanged.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_x & 1 & 0 & 0 \\ sh_y & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$x' = x + sh_x y$
$y' = y$
$z' = z + sh_z y$

- 3D - scaling:

Scaling is used to change the size of the object. The size can be increased or decreased. The scaling three factors are required. S_x, S_y and S_z.

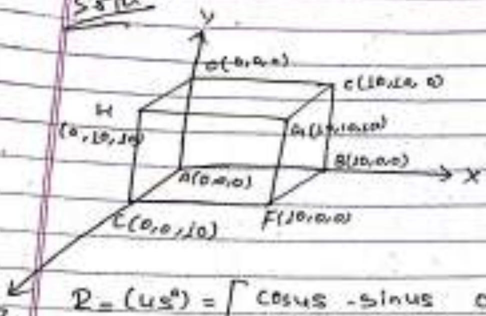
S_x = Scaling factor in x-direction
S_y = Scaling factor in y-direction
S_z = Scaling factor in z-direction

Scaling transformation of a position $P = (x, y, z)$ relative to the coordinate origin can be written as; Matrix represented

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(5) A cube of length 10 units having ~~are~~ one of its corner at origin (0,0,0) and three edges along principal axis. If the cube is to be rotated about z-axis by an angle of 45° in counter clockwise direction, calculate the new position of the point.

Soln



$$R_z(45°) = \begin{bmatrix} \cos 45 & -\sin 45 & 0 & 0 \\ \sin 45 & \cos 45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now,

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{bmatrix} 0 & 10 & 10 & 0 & 0 & 10 & 10 & 0 \\ 0 & 0 & 10 & 10 & 0 & 0 & 10 & 10 \\ 0 & 0 & 0 & 0 & 10 & 10 & 10 & 10 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

(6) What is 3D viewing? Explain the 3D viewing pipeline with block diagram.

Ans. In 3D viewing, we specify a view volume in the world, a projection onto a projection plane, and a viewpoint on the view surface conceptually objects in 3D world are clipped against the 3D view volume and are then projected. The contents of the projection plance, called the window, are then transformed (mapped) in to the viewport for display.

Viewing in 3D involves the following consideration;

- We can view an object from any spatial position eg. in front of an object, Behind the object, in the middle of a group of the objects, inside an object.

- 3D descriptions of objects must be projected on to the flat viewing surface of the output device.

- The clipping windows enclose a volumn of a space.

3D - Viewing Pipeline

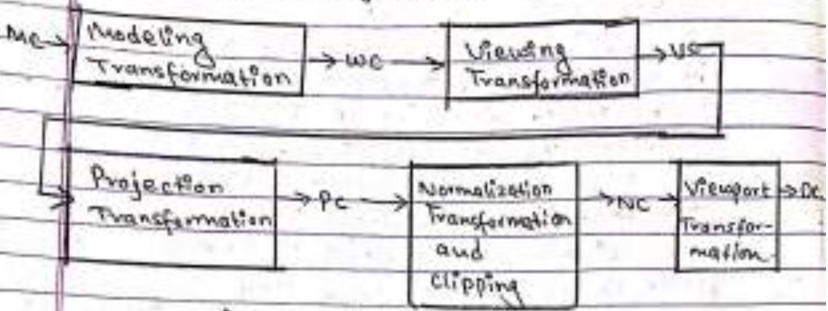


fig: 3D viewing Pipeline.

General three-dimensional transfomation pipeline, from modeling coordinates to world coordinates to viewing coordinates to projection coordinates to normalized coordinates and untimately, to device coordinate.

- Modeling transformation and viewing transformation can be done by 3D transfomation. The viewing coordinate system is used in graphics package as a reference for speofying the observe viewing position as the position of the projection plane. Projection operation converts the viewing coordinate description (3D) to coordinate position on the projection plane (2D) Normalization transformation and clipping and viewport transformation maps the coordinate positions on the projection plane to the output device.

(2.) What is projection? Explain types of projection in detail. And differentiate between Parallel and perspective projection.

Ans-

Projection is array method of mapping three dimensional (3D) objects into two dimensional (2D) view ~~point~~ plane (screen). In general, projection transforms a N dimension points to N-1 dimension. There are two types of projections

(1.) Parallel projection:

In parallel projection, coordinate positions are transfered to view plane along parallel line.

A parallel projection preserves relative proportion ~~sides~~ of an object ~~are obtained by but doesn't give~~ so that accurate views of various sides of an object are obtained but doesn't give realistic representation of 3D objects. Can be used for exact measurement so parallel line remains parallel.

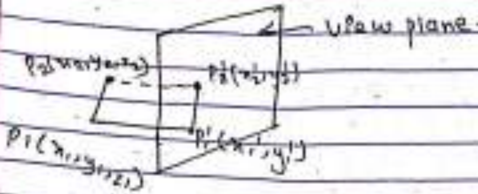
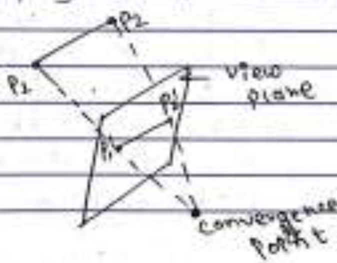


fig: Parallel projection of an object to view plane.

(b) Perspective projection:

In perspective projection, objects position are transformed ~~by~~ to the view plane along lines that coverge to point behind view plane.

A perspective projection produces realistic views but doesnot preserve ~~real~~ relative proportions. Projections of distance objects from view plane are smaller than the projection of objects of the same size that are closer to the projection plane



The difference between parallel and perspective projection are as follows:

Parallel projection	Perspective projection
- Coordinate position of object are transfered into view plane along parallel line.	- Coordinate points are ~~the~~ ~~4g~~ transferred into view plane along lines that converges to a point called convergence point
- Relative proportion of object are maintained.	Relative position of object not maintained.
- It gives accurate view of object.	If view plane is nearest to object image appear larger and if view plane is farther image appear smaller.
- It doesnot give realistic view of object	It gives more realistics perspective view of object as per human n
- Used in engineering and architectural drawing	Used ~~for~~ in building design, rail track design.

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(8) What is the difference between a window and viewpoint. Why is required to map an object from a window to a viewpoint? Explain.

Ans: The difference between window and a viewpoint are:

	Windows	Viewpoint
(1)	World coordinate area selected for display.	Device coordinate area selected for display.
(2)	Region created according to the world coordinate.	Region created acording to the device coordinate.
(3)	Helps to determine the section of the scene to be displayed.	Helps to determine the selection of the scene ~~of th~~ to be displayed.
(4)	What is to be viewed.	Where it is to be viewed.

The process of mapping the world coordinate scene to device coordinate is called viewing transformation or windows ~~transfor~~ to viewport transformation.

It is required to map an object from a window to a viewport because :-

1. By changing the position of the viewport, we can view objects at different positions on the display area of an output.

2. By viewing the size of the viewports. we can change size of displayed objects.

3. Zooming effects can be obtained by successively mapping different sized window on a fixed size newport.

4. Panning effect are produced by moving a fixed size window across the various aspects in a scene.