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Now

$$\frac{d^2y}{dx^2} = \frac{1}{4}\left[0.6 - 0.1 + \frac{11}{12} * (-1.3)\right]$$

$$= -0.1729$$

## Assignment - 5

(1) Estimate the coeff. of $y = ax + b$ from the following data using ~~test~~ least square method.

| $x$ | 0 | 2 | 5 | 7 |
|---|---|---|---|---|
| $y$ | -1 | 5 | 12 | 20 |

Sol$^n$

let the linear eq$^n$ be: $y = ax + b$.

Normal equation to find a and b will be.

$$\Sigma y = nb + a\Sigma x$$

$$\Sigma xy = b\Sigma x + a\Sigma x^2$$

Table to find $\Sigma x$, $\Sigma y$, $\Sigma xy$, $\Sigma x^2$

| $x$ | $y$ | $xy$ | $x^2$ |
|---|---|---|---|
| 0 | -1 | 0 | 0 |
| 2 | 5 | 10 | 4 |
| 5 | 12 | 60 | 25 |
| 7 | 20 | 140 | 49 |
| $\Sigma x = 14$ | $\Sigma y = 36$ | $\Sigma xy = 210$ | $\Sigma x^2 = 78$ |

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Using eqⁿ (i.) and (ii.), $36 = 4b + 14a$

$210 = 14b + 78a$

Solving (i.) and (ii.) we get;

$a = 2.8966$

$b = -1.1379$

$\therefore$ The required linear equation is $= -1.1379 + 2.8966x$.

② Use regression method to fit the circle $y = a + bx + cx^2$ to the data given ~~da~~ below:

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

Solⁿ

Given regression equation is: $y = a + bx + cx^2$.

The normal equations are:

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

Table to find $\Sigma x, \Sigma y, \Sigma x^2, \Sigma x^3, \Sigma x^4, \Sigma xy, \Sigma x^2 y$

| $x$ | $y$ | $x^2$ | $x^3$ | $x^4$ | $xy$ | $x^2y$ |
|---|---|---|---|---|---|---|
| -3 | 1.1 | 9 | -27 | 81 | -3.3 | 9.9 |
| -2 | 1.3 | 4 | -8 | 16 | -2.6 | 5.2 |
| -1 | 1.6 | 1 | -1 | 1 | -1.6 | 1.6 |
| 0 | 2.0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 2.7 | 1 | 1 | 1 | 2.7 | 2.7 |
| 2 | 3.4 | 4 | 8 | 16 | 6.8 | 13.6 |
| 3 | 4.1 | 9 | 27 | 81 | 12.3 | 36.9 |
| $\Sigma x = 0$ | $\Sigma y = 16.2$ | $\Sigma x^2 = 28$ | $\Sigma x^3 = 0$ | $\Sigma x^4 = 196$ | $\Sigma xy = 14.3$ | $\Sigma x^2 y = 69.9$ |

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Substituting values of eq$^n$ (i), (ii) ~~are~~ we get;

$16.2 = 7a + 0b + 28c$ — (iv)

$14.3 = 0a + 28b + 0c$ — (v)

$69.9 = 28a + 0b + 196c$ — (vi)

Solving (iv), (v) and (vi) we get;

$a = 2.0714$

$b = 0.5107$

$c = 0.0607.$

$\therefore$ The required linear equation is: $y = 2.0714 + 0.5107x + 0.0607x^2.$

(3) Fit the following set of data is a curve of the form $y = ae^{bx}$

| | | | | |
|---|---|---|---|---|
| $x$ | 2 | 4 | 6 | 8 |
| $y$ | 25 | 38 | 56 | 84 |

Sol$^n$

Regression eq$^n$ in exponential form is given as

$$y = ae^{bx}.$$

The normal eq$^n$ are:

we have,

$$y = ae^{bx}$$

Taking log on both sides,

$$\log y = \log(ae^{bx})$$

or, $\log y = \log a + \log e^{bx}$

or $\log y + \log a + bx\log e$

let $\log y = Y$

$\log a = A$

$b\log e = B$

when,

$Y = A + Bx$

we find values of A and B using,

$\Sigma Y = nA + B\Sigma x$ —(I)

$\Sigma xY = A\Sigma x + B\Sigma x^2$ —(II)

Table to find $\Sigma x$, $\Sigma Y$, $\Sigma xY$, $\Sigma x^2$

| $x$ | $y$ | $x^2$ | $Y = \log(y)$ | $x \cdot Y$ |
|---|---|---|---|---|
| 2 | 25 | 4 | 1.3979 | 2.7959 |
| 4 | 38 | 16 | 1.5798 | 6.3192 |
| 6 | 56 | 36 | 1.7482 | 10.4892 |
| 8 | 84 | 64 | 1.9243 | 15.3944 |
| $\Sigma x =$ 20 | $\Sigma y =$ 203 | $\Sigma x^2 =$ 120 | $\Sigma Y = 6.6502$ | $\Sigma x.Y = 34.9985$ |

Subsituting values in eqn (I) and (II);

$6.6502 = 4A + 20B$ —(III)

$34.9985 = 20A + 120B$ —(IV)

Solving (III) and (IV) we get;

$A = 1.22565$

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i.e. $\log a = 1.22565$

or, $a = 10^{1.22565}$

$= 16.8132$

$B = 0.08738$

i.e. $b \log e = 0.08738$ $b = \frac{0.08738}{\log e} = 0.2012.$

Required eqn becomes;

$$y = 16.8132\, e^{0.2012x}.$$

(4.) Fit the following set of a data to a curve of the form $y = ax^2 + \frac{b}{x}$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | -1.51 | 0.99 | 8.88 | 7.66. |

Soln:

Given equation;

$$y = ax^2 + \frac{b}{x}$$

The normal equation is given as:

$$\Sigma y = a\Sigma x^2 + b\Sigma \frac{1}{x} \quad \text{— (1.)}$$

$$\frac{\Sigma y}{x^3} = a\Sigma \frac{1}{x} + b\Sigma \frac{1}{x^4} \quad \text{— (11.)}$$

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To find the value of variables following table is used:

| $x$ | $y$ | $x^2$ | $1/x$ | $x^3$ | $y/x^3$ | $x^4$ | $1/x^4$ |
|---|---|---|---|---|---|---|---|
| 1 | -1.51 | 1 | 1 | 1 | -1.51 | 1 | 1 |
| 2 | 0.99 | 4 | 0.5 | 8 | 0.1237 | 16 | 0.0625 |
| 3 | 8.88 | 9 | 0.33 | 27 | 0.3289 | 81 | 0.0123 |
| 4 | 7.66 | 16 | 0.25 | 64 | 0.1196 | 256 | 0.0039 |
| $\Sigma x = 10$ | $\Sigma y =$ 16.02 | $\Sigma x^2 =$ 30 | $\Sigma 1/x =$ 2.08 | $\Sigma x^3 =$ 100 | $\Sigma y/x^3 =$ -0.9377 | $\Sigma x^4 =$ 354 | $\Sigma 1/x^4 =$ 1.0787 |

So equation becomes,

$$16.02 = 30a + 2.08b \quad \text{--- (iii.)}$$

$$-0.9377 = 2.08a + 1.0787b \quad \text{--- (iv.)}$$

Solving eq^n (iii.) and (iv.),

$a = 0.6864$

$b = -2.1949$

$\therefore$ The req^d equation is;

$$y = 0.6864x^2 - \frac{2.1949}{x}$$