# Assignment 6.

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1. find $f'(x)$ and $f''(x)$ at $x=10$.

| $x$ | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| $f(x)$ | 0.25 | 1 | 2.2 | 4 | 6.5 | 8.5 |

Soln

Given interval size : $h = 4-2 = 2$

(i) $\frac{dy}{dx}$ at $x=10$

(ii) $\frac{d^2y}{dx^2}$ at $x=10$

Using Newton's backward interpolation formula is find derivative of $y$ as:-

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \frac{\nabla^5 y_n}{5} + \dots\right.$$

Thus, table to find Newton's backward differences:

| $x$ | $y$ | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ | $\nabla^4 y$ | $\nabla^5 y$ |
|---|---|---|---|---|---|---|
| 2 | 0.25 | [illegible] | — | — | — | — |
| 4 | 1 | 0.75 | — | — | — | — |
| 6 | 2.2 | 1.2 | 0.45 | — | — | — |
| 8 | 4 | 1.8 | 0.6 | 0.15 | — | — |
| 10 | 6.5 | 2.5 | 0.7 | 0.1 | -0.05 | — |
| 12 | 8.5 | 2 | -0.5 | -1.2 | -1.3 | -1.25 |

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for $x_n = 10$, $\nabla y_n = 2.5$, $\nabla^2 y_n = 0.7$, $\nabla^3 y_n = 0.1$, $\nabla^4 y_n = -0.05$

$$\frac{dy}{dx} = \frac{1}{2}\left[2.5 + \frac{0.7}{2} + \frac{0.1}{3} - \frac{0.05}{4}\right]$$

$= 1.4354$

Now,

$$\frac{d^2y}{d^2x} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n\right]$$

$$= \frac{1}{4}\left[0.7 + 0.1 + \frac{11}{12} \times (-0.05)\right]$$

$= 0.18854$

Hence,

$f'(x) = 1.4354$

$f''(x) = 0.18854$ at $x = 10$.

(2) find $y'(x)$ and $y''(x)$ at $x = 0.1$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | 0. | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| $y(x)$ | 30.13 | 31.62 | 32.87 | 33.64 | 33.95 | 33.81 | 33.24 |

Soln

Given interval size: $h = 0.1 - 0 = 0.1$

(1.) $\frac{dy}{dx}$ at $x = 0.1$.

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(11) $\frac{d^2y}{d^2x}$ at $x = 0.1$

Using Newton's forward interpolation formula to find derivative of y as:

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \frac{\Delta^5 y_0}{5} - \dots\right]$$

Table to find Newton's forward difference;

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ | $\Delta^5 y$ | $\Delta^6 y$ |
|---|---|---|---|---|---|---|---|
| 0 | 30.13 | 1.49 | -0.24 | -0.24 | 0.26 | -0.27 | 0.2 |
| 0.1 | 31.62 | 1.25 | -0.48 | 0.02 | -0.01 | 0.02 | — |
| 0.2 | 32.87 | 0.77 | -0.46 | 0.01 | 0.01 | — | — |
| 0.3 | 33.64 | 0.31 | -0.45 | 0.02 | — | — | — |
| 0.4 | 33.95 | -0.14 | -0.43 | — | — | — | — |
| 0.5 | 33.81 | -0.57 | — | — | — | — | — |
| 0.6 | 33.24 | — | — | — | — | — | — |

for $x_0 = 0.1$, $\Delta y_0 = 1.25$, $\Delta^2 y_0 = -0.48$, $\Delta^3 y_0 = 0.02$,
$\Delta^4 y_0 = -0.01$, $\Delta^5 y_0 = 0.02$

$$\frac{dy}{dx} = \frac{1}{0.1}\left[1.25 - \left(\frac{-0.48}{2}\right) + \frac{0.02}{3} + \frac{0.01}{4} + \frac{0.02}{5}\right]$$

$$= 15.0316$$

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Now,

$$\frac{d^2y}{d^2x} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.48 + 0.02 + \frac{11}{12} * -0.01 - \frac{5}{6} * 0.02\right]$$

$$= -52.5833$$

$\therefore y'(x) = 15.0316$

$\therefore y''(x) = -52.5833.$

3. find $y'(1)$ and $y''(1)$

| x | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|---|
| y | 6 | 3 | 2 | 1.2 | 0.8 |

Soln

Given interval size : $h = 1.0 - 0.5 = 0.5$

(i) $\frac{dy}{dx}$ at $x = 1$ (ii) $\frac{d^2y}{dx^2}$ at $x = 1$

Using Newton's forward interpolation formula to find derivative of y as:

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots\right]$$

| $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 0.5 | 6 | -3 | 2 | -1.8 | 2 |
| 1.0 | 3 | -1 | 0.2 | 0.2 | — |
| 1.5 | 2 | -0.8 | 0.4 | — | — |
| 2.0 | 1.2 | -0.4 | — | — | — |
| 2.5 | 0.8 | — | — | — | — |

for $x_0 = 1$, $\Delta y_0 = -1$, $\Delta^2 y_0 = 0.2$, $\Delta^3 y_0 = 0.2$,

$$\therefore \frac{dy}{dx} = \frac{1}{0.5}\left[-1 - \frac{0.2}{2} + \frac{0.2}{3}\right] = -2.0667$$

$$\therefore \frac{d^2y}{d^2x} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0\right] = \frac{1}{(0.5)^2}\left[0.2 - 0.2\right] = 0$$

$\therefore y'(1) = -2.0667$

$\therefore y''(1) = 0.$

(4.) find $f'(3)$ from the following table:-

| $x$ | 2 | 4 | 8 | 12 | 16 |
|---|---|---|---|---|---|
| $f(x)$ | 20 | 23 | 30 | 35 | 40 |

Sol: Newton's divided difference table is given as follows:-

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| $x$ | $y=f(x)$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 2 | 20 | 1.5 | 0.0416 | -0.0104 | 0.0011 |
| 4 | 23 | 1.75 | -0.0625 | 0.0052 | — |
| 8 | 30 | 1.25 | 0 | — | — |
| 12 | 35 | 1.25 | — | — | — |
| 16 | 40 | — | — | — | — |

Newton's divided difference interpolation polynomial is:-

$$f(x) = y_0 + (x-x_0)y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + (x-x_0)(x-x_1)(x-x_3)\Delta^4 y_0$$

$$= 20 + (1.5x-3) + (x^2-6x+8) * 0.0416 + (x^2-6x+8)(x-8) * (-0.0104) + (x^2-6x+8) * (x^2-8x-12x+96) * 0.0011$$

$$= 20 + (1.5x-3) + (0.0416x^2 - 0.2496x + 0.3328) + (x^3-6x^2+8x+48x-64) * -0.0104 + (x^4-6x^3+8x^3+48x^2-64x-12x^3+72x^2-96x+96x^2-576x+768) * 0.0011$$

$$= 20 + (1.5x-3) + (0.0416x^2 - 0.2496x + 0.3328) + (x^3-14x^2+56x-64) * (-0.0104) + (x^4-26x^3+224x^2-736x+768) * 0.0011$$

$$= 20 + (1.5x-3) + (0.0416x^2 - 0.2496x + 0.3328) - 0.0104x^3 + 0.1456x^2 - 0.5824x + 0.6656 + 0.0011x^4 - 0.0286x^3 + 0.2464x^2 - 0.8096x + 0.8448$$

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$= 0.0011x^4 - 0.039x^3 + 0.4336x^2 - 0.1416x + 18.8432$

Now

$f'(x) = 0.0044x^3 - 0.117x^2 + 0.8672x - 0.1416$

Substituting, $x = 3$

$f'(3) = 0.0044 \times (3)^3 - 0.117 * (3)^2 + 0.8672 * 3 - 0.1416$

$= 0.1188 - 1.053 + 2.6016 - 0.1416$

$= 1.5258$ #