

# Assignment 1

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## 1 Scheme Testing

All four schemes were tested using the code `hwk1sol.m` by Dr. Daniel Appelo. The first trial used  $k = 0.5h$ . Schemes 1 and 3 were found to be unstable, and the other two were found to be stable. The second trial used  $k = 0.5h^2$ . In this trial, Scheme 1 was found to be unstable and Scheme 2 was found to be stable, as before. Scheme 3 did not blow up at  $t = 2$ , but some oscillations were introduced which became larger in amplitude with time, leading to the conclusion that the scheme was unstable. Scheme 4 remained stable but was very dissipative. These results are summarized in the plots below.

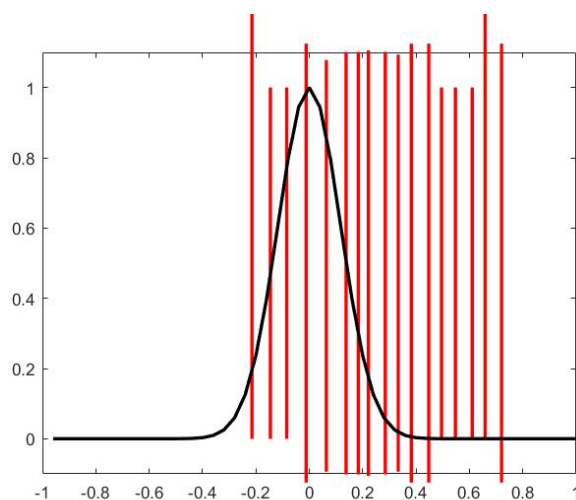


Figure 1: Forward Difference Scheme with  $k = 0.5h$

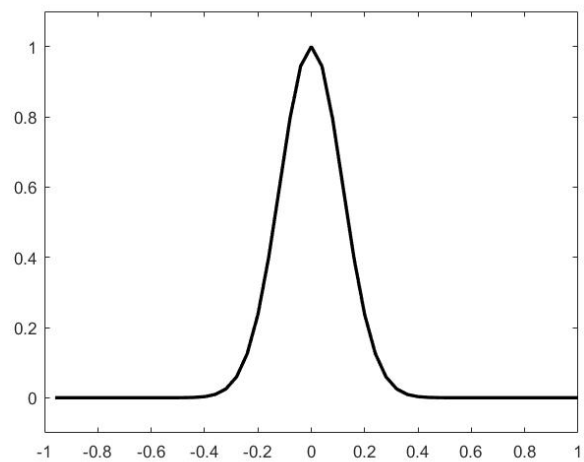


Figure 2: Forward Difference Scheme with  $k = 0.5h^2$

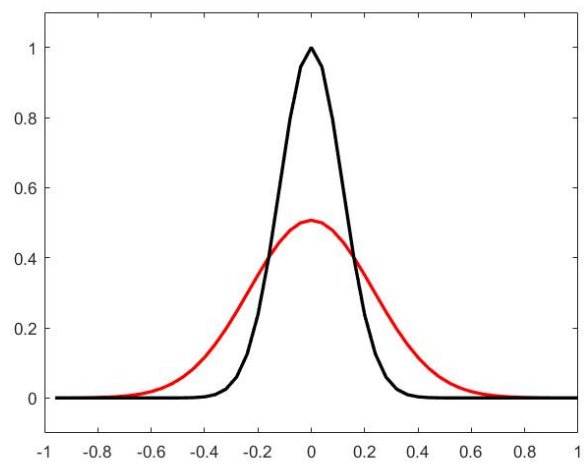


Figure 3: Backward Difference Scheme with  $k = 0.5h$

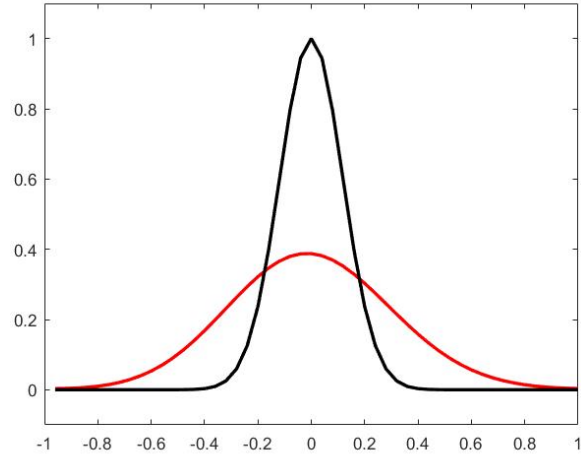


Figure 4: Backward Difference Scheme with  $k = 0.5h^2$

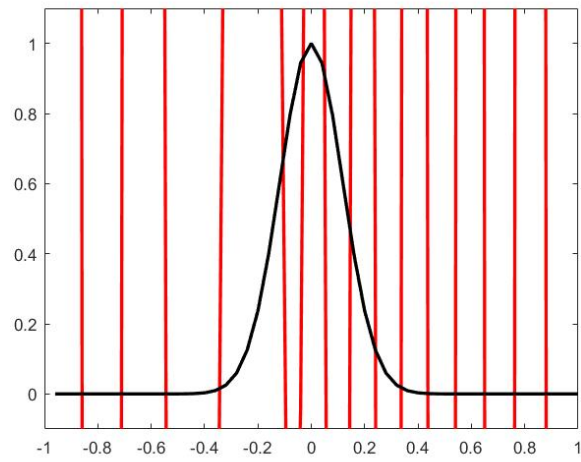


Figure 5: Central Difference Scheme with  $k = 0.5h$

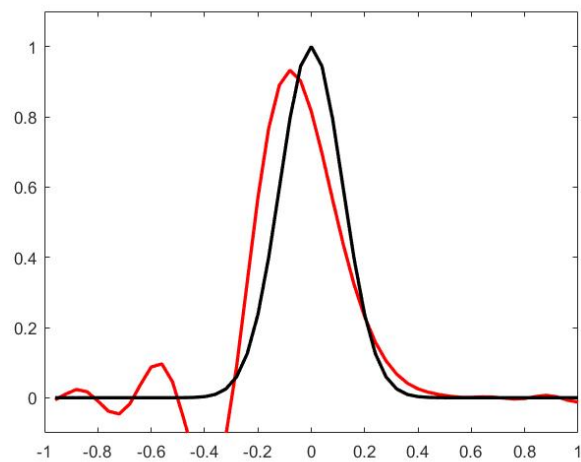


Figure 6: Central Difference Scheme with  $k = 0.5h^2$

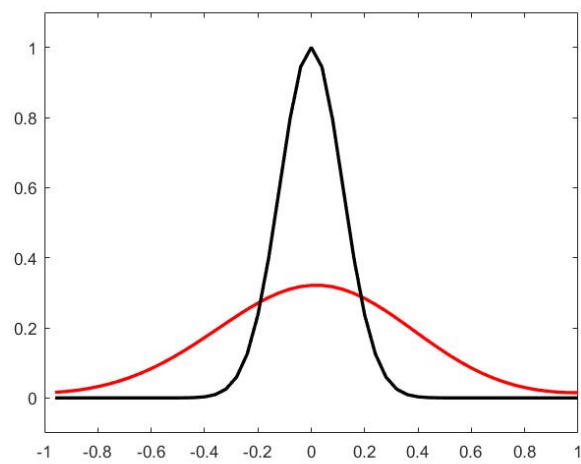


Figure 7: Lax-Friedrichs Scheme with  $k = 0.5h$

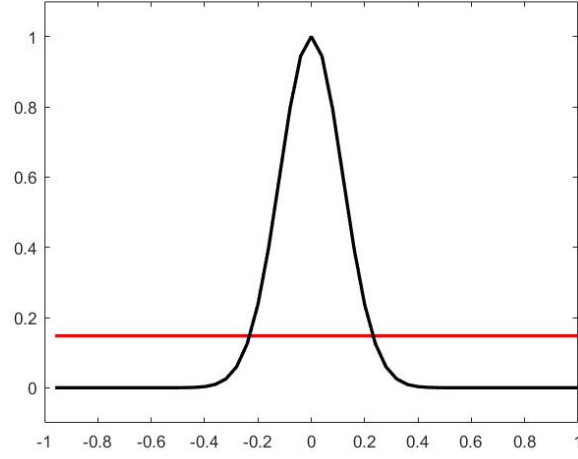


Figure 8: Lax-Friedrichs Scheme with  $k = 0.5h^2$

## 2 Random Choice Methods

The first random choice scheme (Scheme 5) was a 75-25 mix of Schemes 2 and 1 (Implemented in `hwk1sol-rand.m`). This scheme was found to be stable, as the following plot shows.

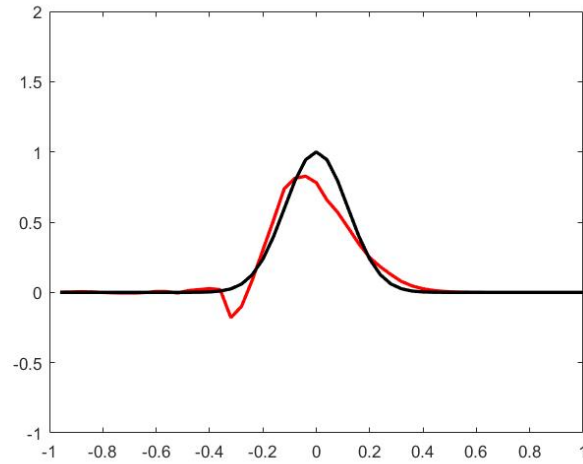


Figure 9: Random Choice Scheme where Scheme 2 is selected 75 percent of the time, and Scheme 1, 25 percent of the time, with  $k = 0.5h$

We plot the error of this scheme in the uniform norm as a function of  $h$ , below. The error appears to decrease, roughly, like  $h$ , indicating first order convergence.

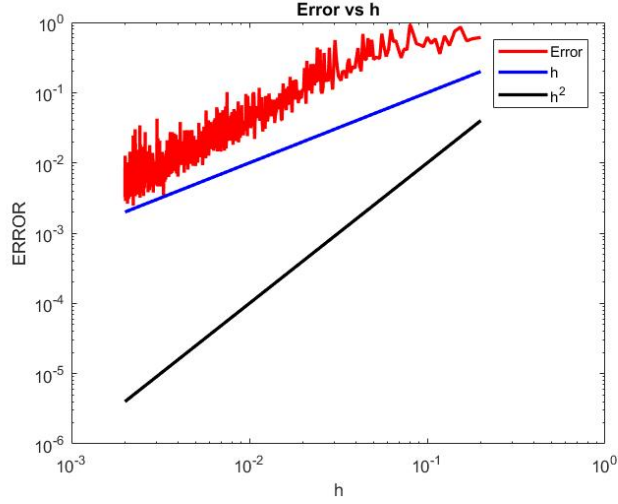


Figure 10: Error vs Space Step Size for Scheme 5

For comparison, an error plot for Scheme 2. The red line appears to be approximately parallel to the blue line, indicating first order convergence in space.

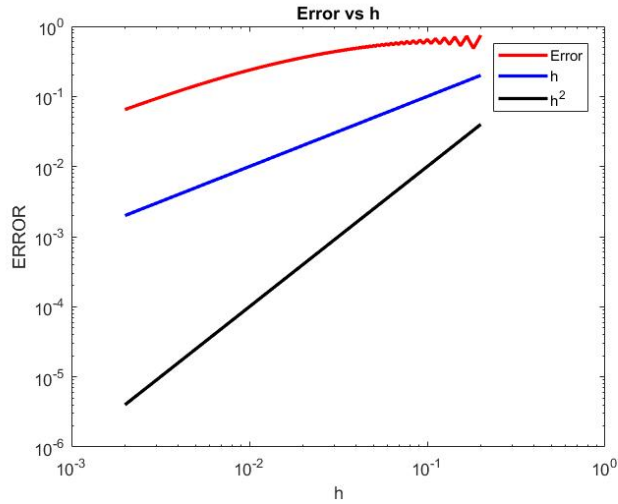


Figure 11: Error vs Space Step Size for Scheme 2

The second random choice method (Scheme 6), a 50-50 mix of Schemes 2 and 3 (hwk1sol-rand2.m), is stable and appears to have roughly second-order convergence. (Although I can't figure out why.)

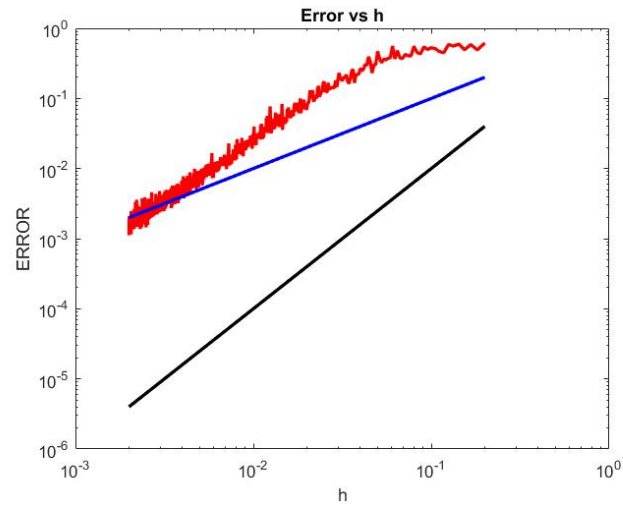


Figure 12: Error vs Space Step Size for Scheme 6