APPM 6610, HW3

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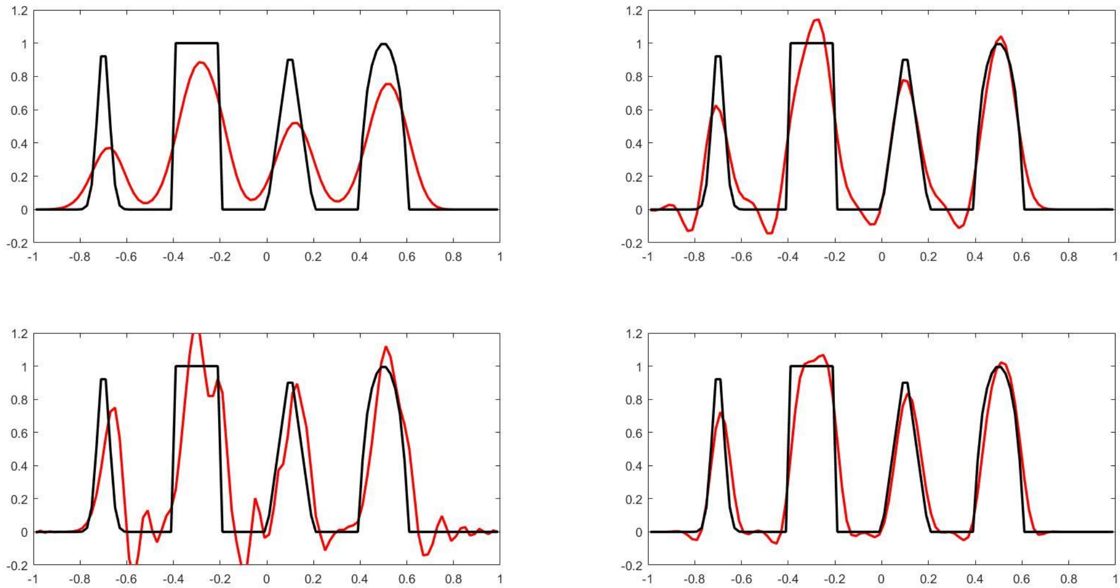
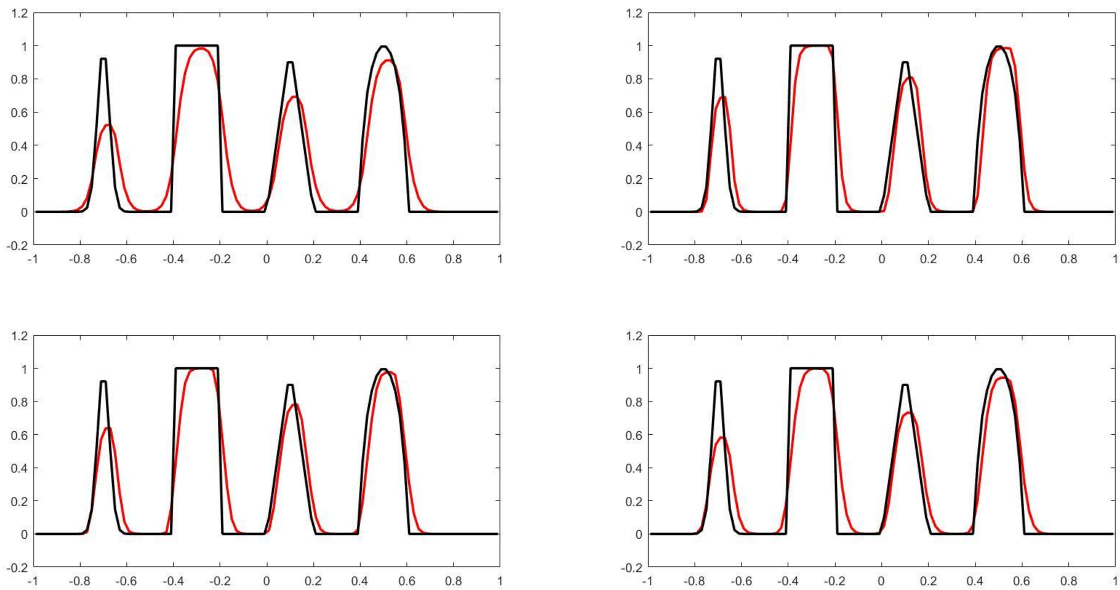
1. FV SCHEMES FOR THE ADVECTION EQUATION

All the schemes below use step sizes of h = 0.020 for space and k = 0.018 for time. The first four pictures show, in order plots after t = 2 for Upwind, Lax Wendroff, Beam-Warming, and Fromm Schemes. The next four pictures show similar plots for Minmod, Superbee, MC, and Van Leer Schemes.

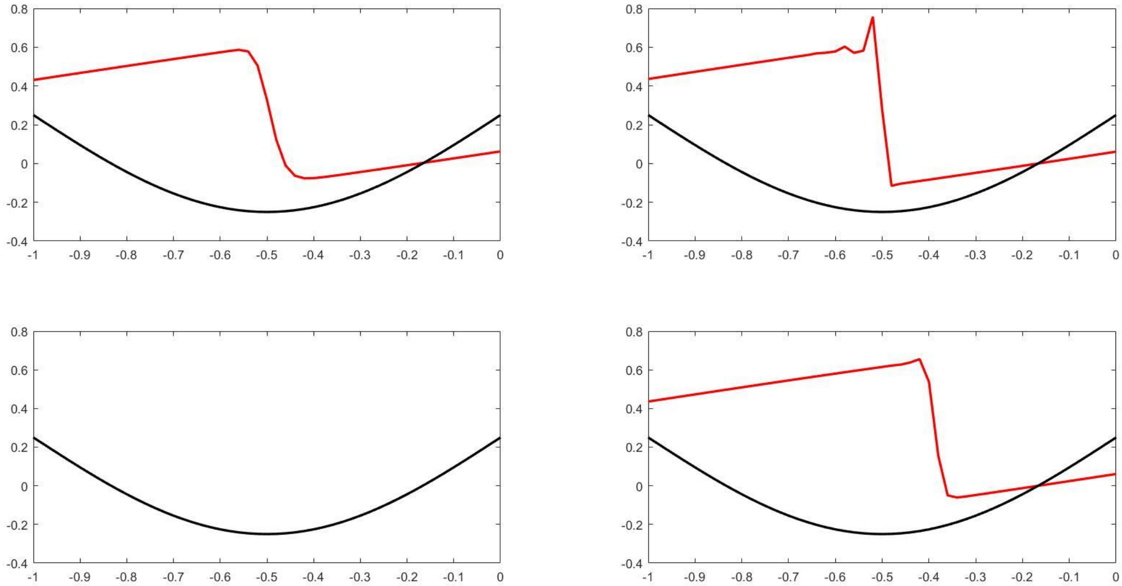


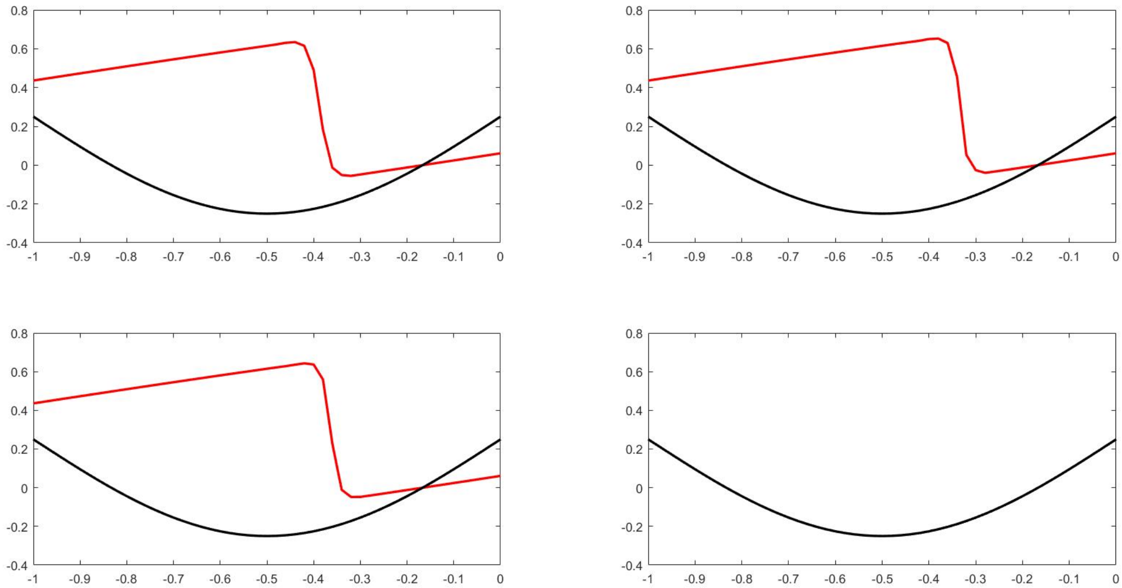


SCHEME	L1 - ERROR	L2 - ERROR	SCHEME	L1-ERROR	L2-ERROR
UPWIND	15.0504	2.0799	MINMOD	8.8221	1.5042
LW	8.5674	1.2908	SUPERBEE	6.4699	1.3076
BEAM - WAR	14.5102	2.2252	MC	6.9412	1.3366
FROMM	7.1283	1.2618	VANLEER	7.2010	1.3499

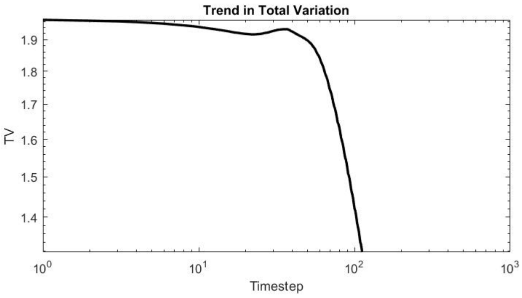
2. FV SCHEMES FOR BURGERS' EQUATION

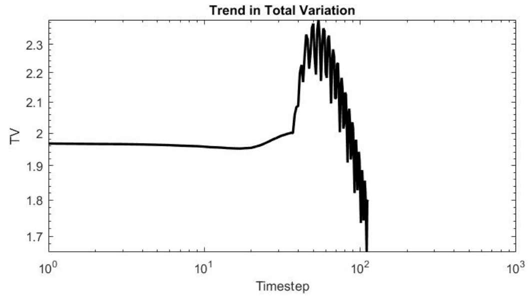
The schemes below are plotted in the same order as those on the previous page, using the same space-time grids. Initial data is taken from Example 1 in *Shu and Osher*. The solution gradually becomes a shock that travels rightward. For this problem, the Beam Warming and Van-Leer Schemes appear to be unstable.

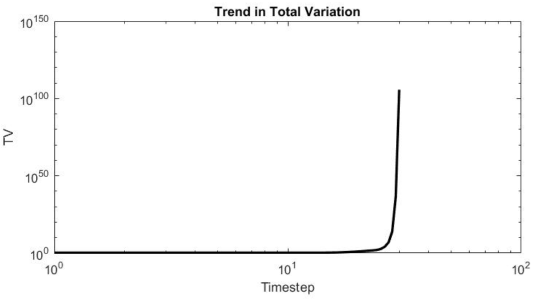


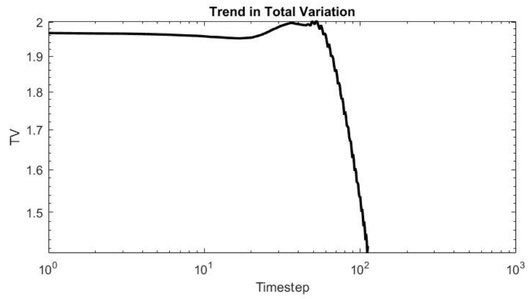


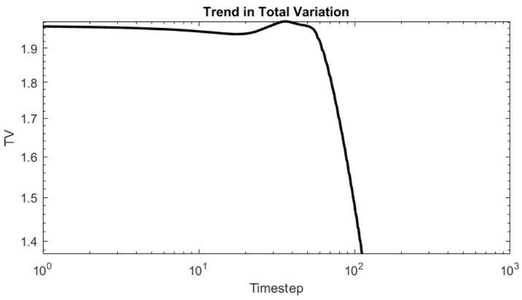
Exact solutions to this problem require iteratively solving a rootfinding problem for u at t = 2, however, we may gain some understanding of how well a scheme is working by looking at the total variation of the solution over time. Below are total variation plots for all eight schemes in the previous order. As expected from the numerical solution, the TVs for Beam-Warming and Van Leer Schemes increase very fast.

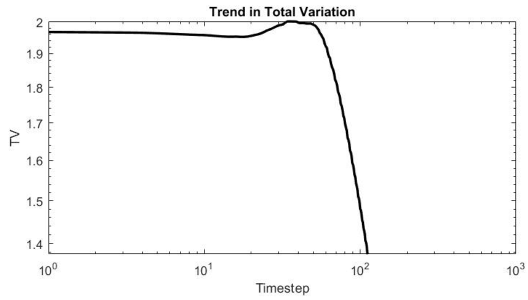


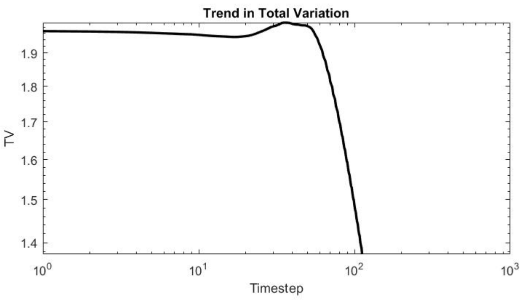


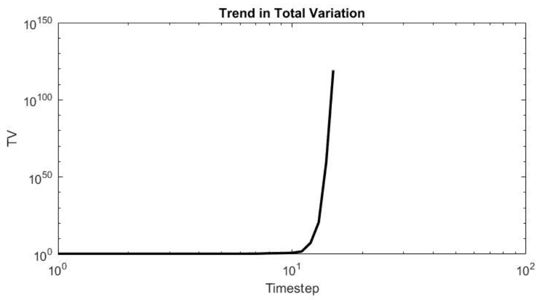












3. FIFTH ORDER WENO SCHEME FOR THE ADVECTION EQUATION

I attempted to implement a fifth order WENO scheme for the advection equation following the code in J. Hesthaven's book, but the scheme turned out to be unstable due to an as yet unresolved bug. The solution blows up from the get go and is not transported at all, which makes me think I should look at the flux terms again and also make sure I've specified the proper boundary conditions.