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M.E./M.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Computer Science and Engineering

MA 4151 – APPLIED PROBABILITY AND STATISTICS FOR COMPUTER SCIENCE ENGINEERS

(Common to M.E. Computer Science and Engineering (With Specialization in Artificial Intelligence and Machine Learning)/M.E. Computer Science and Engineering (With Specialization in Networks)/M.E. Multimedia Technology/Master of Computer Applications (2 Years))

(Regulations 2021)

Time : Three hours Maximum : 100 marks

(Use of Statistical Table is permitted)

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Show that $X = [1, 0, 0]^T$ is generalized eigenvector of rank 2 corresponding to the eigen value $\lambda = 3$ for the matrix $A = \begin{bmatrix} -7 & -25 & 1 \\ 4 & 13 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

2. What is meant by pseudo inverse of a matrix A?

3. If A and B are independent events, then prove that $\overline{A}$ and $\overline{B}$ are also independent events.

4. It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that 2 of 100 books bound by this bindery will have defective bindings.

5. The conditional p.d.f. of X given $Y = y$ is given by

$f\left(\frac{x}{y}\right) = \frac{x+y}{1+y}e^{-x}$, $0 < x < \infty$, $0 < y < \infty$, find $P[X < 1/Y = 2]$.

6. The regression lines between two random variables X and Y is given by $3X + Y = 10$ and $3X + 4Y = 12$. Find the correlation between X and Y.

7. What are the Type I and Type II errors?

8. Write any two properties of F-test.

9. Define : Mean vectors and covariance matrices.

10. Let X be $N_3(\mu, \Sigma)$ with $\mu^T = (2, -3, 1)$ and $\sum = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$. Find the distribution of $3X_1 - 2X_2 + X_3$.

PART B — (5 × 13 = 65 marks)

11. (a) (i) Determine a canonical basis for $A = \begin{bmatrix} 3 & 5 \\ -2 & -4 \end{bmatrix}$. (5)

(ii) Find a singular value decomposition for $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$. (8)

Or

(b) Construct a QR decomposition for the matrix $A = \begin{bmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{bmatrix}$.

12. (a) (i) A and B alternately throw a pair of dice. A wins if he throws the sum of dice as 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is 30/61. (6)

(ii) An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball? (7)

Or

(b) (i) Out of 800 families with 4 children in each family how many families would be expected to have (1) 2 boys and 2 girls (2) atleast 1 boy (3) atmost 2 girls (4) children of both. Assume equal probabilities for boys and girls. (7)

(ii) The marks obtained by a number of students in a certain subject are normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least 1 of them would have scored above 75? (6)

13. (a) (i) The joint pdf of (X, Y) is given by $f(x, y) = 24xy;\ x > 0, y > 0$, $x + y \leq 1$ and $f(x, y) = 0$ elsewhere, find the conditional mean and variance of Y given X. (6)

(ii) Compute the coefficient of correlation between X and Y using the following data : (7)

X :	1	3	5	7	8	10
Y :	8	12	15	17	18	20

Or

(b) In an analysis of a correlation data, the following results only are legible : Variance of $X = 1$ Regression equations $3x + 2y = 26$ and $6x + y = 31$.

(i) What are the mean values of X and Y (3)

(ii) The standard deviation of X and Y (4)

(iii) The coefficient of correlation between X and Y (6)

14. (a) In a random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favor of the proposal. Test the hypothesis that the proportion of men and women in favor of the proposal are the same at 5% level?

Or

(b) The following table gives the number of aircraft accidents that occur during the various days of a week. Find whether the accidents are uniformly distributed over the week.

Days :	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents :	15	19	13	12	16	15

15. (a) Let $X = [X_1 \quad X_2]^T$ be a multivariate normal random vector with mean $\mu = [1 \quad 2]^T$ and covariance matrix $V = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ prove that the random variable $Y = X_1 + X_2$ has a normal distribution with mean equal to 3 and variance equal to 7.

Or

(b) Explain : Determining the number of Principal components in Multivariate Analysis.

PART C — $(1 \times 15 = 15$ marks)

16. (a) X and Y are two R.V.'s having joint density function

$$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y); \; 0 < x < 2, \; 2 < y < 4 \\ 0 \qquad\qquad\qquad \textit{otherwise} \end{cases}$$

Find (i) $P(X < 1 \cap Y < 3)$ (ii) $P(X + Y < 3)$ and (iii) $P(X < 1 / Y < 3)$.

Or

(b) (i) Two independent samples of 8 and 7 items respectively had the following values :

Sample I :	9	11	13	11	15	9	12	14
Sample II :	10	12	10	14	9	8	10	

Is the difference between the means of sample significance? (6)

(ii) Two random samples gave the following data :

Sample	Size	Sample mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population at 5% L.O.S. (9)