Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 30151

M.C.A. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

(Bridge Course)

BX 4005 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

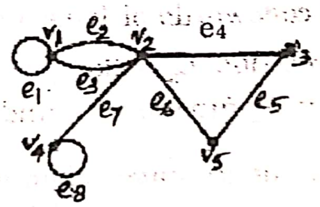
1. Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

2. Is $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent?

3. Let P (n) be the statement that $n! < n^n$, where n is an integer greater than 1. What is the inductive hypothesis?

4. There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

5. Represent the pseudograph shown in Figure using an incidence matrix.



6. What is a path? What is the difference between path and trail?

7. Let $(Z, *)$ be an algebraic structure, where Z is the set of integers and the operation $*$ is defined by $n * m$ = maximum of (n, m). Show that $(Z, *)$ is a semi group.

8. Define Field. Give an example.

9. Define partial order in lattice.

10. Let C be a collection of sets closed under the set operations of union, intersection, and complement. Is C a Boolean Algebra? If yes, what is the zero element and unit element?

PART B — (5 × 13 = 65 marks)

11. (a) (i) Use predicates and quantifiers to express the system specifications "Every mail message larger than one megabyte will be compressed" and "If a user is active, at least one network link will be available". (6)

(ii) Let $Q(x, y)$ denote "$x + y = 0$." What are the truth values of the quantifications $\exists y \forall x Q(x, y)$ and $\forall x \exists y Q(x, y)$, where the domain for all variables consists of all real numbers? (7)

Or

(b) (i) Using method of logic simplify the Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$. (6)

(ii) Check the validity of the following argument." Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised". (7)

12. (a) (i) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? (6)

(ii) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department? (7)

Or

(b) (i) Suppose that a valid codeword is an n-digit number in decimal notation containing an even number of 0s. Let a_n denote the number of valid codewords of length n. The sequence $\{a_n\}$ satisfies the recurrence relation $a_n = 8a_{n-1} + 10^{n-1}$ and the initial condition a₁ 99. Use generating functions to find an explicit formula for a_n. (6)

(ii) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages? (7)

13. (a) (i) Determine whether the graphs G and H displayed in Figure 1 are isomorphic. (6)

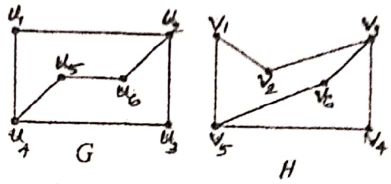


Figure. 1

(ii) What does the degree of a vertex in a niche overlap graph represent? Which vertices in this graph are pendant and which are isolated? Use the niche overlap graph shown in Figure to interpret your answers.

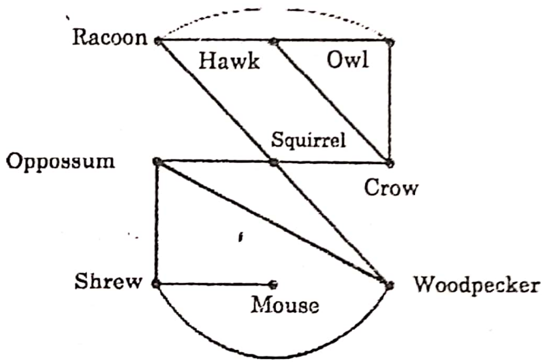


Figure 2

Or

(b) (i) Show that K_n has a Hamilton circuit whenever $n \geq 3$. (6)

(ii) Find the vertex, connectivity and edge connectivity for the graph in Figure 3. (7)

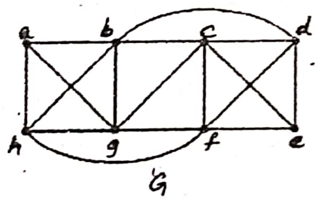


Figure 3

14. (a) (i) If $(G,*)$ is a finite group, H is a non-empty subset of G, verify that H is closed under the operation * and H is a subgroup of G. (6)

(ii) State and prove Lagrange theorem. (7)

Or

(b) (i) Suppose H is a subgroup of G. Let x , $y \in G$. Then prove that either $xH = yH$ or xH and yH have no elements in common. (6)

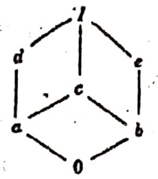
(ii) Consider $\phi : Z_2^2 \to Z_2^3$ defined by $\phi(a,b) = (a, b, a +_2 b)$. If (a_1, b_1), $(a_2, b_2) \in Z_2^2$. Find $\phi((a_1, b_1) + (a_2, b_2))$. (7)

15. (a) Consider the Boolean algebra D210.

(i) List its elements and draw its diagram. (ii) Find the set A of atoms. (iii) Find two subalgebras with eight elements. (iv) Is $X = \{1, 2, 6, 210\}$ a sublattice of D_{210} and a subalgebra. (v) Is $Y = \{1, 2, 3, 6\}$ a sublattice of D_{210} and a subalgebra?

Or

(b) Consider the lattice L in Figure.



(i) Which nonzero elements are join irreducible? (b) Which elements are atoms?

(ii) Which elements are atoms?

(iii) Which of the following are sublattices of L : $L_1 = \{0, a, b, I\}$, $L_2 = \{0, a, e, I\}$, $L_3 = \{a, c, d, I\}$, $L_4 = \{0, c, d, I\}$.

(iv) Is L distributive?

(v) Find complements, if they exist, for the elements, a, b and c.

(vi) Is L a complemented lattice?

PART C — (1 × 15 = 15 marks)

16. (a) (i) Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." (8)

(ii) Prove that if $n = ab$, where a and b are positive integers, then $a \le \sqrt{n}$ or $b \le \sqrt{n}$; Using mathematical logic. (7)

Or

(b) (i) Prove that $\sqrt{2}$ is irrational by using proof by contradiction. (7)

(ii) Use proof by cases to show that $|xy| = |x||y|$, where x and y are real numbers. (Recall that $|a|$, the absolute value of a, equals a when $a \ge 0$ and equals $-a$ when $a \le 0$). (8)