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# Question Paper Code : 60292

M.C.A DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Elective

(Bridge Course)

BX 4005 – MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations – 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Show that $(p \wedge q) \rightarrow p$ is a tautology.

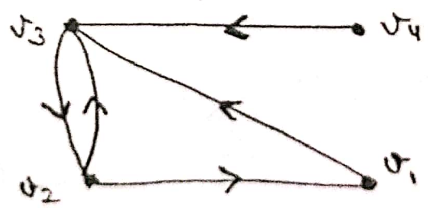
2. Symbolise: "All the world loves a love".

3. State the Pigeonhole principle.

4. Determine they number of triangles that are formed by selecting points from a set of 15 points out of which 8 are collinerar.

5. Prove that the sum of the degrees of all the vertices in a graph $G$ is equal to twice the number of edges.

6. Find the complement (simple graph) of the following graph.




7. Prove that the only idempotent element of a group is its identity element?

8. Give an example of ring which is not a field.

9. Define a Poset.

10. State isotonicity law.

PART B — (5 × 13 = 65 marks)

11. (a) (i) Show that $(\sim p\wedge(\sim q\wedge r))\vee(q\wedge r)\vee(p\wedge r)\Leftrightarrow r$. (6)

(ii) Show that the following argument is valid: If the band could play rock music or the refreshments were not delivered on time then the new year's party would have been canceled and Alicia would have been angry. If the party were canceled, then refunds would have had to be made. No refunds were made. Therefore, the band could play rock music. (7)

Or

(b) (i) Show that $p\rightarrow(q\rightarrow r)\Leftrightarrow p\rightarrow(\sim q\vee r)\Leftrightarrow(p\wedge q)\rightarrow r$. (6)

(ii) Give an argument which will establish the validity for the following inference.

All integers are rational numbers

Some integers are power of 2

Therefore, some rational number are power of 2 (7)

12. (a) (i) Use mathematical inducation, show that $1^2+2^2+3^2\ldots+n^2=\dfrac{n(n+1)(2n+1)}{6}$. (6)

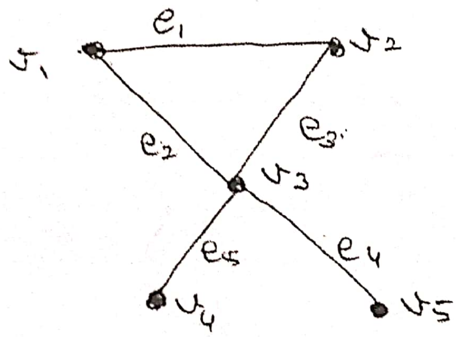
(ii) Find the recurrence relation, satisfying $y_n = A.3^n + B.)-2)^n$. (7)

Or

(b) (i) Use the principle of inclusion and exculsion, find the number of integers between 1 and 200 that are not divisible by any of the integers 2, 3 and 5. (6)

(ii) Find the generating function of Fibonacci sequence. (7)

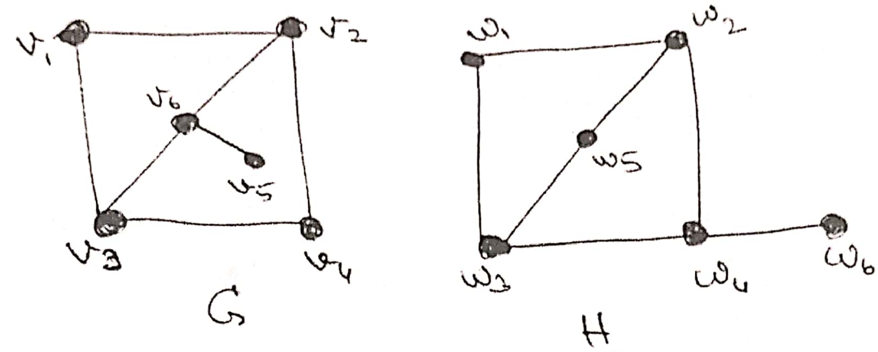
13. (a) (i) Write the incidence and adjacency matrices for the following graph: (6)




(ii) If $G$ is Hamiltonion, prove that for every non-empty proper subset $S$ of $V,\omega(G-S)\le|S|$. (7)

Or

(b) (i) Determine whether the following graphs $G$ and $H$ are isomorphic. (6)




(ii) Let $D$ be a connected directed graph. Prove that $D$ is Eulerian if and only if $d^{+}(v) = d^{-}(v), \forall v \in G$. (7)

14. (a) (i) Prove that fourth roots of unity namely $\{1,-1, i,-i\}$ is an abelian group under multiplication of complex numbers. (6)

(ii) State and prove the fundamental theorem of homomorphism. (7)

Or

(b) (i) Prove that $A$ group $(G,*)$ is abelian if and only if $(a * b)^2 = a^2 * b^2$. (6)

(ii) Prove that the intersection of two normal subgroups of a group of $(G,*)$ is a normal subgroup of $(G,*)$. (7)

15. (a) (i) Let $A$ be the set of factors of a particular positive integer $m$ and $e,\leq$ be the relation "divides". Draw Hasses diagrams for (1) $m = 12$ (2) $m = 30$ (3) $m = 45$. (6)

(ii) In any Boolean algebra, show that $ab' + bc' + ca' = a'b + b'c + c'a$. (7)

Or

(b) (i) Show that DeMorgan's laws hold in a complemented distributive lattice. (6)

(ii) Simplify the Boolean function $f(x,y,z) = \pi(0,2,4,5)$. (7)

PART C — ($1 \times 15 = 15$ marks)

16. (a) (i) Solve: $T(K) - 7T(K-1) + 10T(K-2) = 6 + 8K$, with $T(0) = 1$ and $T(1) = 2$. (8)

(ii) Solve: $y_{n+2} - 4y_{n+1} + 3y_n = 0$, $y_0 = 2$, $y_1 = 4$ by generating function. (7)

Or

(b) (i) State and prove the Lagrange's Theorem. (7)

(ii) Determine whether $H_1 = \{0, 5, 10\}$ and $H_2 = \{0, 4, 12\}$ are subgroups of $Z_{15}$. (8)