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Question Paper Code : 12324

M.C.A. DEGREE EXAMINATIONS, JANUARY 2022.

First Semester

MA 4151 — APPLIED PROBABILITY AND STATISTICS FOR COMPUTER SCIENCE ENGINEERS

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Given that the system is consistent, find the least square solution of the system of equations $2x + y = 6,\ x + y = 8, -2x + y = 11, -x + y = 8,\ 3x + y = 4$.

2. Define an inner product space.

3. State any two properties of moment generating function.

4. Find the mean of the Binomial distribution.

5. If the joint probability density function of (X, Y) is $f(x,y) = 6e^{-2x-3y}$, $x \geq 0, y \geq 0$, find the marginal density function of X.

6. Prove that correlation co-efficient between X and Y lies between -1 and 1.

7. Define Type I and Type II errors.

8. Write any two uses of chi-square distribution.

9. Define generalized eigen vector.

10. State the objectives of principal component analysis.

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11\. (a) (i) If V is the vector space of real or complex continuous functions on the real interval $a \le t \le b$, prove that

$$< f, g > = \begin{cases} \int_a^b f(t)g(t)dt \text{ for real functions} \\ \int_a^b f(t)\overline{g(t)}dt \text{ for complex functions} \end{cases} \text{ is an inner product}$$

space on V. (7)

(ii) Find the pseudo inverse of $A = (-1, 2, 2)$ by using singular value decomposition. (6)

Or

(b) (i) Find the eigen values of $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ by QR factorization. (7)

(ii) Find if the generalized eigenvector corresponding to the eigen value $\lambda = 9$ for the matrix $A = \begin{pmatrix} 4 & 2 & 1 \\ 2 & 7 & 2 \\ 1 & 2 & 4 \end{pmatrix}$ exists. State the reason for your conclusion. (6)

12\. (a) (i) In a coin tossing experiment, if the coin shows head, 1 die is thrown and the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2? (7)

(ii) Find the moment generating function of exponential distribution and hence find its mean. (6)

Or

(b) (i) The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1) without breakdown, (2) with only one break down. (6)

(ii) A continuous random variable X has a probability density function $f(x) = kx^2e^{-x}$, $x > 0$. Find k, mean and variance. (7)

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13. (a) (i) If the joint probability density function of a random variable (X, Y) is given by $f(x,y)=\begin{cases}2; & 0<x<1,\ 0<y<x\\ 0; & otherwise\end{cases}$. Find the marginal density functions of X and Y. (7)

(ii) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the joint probability density function of $U = X - Y$. (6)

Or

(b) If the joint probability density function of (X, Y) is given by $f(x,y)=\begin{cases}x+y; & 0\le x\le 1,\ 0\le y\le 1\\ 0; & otherwise\end{cases}$ find the coefficient of correlation between the variable X and Y. (13)

14. (a) Random samples of specimens of coal from two mines A and B are drawn and their heat-producing capacities (in millions of calories/ton) were measured yielding the following results. (13)

Mine A	8350	8070	8340	8130	8260	
Mine B	7900	8140	7920	7840	7890	7950

Is there significant difference between the means of these two samples at 0.01 level of significance?

Or

(b) (i) Two random sample gave the following data.

	Size	Mean	Variance
Sample I	8	9.6	1.2
Sample II	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population? (7)

(ii) A total number of 3759 individuals were interviewed in a public opinion survey on a political proposal. Of them, 1872 were men and the rest women. 2257 individuals were in favour of the proposal and 917 were opposed to it. 243 men were undecided and 442 women were opposed to the proposal. Do you justify or contradict the hypothesis that there is no association between sex and attitude? (6)

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15. (a) Find the mean vector and covariance matrix for the two discrete random variable X_1 and X_2 whose joint probability distribution is given below. (13)

X_2 \ X_1	0	1	2	3
0	1/21	3/14	1/7	1/84
1	1/7	2/7	1/14	0
2	1/21	1/28	0	0

Or

(b) If the covariance matrix of $X = (X_1, X_2, X_3)^T$ is $\sum = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$, find the principal components of X_1, X_2, X_3 using standardized variables.

PART C — (1 × 15 = 15 marks)

16. (a) In 250 digits from the lottery numbers, the frequencies of the digits were as following :

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	23	25	20	23	23	22	29	25	33	27

Test the hypothesis that the digits were randomly drawn. (15)

Or

(b) The chances of A, B, C becoming the general manger of a certain company are in the ratio 4:2:3. The probabilities that the bonus scheme will be introduced in the company if A, B, C become general manager are 0.3, 0.7, 0.8 respectively. If the bonus scheme has been introduced, what is the probability that A has been selected as general manager? (15)

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