Reg. No. :

Question Paper Code : 30909

M.E./M.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

First Semester

Computer Science and Engineering

MA 4151 – APPLIED PROBABILITY AND STATISTICS FOR COMPUTER SCIENCE ENGINEERS

(Common to: M.E. Computer Science and Engineering (With Specialization in Artificial Intelligence and Machine Learning/M.E. Computer Science and Engineering (With Specialization in Networks)/M.E. Multimedia Technology/ Master of Computer Applications)

(Use of statistical tables may be permitted)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What is a basis of a vector space?

2. Let (u,v) be the Euclidean inner product on R^2 and let $u=(3,2)$ and $v=(4,5)$. Verify $(u,v)=(v,u)$.

3. In a coin tossing experiment, if the coin shows head, 1 die is thrown and the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2?

4. If X is uniformly distributed with mean 1 and variance 4/3, find $P(-1<X<2)$.

5. If the joint probability density function of (X,Y) is $f(x,y)=\frac{1}{4}$, $0 \le x, y \le 2$. Find $P(X+Y) \le 1$.

6. In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible. Regression equations: $8x-10y+66=0$, $40x-18y-214=0$, Find the correlation coefficient between X and Y.

7. Define null hypothesis and alternative hypothesis.

8. Write the confidence interval for large sample single mean test.

9. Define a random vector and a random matrix.

10. Is the statement "If two random vectors are independent covariance between these vectors are zero." Correct? If so why? If not why not? Is the converse of this statement true?

PART B — (5 × 13 = 65 marks)

11. (a) (i) Find the distance between X and Y with respect to (I) Euclidean norm and (II) inner product norm with respect to W, where $X=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $Y=\begin{bmatrix}4\\5\\6\end{bmatrix}$ and $W=\begin{bmatrix}1&1&0\\0&1&1\\1&0&1\end{bmatrix}$. (6)

(ii) Find a generalized eigenvector of rank 3 corresponding to the eigenvalue $\lambda=7$ for the matrix $\begin{bmatrix}7&1&2\\0&7&1\\0&0&7\end{bmatrix}$. (7)

Or

(b) Construct a QR-decomposition for the matrix $A=\begin{bmatrix}-4&2&2\\3&-3&3\\6&6&0\end{bmatrix}$. (13)

12. (a) (i) Customers are used to evaluate preliminary product designs. In the past 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

(1) What is the probability that a product attains a good review?

(2) If a new design attains a good review, what is the probability that it will be a highly successful product?

(3) If a product does not attain a good review, what is the probability that it will be a highly successful product? (7)

(ii) An electronic product contains 40 integrated circuits. The probability that any integrated circuit is defective is 0.01, and the integrated circuits are independent. The product operates only if there are no defective integrated circuits. What is the probability that the product operates? (6)

Or

(b) (i) The daily consumption of milk in a city in excess of 20,000 gallons is approximatêly distributed as Gamma variate with parameters $\upsilon = 2$ and $\lambda = \dfrac{1}{10000}$. The city has a daily stock of 30,000 gallons. What is the probability that the stock is insufficient on a particular day? (6)

(ii) The life of a semiconductor laser at a constant power is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours.

(1) What is the probability that a laser fails before 5000 hours?

(2) What is the life in hours that 95% of the lasers exceed?

(3) If three lasers are used in a product and they are assumed to fail independently, what is the probability that all three are still operating after 7000 hours? (7)

13. (a) Let X and Y are two random variables having the joint probability function $f(x, y) = k(2x + 3y)$ where X and Y can assume the integer values 0, 1, 2 and 1, 2, 3 respectively. Find (13)

(i) The value of k.

(ii) The marginal distributions of X and Y

(iii) The conditional distributions

(iv) E(X), E(Y), E(XY)

(v) Cov (X, Y)

(vi) Correlation between X and Y

Or

(b) (i) If X and Y are uncorrelated random variables with variances 16 and 9, find the correlation coefficients between $X + Y$ and $X - Y$. (6)

(ii) A fair coin is tossed four times. Let X denote the number of heads occurring and let Y denote the longest string of heads occurring. Find Cov (X, Y). (7)

14. (a) (i) Experience has shown that 20 percent of a manufactured product is of top quality. In one day's production of 400 articles, only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20 percent was wrong. Based on the particular day's production find also the 95 percent confidence limits for the percentage of top-quality product. (7)

(ii) To test the claim that the resistance of electric wire can be reduced by more than 0.050 *ohm* by alloying, 32 values obtained for standard wire yielded $\overline{x} = 0.136$ ohm and $s_1 = 0.004$ *ohm*, and 32 values obtained for alloyed wire yielded $\overline{y} = 0.083$ *ohm* and $s_2 = 0.005$ *ohm*. At the 0.05 level of significance, does this support the claim? (6)

Or

(b) (i) Two samples of size nine and eight gave the sums of squares of deviation from their respective means equal to 160 and 91 respectively. Can they be regarded as drawn from the same normal population? (6)

(ii) Fit a Poisson distribution for the following distribution and also test the goodness of fit. (7)

X	0	1	2	3	4	5
Frequency f	142	156	69	27	5	1

15. (a) (i) Find the covariance matrix for the two random variables X_1 and X_2 when their joint probability function $f(x_1, x_2)$ is given by (7)

x_1 x_2	0	1
-1	.24	.06
0	.16	.14
1	.40	.0

(ii) Let X be a 3×1 matrix vector belongs to $N_3(\mu, \Sigma)$ with $\begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

Are X_1 and X_2 independent? What about (X_1, X_2) and (X_3)? (6)

Or

(b) Suppose the random variables X_1, X_2 and X_3 have the covariance matrix $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. Calculate the population principal components for the random variables. (13)

PART C — $(1 \times 15 = 15$ marks)

16. (a) The distance between major cracks in a highway follows an exponential distribution with a mean of 5 miles.

(i) What is the probability that there are no major cracks in a 10-mile stretch of the highway?

(ii) What is the probability that there are two major cracks in a 10-mile stretch of the highway?

(iii) What is the standard deviation of the distance between major cracks?

(iv) What is the probability that the first major crack occurs between 12 and 15 miles of the start of inspection?

(v) What is the probability that there are no major cracks in two separate 5-mile stretches of the highway?

(vi) Given that there are no cracks in the first 5 miles inspected, what is the probability that there are no major cracks in the next 10 miles inspected?

Or

(b) If X and Y are two random variables having the joint probability density function $f(x,y)=2$ for $0<y<x$, $0<x<1$ Find

(i) Marginal density function of X and Y

(ii) Conditional densities $f\left(\frac{x}{y}\right)$ and $f\left(\frac{y}{x}\right)$

(iii) Conditional variance of X given $Y=2$

(iv) Check whether X and Y are independent

(v) Find Cov (X, Y)

(vi) Correlation between X and Y
