Bagging Based Hybrid Time Series Models: A Case Study of Indian Economic Indicators

By

Adhiti. U

21MSCS08

A dissertation submitted to

Yenepoya (Deemed to be University)

In Partial fulfillment for the award of degree of

Masters of Science in

STATISTICS

Under the guidance of

Ms. Yashaswini

RABBI ZIDNI 'ILMA

YENEPOYA

(DEEMED TO BE UNIVERSITY)

Recognized under Sec 3(A) of the UGC Act 1956

Accredited by NAAC with 'A+' Grade

Yenepoya (Deemed to be University)

Mudipu, Managalore- 574153

2021-2023

CERTIFICATE

This is to certify that, Miss Adhiti. U (21MSCS08), a student of M.Sc. Statistics from the Department of Statistics, Yenepoya (Deemed to be University), Mudipu has completed her semester project under my guidance from April 2023 to August 2023. The project work entitled Bagging Based Hybrid Time Series Models: A Case Study of Indian Economic Indicators embodies the novel work done by her.

Name and Signature of Guide **Name and signature of HOD**

DECLARATION

I hereby declare that the project work entitled "Bagging Based Hybrid Time Series Models: A Case Study of Indian Economic Indicators" submitted to the Department of Statistics, Yenepoya (Deemed to be University), Mudipu, is a record of the work done by me under the guidance of Ms. Yashaswini.This project report is submitted in partial fulfillment of the requirements for the award of degree of Master of Science in Statistics.

The result embodied in this thesis has not been submitted to any other Institute or University for the award of any degree/certificate.

Date:

Place: Mudipu

Name and signature

ACKNOWLEDGEMENT

I am delighted to share the report detailing the project work conducted during my Master’s Degree in Statistics I would like to express my sincere gratitude and appreciation to all those who have supported me throughout the course of this project. Without their help, guidance, and encouragement, this project would not have been possible.

I extend my heartfelt gratitude to Ms. Yashaswini, my mentor throughout the journey of this project. Her guidance and support have been instrumental in shaping the outcome of this Project.

I would like to express my heartfelt gratitude to Dr. Ismail B, Professor, and head of the Department for his unwavering support and encouragement throughout the course of this project.

I would also wish to convey my Thanks to all faculties and all the students, Department of statistics, Yenepoya (Deemed To be University).

I would like to express my heartfelt appreciation to my grandmother Padmavathi U and my parents Vidya and U Durga Prasad, for their unwavering support, affection and motivation throughout this entire project. Thank you for your love, concern and guidance that has been truly irreplaceable.

I want to express my sincere Thanks to all my Friends for helping to develop this project and for the encouragement throughout the project.

Thank you all for being part of this remarkable experience

Adhiti. U

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 Introduction and Background

Time Series analysis is extremely important due to the massive production of time-oriented data in many fields ranging from finance and economics to managing production operations etc. As continuous monitoring and collecting the data become more common due to its exponential growth, the need for competent time series analysis with both statistical and machine learning techniques increases drastically. The choice of applying machine learning models and time series models for forecasting is driven by a variety of factors each catering to specific aspects of the data and the forecasting problem. The time series models excel in capturing patterns like seasonality, trend and cyclic variations which are common in the data. The models rely on past observations to predict the future values making them short term forecasted values. On the other hand, machine learning models offer a broader range of capabilities. They deal with the multi-dimensional and non-linear data, where the traditional time series model falls short.

The forecasting-based time series techniques play a very important role in economy. The time series forecasting can help individuals, business, and governments to plan. For example, a government might use forecasted tax revenues to make policy decisions. The inflation forecasts may be used to decide whether to raise or to lower the inflation rates.

The concept of time series dates to early 19th century. George Box was a pioneering Statistician who developed the time series model. The Autoregressive Integrated Moving Average (ARIMA) model was introduced by George E. P. Box and Gwilym M Jenkins. In continuation with their earlier work on time series they published the book "Time Series Analysis: Forecasting and Control" in the year 1970. Since then, ARIMA models are applied to various types of time series data with the objective of forecasting. One such example is from the study by Mehta et al (1975), who applied this model to forecast the water quality of Passaic River. The Seasonal Autoregressive Integrated Moving Average (SARIMA) is an extension of ARIMA model with the additional parameters to capture the seasonal variations in the time series. There were many notable studies which applied SARIMA model for forecasting. Beilock and Dunn (1980) used SARIMA model to predict the end of month stock of frozen French fries.

Even though ARIMA models are widely used, they are not without the limitations. These models assume linear relationships between dependent variable and its lagged values. When the time series has non-linear pattern the ARIMA model's forecasting performance may not be good. The large amount of studies in the literature of time series techniques have concentrated on the building model with accurate forecasts. Hybridization of existing models is one such technique that has the history of more than 20 years. One model may fail to capture all the complexity in the data. A hybrid model is the one which combines several models and provides the forecasts. Many researchers have developed the hybrid models over the years. Bates and Granger (1969) discussed the methods of combining the forecasts from two models and showed that the combined forecast is better than the individual ones and Markakis (1983) fitted 10 different forecasting models to 1001 time series times and empirically showed that the hybrid model with weighted average method of aggregating was superior to individual models. The adaptation of hybrid model reduces the risk of inaccurate forecasts due to inappropriate prediction. The hybrid models are known for their improved accuracy in forecasting.

The bootstrap aggregating or bagging is method proposed by Breiman (1996). It is a popular method used in machine learning for improving the forecasting/prediction accuracy by addressing the issues like data uncertainty, parameter uncertainty, model selection uncertainty. An ensemble of forecasts is generated by using several bootstrap samples and these ensembles are aggregated using mean, median, trimmed mean, weighted mean etc. The point forecasts thus obtained are proved to be more accurate. The present study aims to build the better prediction model in two phases. The hybrid model is built in the first phase and is bagged in the second phase using existing as well as a new aggregating/ensemble method. This model is called as bagging based hybrid model and its performance is testing using the time series on Indian economic indicators.

The time series forecasting is to use these models to predict the future values based on the knowledge of the historical observations. When dealing with particular dataset and aiming to achieve accurate results it is essential to consider various approaches and techniques. In this project, the methods included are Base, Hybridization and Bagging (Bootstrap Aggregating) time series models and these techniques enhances performance and generalizations on the time series and machine learning models. To evaluate those models, we use various statistical measures such as Root mean squared error (RMSE), Mean absolute error (MAE), Akaike information criteria (AIC) etc in order to find the best fit for the given time series data. In this project the primary focus is on RMSE for assessing the accuracy and precision of the given time series model.

1.2 Motivation

The study aims to compare the performance of the proposed model that is bagging-based hybrid time series models with other time series models such as linear, non-linear and hybrid models to forecast Indian economic indicators with higher accuracy. The study includes exploratory techniques such as graphical analysis, times series decomposition and correlation analysis.

The datasets used in this study is economic indicators collected from 2009 to 2022 which is further divided into train and test datasets. The trained dataset will be used to train the time series models while the test dataset will be used to validate the performance of the models.

The study findings are expected to provide valuable insights into the forecasting of Indian economic indicators which can help the investors and the traders in better decision making. Additionally, the accuracy of the proposed model can potentially lead to better forecasts of future economic trends which can help in bringing positive impact in economic growth in India.

1.3 Terminology

In this study the key concepts collaborate together to form the foundation of the research, starting with the economic indicators which is used as the time series dataset in this study. Forecasting these indictors offers insights into the future direction of the economy. By analysing their historical patterns and relationships, analysts can make informed decisions about the future economic conditions. Different approaches of time series models are used in this study and compared in order to identify the best approach. Different machine learning and time series models are used as they have distinct strengths and weaknesses and their suitability depends on the specific characteristics of the data and the nature of the forecasting problem. The following main techniques are used in this project work:

- Base time series models
- Hybrid time series models
- Weighing method/ Aggregating Methods

- Bagged base time series models
- Bagged hybrid models
- Performance of the model
- Forecasting accuracy

The base time series models includes both statistical time series models such as SARIMA, Exponential smoothing and the machine learning models like multi-layered Perceptron Neural Network Auto regression and the combination of decomposition techniques with statistical time series models like STL+SARIM and STL+ETS are used in this study. Key measures like Root mean square error are used to see how close the forecasts are to the actual values and to get the best model that fits the majority of the indicators we use the measure called Relative rate (%). Exploratory analysis is utilized to gain fundamental understanding of the time series data and its underlying patterns before applying the modelling techniques. Time profiles are plotted to identify trends and seasonality in the data.

1.4 Literature Review

(i) Evolution Hybrid Model:

The Autoregressive Integrated Moving Average (ARIMA) models was introduced by George E. P. Box and Gwilym M Jenkins. In continuation with their earlier work on time series they published the book "Time Series Analysis: Forecasting and Control" in the year 1970. Since then, ARIMA models are applied to various types of time series data with the objective of forecasting. One such example is from the study by Kumar Manoj (2014), who applied this model to forecast the Sugarcane Production in India. Mohammed A Quddus (2008) introduced the variant of ARIMA model for analysing the count data. The Seasonal Autoregressive Integrated Moving Average (SARIMA) is an extension of ARIMA model with the additional parameters to capture the seasonal variations in the time series. There were many notable studies which applied SARIMA model for forecasting. Feng Xu et al (2021) generated the forecasts of fish migration caused by Ocean Warming using SARIMA model. Ngoc-Tri et al(2022) proposes a hybrid artificial intelligence model for forecasting one – day ahead time series energy consumption in buildings. Hybrid of Seasonal Autoregressive integrated Moving average (SARIMA), The firefly-inspired Optimization algorithm and the support vector regression named (SAMFOR) was evaluated which gave the highest accuracy compared to SARIMA, Support vector regression (SVR), Random Forest (RF) models, SARIMA-SVR alone. In recent years, hybrid models that combine ARIMA and neural networks have become increasingly popular. One of the notable work is due to Nielson and Madsen (1999) where they combined Autoregressive Integrated Moving Average (ARIMA) model with Artificial Neural Networks (ANN). G Peter Zhang (1999) proposes a model that combines an ARIMA model with a neural network to capture both linear and non-linear dependencies in the time series data. The prediction of fuel wood prices is an important problem for many households and business in Greece. To address this problem Theodoros Koutroumanidis et al(2009) approaches ARIMA models, artificial neural networks(ANN) and hybrid ARIMA –ANN models. These models captured linear trends and seasonality in the data and ANN captures the non-linear patters making them suitable for forecasting fuel wood prices over time.

(ii) **Evolution of Bagging models:**

The concept of Bagging was first introduced by Leo Breiman in 1966 as a way to improve decision tree models on Breiman's original paper on bagging, Over the years researchers have proposed several variations and extensions of bagging algorithms to further improve its performance. One such study is proposed by Rob J Hyndman "Bagging exponential smoothing methods using STL decomposition and Box-Cox transformation (2010), the proposed approach improved the accuracy of the time series forecasts with non-linear trends and seasonality forecasting international tourists' arrivals to Thailand. The paper "How useful is Bagging in forecasting Economic time series? A Case study of U.S. Consumer Price inflation "by Atsushi Inoue and Lutz Kilian evaluates the importance of bagging in forecasting U.S. consumer price inflation. This article focuses on problem of whether the inclusion of indicators of real economic activity lowers the prediction mean squared error of forecasting models of U.S. Consumer price inflation. The authors compare the performance of bagging with several other commonly used forecasting methods including ARIMA, and neural networks

1.4 Aim and Objective

Aim:

To develop a bagging-based hybrid time series model to forecast Indian economic indicators which has better forecasting capacity than the existing hybrid model.

Objectives:

- To examine the performance of selected base models in forecasting Indian economic indicators.
- To examine the performance of parallel hybrid models with various weighting methods in forecasting Indian economic indicators.
- To examine the performance of bagged base models in forecasting Indian economic indicators
- To examine the performance of a bagging-based parallel hybrid models with various weighting methods in forecasting Indian economic indicators.

1.5 Source of the Data

- **Clearing Corporation Of India Limited (CCIL):** The Clearing Corporation of India Limited (CCIL) website provides clearing and settlement services, for various financial instruments such as government securities, treasury bills corporate bonds and money market instruments traded in India. It was incorporated in April 2001 under Companies Act, 1956 and is regulated by Reserve Bank of India (RBI) and the Securities and Exchange Board Of India (SEBI). The data CCIL Treasury Bill Index from website's index page in the statistical section for the year 2009-2022 is extracted for the present study.
  Link to the dataset:
  https://www.ccilindia.com/Research/Statistics/Pages/CCILTBILLIndex.aspx
- **Asian Development Bank (ADB) Website**: The Asian Development Bank (ABD) is a reputable international institution that was established in 1966 to promote economic development and reduce poverty in Asia and the Pacific. The information and data provided on the ADB's website are based on rigorous research and analysis and are often cited by other reputable organizations and institutions. The data on

various economic indicators such as GDP, inflation rate for India are collected from the website's indicators page.

Link to the dataset:

https://aric.adb.org/database/economic-financial-indicators

1.6 Software Used

- Microsoft Excel (2019) is used for data aggregation and management
- Jamovi 2.3.26 is used to generate the tables.
- R studio 4.3.0 is used for advanced data

CHAPTER 2

MATERIALS AND METHODOLOGY

This chapter provides a comprehensive overview of the data, tools, and techniques used in the study, ensuring transparency, reproducibility, and a clear understanding of the research process.

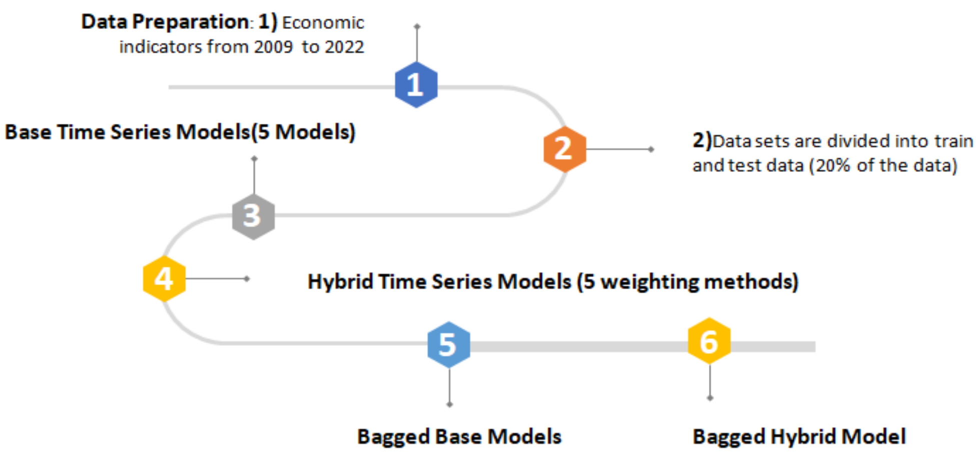
The chapter is structured as follows: the initial section presents an overview of the project. In section 2.2, the theoretical background for the different time series models employed in the study is provided by exploring its descriptive statistics. Section 2.3 delves into a comprehensive explanation of the Base time series models and Section 2.4 explores the hybrid time series models and the various weighing methods used. Further details on bagging in the context of time series analysis and its application to both base and hybrid models are covered in section 2.5. The proposed bagged hybrid model is presented in section 2.6. Finally, section 2.7 outlines the mechanism used to evaluate the performance of all the models.

2.1 Overview

The study focuses on employing time series model techniques to forecast the Indian Economic indicators spanning from 2009 to 2022. The dataset includes various metrics such as GDP, inflation rates and more. By involving both statistical and machine learning models the study aims to enhance accuracy and precision in economic forecasts. The study explores various approaches including Base time series models, hybrid models and bagging techniques.

It starts by dividing the dataset into two distinct subsets, the training dataset and the testing dataset. The splitting of data is performed chronologically by maintaining the integrity of the time series data. Approximately 80%of the data is considered as the training dataset and the remaining 20 % is assigned to the test dataset. The figure 2.1.1 represents the overall summary of the study.

Figure 2.1 : Flow chart representing the overall summary of the study



The training dataset serves as a foundation for the model development. Different time series models such as base, hybrid and bagged based models are trained on this set. The models capture various patterns, trends and relationship within the economic indicators. The test dataset plays a important role on model validation and performance assessment.

The forecasted results are compared against the actual values present in the training dataset. This comparison is facilitated by evaluation metrics such as Root Mean Error (RMSE) and Relative Range (RR).

2.2 Exploratory Analysis

The primary objective of exploratory analysis is to gain insights on the characteristics of the time series data by exploring the data using simple statistical and graphical tools. The study involves

Descriptive statistics: Such as Mean of the time series data that provides insights into its central values and standard deviation which highlights the variability or dispersion of data points around its mean.

Kurtosis and Excess Kurtosis: Kurtosis measures the shape of the data distribution and indicates the presence of heavy tails or outliers. Excess kurtosis compares the kurtosis coefficient with that of a normal distribution. Most normal distributions are assumed to have a kurtosis of three, so excess kurtosis would be more or less than three.

Shapiro – Wilk Test: The Shapiro-Wilk test assesses the normality of the data distribution. The null and alternative hypothesis are given as

H_0: The sample has been generated from a normal distribution.

$H_{1:}$ The sample is not generated from a normal distribution**.**

If the p-value is less than 0.05, the result is statistically significant that is we reject the null hypothesis and conclude that the data is not normal.

Mann-Kendal Trend Test: A **Mann-Kendall Trend Test** is used to determine whether or not a trend exists in time series data. It is a non-parametric test, meaning there is no underlying assumption made about the normality of the data. The hypotheses for the test are as follows:

H_0 (null hypothesis): There is no trend present in the data.

H_1 (alternative hypothesis): A trend is present in the data. (This could be a positive or negative trend)

If the p-value of the test is lower than some significance level (say 0.05), then there is statistically significant evidence that a trend is present in the time series data.

Kruskal -Wallis test -The Kruskal-Wallis test is a non-parametric test used for testing whether samples originate from the same distribution. The null hypothesis states that all months (or quarters, respectively) have the same mean. When this hypothesis is rejected, it is assumed that time series values differ significantly between periods

2.3 Base Time Series Models

A time series is a chronological sequence of observations on variable of interest and is denoted as X_t where the subscript t represents time it can also be represented as $\{X_t, t = 1,2 \ldots.\}$.The complete set of time $t = 1,2..,T$ will often be referred to as the observation period. The observations are measured at equal intervals such as in days, months, years etc.

Time series data exhibits a variety of patterns and is described in terms of three main components namely trend, seasonality and cyclic components. If we assume an additive decomposition of time series data say X_t, then we can represent the give time series data as

$X_t \ = S_t + T_t + C_t \ + R_t$, where

- $\boldsymbol{S_t}$, **Seasonal Component:** The trend component represents the long-term movement or systematic increase or decrease in the time series data over time. It shows the overall direction in which the data is moving. Trends can be linear, nonlinear, or even undefined (in the case of stationary data). A time series with a trend is often referred to as non-stationary.

- $\boldsymbol{T_t}$**, Trend Component:** The seasonal component reflects the regular and predictable patterns that repeat at fixed intervals over time. These patterns may occur daily, monthly, quarterly, or at other regular intervals. Seasonality is often associated with factors like weather, holidays, or other recurring events. Seasonal effects cause data to exhibit periodic fluctuations.
- $\boldsymbol{C_t}$**, Cyclic Component:** The cyclical component represents the non-periodic, medium to long-term fluctuations in a time series that do not follow a fixed pattern. Unlike seasonality, cyclical patterns do not repeat at fixed intervals. Economic cycles, business cycles, and other long-term fluctuations often contribute to the cyclical component.
- $\boldsymbol{R_t}$, **Irregular Component:** The irregular or residual component accounts for the random fluctuations or noise present in the time series data that cannot be explained by the trend, seasonal, or cyclical patterns. These random variations may arise due to measurement errors, unforeseen events, or other unpredictable factors.

2.3.1 Key Concepts

Some of the very important key concepts in time series analysis are listed below:

- **Stationarity:** Stationarity in the context of a time series refers to a statistical property where the characteristics of the data remain constant over time. A stationary time series is one in which the mean, variance, and autocorrelation structure do not change with time. In other words, the statistical properties of the time series are time-invariant. The concept of stationarity is essential in time series analysis because many statistical models and forecasting techniques assume stationarity or work best with stationary data. When a time series is stationary, it simplifies the modelling process and allows for more reliable and interpretable results.

There are two main types of stationarity:

1. Strict Stationarity (Strong Stationarity): A time series is strictly stationary if the joint distribution of any collection of time points is invariant to shifts in time. In other words, the statistical properties of the time series remain the same regardless of where we start or how far we move along the time axis.

2. Weak Stationarity: Weak stationarity is a less stringent form of stationarity. A time series is considered weakly stationary if the mean, variance, and autocorrelation structure remain constant over time, but individual observations can have different distributions at different time points. Weak stationarity is the most commonly used concept in practice. In practice, stationarity can be checked visually through time series plots, such as line plots or scatter plots, or through formal statistical tests. Common methods to achieve stationarity in a non-stationary time series include:

- **Logarithmic Transformation**: It is a data pre-processing technique used in time series analysis to stabilize variance and improve the interpretability of the given data. It applies natural logarithm to the original time series data.
  It is represented as, $Xt = logXt$

- **Differencing**: Differencing is used in order to completely eliminate the trend and seasonality of the given time series data for making it stationary time series. There are two types of differencing for eliminating trend and seasonal patterns

  - **Trend Difference (De-trending)**: It involves calculating the difference between consecutive observations to eliminate the influence of a linear or non-linear trend in the data. It is represented as,
    $\mathrm{Xt} - Xt - 1 = (1 - B)\,\mathrm{Xt}$, where $Xt - 1 = BXt$

  - **Seasonal Difference (Seasonal Adjustment):** It is a technique used to eliminate the seasonal patterns in the time series data. It involves in calculating the difference between a data point and the corresponding data point from the same season in the previous year. It can be represented as,

$Xt - s = (1 - B_S)Xt$,where s=12 (As we have monthly data)

Autocorrelation (ACF) and Partial Autocorrelation (PACF): These are the statistical tools that measure the correlation between a time series data and its values. The plots of ACF and PACF are crucial diagnostic tools for selecting suitable parameters for the identified time series models such As ARIMA and its variations. For a given x_t,autocorrelation at lag k is the correlation between the pair $(x_{t,} x_{t-k})$Sample autocorrelation function is the consistent estimator of the ACF. It is defined as

The ACF is given as,

$$\mathrm{Rk} = \mathrm{CK/CO} = \frac{\sum_{t=1}^{n-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{n}(x_t - \bar{x})^2}$$

Partial autocorrelations are used to measure the degrees of association between $x_t, x_{t-p}$when x effects at other time lags 1,2,3, …. p-1 are removed

- **Decomposition of a Time Series:** The decomposition of the time series is the method where the given time series data is decomposed into components such as trend, seasonality, cyclic and residuals (noise). The goal of decomposition is to gain clear insight of various patterns and fluctuations present within the data.
- **Model Fitting and Diagnostic Checking:** Model fitting and diagnostic checking is the most integral part of time series analysis. It involves in identifying and applying an appropriate mathematical model (time series model) that best captures the underlying patterns of the time series. Once the model of fitted, diagnostic checking is essential to assess the model's adequacy and validity.

- **Model Evaluation:** This step involves various techniques and metrics to evaluate how well the fitted times series model captures the underlying patterns of the time series data and determines its effectiveness in forecasting.

2.3.2 SARIMA

Seasonal Autoregressive Integrated Moving Average (SARIMA): This model works by modelling the relationship between past and present values of the series and identifying patterns in the data, SARIMA uses the combination of autoregression (AR)and moving average (MA) models, as well as differencing to capture the patterns and seasonality in the data. The term seasonality refers to the regular and predictable variations in the data that occur over a certain period time such as monthly cycles and by incorporating seasonality to the model, SARIMA can make more accurate predictions of the future values.

For Seasonal Autoregressive Integrated Moving Average (SARIMA) models, they are generally denoted as $SARIMA(p, d, q)(P, D, Q, m)$ where,

P - Number of seasonal autoregressive terms (SAR)

D - Number of seasonal differences

Q - Number of seasonal moving average terms (SMA)

p - non-seasonal AR order

q – non-seasonal MA order

d - non-seasonal differencing

and it is represented as

$$\phi(B)\phi(B^s)(1-B)^d(1-B^s)^D X_t = \theta(B)\theta(B^s)e_t$$

$$\phi(B) = 1 - \beta_1 B - \cdots .. - \beta_p B^p$$

$$\theta(B) = 1 - \alpha_1 B - \cdots .. - \alpha_q B^q$$

$$\phi(B^s) = 1 - \phi_1(B^s) - \cdots .. - \phi_p B^{sp}$$

$$\theta(B^s) = 1 - \theta_1(B^s) - \cdots .. - \theta_Q B^{sQ}$$

2.3.3 Exponential Smoothing Method (ETS method)

In exponential smoothing methods the forecast are made considering the weighted averages of the past observations. The latest observations are given the exponentially more weight than the older observations. Exponential Smoothing Methods combine Error, Trend, and Seasonal components in a smoothing calculation. Each term can be combined either additively, multiplicatively, or be left out of the model. These three terms (Error, Trend, and Season) are referred to as ETS. It divides the series into three parts: error, trend, and seasonality when dealing with time-series data.

In its simplest form, an exponential smoothing of time series data allocates the exponentially decaying weights from newest to oldest observations, analyzing data from a specific period of time via providing more importance to recent data and less importance to former data. This method produces "smoothed data", the data that has a noise removed, and allows trends and patterns to be more clearly visible. The main purpose of exponential smoothing is to smooth the original sequence and then make use of the smoothed sequence to predict upcoming values of the variables of concern. This process is mainly helpful when the parameters related to the time series varies gradually over time.

Exponential Smoothing Methods can be defined in terms of an ETS framework, in which the components are calculated in a smoothing fashion. As we can see, combining Error, Trend, and Season in at least three different ways gives us a lot of combinations. The first, second and the third letter represents the Error type, Trend type and the seasonal type respectively.

- “A” stands for Additive Error/seasonality/trend
- “M: stands for Multiplicative Error/seasonality/trend
- “Z” stands for Automatically Selected Error/Seasonality/trend
- “N “ stands for No Seasonality / No trend

There are three types of exponential smoothing based on mainly three types of configurations each handling different levels of trend and seasonality

1. **Simple Exponential Smoothing (SES):**
   In terms of ETS configuration or state space model is given by “ANN” which refers to Simple Exponential smoothing with Additive errors, No trend and No seasonality
2. **Holt’s Linear Exponential Smoothing:**
   In terms of ETS configuration “AAN” refers to Holt’s Linear Exponential Smoothing with Additive errors, Additive trend and No seasonality.
3. **Holt-Winter’s Exponential Smoothing:**
   Additive Holt–Winters method is obtained by model “AAA” and the multiplicative Holts Winter is denoted as “MAM

We propose an automatic forecasting procedure that tries each of the 24 state space models on a given time series and selects the best method using AIC and other criteria.

2.3.4 STL+SARIMA

STL is a versatile and robust method for decomposing time series. STL is an acronym for “Seasonal and Trend decomposition using Loess”, while Loess is a method for estimating nonlinear relationships. It is particularly used for time series data with complex seasonal patterns and trends. STL uses a weighted regression technique called Loess to extract the underlying components. The process involves iteratively removing the seasonal and trend components to obtain a de-trended and de-seasonalized remainder series

The combination of STL and SARIMA involves applying the STL decomposition to the original time series data to extract the seasonal and trend components. Then the SARIMA model is applied to the de-sesonalized and de-trended components, remainder series obtained from the STL decomposition.

2.3.5 STL+ETS

Seasonal Trend decomposition using Loess decomposes a time series into three components, Seasonal, trend and residuals. It does it by iteratively applying weighted average (Loess) to capture the seasonal and trend patterns leaving behind the residual component. Apply an appropriate ETS model to the residual series. The ETS model captures any remaining patterns, autocorrelations and variations in the data after removing seasonality and trend. By combining STL and ETS we leverage STL 's ability to handle complex seasonality and trend and ETS's flexibility in capturing different patterns of error, trend and seasonality. The combination aims to enhance forecasting accuracy by effectively modelling and capturing the underlying patterns and variations in the data.

2.3.6 Neural Network Autoregression (NNAR) model

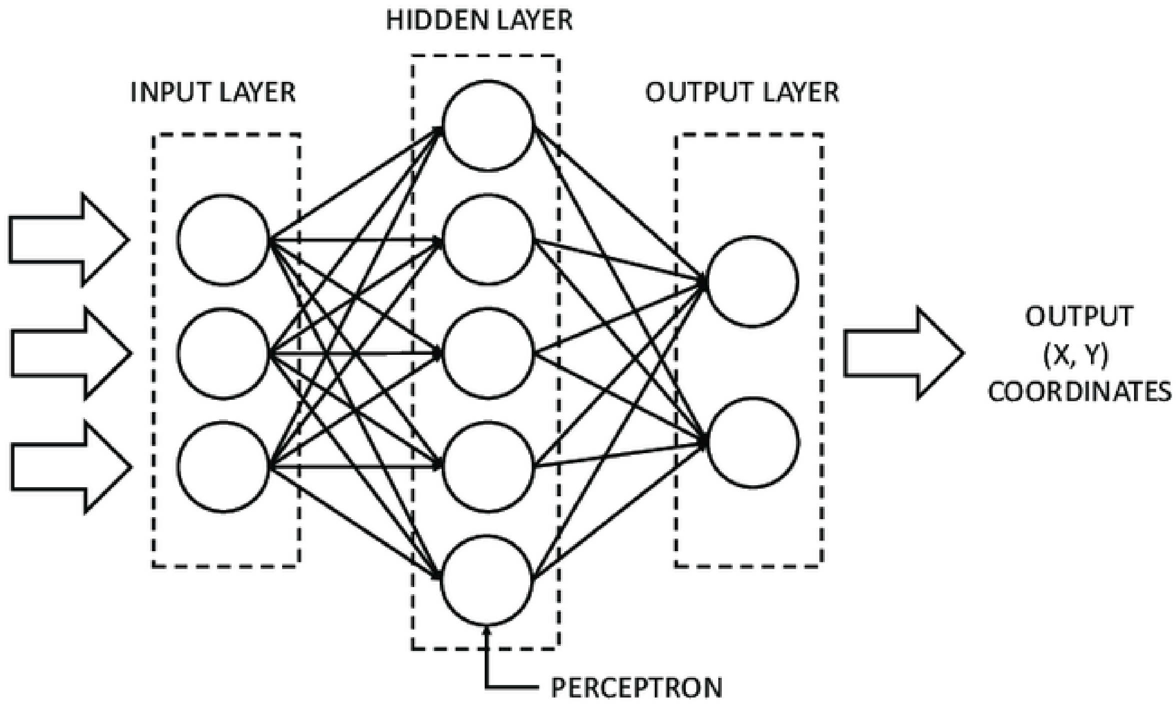
NNAR models can be viewed as a network of neurons or nodes that depict complex nonlinear relationships and functional forms. In a basic neural network framework, the neurons are organized in two layers: (i) the bottom layer identifies the original time series, and (ii) the top layer identifies the predictions. The resulting model is equivalent to a simple linear regression and becomes nonlinear only when an intermediate layer with "hidden neurons" is included. For seasonal data, NNAR models can be described with the notation $NNAR\ (p, P, k)m$, where m is the seasonal period, p denotes the number of nonseasonal lagged inputs for the linear AR process, P represents the seasonal lags for the AR process, and k indicates the number of nodes/neurons in the hidden layer.

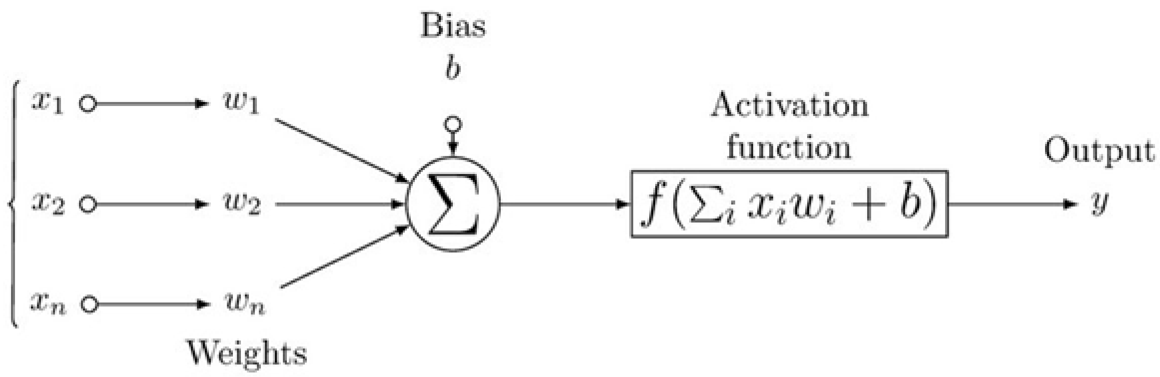
2.3.7 Multi-Layered Perceptron (MLP)

An (artificial) neural network is a ML model inspired by the biological neural networks such as in brains. A simple Artificial Neural Network is only the solution to some problems as it is limited to a linear function and cannot handle complex and massive data. To help in this situation, a Multilayer Perceptron Neural Network can work with non-linear functions. A multi -layered Perceptron (MLP) is a type of artificial neural network that transforms any input dimension to the desired dimension consisting of multi-layers of interconnected nodes or "neuron". It is a foundational architecture of deep learning which is a wildly popular topic in machine learning that involves training artificial neural networks with multiple layers, also known as deep architectures.

It is the commonly used neural network based on supervised learning in which information flows in one direction and has no loops. The main objective is to find the optimized function f() that maps input to the desired output and to learn the optimized bias value (θ) for it. Learning occurs in the MLP using a back propagation algorithm by adjusting the connection weights when there is a deviation between the expected and actual output.

Architecture of MLP





It is a supervised learning algorithm that contains nodes' values, activation functions, inputs, and node weights to calculate the output. The Multilayer Perceptron (MLP) Neural Network works only in the forward direction. All nodes are fully connected to the network. Each node passes its value to the coming node only in the forward direction. The MLP neural network uses a Backpropagation algorithm to increase the accuracy of the training model.

- Multilayer Perceptron falls under the category of feed-forward algorithms, because inputs are combined with the initial weights in a weighted sum and subjected to the activation function, just like in the Perceptron. But the difference is that each linear combination is propagated to the next layer.
- Each layer is feeding the next one with the result of their computation, their internal representation of the data. This goes all the way through the hidden layers to the output layer.
- If the algorithm only computed the weighted sums in each neuron, propagated results to the output layer, and stopped there, it wouldn't be able to *learn* the weights that minimize the cost function. If the algorithm only computed one iteration, there would be no actual learning. This is where back propagation comes into the picture. In Back-propagation algorithm, error between target value and observed value is minimized.

2.4 Hybrid Time Series Models

Hybrid Time series models are a time series forecasting models that combine multiple time series forecasting techniques along with machine learning algorithms to improve forecasting accuracy and performance. The time series data often exhibits complex and non-linear patterns such as irregular fluctuations and varying trends over time and the traditional statistical methods like Autoregressive Integrated Moving Average (ARIMA) and Exponential Smoothing are only effective in capturing linear relationships and certain simple patterns but may fail to model non linearity's in the data ,whereas machine learning algorithms have the ability to capture the non-linear patterns and relationships effectively. The hybrid models may provide better forecasts when dealing with the combination of both linear and non-linear patterns by leveraging the strengths of both statistical methods and machine learning approaches. The statistical component of the hybrid model can capture the linear components of the time series, while the machine learning component can handle the non-linear aspects.

The hybrid time series models can be built by aggregating the forecasts in two different approaches that is parallel and series making it into two different categories namely parallel and series hybrid time series models.

The term 'hybrid model' in the present study refers to the parallel hybrid model. The series hybrid models are omitted as it takes longer time to compute than the parallel hybrid models. In the study parallel hybrid model is obtained using the hybrid of 6 different time series models which generates 12 different forecasted values from each models denoted as f_i where i represents the 12 time series models index

The base models are used generate forecast for T periods. Let $\hat{f}_{it}$ be the forecast from i^{th} base model for t^{th} timepoint, $i = 1,2, \dots b$ $t = 1,2, \dots T$, where b is the total number of base models considered, T is the total number of periods for which the forecasts are desired. The forecast under parallel hybrid model is given by

$$\hat{f}_{ct} = \phi\left(\omega_1 \hat{f}_{1t}, \omega_2 \hat{f}_{2t}, \dots, \omega_b \hat{f}_{bt}\right) \quad t = 1,2, \dots T$$

Where $\phi(.)$ is either linear or non-linear in nature, $\omega_i \hat{f}_{it}$ is the weighted forecast of i^{th} base model at time t, $\omega_i$is the weight for i^{th} base model which is constant for all the forecasts from that model. The linear combination function is the most used method in parallel hybrid model. In linear combination function approach, the final forecasts are obtained by summation of weighted forecasts of all the base models. The method that computes the weights $\{\omega_1, \omega_2, \dots \omega_b\}$ is called as the weighting method.

In the present study the parallel hybrid models with linear combination function with following 6 weighting methods are considered:

- **Simple Average (SA) weighting method**: The Simple average method combines the forecasts from different base models by taking the arithmetic mean of the individual forecasts to create a composite forecast. The final forecast is the result of the simple average of the individual forecasts from all the base models considered in the hybrid model. It is the easiest way for determining component weights which allocates equal weights to each forecasting model.
- **Trimmed Mean (TM) weighting method**: The trimmed mean weighing method uses the trimmed mean as weighting scheme to combine the forecast generated by different forecasting method removing a certain percentage of extreme values from both the ends of the sorted data(In the present study 10 % is trimmed from both the ends).This approach gives less weight to extreme forecasts and focuses on the central tendency of the individual forecasts.

  The formula for trimmed mean can be given as ,

$$TM(i) = \frac{1}{(n-2i)} \sum_{k=i+1}^{n-i} f_k$$

  Where,

  i is the integer and is $0 < i < n/2$ representing the index of the forecasting method

  n is the number of forecasting methods(The number of individual forecasts)

  f_k represents the forecast generated by the k^{th} forecasting method
- **Weighted Mean (WM) weighting method**: The weighted mean method involves combing the forecasts generated by different forecasting methods using weighted averages. Unlike the simple average method where all the individual forecasts are given equal weights, the weighted mean method assigns different weights to each forecast allowing more influential forecast to have a greater impact on the final aggregated forecast. The formula to calculate the weighted mean is given by

$$wM(i) = \frac{1}{n}\left(i\hat{f}_{i+1} + \sum_{k=inf\ 1}^{n=i} \hat{f}_k + \dot{I}\hat{f}_{n=i} \right)$$

- **Simple Weightage Average Method (SWAM):** In this method each individual forecast is assigned a weight and this weight is based on the Root Mean Squared Error (RMSE) of each model and the final aggregated forecast is obtained by taking the weighted average of the individual forecasts. The weights can be calculated as

$$Wi = \frac{i}{\sum_{i=1}^{n} i}, \; i=1 \text{ to } n$$

Where ,

Wi represents the forecast weight for the i^{th} forecasting method

i is the index of the forecasting method for which the weight is being calculated

- **Ordinary Least Squared method for minimum error (OLSME):** The ordinary least square method can be used as minimum error method for assigning weights to the individual forecasts. The goal is to find the optimal weights that minimize the error between the combined forecast and the actual observed values. Linear regression is performed using the fitted values from 6 different time series models which is taken as independent variables to forecast the given time series data (dependent variable). Using ordinary least square method (OLS) method the optimal weights () are estimated that minimize the sum of squared differences between the observed values x and the fitted values from the linear regression model. The estimated coefficients are then used to calculate the combined forecast of the test data.
- **Var-Cov Method**: The Variance – Covariance weighting method is used to calculating the weights for individual models based on the sum of squared residuals. The residuals represent the difference between the actual values and the fitted values obtained from each time series models. Normalization is done by diving each weight by sum of all weights ensuring that the sum of all the normalized weights is equal to 1. The method than uses these errors to determine how much weight to give to each model when combing the forecasts. The models with smaller errors are given higher weights indicating that they have stronger influence on the combined forecast.

$$ycombined, = w1f1, t + w2f2, t + \cdots + wnfn, t(w1 + w2 + \cdots wn = 1)$$

The forecasting error is given by,

$$e\_combined, = yt - \hat{}yt = \sum_{i=1}^{n} wi\, yi - (w1f1, t + w2f2, t + \cdots + wnfn, t)$$

$$= w_1 (y_t - wf_1, t) + w2 (yt - w2f2, t)$$

2.5 Bagged Base Time Series Models

Bagging also known as Bootstrap Aggregating is an ensemble technique used to improve the accuracy and robustness of the time series models. Bagged method aims to train the time series model on different random bootstrap samples of the training data and then combine their predictions. The given time series data is split into multiple random subsets with replacement each called a "bootstrap sample". In the study 5,10,20,30,40 and 50 bootstrap samples are considered for bagging the base models. For each of these bootstrap samples the base time series models are trained which ensures that each model is trained on different data introducing diversity in the model. This method's particularly effective for reducing the variance and improving generalizations

2.6 Bagged Hybrid Time Series Models

A bagged hybrid model is an ensemble technique that combines the principles of bagging (Bootstrap Aggregating) and hybrid modelling. The Bagged hybrid model involves the technique of bootstrap aggregating which randomly selects the samples by re-sampling the subsequence's of the time series and training the model which produces the forecasts. Further combining these individual forecasts using the hybrid techniques like Var-Cov method , trimmed mean etc where each models contribution is determined by its performance . This approach aims to capture different aspects of the time series patterns while reducing the impact of individual model weakness or over-fitting.

2.7 Forecast Performance

Evaluation Metrics:

The main metrics used to compare the performances of the base and hybrid prediction time series models are mean absolute error (MAE), mean absolute percentage error (MAPE), mean square error (MSE) and root mean square error (RMSE). In the study we consider only root mean square error (RMSE) and mean square error (MSE) as they are the most common metrics used to measure the accuracy of a time series forecasting model.

Root Mean Square Error (RMSE): It is calculated by taking the square root of the mean of the squared differences between the actual values and the predictions of the model. RMSE is often reported in units of "error per unit," providing a fair and straight forward comparison of different time series models. The formula for RMSE is given by

$$\text{RMSE}=\sqrt{\frac{1}{n}\sum_{i=1}(y_i-\hat{y}_i)^2}$$

Mean absolute error (MAE): It is another common metric used to measure the accuracy of a time-series forecasting model. It is also called **Mean Absolute Deviation (MAD).**It is calculated by taking the average of the absolute differences between the actual values and the predictions of the model**. The formula for MAE is given by**

MAE$=\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$

Where,

y_i is the original time series data

$\hat{y}_i$ is the forecasted time series

CHAPTER 3

ANALYSIS AND DISCUSSION

Introduction:

This chapter provides a comprehensive time series analysis of 12 chosen Indian economic indicators. The detailed discussion covers the results, interpretation, and conclusions drawn from the evidence. The analysis includes findings from various time series models, including base, hybrid, bagged base, and bagged hybrid models.

The chapter is divided into 6 sections. Section 3.1 gives an overview of the time series by summarising the main characteristics of time series including trends and seasonality. Section 3.2 presents the evaluation of the performance of various base models considered and discusses the accuracy of forecasts generated. Section 3.3 gives out the assessment of the performance of different hybrid models along with an analysis of the accuracy of generated forecasts. The comparison of performance (both in terms of time and forecasting accuracy) of base and hybrid models is presented in section 3.4. The impact of bagging on a base and hybrid model is assessed in section 3.5 and the section 3.6 is provided with the conclusion.

3.1 Overview of Time Series

The Economic indicators are the quantitative measurements that provide insights into various aspects of an economic's performance. It serves as an important benchmark for evaluating economic performance making informed policy decisions and understanding broader economic trends.

The Indian economic indicators are collected from multiple sources. The following table gives the metadata on the time series gathered.

Table 3.1: Representing the metadata of the indicators

Code	Name of the Indicators
E1	Broad Money Growth (%)
E2	Broad Money to Reserves (%)
E3	composite Stock Price Index (monthly average, local index)
E4	Policy Rate, end of period (% per annum)
E5	Exchange Rate, Local Currency per US$ (average)
E6	Headline Inflation Rate
E7	Real Effective Exchange Rates Based on Manufacturing Consumer Price Index for India, Index 2015=100, Monthly, Not Seasonally Adjusted
E8	Consumer Price Index: Total All Items for India, Growth rate same period previous year, Monthly, Not Seasonally Adjusted
E9	Economic Policy Uncertainty Index for India, Index, Monthly, Not Seasonally Adjusted
E10	Production: Manufacturing: Total manufacturing: Total manufacturing for India, Index 2015=100, Monthly, Not Seasonally Adjusted
E11	Real Broad Effective Exchange Rate for India, Index 2020=100, Monthly, Not Seasonally Adjusted
E12	Goods, Value of Exports for India, Dollars, Monthly, Not Seasonally Adjusted* 1000000 dollars

Each of the time series covers the period from January 1, 2006, to December 1, 2022.The data from January 1, 2006, to December 12, 2021, accounting for nearly 95% of the dataset, is utilized as the training data for fitting different time series models. The accuracy of these models is assessed using the test data, which comprises the time series spanning from January 1, 2022, to December 1, 2022.

The following table gives the descriptive statistics (on a monthly basis) of all indicators for the period 2006 to 2022.

Table 3.2 Descriptive Statistics of Indian Economic indicators

Indicator	Mean	Standard deviation	Skewness	Excess Kurtosis	Shapiro-Wilk P -value
E1	13.76309	4.57594	0.49617	-0.8028	9.201e-07
E2	4.83715	0.66015	-0.80592	-0.45392	3.012e-10
E3	27108.18166	13564.8503	0.95656	0.11437	1.187e-08
E4	6.42843	1.38352	-0.32504	-0.93876	2.026e-07
E5	59.17185	12.08179	-0.0425	-1.33668	6.548e-09
E6	6.72439	2.51208	0.27414	-0.58206	0.007594
E7	97.50539	6.35537	-0.39365	-1.10878	6.585e-07
E8	7.12427	2.79845	0.61808	0.40131	0.001145
E9	95.78098	49.14168	1.28901	1.94484	1.533e-09
E10	96.40122	16.8119	-0.47963	-0.10003	0.00431
E11	96.13946	4.83326	-0.62315	-0.52749	2.654e-06
E12	23028.91176	7249.50838	0.05665	-0.04796	4.254e-06

In general, it is often desirable to have a time series with skewness and excess kurtosis values close to zero which would indicate a Gaussian time series. The rule of thumb for symmetry is that coefficient of skewness lies between -0.5 to 0.5. From above table, the indicators EI2, EI3, EI8, EI9 and EI11 are found to have significant skewness. The indicators EI5, EI7 and EI9 have absolute value of excess kurtosis exceeding 1. The Shapiro Wilk test for normality indicates statistically significant (p-value <0.05) across all the time series data which concludes that the given time series data is non-normal. The boxplots of all the indicators are presented below.

The boxplots are used to visualize the distribution of the time series and to identify outliers.

Figure 3.1: Representing the Box plot of the indicators

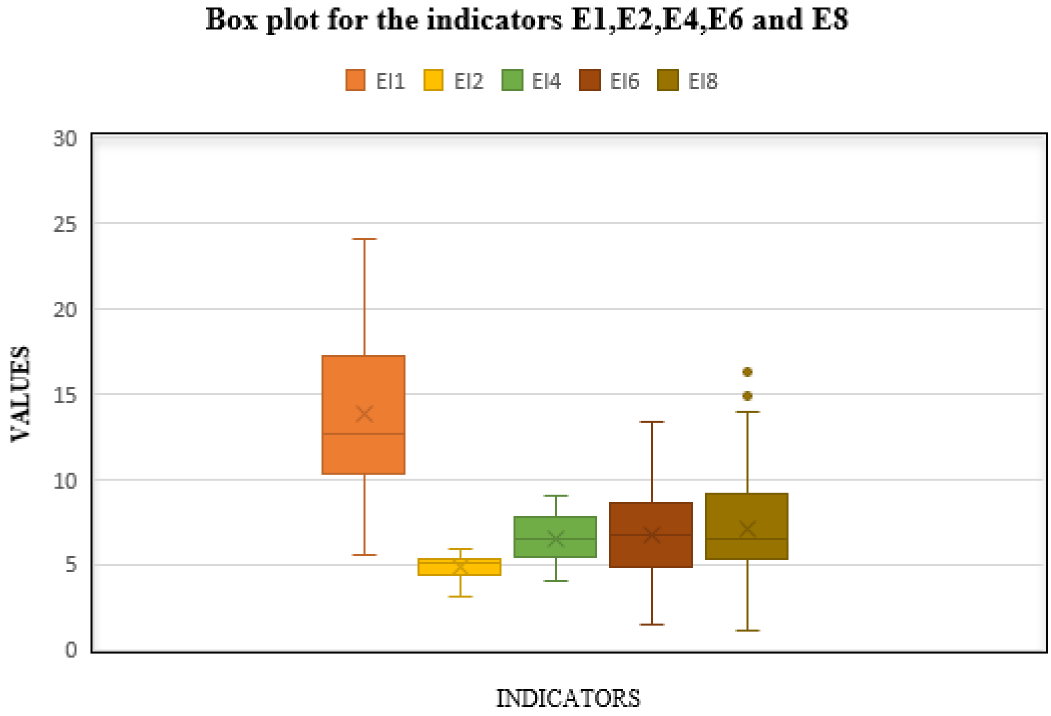


Figure 3.2: Representing the Box plot of the indicators

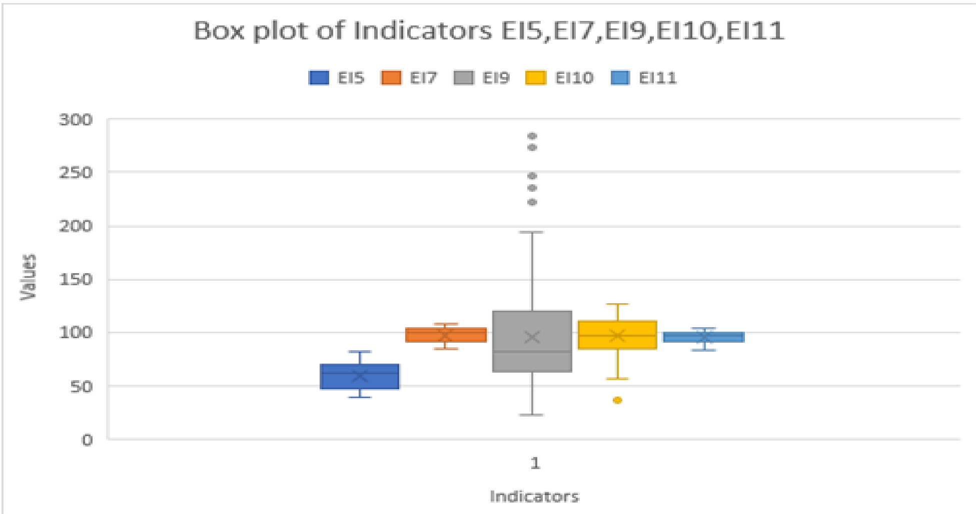
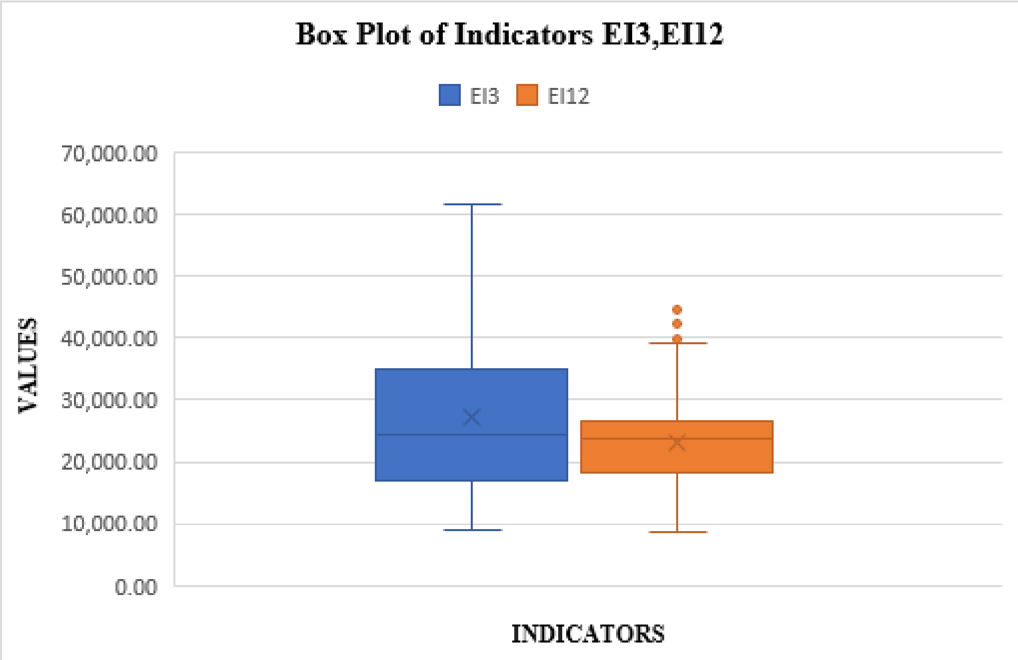


Figure 3.3: Representing the Box plot of the indicators



From the above figures we can conclude that there might be the presence of outliers in the indicators E9, E10 and E12 as there are data points that fall outside the whiskers. And the distribution of the time series considered is not symmetric. This could suggest that the observed economic indicators are skewed towards specific months or seasons.

Time Profiles:

The following figures give the time profile of all the indicators:

Figure 3.4: Representing the time profile of E1

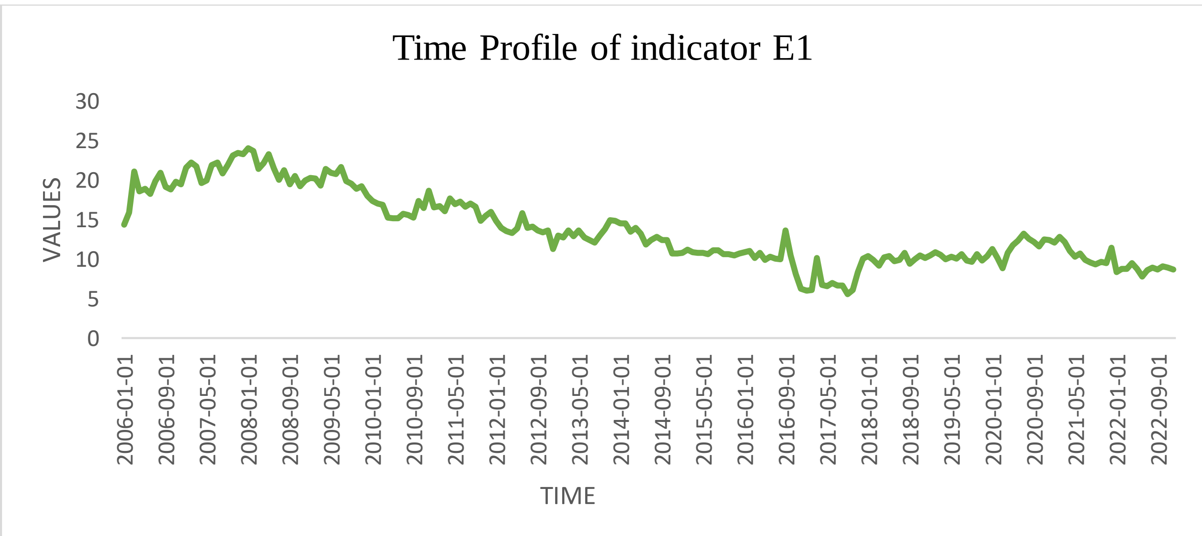


Figure 3.5: Representing the time profile of E2

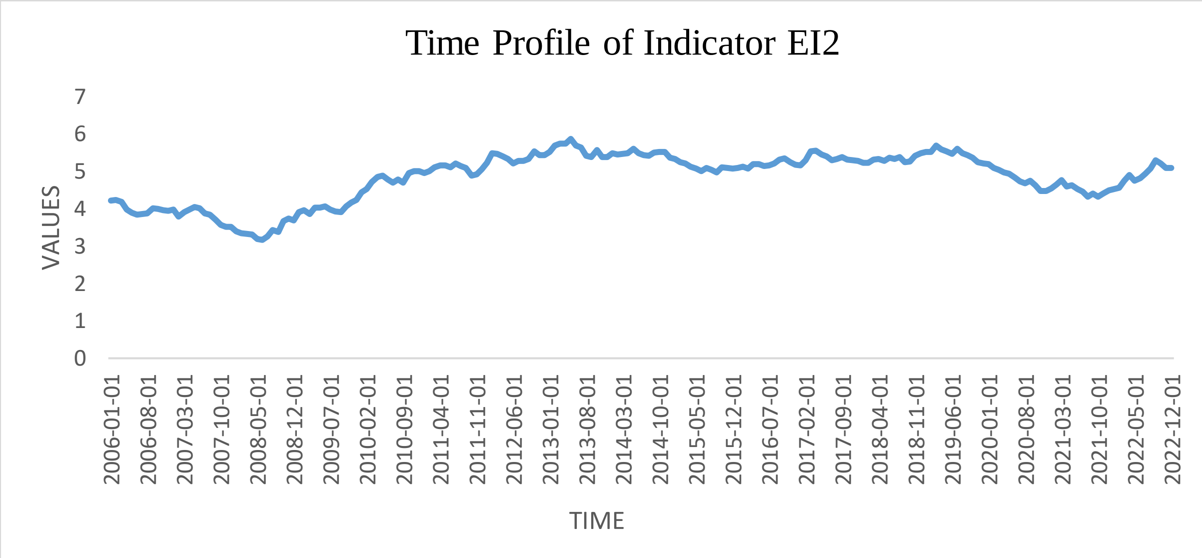


Figure 3.6: Representing the time profile of E3

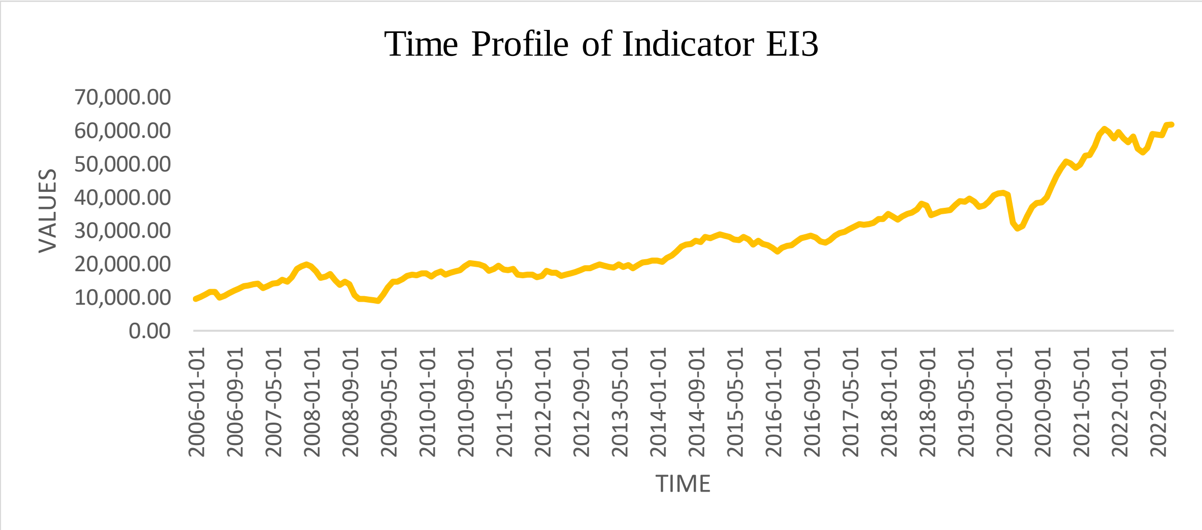


Figure 3.7: Representing the time profile of E4

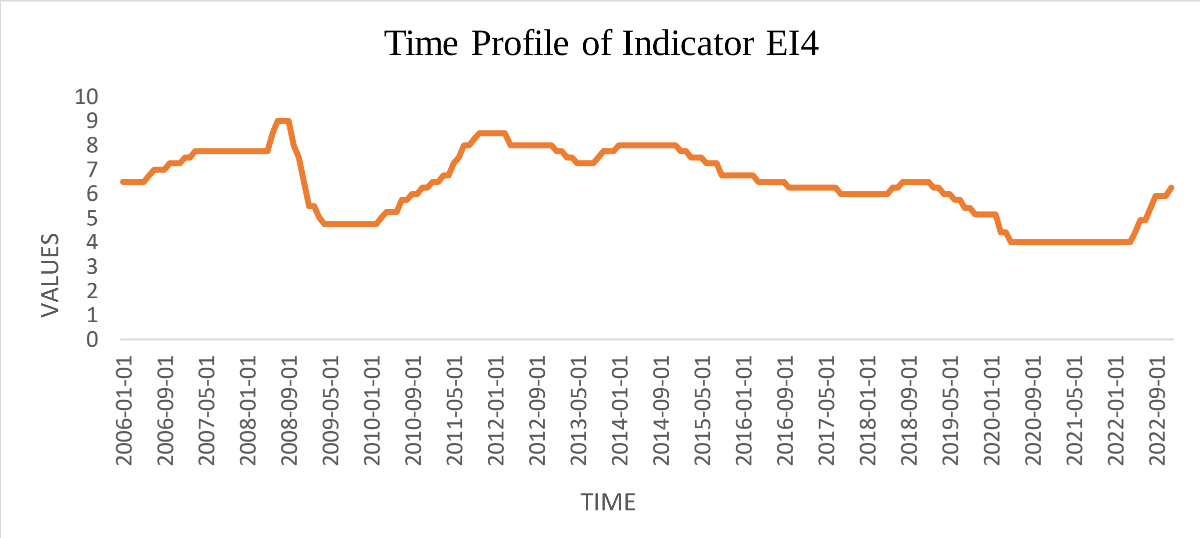


Figure 3.8: Representing the time profile of E5

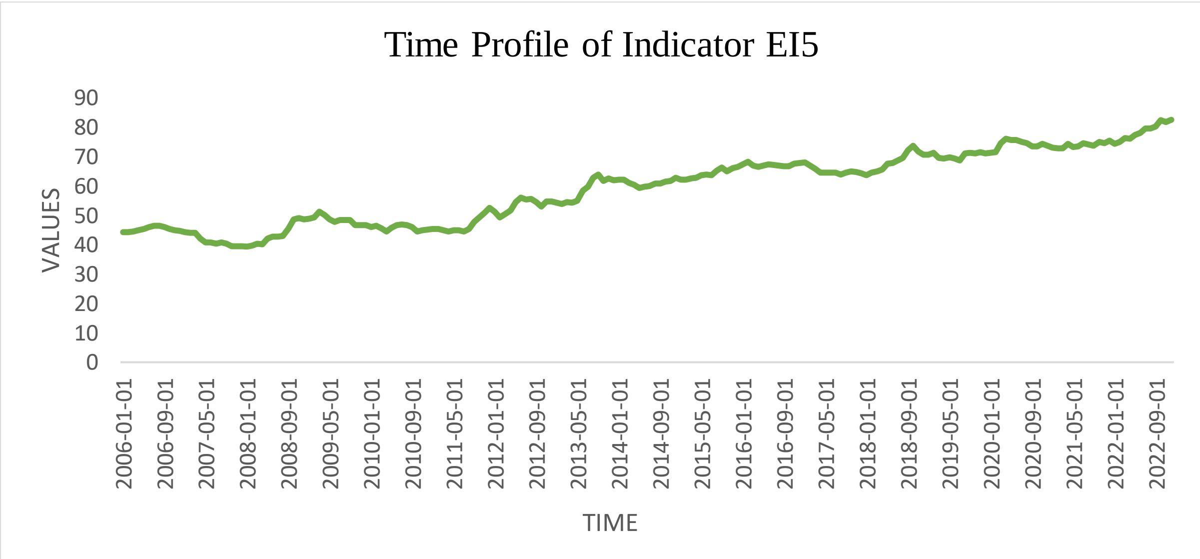


__Figure 3.9: Representing the time profile of E6__

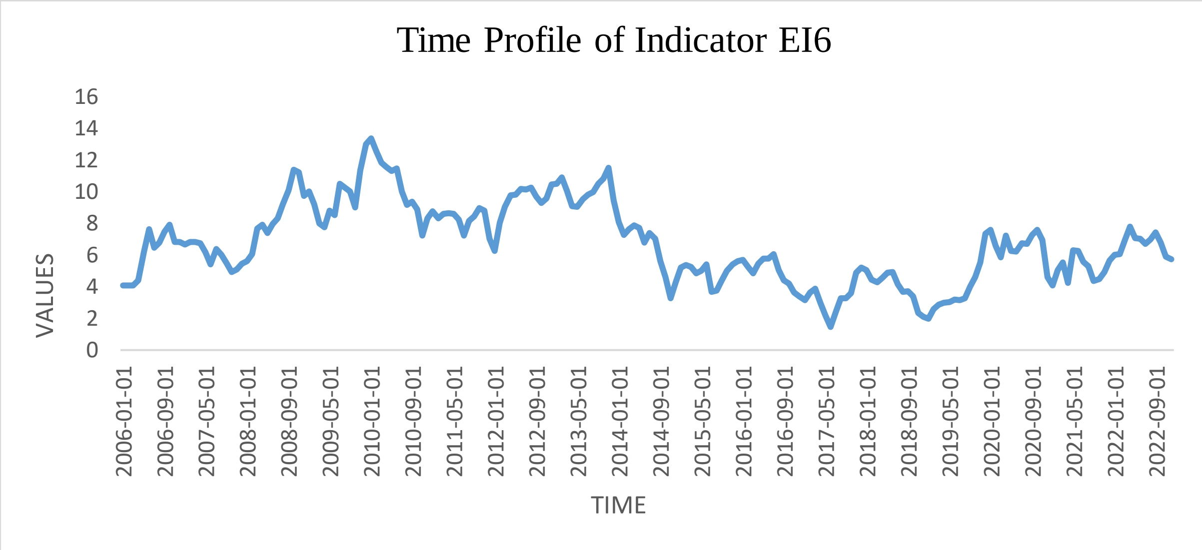


__Figure 3.10: Representing the time profile of E7__

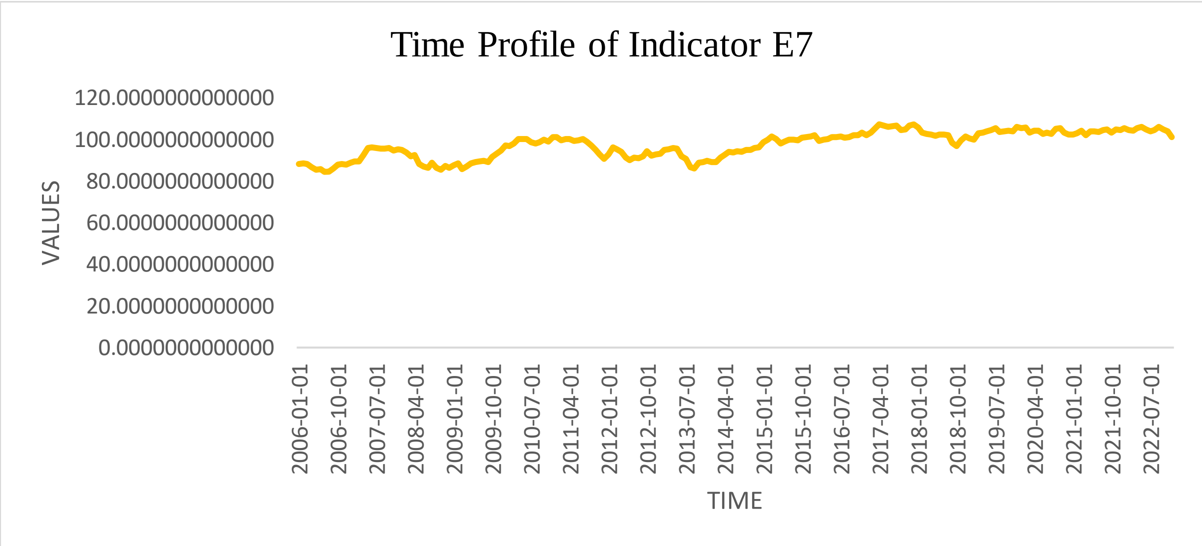


Figure 3.11: Representing the time profile of E8

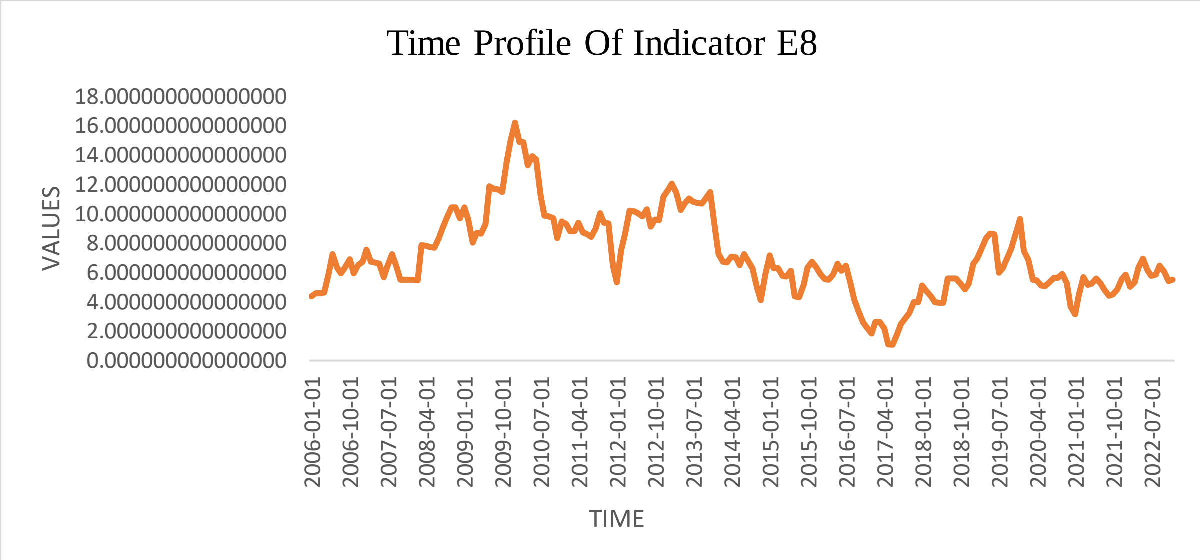


Figure 3.12: Representing the time profile of E9

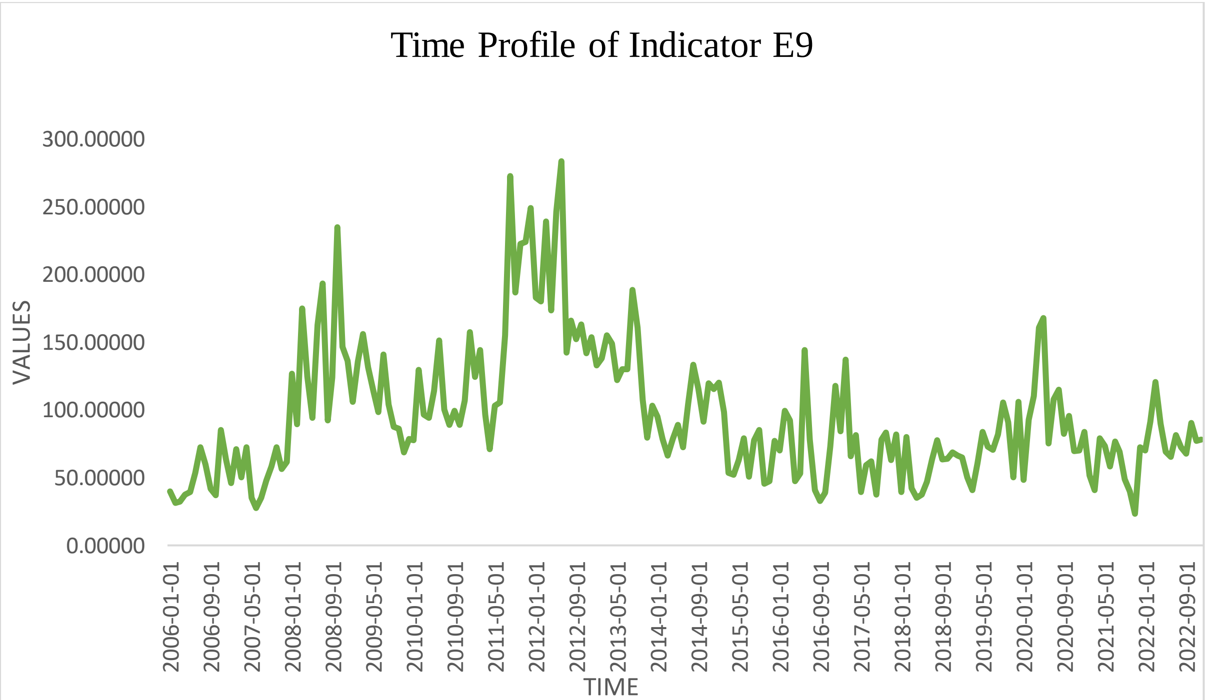


Figure 3.13: Representing the time profile of E10

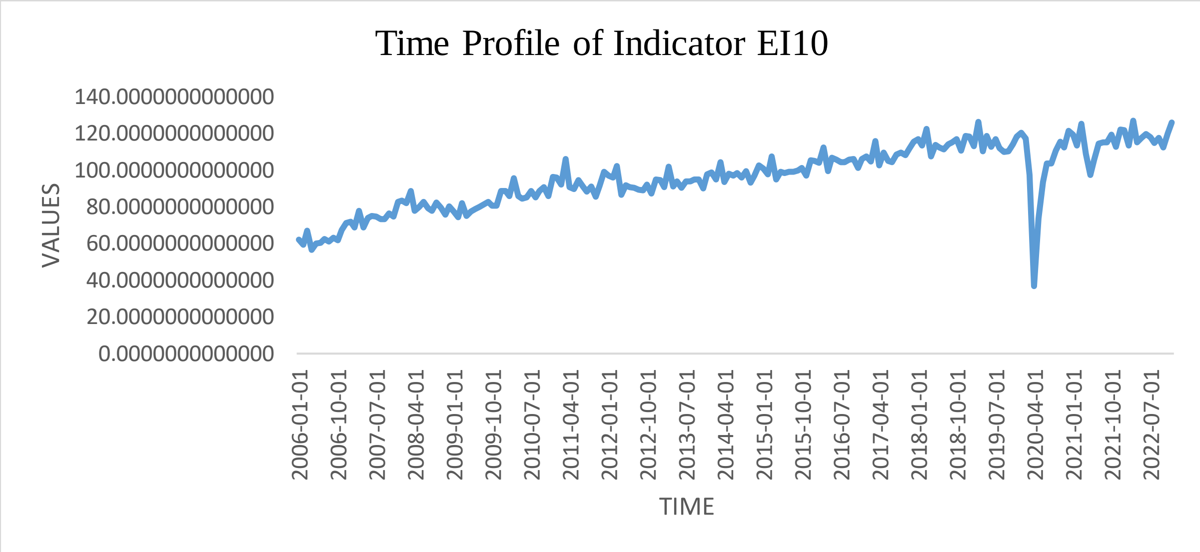


Figure 3.14: Representing the time profile of E11

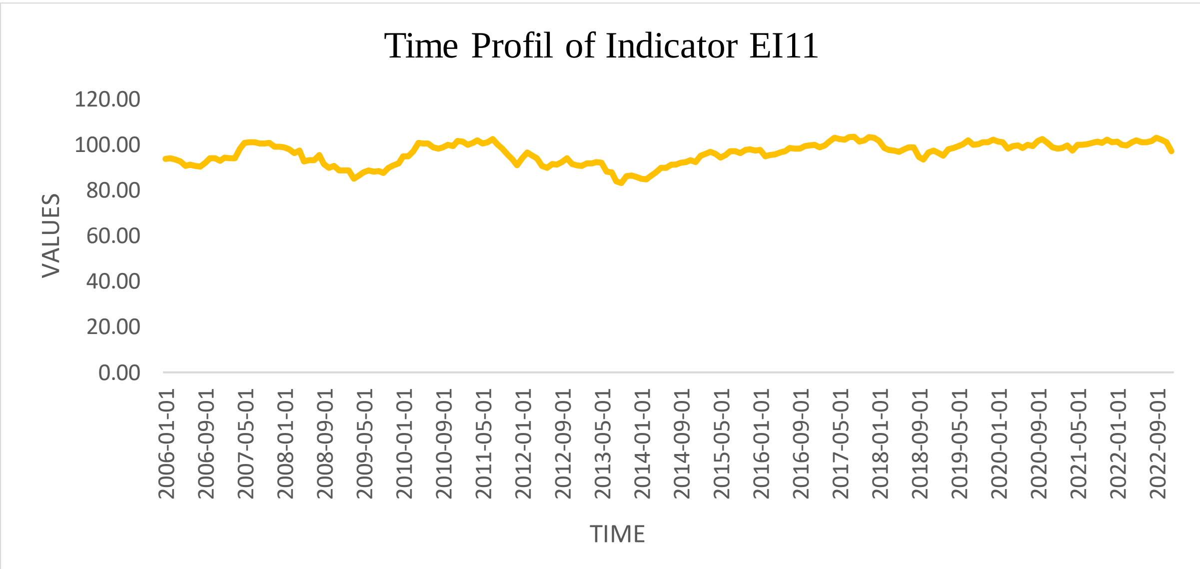
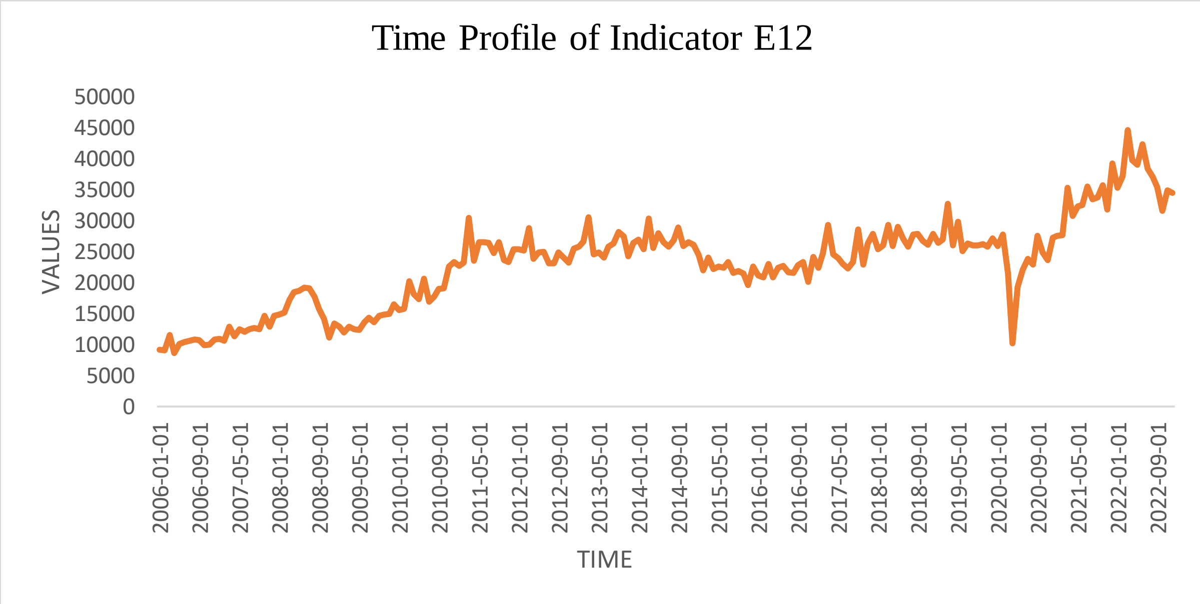


Figure 3.15: Representing the time profile of E12



Interpretation: From the above graphical representation of the times series considered it is evident that the given time series is non-stationary due to the presence of trends (upward or downward) and repeated patterns of seasonality variations. The trend indicates the shifts in the variable over time, while the seasonal variations suggest periodic fluctuations that are influenced by specific time intervals.

Trend and Seasonality Tests:

The following table gives the test statistics and p values of Mann Kendal trend test and Kruskal Wallis seasonality test for all indicators:

Table 3.3 Trend and Seasonality tests for Indian Economic Indicators

Indicator	Mann Kendal trend test		Kruskal Wallis seasonality test	
	Test Statistics	P value	Test Statistics	P value
EI1	-0.673	<2.22e-16	7.76	0.7342634
EI2	0.332	<2.22e-16	16.86	0.1121286
EI3	0.84	<2.22e-16	7.78	0.732521
EI4	-0.389	1.0129e-14	9.93	0.5369927
EI5	0.805	< 2.22e-16	9.76	0.5521431
EI6	-0.326	2.0457e-11	0.91	0.9999689
EI7	0.612	< 2.22e-16	7.39	0.7669223

EI8	-0.302	4.9771e-10	0.94	0.9999623
EI9	-0.136,	0.0049987	16.86	0.1119678
EI10	0.764	< 2.22e-16	114.02	0
EI11	0.311	< 2.22e-16	14.65	0.1991757
EI12	0.565,	< 2.22e-16	86.89	6.750156e-14

Result : From above table, Mann Kendal trend test suggests that there is statistically significant trend present in the time series data (all the p values < 0.05). The trend could be either increasing or decreasing depending on the direction of the Mann-Kendal test statistics.

Kruskal Wallis suggests that there isn't sufficient evidence to conclude that there are significant difference in medians between the time periods that is there is no seasonality (p-value>0.05), whereas for the indicators E11 and E12 it is statistically significant, therefore we conclude that is seasonal variations in these indicators.

3.1 Base Time Series Models

The following base models are considered and their accuracy is evaluated.

- Seasonal Autoregressive Integrated Moving Average (SARIMA)
- Exponential Smoothing State Space (ETS) Model
- Seasonal Trend Decomposing using LOESS (STL) and then applying ARIMA (STL+ARIMA)
- Seasonal Trend Decomposing using LOESS (STL) and then applying ETS (STL+ETS)
- Multi-Layer Perceptron (MLP) model
- Neural Network Autoregression (NNAR) model.

Specification of various models fitted to 12 indicators

Table 3.4: Model Representations of all the 12 Indicators

	SARIMA	ETS	STL+ARIMA	STL+ETS	NNAR
E1	ARIMA (2,1,2)	ETS (A, N, N)	ARIMA (2,1,0)	ETS (A, A, N)	NNAR (2,2)
E2	ARIMA(0,1,0)	ETS (A, N, N)	ARIMA(0,1,1)	ETS (A, A, N)	NNAR(2,2)
E3	ARIMA(0,1,1)	ETS (M, A, N)	ARIMA (0,1,1)	ETS (A, A, N)	NNAR(1,1)
E4	ARIMA(1,1,2)	ETS (M, A, N)	ARIMA(2,1,2)	ETS (M, A, N)	NNAR(3,2)
E5	ARIMA (0,1,1)	ETS (A, A, N)	ARIMA (0,1,1)	ETS (A, A, N)	NNAR(1,1)
E6	ARIMA (0,1,3)	ETS (A, N, N)	ARIMA (0,1,3)	ETS (A, N, N)	NNAR(14,8)
E7	ARIMA (0,1,0)	ETS (A, N,N)	ARIMA (0,1,1)	ETS (A, N,N)	NNAR(2,2)
E8	ARIMA (0,1,1)	ETS(A,N,N)	ARIMA (0,1,1)	ETS(A,N,N)	NNAR(13,7)
E9	ARIMA (0,1,2)	ETS(M,A,N)	ARIMA (1,1,2)	ETS(M,A,N)	NNAR(3,2)
E10	ARIMA (1,1,1)	ETS(A,A,A)	ARIMA (1,1,3)	ETS(M,A,N)	NNAR(15,8)
E11	ARIMA (0,1,0)	ETS(A,N,N)	ARIMA (2,1,2)	ETS(A,N,N)	NNAR(2,2)
E12	ARIMA (0,1,1)	ETS (M, A, M)	ARIMA (0,1,1)	ETS (M, A, N)	NNAR(14,8)

Interpretation:

The order of the models are obtained according to the characteristics of the time series data.

NNAR: For the model NNAR (2,2) we say that average of 20 networks, each of which is a 2-2-1 network with 9 weights are considered, similarly for NNAR(1,1) Average of 20 networks, each of which is a 1-1-1 network with 4 weights, NNAR(3,2) Average of 20 networks, each of which is a 3-2-1 network with 11 weights, NNAR (14,8) Average of 20 networks, each of which is a 14-8-1 network with 129 weights, NNAR(13,7) Average of 20 networks, each of which is a 13-7-1 network with 106 weights, NNAR(15,8) Average of 20 networks, each of which is a 15-8-1 network with 137 weights are considered.

RMSE:

The metrics Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) are used to evaluate the forecasting accuracy of all the base models. Since RMSE and MAE agreed with one another, only RMSE values are reported in the following table:

Table 3.5: RMSE of selected base models

Indicator	SARIMA	ETS	STL+ SARIMA	STL+ ETS	NNAR	MLP	Best Model
EI1	1.9942	2.18297	2.06001	1.97547	2.25319	1.6422	MLP
EI2	0.49021	0.49022	0.46503	0.44836	0.28555	0.46293	NNAR
EI3	2436.343	2645.25	2170.772	2714.921	2917.917	4780.291	STL+SARIMA
EI4	1.2791	1.27908	1.36634	1.39662	1.16274	1.5952	NNAR
EI5	2.86913	3.07816	2.67794	3.43042	4.62545	3.2957	STL+SARIMA
EI6	1.58834	1.20902	1.47993	1.37034	1.3066	1.57181	ETS
EI7	1.22507	1.22507	1.55173	1.47138	1.43223	1.37535	SARIMA
EI8	1.04646	0.62264	0.66878	0.74256	0.9509	1.2287	ETS
EI9	28.82434	21.41196	34.21229	31.34499	19.58322	25.7322	NNAR
EI10	13.35769	5.01087	5.42245	5.36884	5.32329	5.74075	ETS
EI11	1.44802	1.44803	1.73936	1.5766	1.73374	1.5533	SARIMA
EI12	4153.636	3015.434	3702.535	3757.13	6628.942	6705.081	ETS

Result:

The ETS model is found be the best in 33.33% of the cases. Each of the SARIMA, STL+ARIMA, NNAR and MLP models are found be the best in 16.67% of the cases. However, deciding the best model solely based on minimum RMSE would be misleading. Hence, we use the measure Relative Range (RR). Let E_i be the set of all RMSE values for i^{th} indicator

The RR for i^{th} indicator is computed as,

$$RR_i = \frac{Max(E_i) - Min(E_i)}{Max(E_i)}, i = 1,2, \dots .12$$

If the RR for an indicator is at least 25%, then we conclude that corresponding best model has remarkably minimum RMSE when compared to other models for the same indicator.

The following table gives the best model and RR of corresponding indicator which helps us to decide whether the best model has remarkably minimum RMSE.

Table 3.6: RR of the best base models

Indicator	Best Model	RR (%)
EI1	MLP	27.12
EI2	NNAR	41.75
EI3	STL+SARIMA	54.59
EI4	NNAR	27.11
EI5	STL+SARIMA	42.1
EI6	ETS	23.88
EI7	SARIMA	21.05
EI8	ETS	49.33
EI9	NNAR	42.76
EI10	ETS	62.49
EI11	SARIMA	16.75
EI12	ETS	55.03

Result: From the above table we observe that all the base time series models has atleast 25% of RR rate except for the indicators EI7 and EI11 for which SARIMA model is implemented. Therefore, we conclude that SARIMA is not a good fit for the time series data EI7 and EI11.

Overall Conclusion: Based on the findings from 3.1 and 3.2, the descriptive analysis and graphical representation indicates that the given time series data is non- stationary due to the presence of trend and seasonal variations in the data. These characteristics have been further evaluated by application of for non- parametric statistical tests like Mann-Kendal and Kruskal-Wallis tests. The RR (%) measure confirms that the time series model is not a good fit for the indicators EI7 and EI11.

3.2 Hybrid Time Series Models

The following hybrid models are considered:

- Simple Average (SA) weighting method (H-SM)
- Trimmed Mean (TM) weighting method (H-TM)
- Weighted Mean (WM) weighting method (H-WM)

- Simple Weightage Average Method (SWAM) (H-SWAM)
- Ordinary Least Squared method for minimum error (OLSME) (H-OLSME)
- Var-Cov Method (H-Var Cov)

The following table gives the RMSE measures of all the six hybrid models considered:

Table 3.7: RMSE of selected hybrid models

Indicator	H-SM	H-TM	H-WM	H-SWAM	H-OLSME	H-VarCov	Best Model
EI1	2.04548	2.05924	7.46192	2.04332	1.81322	2.04332	H-OLSME
EI2	0.44507	0.46709	1.88672	0.46055	0.29843	0.46055	H-OLSME
EI3	2503.607	2436.334	30831.72	2362.745	4609.361	2362.745	H-SWAM
EI4	1.3264	1.32937	1.31052	1.33028	1.47434	1.33028	H-WM
EI5	3.33853	3.17291	36.06687	3.53309	5.00604	3.53309	H-TM
EI6	1.34384	1.36954	1.77562	1.29538	0.77841	1.29538	H-OLSME
EI7	1.30703	1.32555	51.85625	1.32023	2.07823	1.32023	H-SM
EI8	0.82182	0.80412	2.27839	0.81732	0.95613	0.81732	H-TM
EI9	25.14996	25.21755	25.49602	27.52403	28.04755	27.52403	H-SM
EI10	4.75768	4.1855	60.44571	6.25718	8.11916	6.25718	H-TM
EI11	1.56579	1.53775	50.29614	1.53007	2.32269	1.53007	H-SWAM
EI12	3501.017	3586.194	19329.18	3680.258	4517.432	3680.258	SM

Result: The H-SM, H-TM and H-OLSME models are the best in 25% of the cases, H-SWAM is the best model in 16.7% of the cases, H-WM model is the best in 8.3% of the cases. As evaluating just the minimum value of RMSE is insufficient RR value is given for the indicators as follows. The following table gives the RR for each indicator along with the best model.

Table 3.8: RR of the best hybrid models

Indicator	Best Model	RR (%)
EI1	H-OLSME	75.7
EI2	H-OLSME	84.18
EI3	H-SWAM	92.34
EI4	H-WM	11.11
EI5	H-TM	91.2
EI6	H-OLSME	56.16
EI7	H-SM	97.48
EI8	H-TM	64.71
EI9	H-SM	10.33
EI10	H-TM	93.08
EI11	H-SWAM	96.96
EI12	SM	81.89

Result:

From the above table all indicators except EI4 and EI9 have RR more than 50%. The high value of RR for an indicator implies that the six hybrid models considered have very much variation in the RMSE. For example, the, the best model of EI11 is H-SWAM is 96.96% which indicates that the range of RMSEs is 96.96% of the maximum RMSE reported. The H-WM model gives the maximum RMSE for EI11 which is 50.29614 and Minimum RMSE is 1.53007 which is given by H-SWAM model. Even though H-WM is the worst model for EI11 it is found to be the best model for EI4.

Note:

The weighing methods in the hybrid model play a very crucial role. There is no universal rule to determine the ideal weighing method, as its performance heavily relies on the specific dataset. Investigating multiple weighing methods for the hybrid model, rather than relying on one, is a thoughtful approach.

3.3 Comparison Of Base and Hybrid Time Series Models

Table 3.9 Comparison of base and hybrid time series models

Indicator	Base	hybrid	Best	Overall Best
EI1	1.6422	1.81322	BASE	MLP
EI2	0.28555	0.29843	BASE	NNAR
EI3	2170.772	2362.745	BASE	STL+SARIMA
EI4	1.16274	1.31052	BASE	NNAR
EI5	2.67794	3.17291	BASE	STL+SARIMA
EI6	1.20902	0.77841	HYBRID	OLS Based
EI7	1.22507	1.30703	BASE	SARIMA
EI8	0.62264	0.80412	BASE	ETS
EI9	19.58322	25.14996	BASE	NNAR
EI10	5.01087	4.1855	HYBRID	TM
EI11	1.44802	1.53007	BASE	SARIMA
EI12	3015.434	3501.017	BASE	ETS

Conclusion: From the above table we conclude that the Base time series models are the best approach for the time series data considered.

3.4 Bagged Based and Hybrid Models

In the preceding sections, we constructed both base and hybrid models using the provided time series data. To determine the impact of bagging on forecasting accuracy, we generated bagged time series for each indicator. As there are no established guidelines for selecting the number of bootstrap samples, we fitted the models using different numbers of bootstrapped time series. Specifically, we experimented with k=5, 10, 20, 30, 40, and 50. Larger values of k (greater than 50) were not considered in this study due to time limitations.

The procedure is to evaluate of bagging for various values of k, and then decide the optimal value of k for which the RMSE is minimum.

The forecasted time series data from both base and hybrid time series model along the bagged time series models are given below for comparison.

The table below represents the RMSE values for all the bootstrap samples considered for Bagging time series models and then the minimum RMSE values are highlighted.

Table 3.10: Comparison of base and hybrid time series models

Indicator	EI1	EI2	EI3	EI4	EI5	EI6	EI7	EI8	EI9	EI10	EI11	EI12
SARIMA	1.9942	0.49021	2436.34	1.2791	2.86913	1.58834	1.22507	1.04646	28.8243	13.3577	1.44802	4153.64
(k=5)	1.39822	0.59178	5001.37	1.27663	2.9818	1.38911	1.39788	0.73649	35.5296	5.74542	1.64072	4130.47
(k=10)	1.19684	0.58223	4856	1.29202	3.44548	1.74771	1.28265	0.90189	32.8302	3.90585	1.43445	4311.01
(k=20)	1.0733	0.63438	4614.48	1.27659	3.38231	1.74029	1.42848	0.97506	32.5197	3.76271	1.42661	4142
(k=30)	1.12762	0.61623	6222.52	1.37374	3.76146	1.74244	1.21	0.95577	33.0933	3.64831	1.44455	4525.7
(k=40)	1.05826	0.6208	6174.61	1.27385	3.8089	1.63359	1.21971	0.86909	33.6506	3.46656	1.51614	4192.12
(k=50)	0.92088	0.61469	6266.68	1.28283	3.76082	1.62367	1.23607	0.90624	32.6366	3.42545	1.54221	4229.52
ETS	2.18297	0.49022	2645.25	1.27908	3.07816	1.20902	1.22507	0.62264	21.412	5.01087	1.44803	3015.43
(k=5)	1.62138	0.58578	5976.24	1.2627	3.8245	1.41406	1.37963	0.61333	32.1001	3.74012	1.65696	3343.79
(k=10)	1.38297	0.56242	5301.11	1.27917	3.9221	1.78885	1.29441	0.81307	33.3898	3.81017	1.39829	3249.76
(k=20)	1.29345	0.63361	4743.69	1.26513	4.07758	1.81407	1.41659	0.70298	32.499	3.9133	1.41331	3314.29
(k=30)	1.17566	0.6126	6287.19	1.41546	4.41416	1.79772	1.22274	0.71633	35.1329	3.79431	1.42581	3111.69
(k=40)	1.22732	0.61431	5842.84	1.28073	4.62252	1.69166	1.23477	0.78541	33.6926	3.78412	1.5326	3235.42
(k=50)	1.0174	0.5958	6074.89	1.2893	4.59204	1.63147	1.22193	0.81784	32.408	3.64151	1.55402	3243.54
STL+ SARIMA	2.06001	0.46503	2170.77	1.36634	2.67794	1.47993	1.55173	0.66878	34.2123	5.42245	1.73936	3702.53
(k=5)	1.53019	0.66743	5522.94	1.28181	2.93618	1.48234	1.44189	0.77026	38.6808	4.38844	1.56044	3544.38
(k=10)	1.24338	0.60293	5753.73	1.26768	3.61224	1.72623	1.37134	0.87682	35.7175	4.04542	1.40417	3572.31
(k=20)	1.253	0.68925	5131.9	1.21128	3.68922	1.81049	1.44502	0.84617	34.4887	3.91622	1.34211	3478.6
(k=30)	1.14899	0.65907	7703.68	1.37121	3.94332	1.81962	1.27448	0.85757	37.6611	3.85887	1.43008	3459.65
(k=40)	1.14906	0.66144	6542.08	1.23577	3.85016	1.71332	1.315	0.86646	36.0533	3.59424	1.52367	3422.04
(k=50)	0.99687	0.6405	6588.72	1.25217	3.95482	1.6528	1.30082	0.87771	35.7512	3.71064	1.56719	3455.27
STL+ETS	1.97547	0.44836	2714.92	1.39662	3.43042	1.37034	1.47138	0.74256	31.345	5.36884	1.5766	3757.13
(k=5)	1.58391	0.57195	4647.86	1.28751	3.56717	1.47673	1.467	0.61045	36.9633	4.08103	1.52345	3462.85
(k=10)	1.33972	0.53288	5831.21	1.25188	4.28016	1.72778	1.34454	0.79899	35.3014	4.04689	1.39195	3589.12
(k=20)	1.26209	0.60273	4953.29	1.17404	4.35452	1.81235	1.42185	0.69306	33.4679	4.18644	1.34229	3462.18
(k=30)	1.13707	0.58445	6431.92	1.37365	4.4529	1.80681	1.26884	0.68805	38.8701	3.81209	1.39703	3422.77
(k=40)	1.16092	0.58464	6032.15	1.2276	4.34068	1.69524	1.29037	0.78389	36.7018	3.50922	1.52563	3393.72
(k=50)	0.94266	0.58862	6104.03	1.24372	4.58192	1.61976	1.27917	0.80046	35.4521	3.59868	1.54395	3447.82
NNAR	2.25319	0.28555	2917.92	1.16274	4.62545	1.3066	1.43223	0.9509	19.5832	5.32329	1.73374	6628.94
(k=5)	0.78356	0.65556	4928.83	1.18065	4.43014	1.27748	1.75312	0.881	22.2939	4.45075	2.58108	3841.64
(k=10)	0.84613	0.54028	4160.39	1.1206	4.83939	2.32172	1.35669	1.44275	18.7226	4.44521	1.83296	3833.24
(k=20)	0.93876	0.69917	4145.33	1.00836	4.77873	2.51117	1.24885	1.27252	20.0987	4.98307	1.73466	4486.24
(k=30)	1.15761	0.66775	5352.16	1.19774	4.90002	2.58481	1.32405	1.20416	20.6022	3.87706	1.90614	4083.06
(k=40)	0.94808	0.66676	5715.78	1.07779	4.98156	2.22605	1.29933	0.89501	19.962	3.79209	1.93724	6514.8
(k=50)	0.92319	0.64116	6255.33	1.09147	4.98365	2.08805	1.33939	1.14206	20.2645	3.73578	1.85032	4795.29
MLP	1.6422	0.46293	4780.29	1.5952	3.2957	1.57181	1.37535	1.2287	25.7322	5.74075	1.5533	6705.08
(k=5)	0.78356	0.65556	4928.83	1.18065	4.43014	1.27748	1.75312	0.881	22.2939	4.45075	2.58108	3841.64
(k=10)	0.84613	0.54028	4160.39	1.1206	4.83939	2.32172	1.35669	1.44275	18.7226	4.44521	1.83296	3833.24
(k=20)	0.93876	0.69917	4145.33	1.00836	4.77873	2.51117	1.24885	1.27252	20.0987	4.98307	1.73466	4486.24
(k=30)	1.15761	0.66775	5352.16	1.19774	4.90002	2.58481	1.32405	1.20416	20.6022	3.87706	1.90614	4083.06
(k=40)	0.94808	0.66676	5715.78	1.07779	4.98156	2.22605	1.29933	0.89501	19.962	3.79209	1.93724	6514.8
(k=50)	0.92319	0.64116	6255.33	1.09147	4.98365	2.08805	1.33939	1.14206	20.2645	3.73578	1.85032	4795.29

Result: From the above the model ETS and STL+SARIMA are the best time series model in 50 % of the cases , SARIMA and STL+ETS are the best model in 47 % of the cases , Bagged based model of NNAR with 5 bootstrap sample is the best model in 34% of the cases, bagged based model of SARIMA,ETS and MLP with 5, 50 and 20 bootstrap respectively along with base time series models like NNAR and MLP is the best model in 25 % of the cases and the rest of the models fit best in less than 17% of the cases.

The graph of RMSE verses the bootstrap samples are plotted as follows

Figure 3.16: Representing the graph of RMSE of bagged model for E1

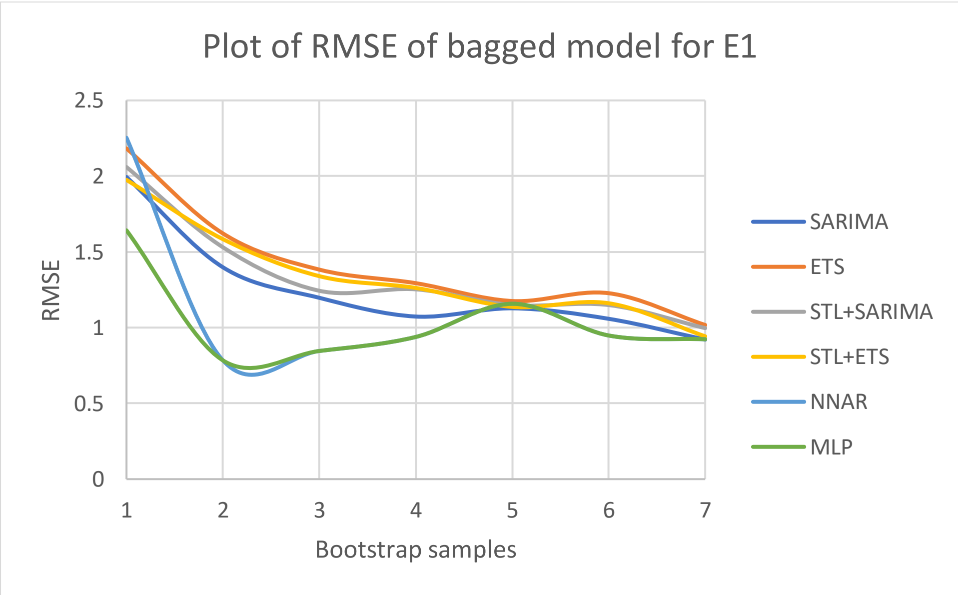


Figure 3.17: Representing the graph of RMSE of bagged model for E2

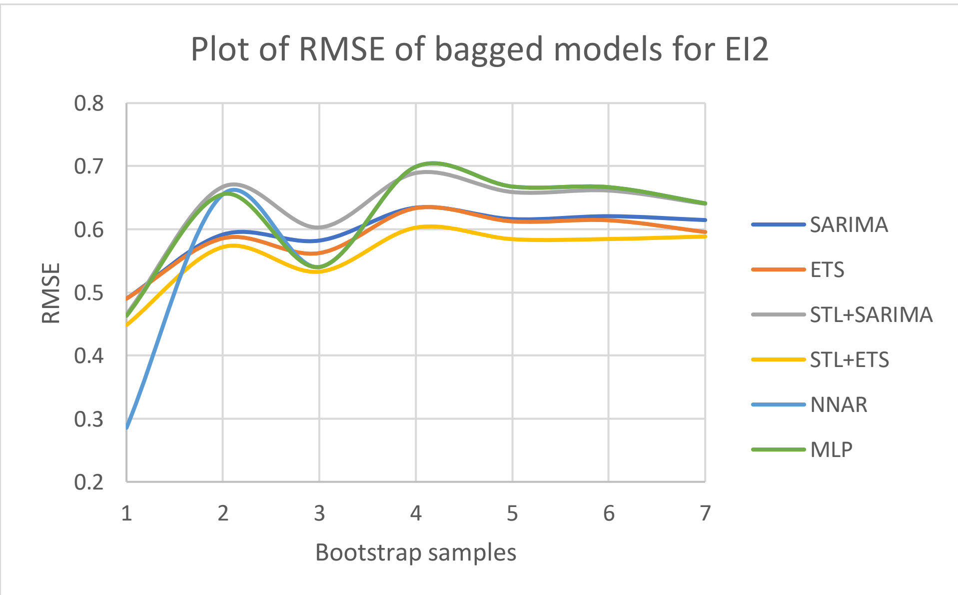


Figure 3.18: Representing the graph of RMSE of bagged model for E3

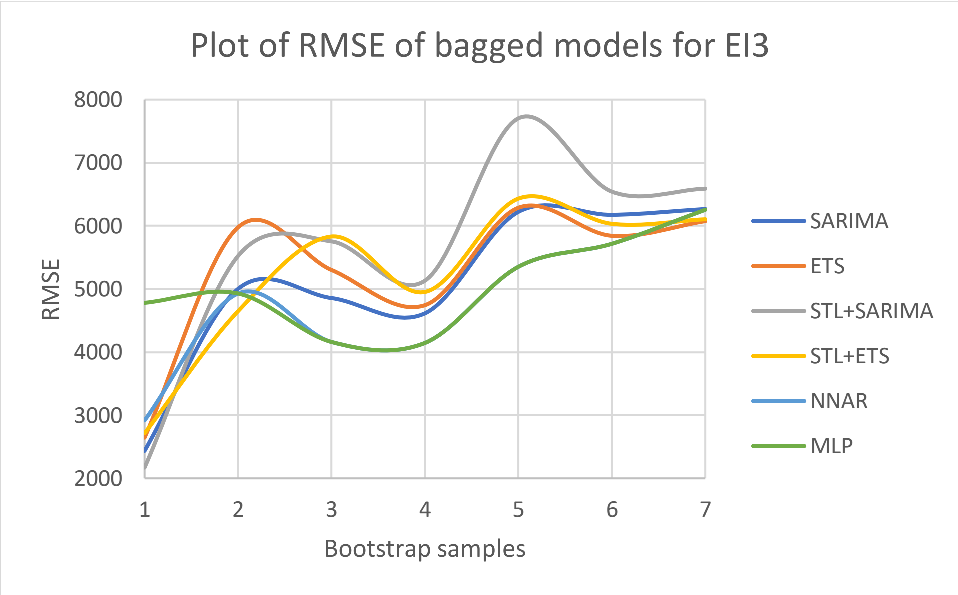


Figure 3.19: Representing the graph of RMSE of bagged model for E4

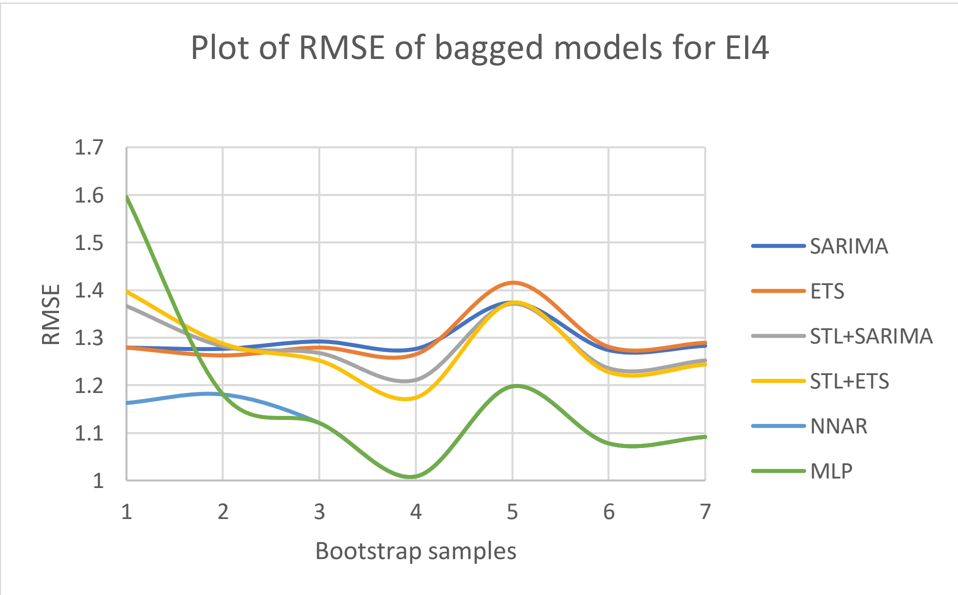


Figure 3.20: Representing the graph of RMSE of bagged model for E5

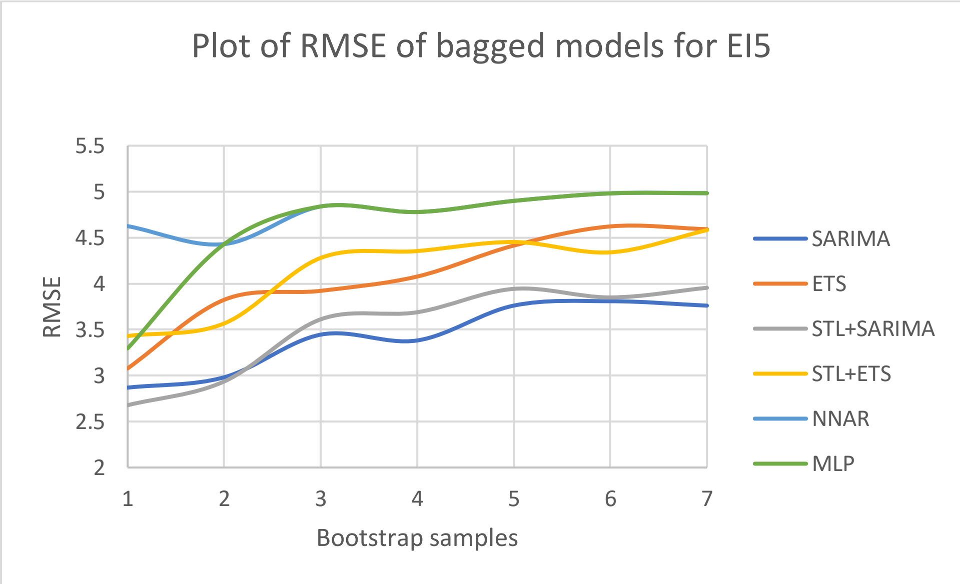


Figure 3.21: Representing the graph of RMSE of bagged model for E6

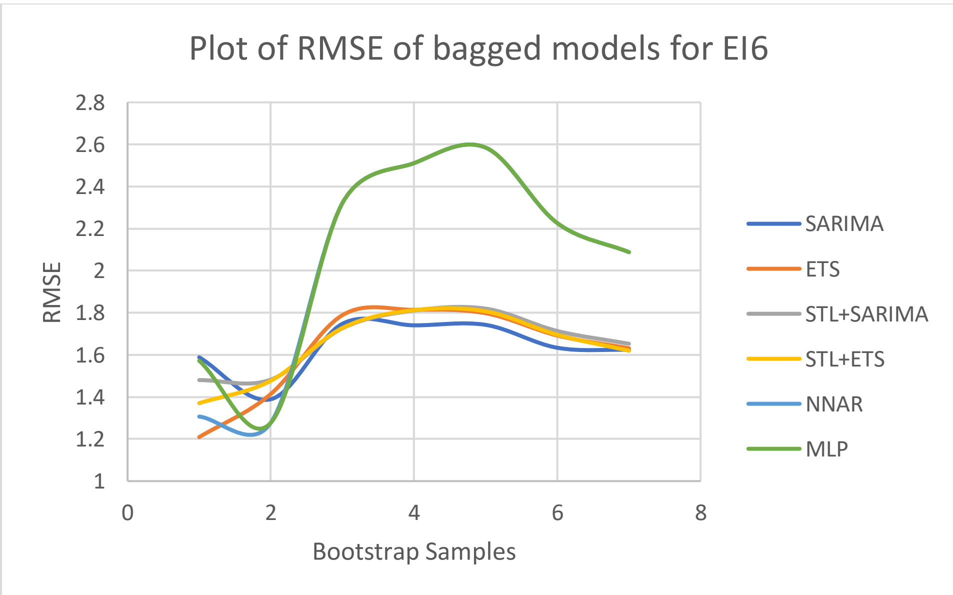


Figure 3.22: Representing the graph of RMSE of bagged model for E7

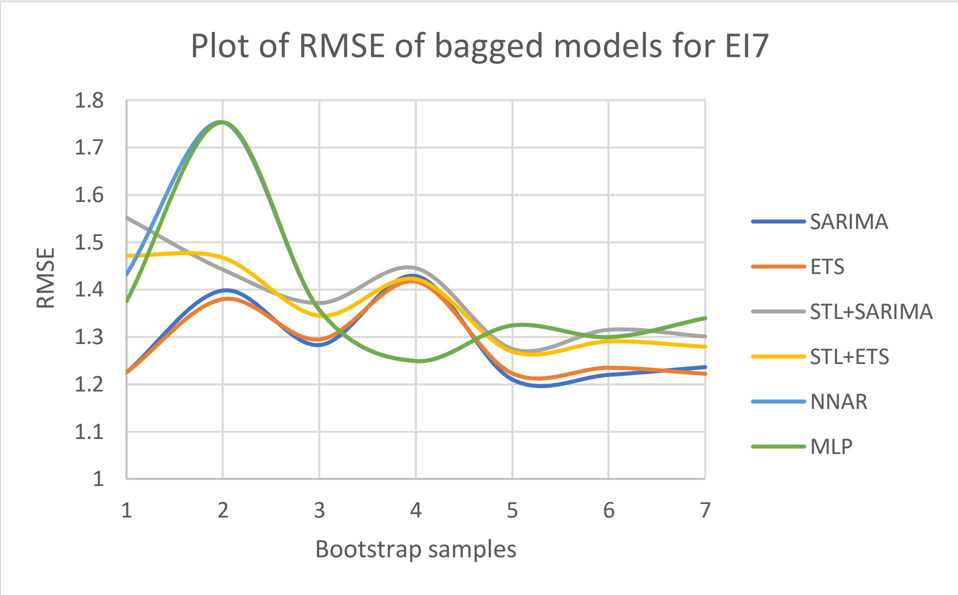


Figure 3.23: Representing the graph of RMSE of bagged model for E8

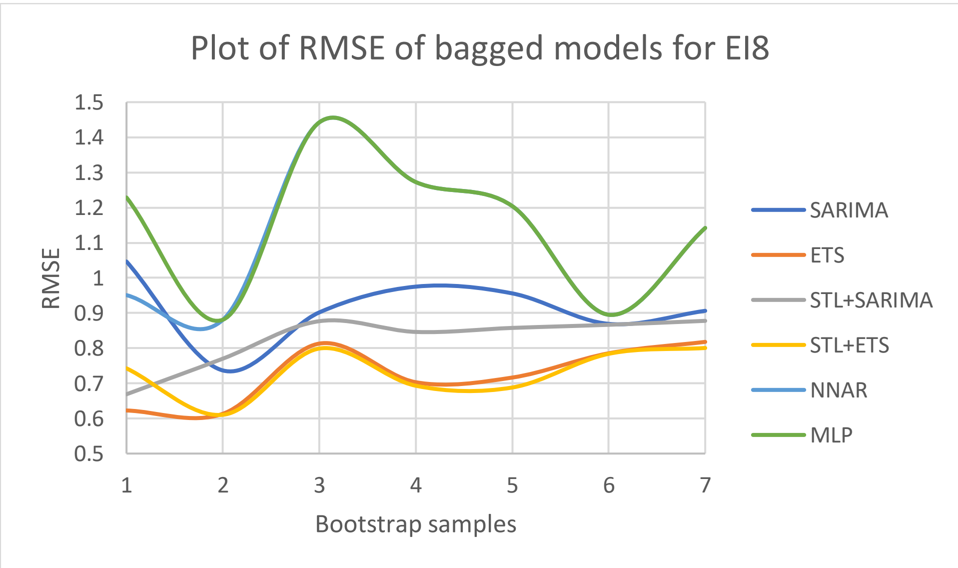


Figure 3.24: Representing the graph of RMSE of bagged model for E9

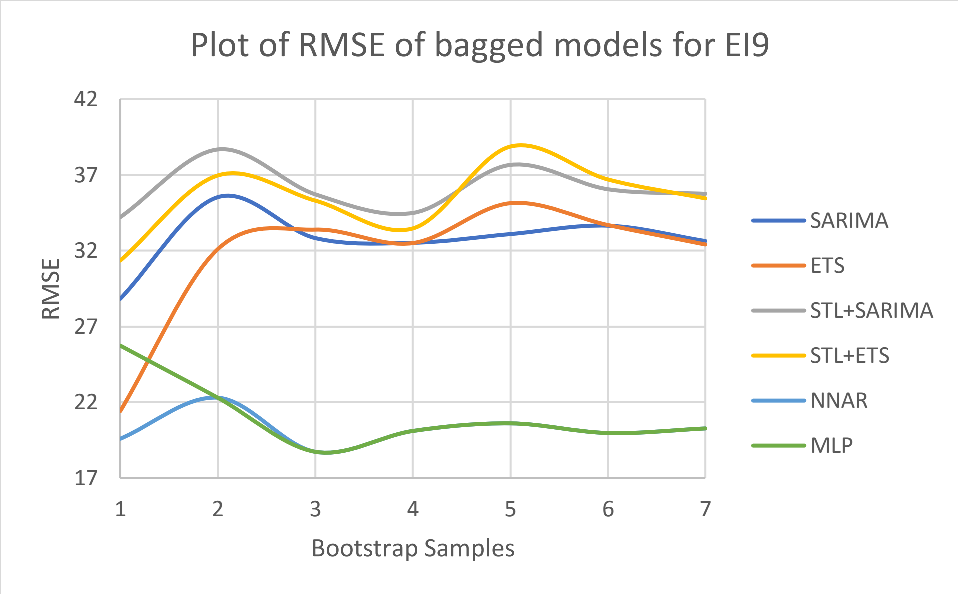


Figure 3.25: Representing the graph of RMSE of bagged model for E10

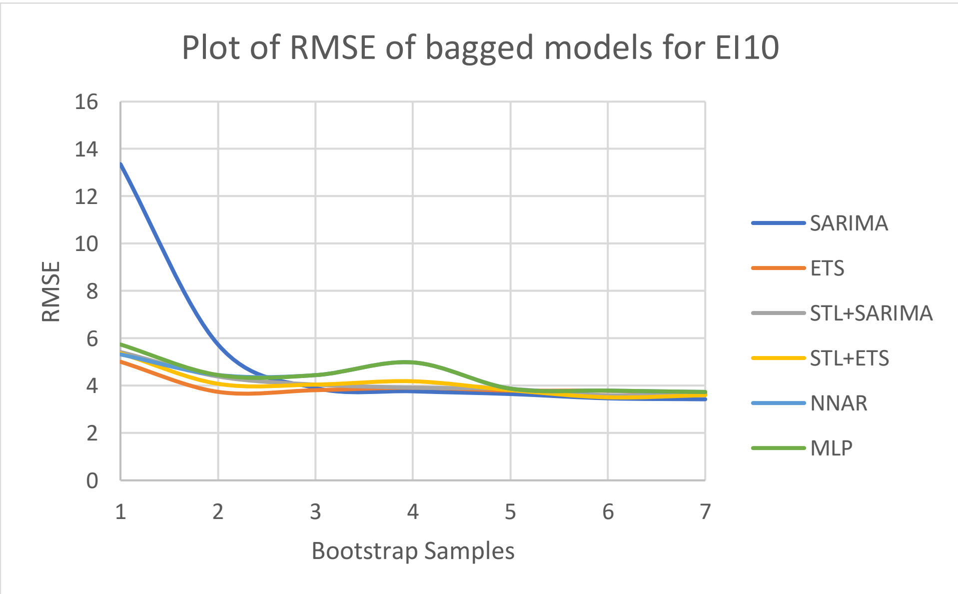


Figure 3.26: Representing the graph of RMSE of bagged model for E11

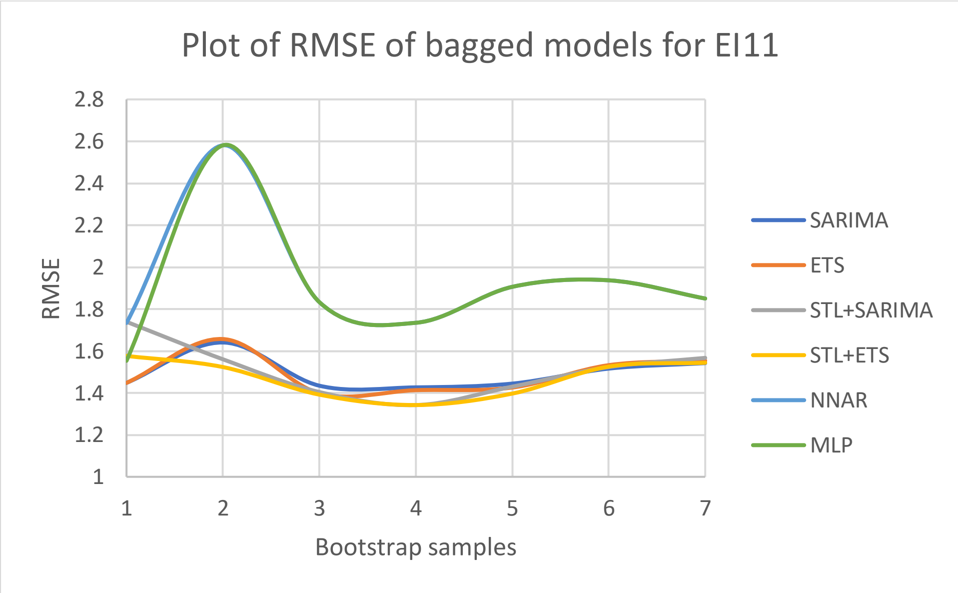
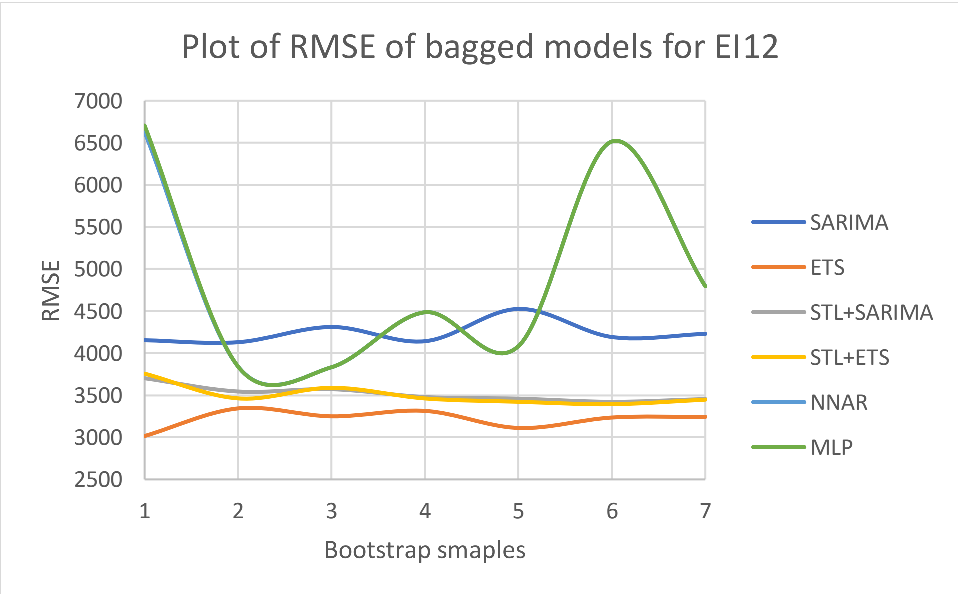


Figure 3.27: Representing the graph of RMSE of bagged model for E12



The table below gives the RMSE of Bagged hybrid time series models

Table 3.11: RMSE of bagged hybrid models

Indicators	(k=5)	(k=10)	(k=20)	(k=30)	(k=40)	(k=50
EI1	1.337178	1.182304	1.129651	1.105232	1.050978	0.900608
EI2	0.629221	0.557393	0.639258	0.619327	0.621751	0.613895
EI3	5195.085	5366.227	4811.415	6286.282	6085.636	6241.552
EI4	1.250414	1.263458	1.219288	1.357661	1.238458	1.248028
EI5	3.39642	3.92095	3.909536	4.178767	4.231352	4.224139
EI6	1.399506	1.850245	1.923631	1.937975	1.790339	1.730785
EI7	1.509639	1.263601	1.378215	1.239845	1.241486	1.241734
EI8	0.670198	0.91063	0.880292	0.863722	0.843562	0.896874
EI9	32.38637	30.56993	29.48457	32.75883	31.5237	30.50899
EI10	3.809269	3.683182	3.805893	3.548091	3.344414	3.452687
EI11	1.794903	1.412528	1.386495	1.404284	1.467901	1.453137
EI12	3616.418	3740.686	3723.042	3683.074	3883.657	3725.527

Result: The bagged hybrid models for the bootstrap sample k=5 and 20 are the best models in 34% of the cases and the remaining models are the best in 8 % of the cases.

3.5 Conclusion

The following table gives the RMSE of all the models considered in the present study and it is observed that bagging gives the better forecasts in 8 out of 12 cases.

Table 3.12: RR of bagged hybrid models

	Best Base Model	Hybrid Model	Best Bagged Base Model	Best Bagged Hybrid Model
EI1	1.6422	1.81322	0.78356	0.900608
EI2	0.28555	0.29843	0.28555	0.557393
EI3	2170.772	2362.745	2170.77	4811.415
EI4	1.16274	1.31052	1.00836	1.219288
EI5	2.67794	3.17291	2.67794	3.39642
EI6	1.20902	0.77841	1.20902	1.399506
EI7	1.22507	1.30703	1.21	1.239845
EI8	0.62264	0.80412	0.61045	0.670198
EI9	19.58322	25.14996	18.7226	29.48457
EI10	5.01087	4.1855	3.42545	3.344414
EI11	1.44802	1.53007	1.34211	1.386495
EI12	1.6422	1.81322	3015.43	3616.418

CHAPTER 4

SUMMARY AND CONCLUSION

The aim of the study was to explore the various time series models, along with their hybrid structures and inspect whether bagging was helpful in improving the forecasting accuracy. Time series and machine learning models with variety of techniques are employed to the trained data of economic indicators collected from 2009 to 2022. The remaining test dataset is used to validate the performance of the models. Strengths and weakness of each models in relation to the data characteristics are observed using graphical representations and metrics

The following conclusions are drawn:

- From the measures of skewness and kurtosis we observe indicators EI2, EI3, EI8, EI9 and EI11 show notable skewness, while indicators EI5,EI7 and EI9 exhibits high excess kurtosis.
- The normality tests results in statistical significance which infers non- normality based on Shapiro Wilk test with p – value<0.05.
- The time profiles of the indicators reveal non- stationarity, attributed to noticeable trends and recurring seasonal patterns.
- The tests for trend confirm the presence of trend in the time series data. whereas the test for seasonality suggests no seasonality in most of the economic indicators except for EI11 and EI12.
- The ETS model outperforms others in 33.33% of cases, while SARIMA ,STL+ARIMA,NNAR and MLP models are each best in 16.67 % of cases. Observing the RR rate all the base time series models have atleast a 25 % RR rate , except for EI7 and EI11 where SARIMA doesn't fit well for EI7 and EI11.
- The Hybrid time series modelling techniques SM, TM and OLSE performs best in 25% of cases, while SWAM is the top model in 16.7 % of cases and WM in 8.3 % of cases.
- In case of Hybrid time series models SWAM is the best technique for EI11 and the least effective model for EI11 but is the best for EI4.
- Findings indicates that ETS and STL+SARIMA models are the top choices in 50% of cases while SARIMA and STL+ETS models excel in 47 % of cases

- Bagged models of SARIMA, ETS and MLP are the best in 25% of cases.
- Bagged hybrid models with bootstrap samples k=5 and k=20 emerge as the best models in 34% of cases. While the remaining models demonstrate the best performance in 8% of cases.
- Overall, the study leads to conclusion that bagging leads to more accurate forecasts in 8 out of 12 cases.
- The Bagging technique requires relatively longer time (approx. 52 min) for computation compared to the computational requirements of base of Hybrid models

References

1.Hajirahimi Z, Khashei M. Hybrid structures in time series modeling and forecasting: A review. Engineering Applications of Artificial Intelligence. 2019 Nov;86:83–106.

2.Yang Y, Fan C, Xiong H. A novel general-purpose hybrid model for time series forecasting. Applied Intelligence. 2021 Jun 5;52(2):2212–23.

3.Perone G. Comparison of ARIMA, ETS, NNAR, TBATS and hybrid models to forecast the second wave of COVID-19 hospitalizations in Italy. The European Journal of Health Economics. 2021 Aug 4;

4.Wilson GT. Time Series Analysis: Forecasting and Control, 5th Edition, by George E. P. Box, Gwilym M. Jenkins, Gregory C. Reinsel and Greta M. Ljung, 2015. Published by John Wiley and Sons Inc., Hoboken, New Jersey, pp. 712. ISBN: 978-1-118-67502-1. Journal of Time Series Analysis. 2016 Mar 27;37(5):709–11.

5.Breiman L. Bagging Predictors Bagging Predictors [Internet]. 1994. Available from: https://www.stat.berkeley.edu/~breiman/bagging.pdf

6.Corchado J, Lees B. A HYBRID CASE-BASED MODEL FOR FORECASTING Abbreviated Title: Hybrid Case-Based Model [Internet]. 2001 Feb [cited 2023 Aug 15]. Available from: https://bisite.usal.es/archivos/2001_rev_aai_007.pdf

7.Inoue A, Kilian L. How Useful Is Bagging in Forecasting Economic Time Series? A Case Study of U.S. Consumer Price Inflation. Journal of the American Statistical Association [Internet]. 2008 [cited 2023 Aug 15];103:511–22. Available from: https://econpapers.repec.org/article/besjnlasa/v_3a103_3ay_3a2008_3am_3ajune_3ap_3a511-522.htm

8.Manoj K, Madhu A. An Application Of Time Series Arima Forecasting Model For Predicting Sugarcane Production In India. Studies in Business and Economics [Internet]. 2014;9(1):81–94. Available from: https://ideas.repec.org/a/blg/journl/v9y2014i1p81-94.html

9.Xu F, Du YA, Chen H, Zhu JM. Prediction of Fish Migration Caused by Ocean Warming Based on SARIMA Model. Wang S, editor. Complexity. 2021 Mar 24;2021:1–9.

10.Khodaverdi M. Forecasting future energy production using hybrid artificial neural Forecasting future energy production using hybrid artificial neural network and arima model network and arima model [Internet]. [cited 2023 Aug 15]. Available from: https://researchrepository.wvu.edu/cgi/viewcontent.cgi?article=5122&context=etd

11.Qureshi M, Daniyal M, Tawiah K. Comparative Evaluation of the Multilayer Perceptron Approach with Conventional ARIMA in Modeling and Prediction of COVID-19 Daily Death Cases. Journal of Healthcare Engineering [Internet]. 2022 Nov 9 [cited 2023 Aug 15];2022:e4864920. Available from: https://www.hindawi.com/journals/jhe/2022/4864920/

12.Saeed W. Frequency-based ensemble forecasting model for time series forecasting. Computational and Applied Mathematics. 2022 Feb 7;41(2).

13.Mishra S, Shaik AG. Performance Evaluation of Prophet and STL-ETS methods for Load Forecasting [Internet]. IEEE Xplore. 2022 [cited 2023 Aug 15]. p. 1–6. Available from: https://ieeexplore.ieee.org/document/9862533

14.Khan FM, Gupta R. ARIMA and NAR based Prediction Model for Time Series Analysis of COVID-19 cases in India. Journal of Safety Science and Resilience. 2020 Jun;

15.Hyndman RJ, Athanasopoulos G. Forecasting : Principles and Practice [Internet]. 2nd ed. Heathmont, Vic.: Otexts; 2018. Available from: https://otexts.com/fpp2/