Formula

Gaussian Integrals

Mathematics

Definition

Gamma Function

Mathematics

Definition

Riemann-Zeta Function

Mathematics

Definition

Fermi and Bose Integrals:

$$\int_0^\infty \frac{x^n}{e^x+1}\mathrm{d}x$$

Mathematics

Theory

Residue Theorem

Mathematics

Formula

Common E&M Integrals

Mathematics

Formula

Geometric (Partial) Series

Mathematics

Definition

Line & Volume Elements

Mathematics

Formula

Stirling's Approximation

Mathematics

Definition

Vector Derivatives: Cartesian

Mathematics

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} \mathrm{d}x$$

For half integers, the Gamma function has a special form

$$\Gamma\left(\frac{n}{2}\right) = \frac{(n-2)!!\sqrt{\pi}}{2^{2(n-1)}}$$

where $n!! = n \times (n-2) \times \cdots \times 3 \times 1$ if n is odd and $n!! = n \times (n-2) \times \cdots \times 4 \times 2$ is n is even and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ and $\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$.

$$I_n(x) = \int_0^\infty x^n e^{-ax^2} dx = \begin{cases} \frac{1}{2}\sqrt{\frac{\pi}{a^{m+1}}}\frac{(2m)!}{4^m m!} & n = 2m \\ \frac{1}{2}\frac{1}{a^{k+1}}k! & n = 2k+1 \end{cases}$$

$$I_0(x) = \frac{1}{2}\sqrt{\frac{\pi}{a}} \qquad I_1(x) = \frac{1}{2a}$$

$$I_2(x) = \frac{1}{4a}\sqrt{\frac{\pi}{a}} \qquad I_3(x) = \frac{1}{2a^2}$$

Solution:

$$\left(1 - \frac{1}{2^n}\right)\Gamma(n+1)\,\zeta(n+1)$$

where $\Gamma(n)$ and $\zeta(n)$ are the Gamma and Riemann-Zeta functions.

$$\zeta(n) = \sum_{k=0}^{\infty} \frac{1}{(k+1)^n}$$

$$\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right) + c$$

$$\int \frac{\mathrm{d}x}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}} + c$$

Let C be a simple closed contour, described in the positive sense. If a function f is analytic inside and on C except for a finite number of singular points z_k $(k = 1, 2, ..., n)$ inside C, then

$$\int_C f(z)\,\mathrm{d}z = 2\pi i \sum_{k=1}^{n} \mathrm{Res}_{z=z_k} f(z)$$

The line elements for planar, spherical and cylindrical geometries are

$$\mathrm{d}\boldsymbol{\ell}_P = \mathrm{d}x\hat{\boldsymbol{x}} + \mathrm{d}y\hat{\boldsymbol{y}} + \mathrm{d}z\hat{\boldsymbol{z}}$$
$$\mathrm{d}\boldsymbol{\ell}_S = \mathrm{d}r\hat{\boldsymbol{r}} + r\mathrm{d}\theta\hat{\boldsymbol{\theta}} + r\sin(\theta)\mathrm{d}\varphi\hat{\boldsymbol{\varphi}}$$
$$\mathrm{d}\boldsymbol{\ell}_C = \mathrm{d}s\hat{\boldsymbol{s}} + s\mathrm{d}\varphi\hat{\boldsymbol{\varphi}} + \mathrm{d}z\hat{\boldsymbol{z}}$$

The volume elements for planar, spherical and cylindrical geometries are $\mathrm{d}\tau = \mathrm{d}x\mathrm{d}y\mathrm{d}z$, $\mathrm{d}\tau = r^2\sin(\theta)\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi$, $\mathrm{d}\tau = s\mathrm{d}s\mathrm{d}\varphi\mathrm{d}z$.

$$\sum_{i=0}^{N} r^i = \frac{1 - r^{N+1}}{1 - r}$$

$$\sum_{i=0}^{\infty} r^i = \frac{1}{1 - r}$$

Gradient: $\boldsymbol{\nabla} f = \partial_x f\hat{\boldsymbol{x}} + \partial_y f\hat{\boldsymbol{y}} + \partial_z f\hat{\boldsymbol{z}}$

Divergence: $\boldsymbol{\nabla} \cdot \mathbf{v} = \partial_x v_x + \partial_y v_y + \partial_z v_z$

Curl: $\boldsymbol{\nabla} \times \mathbf{v} = (\partial_y v_z - \partial_z v_y)\hat{\boldsymbol{x}} + (\partial_z v_x - \partial_x v_z)\hat{\boldsymbol{y}} + (\partial_x v_y - \partial_y v_x)\hat{\boldsymbol{z}}$

Laplacian: $\boldsymbol{\nabla}^2 f = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

$$\ln n! \approx n\ln n - n$$

Definition

Fundamental Theorems of Vector Calculus

Mathematics

Definition

Bernoulli's equation*

Classical Mechanics

Definition

COM and Displacement Coordinates

Classical Mechanics

Definition

Moment of Inertia

Classical Mechanics

Formula

Common Moments of Inertia

Classical Mechanics

Theory

Perpendicular-Axis Theorem for a Plane Lamina

Classical Mechanics

Theory

Parallel-Axis Theorem for Any Rigid Body

Classical Mechanics

Definition

Euler-Lagrange Equations

Classical Mechanics

Definition

Lagrangian: Single Harmonic Oscillator

Classical Mechanics

Definition

Lagrangian: Single Coupled Harmonic Oscillator

Classical Mechanics

The Bernoulli's equation is given as

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant}$$

where p is pressure, ρ is density, g is gravity, and v^2 is velocity of the fluid. Another way to write it is

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

which has a familiar form to classical mechanics.

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}} (\boldsymbol{\nabla} f) \cdot \mathrm{d}\boldsymbol{\ell} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int (\boldsymbol{\nabla} \cdot \mathbf{A}) \mathrm{d}\tau = \oint \mathbf{A} \cdot \mathrm{d}\boldsymbol{a}$

Curl Theorem: $\int (\boldsymbol{\nabla} \times \mathbf{A}) \cdot \mathrm{d}\boldsymbol{a} = \oint \mathbf{A} \cdot \mathrm{d}\boldsymbol{\ell}$

The moment of inertia has the discrete version $I_P = \sum_i m_i r_i^2$. For a continuous rigid body, let $\rho(\mathbf{x})$ be the mass density at each point in the body, $\mathbf{r}$ be a vector perpendicular to the axis of rotation, extending from a point on the rotation axis to a point in the solid, and $\mathrm{d}V$ be the volume of the body Q, then

$$I = \iiint_Q \rho(x, y, z) \left|\mathbf{r}^2\right| \mathrm{d}V$$

The center of mass is given by

$$q_c = \frac{\sum_i m_i q_i}{\sum_i m_i}$$

while the displacement coordinate is given by

$$q_d = q_{ij} = q_i - q_j.$$

Use these to simplify Lagrangian systems.

Let I_i be the moment of inertia about the i-axis, then

$$I_z = I_x + I_y$$

The moment of inertia of any plane lamina about an axis normal to the plane of the lamina is equal to the sum of the moments of inertia about any two mutually perpendicular axes passing through the given axis and lying in the plane of the lamina.

Solid Cylinder: $I = \frac{1}{2}MR^2$
Symmetric Loop: $I = MR^2$
Solid Sphere: $I = \frac{2}{5}MR^2$
Rod about Center: $I = \frac{1}{12}ML^2$
Solid Cylinder about z: $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
Loop about diameter: $I = \frac{1}{2}MR^2$
Thin Spherical Shell: $I = \frac{2}{3}MR^2$
Rod about End: $I = \frac{1}{3}ML^2$

In classical mechanics, the Euler-Lagrange equations are a set of differential equations of second order given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0$$

where $\mathcal{L}(\dot{q}_i, q_i)$ is the Lagrangian of the system depending on the velocities $\dot{q}_i$ and positions q_i.

We may in general say that

$$I = I_{cm} + ml^2$$

The moment of inertia of a rigid body about any axis is equal to the moment of inertia about a parallel axis passing through the center of mass plus the product of the mass of the body and the square of the distance between the two axes.

Let two mass points m_1, m_2 be coupled with a spring k, then the Lagrangian for this system is

$$\mathcal{L} = \frac{1}{2}m_1\dot{q}_1^2 + \frac{1}{2}m_2\dot{q}_2^2 - \frac{1}{2}k(q_2 - q_1)^2$$

where $q_1(t)$ and $q_2(t)$ are the positions of the mass points.

The Lagrangian for this system is

$$\mathcal{L} = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2$$

where m is the mass of the point attached to the spring whose constant is k.

Definition

Lagrangian: Triatomic Model

Classical Mechanics

Definition

Lagrangian: Single Pendulum

Classical Mechanics

Definition

Lagrangian: Heavy Symmetric Top

Classical Mechanics

Definition

Lagrangian: Double Pendulum

Classical Mechanics

Definition

Lagrangian: Coupled Pendulums

Classical Mechanics

Definition

Lagrangian: Thin Circular Loop on an Inclined Plane

Classical Mechanics

Definition

Lagrangian: Central Force & Orbital Conditions

Classical Mechanics

Definition

Differential Scattering Cross Section

Classical Mechanics

Theory

Physical Principles of Electrostatics

Electricity & Magnetism

Definition

Coulomb's Law for N particles

Electricity & Magnetism

The Lagrangian for a single mass m attached to a ceiling with a thin massless rope of length $r = \ell$ moving in a fixed plane is given by

$$\mathcal{L} = \frac{1}{2}m\left(\dot{q}_x^2 + \dot{q}_y^2\right) - mg\left(\ell - q_y\right)$$

In spherical polar coordinates, this becomes

$$\mathcal{L} = \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\left(1 - \cos\theta\right)$$

where θ is taken with respect to the vertical.

Let three mass points $m_1,\ m_2,\ m_3$ be given and two springs with constants k_{12} and k_{13} connect two masses. The Lagrangian for this system is given by

$$\mathcal{L} = \frac{1}{2}m_1\dot{q}_1^2 + \frac{1}{2}m_1\dot{q}_1^2 + \frac{1}{2}m_1\dot{q}_1^2 - \frac{1}{2}k_{12}(q_2 - q_1 - d)^2 - \frac{1}{2}k_{23}(q_2 - q_3 - d)^2$$

where d is the equilibrium distance for the springs.

The Lagrangian for a double pendulum of lengths ℓ_1 and ℓ_2 attached to the ceiling with masses m_1, m_2 making angles θ_1, θ_2 with the vertical is given by

$$\mathcal{L} = \frac{1}{2}M\ell_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2\ell_2^2\dot{\theta}_2^2 + m_2\ell_1\ell_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 - \theta_2) + Mg\ell_1\cos(\theta_1) + m_2g\ell_2\cos(\theta_2)$$

where $M = m_1 + m_2$.

The Lagrangian for a heavy symmetric top with one point fixed on the $x - y$ plane is given by

$$\mathcal{L} = \frac{I_1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2 - Mgl\cos\theta$$

where M is the mass of the top as located at the COM, $\dot{\psi}$ is the rotation of the top about its figure axis z, $\dot{\phi}$ is the precession or rotation of the figure axis z about the vertical z', and $\dot{\theta}$ is the nutation of z with respect to z'. The angular velocity $\boldsymbol{\omega} = \dot{\boldsymbol{\theta}} + \dot{\boldsymbol{\phi}} + \dot{\boldsymbol{\psi}}$.

The Lagrangian for a thin circular loop on an inclined plane which rolls under the influence of gravity is given by

$$\mathcal{L} = \frac{1}{2}\left(m\dot{x}^2 + I\dot{\theta}^2\right) - \left(mg\left(1 - x\right)\sin(\alpha)\right).$$

The Lagrangian for two pendulums coupled with a spring is given by the two pendulum Lagrangians along with another interaction term; $\mathcal{L} = \mathcal{L}_{p1} + \mathcal{L}_{p2} + \mathcal{L}_{int}$ where

$$\mathcal{L}_{int} = -\frac{1}{2}k\left(q_2 - q_1\right)^2.$$

The differential cross section $\sigma\left(\Omega\right)$ is given by

$$\sigma\left(\Omega\right)\mathrm{d}\Omega = \frac{P_{\#}\text{ scattered into solid angle d}\Omega\text{ per unit time}}{\text{incident intensity}}$$

where $P_{\#}$ is the number of particles.

The Lagrangian for a particle under the influence of a central force $V\left(r\right)$ is given by

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\left(\theta\right)\dot{\phi}^2\right) - V\left(r\right)$$

where θ is measured from the z axis and ϕ draws a circle in the x-y plane. Choose the equatorial plane, so $\theta = \pi/2$, and the note that $\ddot{r} = 0$ since $r = a$ has to be constant for orbital motion.

$$\mathbf{F} = Q\mathbf{E} = Q\left(\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{\mathbf{r}_i}\right)$$

where ϵ_0 is the permitivity of free space, $\mathbf{E}$ is the electric field, q_i are the point charges, and r_i are the distances from Q.

Coulomb's law and the principle of superposition constitute the physical input for electrostatics.

Definition

Electric Field (Continuous)

Electricity & Magnetism

Definition

Gauss' Law: Integral & Differential

Electricity & Magnetism

Theory

Properties of Conductor

Electricity & Magnetism

Theory

Electrical Component Voltages

Electricity & Magnetism

Definition

Maxwell's Equations (Guassian)

Electricity & Magnetism

Definition

Maxwell's Equations (SI)

Electricity & Magnetism

Definition

Poynting Vector

Electricity & Magnetism

Theory

Poynting's Theorem

Electricity & Magnetism

Definition

Monopole Moment

Electricity & Magnetism

Definition

Electric Torque & Dipole Moment

Electricity & Magnetism

For any closed surface, the following is true

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{a} = \frac{Q_{enc}}{\epsilon_0}$$
$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho_{enc}}{\epsilon_0}$$

where Q_{enc} (ρ_{enc}) is the total charge (density) enclosed within the surface.

The electric field for a continuous charge distribution is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_\Sigma \frac{\mathrm{d}q_\Sigma}{r^2} \hat{\boldsymbol{r}}$$

where Σ is a n-dimensional spatial manifold, and ϵ_0 is a constant, $r^2 = r_F^2 - r_S^2$ is the radial distance from the field point r_F to the source distribution r_S, and $\mathrm{d}q \in \{\lambda \mathrm{d}\ell, \sigma \mathrm{d}a, \rho \mathrm{d}\tau\}$ depending on the manifold.

$$V = IR \qquad V = \frac{Q}{C}$$
$$V = L\frac{dI}{dt}$$

(*i*) $\mathbf{E} = 0$ inside the conducting material.

(*ii*) $\rho = 0$ which follows from $\boldsymbol{\nabla} \cdot \mathbf{E} = \rho/\epsilon_0$.

(*iii*) The only remaining charge is on the surface.

(*iv*) A conductor has an equipotential surface.

(*v*) The electric field is normal to the surface just outside the conductor.

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_f \qquad \boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \qquad \boldsymbol{\nabla} \times \mathbf{H} = \left(\frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J}\right)$$

$$\boldsymbol{\nabla} \cdot \mathbf{D} = 4\pi\rho_f \qquad \boldsymbol{\nabla} \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$
$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \qquad \boldsymbol{\nabla} \times \mathbf{H} = \frac{1}{c}\left(\frac{\partial \mathbf{D}}{\partial t} + 4\pi \mathbf{J}\right)$$

The rate of energy transfer (per unit volume) from a region of space equals the rate of work done on a charge distribution plus the energy flux leaving that region

$$-\frac{\partial u}{\partial t} = \boldsymbol{\nabla} \cdot \mathbf{S} + \mathbf{J} \cdot \mathbf{E}$$

This is analogous to the work-energy theorem in classical mechanics and similar to the continuity equation.

The directional energy flux of an electromagnetic field. The SI unit is Watt per square meter and is defined by

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

where μ_0 is the vacuum permeability.

The electric dipole moment $\mathbf{p}_d$ is a measure of the system's overall polarity given by

$$\mathbf{p}_d = q\mathbf{d}$$

where $\mathbf{d}$ is the displacement vector pointing from the *negative* charge to the positive charge. This can be used to define the torque as well

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

where $\mathbf{E}$ is the electric field.

The monopole moment is simply given by the sum of charges

$$q_m = \sum_i q_i$$

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Theory

Adiabatic Process

Thermodynamics

Formula

Adiabatic Properties of Ideal Gas

Thermodynamics

Formula

Indistinguishable (Identical) Particle Statistical Distributions

Thermodynamics

Formula

Carnot Efficiency

Thermodynamics

Theory

Carnot Cycle

Thermodynamics

Theory

Equipartition Theorem

Thermodynamics

Theory

Fermi Gases

Thermodynamics

Theory

Gibbs Free Energy

Thermodynamics

Theory

Helmholtz Free Energy

Thermodynamics

Theory

Ideal Gasses

Thermodynamics

$$T_1 V_1^{\gamma-1} = \text{const}$$ $$T_1^{\gamma/(1-\gamma)} P_1 = \text{const}$$ $$P_1 V_1^{\gamma} = \text{const}$$	Also called *isentropic*. $\Delta S = 0$ in the process. Use the thermodynamic identity at constant volume and a systems internal energy equation to derive properties about the entropy of the system.
$$\eta = 1 - \frac{T_l}{T_h}$$	Bose-Einstein: $$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/k_B T} - 1}$$ Fermi-Dirac: $$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/k_B T} + 1}$$
A classical gas's energy gains $\frac{1}{2}k_B T$ for each degree of freedom. An ideal monotomic gas has $U = \frac{3}{2}k_B T$ from three translational degrees of freedom, while an ideal diatomic gas has $U = \frac{5}{2}k_B T$ from an additional two degrees of rotational freedom.	Characterized by alternating stages of isothermal and isentropic expansion and compression. Work done is $$W = (T_h - T_l)(S_H - S_L)$$ where T_l and T_h are the low and high temperatures reached during the cycle and S_L and S_H are the low and high entropies of the working substance.
$$G \equiv U + PV - TS$$	1. High kinetic energy 2. Low heat capacity 3. Low magnetic susceptibility 4. Low interparticle collision rate 5. High pressure 6. Low temperature
$$PV = nRT$$ $$PV = Nk_B T$$ $$Z_N = \frac{Z_1^N}{N!}$$	Acts as effective energy in isothermal changes of volume. $$F \equiv U - TS$$ $$dF = dU - SdT$$

Theory

Ideal Gas (RMS Average Speed)

Thermodynamics

Theory

Ideal Monoatomic Gas

Thermodynamics

Definition

Maxwell Speed Distribution

Thermodynamics

Definition

Mean Free Path

Thermodynamics

Formula

Partition Function Properties

Thermodynamics

Theory

Photon Gases

Thermodynamics

Formula

Planck Distribution Function

Thermodynamics

Formula

Planck Spectral Density (frequency)

Thermodynamics

Formula

Radiant Energy Flux (blackbody)

Thermodynamics

Formula

Stefan-Boltzmann Law (energy density)

Thermodynamics

$$C_V = \frac{3}{2}Nk_B \qquad C_P = \frac{5}{2}Nk_B$$
$$U = \frac{3}{2}Nk_BT \qquad \gamma = \frac{5}{3}$$

Derived by considering a single particle. For translation in three dimensions $KE = \frac{3}{2}k_BT$ and also $KE = \frac{1}{2}mv^2$ so that when combined,

$$\frac{1}{2}mv^2 = \frac{3}{2}k_BT \implies \boxed{v = \sqrt{\frac{3k_BT}{m}}}$$

Note that $m = \sum_i m_i^p$ where m^p is the mass of the proton. For example, a nitrogen molecule, N_2, has the mass which is twice of N_1 which has 14 protons. Thus $m_{N_2} = 28m^p$.

The mean free path is the average distance ℓ an object has traveled before an interaction has occurred. Let $N = n_V V$ be the number of interactions where n is the interaction density. In the case of a cylindrical volume $V = A\ell$, then $\lambda = \frac{\ell}{n_V V} = \frac{1}{n_V A}$. Often, one can use the ideal gas formula to estimate $n_V = N_A\,(n/V) = N_A P/RT$ where N_A is Avogadro's number.

$$f\,(v) = \sqrt{\left(\frac{m}{2\pi k_BT}\right)^3} 4\pi v^2 \exp\left(-\frac{mv^2}{2k_BT}\right)$$

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}} \qquad \langle v\rangle = \sqrt{\frac{8k_BT}{\pi m}}$$

$$U = \sigma_b V T^4$$
$$P = \frac{1}{3}\sigma_b V T^4$$
$$\mu = 0$$

$$Z = \sum_n e^{-\varepsilon_n/k_BT}$$

$$U = k_BT^2\frac{\partial \ln Z}{\partial T} \qquad F = -k_BT\ln Z$$

$$u_\omega = \frac{\hbar}{\pi^2c^3}\frac{\omega^3}{e^{\hbar\omega/k_BT} - 1}$$

$$\langle s\rangle = \frac{1}{e^{\hbar\omega/k_BT} - 1}$$

$$\frac{U}{V} = u = \frac{\pi^2{k_B}^3}{15\hbar^3c^3}T^4$$
$$u = \sigma_B T^4$$

$$u = \frac{4}{c}J_u$$

$$J_u = \frac{\pi^2{k_B}^4}{60\hbar^3c^2}T^4$$
$$J_u = \frac{c}{4}u$$

Definition

Thermodynamic Identity

Thermodynamics

Definition

Planck Formula

Quantum

Formula

Classical Electron Radius Formula

Quantum

Definition

Momentum Operator: Position Basis

Quantum

Definition

Hamiltonian Operator: Time Basis

Quantum

Definition

Angular Momentum Operator: Properties

Quantum

Definition

Spin Operator: Properties

Quantum

Definition

Time-Dependent Schrödinger equation*

Quantum

Theory

Bound State

Quantum

Theory

Scattering State

Quantum

The energy of a photon is proportional to its frequency:

$$E_\gamma = hf = \hbar\omega = pc$$

$$dU = TdS - PdV + \mu dN$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$P = -\left(\frac{\partial U}{\partial V}\right)_S$$

For 1D:

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

For 3D:

$$\hat{p} = \frac{\hbar}{i}\boldsymbol{\nabla}$$

If the electron were a classical solid sphere with angular momentum $L = \frac{1}{2}\hbar$, then the radius is given by

$$r_c = \frac{e^2}{4\pi\epsilon_0 mc^2}$$

.

Def: $\mathbf{L} = \mathbf{r} \times \mathbf{p} = (yp_z - zp_y, zp_x - xp_z, xp_y - yp_x)$.
Commutation: $[L_i, L_j] = i\hbar L_k$.
Magnitude: $\mathbf{L}^2 = \mathbf{L}_x^2 + \mathbf{L}_y^2 + \mathbf{L}_z^2$. $[\mathbf{L}^2, \mathbf{L}] = \mathbf{0}$.
$\pm$ Basis: $L_\pm = L_x \pm iL_y$. $[\mathbf{L}^2, L_\pm] = 0$.
Eigenvalues:
$L_z f_\ell^m = \hbar m f_\ell^m$, where m is the magnetic quantum number.
$\mathbf{L}^2 f_\ell^m = \hbar^2 \ell(\ell+1) f_\ell^m$ and ℓ is the azimuthal quantum number.
$L_\pm f_\ell^m = \hbar\sqrt{\ell(\ell+1) - m(m\pm1)} f_\ell^{m\pm1}$

For any dimension:

$$H = -\frac{\hbar}{i}\frac{\partial}{\partial t}$$

The full time dependent Schrödinger equation is given by

$$i\hbar\frac{\partial\Psi}{\partial t}(\mathbf{r},t) = H\Psi = \left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 + V(\mathbf{r},t)\right)\Psi$$

Def: $\mathbf{S} = I\omega$.
Commutation: $[S_i, S_j] = i\hbar S_k$.
Magnitude: $\mathbf{S}^2 = \mathbf{S}_x^2 + \mathbf{S}_y^2 + \mathbf{S}_z^2$. $[\mathbf{S}^2, \mathbf{S}] = \mathbf{0}$.
$\pm$ Basis: $S_\pm = S_x \pm iS_y$. $[\mathbf{S}^2, S_\pm] = 0$.
Eigenvalues:
$\mathbf{S}^2 |s, m_s\rangle = \hbar^2 s(s+1) |s, m_s\rangle$ and s is the spin quantum number.
$S_z |s, m_s\rangle = \hbar m_s |s, m_s\rangle$, where m_s is the secondary spin quantum number.
$L_\pm |s, m_s\rangle = \hbar\sqrt{s(s+1) - m_s(m_s\pm1)} |s, m_s\rangle$

A **scattering state** occurs when the energy of a particle is always larger than the background potential. Particles in scattering states are usually free, have a continuous energy spectra. The mathematical criteria is $E > 0 \implies$ a scattering state.

A **bound state** occurs when a particle is unable to escape a region defined by the background potential. Particles in bound states oscillate between their turning points and have discrete energy spectra. The mathematical criteria is $E < 0 \implies$ a bound state.

Theory **Quantum Numbers $\|n, \ell, m, \pm\rangle$** Quantum	Theory **Principle Quantum Number: Hydrogen** Quantum
Theory **Azimuthal Quantum Number: Hydrogen** Quantum	Theory **Magnetic Quantum Number: Hydrogen** Quantum
Theory **Primary and Secondary Spin Quantum Number** Quantum	Formula **Bohr Radius** Quantum
Formula **Bohr Energies** Quantum	Theory **Hydrogen Spectrum: Transition Series** Quantum
Formula **Hydrogen Spectrum: Rydberg's Constant** Quantum	Definition **Lorentz Transformations** Relativity

The principal quantum number (n) describes the electron shell, or energy level, of an electron. The value of n ranges from 1 to the shell containing the outermost electron of that atom, that is $n \in \{1, 2, \cdots, n_s\}$ where n_s is the outermost electron shell.

Four quantum numbers (QNs) exactly determine a 3D electron's wavefunction. The **Principle** QN (n) describes the energy level. The **Azimuthal** QN (ℓ) describes the magnitude of the orbital angular momentum. The **Magnetic** QN (m) describes the projection of the orbital angular momentum along a specified axis. The **Spin** QN ($\pm$) describes the spin of the electron within that orbital shell. The **Secondary** spin QN (m_s) describes the projection of the spin of the electron along a specific axis.

The magnetic quantum number (m) describes the projection of the orbital angular momentum along a specified axis and belongs to the set

$$m = -\ell, -\ell + 1, \cdots, \ell - 1, \ell = (-\ell, \ell)_{\mathbb{N}}$$

where ℓ is the azimuthal quantum number.

The azimuthal quantum number (ℓ) describes the magnitude of the orbital angular momentum and has the upper bound

$$\ell = 0, 1, 2, \cdots, n - 1$$

where n is the principle quantum number.

The general formula for the Bohr radius of an electron in a bound state with another positively charged particle is

$$a = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}$$

where $\mu = m_1 m_2 / (m_1 + m_2)$ is the reduced mass.

The **primary** spin quantum number (s) describes the spin orientation of the electron within its orbital shell. Usually $s = \pm\frac{1}{2}$. The **secondary** spin quantum number m_z which ranges from

$$m_z = -s, -s + 1, \cdots, s - 1, s = [-s, s]_{\mathbb{N}}$$

1. **Lyman Series**: Transitions to the ground state ($n_f = 1$), lie in the ultraviolet.
2. **Balmer Series**: Transitions to the first excited state ($n_f = 2$), fall in the visible region.
3. **Paschen Series**: Transitions to the second excited state ($n_f = 3$), fall in the infrared.

The general energy transition formula for two particles in a bound state is

$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right] = \frac{E_1}{n^2}$$

where n denotes the stating energy level and $E_1 = -13.6$ eV is the ground state binding energy.

A Lorentz transformation from the inertial S' frame to the lab S frame is given by

$$\begin{aligned} t &= \gamma\left(t' + \frac{v}{c^2}x\right) \\ x &= \gamma(x' + ct') \\ y &= y' \\ z &= z' \end{aligned}$$

The Rydberg constant is determined by using Planck's formula with the Bohr energies;

$$R = \frac{m}{4\pi c\hbar^3}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 = 1.097 \times 10^7 \mathrm{m}^{-1}$$

FORMULA

Velocity Addition Rule

RELATIVITY

FORMULA

Relativistic Energy/3-Momentum

RELATIVITY

DEFINITION

Relativistic Invariant

RELATIVITY

DEFINITION

(Doppler) Compton Scattering

$\gamma + e^- \to \gamma + e^-$

RELATIVITY

PROBLEM

Contour Integration:

$$\int_0^\infty \frac{\cos(x)}{1+x^2}\mathrm{d}x$$

PROBLEM

Contour Integration:

$$\oint_C \frac{\mathrm{d}z}{z}$$

PROBLEM

Contour Integration:

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{\left(x^2+1\right)^2}$$

PROBLEM

Contour Integration:

$$\int_{-\pi}^{\pi} \frac{\mathrm{d}t}{1+3\left(\cos t\right)^2}$$

PROBLEM

Contour Integration:

$$\int_0^\infty \frac{\log(x)}{\left(1+x^2\right)^2}\mathrm{d}x$$

PROBLEM

Contour Integration:

$$\int_0^{2\pi} \frac{\mathrm{d}\theta}{1+a\cos(\theta)}\mathrm{d}x$$

The relativistic energy formula is given by

$$E = \gamma mc^2 = mc^2 + (\gamma - 1)\, mc^2 = R + T$$

where R denotes the rest energy and T is the kinetic energy. The relativistic momentum is given by

$$\mathbf{p} = m\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t'} = \gamma m\mathbf{v}$$

where $\mathbf{v}$ is the velocity of the object in the lab frame.

Suppose a particle moves a distance $\mathrm{d}x$ in S in time $\mathrm{d}t$. If its velocity is u_S in the S frame, then the velocity in the S' frame is

$$u_{S'} = \frac{\mathrm{d}x'}{\mathrm{d}t'} = \frac{u_S - v}{1 - uv/c^2}$$

where v is the speed of S' with respect to S.

Compton scattering occurs when an incoming photon scatters off of an electron at rest. The wavelength of the photon after scattering is

$$\lambda' = \lambda + \frac{h}{mc}\left(1 - \cos(\theta)\right)$$

If the electron has an initial velocity, then we have to transform into the electron's rest frame. In this case, it's easier to work with energy

The four-momentum is given by $p^\mu = (E/c, \mathbf{p})$. The relativistic invariant constructed from the four-momentum is the modulus (signature $(-,+,+,+)$);

$$p^\mu p_\mu = \left(\frac{E}{c}\right)^2 - (\mathbf{p}\cdot\mathbf{p}) = m^2c^2$$

This is an invariant in any frame and conserved since the energy and 3-momentum are conserved.

Solution:

$$2\pi i$$

Solution:

$$\frac{\pi}{2e}$$

Solution:

$$\pi$$

Solution:

$$\frac{\pi}{2}$$

Solution:

$$\frac{2\pi}{\sqrt{1-a^2}}$$

Solution:

$$-\frac{\pi}{4}$$

Problem

Fermi and Bose Integral: $\int_0^\infty \frac{x^3}{e^x+1}$

Problem

Central Force: $V(r) = r^\alpha$
What is the condition on α for a stable orbit?

Problem

Rigid Rolling Without Slipping: Friction Coefficient

Problem

Ball Rolling off a Hemisphere: Angle

Problem

Threshold Energy
$A + B \to \sum_i C_i$

Problem

Threshold Energy
$A \to B + C$

Definition

Monopole Moment

Electricity & Magnetism

Definition

Electric Torque & Dipole Moment

Electricity & Magnetism

Definition

Conductors, Insulators, and Polarization

Electricity & Magnetism

Definition

Electric Torque

Electricity & Magnetism

Use the central force Lagrangian, find the equations of motion, and then substitute the angular momentum into the radial equation and integrate to find an effective Lagrangian. This effective Lagrangian has an effective potential, V_{eff}. Conditions on stability;

$$\boxed{\left.\frac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}r}\right|_{r_0} = 0} \qquad \boxed{\left.\frac{\mathrm{d}^2V_{\text{eff}}}{\mathrm{d}r^2}\right|_{r_0} > 0}$$

Solution:

$$\frac{\pi^4}{15}$$

Set up a Lagrangian and a constraint equation. Find the two Euler-Lagrange equations carefully and apply the constraints $r = a,\ \dot{r} = 0,\ \ddot{r} = 0$ at the point when the particle leaves the surface. Find when the constraint $\lambda = 0$ and solve for the angle.

Solution: $-ma\dot{\theta}^2 + mg\cos(\theta) = \lambda$ and $\ddot{\theta} = \frac{g}{a}\sin(\theta)$ and the departure angle

$$|\theta_c| = 48.2^\circ$$

Set up a Lagrangian for the system and obtain the Euler-Lagrange equations along the incline plane and normal to the incline plane. The rolling without slipping condition is

$$\dot{x} = R\dot{\theta}$$

where R is the distance from the central axis to the boundary. Use that the sum of the forces must be zero to find the coefficient of friction.

Solution for hoop: $\mu = \frac{1}{2}\tan(\alpha)$ for the incline angle α.

The threshold energy for parent A to source daughters B and C is given by

$$E_B = \left(\frac{m_A^2 + m_B^2 - m_C^2}{2m_A}\right)c.$$

Parent A is at rest, but one can use a cyclic rotation if B or C are at rest instead.

The threshold energy for parents A and B to source daughters C_i is given by

$$E_A = \left(\frac{M^2 - m_A^2 - m_B^2}{2m_B}\right)c$$

where $M = \sum_i m_{C_i}$.

The physical dipole moment $\mathbf{p}_d$ is a measure of the system's overall polarity given by

$$\mathbf{p}_d \equiv \int \mathbf{r}'\rho\left(r'\right)\mathrm{d}\tau', \qquad \mathbf{p}_d = \sum_{i=1}^{n} q_i\mathbf{r}_i'$$

where $\mathbf{r}'$ is the distance between the origin and the charge. The dipole term in the multipole expansion goes as

$$V_{\text{dip}}\left(\mathbf{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$$

The monopole moment is simply given by the sum of charges

$$Q = \sum_i q_i.$$

The monopole term in the multipole expansion is dominated by

$$V_{\text{mon}}\left(\mathbf{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

where Q is the total charge.

Using the electric dipole, we may defined the electric torque as

$$\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$$

where $\mathbf{E}$ is the electric field.

A conductor has an infinite supply of free, unbound, electrons.

An insulator, or dielectric, all charges are attached to specific atoms or molecules, they're on a tight leash. The two mechanisms by which electric fields deform a dielectric atom are *stretching* and *rotating*.

By applying an external electric field, dielectrics atoms attain a new equilibrium configuration which is referred to as "becoming polarized."

Definition

Atomic Polarizability

Electricity & Magnetism

Definition

Dipole Potentials in Bound Charges

Electricity & Magnetism

Definition

General Solution to Laplace's Equation in Spherical Coordinates

Electricity & Magnetism

The dipole potential in a bound charge is given by

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{\sigma_b}{\ell}\mathrm{d}a' + \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho_b}{\ell}\mathrm{d}\tau'$$

where ℓ is the distance between the dipole and the field point, $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$ and $\rho_b = -\boldsymbol{\nabla}\cdot\mathbf{P}$ are the surface and volume bound charges.

An external electric field polarizes an atom and induces a dipole moment. In the weak electric field limit, we may approximate

$$\mathbf{p} = \alpha\mathbf{E} = \alpha_\perp\mathbf{E}_\perp + \alpha_\parallel\mathbf{E}_\parallel$$

Furthermore, the polarization (density) $\mathbf{P}$ is usually given by $\mathbf{p} = \mathbf{P}\mathrm{d}\tau'$.

Laplace's equation is $\boldsymbol{\nabla}^2 V = 0$. The general solution is given by

$$V(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos(\theta)) \qquad (1)$$

where $P_\ell(\cos(\theta))$ are Legendre polynomials.