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Assignment Ques 1-

Sample Size = n

Population mean = θ1
variance = θ2

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \times e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$P(x_1) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \times e^{-\frac{1}{2}\left(\frac{x_1-\mu}{\sigma}\right)^2}$$

$$P(x_2) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \times e^{-\frac{1}{2}\left(\frac{x_2-\mu}{\sigma}\right)^2}$$

$$P(x_N) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \times e^{-\frac{1}{2}\left(\frac{x_N-\mu}{\sigma}\right)^2}$$

$\left(\frac{x_1-\mu}{\sigma}\right)^2 + \left(\frac{x_2-\mu}{\sigma}\right)^2 + \dots \left(\frac{x_N-\mu}{\sigma}\right)^2$

$x_1^2+\mu^2-2\mu x_1$
$x_2^2+\mu^2-2\mu\cdot x_2$
$\vdots$
$x_N^2+\mu^2-2\mu\cdot x_N$

$-2\mu x_1 - 2\mu x_2 \dots - 2\mu x_N$
$-2\mu(x_1+x_2 \dots x_N$

$$L(x_1, x_2 \dots x_N) = \prod (f(x))$$

$$= \left[\frac{1}{\sqrt{2\pi}\cdot\sigma}\right]^n \times e^{-\frac{1}{2}\left[\left(\frac{x_1-\mu}{\sigma}\right)^2 + \left(\frac{x_2-\mu}{\sigma}\right)^2 \dots - \left(\frac{x_N-\mu}{\sigma}\right)^2\right]}$$

$$= \left[\frac{1}{\sqrt{2\pi}\cdot\sigma}\right]^n \times e^{-\frac{1}{2}\left[\frac{\sum_{i=1}^{N} x_i^2 - 2\mu\left(\sum_{i=1}^{n} x_i\right) + \mu^2 \times n}{\sigma^2}\right]}$$

Taking Natural log on both sides

$$\ln(L) = n\cdot\ln\left(\frac{1}{\sqrt{2\pi}\cdot\sigma}\right) - \frac{1}{2}\left(\frac{\sum x_i^2 - 2\mu\cdot\sum x_i + \mu^2\cdot n}{\sigma^2}\right)$$

Now Taking derivative w.r.t μ — ①

$$\frac{dL}{d\mu} = \frac{-1}{2}\left(\frac{-2\times\sum x_i + 2\mu\cdot n}{\sigma^2}\right) = 0$$

$$\therefore \cancel{2}\sum x_i = \cancel{2}\mu\cdot n \Rightarrow \boxed{\mu = \frac{\sum x_i}{n}}$$

Now Taking derivative w.r.t σ^2 — ②

$$\cancel{\frac{dL}{d\sigma^2} = -\frac{1}{2}\left(\frac{\sum x_i^2 - 2\mu\sum x_i + \mu^2\cdot n}{(\sigma^2)^2}\right) = 0}$$

$\sum x_i^2$

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$$\ln(L) = n \cdot [\log 1 - \log(\sqrt{2\pi} \cdot \sigma)] - \frac{1}{2\sigma^2} (\Sigma x_i - \mu)^2$$

$$\Rightarrow -\frac{n}{2} \cdot \log(\sigma^2 \cdot 2\pi) - \frac{1}{2\sigma^2} (\Sigma x_1 - \mu)^2$$

$$\Rightarrow -\frac{n}{2}\left[\log(\sigma^2) + \log 2\pi\right] - \frac{1}{2\sigma^2} (\Sigma x_1 - \mu)^2$$

Now Taking derivative w.r.t σ^2 — ②

$$\frac{dL}{d\sigma^2} \Rightarrow -\frac{n}{2} \times \left[\frac{1}{\cancel{\sigma^2}} \times 2\cancel{\sigma}\right] + \frac{1}{2\sigma^3} (\Sigma x_1 - \mu)^2 = 0$$

$$\Rightarrow -\frac{n}{\cancel{2}} \cdot \frac{n}{\sigma} = \frac{(\Sigma x_1 - \mu)^2}{2\sigma^3}$$

$$\boxed{\sigma^2 = \frac{\Sigma x_1 - \mu}{n}}$$

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Assignment Ques 2

PMF of Binomial Distribution = ${}^{n}C_{x} \cdot p^{x} \cdot (1-p)^{n-x}$

$P(x_1) = {}^{n}C_{x} \cdot p^{x_1} \cdot (1-p)^{n-x_1}$

$P(x_2) = {}^{n}C_{x} \cdot p^{x_2} \cdot (1-p)^{n-x_2}$

$\vdots$

$P(x_N) = {}^{n}C_{x} \cdot p^{x_N} (1-p)^{n-x_N}$

$n - x_1 + n - x_2 \ldots$
$n - x_N$
nC

Joint Probability

$L(x_1, x_2 \cdots x_N) = ({}^{n}C_{x})^{n} \cdot p^{\Sigma x_i} \cdot (1-p)^{n^2 - \Sigma x_i}$

Taking log on both sides

$$\ln(L) = n \cdot \ln\left[{}^{n}C_{x}\right] + \Sigma x_i \ln p + (n^2 - \Sigma x_i) \cdot \ln(1-p)$$

$$\frac{dL}{dp} = \frac{\Sigma x_i}{p} - \frac{(n^2 - \Sigma x_i)}{(1-p)} = 0$$

$\Rightarrow$ ~~$\Sigma x_i (1-p) = -p(n^2 - \Sigma x_i)$~~

$\Sigma x_i - \Sigma x_i \times p = -pn^2 + p \cdot \Sigma x_i$

$\Sigma x_i - 2\Sigma x_i p = -pn^2$

~~$\Sigma x_i (1 - 2p) = -n^2 p$~~

$$\frac{\Sigma x_i (1-p) - (n^2 - \Sigma x_i) \cdot p}{p(1-p)} = 0$$

$$\Sigma x_i - \cancel{\Sigma x_i \times p} - n^2 p + \cancel{\Sigma x_i \cdot p} = 0$$

$$\boxed{p = \frac{\Sigma x_i}{n^2}}$$