Thapar Institute of Engineering and Technology, Patiala
School of Mathematics
Probability and Statistics (UCS410)
Practice Sheet-2

1. A die is loaded (not all outcomes are equally likely) such that the probability that the number *i* shows up is K*i*, *i* = 1, 2, ..., 6, where K is a constant. Find (a) the value of K and (b) the probability that a number greater than 3 shows up.

2. In how many different ways can a true-false test consisting of 9 questions be answered?

3. (a) How many three-digit numbers can be formed from the digits 0, 1, 2, 3, 4, 5, and 6 if each digit can be used only once?
(b) How many of these are odd numbers?
(c) How many are greater than 330?

4. A subway station in a large city has 12 gates, six inbound (entering into the subway station) and six outbound (exiting the subway station). The number of gates open in each direction is observed at a particular time of day. Assume that each outcome of the sample space is equally likely.
(a) Define a suitable sample space.
(b) What is the probability that at most one gate is open in each direction?
(c) What is the probability that at least one gate is open in each direction?
(d) What is the probability that the number of gates open is the same in both directions?
(e) What is the probability of the event that the total number of gates open is six?

5. A box contains 500 envelopes, of which 75 contain \$100 in cash, 150 contain \$25, and 275 contain \$10. An envelope may be purchased for \$25. What is the sample space for the different amounts of money? Assign probabilities to the sample points and then find the probability that the first envelope purchased contains less than \$100.

6. Prove that (a) $P(A \cap B) \geq P(A) + P(B) - 1$ (b) $P(A \cup B) \leq P(A) + P(B)$

7. If $P(A) = 0.24$, $P(B) = 0.67$, and $P(A \cap B) = 0.09$, find (a) $P(A \cup B)$ (b) $P((A \cup B)^c)$ (c) $P(A^c \cup B^c)$ (d) $P((A \cap B)^c)$ (e) $P(A^c \cap B^c)$

8. In the game of poker, 5 cards are drawn from a pack of 52 well-shuffled cards. Find the probability that (a) 4 are aces, (b) 4 are aces and 1 is a king, (c) 3 are tens and 2 are jacks, (d) 3 are of any one suit and 2 are of another, and (e) at least 1 ace is obtained.

9. A fruit basket contains 25 apples and oranges, of which 20 are apples. If two fruits are randomly picked in sequence, what is the probability that both the fruits are apples?

10. An automobile manufacturer is concerned about a possible recall of its best-selling four-door sedan. If there were a recall, there is a probability of 0.25 of a defect in the brake system, 0.18 of a defect in the transmission, 0.17 of a defect in the fuel system, and 0.40 of a defect in some other area.
(a) What is the probability that the defect is the brakes or the fueling system if the probability of defects in both systems simultaneously is 0.15?
(b) What is the probability that there are no defects in either the brakes or the fueling system?

11. A fair die is tossed twice. Find the probability of getting a 4, 5, or 6 on the first toss and 1, 2, 3, or 4 on the second toss.

12. Three balls are drawn successively from a box containing 6 red balls, 4 white balls, and 5 blue balls. Find the probability that they are drawn in the order red, white, and blue if each ball is (a) replaced, (b) not replaced.

13. If $P(A) > 0$, $P(B) > 0$ and $P(A) < P(A|B)$, show that $P(B) < P(B|A)$.

14. A random sample of 200 adults are classified below by sex and their level of education attained.

Education	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

If a person is picked at random from this group, find the probability that
(a) the person is a male, given that the person has a secondary education.
(b) the person does not have a college degree, given that the person is a female.

15. Assume that a noisy channel independently transmits symbols, say 0s 60% of the time and 1s 40% of the time. At the receiver, there is a 1% chance of obtaining any particular symbol distorted. What is the probability of receiving a 1, irrespective of which symbol is transmitted?

16. The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?

17. Suppose a statistics class contains 70% male and 30% female students. It is known that in a test, 5% of males and 10% of females got an "A" grade. If one student from this class is randomly selected and observed to have an "A" grade, what is the probability that this is a male student?

18. A paint-store chain produces and sells latex and semi-gloss paint. Based on long-range sales, the probability that a customer will purchase latex paint is 0.75. Of those that purchase latex paint, 60% also purchase rollers. But only 30% of semi-gloss paint buyers purchase rollers. A randomly selected buyer purchases a roller and a can of paint. What is the probability that the paint is latex?