Show that the following matrices are not similar to diagonal matrices.

(i) $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

Show that the following matrices are similar to diagonal matrices. Find the diagonal and modal matrix in each case.

(i) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} -17 & 18 & -6 \\ -18 & 19 & -6 \\ -9 & 9 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ (iv) $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

Ans. :

(i) $D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}, P = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

(ii) $D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$

(iii) $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$P = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -2 & 1 \\ 3 & 3 & -2 \end{bmatrix}$

(iv) $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, P = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

3. Determine diagonal matrices orthogonally similar to the following real symmetric matrices. Also, find modal matrix in each case.

(i) $\begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & 8 \end{bmatrix}$

(ii) $\begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$

Ans. : (i) $D = \begin{bmatrix} 9 & 0 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & -9 \end{bmatrix}$,

$P = \begin{bmatrix} \frac{4}{\sqrt{18}} & 0 & \frac{1}{3} \\ \frac{1}{\sqrt{18}} & \frac{1}{\sqrt{2}} & -\frac{2}{3} \\ -\frac{1}{\sqrt{18}} & \frac{1}{\sqrt{2}} & \frac{2}{3} \end{bmatrix}$

(ii) $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 9 \end{bmatrix}$,

$P = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$

4. Find the symmetric matrix A having eigen values $\lambda_1 = 0$, $\lambda_2 = 3$ and $\lambda_3 = 15$ with the corresponding eigen vectors $X_1 = [1, 2, 2]^T$, $X_2 = [-2, -1, 2]^T$ and X_3.

Ans. : $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Solve the following differential equations:

1. $y^2\frac{dy}{dx}+x^2=0$.

[**Ans.** : $x^3+y^3=c$]

2. $(1+x)y-(1+y)x\frac{dy}{dx}=0,\ x>0,\ y>0.$

[**Ans.** : $x-y+\log\left(\frac{x}{y}\right)=c$]

3. $(e^x+1)\cos x\,dx+e^x\sin x\,dy=0.$

[**Ans.** : $(e^x+1)\sin x=c$]

4. $y\frac{dy}{dx}=xe^{-x}\sqrt{1-y^2}.$

[**Ans.** : $\sqrt{1-y^2}=(x+1)e^{-x}+c$]

5. $x(e^{4y}-1)\frac{dy}{dx}+(x^2-1)e^{2y}=0,\ x>0.$

[**Ans.** : $\cosh(2y)=\log x-\frac{x^2}{2}+c$]

6. $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}.$

[**Ans.** : $y\sin y=x^2\log x+c$]

7. $\frac{dy}{dx}=\frac{\sin x+\frac{\log x}{x}}{\cos y-\sec^2 y}.$

[**Ans.** : $\sin y-\tan y=-\cos x+\frac{1}{2}(\log x)^2+c$]

8. $y\sec^2 x+(y+7)\tan x\frac{dy}{dx}=0.$

[**Ans.** : $y^7\tan x=ce^{-y}$]

9. $(x+1)\left(\frac{dy}{dx}-1\right)=2(y-x).$

[**Ans.** : $y-x=c(x+1)^2$]

10. $\cos(x+y)dy=dx.$

[**Ans.** : $y-\tan\left(\frac{x+y}{2}\right)=c$]

11. $\frac{dy}{dx}=\frac{y-x}{y-x+2}.$

[**Ans.** : $(y-x)^2=c-4y$]

12. $x\frac{dy}{dx}=y+x^2\tan\left(\frac{y}{x}\right)$

[**Ans.** : $e^{y/x}=x+c$]

14. $(1+x^3)dy-x^2y\,dx=0,\ y(1)=2.$

[**Ans.** : $y^3=4(1+x^3)$]

15. $\frac{dy}{dx}=\frac{x}{y}-\frac{x}{1+y},\ y(0)=2.$

[**Ans.** : $3y^2+2y^3=3x^2+28$]

16. $\frac{dy}{dx}+2y=x^2y,\ y(0)=1.$

[**Ans.** : $y=e^{\frac{x^3}{3}-2x}$]

17. $e^y\left(\frac{dy}{dx}+1\right)=1,\ y(0)=1.$

[**Ans.** : $e^y=1-(1-e)e^{-x}$]

18. $\frac{dy}{dx}=2y\sin^2 x,\ y\left(\frac{\pi}{2}\right)=1.$

[**Ans.** : $\log y=x-\frac{1}{2}\sin 2x-\frac{\pi}{2}$]

19. $\cos y\,dx+(1+e^{-x})\sin y\,dy=0,$

$y(0)=\frac{\pi}{4}.$

[**Ans.** : $\sqrt{2}$]

Solve the following differential equations:

1. $x(y-x)\frac{dy}{dx} = y(y+x)$.

$\left[\text{Ans.}: \frac{y}{x} - \log xy = c\right]$

2. $\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$.

$[\text{Ans.}: 3x + y\log x + cy = 0]$

3. $x\frac{dy}{dx} = y(\log y - \log x + 1)$.

$\left[\text{Ans.}: \log\frac{y}{x} = cx\right]$

4. $y\,dx + x\log\frac{y}{x}dy - 2x\,dy = 0$.

$\left[\text{Ans.}: y = c\left(1 + \log\frac{x}{y}\right)\right]$

5. $\left(xe^{\frac{y}{x}} - y\sin\frac{y}{x}\right)dx + x\sin\frac{y}{x}dy = 0$.

$\left[\text{Ans.}: \log x^2 - e^{-\frac{y}{x}}\left(\sin\frac{y}{x} + \cos\frac{y}{x}\right) = c\right]$

6. $x\sin\frac{y}{x}dy = \left(y\sin\frac{y}{x} - x\right)dx$.

$\left[\text{Ans.}: \cos\frac{y}{x} = \log x + c\right]$

7. $x\frac{dy}{dx} = y + x\sec\left(\frac{y}{x}\right)$.

$\left[\text{Ans.}: \sin\frac{y}{x} = \log(cx)\right]$

8. $\left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x}\right)dx + x\sec^2\frac{y}{x}dy = 0$.

$\left[\text{Ans.}: x\tan\frac{y}{x} = c\right]$

9. $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$.

$\left[\text{Ans.}: x + ye^{\frac{x}{y}} = c\right]$

10. $(3xy + y^2)dx + (x^2 + xy)dy = 0$, $y(1) = 1$.

$[\text{Ans.}: x^2y\,(2x+y) = 3]$

11. $2x(x+y)\frac{dy}{dx} = 3y^2 + 4xy$, $y(1) = 1$.

$\left[\text{Ans.}: y^2 + 2xy = 3x^3\right]$

12. $3x\frac{dy}{dx} - 3y + (x^2 - y^2)^{\frac{1}{2}} = 0$, $y(1) = 1$.

$\left[\text{Ans.}: 3\cos^{-1}\left(\frac{y}{x}\right) - \log x = 0\right]$

13. $(x^3 - 3xy^2)dx + (y^3 - 3x^2y)dy = 0$, $y(0) = 1$.

$\left[\text{Ans.}: x^4 - 6x^2y^2 + y^4 = 1\right]$

14. $xy\log\frac{x}{y}dx + \left(y^2 - x^2\log\frac{x}{y}\right)dy = 0$, $y(1) = e$.

$\left[\text{Ans.}: \frac{x^2}{2y^2}\log\frac{x}{y} - \frac{x^2}{4y^2} + \log y = 1 - \frac{3}{4e^2}\right]$

ve the following differential equations:

1. $(x+2y)dx+(3x+6y+3)dy=0.$

[Ans.: $x+3y-3\log|x+2y+3|=c$]

2. $(4x-4y+1)dy-(3x-2y+1)dx=0.$

[Ans.: $4x-8y-\log(12x-8y+1)=c$]

3. $(x+y+3)dy=(x+y-3)dx.$

[Ans.: $-x+y-3\log(x+y)=c$]

4. $(x+y+3)dx-(2x+2y-1)dy=0.$

[Ans.: $-3x+6y-7\log|3x+3y+2|=c$]

5. $(2x+6y+1)dy-(x+3y-2)dx=0.$

[Ans.: $-x+2y+\log|x+3y-1|=c$]

6. $(y-x+2)dy=(y-x)dx.$

[Ans.: $(y-x)^2+4y=c$]

7. $(4x+2y+5)dy-(2x+y-1)dx=0.$

[Ans.: $10y-5x+7\log|10x+5y+9|=c$]

8. $(2x-4y+5)dy-(x-2y+3)dx=0.$

[Ans.: $x^2-4xy+4y^2+6x-10y=c$]

9. $\frac{dy}{dx}=-\frac{2x-y+1}{x+y}.$

[Ans.: $\log\left[2\left(x+\frac{1}{3}\right)^2+\left(y-\frac{1}{3}\right)^2\right]+\sqrt{2}\tan^{-1}\left[\frac{3y-1}{\sqrt{2}(3x+1)}\right]=c$]

10. $(x+y-1)dx-(x-y-1)dy=0.$

[Ans.: $\log[(x-1)^2+y^2]-2\tan^{-1}\left(\frac{y}{x-1}\right)=c$]

11. $(3x-2y+4)dx-(2x+7y-1)dy=0.$

[Ans.: $3x^2-4xy-7y^2+8x+2y=c$]

12. $(x-y-1)dx+(4y+x-1)dy=0.$

[Ans.: $\log[4y^2+(x-1)^2+\tan^{-1}\left(\frac{2y}{x-1}\right)=c$]

13. $(x-y-1)dx+(x+y+5)dy=0.$

[Ans.: $\log\left[(y+3)^2+(x+2)^2\right]+2\tan^{-1}\left(\frac{y+3}{x+2}\right)=c$]

14. $(y-x+2)dx+(x+y+6)dy=0.$

[Ans.: $(y+4)^2+2(x+2)(y+4)-(x+2)^2=c$]

15. $\frac{dy}{dx}=\frac{y+x-2}{y-x-4}.$

[Ans.: $(x+1)^2-(y-3)^2+2(x+1)(y-3)=c$]

16. $\frac{dy}{dx}=\frac{2x+9y-20}{6x+2y-10}.$

[Ans.: $(2x-y)^2=c(x+2y-5)$]

17. $(3x+2y+3)dx-(x+2y-1)dy=0,$
$y(-2)=1.$

[Ans: $(2x+2y+1)(3x-2y+9)^4=-1$]

[Ans.: $\log\left[(x-1)^2+(y+3)^2\right]+2\tan^{-1}\left(\frac{x-1}{y+3}\right)=2\log$

18. $(x+y+2)dx-(x-y-4)dy=0,$
$y(1)=0.$

Solve the following differential equations:

1. $(2x^3+3y)\,dx+(3x+y-1)\,dy=0.$

[**Ans.:** $x^4+6xy+y^2-2y=c$]

2. $(1+e^x)\,dx+y\,dy=0.$

[**Ans.:** $x+e^x+\frac{y^2}{2}=c$]

3. $\sinh x\cos y\,dx-\cosh x\sin y\,dy=0.$

[**Ans.:** $\cosh x\cos y=c$]

4. $xe^{x^2+y^2}\,dx+y(1+e^{x^2+y^2})dy=0,$
$y(0)=0.$

[**Ans.:** $y^2+e^{x^2+y^2}=1$]

5. $\left(4x^3y^3+\frac{1}{x}\right)dx+\left(3x^4y^2-\frac{1}{y}\right)dy=0,$
$y(1)=1.$

[**Ans.:** $x^4y^3+\log\left(\frac{x}{y}\right)=1$]

6. $(4x^3y^3dx+3x^4y^2dy)$
$-(2xy\,dx+x^2dy)=0.$

[**Ans.:** $x^4y^3-x^2y=c$]

7. $2x(ye^{x^2}-1)dx+e^{x^2}dy=0.$

[**Ans.:** $ye^{x^2}-x^2=c$]

8. $\left(1+x^2\sqrt{y}\right)y\,dx+\left(x^2\sqrt{y}+2\right)x\,dy=0.$

[**Ans.:** $2xy+\frac{2}{3}x^3y^{\frac{3}{2}}=c$]

9. $(e^x+1)\cos x\,dx+e^x\sin x\,dy=0.$

[**Ans.:** $\sin x(e^x+1)=c$]

10. $(x^2+1)\frac{dy}{dx}=x^3-2xy+x.$

[**Ans.:** $x^4-4x^2y+2x^2-4y=c$]

11. $\frac{dy}{dx}=\frac{x^2-2xy}{x^2-\sin y}.$

[**Ans.:** $x^3-3(x^2y+\cos y)=c$]

12. $\frac{dy}{dx}=\frac{y+1}{(y+2)e^y-x}.$

[**Ans.:** $(y+1)(x-e^y)=c$]

13. $(x-y\cos x)dx-\sin x\,dy=0,$
$y\left(\frac{\pi}{2}\right)=1.$

[**Ans.:** $x^2-2y\sin x=\frac{\pi^2}{4}-2$]

14. $(2xy+e^y)\,dx+(x^2+xe^y)dy=0,$
$y(1)=1.$

[**Ans.:** $x^2y+xe^y=e+1$]

Exercise 10.6

the following differential equations:

$\frac{dy}{dx} = 3x^2 - 2xy + 1$

$\left[\text{Ans.}: y = \frac{c}{x^2} + x + \frac{1}{x}\right]$

$(2y - 3x)dx + x\,dy = 0$

$\left[\text{Ans.}: x^2 y = x^3 + c\right]$

$(x+1)\frac{dy}{dx} - 2y = (x+1)^4$

$\left[\text{Ans.}: y = \left(\frac{x^2}{2} + x + c\right)(x+1)^2\right]$

$\frac{dy}{dx} + y\cot x = \cos x$

$\left[\text{Ans.}: y\sin x = \frac{\sin^2 x}{2} + c\right]$

$\frac{1}{x}\frac{dy}{dx} + 2y = e^{-x^2}$

$\left[\text{Ans.}: ye^{x^2} = \frac{x^2}{2} + c\right]$

$(y+1)dx + [x - (y+2)e^y]dy = 0$

$\left[\text{Ans.}: (y+1)(x - e^y) = c\right]$

$dx + x\,dy = e^{-y}\sec^2 y\,dy$

8. $(1+x)\frac{dy}{dx} - y = e^x(x+1)^2$

$\left[\text{Ans.}: y = (1+x)(e^x + c)\right]$

9. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$\left[\text{Ans.}: ye^{2\sqrt{x}} = 2\sqrt{x} + c\right]$

10. $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$

$\left[\text{Ans.}: xy = \sin x + c\cos x\right]$

11. $\cos^2 x\frac{dy}{dx} + y = \tan x$

$\left[\text{Ans.}: y = \tan x - 1 + ce^{-\tan x}\right]$

12. $(2x + y^4)\frac{dy}{dx} = y$

$\left[\text{Ans.}: \frac{2x}{y^2} = y^2 + c\right]$

13. $\sqrt{a^2 + x^2}\frac{dy}{dx} + y = \sqrt{a^2 + x^2} - x$

$\left[\text{Ans.}: \left(x + \sqrt{x^2 + a^2}\right)y = a^2 x + c\right]$

14. $\frac{dy}{dx} = \frac{1}{x + e^y}$

$\left[\text{Ans.}: xe^{-y} = c + y\right]$

15. $\frac{dy}{dx} - \left(\frac{3}{x}\right)y = x^3,\ y(1) = 4$

$\left[\text{Ans.}: y = x^3(x+3)\right]$

16. $(1+x^2)\frac{dy}{dx} - 2xy = 2x(1+x^2),$
$y(0) = 1$

$\left[\text{Ans.}: y = (1+x^2)[1 + \log(1+x^2)]\right]$

17. $x\frac{dy}{dx} - 3y = x^4(e^x + \cos x) - 2x^2,$
$y(\pi) = \pi^3 e^\pi + 2\pi^2$

$\left[\text{Ans.}: y = 2x^2 + (e^x + \sin x)x^3\right]$

18. If $\frac{dy}{dx} + 2y\tan x = \sin x,\ y\left(\frac{\pi}{3}\right) = 0$

show that maximum value of y is $\frac{1}{4}$.

19. $\frac{dy}{dx} + \frac{y}{x} = \log x,\ y(1) = 1$

$\left[\text{Ans.}: y = \frac{x\log x}{2} - \frac{x}{4} + \frac{5}{4x}\right]$

20. $\frac{dy}{dx} + 2xy = xe^{-x^2}$

$\left[\text{Ans.}: ye^{x^2} = \frac{x^2}{2} + c\right]$

ve the following differential equations:

$\frac{dy}{dx} = x^3y^3 - xy$

$\left[\text{Ans.}: \frac{1}{y^2} = x^2 + 1 + ce^{x^2}\right]$

$x^2y - x^3\frac{dy}{dx} = y^4\cos x$

$\left[\text{Ans.}: x^3 = y^3(3\sin x - c)\right]$

$(1+2y^2)dx + 2y(1+x^2)dy = 0$

$\left[\text{Ans.}: y^2(1+x^2) = -x^3 + c\right]$

$x(1-3x^2y^2)dy = 0$

$\left[\text{Ans.}: y^6 = ce^{-\frac{1}{x^2y^2}}\right]$

$[y + xy^3(1+\log x)]dx = 0$

$\left[\text{Ans.}: x^2 = -\frac{2}{3}x^3y^2\left(\frac{2}{3} + \log x\right) + cy^2\right]$

6. $\frac{dy}{dx} + y = y^2e^x$

$\left[\text{Ans.}: -\frac{e^{-x}}{y} = x + c\right]$

7. $x\,dy + y\,dx = x^3y^3\,dx$

$\left[\text{Ans.}: \frac{2}{y^2} = 5x^3 + cx^4\right]$

8. $x\frac{dy}{dx} + y = y^3x^{n+1}$

$\left[\text{Ans.}: \frac{n-1}{y^2} = cx^2 - 2x^{n+1}\right]$

9. $xy(1+x^2y^2)\frac{dy}{dx} = 1$

$\left[\text{Ans.}: \frac{1}{x^2} = ce^{-y^2} - y^2 + 1\right]$

10. $x^2y^3\,dx + (x^3y - 2)\,dy = 0$

$\left[\text{Ans.}: x^3 = \frac{2}{y} + \frac{2}{3} + ce^{\frac{3}{y}}\right]$

11. $y\frac{dx}{dy} = x - yx^2\cos y$

$\left[\text{Ans.}: \frac{y}{x} = y\sin y + \cos y + c\right]$

12. $\frac{dy}{dx} = \frac{e^x}{x^2} - \frac{1}{x}$

$\left[\text{Ans.}: 2xe^{-y} = 1 + 2cx^2\right]$

13. $y\frac{dy}{dx} + \frac{4}{3}x - \frac{y^2}{3x} = 0$

$\left[\text{Ans.}: y^2x^{-\frac{2}{3}} + 2x^{\frac{4}{3}} = c\right]$

14. $\frac{dy}{dx} + (2x\tan^{-1}y - x^3)(1+y^2) = 0$

$\left[\text{Ans.}: 2\tan^{-1}y = (x^2-1) + ce^{-x^2}\right]$

15. $\tan y\frac{dy}{dx} + \tan x = \cos y\cos^2 x$

$\left[\text{Ans.}: \sec y\sec x = \sin x + c\right]$

16. $(y + e^x - e^{-x})dx + (1+e^y)dy = 0$

$\left[\text{Ans.}: y + e^y = (x+c)e^{-x}\right]$

17. $x^2\cos y\frac{dy}{dx} = 2x\sin y - 1$

$\left[\text{Ans.}: 3x\sin y = cx^3 + 1\right]$

18. $4x^2y\frac{dy}{dx} = 3x(3y^2+2) + 2(3y^2+2)$

$\left[\text{Ans.}: 4x^9 = (3y^2+2)^2(-3x^3+c)\right]$

19. $\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\tan y\sin y$

$\left[\text{Ans.}: \operatorname{cosec} y = 1 + cx\right]$

20. $x\frac{dy}{dx} + 3y = x^4e^{\frac{1}{x^2}}y^3$

$\left[\text{Ans.}: \frac{1}{y^2} = \left(e^{\frac{1}{x^2}} + c\right)x^4\right]$

21. $x^2\frac{dy}{dx} = \sin^2 y - (\sin y\cos y)x$

$\left[\text{Ans.}: \cot y = \frac{1}{2x} + c\right]$

22. $\frac{dr}{d\theta} = \frac{r\sin\theta - r^2}{\cos\theta}$

$\left[\text{Ans.}: \frac{1}{r} = c\cos\theta + \sin\theta\right]$

23. $\cos x\frac{dy}{dx} + 4y\sin x = 4\sqrt{y}\sec x$

$\left[\text{Ans.}: \sqrt{y}\sec^2 x = 2\left(\tan x + \frac{\tan^3 x}{3}\right) + c\right]$

24. $\sin y\frac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$

$\left[\text{Ans.}: 4\cos y = 2\sin^2 x - 2\sin x + 1 - 4ce^{-2\sin x}\right]$

25. $e^y\left(\frac{dy}{dx} + 1\right) = e^x$

$\left[\text{Ans.}: e^{x+y} = \frac{e^{2x}}{2} + c\right]$

Exercise 10.8

Solve the following differential equations:

1. $(D^2+D-2)y=0.$

[**Ans.** : $y=c_1e^{-2x}+c_2e^{x}$]

2. $(4D^2+8D-5y)=0.$

[**Ans.** : $y=c_1e^{\frac{x}{2}}+c_2e^{-\frac{5x}{2}}$]

3. $(D^2-4D-12)y=0.$

[**Ans.** : $y=c_1e^{6x}+c_2e^{-2x}$]

4. $(D^2+2D-8)y=0.$

[**Ans.** : $y=c_1e^{2x}+c_2e^{-4x}$]

5. $(D^2+4D+1)y=0.$

[**Ans.** : $y=c_1e^{(-2+\sqrt{3})x}+c_2e^{(-2-\sqrt{3})x}$]

6. $(4D^2+4D+1)y=0.$

[**Ans.** : $y=(c_1+c_2x)e^{-\frac{x}{2}}$]

7. $(D^2+2\pi D+\pi^2)y=0.$

[**Ans.** : $y=(c_1+c_2x)e^{-\pi x}$]

8. $(9D^2-12D+4)y=0.$

[**Ans.** : $y=(c_1+c_2x)e^{\frac{2x}{3}}$]

9. $(25D^2-20D+4)y=0.$

[**Ans.** : $y=(c_1+c_2x)e^{\frac{2x}{5}}$]

10. $(9D^2-30D+25)y=0.$

[**Ans.** : $y=(c_1+c_2x)e^{\frac{5x}{3}}$]

11. $(D^2-6D+25)y=0.$

[**Ans.** : $y=e^{3x}(c_1\cos 4x+c_2\sin 4x)$]

12. $(D^2+6D+11)y=0.$

[**Ans.** : $y=e^{-3x}(c_1\cos\sqrt{2}x+c_2\sin\sqrt{2}x)$]

13. $[D^2-2aD+(a^2+b^2)y]=0.$

[**Ans.** : $y=e^{ax}(c_1\cos bx+c_2\sin bx)$]

14. $(D^3-9D)y=0.$

[**Ans.** : $y=c_1+c_2e^{3x}+c_3e^{-3x}$]

15. $(D^3-3D^2-D+3)y=0.$

[**Ans.** : $y=c_1e^{-x}+c_2e^{x}+c_3e^{3x}$]

16. $(D^3-6D^2+11D-6)y=0.$

[**Ans.** : $y=c_1e^{x}+c_2e^{2x}+c_3e^{3x}$]

17. $(D^3-6D^2+12D-8)y=0.$

[**Ans.** : $y=(c_1+c_2x+c_3x^2)e^{2x}$]

18. $(D^3+D)y=0.$

[**Ans.** : $y=c_1+c_2\cos x+c_3\sin x$]

19. $(D^3+5D^2+8D+6)y=0.$

[**Ans.** : $y=c_1e^{-3x}+e^{-x}(c_2\cos x+c_3\sin x)$]

20. $(8D^4-6D^3-7D^2+6D-1)y=0.$

[**Ans.** : $y=c_1e^{\frac{1}{2}}+c_2e^{\frac{1}{2}}+c_3e^{x}+c_4e^{-x}$]

21. $(D^4-2D^3+D^2)y=0.$

[**Ans.** : $y=c_1+c_2x+(c_3+c_4x)e^{x}$]

22. $(D^4-3D^3+3D^2-D)y=0.$

[**Ans.** : $y=c_1+(c_2+c_3x+c_4x^2)e^{x}$]

23. $(D^4+8D^2-9)y=0.$

[**Ans.** : $y=c_1e^{x}+c_2e^{-x}+c_3\cos 3x+c_4\sin 3x$]

24. $(D^4+D^3+14D^2+16D-32)y=0.$

[**Ans.** : $y=c_1e^{x}+c_2e^{-2x}+c_3\cos 4x+c_4\sin 4x$]

25. $(D^4+2D^3-9D^2-10D+50)y=0.$

[**Ans.** : $y=e^{2x}(c_1\cos x+c_2\sin x)+e^{-3x}(c_3\cos x+c_4\sin x)$]

26. $(D^4+18D^3+81)y=0.$

[**Ans.** : $y=(c_1+c_2x)\cos 3x+(c_3+c_4x)\sin 3x$]

27. $(D^4-4D^3+14D^2-20D+25)y=0.$

[**Ans.** : $y=e^{x}[(c_1+c_2x)\cos 2x+(c_3+c_4x)\sin 2x]$]

28. $(D^2+D-2)y=0, y(0)=4, y'(0)=-5.$

[**Ans.** :

29. $(4D^2+12D+9)y=0,$
$y(0)=-1, y'(0)=2$

[**Ans.** : $y=\left(\frac{x}{2}\right.$

30. $(D^2-4D+5)y=0,$
$y(0)=2, y'(0)=-1$

[**Ans.** : $y=e^{2x}(2\cos x-5\sin$

31. $(9D^2-6D+1)y=0,$
$y(1)=e^{\frac{1}{3}}, y(2)=1.$

[**Ans.** :
$y=\left[\left(e^{-\frac{2}{3}}-1\right)x+\left(2-e^{\frac{2}{3}}\right.\right.$

32. $(4D^3-4D^2-9D+9)y=0,$
$y(0)=1, y'(0)=0, y''(0)=0$

[**Ans.** : $y=\frac{1}{5}\left(9e^{x}-5e^{\frac{3x}{2}}+e^{\right.$

33. $(D^3+D^2-2)y=0, y(0)=2,$
$y'(0)=2, y''(0)=-3.$

[**Ans.** : $y=e^{x}+e^{-x}(\cos x+2\sin$

34. $(D^4-3D^3)=0, y(0)=2,$
$y'(0)=5, y''(0)=15, y'''(0)=$

[**Ans.** : $y=1+2x+3x^2+$

35. $(D^4-3D^3+2D^2)y=0, y(0)=$
$y'(0)=0, y''(0)=2, y'''(0)=$

[**Ans.** : $y=2(e^{x}-$

Solve the following differential equations using variation of parameter method

1. $(D^2+1)y = \tan x$.

[**Ans.** : $y = c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x)$]

2. $(D^2+4)y = \sec^2 2x$.

[**Ans.** : $y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4} + \frac{\sin 2x}{4} \log(\sec 2x + \tan 2x)$]

3. $(D^2+1)y = \text{cosec}\cot x$.

[**Ans.** : $y = c_1 \cos x + c_2 \sin x - \cos x \log|\sin x| - x \sin x - \sin x \cot x$]

4. $(D^2+1)y = \frac{1}{1+\sin x}$.

[**Ans.** : $y = c_1 \cos x + c_2 \sin x - (1 - \sin x + x\cos x) + \sin x \log(1+\sin x)$]

5. $(D^2-1)y = \frac{2}{1-e^x}$.

[**Ans.** : $y = c_1 e^x + c_2 e^{-x} + e^x \log(1+e^{-x}) - e^x - 1 - e^{-x}\log(1+e^x)$]

6. $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$.

[**Ans.** : $y = (c_1 + c_2 x)e^{3x} - (1+\log x)e^{3x}$]

7. $(D^2-1)y = 2(1-e^{-2x})^{-\frac{1}{2}}$.

[**Ans.** : $y = c_1 e^x + c_2 e^{-x} - e^x \sin^{-1}(e^{-x}) - (e^{2x}-1)^{\frac{1}{2}} e^{-x}$]

8. $(D^2-2D)y = e^x \sin x$.

[**Ans.** : $y = c_1 + c_2 e^{2x} - \frac{e^x}{2}\sin x$]

9. $(D^2+3D+2)y = e^x + x^2$.

[**Ans.** : $y = c_1 e^{-x} + c_2 e^{-2x} + \frac{e^x}{6} + \left(\frac{x^2}{2} - \frac{3x}{2} + \frac{7}{4}\right)$]

10. $(D^2-2D+1)y = x^{\frac{3}{2}} e^x$.

[**Ans.** : $y = (c_1 + c_2 x)e^x + \frac{4}{35}x^{\frac{7}{2}}e^x$]

11. $(D^2-3D+2)y = xe^x + 2x$.

[**Ans.** : $y = c_1 e^x + c_2 e^{2x} - \frac{x^2}{2}e^x - xe^{-x} + x + \frac{3}{2}$]

12. $(D^2+1)y = x\cos 2x$.

[**Ans.** : $y = c_1 \cos x + c_2 \sin x - \frac{x}{2}\cos 2x + \frac{4}{9}\sin 2x$]

13. $(D^2+1)y = \log\cos x$.

[**Ans.** : $y = c_1 \cos x + c_2 \sin x + (\log\cos x - 1) + \sin x \log(\sec x + \tan x)$]

14. $(D^2+4D+8)y = 16e^{-2x}\sec^2 2x$

[**Ans.** : $y = e^{-2x}(c_1 \cos 2x + c_2 \sin 2x) + 4e^{-2x}\cos 2x \log|\text{cosec}\, 2x + \cot 2x| - 4e^{-2x}$]

Solve the following differential equations:

1. $\frac{dx}{dt} = 3x + 8y, \qquad \frac{dy}{dt} = -x - 3y$

[**Ans.** : $x = -4c_1e^{t} - 2c_2e^{-t}$, $y = c_1e^{t} - c_2e^{-t}$]

2. $\frac{dx}{dt} = 2y - 1, \qquad \frac{dy}{dt} = 1 + 2x$

[**Ans.** : $x = c_1e^{2t} + c_2e^{-2t} - \frac{1}{2}$, $y = c_1e^{2t} - c_2e^{-2t} + \frac{1}{2}$]

3. $(D + 6)y - Dx = 0, (3 - D)x - 2Dy = 0$ with $x = 2, y = 3$ at $t = 0$

[**Ans.** : $x = 4e^{2t} - 2e^{-3t}$, $y = e^{2t} + 2e^{-3t}$]

4. $\frac{dx}{dt} + y - 1 = \sin t, \quad \frac{dy}{dt} + x = \cos t$

[**Ans.** : $x = c_1e^{t} + c_2e^{-t}$, $y = 1 + \sin t - c_1e^{t} + c_2e^{-t}$]

5. $(D + 5)x + (D + 7)y = 2e^{t}$,
$(2D + 1)x + (3D + 1)y = e^{t}$

[**Ans.** : $x = \frac{1}{1+5t}\left\{(2 - 8c_2)e^{t} + \frac{5}{2}c_2e^{-t}\right.$, $y = c_1e^{-2t} + c_2e^{t}$]

6. $\frac{d^2x}{dt^2} + y = \sin t, \quad \frac{d^2y}{dt^2} + x = \cos t$

[**Ans.** : $x = c_1e^{t} + c_2e^{-t} + c_3\cos t + c_4\sin t - \frac{t}{4}\cos t + \frac{t}{4}\sin t$, $y = -c_1e^{t} - c_2e^{-t} + c_3\cos t + c_4\sin t + \frac{1}{4}(2 + t)(\sin t - \cos t)$]

7. $D^2x + 3x - 2y = 0$, $D^2x + D^2y - 3x + 5y = 0$ with $x = 0, y = 0, Dx = 3, Dy = 2$ when $t = 0$

[**Ans.** : $x = \frac{1}{4}(11\sin t + \frac{1}{3}\sin 3t)$, $y = \frac{1}{4}(11\sin t - \sin 3t)$]

$(D^2+3D+2)y = 12e^{-x}\sin^3 x.$

$\left[\text{Ans.: } y = c_1e^{-x} + c_2e^{-2x} + \frac{e^{-x}}{10}[(\cos 3x + 3\sin 3x) - 45(\cos x + \sin x)]\right]$

$(D^2+4D+3)y = 6e^{-x}.$

$\left[\text{Ans.: } y = c_1e^{-x} + c_2e^{-3x} + 3xe^{-x}\right]$

$(D^2-D-6)y = 5e^{-2x} + 10e^{3x}.$

$\left[\text{Ans.: } y = c_1e^{3x} + c_2e^{-2x} + 2xe^{3x} - xe^{-2x}\right]$

$(D^2+16)y = 16\sin 4x.$

$\left[\text{Ans.: } y = c_1\cos 4x + c_2\sin 4x - 2x\cos 4x\right]$

$(D^2+25)y = 50\cos 5x + 30\sin 5x.$

$\left[\text{Ans.: } y = c_1\cos 5x + c_2\sin 5x - x(3\cos 5x - 5\sin 5x)\right]$

$(D^3-2D^2+4D-8)y = 8(x^2+\cos 2x)$

$\left[\text{Ans.: } y = c_1e^{2x} + c_2\cos 2x + c_3\sin 2x - (x^2+x) - \frac{x}{2}(\cos 2x + \sin 2x)\right]$

$(D^2-4D+5)y = 16e^{2x}\cos x.$

$\left[\text{Ans.: } y = e^{2x}(c_1\cos x + c_2\sin x) + 8xe^{2x}\sin x\right]$

$(D^2-6D+13)y = 6e^{3x}\sin x\cos x.$

$\left[\text{Ans.: } y = e^{3x}(c_1\cos 2x + c_2\sin 2x) - \frac{3x}{4}e^{3x}\cos 2x\right]$

$(D^3+2D^2-D-2)y = e^x + x^2.$

$\left[\text{Ans.: } y = c_1e^x + c_2e^{-x} + c_3e^{-2x} + \frac{1}{6}xe^x - \frac{x^2}{2} + \frac{x}{2} - \frac{5}{4}\right]$

22. $(D^2-4D+4)y = x^3e^{2x} + xe^{2x}.$

$\left[\text{Ans.: } y = (c_1+c_2x)e^{2x} + \left(\frac{x^5}{20} + \frac{x^3}{6}\right)e^{2x}\right]$

23. $(D^2-3D+2)y = xe^{2x} + \sin x.$

$\left[\text{Ans.: } y = c_1e^x + c_2e^{2x} + \left(\frac{x^2}{2} - x\right)e^{2x} + \frac{1}{10}\sin x + \frac{3}{10}\cos x\right]$

24. $(D^2+1)y = \sin^3 x.$

$\left[\text{Ans.: } y = c_1\cos x + c_2\sin x + \frac{1}{32}\sin 3x - \frac{3}{8}x\cos x\right]$

25. $(D^2+2D+1)y = x^2e^{-x}.$

$\left[\text{Ans.: } y = (c_1+c_2x)e^{-x} + \frac{x^4}{12}e^{-x}\right]$

26. $(D^3-D^2-4D+4)y = 2x^2 - 4x - 1 + 2x^2e^{2x} + 5xe^{2x} + e^{2x}.$

$\left[\text{Ans.: } y = c_1e^x + c_2e^{2x} + c_3e^{-2x} + \frac{x^2}{2} + \frac{x^3}{6}e^{2x}\right]$

27. $(D^2-5D-6)y = e^{3x},$
$y(0) = 2, \quad y'(0) = 1$

$\left[\text{Ans.: } y = \frac{10}{21}e^{6x} + \frac{45}{28}e^{-x} - \frac{1}{12}e^{3x}\right]$

28. $(D^2-5D+6)y = e^x(2x-3),$
$y(0) = 1,\ y'(0) = 3.$

$\left[\text{Ans.: } y = e^{2x} + xe^x\right]$

29. $(D^3-D)y = 4e^{-x} + 3e^{2x},$
$y(0) = 0,\ y'(0) = -1,\ y''(0) = 2.$

Solve the following differential equations using method of undetermined coefficients.

1. $(D^2+6D+8)y = e^{-3x}+e^{x}$.

[**Ans.** : $y = c_1e^{-2x}+c_2e^{-4x}-e^{-3x}+\frac{e^{x}}{15}$]

2. $(4D^2-1)y = e^{x}+e^{3x}$.

[**Ans.** : $y = c_1e^{\frac{x}{2}}+c_2e^{-\frac{x}{2}}+\frac{1}{105}(35e^{x}+3e^{3x})$]

3. $(D^2+D-6)y = 39\cos 3x$.

[**Ans.** : $y = c_1e^{2x}+c_2e^{-3x}+\frac{1}{2}(\sin 3x-5\cos 3x)$]

4. $(D^2+2D+5)y = 6\sin 2x+7\cos 2x$.

[**Ans.** : $y = e^{-x}(c_1\cos 2x+c_2\sin 2x)+2\sin 2x-\cos 2x$]

5. $(D^2+4D-5)y = 34\cos 2x-2\sin 2x$.

[**Ans.** : $y = c_1e^{x}+c_2e^{-5x}+2(\sin 2x-\cos 2x)$]

6. $(D^3-D^2+D-1)y = 6\cos 2x$.

[**Ans.** : $y = c_1e^{x}+c_2\cos x+c_3\sin x+\frac{2}{5}(\cos 2x-2\sin 2x)$]

7. $(2D^3-D-3)y = x^3+x+1$.

[**Ans.** : $y = c_1e^{-x}+c_2e^{\frac{3x}{2}}-\frac{1}{27}(9x^3-9x^2+51x-23)$]

8. $(D^2+4)y = 8x^2$.

[**Ans.** : $y = c_1\cos 2x+c_2\sin 2x+2x^2-1$]

9. $(3D^2+2D-1)y = e^{-2x}+x$.

[**Ans.** : $y = c_1e^{-x}+c_2e^{\frac{x}{3}}+\frac{1}{7}(e^{-2x}-7x-14)$]

10. $(D^2-2D+3)y = x^2+\sin x$.

[**Ans.** : $y = e^{x}\left(c_1\cos\sqrt{2}x+c_2\sin\sqrt{2}x\right)+\frac{1}{27}(9x^2+6x-8)+\frac{1}{4}(\sin x+\cos x)$]

11. $(D^4-1)y = x^4+1$.

[**Ans.** : $y = c_1e^{x}+c_2e^{-x}+c_3\cos x+c_4\sin x-x^4-25$]

12. $(D^2-1)y = e^{3x}\cos 2x-e^{2x}\sin 2x$.

[**Ans.** : $y = c_1e^{x}+c_2e^{-x}+\frac{1}{30}e^{2x}(2\cos 3x+\sin 3x)+\frac{1}{40}e^{3x}(\cos 2x+3\sin 2x)$]