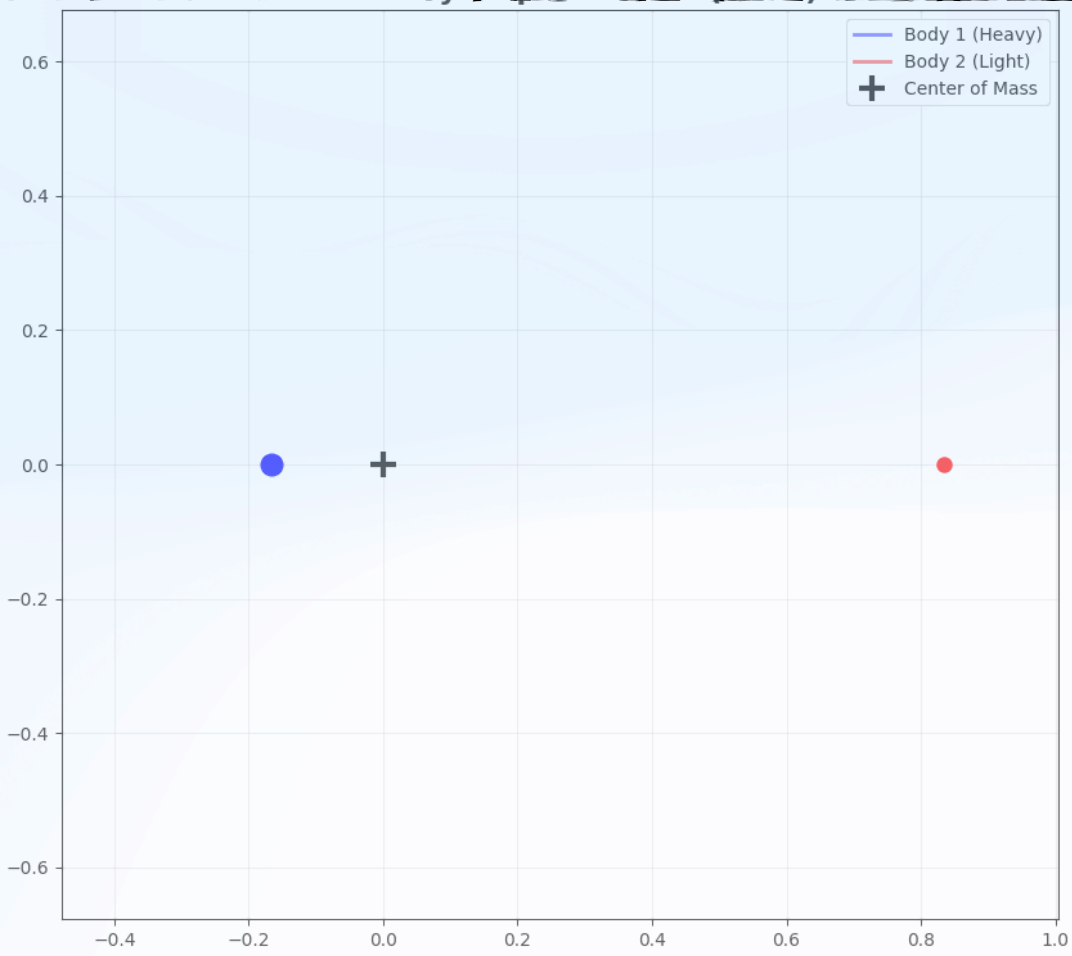
Two-Body Orbital Simulation
RK4 v/s RK45



Problem Statement!

- How do different numerical integration methods perform for long-term orbital simulations?

- Where in this case we just care about about (non-relativistic)Two body motion —under influence of gravitation!

Overview

Here I am.....

- Simulating two-body gravitational motion
- Comparing RK4 (fixed-step) and RK45 (adaptive-step)
- Finally….plotting trajectories, energy, separation, and efficiency

Physics Background

- Two bodies attract via Newton's law of gravitation

$$\frac{d^2\vec{r_1}}{dt^2} = \frac{Gm_2(\vec{r_2} - \vec{r_1})}{|\vec{r_2} - \vec{r_1}|^3}$$

$$\frac{d^2\vec{r_2}}{dt^2} = \frac{Gm_1(\vec{r_1} - \vec{r_2})}{|\vec{r_1} - \vec{r_2}|^3}$$

- Equations reduce to 8 coupled first-order ODEs
- Conserved quantities: total energy and momentum (both angular and linear)
- Orbital dynamics depend on initial separation and eccentricity

Numerical Methods

RK4:

- Fourth-order solver
- Fixed time step (dt = 0.005)

RK45 (Cash–Karp):

- Embedded 4th/5th order
- Adaptive step control using error estimates
- Accept–reject mechanism

Fixed Step Logic (RK4)

$$k_1 = f(t_n, y_n)$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Adaptive Step Logic (RK45)

Uses two estimates —4th and 5th order

Error = $|y_5 - y_4|$

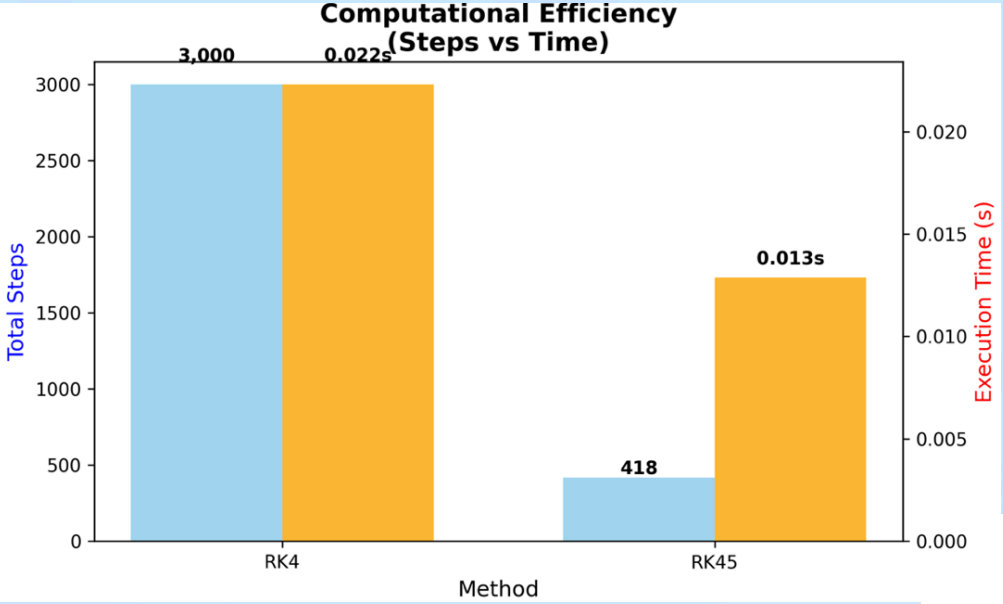
If (error < tolerance): → accept step

Else: → reject and reduce time step

Ensures accuracy while reducing computation cost.

Code Architecture

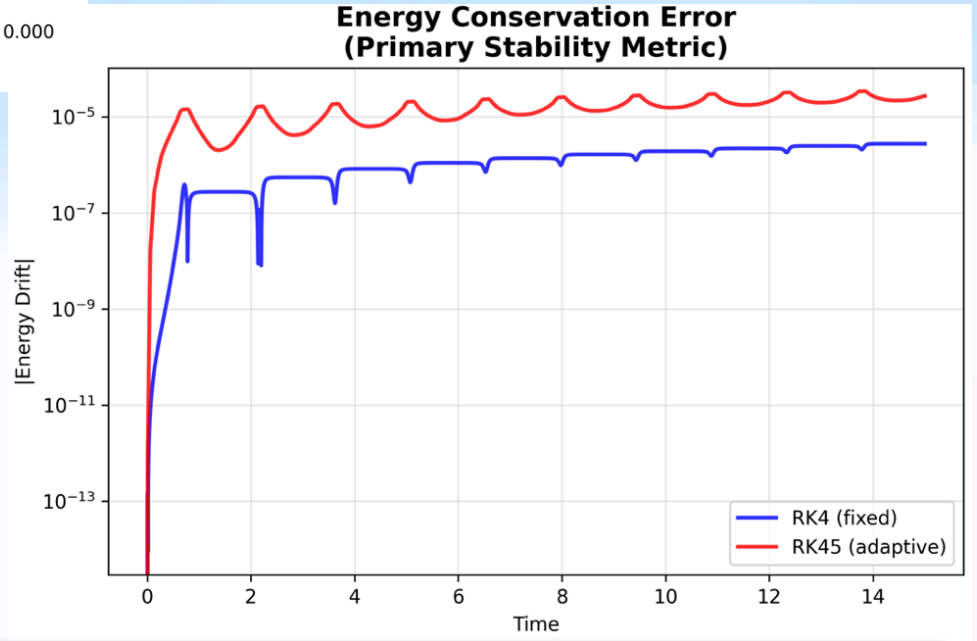
- Class TwoBody encapsulates physics and solvers
- Key functions:
  - deriv(): returns time derivatives
  - rk4(): fixed-step integrator
  - rk45(): adaptive-step integrator
  - run_rk4() / run_rk45(): simulation loops
  - energy() and energy_stats(): diagnostics



Clearly,……RK4 takes lesser steps than RK4(almost 7.2 times fewer!)

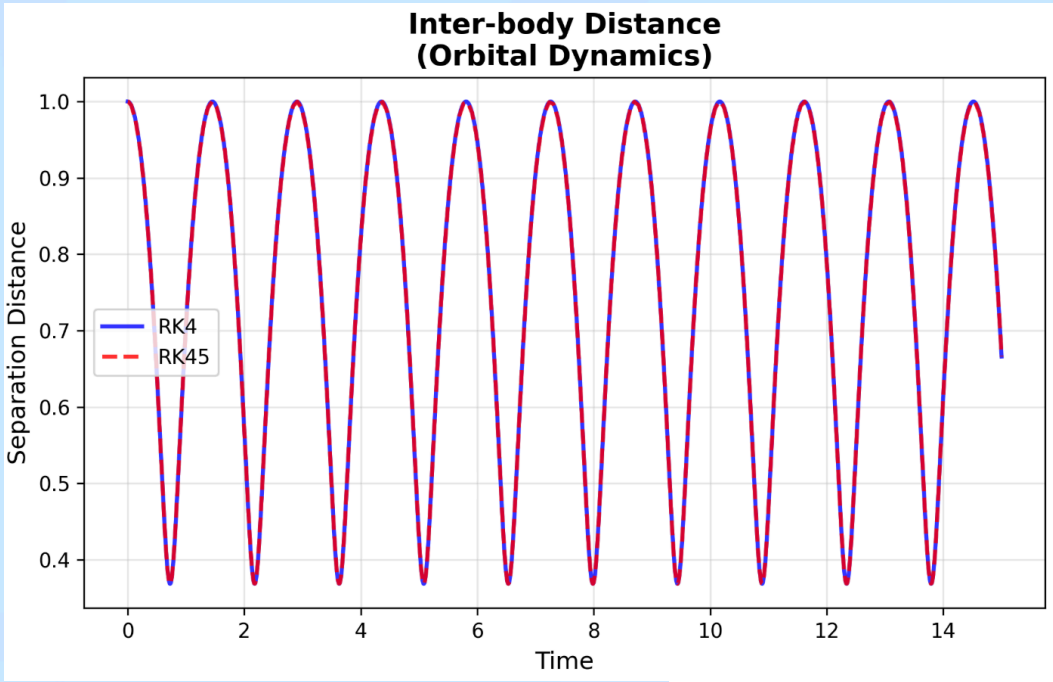
But….!!

After certain amount of periods……RK4 appears to be more precise than RK45!!



HENCE

RK4's smaller, uniform time steps provide more consistent accuracy, while RK45 trades some precision for computational efficiency through larger adaptive steps.



Remarks:

Just a plot of " |r| v/s. t "…….

…clearly there there not much significant difference in both for different methods

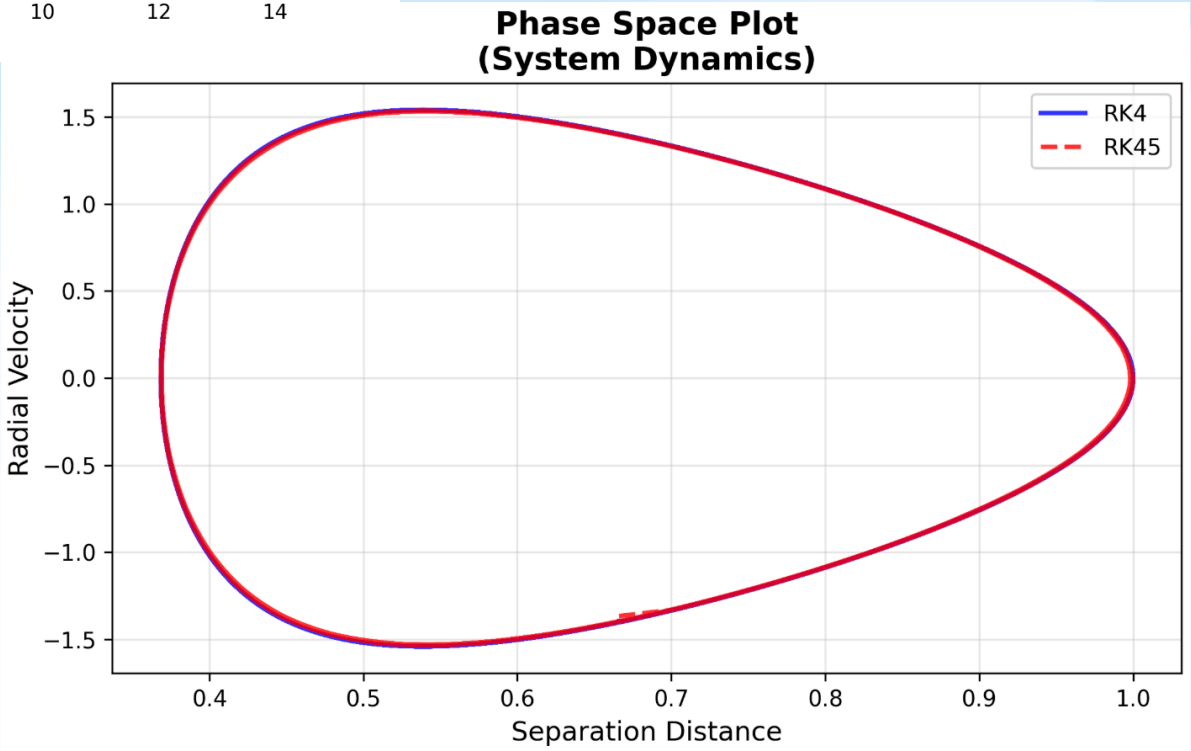
Remarks:

The Phase-Space-Plot ……

q—>|r| and

p—> (change in |r| wrt t)

This depicts an egg like shape which is deviation from ellipse (reason being e≠0 instead e=0!!)



Thank You